Работа с ориентированным графами

Полная документация http://www.bnlearn.com/ (http://www.bnlearn.com/)

Установите необходимые библиотеки

```
install.packages('bnlearn')
install.packages('visNetwork')
```

Еще несколько из репозитория пакетов для биоинформатики

```
if (!requireNamespace("BiocManager", quietly = TRUE))
    install.packages("BiocManager")
BiocManager::install()
```

```
BiocManager::install(c("graph", "RBGL", "Rgraphviz"))
```

А вот теперь можно поставить еще один пакет из основного репозитория

```
install.packages("gRain")
```

Подключим все пакеты

In [1]:

```
1  library('visNetwork')
2  library('Rgraphviz')
3  library('gRain')
4  library('compiler')
5  library('bnlearn')
```

started 17:24:08 2020-05-13, finished in 1.26s

```
Loading required package: graph

Loading required package: BiocGenerics

Loading required package: parallel


Attaching package: ‘BiocGenerics’


The following objects are masked from ‘package:parallel’:

    clusterApply, clusterApplyLB, clusterCall, clusterEvalQ,
    clusterExport, clusterMap, parApply, parCapply, parLapply,
    parLapplyLB, parRapply, parSapply, parSapplyLB


The following objects are masked from ‘package:stats’:

    IQR, mad, sd, var, xtabs


The following objects are masked from ‘package:base’:

    anyDuplicated, append, as.data.frame, basename, cbind, colnames,
    dirname, do.call, duplicated, eval, evalq, Filter, Find, get, gre
p,
    grepl, intersect, is.unsorted, lapply, Map, mapply, match, mget,
    order, paste, pmax, pmax.int, pmin, pmin.int, Position, rank,
    rbind, Reduce, rownames, sapply, setdiff, sort, table, tapply,
    union, unique, unsplit, which, which.max, which.min


Loading required package: grid

Loading required package: gRbase


Attaching package: ‘compiler’


The following object is masked from ‘package:gRbase’:

    compile



Attaching package: ‘bnlearn’


The following objects are masked from ‘package:gRbase’:
```

```
    ancestors, children, parents

The following objects are masked from ‘package:BiocGenerics’:

    path, score

The following object is masked from ‘package:stats’:

    sigma
```

In [2]:

```r
options(repr.plot.width = 10, repr.plot.height = 8)
```

started 17:24:08 2020-05-13, finished in 1.39s

Вспомогательная функция, для красивой визуализации графов

In [3]:

```r
plot.network <- function(structure, ht = "400px"){
  nodes.uniq <- unique(c(structure$arcs[,1], structure$arcs[,2]))
  nodes <- data.frame(id = nodes.uniq,
                      label = nodes.uniq,
                      color = "darkturquoise",
                      shadow = TRUE)

  edges <- data.frame(from = structure$arcs[,1],
                      to = structure$arcs[,2],
                      arrows = "to",
                      smooth = TRUE,
                      shadow = TRUE,
                      color = "black")

  return(visNetwork(nodes, edges, height = ht, width = "100%"))
}
```

started 17:24:08 2020-05-13, finished in 1.40s

Структура графов

Загрузим данные об оцентах 88 студентов по механике, векторам, алгебре, анализу и статистике. Все значения лежат в пределах от 0 до 100.

In [4]:

```
data(marks)
str(marks)
```

started 17:24:08 2020-05-13, finished in 1.42s

```
'data.frame':   88 obs. of  5 variables:
 $ MECH: num  77 63 75 55 63 53 51 59 62 64 ...
 $ VECT: num  82 78 73 72 63 61 67 70 60 72 ...
 $ ALG : num  67 80 71 63 65 72 65 68 58 60 ...
 $ ANL : num  67 70 66 70 70 64 65 62 62 62 ...
 $ STAT: num  81 81 81 68 63 73 68 56 70 45 ...
```

In [5]:

```
head(marks)
```

started 17:24:08 2020-05-13, finished in 1.45s

A data.frame: 6 × 5

MECH	VECT	ALG	ANL	STAT
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
77	82	67	67	81
63	78	80	70	81
75	73	71	66	81
55	72	63	70	68
63	63	65	70	63
53	61	72	64	73

Зададим матрицу ориентированных ребер, из которых далее построим граф

In [6]:

```
edges <- matrix(
 c("VECT", "MECH", "ALG", "MECH", "ALG", "VECT",
 "ANL", "ALG", "STAT", "ALG", "STAT", "ANL"),
 ncol = 2, byrow = TRUE,
 dimnames = list(c(), c("from", "to")))

edges
```

started 17:24:08 2020-05-13, finished in 1.47s

A matrix: 6 × 2 of
type chr

from	to
VECT	MECH
ALG	MECH
ALG	VECT
ANL	ALG
STAT	ALG
STAT	ANL

Зададим пустой граф, состоящий только из вершин, соответствующим нашим признакам. Далее установим ребра в графе

