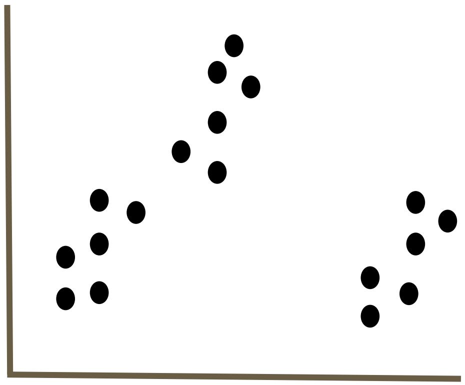
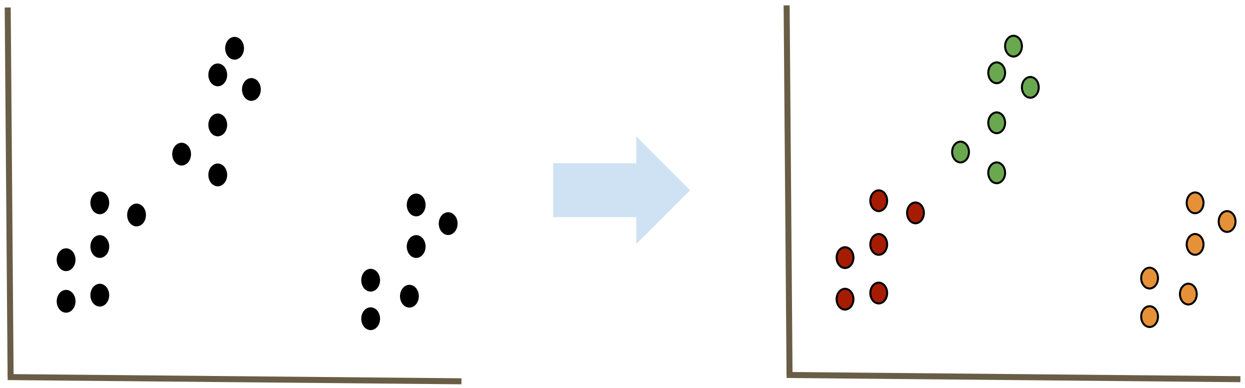
Clustering - Kmeans

Boston University CS 506 - Lance Galletti

What is a Clustering



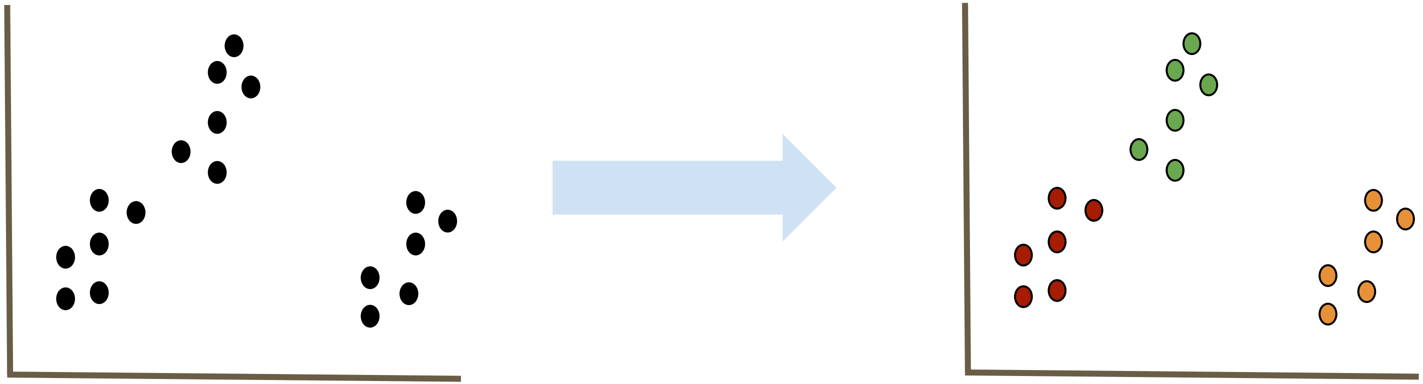
What is a Clustering



What is a Clustering

A clustering is a grouping / assignment of objects (data points) such that objects in the same group / cluster are:

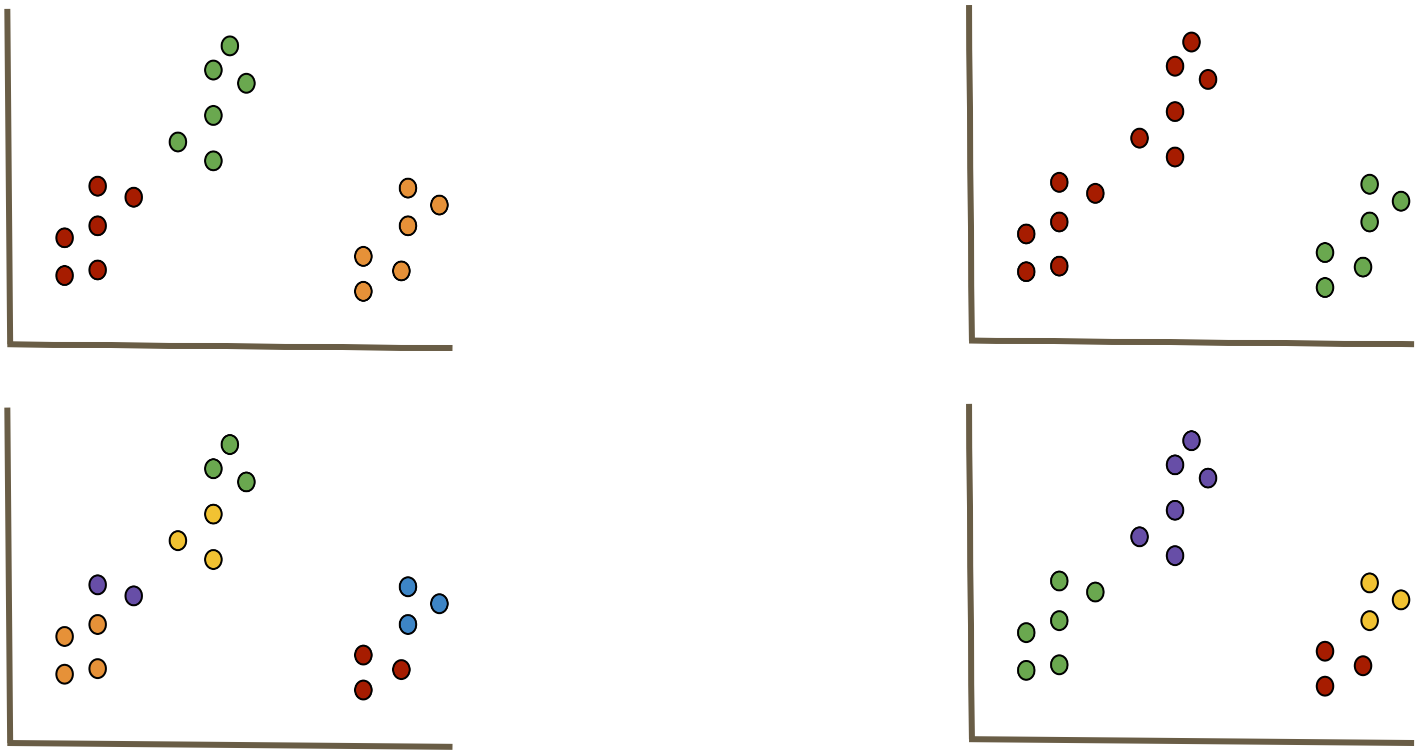
- similar to one another
- dissimilar to objects in other groups



Applications

- Outlier detection / anomaly detection
  - Data Cleaning / Processing
  - Credit card fraud, spam filter etc.
- Feature Extraction
- Filling Gaps in your data
  - Using the same marketing strategy for similar people
  - Infer probable values for gaps in the data (similar users could have similar hobbies, likes / dislikes etc.)

Clusters can be Ambiguous



Types of Clusterings

Partitional
Each object belongs to exactly one cluster

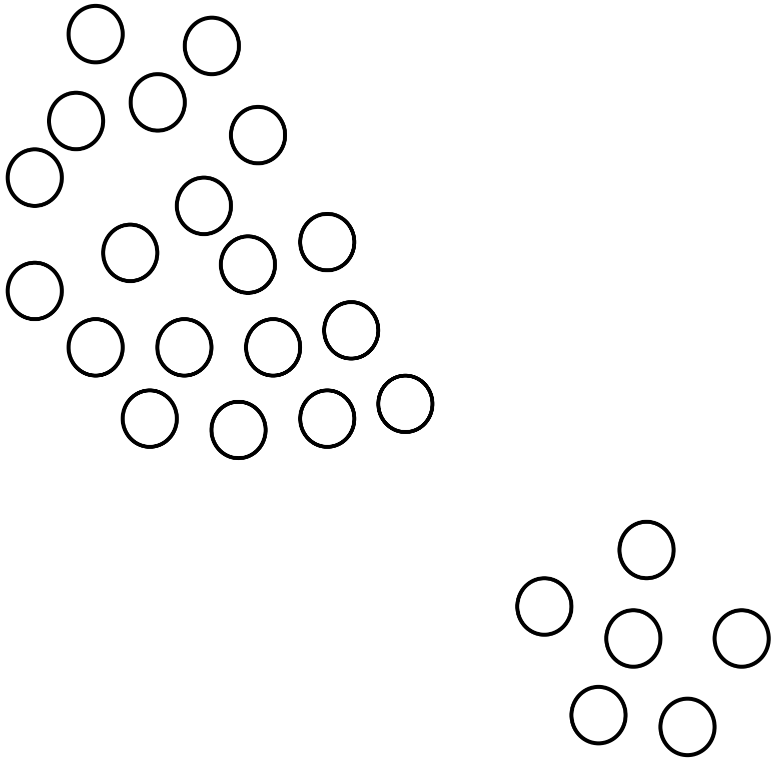
Hierarchical
A set of nested clusters organized in a tree

Density-Based
Defined based on the local density of points

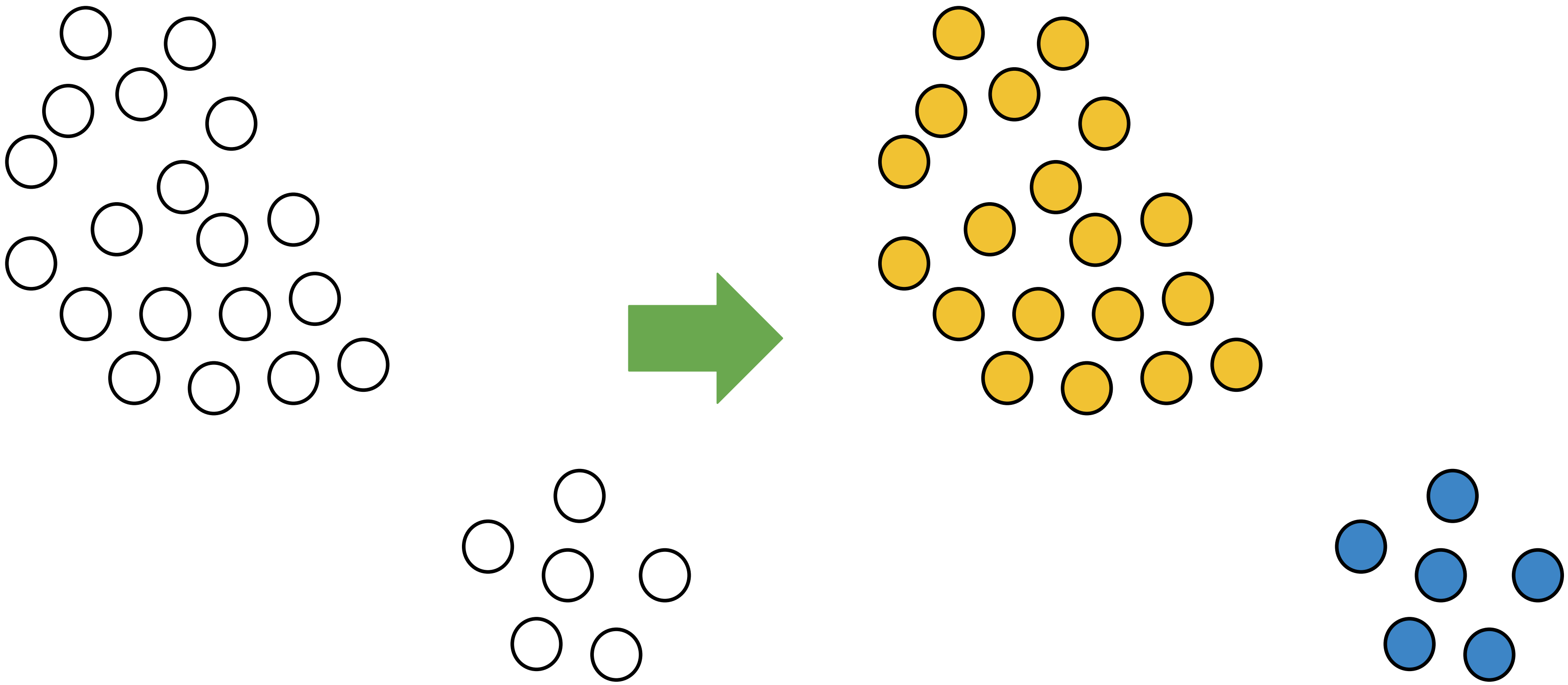
Soft Clustering
Each point is assigned to every cluster with a certain probability

