[컴사] 유클리드 호제

★ $GCD(a,b)=d \iff a\cdot x + b\cdot y = d$
(Bezout's Identity)

① $a = b\cdot q_0 + r_1$
② $b = r_1\cdot q_1 + r_2$
$r_1 = r_2\cdot q_2 + r_3$
$r_2 = r_3\cdot q_3 + r_4$
⋮
③ $r_{i-1} = r_i\cdot q_i + r_{i+1}$
④ if $r_i = s_i\cdot a + t_i\cdot b$
then $r_{i+1} = s_{i+1}\cdot a + t_{i+1}\cdot b$
$r_{i-1} = s_{i-1}\cdot a + t_{i-1}\cdot b$

④→③ 대입
$s_{i-1}\cdot a + t_{i-1}\cdot b = (s_i\cdot a + t_i\cdot b)\cdot q_i$
$+ s_{i+1}\cdot a + t_{i+1}\cdot b$
(정리). $s_{i+1}\cdot a + t_{i+1}\cdot b = (s_{i-1} - s_i q_i)a$
$+ (t_{i-1} - t_i q_i)b$

$\therefore$ ⑤ $\begin{cases} s_{i+1} = s_{i-1} - s_i\cdot q_i \\ t_{i+1} = t_{i-1} - t_i\cdot q_i \end{cases}$

$\begin{cases} r_0 = a,\ r_1 = b \ \cdots ③ \\ s_0 = 1 \quad s_1 = 0 \\ t_0 = 0 \quad t_1 = 1 \end{cases}$ … ④

$q_i = r_i / r_{i+1}$

```
while ( r1 > 0 ) {
    q = r0 / r1
    temp = r0
    r0 = r1
    r1 = temp - r1 · q
    temp = s0
    s0 = s1
    s1 = temp - s1 · q
    temp = t0
    t0 = t1
    t1 = temp - t1 · q
}
```

$\boxed{★★★\ a s_0 + b t_0 = d}$

① $p=7,\ q=11,$

② $N=p\cdot q=77$

③ $\phi(N)=(p-1)(q-1)=60$

④ $\phi(N)>e,\ gcd(\phi(N),e)=1$

$e=\{7,11,13,17,19,23,29,$
$31,37,41,43,47,49,$
$53,59\}$

$e=59,$ 하나를 선택한다.

⑤ $d\cdot e\equiv 1\ (mod\ \phi(n))\ (d\neq e)$

$gcd(d,e)=1$

※B.I. $\boxed{d\cdot e\cdot x+\phi(n)\cdot y=1}$

$119\times 59\cdot(+1)+60\cdot(-117)=1$

$\therefore\ d=119$

⑥ 암호화

$A\rightarrow 65^{59}\ (mod\ 77)\rightarrow 32$

$C\rightarrow 67^{59}\ (mod\ 77)\rightarrow 23$

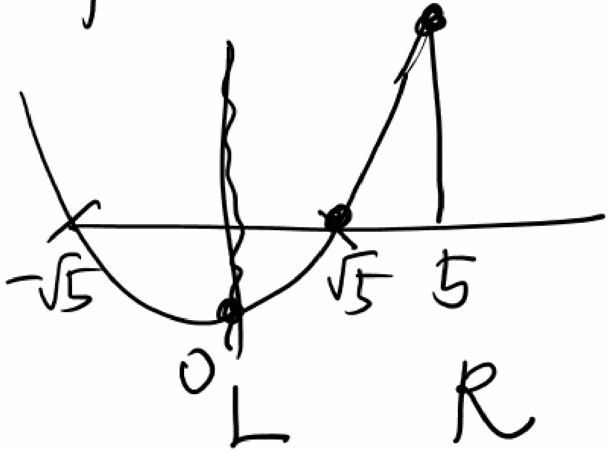
⑦ 복호화 $32^{119}\ (mod\ 77)\rightarrow 65\ A$

$23^{119}\ (mod\ 77)\rightarrow 67\ C$

[컴사] 중간값정리

$x^2 = 5$

$f(x) = x^2 - 5$



$f(L) \times f(R) < 0$

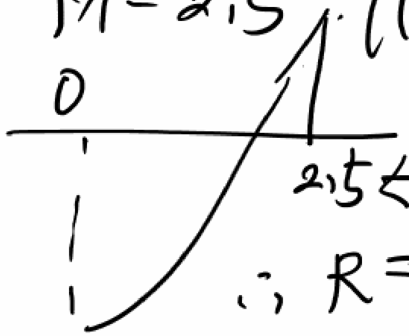
$\Rightarrow M = \frac{L+R}{2}$

ex) $L = 0,\ R = 5$

① $f(0) = -5,\ f(5) = 20$

$\therefore f(0) * f(5) < 0$

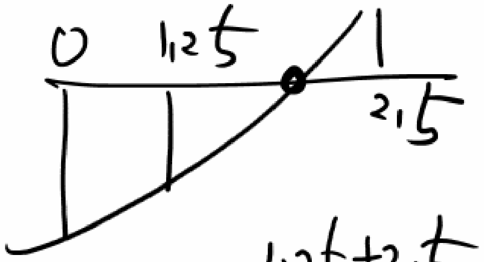
$M = 2.5,\ f(M) = f(2.5) > 0$



$\therefore R = M$

② $M = \frac{0+2.5}{2} = 1.25,\ f(1.25) < 0$

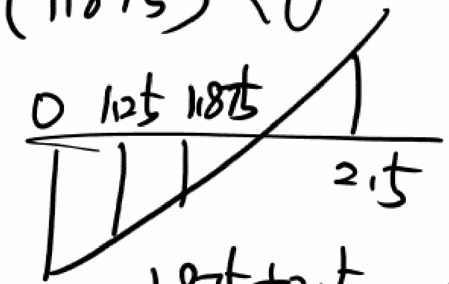
②



Let $M = \frac{1.25+2.5}{2} = 1.875$

$f(1.875) < 0$

③



$M = \frac{1.875+2.5}{2} = 2.1875$

$f(2.1875) < 0$

$M = \frac{2.1875+2.5}{2} = 2.34375$

[컴사] 페르마 소정리 (수식유도)