In [7]:

```
dag <- empty.graph(names(marks))
arcs(dag) <- edges
```

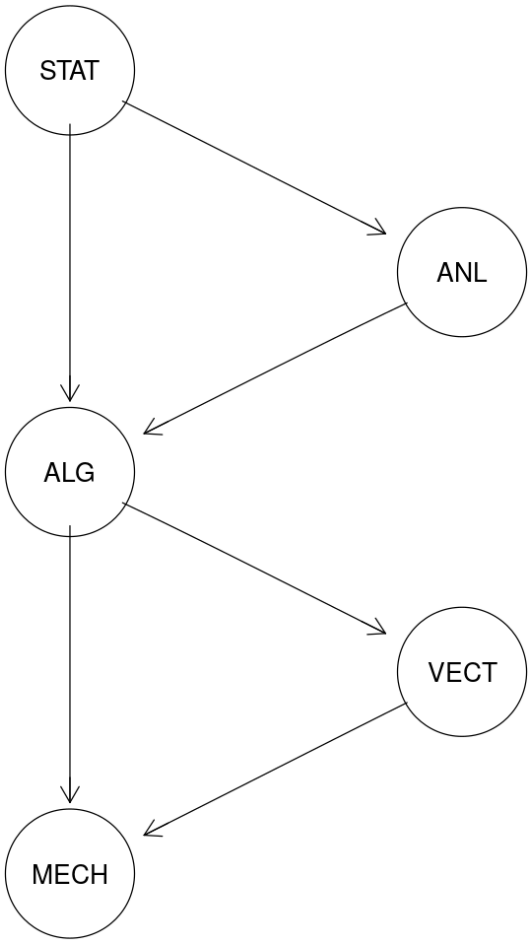
started 17:24:08 2020-05-13, finished in 1.48s

Визуализация графа

In [8]:

```
1  graphviz.plot(dag)
```

started 17:24:08 2020-05-13, finished in 1.64s



Посмотрим, как хранится информация об узлах графа.

Для каждого узла тут содежится следующая информяация:

- `parents` -- родители;
- `children` -- дети;
- `nbr` -- соседи (родители и дети);
- `mb` -- Markov blanket -- марковское ограждение узла. Это набор переменных, для которых верно, что переменная узла условно независима от всех остальных переменных в графе при условии марковского ограждения. Например, в примере ниже переменная MECH условно не зависит от всех остальных переменных графа (т.е. STAT, ANL) при условии VECT и ALG. Этот набор может не совпадать с соседями, если дети образуют коллайдр. Еще пример (https://ru.wikipedia.org/wiki/Марковское_ограждение#/media/File:Diagram_of_a_Markov_blanket.svg);

In [9]:

```
1  dag$nodes
```

started 17:24:08 2020-05-13, finished in 1.66s

$MECH
$mb
'VECT' 'ALG'

$nbr
'VECT' 'ALG'

$parents
'VECT' 'ALG'

$children

$VECT
$mb
'MECH' 'ALG'

$nbr
'MECH' 'ALG'

$parents
'ALG'
$children
'MECH'

$ALG
$mb
'MECH' 'VECT' 'ANL' 'STAT'

$nbr
'MECH' 'VECT' 'ANL' 'STAT'

$parents
'ANL' 'STAT'

$children
'MECH' 'VECT'

$ANL
$mb
'ALG' 'STAT'

$nbr
'ALG' 'STAT'

$parents
'STAT'
$children
'ALG'

$STAT
$mb
'ALG' 'ANL'

$nbr

'ALG' 'ANL'

$parents
$children
'ALG' 'ANL'

Так можно извлечь информацию о конкретном узле. Чтобы извлечь, например, его родителей, нужно аналогично написать `dag$nodes$MECH$parents` .

In [10]:

```
1  dag$nodes$MECH
```

started 17:24:08 2020-05-13, finished in 1.67s

$mb
'VECT' 'ALG'

$nbr
'VECT' 'ALG'

$parents
'VECT' 'ALG'

$children

Извлечении матрицы ребер графа

In [11]:

```
1  dag$arcs
```

started 17:24:08 2020-05-13, finished in 1.69s

A matrix: 6 × 2 of type chr

from	to
VECT	MECH
ALG	MECH
ALG	VECT
ANL	ALG
STAT	ALG
STAT	ANL

Есть и более цивилизованный способ извлечения информации о структуре графа.

Например, так можно вывести всех родителей вершины ALG.

Внимание! *Если у вас при выполнении следующего кода возникает ошибка, возможная проблема может быть в том, что при подключении другого пакета он переопределил имя функции. В таком случае явно укажите пакет* `bnlearn::parents` .

In [12]:

```
1  parents(dag, 'ALG')
```

started 17:24:08 2020-05-13, finished in 1.70s

'ANL' 'STAT'

Код, проверяющий, является ли ANL родителем вершины ALG

In [13]:

```
1  'ANL' %in% parents(dag, 'ALG')
```

started 17:24:08 2020-05-13, finished in 1.72s

TRUE

Потомки вершины ANL

In [14]:

```
1  descendants(dag, 'ANL')
```

started 17:24:08 2020-05-13, finished in 1.73s

'ALG' 'VECT' 'MECH'

Аналогичные операции существуют для ребер графа. Например, все исходящие ребра вершины ALG

In [15]:

```
1  outgoing.arcs(dag, 'ALG')
```

started 17:24:08 2020-05-13, finished in 1.74s

A matrix: 2 × 2

of type chr

from	to
ALG	MECH
ALG	VECT

Полный список операций со структурой графа

```
 1 ## nodes
 2 mb(x, node)
 3 nbr(x, node)
 4 parents(x, node)
 5 parents(x, node, debug = FALSE) <- value
 6 children(x, node)
 7 children(x, node, debug = FALSE) <- value
 8 spouses(x, node)
 9 ancestors(x, node)
10 descendants(x, node)
11 in.degree(x, node)
12 out.degree(x, node)
13 root.nodes(x)
```

```
leaf.nodes(x)
nnodes(x)

## arcs
arcs(x)
arcs(x, check.cycles = TRUE, check.illegal = TRUE, debug = FALSE) <- value
directed.arcs(x)
undirected.arcs(x)
incoming.arcs(x, node)
outgoing.arcs(x, node)
incident.arcs(x, node)
compelled.arcs(x)
reversible.arcs(x)
narcs(x)

## adjacency matrix
amat(x)
amat(x, check.cycles = TRUE, check.illegal = TRUE, debug = FALSE) <- value

## graphs
nparams(x, data, effective = FALSE, debug = FALSE)
ntests(x)
whitelist(x)
blacklist(x)

## shared with the graph package.
# these used to be a simple nodes(x) function.
## S4 method for signature 'bn'
nodes(object)
## S4 method for signature 'bn.fit'
nodes(object)
# these used to be a simple degree(x, node) function.
## S4 method for signature 'bn'
degree(object, Nodes)
## S4 method for signature 'bn.fit'
degree(object, Nodes)
# re-label the nodes.
## S4 replacement method for signature 'bn'
nodes(object) <- value
## S4 replacement method for signature 'bn.fit'
nodes(object) <- value
```

