APPLICATION OF TRANSPORTATION MODEL TO DETERMINE THE OPTIMAL TRANSPORTATION ROUTE USING PYTHON PROGRAMMING LANGUAGE

(A CASE STUDY OF COCA-COLA COMPANY IN MINNA METROPOLIS)

BY

ODIANOSEN, Hope

2017/1/69068PM

DEPARTMENT OF MATHEMATICS

FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA

JANUARY, 2024

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DECLARATION

I declare that this project titled "Application Of Transportation Model To Determine The Optimal Transportation Route Using Python Programming Language (A Case Study Of Coca-Cola Company In Minna Metropolis)" was conducted by me in the mathematics department, under the guidance of Dr. Lawal Adamu. The information presented here is the result of my own research and has not been submitted for any other academic purpose. Any external sources utilized in this work, whether published or unpublished, have been acknowledged.

______________________________ ______________________________

HOPE ODIANOSEN **DATE**

2017/1/69068PM

CERTIFICATION

This project titled: **Application Of Transportation Model To Determine The Optimal Transportation Route Using Python Programming Language (A Case Study Of The Coca-Cola Company In Minna Metropolis)** with matric number **2017/1/69068PM** meets the regulations governing the award of the degree of Bachelor of Technology (B. Tech). Federal University of Technology, Minna and it is approved for its contribution to scientific knowledge and literary presentation.

DR. LAWAL ADAMU ______________________________

Project Supervisor Signature/ Date

PROF A.I ENAGI ______________________________

Head of Department Signature/ Date

PROF. M. YISA ______________________________

Dean of School of Physical Sciences Signature/ Date

______________________________ ______________________________

External Supervisor Signature/Date

DEDICATION

I dedicate this project to Almighty God, The Creator of all that exists for His guidance, protection, provision, sustenance, love, mercy, and goodness, I would not have made it this far without Him, all Glory belongs to Him. And to my precious parents, Mr. Andrew Ebhohimen and Mrs. Mercy Ebhohimen, who gave me the opportunity and support to achieve this milestone.

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ABSTRACT

Transportation problem is always concerned with determining the best or optimal routes in distributing products to minimize cost and time. Transportation problem is widely used as a decision-making tool in various fields. A company desires for maximum profit and minimize any cost. Among all of the costs, transportation cost has seen as major issue. So, it has become a priority for companies to minimize the transportation cost. This project presents a transportation model for determining the ideal routes for Coca-Cola delivery in the Minna metropolitan. The transportation model was created using data gathered from the company. Three mathematical strategies were utilized to solve the formulated problem: the Norh West Comer Rule NWCR, the Least Cost Method LCM, and Vogel's Approximation Method VAM. The Modified Distribution Method (MoDi) was used to conduct an optimization test on VAM. The VAM was shown to provide an optimal solution. Also, a computer programming software was used to solve the problem, the solutions were compared and it proved to be an effective and efficient tool for solving transportation problems especially when you have to work with large data set. The project's findings are critical for Coca-Cola in the Minnesota metropolitan for determining the best transit routes and making decisions.

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CHAPTER ONE

1.0. INTRODUCTION

1.1. Background to the Study

It is an established fact, that the aim of every business is to maximise profit and minimize losses in every possible way, hence, the Business environment is faced with challenges when it comes to the transportation of products from the sources (e.g. factory) to their destinations (e.g. Customers).

The transportation problem is one of the fundamental problems of network flow problem which is usually used to minimize the transportation cost for industries to transport goods, which deals with shipping commodities from sources to destinations. Transportation model plays a vital role to ensure the efficient movement and in – time availability of raw materials and finished goods from sources to destinations.

It is a linear programming stemmed from a network structure consisting of a finite number of nodes and arcs attached to them. In the application of linear programming techniques, the transportation problem was probably one of the first significant problems studied.

The objective of transportation problem is to determine the shipping schedule that minimizes the total shipping cost while satisfying the demand and supply limit. If we are able to minimize the transportation time, transportation cost comes down naturally. (Kamba *et al*., 2020)

In the 1930s, Tolstoĭ was one of the first to study the transportation problem mathematically. The standard form of the problem was first formulated, along with a constructive solution, by Hitchcock in 1941. His work entitled ‘The Distribution of a Product from several sources to Numerous Localities’ gives the origin for the Hitchcock Transportation Problem analogous to Monge’s formulation.