$2^1 \equiv 2 \pmod 5$

$2^2 \equiv 4 \pmod 5$

$2^3 \equiv 8 \equiv 3 \pmod 5$ (추론) $a^{p-2} \equiv \frac{1}{a} \pmod p$

$\Uparrow$

$2^4 \equiv 16 \equiv 1 \pmod 5 \Longleftrightarrow a^{p-1} \equiv 1 \pmod p$

$2^5 \equiv 32 \equiv 2 \pmod 5 \Longleftrightarrow a^p \equiv a \pmod p$

ex) $_7C_5 = \frac{7!}{2!5!} \equiv \frac{7!}{240} \equiv \frac{7!}{p} \pmod{11} \Rightarrow 7! \cdot p^{11-2}$

$_7C_5 = \frac{7!}{2!5!} = \frac{7!}{2 \times 120} \equiv \frac{7!}{2 \times 10} \pmod{11} \Rightarrow 7! \cdot 2^p \cdot 10^p \pmod{11}$

Convex Hull

1. 주어진 점들 중 y좌표가 가장 작거나 혹은 가장 작은 점이 둘 이상이라면 x좌표가 가장 작은 점을 선택한다.

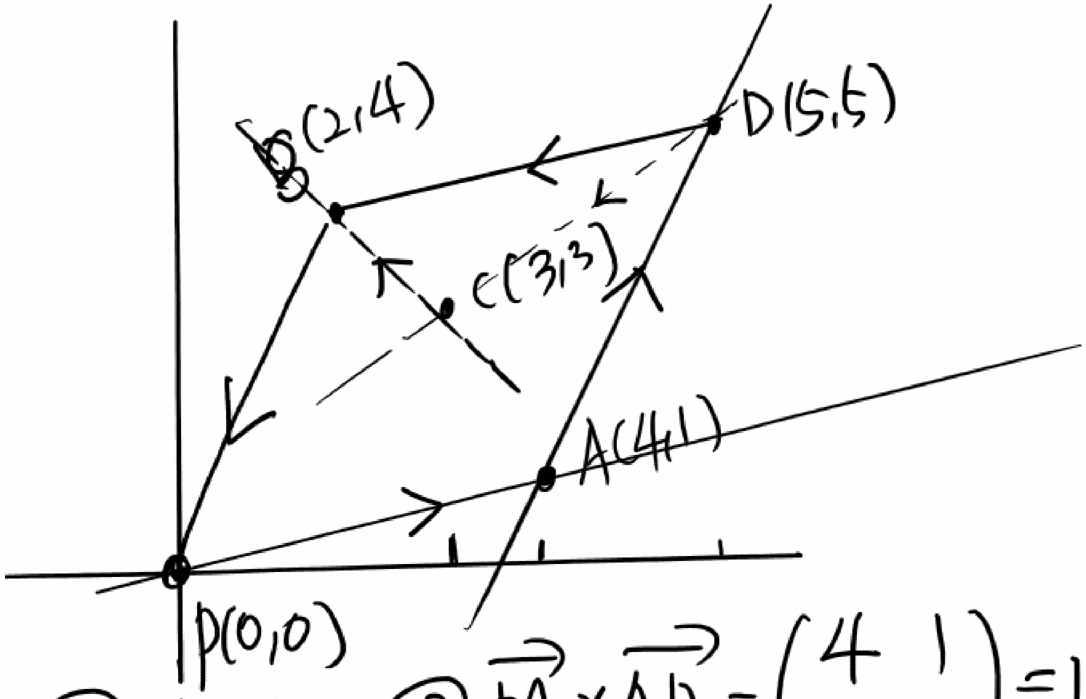
2. 선택한 점을 기준으로 나머지 점들을 반시계 반향으로 정렬(각도+거리)

3. 그라함 스캔 알고리즘 적용

Graham's Scan Algorithm

1. 제일 처음 선택한 점을 스택에 먼저 넣고 정렬된 점들을 차례대로 스택에 넣는다.
2. 새로운 점을 스택에 push할 때, 만약 스택에 두개 이상의 점이 있다면 가장 최근에 push된 두 점을 이은 직선을 기준으로 새로운 점이 왼쪽에 있다면 push, 오른쪽에 있다면 스택의 가장 위의 점을 pop.

