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a) $T(n) = \begin{cases} O(1), & n = 0 \\ T(n-1) + O(1), & n \geq 1 \end{cases}$

$T(n) \leq T(n-1) + C \leq (T(n-2) + C) +$

$+ C = T(n-2) + C(1+1) \leq \dots$

$\leq T(n-n) + C \cdot n = O(n)$

d) $T(n) = \begin{cases} O(1), & n \leq a, \ a > 1 \\ aT(n-a) + O(1), & n > a \end{cases}$

$T(n) \leq aT(n-a) + C \leq$

$$\leq a\left(a\left(T(n-1-a)+C\right)+C\right)+C =$$

$$= a^2 T(n-1-a) + C(a+1) \leq$$

$$\leq a\left(a^2 T(n-2-a)+C\right)+C(a+1)$$

$$= a^3 T(n-2-a) + C(2a+1) \leq \dots$$

$$\leq a^{a} T = a^3 T(n-(a+2)) + C(2a+1)$$

$$\leq \dots \leq a^{(n-a)} T(n-a+n-a)) +$$

$$+ C(a(n-a)+1) =$$

$$= a^{n-a} \cdot C + C(-a^2+an+1) =$$

$$= a^{n-a} C + anC - a^2 C + C =$$

$$= O(a^{n-a})$$

g) $T(n) \begin{cases} O(1), & n = 1 \\ aT(\lceil n/a \rceil) + O(1), & n \geq 2, \ a \geq 2 \end{cases}$

$n = a^m \Rightarrow m = \log_a(n)$

$T(n) \leq aT(\lceil n/a \rceil) + C$

$\leq a(aT(\ldots$

$\Rightarrow T(n) \leq aT(a^{m-1}) + C \leq$

$\leq a(aT(a^{m-2}) + C) + C =$

$= a^2 T(a^{m-2}) + C(a+1) \leq \ldots$

$\leq a^m T(a^{m-m}) + C(am+1) =$

$= a^m C + C(am+1) = O(a^m) =$

$= O(a^{\log_a n}) = O(n)$

h) $T(n) = \begin{cases} O(1), & n = 1 \\ aT(\lfloor n/a \rfloor) + O(n), & n \geq 2, \ a \geq 2 \end{cases}$

$n = a^m \Rightarrow m = \log_a n$

$T(n) \leq aT(a^{m-1}) + Cn =$

$= aT(a^{m-1}) + a^m C \leq$

$\leq a(aT(a^{m-2}) + a^m C) + a^m C =$

$= a^2 T(a^{m-2}) + a^m C(a+1) \leq$

$\leq a(a^2 T(a^{m-3}) + a^m C) +$

$+ a^m C(a+1) = a^3 T(a^{m-3}) +$

$+ a^m C(2a+1) \leq \ldots \leq$

$\leq a^m T(a^{m-m}) + Ca^m(ma+1) =$

$= Ca^m + Ca^{m+1}\left(m + \frac{1}{a}\right) =$

$= O(a^{\log_a n} \cdot (a \log_a n + 1)) =$

$= O(n a \log_a n + n)$