In 1947, Koopmans also presented his historic study based on his World War II

time experience called ‘Optimum Utilization of the Transportation System’. Because of this and the work done earlier by Hitchcock, the classical case is often referred to as the ‘Hitchcock-Koopmans Transportation Problem’. Solving transportation problems where products are to be supplied from one side (Sources) to another (demands) with a goal to minimize the overall transportation cost. (Uddin *et al*., 2021)

As an example, to paint a scenario of what TP is basically about

Assuming you work as a supply chain manager for a beverage company like Coca Cola that produces varieties of soda. Your company has several manufacturing plants (origins) located in different parts of the country, and you need to distribute your soda products to various deports(destinations) in those regions. Each manufacturing plant has different production capacities, and each deport has specific demand for your soda products. Your main objective now is to minimize the transportation costs while ensuring that each retailer receives the required quantity of soda products, maintaining high customer satisfaction levels.

Using transportation optimization techniques, you can create an efficient distribution plan. This plan assigns the optimal quantity of soda products from each manufacturing plant to each deport, taking into account factors such as transportation costs, production capacities, and demand. By doing this, you can reduce the overall cost of transporting your products while meeting customer demand effectively.

1.2. Background of Company

The Coca-Cola company, which is one of the biggest beverages producing companies in Nigeria and the world at large will be the case study for this work.

The Coca-Cola Company is an American multinational corporation founded in 1892, It is best known as the producer of Coca-Cola. The drink industry company also manufactures, sells, and markets other non-alcoholic beverage concentrates and syrups, and alcoholic beverages too. The company's stock is listed on the NEW YORK STOCK EXCHANGE (NYSE)

The soft drink was developed in 1886 by pharmacist John Stith Pemberton. At the time it was introduced, the product contained cocaine from coca leaves and caffeine from kola nuts which together acted as a stimulant. The coca and the kola are the source of the product name, and led to Coca-Cola's promotion as a "healthy tonic". Pemberton had been severely wounded in the American Civil War, and had become addicted to the pain medication morphine. He developed the beverage as a patent medicine in an effort to control his addiction.

In 1889, the formula and brand were sold for $2,300 (roughly $71,000 in 2022) to Asa Griggs Candler, who incorporated the Coca-Cola Company in Atlanta in 1892. The company has operated a franchised distribution system since 1889. The company largely produces syrup concentrate, which is then sold to various bottlers throughout the world who hold exclusive territories. The company owns its anchor bottler in North America, Coca-Cola Refreshments.

In 1951, Coca-Cola made its way to Nigeria, where it quickly became popular with consumers. Currently, the company serves around 108,000,000 people in Nigeria. The franchise for A.G. Leventis to market the product in Nigeria was acquired. (Wikipedia, 2023)

Nigerian Bottling Company is a beverage firm that is the franchise bottler of Coca-Cola in Nigeria. The firm has also owned the Nigerian franchise to market Fanta, Sprite, Schweppes, Ginger Ale, Limca, Krest, Parle Soda and Five Alive.

Nigerian Bottling Company also known has NBC, started production in 1953 at the basement facilities of the mainland Hotel, Ebute Meta in Apapa Lagos, it was owned by the Leventis Group, producing Coke licensed from Coca Cola Company. In 1960, NBC introduced Fanta orange drink into the market and later Sprite lemon drink.

The Nigeria Bottling Company was founded in 1953 as a subsidiary of Leventis, with headquarters at Ebute Meta in Apapa, Lagos, to increase Coca-Cola bottle production. Today, the Coca-Cola Company has eleven (11) plants across Nigeria's thirty-six states, employing approximately three thousand five hundred and fifty (3550) people.

In Minna Niger state, the Coca-Cola Company has its main warehouse/main Depot situated in Chachanga, where all other mini-Depots purchase from. And there are several mini-Depots scattered around Minna metropolis, but we will be focusing mainly on the main ones which consist Dutsen-Kura, and Maikukele.

1.3. Statement of Problem

The price of shipping a single product (beverages) from the manufacturer or warehouse to the retail location (demand centres) affects the cost of the product per unit. Given that consumers are constantly price sensitive, the manufacturer and the consumers are both impacted by the higher price of the products as a result of higher transportation costs. So, determining the optimal routes to take in order to reduce the cost of shipping the product is therefore essential and significant.