ConvexHull(1708, 4181)



①

A
P

② $\vec{PA} \times \vec{AD} = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} = 15 > 0$ CCW

$\vec{PA} = \vec{OA} - \vec{OP} = (4,1)$

$\vec{AD} = (\vec{OD} - \vec{OA}) = (1,4)$

D
A
P

③ $\vec{AD} \times \vec{DC} = \begin{pmatrix} 1 & -2 \\ 4 & -2 \end{pmatrix} = 6 > 0$ CCW

$\vec{OD} - \vec{OA} = (1,4)$

$\vec{DC} = \vec{OC} - \vec{OD} = (-2,-2)$

C
D
A
P

④ $\vec{DC} \times \vec{CB} = \begin{pmatrix} -2 & -1 \\ -2 & 1 \end{pmatrix} = -4 < 0$ CW

$\vec{DC} = (-2,-2)$

$\vec{CB} = \vec{OB} - \vec{OC} = (-1, 1)$

C 버림

B
D
A
P

⑤ $\vec{DB} \times \vec{BP} = \begin{pmatrix} -3 & -2 \\ -1 & -4 \end{pmatrix} = 10$ CCW

$\vec{DB} = \vec{OB} - \vec{OD} = (-3,-1)$

P
B
D
A
P

정답

1. $a_{n+1} = a_n + r \cdot a_n$, $a_1 = A$

$\therefore a_{n+1} = (1+r)a_n .$

$a_n = a_1 \cdot (1+r)^{n-1}$

$\therefore \underline{a_n = A \cdot (1+r)^{n-1}}$

2. $S_n = 1+2+3+ \cdots +n$

$= S_{n-1} + n$

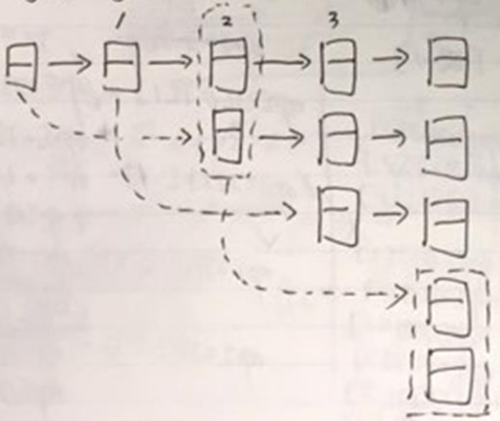
$\therefore \underline{S_n = S_{n-1} + a_n} .$ $\boxed{S_1 = 1 . \ a_1 = 1}$

$a_n = a_1 + (n-1)d$

$\underline{a_n = n}$

3. Leonardo di Pisa (Liberabaci)

$f_{n+2} = f_{n+1} + f_n$ (피보나치)



4. Edoudard Lucas Hanoi

$H_{n+1} = 2H_n + 1$, $H_1 = 1$.

$\alpha = 2\alpha + 1$. $\alpha = -1$

$\therefore (H_{n+1} + 1) = 2(H_n + 1)$

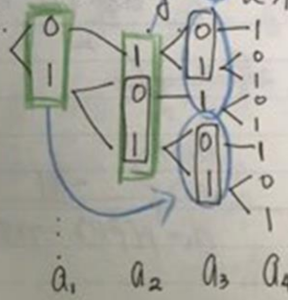
$b_{n+1} = 2 b_n$. $b_1 = H_1 + 1 = 2$.

$\therefore b_n = H_n + 1 = 2^{n-1} \cdot 2$

$\therefore \underline{H_n = 2^n - 1}$

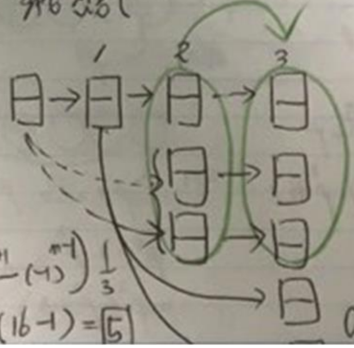
Bit string 연속 0 불가능.

5.



$\therefore \underline{a_{n+2} = a_{n+1} + a_n}$

6. 화성 침공 (Doom's Day)



$(a-2)(a+1)$
$= a^2 - a - 2 = 0$.

$a_{n+2} = a_{n+1} + 2a_n$. $a_1 = 1$

$\alpha^2 - \alpha - 2 = 0$ $a_2 = 3$.

$a_{n+2} - 2a_{n+1} = -1(a_{n+1} - 2a_n)$

$C_{n+1} = -1 \cdot C_n$

$C_1 = a_2 - 2a_1$

$= 1$

$C_n = C_1 \cdot (-1)^{n-1}$.

$a_{n+1} - 2a_n = (-1)^{n+1}$

$a_{n+2} + a_{n+1} = 2(a_{n+1} + a_n)$

$B_{n+1} = 2 B_n$.

$B_1 = a_2 + a_1$

$= 4$.

$B_n = B_1 \cdot 2^{n-1}$

$a_{n+1} + a_n = 2^{n+1}$

$a_n = \left(2^{n+1} - (-1)^{n-1}\right) \frac{1}{3}$

$a_3 = \frac{1}{3}(16 - 1) = \boxed{5}$

$$\left[{}_nC_r = \frac{n!}{(n-r)!\,r!},\ {}_nP_r = \frac{n!}{(n-r)!},\ {}_nH_r = {}_{n+r-1}C_r \right]$$