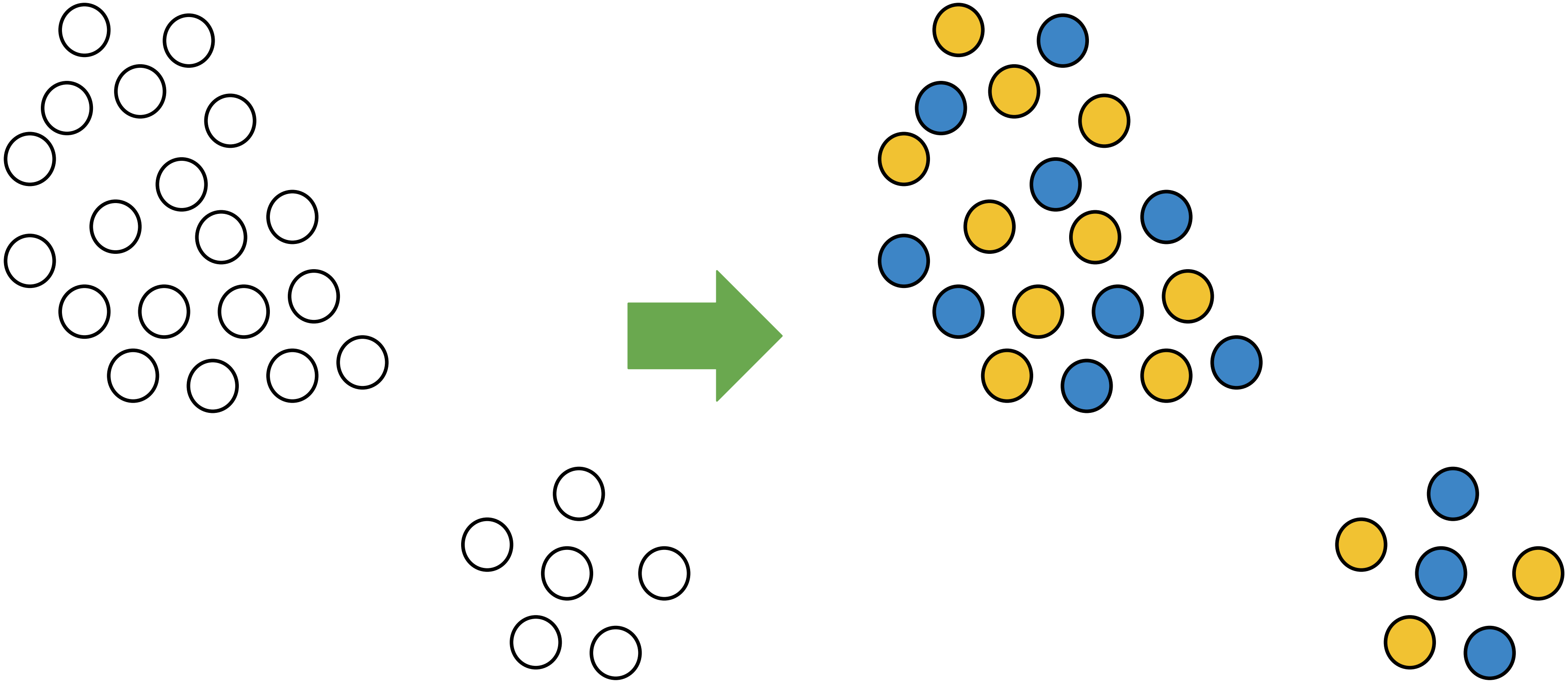
Partitional Clustering

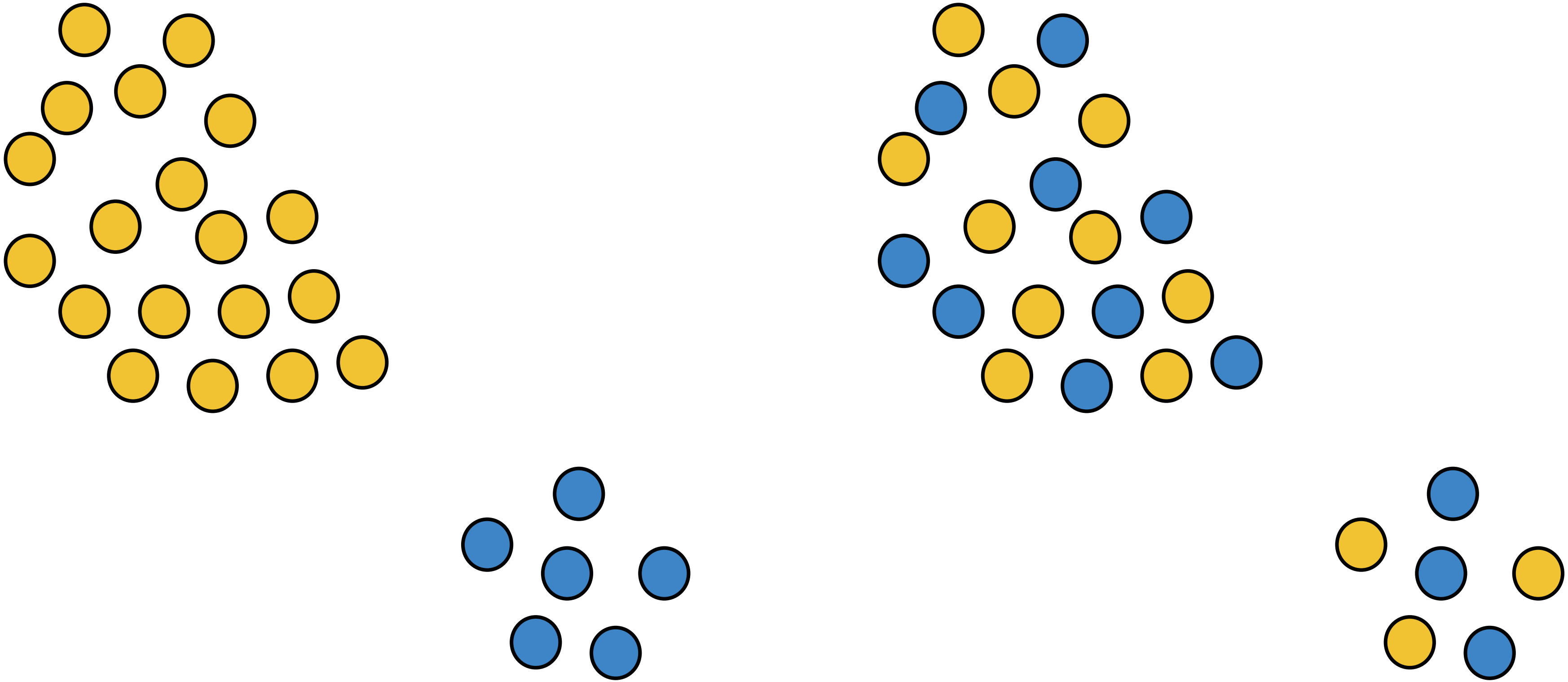
Partitional Clustering

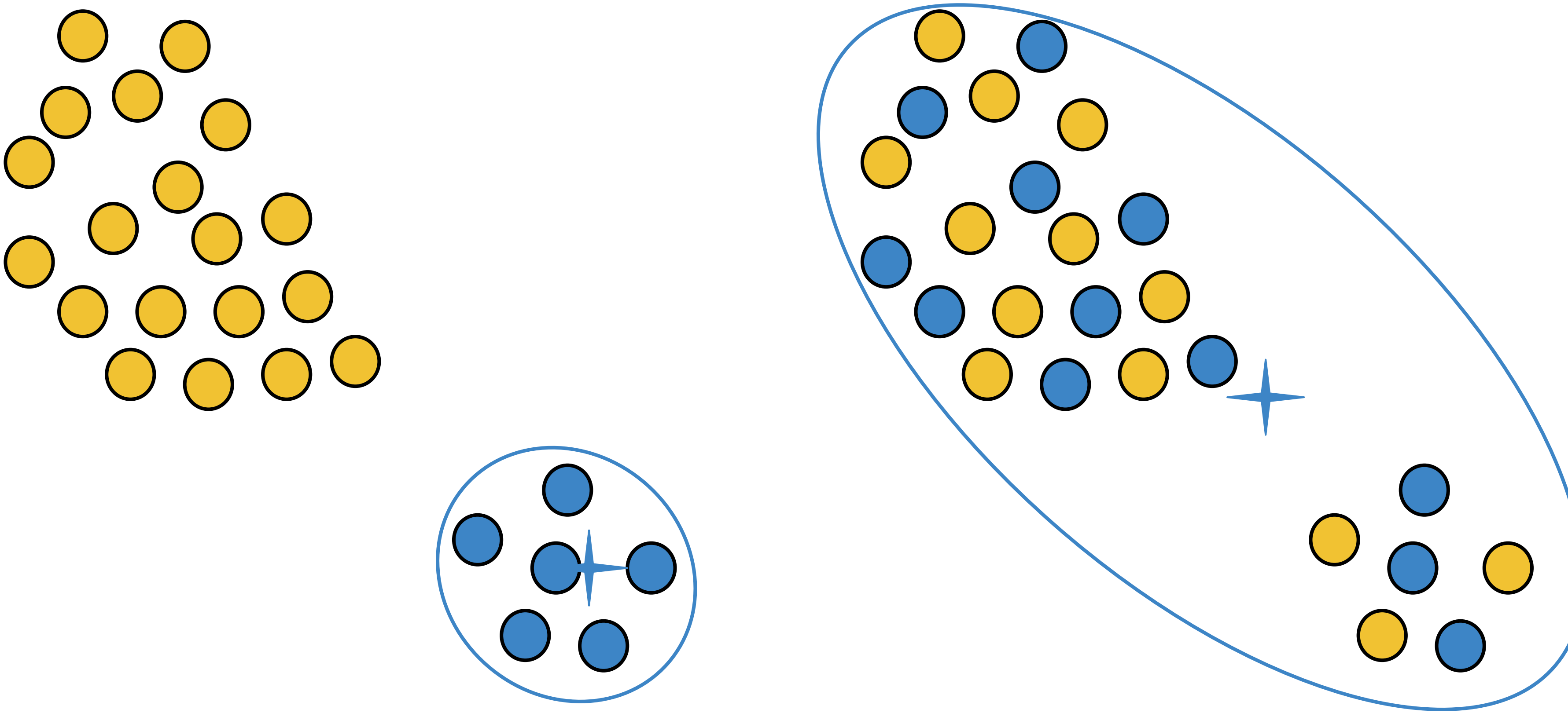


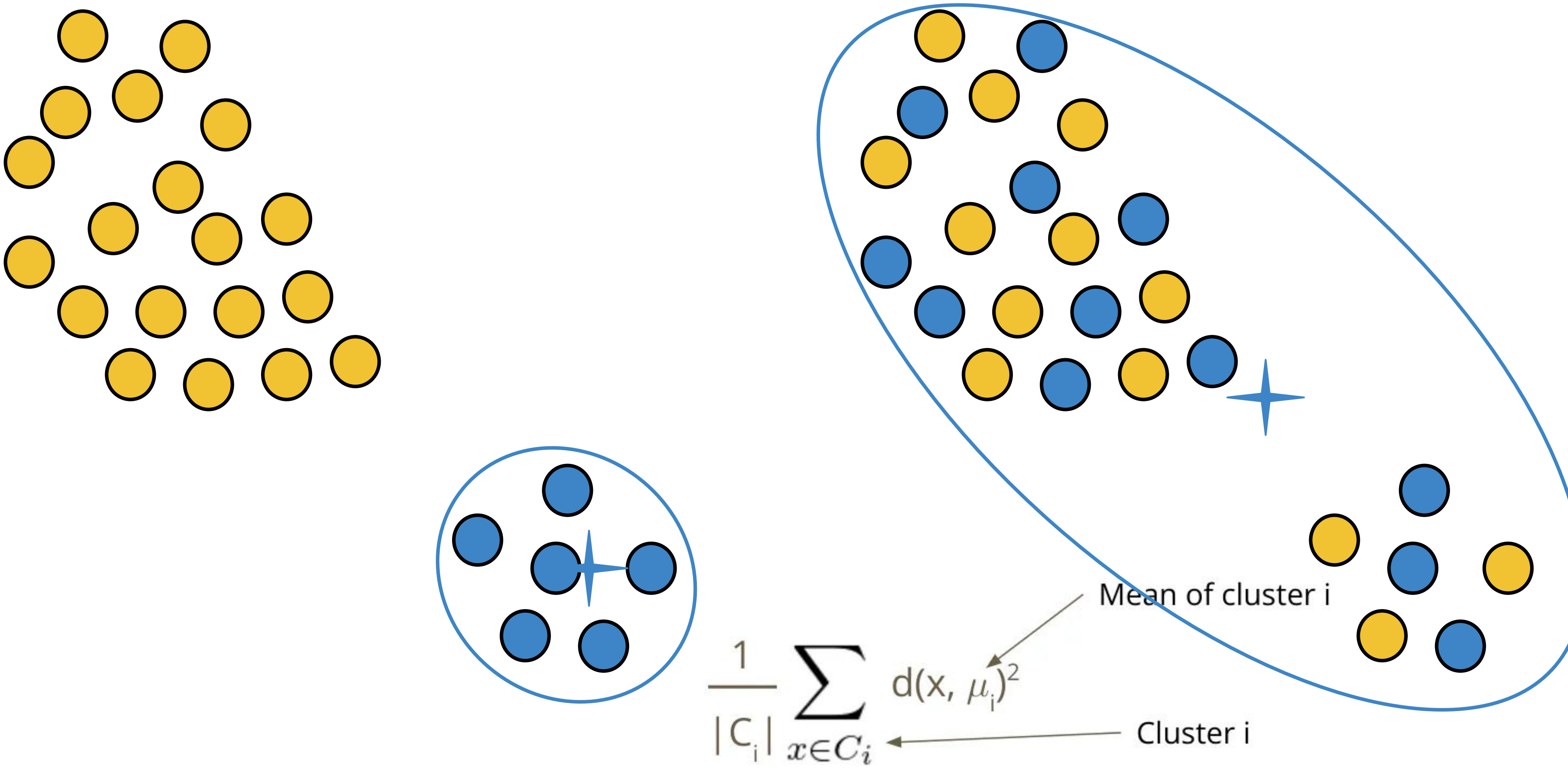
Goal: partition dataset into k partitions










Mean of cluster i
$\frac{1}{|C_i|}\sum_{x\in C_i} d(x, \mu_i)^2$
Cluster i

Cost Function

$$\sum_{i}^{k} \sum_{x \in C_i} d(x, \mu_i)^2$$

- Way to evaluate and compare solutions
- Hope: can find some algorithm that find solutions that make the cost small

K-means

Given **X = {x_1, … , x_n}** our dataset, **d** the euclidean distance, and **k**

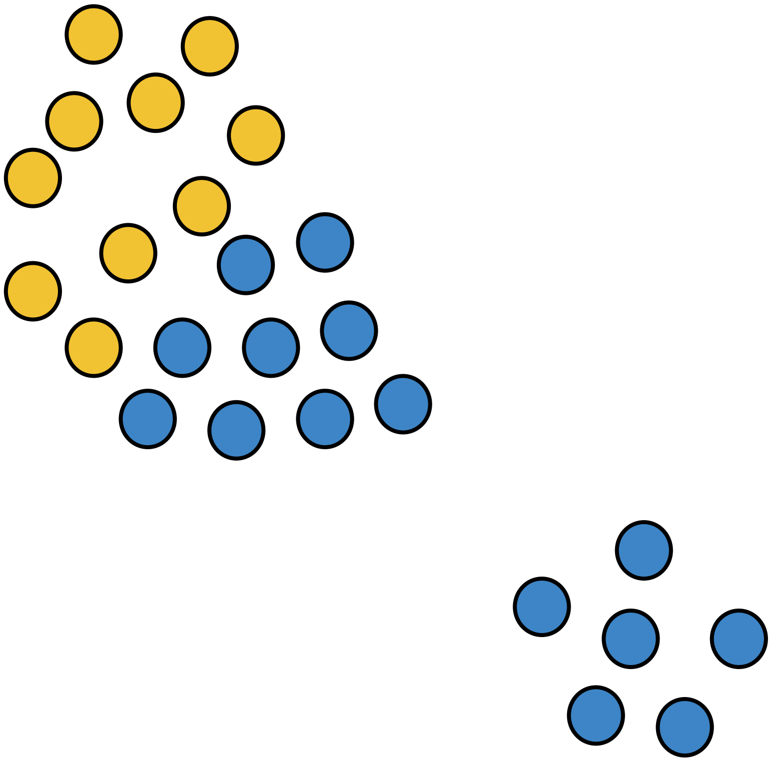
Find **k** centers **{μ_1, … , μ_k}** that minimize the **cost function**:

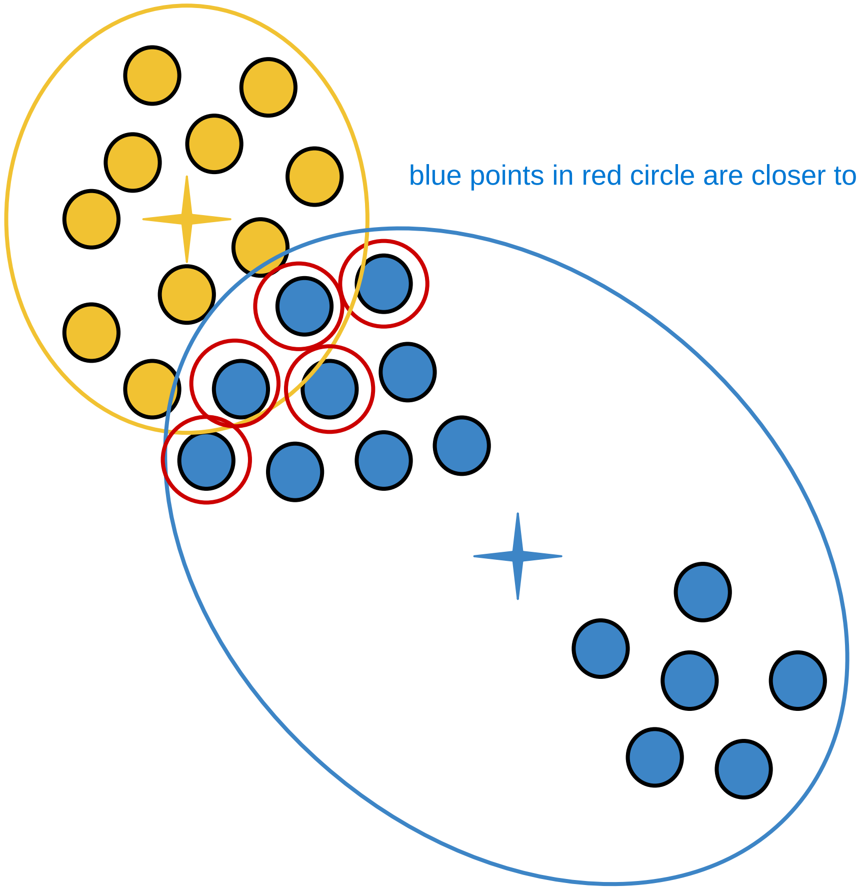
$$\sum_{i}^{k} \sum_{x \in C_i} d(x, \mu_i)^2$$

When **k=1** and **k=n** this is easy. Why?

