Домашняя работа № 1
по КвМех

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№1

$$\psi(x) = \frac{A}{x^2+a^2}$$

усл-е нор-ки:

$$\int_{-\infty}^{+\infty} |\psi(x)|^2 \, dx = 1 = A^2 \int_{-\infty}^{+\infty} \frac{dx}{(x^2+a^2)^2} =$$

$$= A^2 \left| \begin{matrix} u = \frac{1}{x} & du = -x^{-2} dx \\ dv = x(x^2+a^2)^{-2} dx & V = -\frac{(x^2+a^2)^{-1}}{2} \end{matrix} \right| =$$

$$= \cancel{\int_{-\infty}^{+\infty} \frac{A^2}{2x(x^2+a^2)} \Bigg|_{-\infty}^{+\infty}} \to 0 \; - \frac{A^2}{2} \int_{-\infty}^{+\infty} \frac{dx}{x^2(x^2+a^2)} =$$

$$= \frac{A^2}{2a^2} \cancel{\frac{x}{x}} \Bigg|_{-\infty}^{+\infty} \to 0 \; + \frac{A^2}{2a^3} \operatorname{arctg}\left(\frac{x}{a}\right) \Bigg|_{-\infty}^{+\infty} =$$

$$= \frac{A^2}{2a^3} \pi = 1 \Rightarrow A = \sqrt{\frac{2a^3}{\pi}} = \boxed{\frac{a\sqrt{2a}}{\pi}}$$

№ 2

$$\varphi(x) = \frac{B}{x+iB}$$

$$\int_{-\infty}^{+\infty} \frac{B}{x+iB}\,dx = \int \frac{B(x-iB)}{x^2+b^2}\,dx$$

$$\langle\varphi||\varphi\rangle = \frac{B(x+iB)}{(x^2+B^2)} \cdot \frac{B(x-iB)}{(x^2+B^2)} = \frac{B^2\cancel{(x^2+B^2)}}{(x^2+B^2)^{\cancel{2}}}$$

bra ket

$$B^2\int_{-\infty}^{+\infty}\frac{dx}{x^2+b^2} = \frac{B^2}{B}\,\mathrm{arctg}\,\frac{x}{B}\Big|_{-\infty}^{+\infty} =$$

$$= \frac{B^2}{B}\pi = 1 \Rightarrow B = \sqrt{\frac{B}{\pi}} = \cancel{\ }$$

№ 3

$$\langle\varphi|\varphi\rangle = \int_{-\infty}^{+\infty}\varphi^*\varphi\,dx = \int_{-\infty}^{+\infty}\left(\frac{B}{x-iB}\right)\cdot\left(\frac{A}{x^2+a^2}\right)dx$$

$$\int_{-\infty}^{+\infty}\frac{AB\,dx}{(x-iB)(x-ia)(x+ia)}$$

$$-\left(\cancel{-a-b}\ 1\right.$$

$$-\left(a+b+a+b\right)$$

$$-a^2 b\ i$$

$+$ no $\vec{j}$ $\quad + (a+b)\, i a^2$

ko $\vec{n}$ $\quad (b-a)\, i a^2$

$$2 b\, i a^2$$

N 5

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\frac{\pi x}{a} \quad \boxed{x \in [0; a]}$$

$$f_1(x) = d_1 e^{\frac{i\pi x}{a}}, \quad f_2 = d_2 e^{-\frac{\pi x}{a}}$$

a) $\langle f_1 | f_2 \rangle = 0$

$$d_1 d_2 \int_0^a e^{\frac{-i\pi x}{a}} e^{\frac{-i\pi x}{a}}\, dx = \underbrace{d_1 d_2 \cdot \left(-\frac{a}{2i\pi}\right)}_{\text{const}} e^{-\frac{2i\pi x}{a}} \Bigg|_0^a =$$

$$= \underline{\text{const}} \left(\overbrace{\underbrace{\cos 2\pi}_{1} - \underbrace{\sin 2\pi}_{0}}^{1-1=0} - 1 \right) = 0$$

б) α_1, α_2 — ?