① ${}_nC_r = {}_nC_{n-r}$

② ${}_nC_1 = n$

③ ${}_{n+1}C_2 = \frac{n(n+1)}{2}$

④ ${}_nC_r = \frac{n}{r} \cdot {}_{n-1}C_{r-1}$

⑤ ${}_nC_r = \frac{n}{n-r} \cdot {}_{n-1}C_r$ $(n>r,$ if $n==r, {}_nC_r=1)$

⑥ ${}_nC_r = \frac{n-r+1}{r}\, {}_nC_{r-1}$

⑦ ${}_nC_r = \frac{1}{r!} \cdot {}_nP_r$

⑧ ${}_nC_r = {}_{n-1}C_{r-1} + {}_{n-1}C_r$

⑨ ${}_nH_r = {}_nH_{r-1} + {}_{n-1}H_r$

⑩ ${}_nP_r = (n-r+1) \cdot {}_nP_{r-1}$

⑪ ${}_nP_r = n \cdot {}_{n-1}P_{r-1}$

⑫ ${}_nC_0 + {}_nC_1 + {}_nC_2 + \cdots + {}_nC_n = 2^n$

⑬ ${}_nC_0 - {}_nC_1 + {}_nC_2 - {}_nC_3 + \cdots\ {}_nC_n = 0$

⑭ ${}_nC_1 + 2\,{}_nC_2 + 3\,{}_nC_3 + \cdots + n \cdot {}_nC_n = n \cdot 2^{n-1}$

① $_nC_r = \frac{n!}{(n-r)!\,r!}$, $_nC_{n-r} = \frac{(n)!}{(n-n+r)!\,(n-r)!} = \frac{n!}{r!\,(n-r)!}$

$$\therefore {}_nC_r = {}_nC_{n-r}$$

② $_nC_1 = \frac{n!}{(n-1)!\,1!} = \frac{n(n-1)!}{(n-1)!} = n$

③ $_{n+1}C_2 = \frac{(n+1)!}{(n-1)!\,2!} = \frac{(n+1)n\cancel{(n-1)!}}{\cancel{(n-1)!}\,2!} = \frac{n(n+1)}{2}$

④ $_{n-1}C_{r-1} = \frac{(n-1)!}{(n-r)!\,(r-1)!} = \frac{(n-1)!\cdot n \cdot r}{(n-r)!\,(r-1)!\,r\cdot n}$

$$= \frac{n!}{(n-r)!\,r!}\cdot\frac{r}{n} = {}_nC_r\cdot\frac{r}{n}$$

$$\therefore {}_nC_r = \frac{n}{r}\cdot {}_{n-1}C_{r-1}$$

⑤ $_{n-1}C_r = \frac{(n-1)!}{(n-1-r)!\,r!} = \frac{(n-1)!\,n\cdot(n-r)}{(n-1-r)!\,(n-r)\cdot n\cdot r!}$

$$= \frac{n!\,(n-r)}{(n-r)!\,r!\,n} = \frac{n-r}{n}\cdot {}_nC_r$$

$$\therefore {}_nC_r = \frac{n}{n-r}\,{}_{n-1}C_r$$

⑥ $_nC_{r-1} = \frac{n!}{(n-r+1)!\,(r-1)!} = \frac{n!\cdot r}{(n-r+1)(n-r)!\,(r-1)!\,r}$

$$= \frac{r\cdot n!}{(n-r+1)(n-r)!\,r!} = \frac{r}{(n-r+1)}\cdot {}_nC_r$$

$$\therefore {}_nC_r = \frac{n-r+1}{r}\cdot {}_nC_{r-1}$$

⑦ $_{n}P_{r} = \frac{n!}{(n-r)!} = \frac{n!\,r!}{(n-r)!\,r!} = {}_{n}C_{r}\cdot r!$

$\therefore {}_{n}C_{r} = \frac{{}_{n}P_{r}}{r!}$

⑧ $_{n-1}C_{r-1} + {}_{n-1}C_{r} = ④ + ⑤$

$= \frac{r}{n}\cdot{}_{n}C_{r} + \frac{n-r}{n}\cdot{}_{n}C_{r}$

$= {}_{n}C_{r}$

⑨ $_{n}H_{r} = {}_{n}H_{r-1} + {}_{n-1}H_{r}$

$_{n+r-1}C_{r} = {}_{n+r-2}C_{r-1} + {}_{n+r-2}C_{r}$ 임을 보임

⑧에 의하여 증명

⑩ $_{n}P_{r-1} = \frac{n!}{(n-r+1)!} = \frac{1}{(n-r+1)}\cdot\frac{n!}{(n-r)!} = \frac{1}{n-r+1}\,{}_{n}P_{r}$

$\therefore {}_{n}P_{r} = (n-r+1)\cdot{}_{n}P_{r-1}$

⑪ $_{n-1}P_{r-1} = \frac{(n-1)!}{(n-r)!} = \frac{(n!)\,n}{(n-r)!\,n} = \frac{1}{n}\cdot{}_{n}P_{r}$

$\therefore {}_{n}P_{r} = n\cdot{}_{n-1}P_{r-1}$

⑫
if $(x+1)^n = \sum_{k=0}^{n} {}_nC_k \cdot x^k = {}_nC_0 + {}_nC_1 \cdot x^1 + {}_nC_2 \cdot x^2 + \cdots + {}_nC_n x^n$