Коллайдры графа

In [16]:

```
vstructs(dag, moral = TRUE)
```

started 17:24:08 2020-05-13, finished in 1.70s

A matrix: 2 × 3 of type
chr

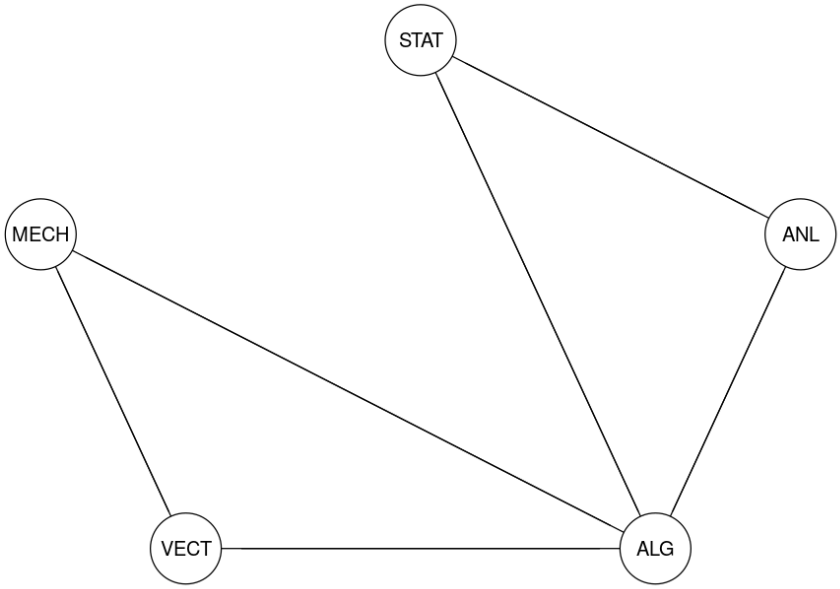
X	Z	Y
VECT	MECH	ALG
ANL	ALG	STAT

Скелет графа (удаление ориентации ребер)

In [17]:

```
1  ug <- moral(dag)
2  plot(ug)
```

started 17:24:08 2020-05-13, finished in 1.79s

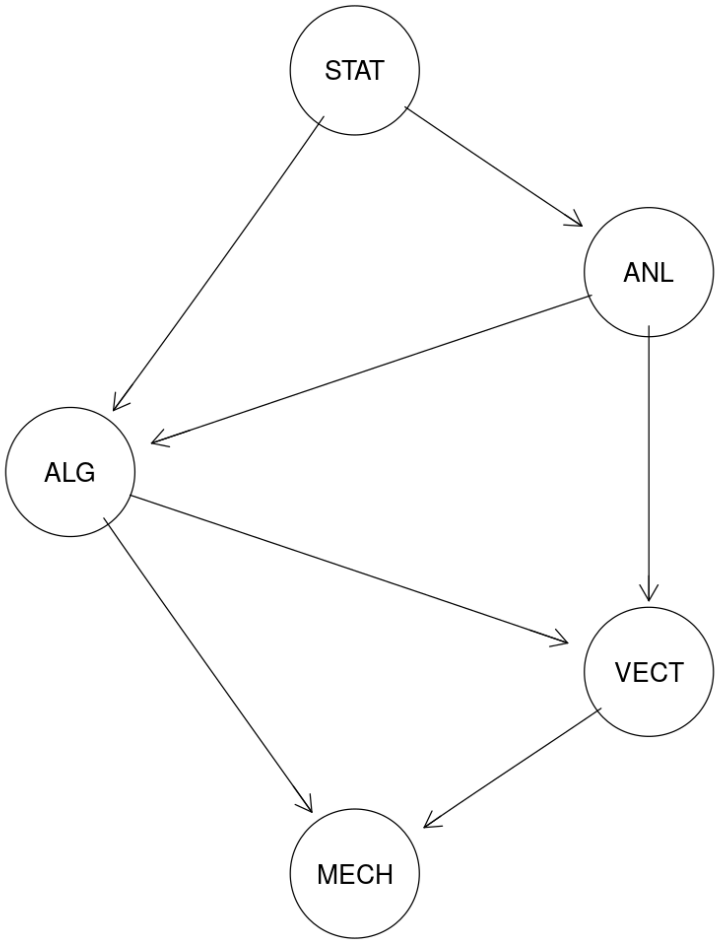


Граф можно задать по одному ребру

In [18]:

```
dag_1 <- empty.graph(nodes(dag))
dag_1 <- set.arc(dag_1, "VECT", "MECH")
dag_1 <- set.arc(dag_1, "ALG", "MECH")
dag_1 <- set.arc(dag_1, "ALG", "VECT")
dag_1 <- set.arc(dag_1, "ANL", "ALG")
dag_1 <- set.arc(dag_1, "STAT", "ALG")
dag_1 <- set.arc(dag_1, "STAT", "ANL")
dag_1 <- set.arc(dag_1, "ANL", "VECT")
graphviz.plot(dag_1)
```

started 17:24:08 2020-05-13, finished in 1.94s



Проверка на равенство графов

In [19]:

```
1  all.equal(dag, dag_1)
```

started 17:24:08 2020-05-13, finished in 1.95s

'Different number of directed/undirected arcs'

Оценка распределений в построенном графе

В функцию можно передать также параметр `method` -- метод оценки распределений. По умолчанию он равен `mle` (оценка максимального правдоподобия). Для дискретных данных можно указать `bayes` .