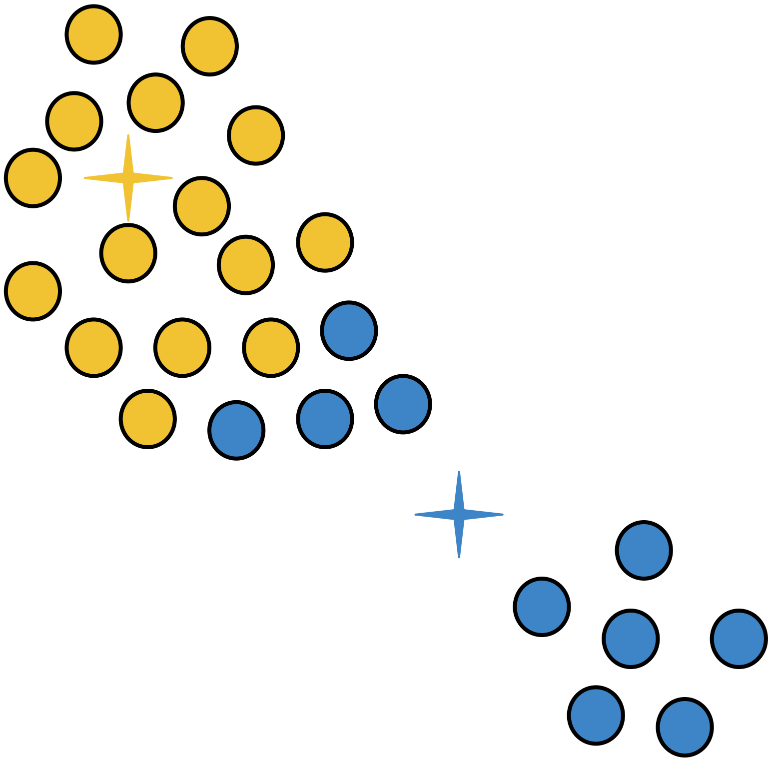
1 cluster including all points; each point has a cluster

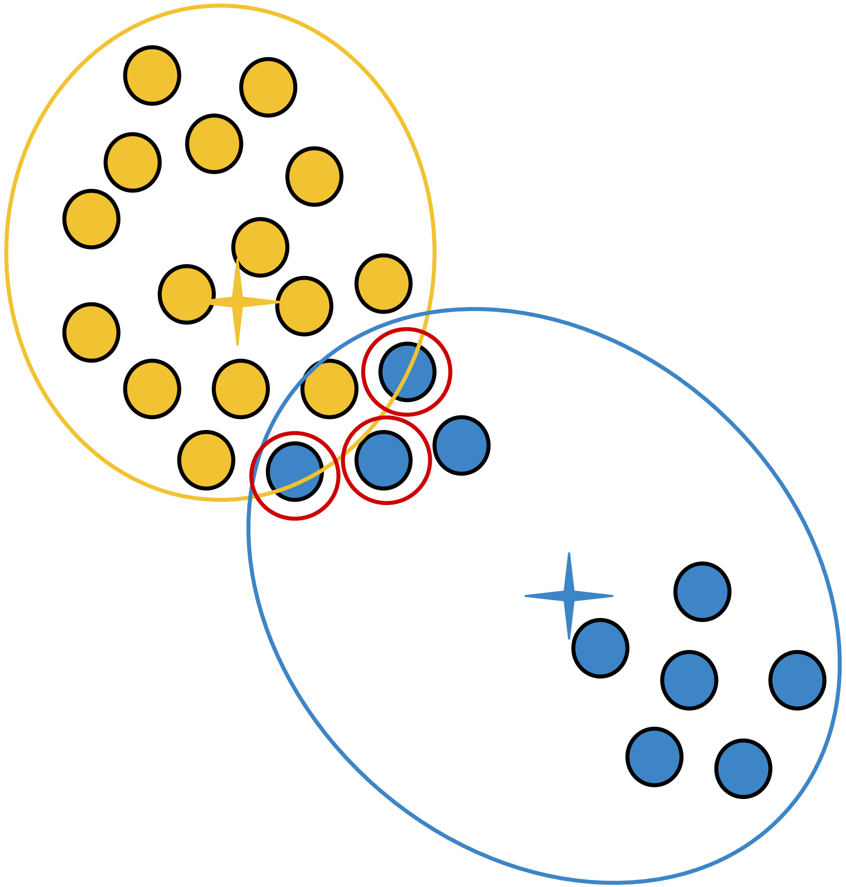
When **x_i** lives in more than 2 dimensions, this is a very difficult (**NP-hard**) problem

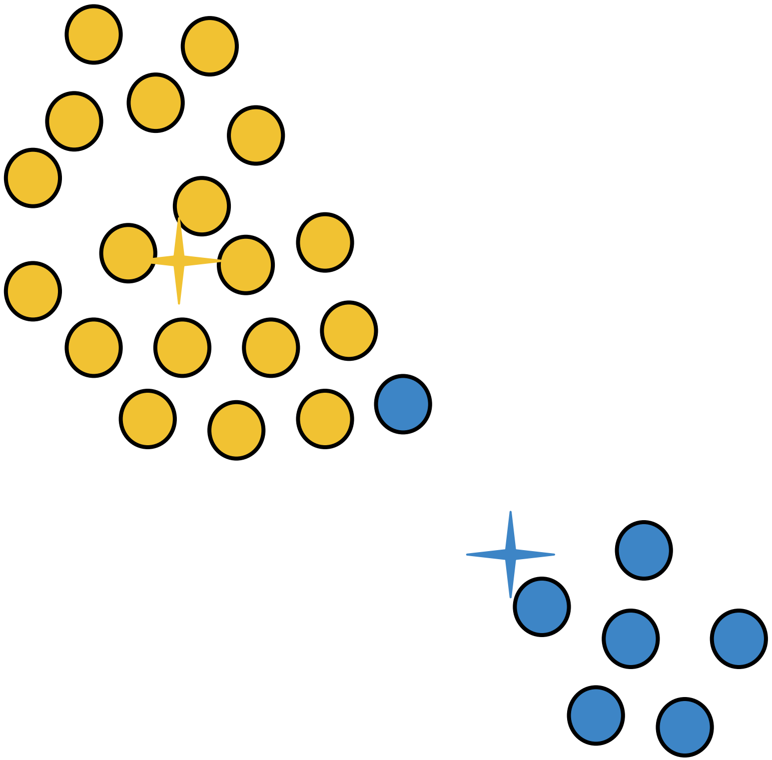


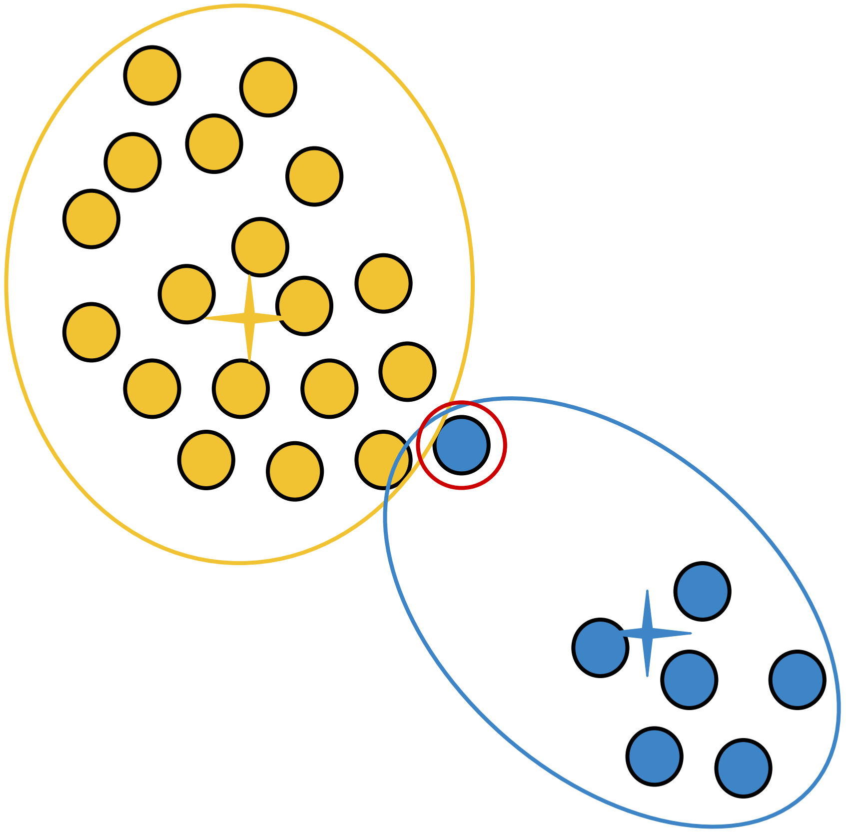


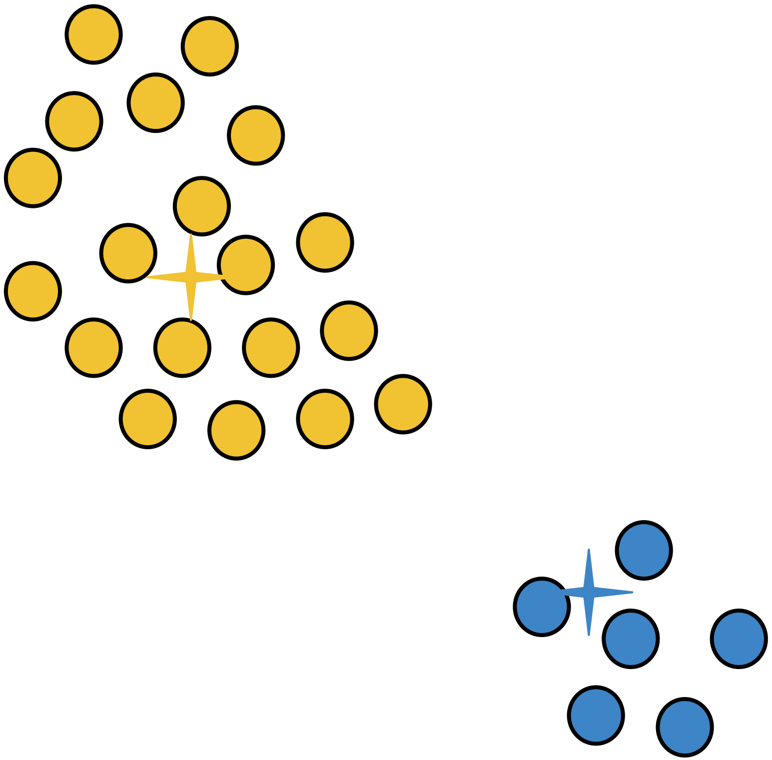
blue points in red circle are closer to the mean of yellow cluster