$\therefore f(x) = (x+1)^n \quad , f(1) = 2^n$

$\therefore 2^n = {}_nC_0 + {}_nC_1 + {}_nC_2 + \cdots + {}_nC_n$

⑬
if $f(-1) = 0 = {}_nC_0 - {}_nC_1 + {}_nC_2 - {}_nC_3 + \cdots\cdots {}_nC_n$

$f(x) = (x+1)^n \; , \; f'(x) = n \cdot (x+1)^{n-1}$

⑭
$f'(1) = n \cdot 2^{n-1}$

$f(x) = {}_nC_0 + {}_nC_1 x^1 + {}_nC_2 x^2 + \cdots + {}_nC_n x^n$

$f'(x) = {}_nC_1 + 2 \cdot {}_nC_2 x + 3 \cdot {}_nC_3 x^2 + \cdots n \cdot {}_nC_n x^{n-1}$

$\therefore f'(1) = {}_nC_1 + 2 \, {}_nC_2 + 3 \, {}_nC_3 + \cdots n \, {}_nC_n$

$\therefore \sum_{k=1}^{n} k \cdot {}_nC_k = n \cdot 2^{n-1}$

⑭
or $\quad {}_nC_r = \frac{n}{r} \cdot {}_{n-1}C_{r-1}$ ④의 1/5

$\therefore r \cdot {}_nC_r = n \, {}_{n-1}C_{r-1}$

양변 Σ

$\sum k \cdot {}_nC_k = n \cdot \sum {}_{n-1}C_{k-1} = n \cdot 2^{n-1}$

스트레이트 플러쉬 (로티플 포함)

① 5개 모두 무늬동일. 연속된 숫자

$_4C_1 \times 10 = 40$

무늬선택 시작할수있는 숫자

② 포카 동일숫자 4개 + 임의1개

$_{13}C_1 \times {}_4C_4 \times {}_{48}C_1 = 624$

숫자선택 무늬4개 52-4

③ 풀하우스 33344. 99966

$_{13}C_1 \times {}_4C_3 \times {}_{12}C_1 \times {}_4C_2 = 3744$

숫자선택 무늬선택 남은숫자中 선택

④ 플러쉬 모두같은 무늬

$_4C_1 \times {}_{13}C_5$ − 스티플 = 5108

무늬선택 숫자선택

⑤ 스트레이트 연속숫자

$10 \times {}_4C_1 \times {}_4C_1 \times {}_4C_1 \times {}_4C_1 \times {}_4C_1$ − 스티플 $= 10200$

시작할수있는숫자 각카드무늬선택

⑥ 트리플 동일숫자3. 나가리 2장

$_{13}C_1 \cdot {}_4C_3 \cdot {}_{12}C_2 \cdot {}_4C_1 \cdot {}_4C_1$

숫자선택 무늬선택 나가리숫자 나가리무늬

$= 54912$

⑦ 투페어 동일숫자2. 동일숫자2 다른수 1

$_{13}C_2 \times {}_4C_2 \times {}_4C_2 \times {}_{44}C_1 = 123552$

숫자2개 지정 무늬 선택 나머지 다른수 52-8

⑧ 원페어 동일 숫자2. 나머지 각각 다른수 3

$_{13}C_1 \times {}_4C_2 \times {}_{12}C_3 \times {}_4C_1 \times {}_4C_1 \times {}_4C_1 = 1098240$

⑩ 전체 $_{52}C_5 = 2598960$

⑨ 나가리, 하이, 탑, 꽝

⑩ − (① + ② + ③ + ④ + ⑤ + ⑥ + ⑦ + ⑧)

$= 1302540$

1st straight flush

4suit (무늬) HDSC

A2345, 23456, 34567, 45678, 56789, 6789T, 789TJ, 89TJQ, 9TJQK, TJQKA

$\Rightarrow 4 \times 10 = 40$

2nd Four of a Kind

H	A	2	3	4	5	6	7	8	9	T	J	Q	K
D	A												
S	A												
C	A												

선택 가정. 나머지 값 12개 × 4종류

$\underbrace{{}_4C_4 \times {}_{13}C_1}_{\text{A 선택 4개.}} \times \underbrace{{}_{48}C_1}_{\text{1개 (52개-4개A 제외한 후)}} = 624$

3rd Full House

A	2	3	4	5	6	7	8	9	T	J	Q	K
A	2											
A	2											
A	2											

$\underline{{}_{13}C_1 \times {}_4C_3} \times \underline{{}_{12}C_1 \times {}_4C_2} = 3744$

4th. Flush 모두 같은 무늬.

H	A	2	3	4	5	6	7	8	9	T	J	Q	K
D	A												
S	A												
C	A												

H만 선택 ${}_4C_1 \times ({}_{13}C_5 - 10) = 5108$

13종류 값

연속 23456, … 제외

5th. Straight : 적어도 하나는 다른무늬 & 연속

= Straight 전체 (연속) − 모두 같은무늬 (straight Flush)

H $\boxed{A}$ 2 3 $\boxed{4}$ 5 6 7 8 9 10 J Q K
D A 2 $\boxed{5}$
S A $\boxed{3}$
C A

A 2 3 4 5 ← straight

$_4C_1 \times {}_4C_1 \times {}_4C_1 \times {}_4C_1 \times {}_4C_1 \times 10$ − straight Flush (4×10)

$= ({}_4\Pi_5 - {}_4C_1) \times 10 = 10200$

6th. Three of a kind

H $\boxed{A}$ $\boxed{2}$ 3 4 5 6 7 8 9 T J Q K
D $\boxed{A}$ 2 $\boxed{3}$
S $\boxed{A}$ 2 3
C A 2 3