In [20]:

```
1  fitted <- bn.fit(dag, data = marks)
```

started 17:24:08 2020-05-13, finished in 1.96s

Посмотрим на то, что получилось.

Поскольку наши данные непрерывны, распределения по умолчанию считаются гауссовскими. Информация ниже о переменной MECH говорит о том, что условное распределение MECH при условии VECT=x, ALG=y является нормальным со средним -12.36 + 0.47x + 0.55y и дисперсией 13.97^2.

In [21]:

```
1  fitted
```

started 17:24:08 2020-05-13, finished in 1.97s

```
  Bayesian network parameters

  Parameters of node MECH (Gaussian distribution)

Conditional density: MECH | VECT + ALG
Coefficients:
(Intercept)         VECT          ALG
-12.3647583    0.4658693    0.5484053
Standard deviation of the residuals: 13.97432

  Parameters of node VECT (Gaussian distribution)

Conditional density: VECT | ALG
Coefficients:
(Intercept)          ALG
 12.4183094    0.7543653
Standard deviation of the residuals: 10.48167

  Parameters of node ALG (Gaussian distribution)

Conditional density: ALG | ANL + STAT
Coefficients:
(Intercept)          ANL         STAT
 24.7254768    0.3482454    0.2273881
Standard deviation of the residuals: 6.871428

  Parameters of node ANL (Gaussian distribution)

Conditional density: ANL | STAT
Coefficients:
(Intercept)         STAT
 24.5824229    0.5223601
Standard deviation of the residuals: 11.86392

  Parameters of node STAT (Gaussian distribution)

Conditional density: STAT
Coefficients:
(Intercept)
   42.30682
Standard deviation of the residuals: 17.25559
```

Логарифм правдоподобия, AIC и BIC по обученно модели

In [22]:

```
logLik(fitted, data = marks)
AIC(fitted, data = marks)
BIC(fitted, data = marks)
```

started 17:24:08 2020-05-13, finished in 1.99s

-1695.58853429146

-1711.58853429146

-1731.40722880728

In [23]:

```
data(alarm)
```

started 17:24:08 2020-05-13, finished in 2.00s

Рассмотрим данные о мониторинговой системе за пациентами, состоящую из следующих датчиков:

- `CVP` (central venous pressure): a three-level factor with levels `LOW`, `LOW` and `HIGH`.
- `PCWP` (pulmonary capillary wedge pressure): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `HIST` (history): a two-level factor with levels `TRUE` and `FALSE`.
- `TPR` (total peripheral resistance): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `BP` (blood pressure): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `CO` (cardiac output): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `HRBP` (heart rate / blood pressure): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `HREK` (heart rate measured by an EKG monitor): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `HRSA` (heart rate / oxygen saturation): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `PAP` (pulmonary artery pressure): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `SAO2` (arterial oxygen saturation): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `FIO2` (fraction of inspired oxygen): a two-level factor with levels `LOW` and `NORMAL`.
- `PRSS` (breathing pressure): a four-level factor with levels `ZERO`, `LOW`, `NORMAL` and `HIGH`.
- `ECO2` (expelled CO2): a four-level factor with levels `ZERO`, `LOW`, `NORMAL` and `HIGH`.
- `MINV` (minimum volume): a four-level factor with levels `ZERO`, `LOW`, `NORMAL` and `HIGH`.
- `MVS` (minimum volume set): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `HYP` (hypovolemia): a two-level factor with levels `TRUE` and `FALSE`.
- `LVF` (left ventricular failure): a two-level factor with levels `TRUE` and `FALSE`.
- `APL` (anaphylaxis): a two-level factor with levels `TRUE` and `FALSE`.
- `ANES` (insufficient anesthesia/analgesia): a two-level factor with levels `TRUE` and `FALSE`.
- `PMB` (pulmonary embolus): a two-level factor with levels `TRUE` and `FALSE`.
- `INT` (intubation): a three-level factor with levels `NORMAL`, `ESOPHAGEAL` and `ONESIDED`.
- `KINK` (kinked tube): a two-level factor with levels `TRUE` and `FALSE`.
- `DISC` (disconnection): a two-level factor with levels `TRUE` and `FALSE`.
- `LVV` (left ventricular end-diastolic volume): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `STKV` (stroke volume): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `CCHL` (catecholamine): a two-level factor with levels `NORMAL` and `HIGH`.
- `ERLO` (error `LOW` output): a two-level factor with levels `TRUE` and `FALSE`.
- `HR` (heart rate): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `ERCA` (electrocauter): a two-level factor with levels `TRUE` and `FALSE`.
- `SHNT` (shunt): a two-level factor with levels `NORMAL` and `HIGH`.
- `PVS` (pulmonary venous oxygen saturation): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.
- `ACO2` (arterial CO2): a three-level factor with levels `LOW`, `NORMAL` and `HIGH`.

- `VALV` (pulmonary alveoli ventilation): a four-level factor with levels `ZERO`, `LOW`, `NORMAL` and `HIGH`.
- `VLNG` (lung ventilation): a four-level factor with levels `ZERO`, `LOW`, `NORMAL` and `HIGH`.
- `VTUB` (ventilation tube): a four-level factor with levels `ZERO`, `LOW`, `NORMAL` and `HIGH`.
- `VMCH` (ventilation machine): a four-level factor with levels `ZERO`, `LOW`, `NORMAL` and `HIGH`.