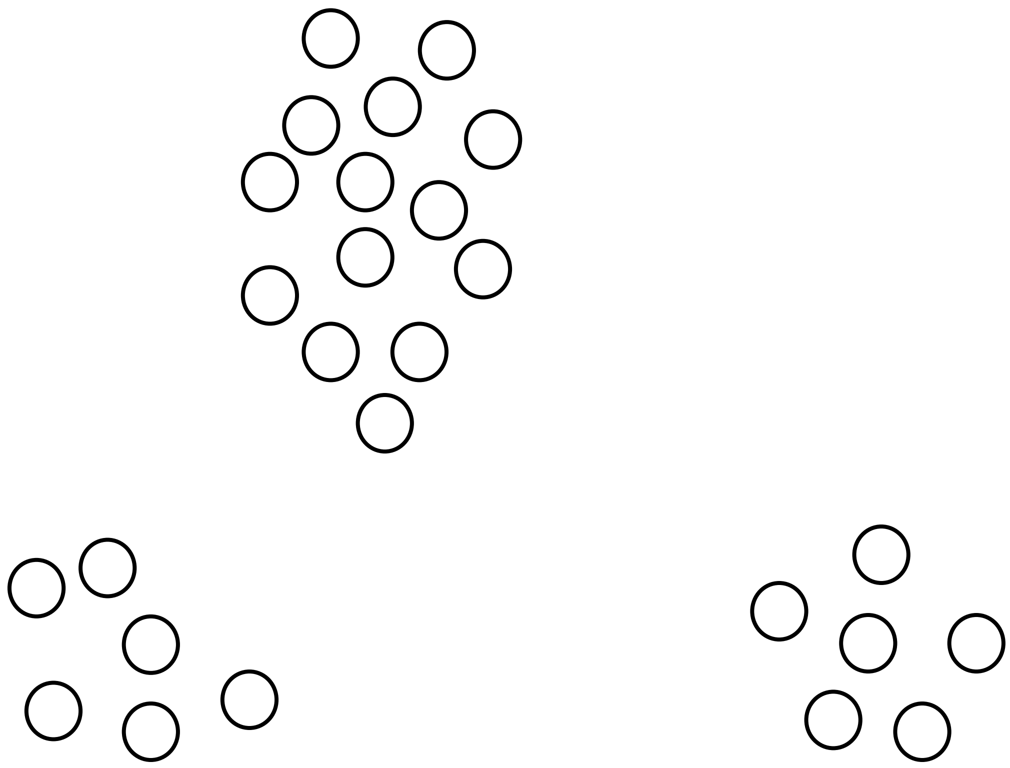


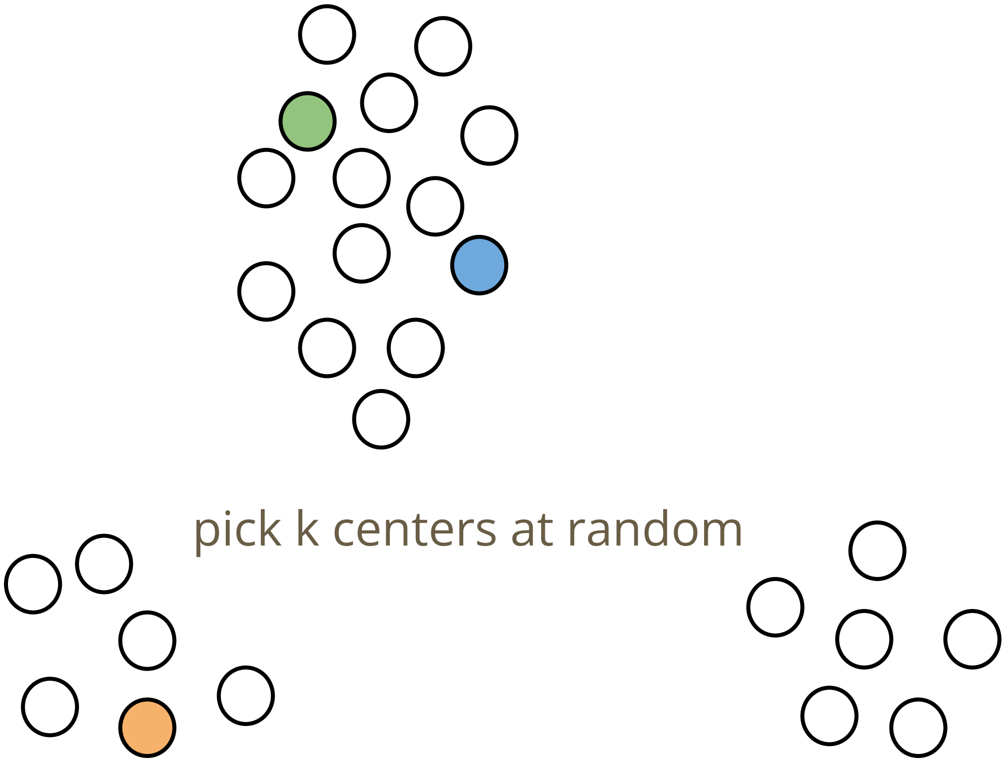


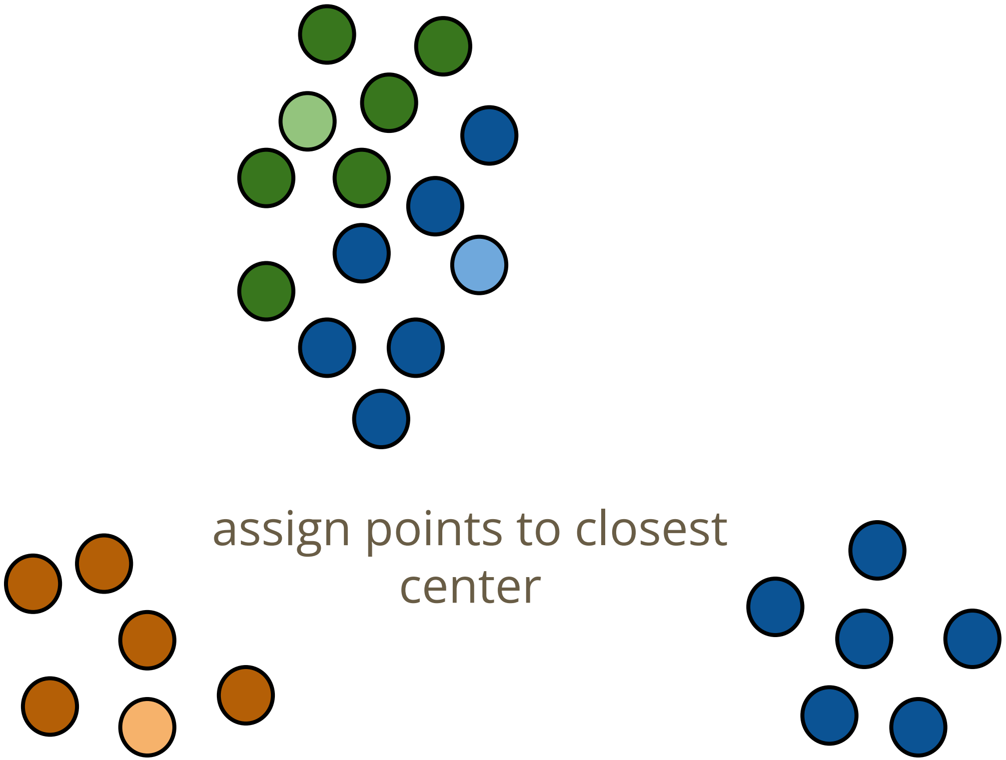


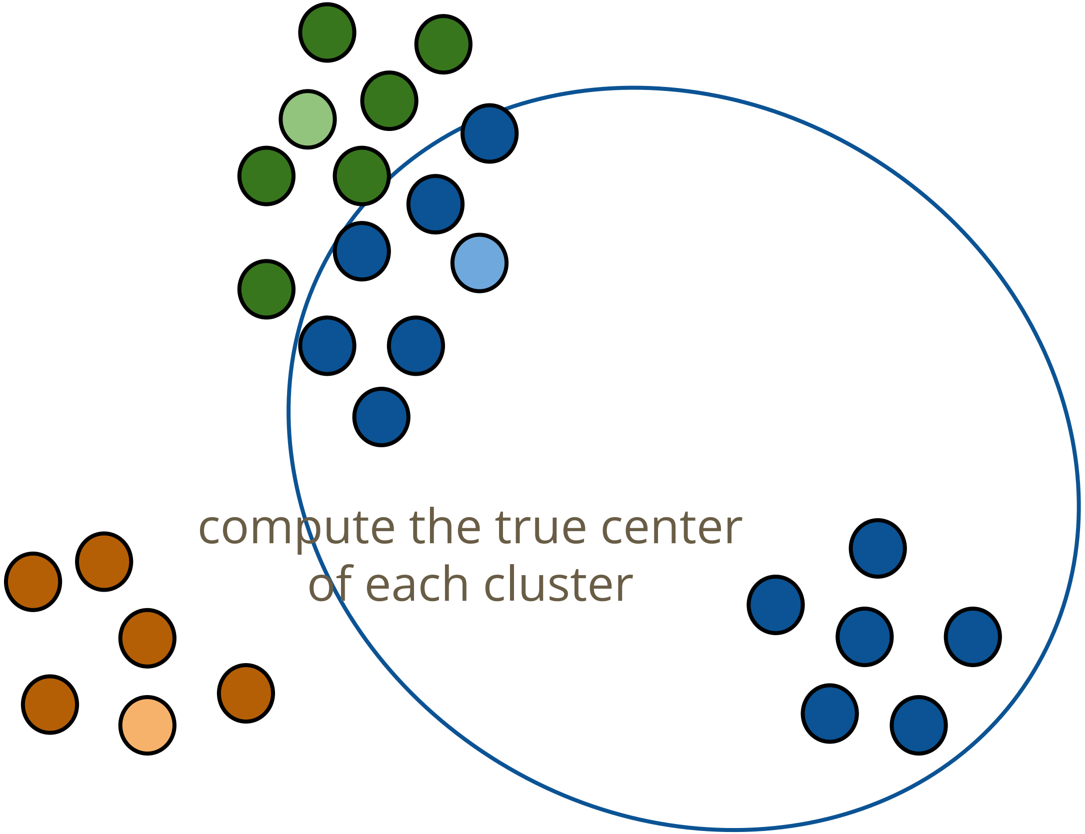
K-means - Lloyd's Algorithm

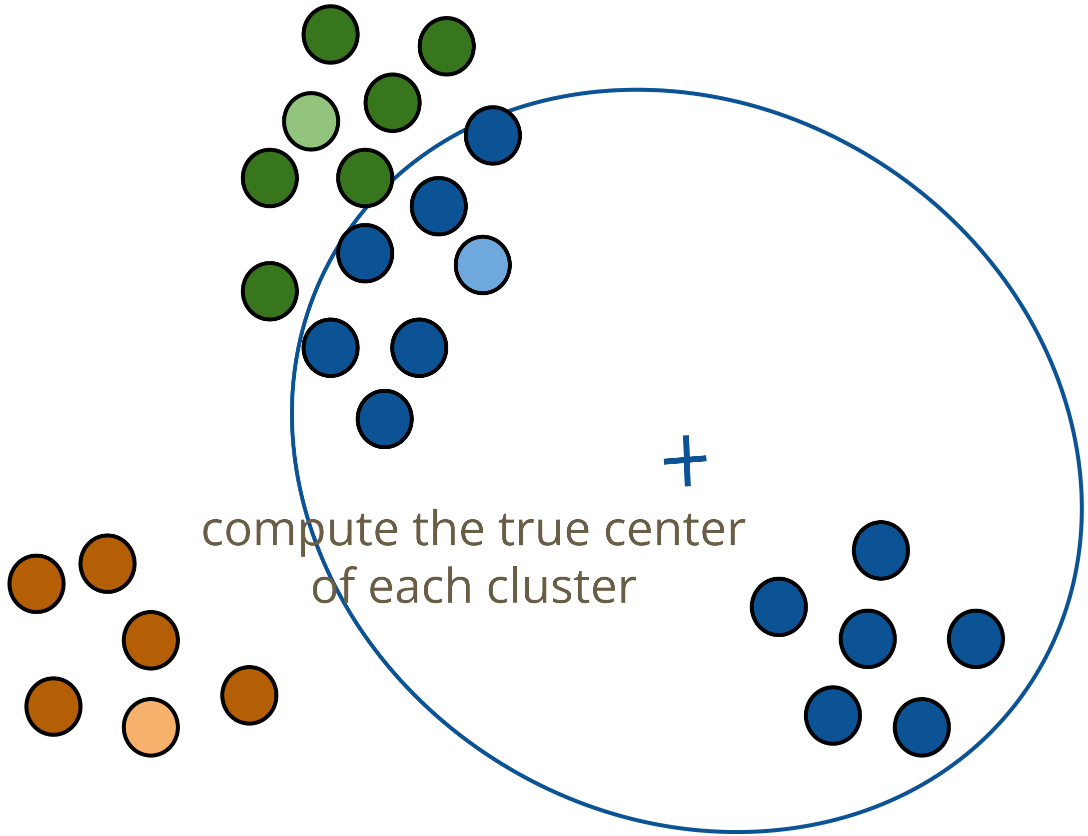
1. Randomly pick **k** centers $\{\mu_1, \ldots, \mu_k\}$
2. Assign each point in the dataset to its closest center
3. Compute the new centers as the means of each cluster
4. Repeat 2 & 3 until convergence




pick k centers at random


assign points to closest
center


compute the true center
of each cluster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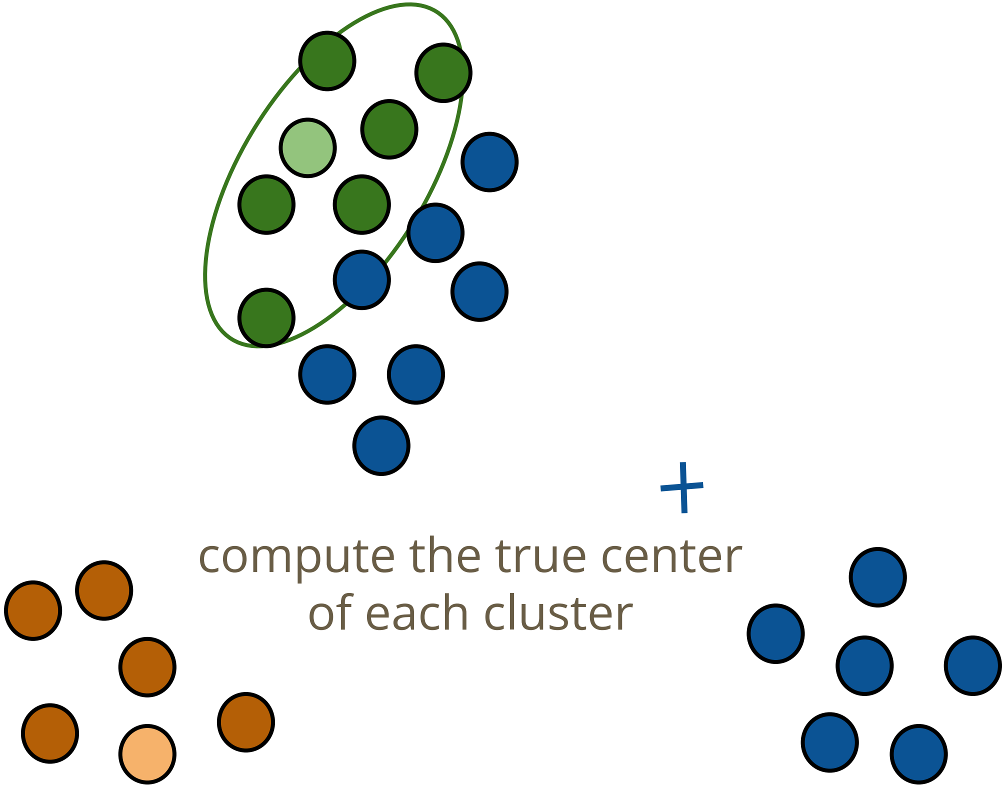
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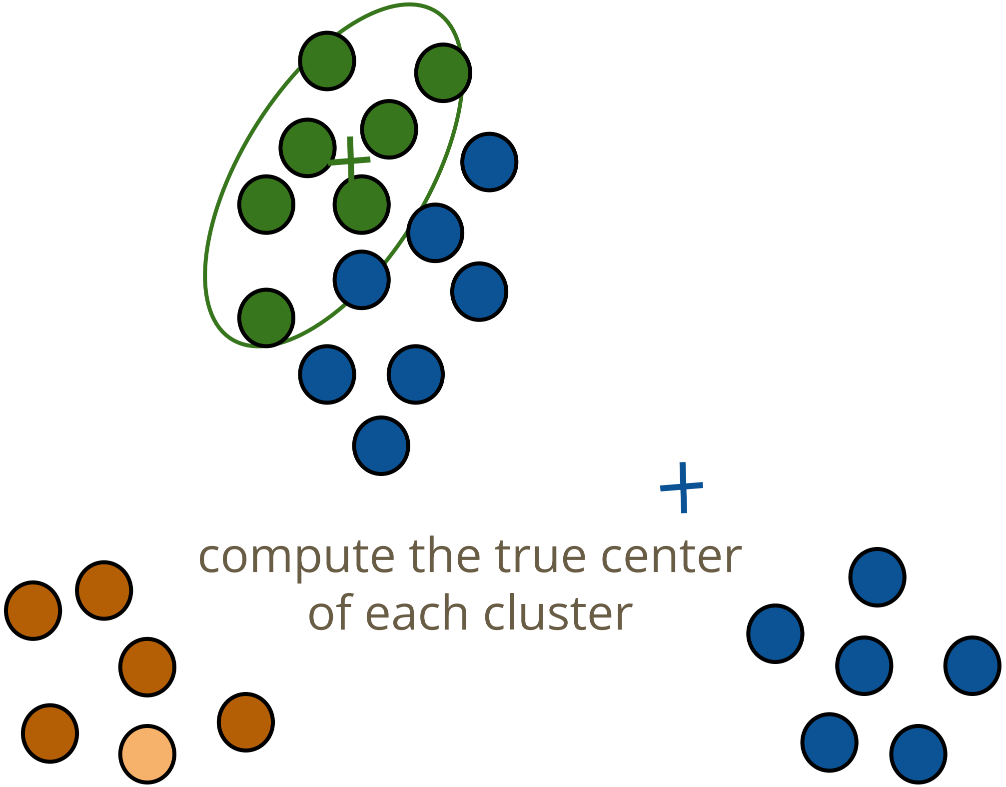