$\underbrace{{}_{13}C_1 \times {}_4C_3}_{\text{13개중 하나값 × 3개무늬}} \times \underbrace{{}_{12}C_1 \times {}_4C_1}_{\text{A제외 하나의 무늬}} \times \underbrace{{}_{11}C_1 \times {}_4C_1}_{\text{A,2제외 하나의 무늬}} \times \underbrace{\frac{1}{2!}}_{\text{어려움 2,3 / 3,2는 같음}}$

or $\underbrace{{}_{13}C_1 \times {}_4C_3}_{\text{AAA: 13개중 하나값 × 3개무늬}} \times \left(\underbrace{{}_{48}C_2}_{\text{23,… : A제외 (4개)}} - \underbrace{{}_{12}C_1 \times {}_4C_2}_{\text{55 같은: A제외 무늬 같은값 2개 선택}} \right)$

$= 54912$

10. All $_{52}C_5 = 2598960$

7. Two pairs

A 2 3 4 5 6 7 8 9 T J Q K
A 2 3
A 2 :
A 2 :

· A22A 같다

$$ {}_{13}C_1 \times {}_4C_2 \times {}_{12}C_1 \times {}_4C_2 \times \frac{1}{2!} \times {}_{44}C_1 = 123552 $$

(52 − A 4장 − 2 4장) = 52 − 4 − 4
= 44

8. One Pair

A 2 3 4 5 6 7 8 9 T J Q K
A 2 2
A 2 3
A 3 4

2번카드 3번카드 4번카드

$$ {}_{13}C_1 \times {}_4C_2 \times ({}_{12}C_1 \times {}_4C_1) \times ({}_{11}C_1 \times {}_4C_1) \times ({}_{10}C_1 \times {}_4C_1) \times \frac{1}{3!} $$

AA234
AA324
⋮
모두동일

아 $= \underbrace{{}_{13}C_1 \times {}_4C_2}_{\text{A카드}} \times \left({}_{48}C_3 - (\text{Full House} + \text{Two pairs})\right)$

A카드 제외

One pair 가 [illegible]

$= 1098240$

9. Nothing (Top, Head) $= {}_{52}C_5 - (①+②+③+④+⑤+⑥+⑦+⑧)$
$= 1302540$

1) 1 2 3 4 5 6 7 8 ⋯ n

$$\left.\begin{aligned} a_n &= a_1 + (n-1)d = 1 + (n-1) = n \\ a_n &= a_2 + (n-2)d = 2 + (n-2) = n \end{aligned}\right\}$$

2) 1 3 5 7 9 ⋯

$$\left.\begin{aligned} a_n &= a_1 + (n-1)d = 1 + 2(n-1) = 2n-1 \\ &= a_2 + (n-2)d = 3 + 2(n-2) = 2n-1 \end{aligned}\right\}$$

3) 1 2 4 8 16 ⋯

$$a_n = a_1 \cdot r^{n-1} = 1 \cdot 2^{n-1}$$

4) $F(n) = F(n-1) + 3,\ F(1) = 1.$

등차수열 $\therefore F(n) = F(1) + (n-1) \cdot 3$

$$= 1 + (n-1) \cdot 3$$

$$= 3n-2$$

5) $F(n) = 4F(n-1) + P,\ F(1) = 1.$

특성방 $\alpha = 4\alpha + P.\ \alpha = -3$

$\therefore F(n) + 3 = 4(F(n-1) + 3)$

Let $b_{n+1} = 4b_n.\ b_1 = F_1 + 3 = 4.$

등비. $b_n = 4 \cdot 4^{n-1} = 4^n$

$\hookrightarrow F(n) = 4^n - 3$

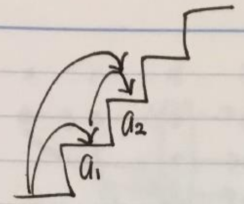
6) 1 2 4 7 11 16

1 2 3 4 5 계차 ⇒ $b_n = 1 + (n-1) = n$

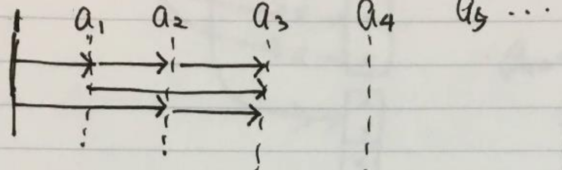
1 1 1 1

$$a_n = a_1 + \sum b_k = 1 + \sum_{k=1}^{n-1} k = \frac{n(n-1)+2}{2}$$

7.



$a_1=1$
$a_2=2$



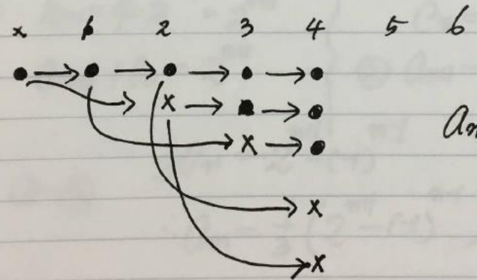
$a_4 = a_3 + a_2$

에서 한칸　에서

$\therefore a_{n+2}=a_{n+1}+a_n$
$a_1=1,\ a_2=2$

~~8.~~ $a_3=a_1+a_2=3,\ a_4=a_3+a_2=5,\ a_5=a_4+a_3=8,\ a_6=a_5+a_4=13$
$a_7=13+8=21,\ a_8=21+13=34,\ a_9=34+21=\underline{\underline{55}},\ a_{10}=89$

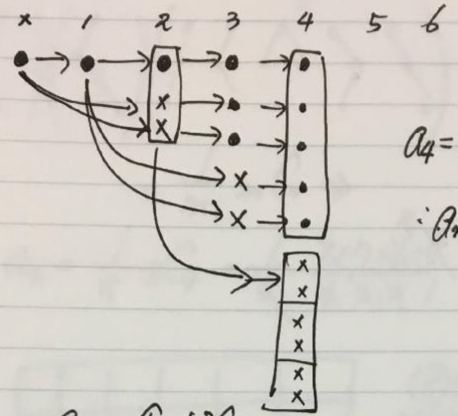
8.