In [24]:

```
1  head(alarm)
```

started 17:24:08 2020-05-13, finished in 2.03s

A data.frame: 6 × 37

CVP	PCWP	HIST	TPR	BP	CO	HRBP	HREK	HRSA	PAP	⋯
<fct>	<fct>	<fct>	<fct>	<fct>	<fct>	<fct>	<fct>	<fct>	<fct>	⋯
NORMAL	NORMAL	FALSE	LOW	NORMAL	HIGH	HIGH	HIGH	HIGH	NORMAL	⋯
NORMAL	NORMAL	FALSE	NORMAL	LOW	LOW	HIGH	HIGH	HIGH	NORMAL	⋯
NORMAL	HIGH	FALSE	NORMAL	NORMAL	HIGH	HIGH	HIGH	HIGH	NORMAL	⋯
NORMAL	NORMAL	FALSE	LOW	LOW	HIGH	HIGH	HIGH	HIGH	NORMAL	⋯
NORMAL	NORMAL	FALSE	LOW	LOW	NORMAL	HIGH	HIGH	HIGH	NORMAL	⋯
NORMAL	NORMAL	FALSE	LOW	NORMAL	HIGH	HIGH	HIGH	HIGH	NORMAL	⋯

In [25]:

```
1  options(repr.plot.width = 20, repr.plot.height = 20)
```

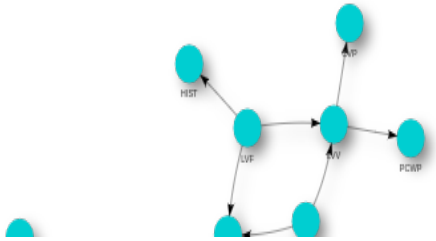
started 17:24:08 2020-05-13, finished in 2.03s

Зададим граф вручную через строку, в которой перечислим все непосредственные причинно-следственные связи. В текстовой строке каждой вершине должен соответствовать один блок из квадратных скобок. На первом месте ставится имя вершины. После вертикальной черты ставятся все родители вершины через двоеточие. Например, подстрока `[HYP]` соответствует вершине `HYP`, у которой нет родителей. Подстрока `[HRBP|ERLO:HR]` соответствует вершине `HRBP`, у которой двое родителей `ERLO` и `HR`.

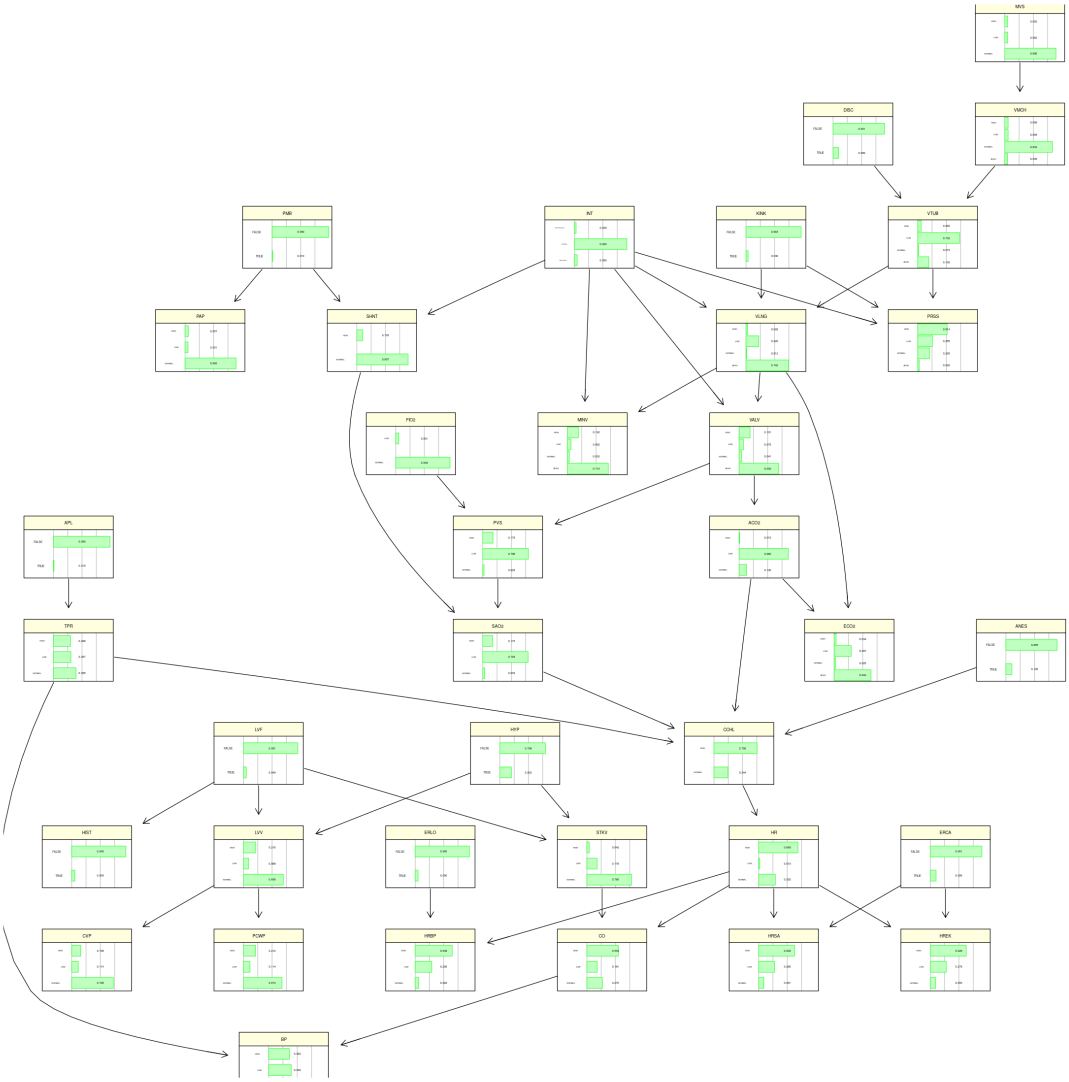
In [26]:

```
modelstring <- paste("[HIST|LVF][CVP|LVV][PCWP|LVV][HYP][LVV|HYP:LVF][LVF]",
  "[STKV|HYP:LVF][ERLO][HRBP|ERLO:HR][HREK|ERCA:HR][ERCA][HRSA|ERCA:HR][ANES]"
  "[APL][TPR|APL][ECO2|ACO2:VLNG][KINK][MINV|INT:VLNG][FIO2][PVS|FIO2:VALV]",
  "[SAO2|PVS:SHNT][PAP|PMB][PMB][SHNT|INT:PMB][INT][PRSS|INT:KINK:VTUB][DISC]"
  "[MVS][VMCH|MVS][VTUB|DISC:VMCH][VLNG|INT:KINK:VTUB][VALV|INT:VLNG][ACO2|VAL
  "[CCHL|ACO2:ANES:SAO2:TPR][HR|CCHL][CO|HR:STKV][BP|CO:TPR]", sep = "")
dag = model2network(modelstring)

# интерактивная визуализация
plot.network(dag, ht = "600px")

# оценка распределений
fitted = bn.fit(dag, alarm)

# визуализация
graphviz.chart(fitted, type = "barprob", grid = TRUE,
               bar.col = "green", strip.bg = "lightyellow")
```