$\langle f_1 | f_1 \rangle = 1$

$$\alpha_1^2 \int_0^a e^{\frac{i\pi x}{a} - \frac{i\pi x}{a}} dx = \alpha_1^2 \int_0^a e^0 \, dx = \alpha_1^2 a = 1$$

$$\alpha_1 = \sqrt{\frac{1}{a}}$$

с α_2 ~~аналогично~~ симметрично

в) $|\psi\rangle = C_1 |f_1\rangle + C_2 |f_2\rangle$

$$\langle f_1 | \psi \rangle = \alpha_1 \int_0^a e^{-\frac{i\pi x}{a}} \sqrt{\frac{2}{a}} \sin\frac{\pi x}{a} \, dx$$

$x = \frac{at}{\pi}$ $\quad a = \frac{\pi x}{t}$

$dx = \frac{a\,dt}{\pi}$

$t = \frac{\pi x}{a}$

$$= \left| \begin{array}{l} \frac{\pi x}{a} = t \\ \sqrt{\frac{2}{a}} \frac{\alpha_1 a}{\pi} = const \\ \frac{\sqrt{2a}\,\alpha_1}{\pi} = const \end{array} \right| = const \int_0^a e^{-it} \sin t \, dt =$$

$$= const \left(\int_0^a \cos t \, d(-\cos t) - i \int_0^a \sin^2 t \, dt \right) =$$

$$= const \left(\frac{\cos^2 t}{2} \Big|_a^0 - i \int_0^a \frac{1 - \cos 2t}{2} dt \right) =$$

$$= \frac{const}{2}\left(\frac{1}{2} - \frac{\frac{\sin^2 a}{\cos^2 a}}{2} - i\left(\frac{t}{2} - \frac{\sin 2t}{2}\right)\Big|_0^a\right) =$$

$$= \frac{const}{2}\left(\sin^2 a - i\left(a - \frac{\sin 2a}{2}\right)\right)$$

$$= \frac{\sqrt{2a}\,\alpha_1}{2\pi}\left(\sin^2 a + i\left(\frac{\sin 2a}{2} - a\right)\right)$$

$$\sqrt{\frac{2}{a}}\,\alpha_1 \int_0^a e^{-i\frac{\pi t}{a}} \sin\frac{\pi t}{a}\,dt =$$

$$= \alpha_1\sqrt{\frac{2}{a}}\left(\int_0^a \cos\left(\frac{\pi t}{a}\right)\sin\left(\frac{\pi t}{a}\right)dt - i\int_0^a \sin\frac{\pi t}{a}\sin\frac{\pi t}{a}\,dt\right.$$

$$= \sqrt{\frac{2}{a}}\,\alpha_1\left(\frac{a}{\pi}\frac{\sin^2\frac{\pi t}{a}}{2}\Big|_0^a - i\int_0^a \frac{1}{2} - \frac{\cos\frac{2\pi t}{a}}{2}\right)dt$$

$$= \frac{\alpha_1\sqrt{\frac{2}{a}}\cdot i}{2}\left(\frac{a}{2\pi}\sin\frac{2\pi t}{a}\Big|_0^a - t\right)\Big|_0^a$$

$$= \frac{\alpha_1\sqrt{\frac{2}{a}}}{2}\, i\,(-a) \Rightarrow \sqrt{\frac{a}{2}} \quad \left(\alpha_1 = \frac{1}{\sqrt{a}}\right)$$

$$c_1 = -\frac{i}{\sqrt{2}}$$

$\langle \phi_2 | \psi \rangle$ = аналогично, как $\langle \phi_1$

будет с " + "

$c_2 = \frac{i}{\sqrt{2}}$

$$\boxed{\text{Тогда: } |\psi_0\rangle = -\frac{i}{\sqrt{2}} |\phi_1\rangle + \frac{i}{\sqrt{2}} |\phi_2\rangle}$$