1.4. Aim and Objectives

The aim of this work is to determine the best or optimal routes that will minimize the cost of transporting and distributing Coca-Cola beverages around Minna metropolis

The objectives are;

i. To find the best and cost minimizing routes for distributing Coca-Cola products in Minna Metropolis.
ii. To develop a transportation model for distributing and carrying Coca-Cola products in Minna Metropolis.
iii. To determine the cost of transportation using the North West Corner Rule, the Least Cost Method, and the Vogel Approximation Method.
iv. Solve the problem using Python computer programming language

1.5. Significance of Study.

If the best routes are identified or determined, the profit will increase while the cost of transportation will reduce. The Coca-Cola Company values this study much because it has the potential to lower the cost of shipping the beverage from its source to its final destinations.

1.6. Limitation of Study

The limitation of this study is that, it is focused only on determining the best/ optimal route for distributing coca cola beverage around Minna, Niger state. And it is restricted to the mini depot(Agents) of the Coca-Cola depot in Chachanga which is also called Super-agent, Minna metropolis

CHAPTER TWO

2.0. LITERATURE REVIEW

2.1. Transportation problem overview

Nowadays, mathematical methods have been used to solve many problems of operational planning for transportation. The transportation problem (TP) is an important component of the Linear Programming (LP) paradigm that occurs in a number of situations and has received a lot of attention in the literature. The transportation problem is undoubtedly one of the most important specific linear programming problems in terms of its relative frequency in applications, as well as the simplicity of the approach established to solve it. The following aspects of the transportation challenge are seen to be the most essential. The TP were the first linear programs discovered to contain entirely unimodular matrices and integral extreme points, resulting in a significant simplification of the simplex approach. Quantitative models and mathematical tools such as linear programming allows a better result. Modern computing equipment such as (Python programming language, Microsoft Excel, MATLAB etc) can be used in this case.

Many scientific fields have contributed to the analysis of transportation challenges, including operations research, economics, engineering, Geographic Information Science, and geography. It is thoroughly investigated in the mathematical programming and engineering literatures. Also known as the facility location and allocation dilemma. The transportation optimization problem can be represented as a large-scale mixed integer linear programming problem.

2.2. History, and Some of the works' others have done on Transportation problem:

Hitchcock (1941) discussed the origins of transportation, as well as a study titled "Distribution of a Product from Several Sources to Numerous Localities". This presentation is thought to be the first significant contribution to the solution of transportation difficulties. Koopmans (1947) submitted an independent research, unrelated to Hitchcock's, titled "Optimum Utilization of the Transportation System". These two innovations aided in the development of transportation technologies that use a variety of shipping sources and destinations. The transportation problem got its name from the fact that many of its applications involve calculating how to deliver products most efficiently.

However, it was not optimally solved as a solution to a complicated business problem until 1951, when George B. Dantzig used the notion of Linear Programming to solve the Transportation models.

Dantzig (1963) then applies the simplex method to the transportation problem as the primary simplex transportation method. The algorithm employs the decomposition approach, iterating between a linear programming transportation problem that allocates previously set plant production quantities to various markets and a routine that optimally sets plant production quantities to equate total marginal production costs, including a shadow price that represents a relative location cost derived from the transportation problem.

In these generalizations, the scenario when the costs are piecewise linear convex functions is considered. He decomposed the problem into a purely linear program.

Furthermore, he claimed that the two problems are the same using a theorem he dubbed the REDUCTION THEOREM. He provides an approach to tackle the problem that is a version of the simplex method with "generalized pricing operation". It ignores the integer solution property of the transportation problem, allowing it to solve some problems that are not truly transportation in nature and may not have an integer solution property.

Shetty (1959) has developed an algorithm for solving transportation problems using nonlinear costs. Some techniques to solving the concave transportation problem are discussed below.

The Branch and Bound algorithm approach is based on using a convex approximation to the concave cost functions. It is equivalent to the solution of a finite sequence of transportation problems. The algorithm was developed as a particular case of the simplified algorithm for minimizing separable concave functions over linear polyhedral as Falk and Soland.

Soland (1971) presented a branch and bound algorithm to solve concave separable transportation problem which he called it the "Simplified algorithm" in comparison with similar algorithm given by Falk and himself in 1969.

(Ahmed et al., 2017) proposed a new method to obtain an initial basic feasible solution for the transportation problems. The proposed method was illustrated with numerical examples and also a comparative study was done to justify the performance of the proposed method. It was noticed that the performance of proposed method was suitable for solving transportation problems.

Abdul *et al*. (2012) has shown a new method named ASM Method for finding an optimal solution for many transportation problems. By this method was established a numerical illustration and was yielded the best the result. This method required very simple arithmetical and logical calculation. This method was explored for those decision makers dealing with logistics and supply chain related issues.