started 17:24:08 2020-05-13, finished in 2.96s




```
Warning message in as.grain.bn.fit(fitted):
“NaN conditional probabilities in CCHL, replaced with a uniform distri
bution.”
Warning message in as.grain.bn.fit(fitted):
“NaN conditional probabilities in ECO2, replaced with a uniform distri
bution.”
Warning message in as.grain.bn.fit(fitted):
“NaN conditional probabilities in PRSS, replaced with a uniform distri
bution.”
Warning message in as.grain.bn.fit(fitted):
“NaN conditional probabilities in VLNG, replaced with a uniform distri
bution.”
```



Посмотрим, как обучилась модель. Поскольку все величины категориальны, в качестве распределения используется мультиномиальное распределение. В таблицах записаны условные вероятности. Например P($BP = HIGH \mid TPR = HIGH, CO = HIGH$) = 0.8977

In [27]:

```
1  fitted
```

started 17:24:08 2020-05-13, finished in 2.97s

```
  Bayesian network parameters

  Parameters of node ACO2 (multinomial distribution)

Conditional probability table:

        VALV
ACO2             HIGH         LOW      NORMAL        ZERO
  HIGH   0.008778931 0.015013055 0.001212121 0.010011524
  LOW    0.906092046 0.885117493 0.073939394 0.892250072
  NORMAL 0.085129024 0.099869452 0.924848485 0.097738404

  Parameters of node ANES (multinomial distribution)

Conditional probability table:
   FALSE    TRUE
0.89535 0.10465

  Parameters of node APL (multinomial distribution)
```

Условные вероятности

По построенному графу можно посчитать вероятности с помощью функции `cpquery` по принципу `cpquery(fitted, event, evidence)` есть вероятность P($event \mid evidence$) по графу `fitted`. Например, вероятность P($CCHL = HIGH \mid VALV = HIGH$) считается запросом ниже.

Вероятности вычисляются с помощью семплирования n раз, поэтому для получения более точных результатов нужно выставить большое значение параметра `n`.

In [28]:

```
1  cpquery(fitted,
2          event = (CCHL == 'HIGH'),
3          evidence = (VALV == 'HIGH'),
4          n = 10^7)
```

started 17:24:08 2020-05-13, finished in 5.94s

0.693490794337525

События можно комбинировать с помощью логических операторов `&` (и) и `|` (или). Например, условная вероятность $\mathrm{P}\left(\{CCHL = HIGH\} \cap \{HRSA = LOW\} \,\middle|\, \{VALV = HIGH\} \cup \{HR = LOW\}\right)$ считается запросом ниже.

In [29]:

```
cpquery(fitted,
        event = (CCHL == 'HIGH') & (HRSA == "LOW"),
        evidence = (VALV == 'HIGH') | (HR == "LOW"),
        n = 10^7)
```

started 17:24:08 2020-05-13, finished in 9.36s

0.0728257757721329

In [30]:

```
cpquery(fitted,
        event = (HRSA == "LOW"),
        evidence = (HR == "LOW"),
        n = 10^7)
```

started 17:24:08 2020-05-13, finished in 12.3s

0.345755071374906

Точные вероятности можно посчитать с помощью пакета `grain`, предварительно переведя граф в формат этого пакета

In [31]:

```
jtree = as.grain(fitted)
jprop = setFinding(jtree, nodes = "VALV", states = "HIGH")
querygrain(jprop, nodes = 'CCHL')
```

started 17:24:08 2020-05-13, finished in 12.3s

```
Warning message in as.grain.bn.fit(fitted):
“NaN conditional probabilities in CCHL, replaced with a uniform distri
bution.”
Warning message in as.grain.bn.fit(fitted):
“NaN conditional probabilities in ECO2, replaced with a uniform distri
bution.”
Warning message in as.grain.bn.fit(fitted):
“NaN conditional probabilities in PRSS, replaced with a uniform distri
bution.”
Warning message in as.grain.bn.fit(fitted):
“NaN conditional probabilities in VLNG, replaced with a uniform distri
bution.”
```