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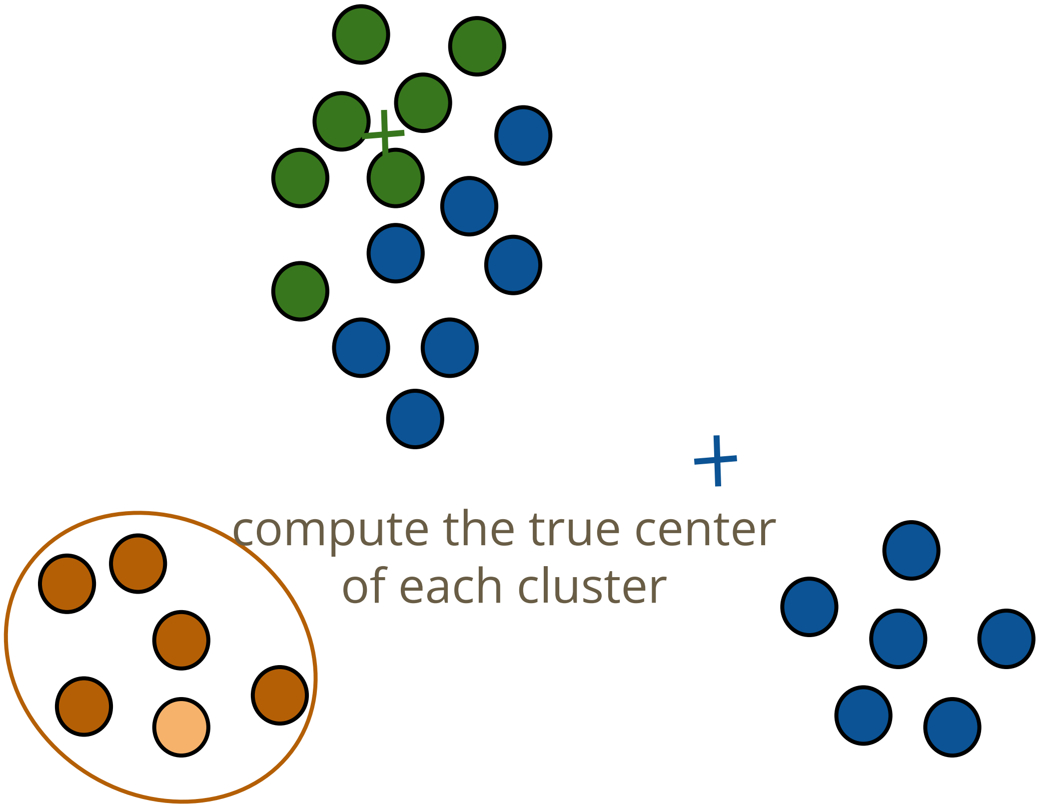
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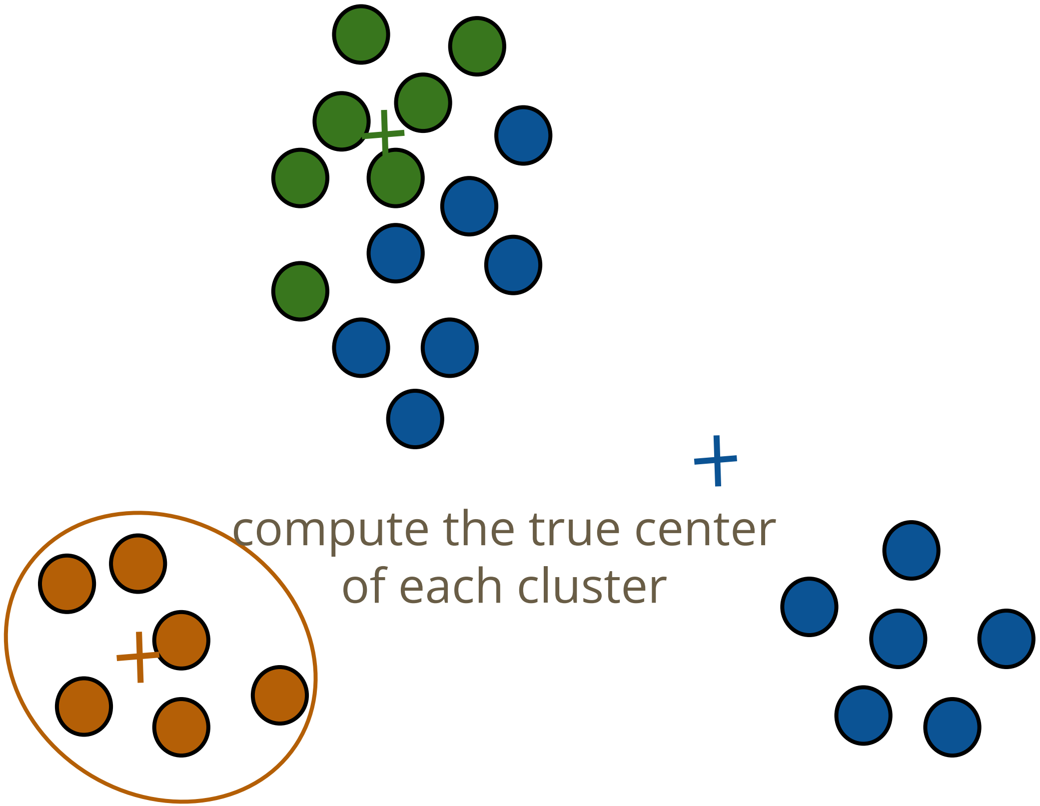
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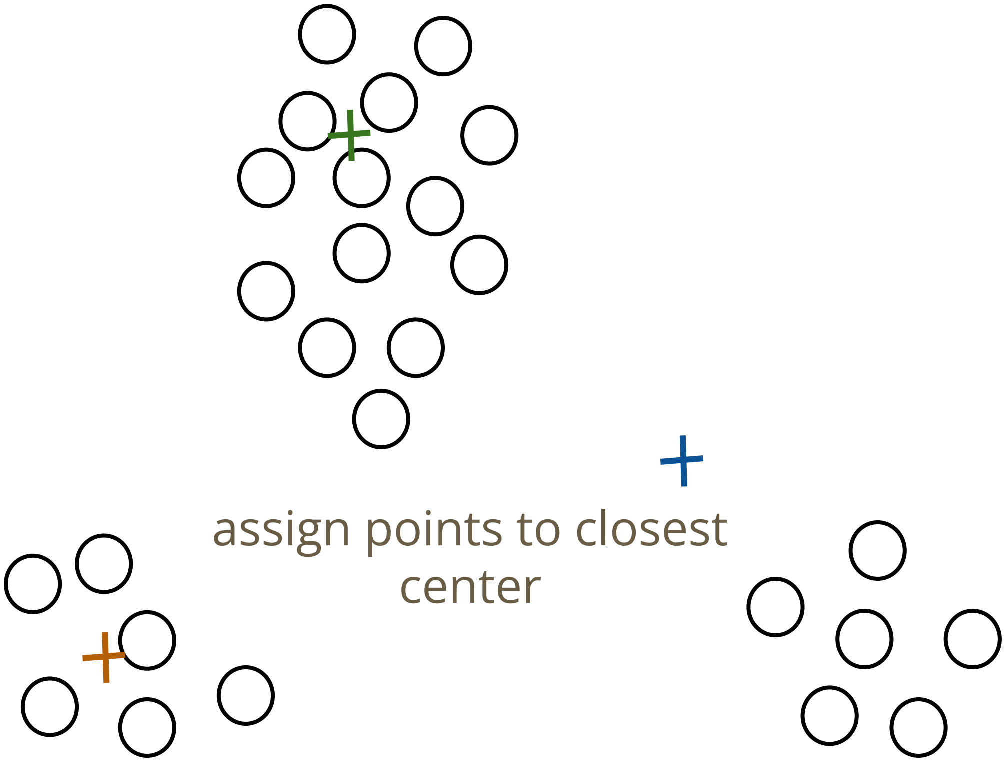
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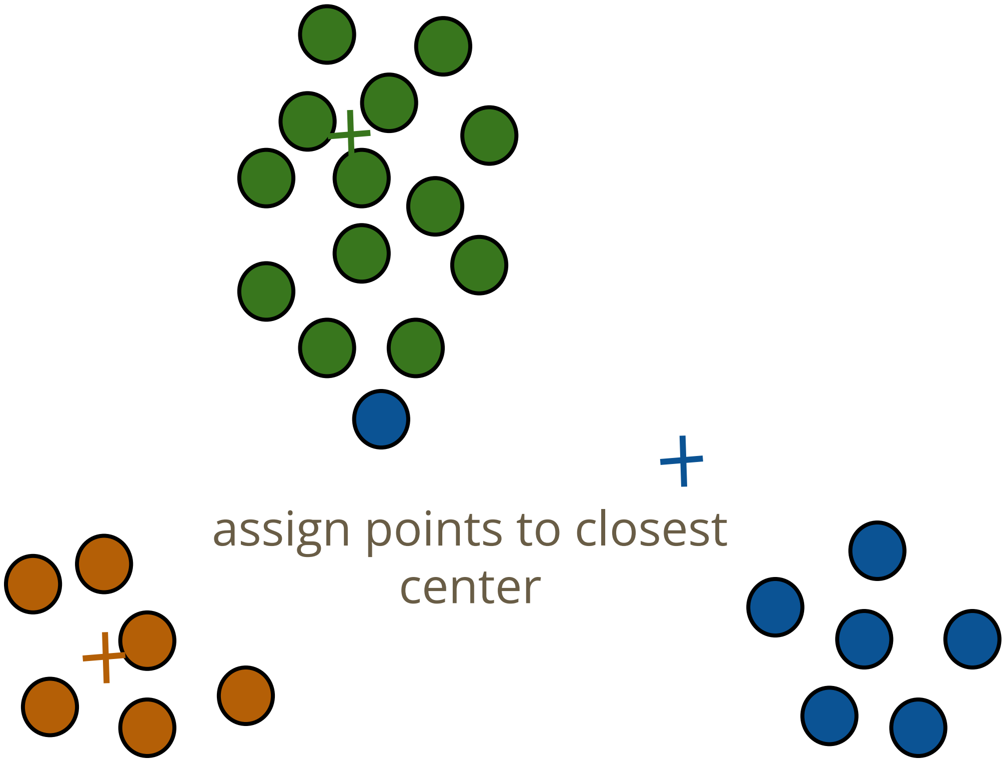
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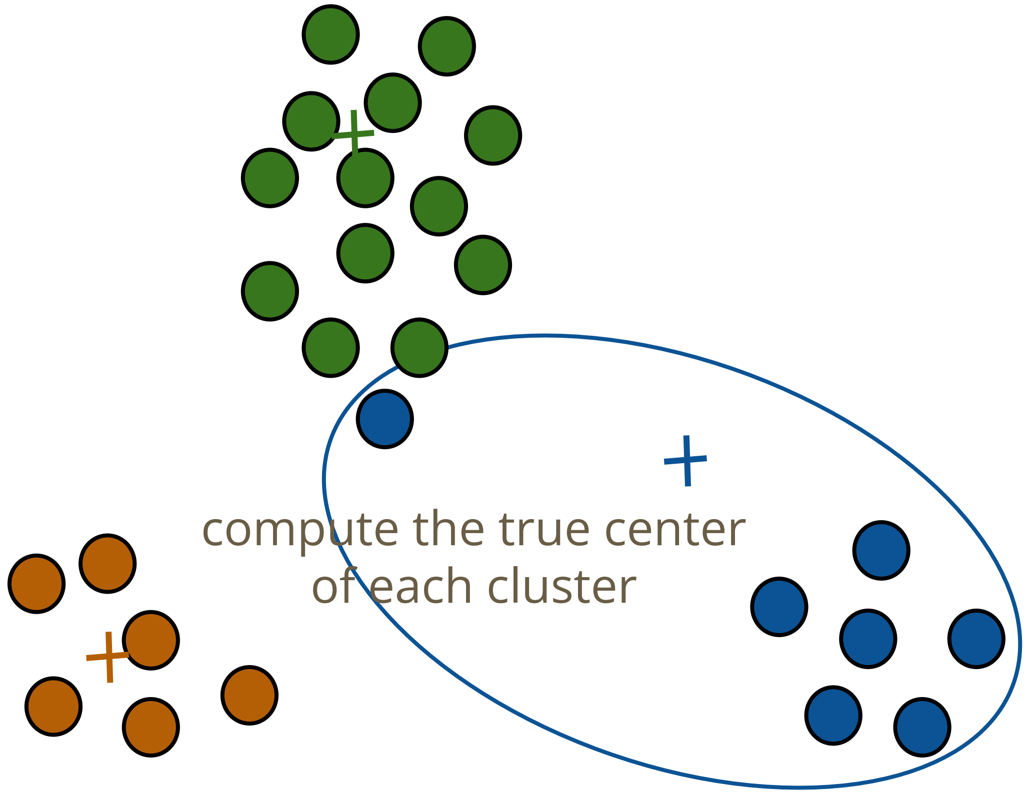
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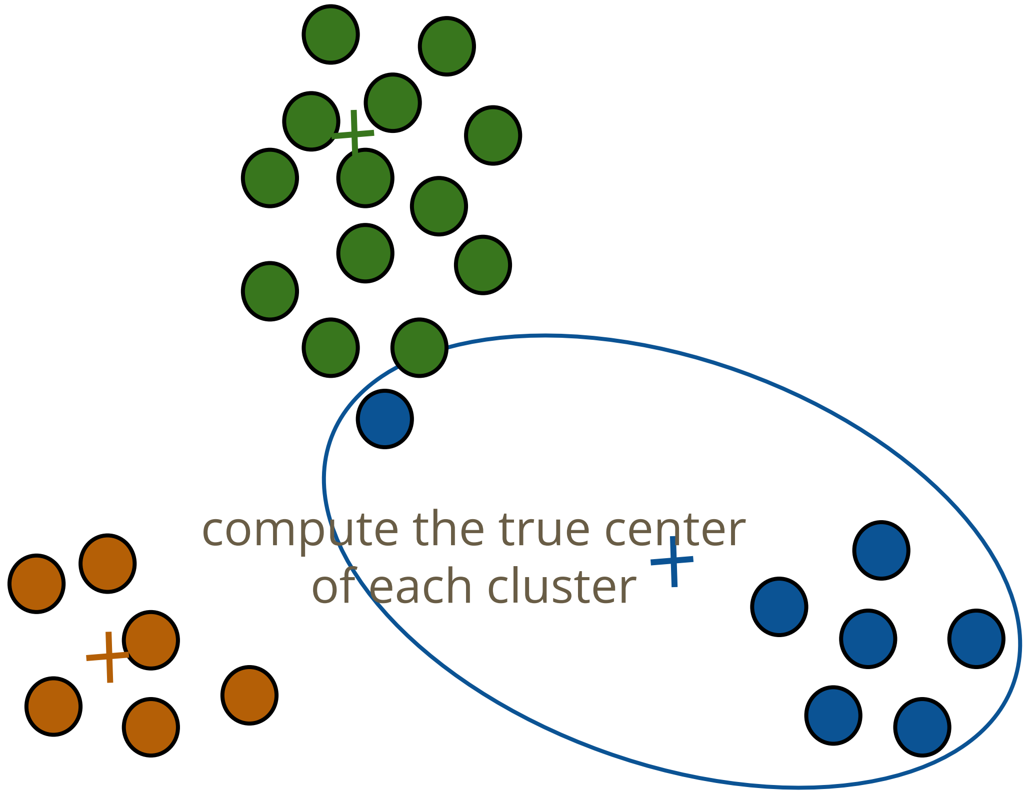