Islam, (2012) has proposed an algorithm called Incessant Allocation Method to obtain a starting basic feasible solution for the transportation problems. Were solved a several numbers of numerical problems in order to justify the method and the results have shown that the proposed algorithm was effective in solving the problems of transportation.

Teklehaymanot *et al*. (2022), analysed Vogel's Approximation Method and its modification due to Shimshak and Goyal in finding an initial solution to an unbalanced transportation problem. They have suggested a heuristic approach for balancing the unbalanced transportation problem and improving the Vogels Approximation Method.

Muztoba (2014) has examined a real-world application of a transportation problem that involves transporting mosquito coil from company's warehouse to distributor's warehouse is modelled using linear programming in order to find the optimal transportation cost. (*Prifti et al., 2020)*

Sasieni *et al*., (1959) noted that problems of allocation arise whenever there are a number of activities to perform, but limitations on either the amount of resources or the way they can be spent prevent us from performing each separate activity in the most effective way conceivable. In such situations, we wish to allot the available resources to the activities in a way that will optimize the total effectiveness.

To model the transportation problem of ABL, a quantitative model was set up. The model sought to determine the transportation (shipping) plan of ABL from two sources to ten destinations in Ghana, subject to the required demands at the various destinations and capacities at the sources.

An Initial Basic Feasible Solution (IBFS) is required to be the starting point to solve ABL's transportation problem and this was obtained by the use of Vogel's Approximation Method (VAM) which is an analytical method of solving transportation problems.

Two other methods, the least – cost and northwest – corner rule can also be used to find the IBS of the transportation problem but the quality of the least – cost starting solution is better than that of the North West – corner rule because it yields a smaller value in the same example.(Ablordeppey, 2012)

CHAPTER 3

3.0. MATHEMATICAL MODELS OF SOLVING TRANSPORTATION PROBLEMS

3.1 Mathematical models for solving transportation problems

They are two stages of obtaining the solution to a transportation problem, which are:

a) The initial feasible solution

b) The optimal solution.

a) The first solution of a transportation problem can be obtained using any of the following mathematical methods. methods, and these are the three main methods for solving transportation problem;

1. North-West corner Rule (NWCR)
2. Least Cost Method and
3. Vogel Approximation Method (VAM)

3.1.1 North-West Corner Rule (NWCR)

The North-West Corner method (or upper left-hand corner) is a systematic methodology that is used to a special form of Linear Programming problem structure called the Transportation Model. It ensures that there is an initial fundamental workable solution (non-artificial).. This method is considered oldest known and had been used. We consider the following steps when using this method.

i. To begin, we select the northwest cell in the transportation table.

ii. Allocate as much space as possible in that cell until the demand in the first column or supply in the first row is met.
iii. If the demand is fulfilled, move to the second column's horizontally to the right cell and assign as much as possible.
iv. If the supply is fulfilled, then move to the cell vertically down located at second row and allocate as much as possible.
v. If both supply and demand are fulfilled, then move one cell diagonally and
vi. allocate as much as possible.
vii. This procedure will continue until all the allocations are over.

3.1.2 Least Cost Method

The Least cost method is also known as minimum cost method. In this method the cheapest route is firstly allocated among all the routes.

i. First, we determine the cell with the least (minimum) cost in the transportation table.
ii. Give the cell the highest possible volume.
iii. The we exclude the row or column where an allocation is made.
iv. Repeat the above steps until all the allocations are made

3.1.3 The Vogel Approximation Method (VAM)

The Vogel Approximation Method which is popularly called VAM. Vogel approximation method is considered as the improved version of the least cost method, and is also known as the penalty method. The concept of VAM is cantered on minimizing opportunity (or penalty) costs. These are the steps involved

i. Calculate the penalties for each row and each column.
ii. Select the column or row with the biggest penalty.
iii. In the named row or column, allocate the maximum attainable volume to the cell with the minimal cost.
iv. cross out those row or column where all the allocations are made.
v. Repeat the procedure until all the allocations are covered.

Now proceed to get the met demands and supplies of the destinations and sources respectively.