$CCHL =

HIGH

0.694003864731633

NORMAL

0.305996135268367

Методы построения графов по данным

Еще раз посмотрим на датасет с оценками

In [32]:

```
1  head(marks)
```

started 17:24:08 2020-05-13, finished in 12.3s

A data.frame: 6 × 5

MECH <dbl>	VECT <dbl>	ALG <dbl>	ANL <dbl>	STAT <dbl>
77	82	67	67	81
63	78	80	70	81
75	73	71	66	81
55	72	63	70	68
63	63	65	70	63
53	61	72	64	73

Обучение производится следующими методами:

- Grow-Shrink (GS)
- Incremental Association (IAMB)
- Fast Incremental Association (Fast-IAMB)
- Interleaved Incremental Association (Inter-IAMB)
- Max-Min Parents and Children (MMPC)
- Semi-Interleaved HITON-PC

Параметры:

- `x` -- данные;
- `whitelist` -- таблица с двумя колонками, содержащая ребра, которые точно должны быть включены в граф;
- `blacklist` -- таблица с двумя колонками, содержащая ребра, которые точно не должны быть включены в граф;
- `test` -- критерий условной независимости: хи-квадрат, частная корреляция, Джонкхиера и другие;
- `alpha` -- уровень значимости критериев.

Подробное описание всех методов и условий их применимости можно почитать в справке к пакету:

In [33]:

```
1  ?bnlearn
```

started 17:24:08 2020-05-13, finished in 12.4s

Интерфейс функций следующий

```
1  pc.stable(x, cluster = NULL, whitelist = NULL, blacklist = NULL, test = NULL,
2    alpha = 0.05, B = NULL, max.sx = NULL, debug = FALSE, undirected = FALSE)
3  gs(x, cluster = NULL, whitelist = NULL, blacklist = NULL, test = NULL,
4    alpha = 0.05, B = NULL, max.sx = NULL, debug = FALSE, optimized = FALSE,
5    strict = FALSE, undirected = FALSE)
6  iamb(x, cluster = NULL, whitelist = NULL, blacklist = NULL, test = NULL,
```

```
  alpha = 0.05, B = NULL, max.sx = NULL, debug = FALSE, optimized = FALSE,
  strict = FALSE, undirected = FALSE)
fast.iamb(x, cluster = NULL, whitelist = NULL, blacklist = NULL, test = NULL,
  alpha = 0.05, B = NULL, max.sx = NULL, debug = FALSE, optimized = FALSE,
  strict = FALSE, undirected = FALSE)
inter.iamb(x, cluster = NULL, whitelist = NULL, blacklist = NULL, test =
NULL,
  alpha = 0.05, B = NULL, max.sx = NULL, debug = FALSE, optimized = FALSE,
  strict = FALSE, undirected = FALSE)
mmpc(x, cluster = NULL, whitelist = NULL, blacklist = NULL, test = NULL,
  alpha = 0.05, B = NULL, max.sx = NULL, debug = FALSE, optimized = FALSE,
  strict = FALSE, undirected = TRUE)
si.hiton.pc(x, cluster = NULL, whitelist = NULL, blacklist = NULL, test =
NULL,
  alpha = 0.05, B = NULL, max.sx = NULL, debug = FALSE, optimized = FALSE,
  strict = FALSE, undirected = TRUE)
```

Обучим граф на наших данных

In [34]:

```
bn.gs <- gs(marks)
bn.gs
```

started 17:24:08 2020-05-13, finished in 12.4s

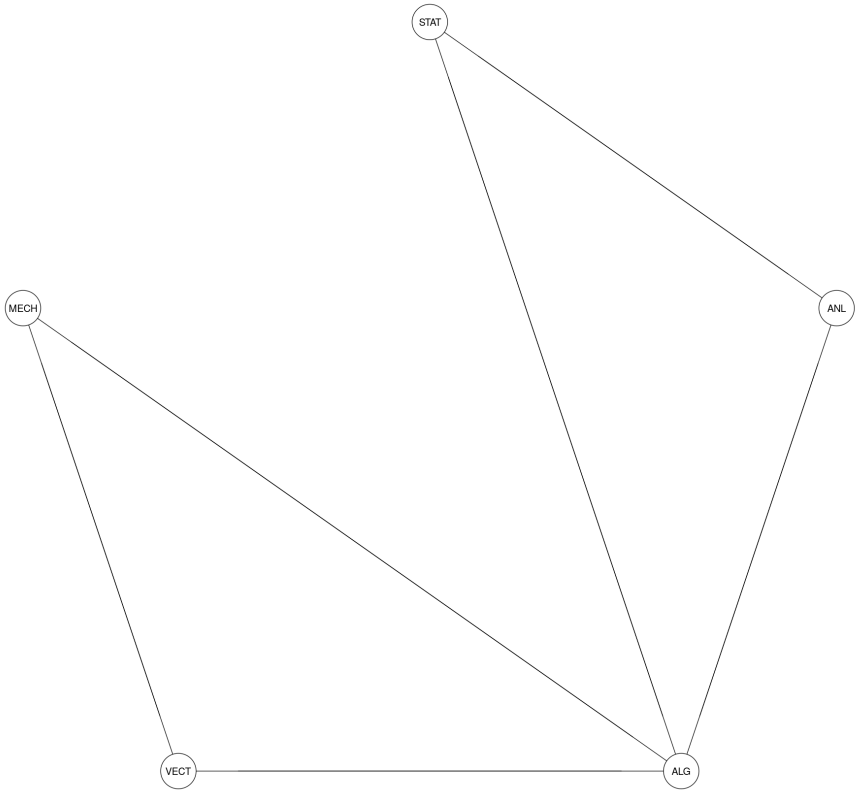
```
  Bayesian network learned via Constraint-based methods

  model:
    [undirected graph]
  nodes:                                5
  arcs:                                 6
    undirected arcs:                    6
    directed arcs:                      0
  average markov blanket size:          2.40
  average neighbourhood size:           2.40
  average branching factor:             0.00

  learning algorithm:                   Grow-Shrink
  conditional independence test:        Pearson's Correlation
  alpha threshold:                      0.05
  tests used in the learning procedure: 80
```