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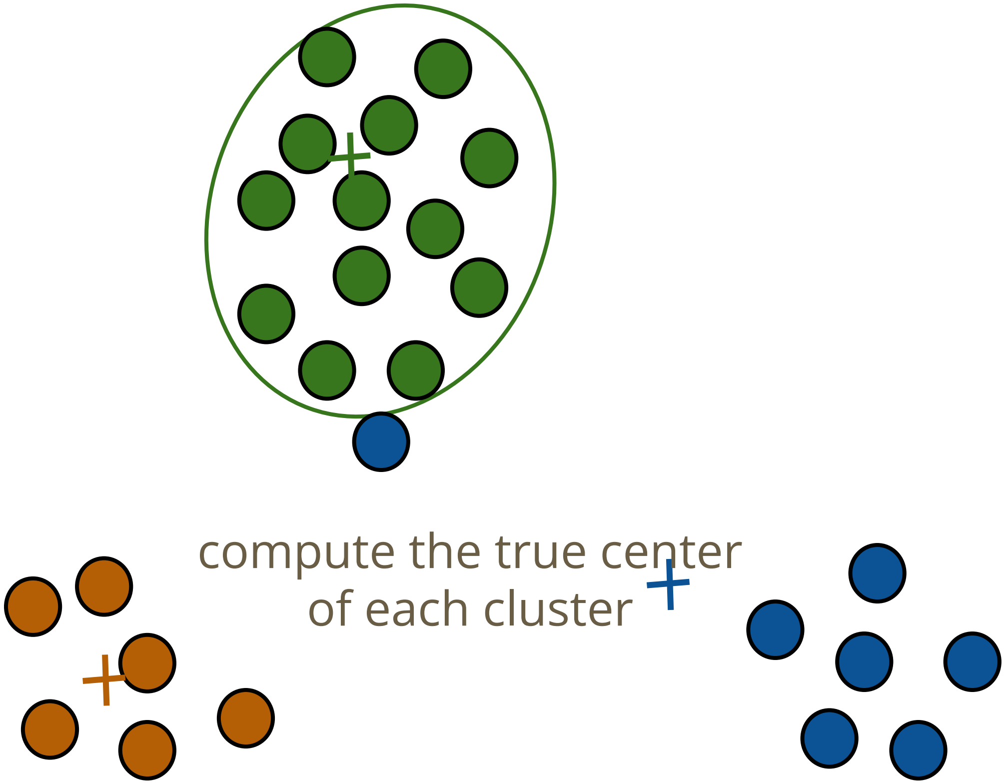
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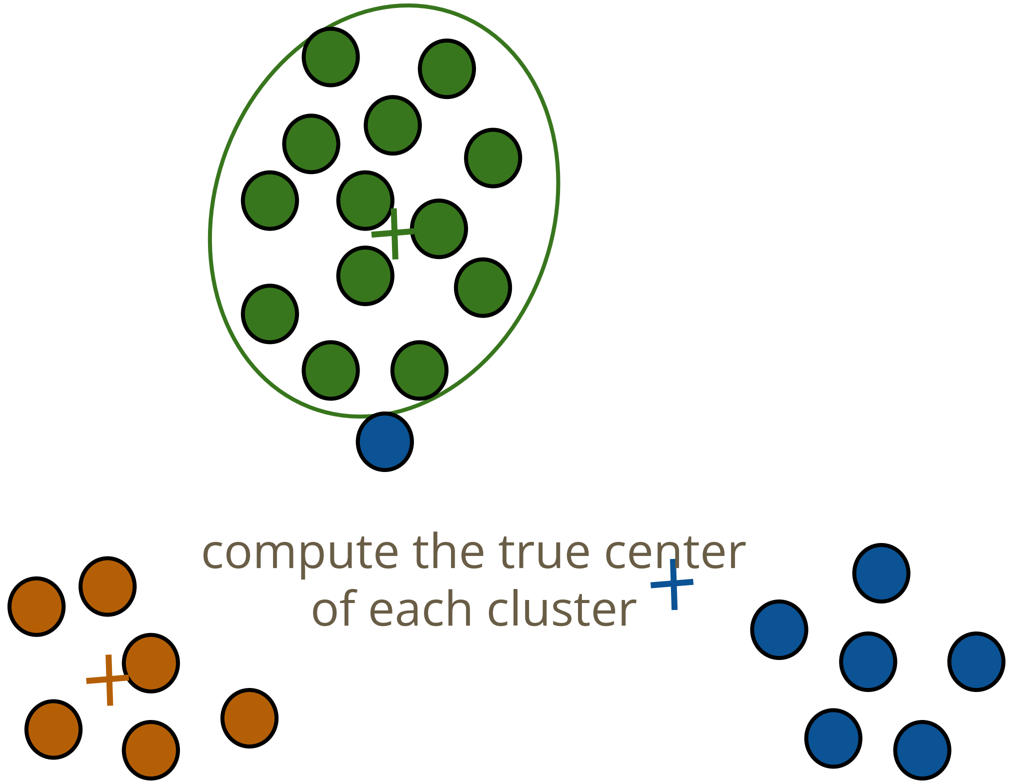
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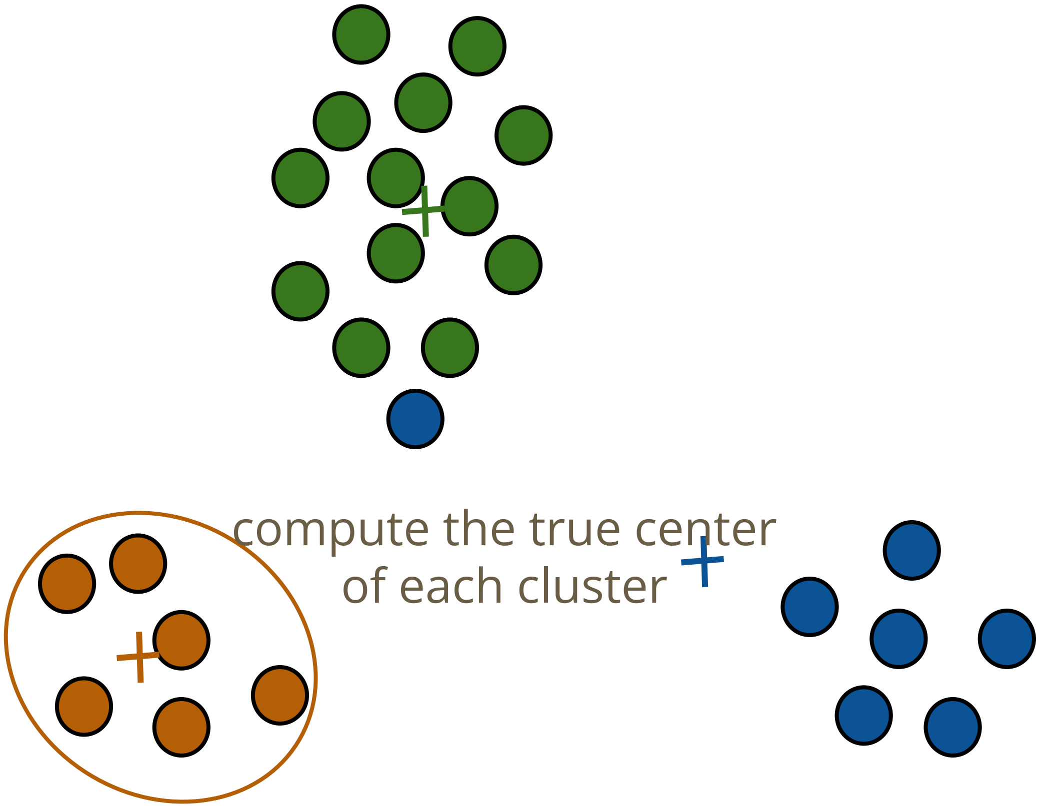
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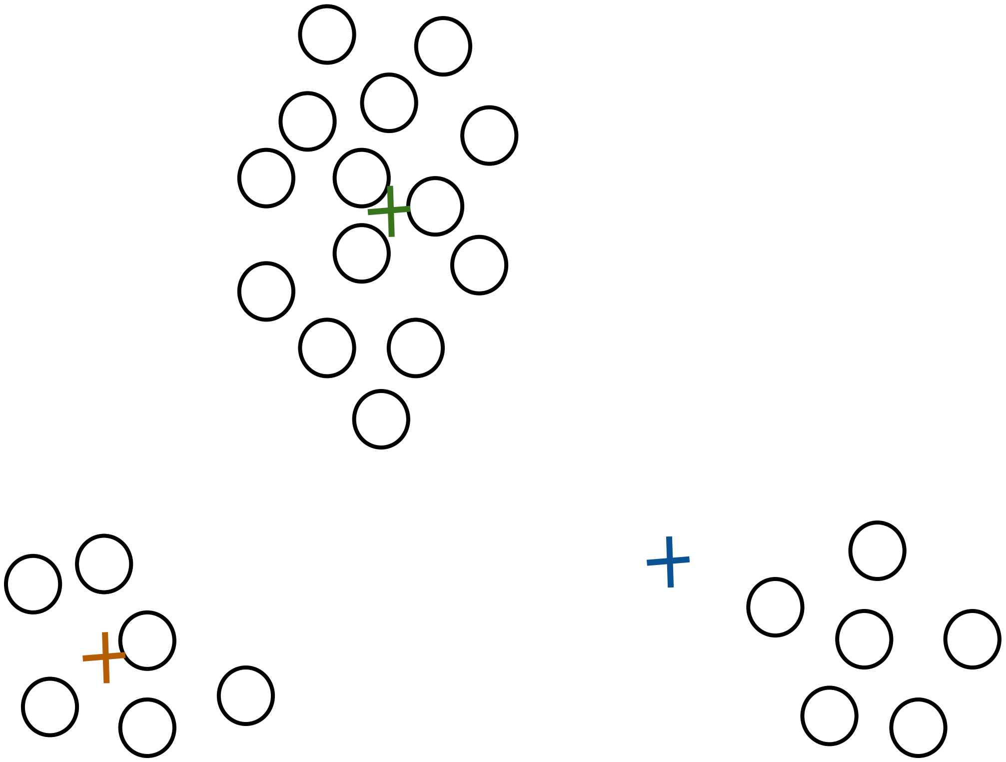
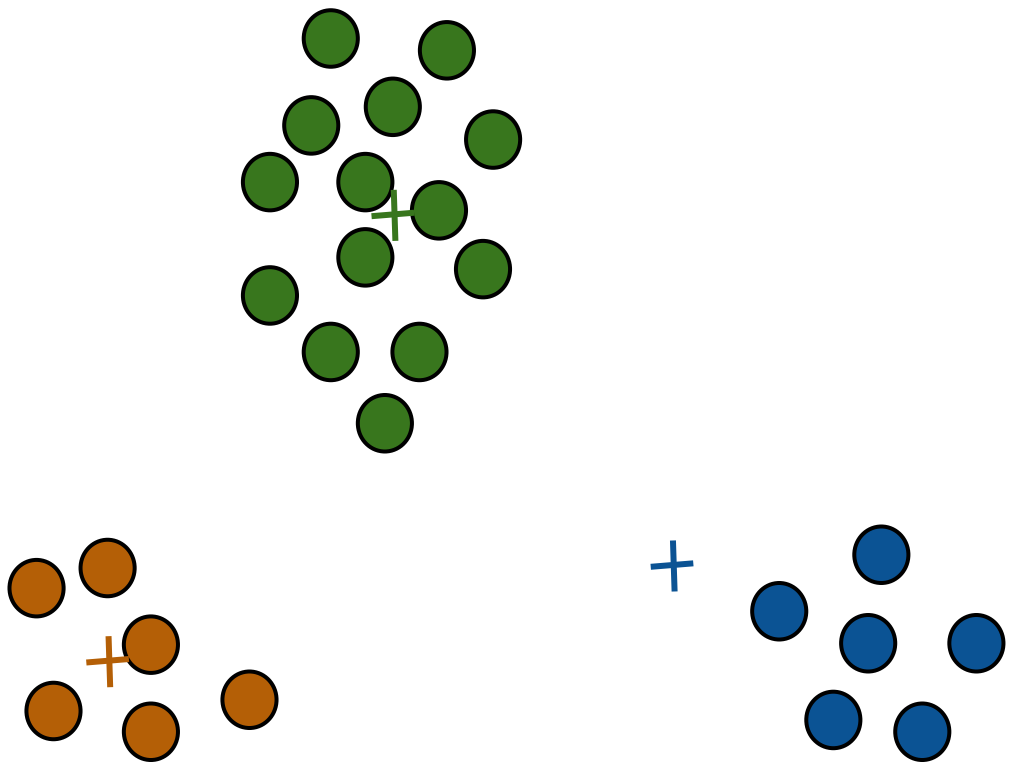
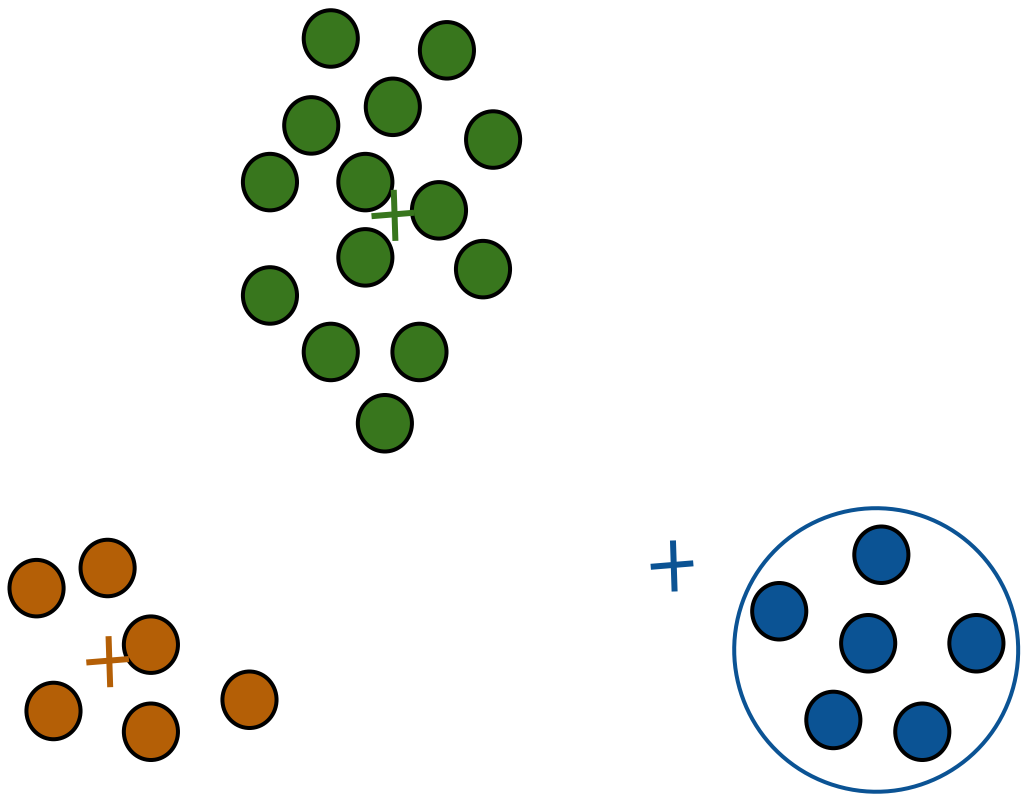
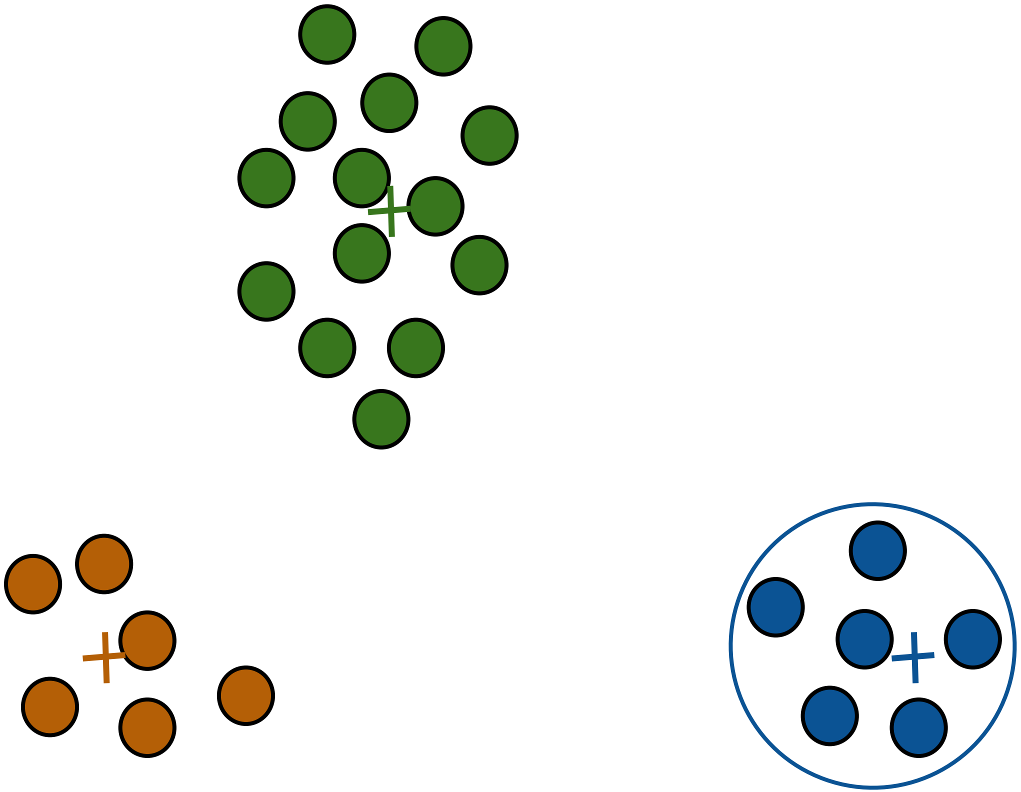
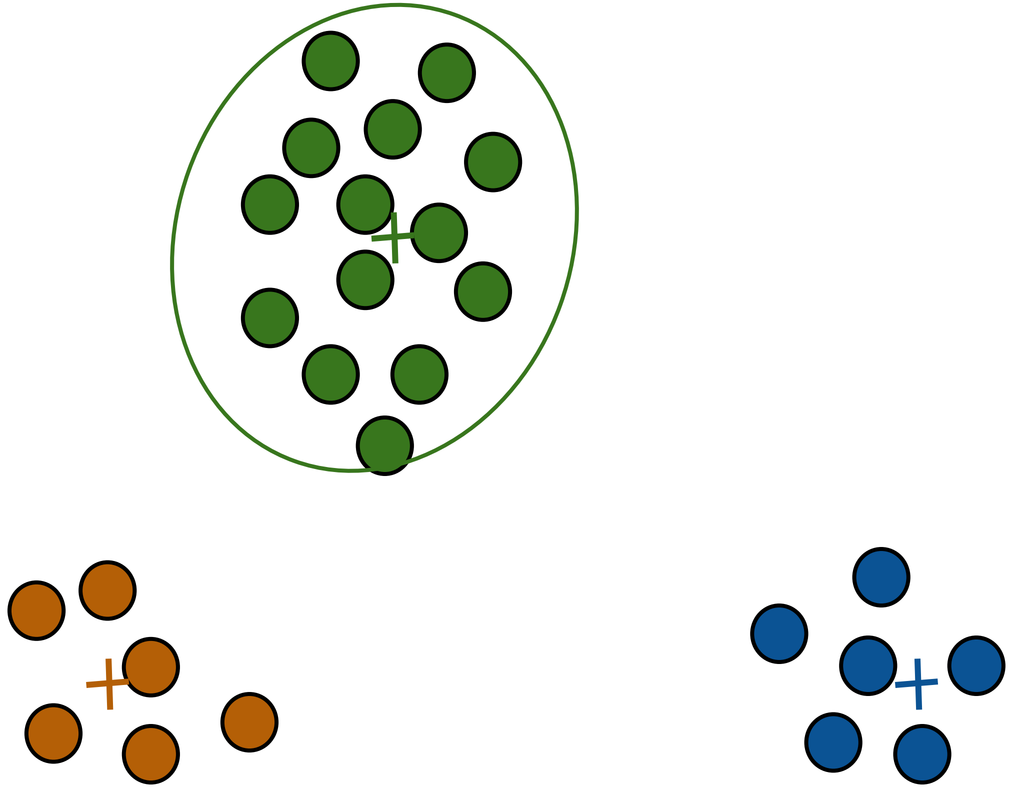
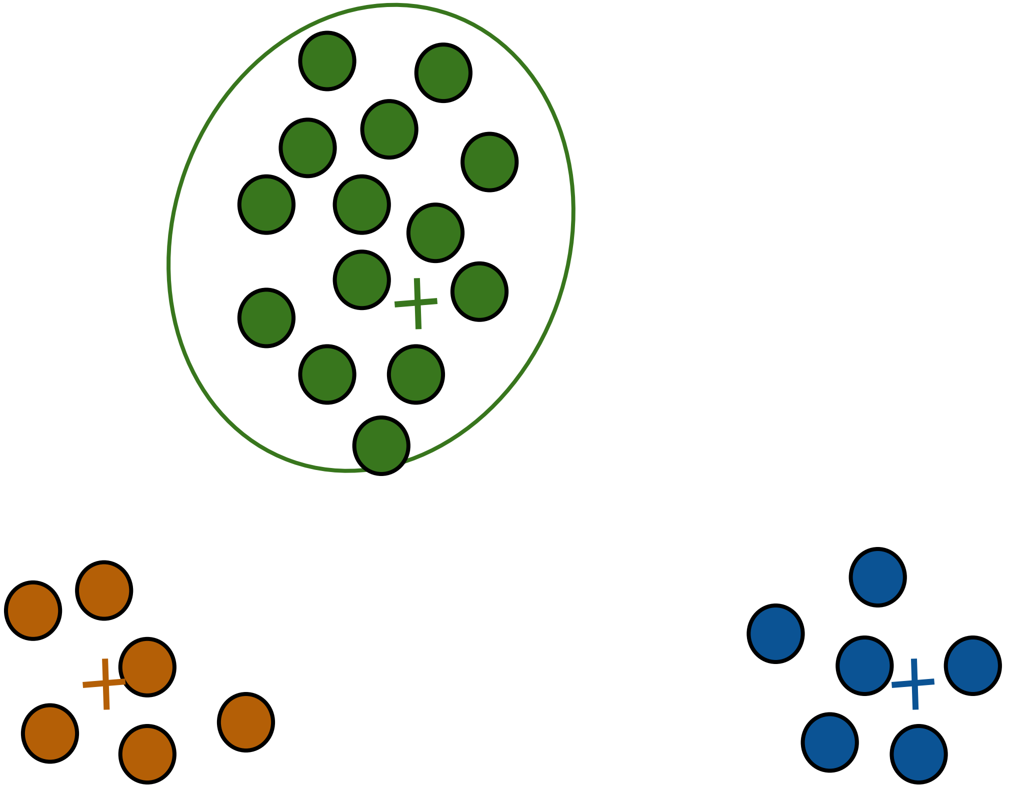