$a_{n+2}=a_{n+1}+a_n$
$a_2=1,\ a_1=1.$

$a_3=2,\ a_4=3,\ a_5=5,\ a_6=8,\ \cdots\ a_{10}=55$

9.



$a_4 = a_3 + 2a_2$

$\therefore a_{n+2} = a_{n+1} + 2a_n$

$a_1 = 1,\ a_2 = 3$

$a_{n+2} = a_{n+1} + 2a_n.$

풀이. $\alpha^2 - \alpha - 2 = 0.\quad (\alpha-2)(\alpha+1) = 0.\quad \alpha = 2,\ \beta = -1.$

$a_{n+2} + a_{n+1} = 2(a_{n+1} + a_n)$

Let $b_{n+1} = 2b_n.\ b_1 = a_2 + a_1 = 4$

$\therefore b_n = 4 \cdot 2^{n-1} = 2^{n+1}$

① $a_{n+1} + a_n = 2^{n+1}$

$a_{n+2} - 2a_{n+1} = -1(a_{n+1} - 2a_n)$

Let $c_{n+1} = -1 \cdot c_n\quad c_1 = a_2 - 2a_1 = 1$

$c_n = (-1)^{n-1}$

② $a_{n+1} - 2a_n = (-1)^{n-1}$

①−② $3a_n = 2^{n+1} - (-1)^{n-1}$

$\therefore a_n = \frac{1}{3}\left(2^{n+1} - (-1)^{n-1}\right)$

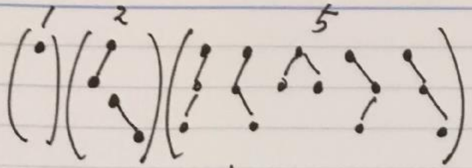
$a_3 = a_2 + 2a_1 = 5$

$a_3 = \frac{1}{3}\left(2^4 - (-1)^2\right) = \frac{1}{3}(16-1) = 5$ 동일

$a_4 = a_3 + 2a_2 = 5 + 6 = 11$

$a_4 = \frac{1}{3}\left(2^5 - (-1)^3\right) = \frac{1}{3}(33) = 11$ 동일

$a_{10} = \frac{1}{3}\left(2^{11} - (-1)^9\right) = \frac{1}{3}(2048+1) = 683$

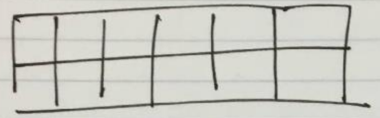
10. 

1, 2, 5, 14

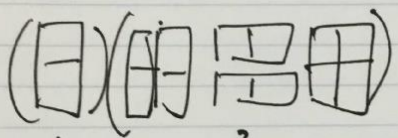
$\Rightarrow$ Catalan !!

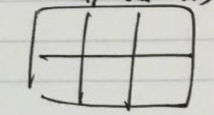
$$= \frac{1}{n+1} {}_{2n}C_n \Rightarrow$$

$$a_4 = \frac{1}{5} \cdot {}_8C_4 = \frac{1}{\cancel{5}} \cdot \frac{\cancel{8} \times 7 \times \cancel{6} \times \cancel{5}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 14$$

11. 

⑦과 같은 맥락!!!

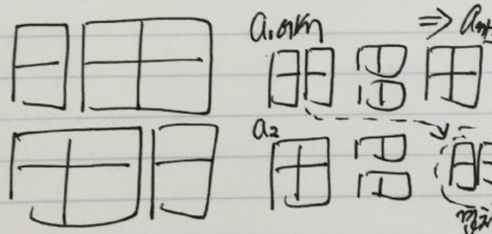




1 3

$a_3 = a_2 + 2a_1 = 5$

$\Rightarrow a_{n+2} = a_{n+1} + 2a_n$ (점화식)

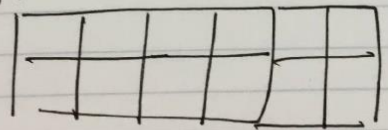


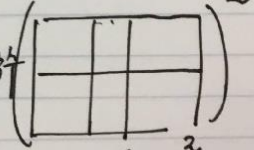
$a_4 = 5 + 6 = 11.$

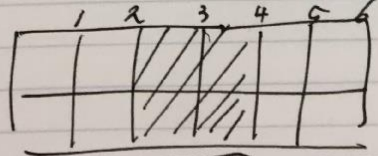
$\hookrightarrow 4.$

겹치니까제외.

or.