Нарисуем граф. Как видим, ни одно ребро ориентировать не получилось. Причина -- очень мало данных, в то время как для хорошего обучения требуется десятки тысяч наблюдений.

In [35]:

```
1  plot(bn.gs)
```

started 17:24:08 2020-05-13, finished in 12.6s



Построим граф по данным о мониторинговой системе

In [36]:

```
bn.iamb <- iamb(alarm)
bn.iamb
```

started 17:24:08 2020-05-13, finished in 13.2s

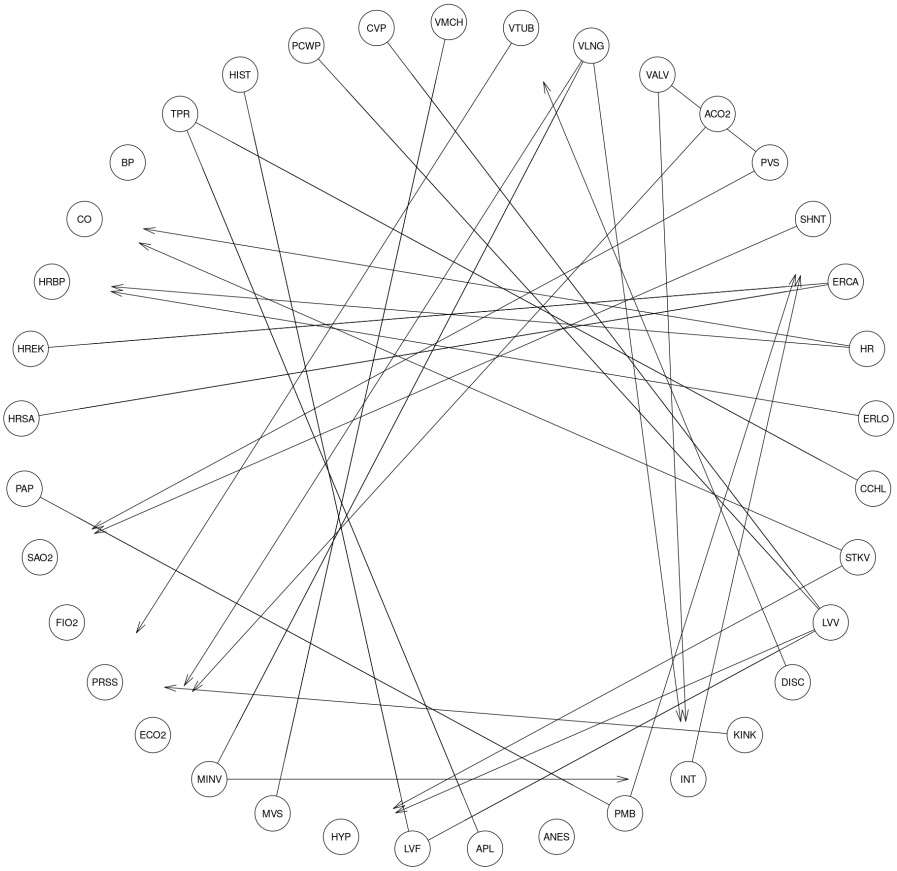
```
Warning message in vstruct.apply(arcs = arcs, vs = vs, nodes = nodes,
debug = debug):
“vstructure BP -> TPR <- CCHL is not applicable, because one or both a
rcs are oriented in the opposite direction.”
Warning message in vstruct.apply(arcs = arcs, vs = vs, nodes = nodes,
debug = debug):
“vstructure BP -> CO <- STKV is not applicable, because one or both ar
cs are oriented in the opposite direction.”
Warning message in vstruct.apply(arcs = arcs, vs = vs, nodes = nodes,
debug = debug):
“vstructure BP -> CO <- HR is not applicable, because one or both arcs
are oriented in the opposite direction.”
Warning message in vstruct.apply(arcs = arcs, vs = vs, nodes = nodes,
debug = debug):
“vstructure HYP -> LVV <- LVF is not applicable, because one or both a
rcs are oriented in the opposite direction.”


  Bayesian network learned via Constraint-based methods

  model:
    [partially directed graph]
  nodes:                                 37
  arcs:                                  34
    undirected arcs:                     13
    directed arcs:                       21
  average markov blanket size:           2.43
  average neighbourhood size:            1.84
  average branching factor:              0.57

  learning algorithm:                    IAMB
  conditional independence test:         Mutual Information (disc.)
  alpha threshold:                       0.05
  tests used in the learning procedure:  6763
```

Данных много, поэтому результат неплохой

In [37]:

```
1  plot(bn.iamb)
```

started 17:24:08 2020-05-13, finished in 13.5s



Красивая визуализация

In [38]:

```
1  plot.network(bn.iamb, ht = "600px")
```

started 17:24:08 2020-05-13, finished in 13.6s