3.2 Test For Optimality

A feasible solution is said to be optimal if it minimizes the transportation cost and maximize the profit. The solutions obtained from the three methods discussed above are all feasible but not the optimal. Hence, we improve them and make them optimal by employing the Modified Distribution Method (MODI); Before employing the modified distribution method, and Optimality test must be performed to first check if the solution is optimal or not. (Kalita, 2006)

We say a solution is optimal if these two conditions are satisfied:

1. There are m + n – 1 allocation, whose **m** is number of rows, **n** is number of columns. For example, the number of allocations is five, if m + n – 1 =6. Then the solution is not optimal

2. These m + n – 1 allocation should be at independent positions. i.e No allocation should be able to be increased or decreased without also changing their position or going against the limits on the row or column. (https://www.yourarticlelibrary.com)

3.3 Transportation Problem Tableau

Destination → Supply ↓		D_1	D_2	…D_j…..	D_n	Availability
S_1		C_{11} X_{11}	C_{12} X_{22}		C_{1n} X_{1n}	S_1
S_2		C_{21} X_{21}	C_{22} X_{22}		C_{2n} X_{2n}	S_2
......S_i……					X_{mn}	……S_i……
S_m		C_{m1} X_{m1}	C_{m2} X_{m2}		C_{mn} X_{mn}	S_m
Destination Requirements		D_1	D_1	……D_j…..	D_n	

TABLE 4.1 Transportation Problem Tableau

3.4 MATHEMATICAL FORMULATION

Supposed a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouses to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation cost between every pair of warehouse and outlet, and these costs are assumed to be linear.

More explicitly, the mathematical formulation is given as:

Minimize

$$Z = \sum_{j-1}^{n} \sum_{i-1}^{m} C_{ij} X_{ij} \qquad (2.1)$$

Subject to

$$\sum_{j-1}^{n} C_{ij} X_{ij} \leq S_i \quad for\ i = 1,2,3 \ldots m \ \ (Supply)$$

(2.2)

$$\sum_{i-1}^{m} C_{ij} X_{ij} \geq D_j \quad for\ i = 1,2,3 \ldots m \ \ (Demand) \qquad (2.3)$$

$$Xij \geq 0 \ \ " \ \ i,j \ \ (i=1,2,3\ldots,m;\ j=1,2,3..,n) \qquad (2.4)$$

Here:

m = number of sources

n = Number of destinations

S_i = amount of supply at sources i

D_j = amount of demand at destination j

C_{ij} = cost of transporting a unit between the source i and destination j

X_{ij} = amount of homogeneous product transported from source i to destination j.

3.5 TYPES OF TRANSPORTATION PROBLEM

There are two types of transportation problem

- **Balanced transportation problem.**

Balanced transportation problem is the one which has total quantity of supply equal to the total quantity of demand. That is;

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j \qquad (2.5)$$

- **Unbalanced transportation problem**

Unbalanced transportation problem is the one which has total quantity of supply not equal to the total quantity of demand. That is;

$$\sum_{j=1}^{m} S_i \neq \sum_{i=1}^{n} D_j \qquad (2.6)$$

Hence, a dummy (fictitious) source or destination is added depending on the one that is greater

Dummy = $\sum_{j=1}^{m} S_i - \sum_{i=1}^{n} D_{j\ or} \sum_{i=1}^{m} D_j - \sum_{j=1}^{n} S_i$ (2.7)

3.6 The Transport problem network

The Transportation problem also have a graphical representation. Graphically, this problem is often seen as a network of m sources and n destinations, and a set of m, n "directed arcs". This network of transport problems is shown in the following figure.