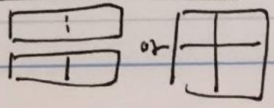
짝수 



$a_n = (a_{\frac{n}{2}})^2$

$+$

$a_n = 2(a_{\frac{n}{2}-1})^2$



$\therefore a_n = (a_{\frac{n}{2}})^2 + 2(a_{\frac{n}{2}-1})^2$

짝수

1 2 4 8

DATE $A = \{1, 2, 3, 4\}$ 2^n 1<<4 subset Max Number = 1<<4 - 1

{ }	0	
{1}	$\boxed{1}$	$\boxed{1}$
{2}	$\boxed{2}$	$\boxed{2}$
{1,2}	3	$\boxed{1}\boxed{2}$
{3}	$\boxed{4}$	$\boxed{4}$
{1,3}	5	$\boxed{1}\boxed{4}$
{2,3}	6	$\boxed{2}\boxed{4}$
{1,2,3}	7	$\boxed{1}\boxed{2}\boxed{4}$
{4}	$\boxed{8}$	$\boxed{8}$
{1,4}	9	$\boxed{1}\boxed{8}$
{2,4}	10	$\boxed{2}+\boxed{8}$
{1,2,4}	11	$\boxed{1}+\boxed{2}+\boxed{8}$
{3,4}	12	$\boxed{4}\boxed{8}$
{1,3,4}	13	$\boxed{1}\boxed{4}\boxed{8}$
{2,3,4}	14	$\boxed{2}\boxed{4}\boxed{8}$
{1,2,3,4}	15	$\boxed{1}\boxed{2}\boxed{4}\boxed{8}$

			& 1과 3이 있나 0101
{1}	1	0001	0001
{1,2}	3	0011	0001
{1,3}	5	0101	$\boxed{0101}$
{1,2,3}	7	0111	$\boxed{0101}$
{1,4}	9	1001	0001
{1,2,4}	11	1011	0001
{1,3,4}	13	1101	$\boxed{0101}$
{1,2,3,4}	15	1111	$\boxed{0101}$

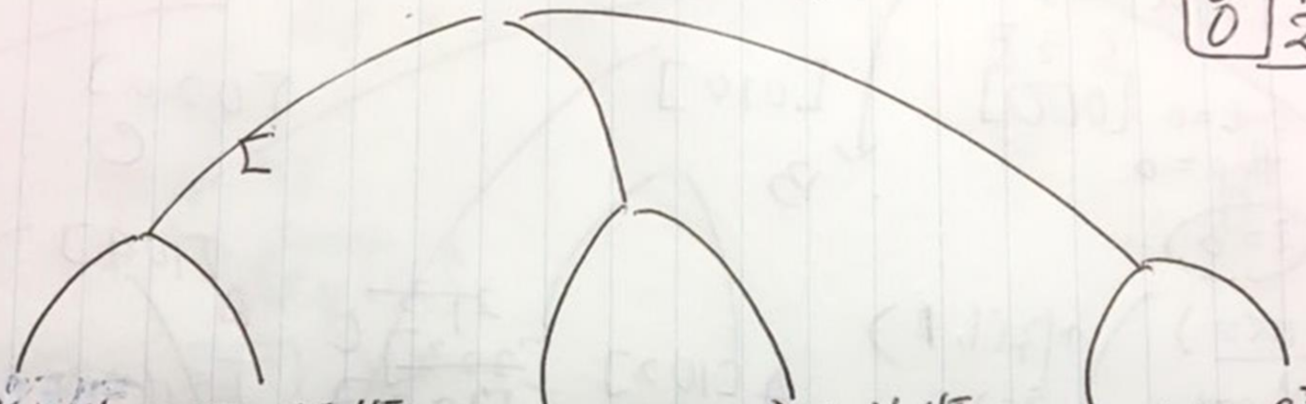
8 {
only 1 {1}
1과 {1, △} $_3C_1$
{1, △, ○} $_3C_2$
{1, 2, 3, 4} $_3C_3$
}

0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
1	10	0	0	10	10	0	0	10	10	0	0	10	10	0	0	10	10
2	100	0	0	0	0	100	100	100	100	0	0	0	0	100	100	100	100
3	1000	0	0	0	0	0	0	0	0	1000	1000	1000	1000	1000	1000	1000	1000
			1		1		1		1		1		1		1		1
	subset			2	2			2	2			2	2			2	2
						3	3	3	3					3	3	3	3
										4	4	4	4	4	4	4	4

{A, B, C}

$\boxed{A>B}$

A	B	C	N	z
2	0	0	2	0
			0	1
			0	2



C	z	F	N	NF
0	0	0	2	0
1	1	1	0	0
2	2	3	0	0

C	z	F	N	NF
0	0	0	2	0
1	2	1	0	0
2	1	5	0	0

C	z	F	N	NF
0	1	0	0	0
1	0	2	2	2
2	2	3	0	0

$\boxed{BA}$C

①

C	z	F	N	NF
0	1	0	0	0
1	2	2	0	0
2	0	6	2	2

BCA

②

C	z	F	N	NF
0	2	0	0	0
1	0	4	2	0
2	1	5	0	0

C	z	F	N	NF
0	2	0	0	0
1	1	4	0	0
2	0	6	2	2

C$\boxed{BA}$

③

dfs(7)

① m[0] += m[2] += m[3] += = 1

② m[0] += m[2] += $\boxed{\text{m[6] += dfs(7) = 1}}$ ☆

③ m[0] += m[4] += $\boxed{\text{m[6] += dfs(n) = 1}}$

☆반복.