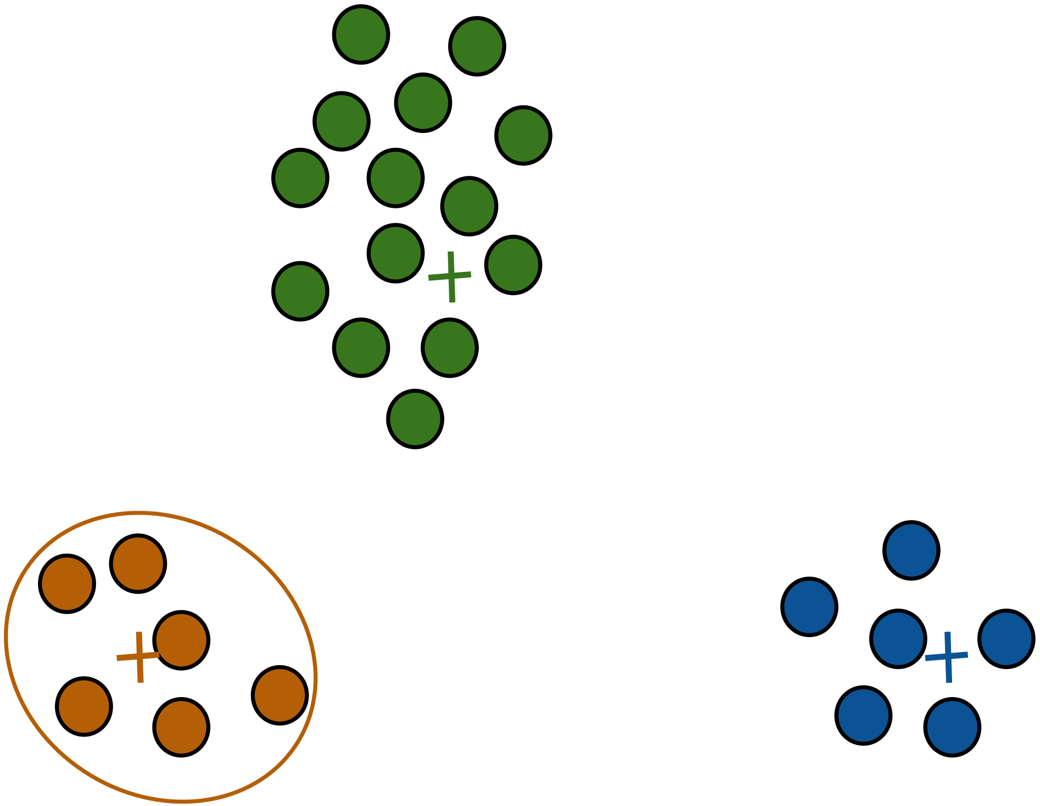


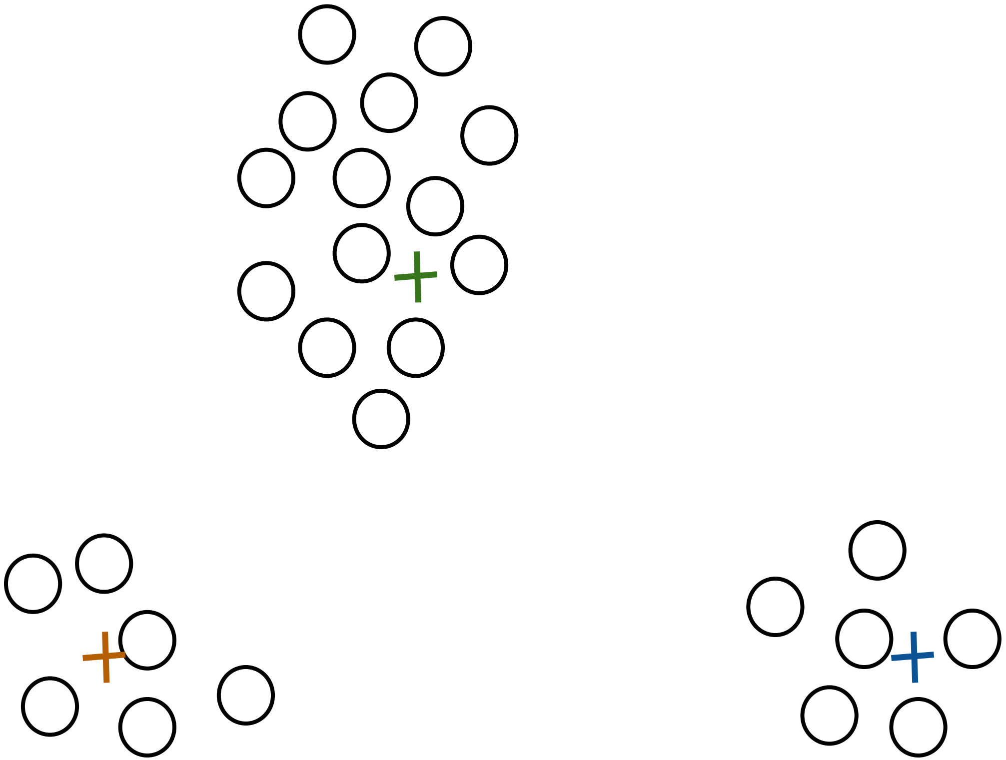


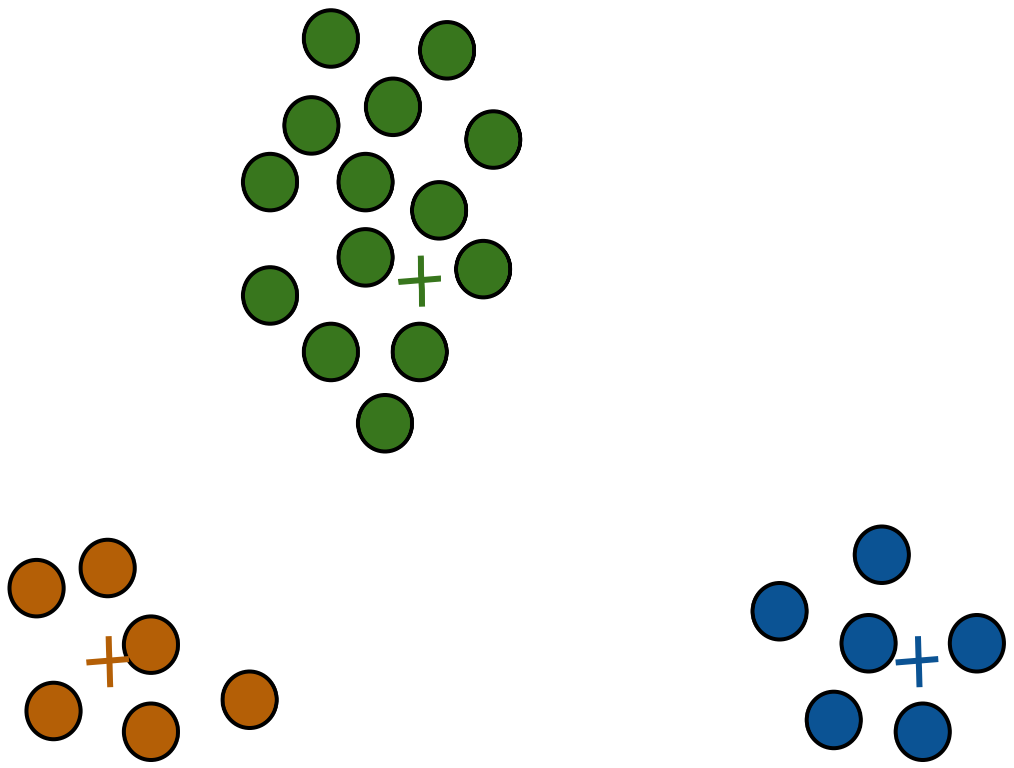


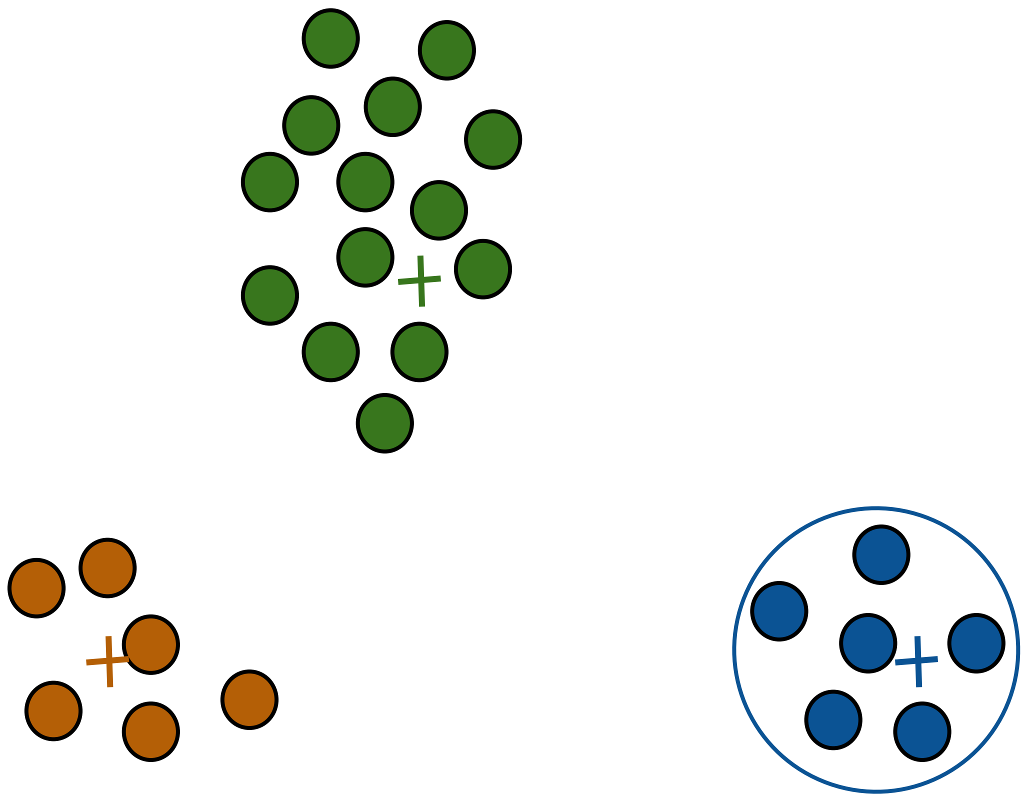


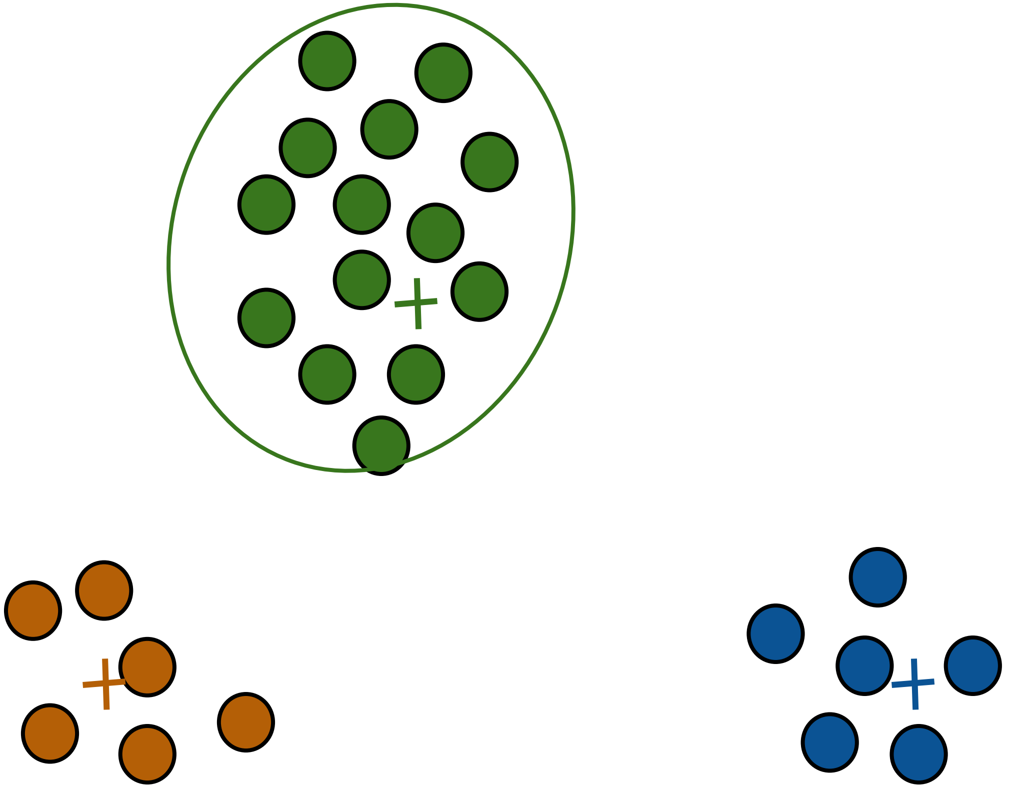


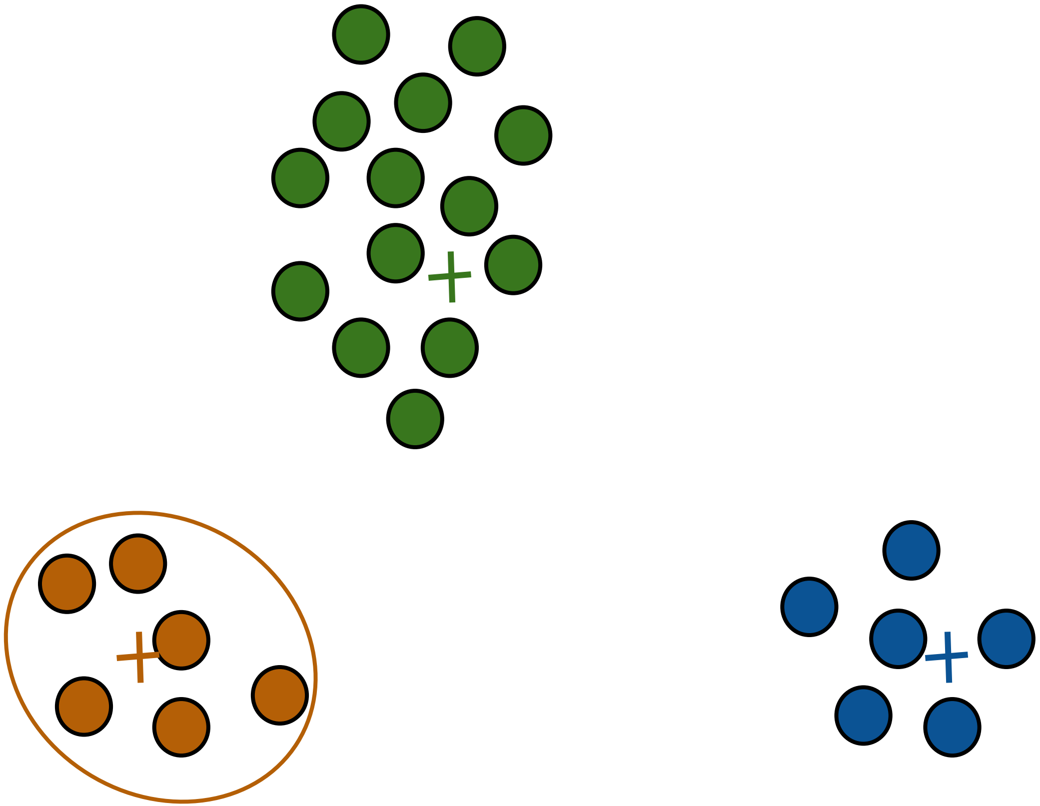


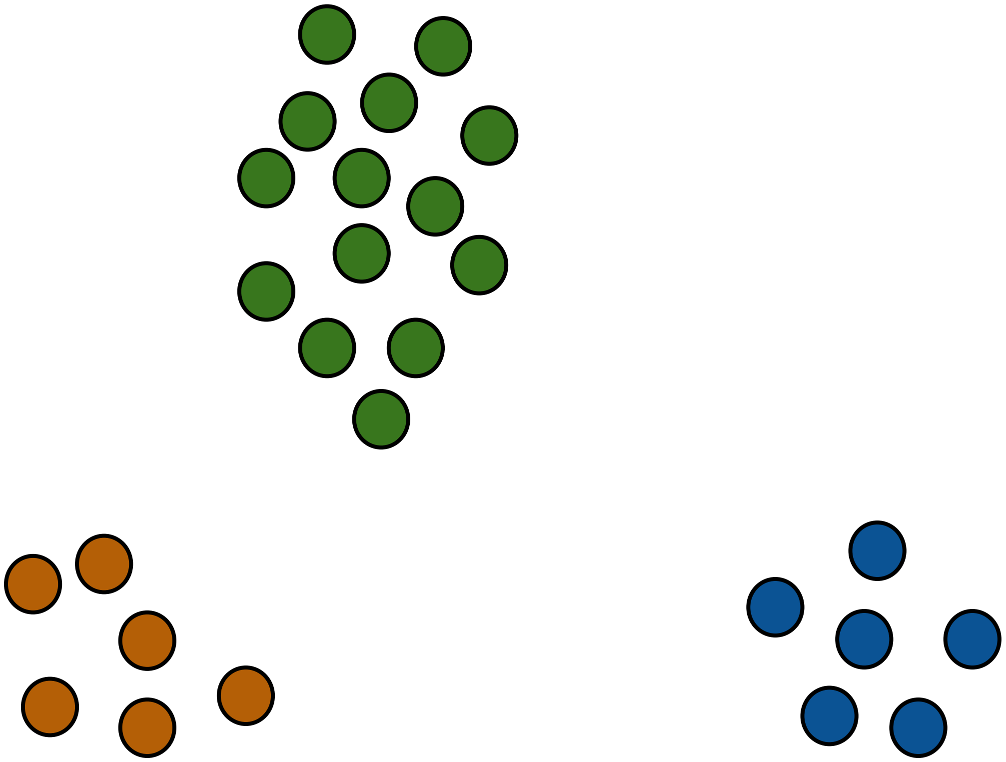






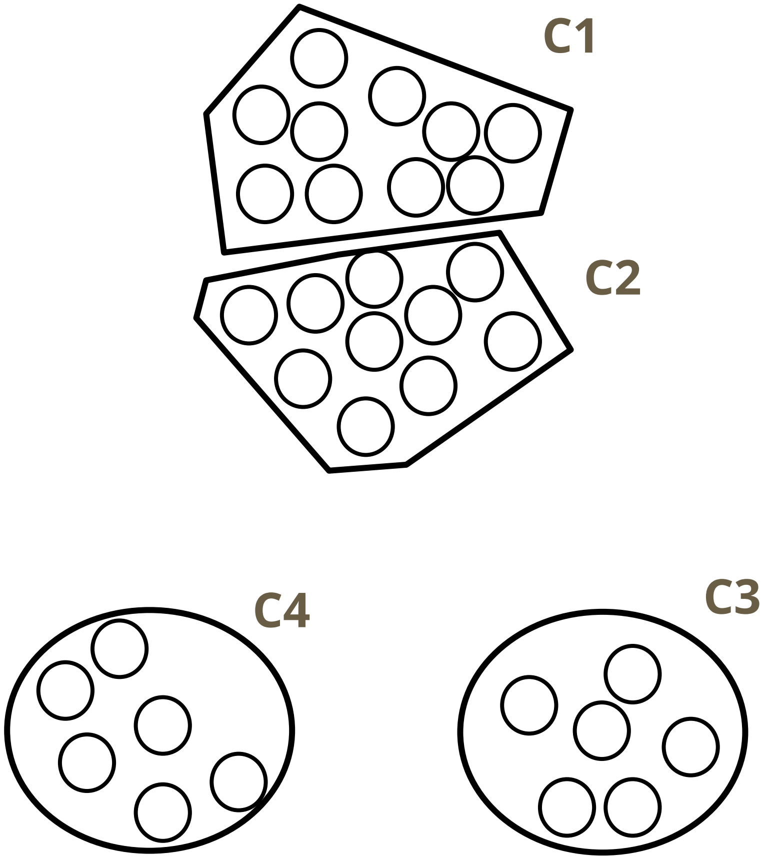