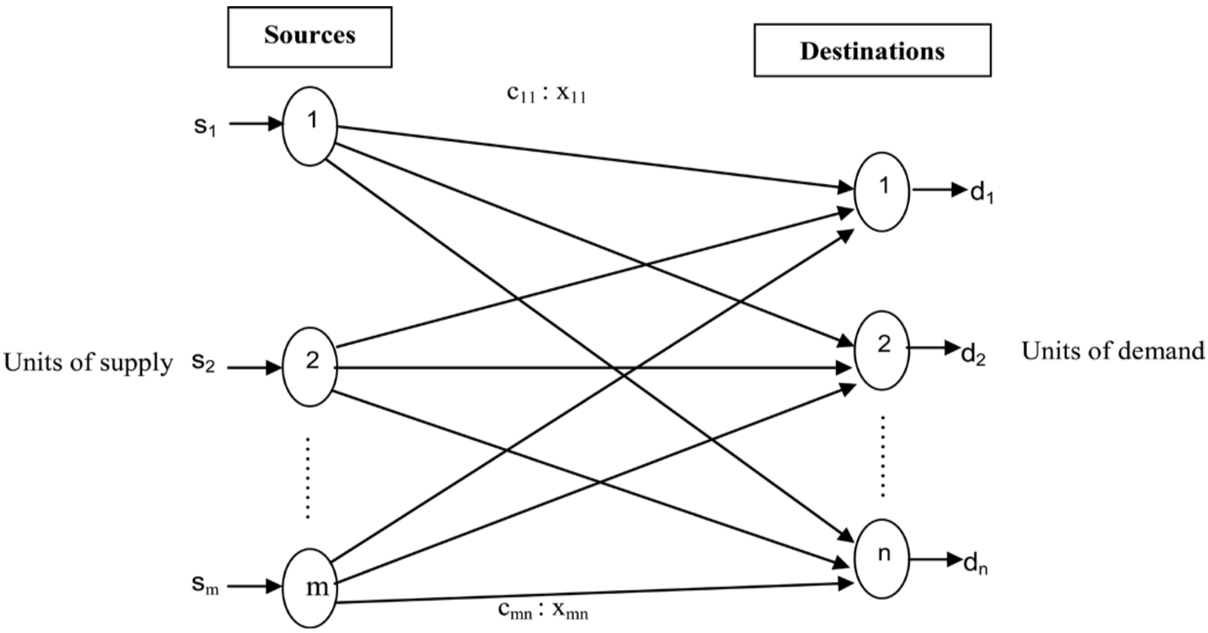


Figure 3.1: Network Representation of Transportation Problem

S1… …Sn → origins

d1… …dn → destination

C1… …Cmn → cost of transport

The arrows indicate the transport flows from source to destination.

CHAPTER FOUR

4.0 DISCUSSIONS AND RESULTS

4.1 Data Collection and Gathering Technique

For the purpose of this study data was collected from some coca cola mini deports in Bosso metropolis. The two mini deports are located in Maikunkele, and Dutsen Kura Gwari.

The data used for the analysis was collected from the logistics manager of the coca cola depots. The data include the transport cost per full Canta (truck used to distributes the product) of 100 coca cola crates from the mini depot to the to the various Destinations, depending on the quantity demanded by the customers and capacities for the two mini depots. These mini depots supply four different units which represents the various supply chain. These locations are namely;

Destination 1= Bosso Emir Junction

Destination 2= Kasua Gwari

Destination 3= Maikunkele Junction

Destination 4= Dutsen-Kura Hausa

Each mini depot is able to supply the following quantity (crates) per week:

MIAKUNKELE = 1800 (Depot A)

DUTSEN KURA GWARI = 1050 (Depot B)

Total = 2,850

Each destination demands the following crates of the Coca-Cola product per week:

Destination 1= 300

Destination 2= 900

Destination 3= 500

Destination 4= 550

Total = 2,250

The transportation cost (Naira per unit) and supply and demand (in units per week) are tabulated below:

Table 4.1 Unbalanced transportation problem

	Destination 1	**Destination 2**	**Destination 3**	**Destination 4**	**Supply**
Depot A	400	600	100	500	1800
Depot B	150	250	400	100	1050
Demand	300	900	500	550	2850 / 2250

According to the data gathered, total supply (2,460 crates) does not equal total demand (2,520 crates). In this case, to get an initial feasible solution, an extra (dummy) destination unit transportation cost will be added.

For simplicity the following terms will be used:

Destination 1 **= D1**

Destination 2 = **D2**

Destination 3 = **D3**

Destination 4 = **D4**

And

Depot A = **Deport A**

Depot B = **Deport B**

The objective here is to calculate how many crates of Coca-Cola product to transport weekly from each mini depot to each destination in order to minimize total transportation costs. The linear programming model for this unbalanced problem is written as follows:

$$Minimize\ Z = 400x_{1D_1} + 600x_{1D_2} + 100x_{1D_3} + 500x_{1D_4} + 150x_{2D_1} + 250x_{2D_2} + 400x_{2D_3} + 100x_{2D_4}$$

Subject to:

$$x_{1D_1} + x_{1D_2} + x_{1D_3} + x_{1D_4} = 1800$$
$$x_{2D_1} + x_{2D_2} + x_{2D_3} + x_{2D_4} = 1050$$
$$x_{1D_1} + x_{2D_1} + 0x_{3D_1} + 0x_{4D_1} = 300$$
$$x_{1D_2} + x_{2D_2} + 0x_{3D_2} + 0x_{4D_2} = 900$$
$$x_{1D_3} + x_{2D_3} + 0x_{3D_3} + 0x_{4D_3} = 500$$
$$x_{1D_4} + x_{2D_4} + 0x_{3D_4} + 0x_{2D_4} = 550$$
$$x_{1D_5} + x_{2D_4} + 0x_{3D_4} + 0x_{2D_4} = 550$$