Questions

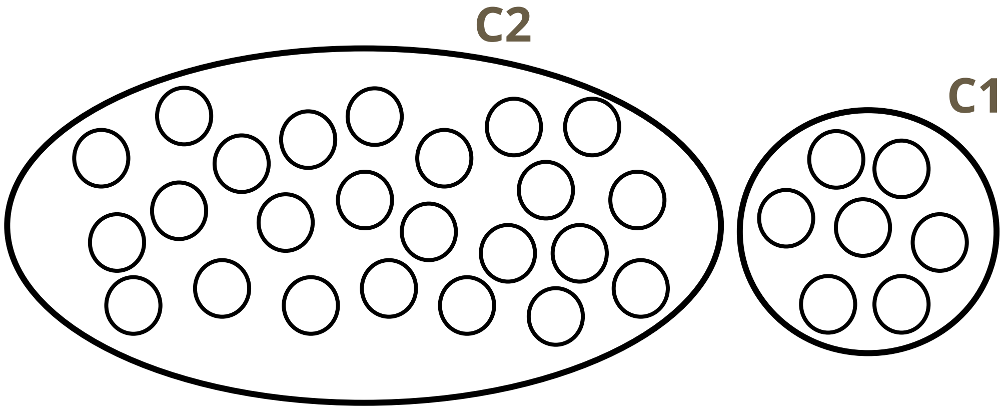
do they converge?



yes

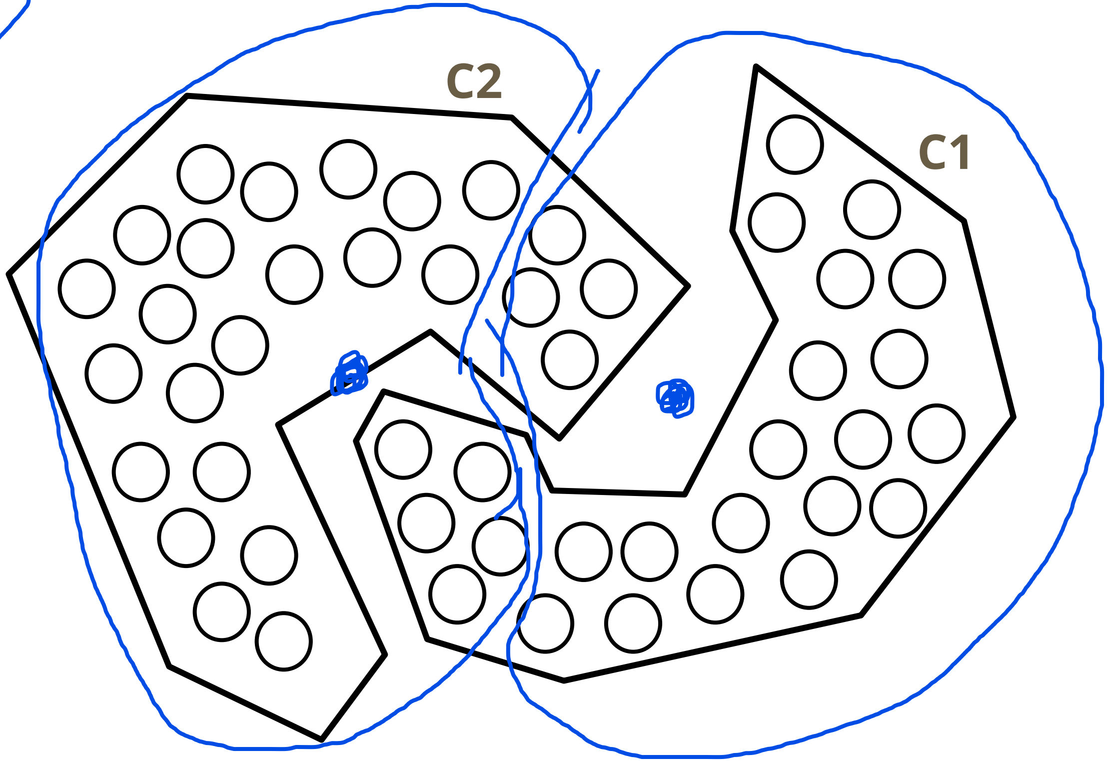
2

no



3

no



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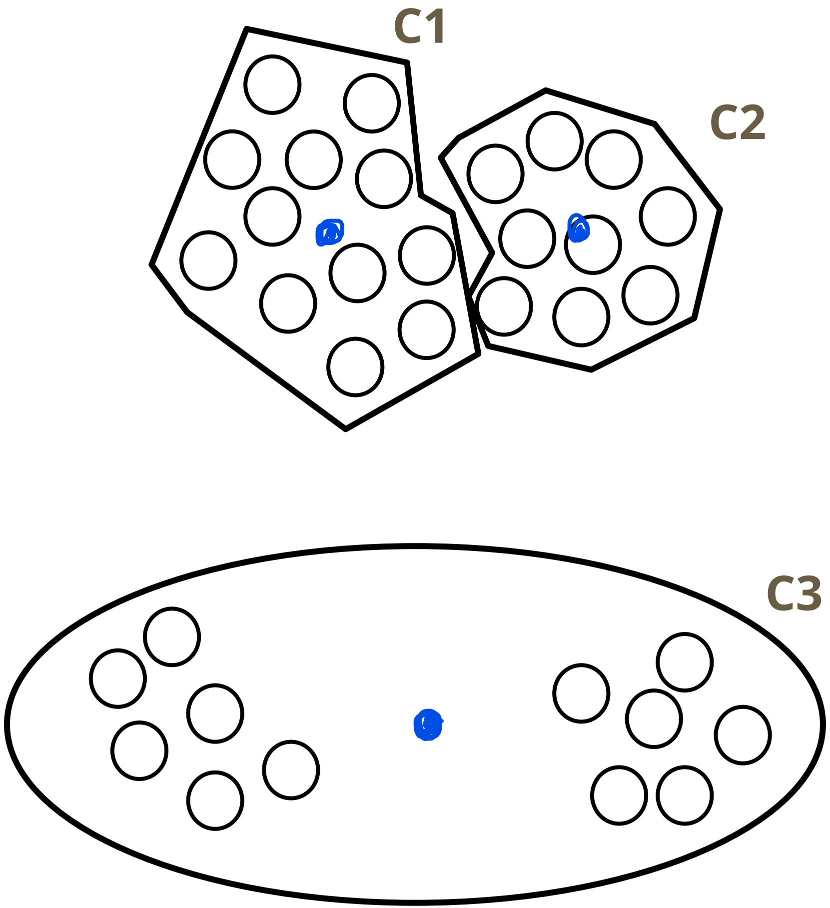
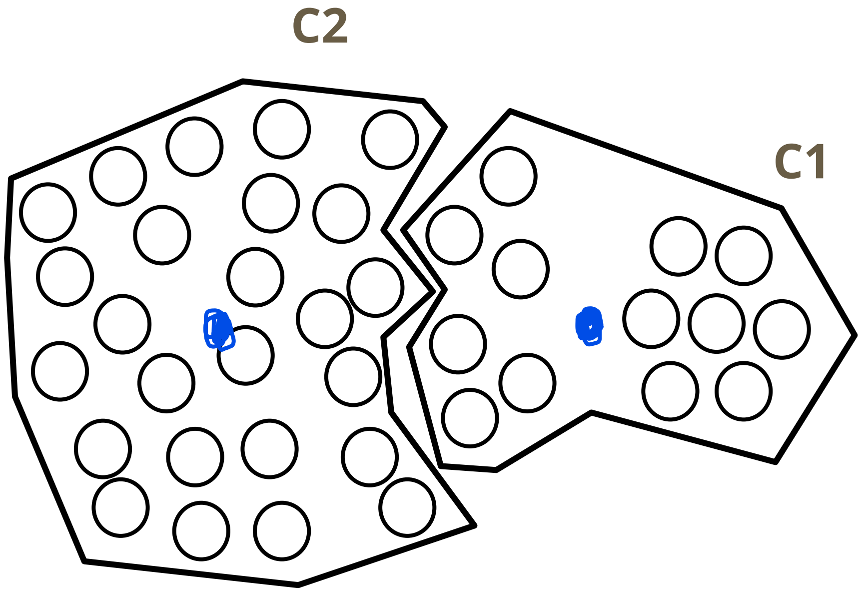
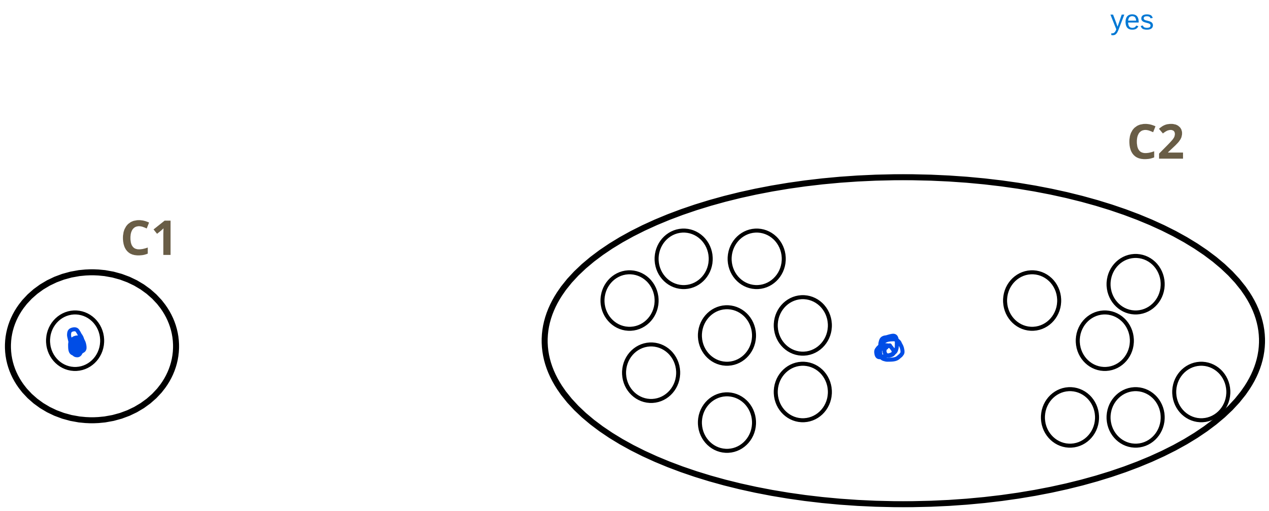


yes

5

yes




yes
C2
C1