Table 4.2 Balanced transportation problem

To / From	D1	D2	D3	D4	Dummy	Supply
Depot A	400	600	100	500	0	1800
Depot B	150	250	400	100	0	1050
Demand	300	900	500	550	600	2850

Now this problem would be solved to obtain the minimum transportation cost, the three main methods will be used, which was stated earlier in chapter 2, which are the North-West Corner Rule (NWCR), Least Cost Method, and The Vogel's Approximation Method.

4.1 The North-West Corner Rule (NWCR)

In the northwest corner method, the largest possible allocation is made to the cell in the upper left-hand corner of the tableau, followed by allocations to adjacent feasible cells.

Table 4.3: First iteration table

To / From	D1	D2	D3	D4	Dummy	Supply
Depot A	400 300	600	100	500	0	1800
Depot B	150	250	400	100	0	1050
Demand	300	900	500	550	600	2850

Note: In North-West Corner rule, allocation starts from the North-west corner, i.e., from (x_{1D_1}) position. Thus, the maximum possible units(crates) that can be allocated to this position is 300 as shown in the Table 4.3 above.

Table 4.4: 2nd iteration table

To / From	D1	D2	D3	D4	Dummy	Supply
Depot A	400 300	600 900	100	500	0	1800
Depot B	150	250	400	100	0	1050
Demand	300	900	500	550	600	2850

To / From	D1	D2	D3	D4	Dummy	Supply
Depot A	400 300	600 900	100 500	500	0	1800
Depot B	150	250	400	100	0	1050
Demand	300	900	500	550	600	2850

Table 4.5: 3rd iteration table

Table 4.6 4th Iteration table:

To / From	D1	D2	D3	D4	Dummy	Supply
Depot A	400 300	600 900	100 500	500 100	0	1800
Depot B	150	250	400	100	0	1050
Demand	300	900	500	550	600	2850

Table 4.7 5th Iteration table:

To / From	D1	D2	D3	D4	Dummy	Supply
Depot A	400 300	600 900	100 500	500 100	0	1800
Depot B	150	250		100 450	0	1050
Demand	300	900	500	550	600	2850

Table 4.8 6th Iteration table:

To / From	D1	D2	D3	D4	Dummy	Supply
Depot A	400 300	600 900	100 500	500 100	0	1800
Depot B	150	250		100 450	0 600	1050
Demand	300	900	500	550	600	2850

From the above table, calculate the cost of transportation. The transportation cost is computed by substituting the cell allocations (i.e., the amounts transported)

$x_{1D_1} = 300$

$x_{1D_2} = 960$

$x_{2D_2} = 500$

$x_{2D_3} = 100$

$x_{2D_4} = 450$

$x_{2D_4} = 600$

Now, substituting into the objective function:

$Z = (400 * 300) + (600 * 900) + (100 * 500) + (500 * 100) + (100 * 450) + (0 * 600)$

=120000 + 540000 + 50000 + 50000 + 45000

The Total Transportation Cost = ₦ 805,000

The initial solution is therefore

Cost = ₦ 805,000

4.2.The Least Cost Method

Initial feasible least-cost method solution:

The least-cost method identifies the least unit cost in the transportation tableau and allocates as much as possible to its cell without violating any of the supply or demand constraints. The satisfied row or column is then deleted (crossed out). The next least weight cost is identified and as much as possible is allocated to its cell, without violating any of the supply and demand constraints. The satisfied row or column is deleted (crossed out). This procedure is continued until all rows and columns have been deleted

Table 4.9 1st Iteration table:

To / **From**	**D1**	**D2**	**D3**	**D4**	**D5 Dummy**	**Supply**
Depot A	400	600	100	500	0 600	1800
Depot B	150	250	400	100	0	1050
Demand	300	900	500	550	600	2850

In the above transportation table, cells x_{1D_5} and x_{2D_5} have the least costs. Since they appear in the same column, the cell where the maximum allocation can made is selected, which is x_{2D_5}.

To / From	D1	D2	D3	D4	D5 Dummy	Supply
Depot A	400	600	100	500	0 600	1800
Depot B	150	250	400	100 550	0	1050
Demand	300	900	500	550	600	2850

Table 4.10 2nd Iteration table:

To / From	D1	D2	D3	D4	D5 Dummy	Supply
Depot A	400	600	100 500	500	0 600	1800
Depot B	150	250	400	100 550	0	1050
Demand	300	900	500	550	600	2850

Table 4.11 3rd Iteration table:

Table 4.12 4th Iteration table

To / From	D1	D2	D3	D4	D5 Dummy	Supply
Depot A	400	600	100 500	500	0 600	1800
Depot B	150 300	250	400	100 550	0	1050
Demand	300	900	500	550	600	2850

Table 4.13 5th Iteration table:

To / From	D1	D2	D3	D4	D5 Dummy	Supply
Depot A	400	600	100 500	500	0 600	1800
Depot B	150 300	250 200	400	100 550	0	1050
Demand	300	900	500	550	600	2850

:

Table 4.14 6th Iteration table

To / From	**D1**	**D2**	**D3**	**D4**	**D5 Dummy**	**Supply**
Depot A	400	600 700	100 500	500	0 600	1800
Depot B	150 300	250 200	400	100 550	0	1050
Demand	300	900	500	550	600	2850

$x_{1D_2} = 700$

$x_{1D_3} = 500$

$x_{1D_5} = 600$

$x_{2D_1} = 300$

$x_{2D_2} = 200$

$x_{2D_4} = 550$

Now, substituting into the objective function:

$$Z = (600 * 700) + (100 * 500) + (150 * 300) + (200 * 250) + (100 * 550) + (0 * 600)$$

=420000 + 50000 + 45000 + 50000 + 55000

The Total Transportation Cost = ₦ 620,000

4.3. Vogel's approximation method (VAM)

Vogel's approximation model (also called VAM), is based on the concept of penalty cost or regret. It is based upon the concept of minimizing opportunity cost for a given supply row or demand column. It is therefore defined as the difference between the lowest cost and the second lowest alternative.

A penalty cost is the difference between the largest and the next largest cell cost in a row (or column).

VAM allocates as much as possible to the minimum cost cell in the row or column with the largest penalty cost.

Table 4.15: First iteration table

To / From	D1	D2	D3	D4	D5 Dummy	Supply	PENALTY
Depot A	400	600	100	500	0	1800	100
Depot B	150	250	400	100 550	0	1050	100
Demand	300	900	500	550	600	2850	
PENALTY	250	350	300	400	0		

From the above table, it is obvious that the largest penalty is 400 in column D4. Therefore 550 allocated to cell x_{2D_4}because it contains the least transportation cost. The units allocated satisfies the number of crates demanded.

Table 4.16: Second iteration table

To / From	D1	D2	D3	D4	D5 Dummy	Supply	PENALTY	
Depot A	400	600	100	500	0	1800	100	300
Depot B	150	250	400 500	100 550	0	1050	100	50
Demand	300	900	500	550	600	2850		
PENALTY	250 250	350 350	300 300	400 400	0 0			

Table 4.17: Third iteration table

To / From	D1	D2	D3	D4	D5 Dummy	Supply	PENALTY		
Depot A	400	600 400	100	500	0	1800	100	300	300
Depot B	150	250 500	400	100 550	0	1050	100	50	100
Demand	300	900	500	550	600	2850			
PENALTY	250 250 400	350 350 600	300 300 100	400 400 -	0 0 0				

Table 4.18: Fourth iteration table

To / From	**D1**	**D2**	**D3**	**D4**	**D5 Dummy**	**Supply**	**PENALTY**			
Depot A	400 300	600 400	100	500	0	1800	100	300	300	300
Depot B	150	250 500	400	100 550	0	1050	100	50	100	-
Demand	300	900	500	550	600	2850				
PENALTY	250	350	300	400	0					
	250	350	300	400	0					
	400	600	100	-	0					
	400	-	100	-	0					

Table 4.18: Fifth iteration table

To / From	**D1**	**D2**	**D3**	**D4**	**D5 Dummy**	**Supply**	**PENALTY**				
Depot A	400 300	600 400	100 500	500	0	1800	100	300	300	300	100
Depot B	150	250 500	400	100 550	0	1050	100	50	100	-	-
Demand	300	900	500	550	600	2850					
PENALTY	250	350	300	400	0						
	250	350	300	400	0						
	400	600	100	-	0						
	400	-	100	-	0						
	-	-	100	-	0						

To / From	D1	D2	D3	D4	D5 Dummy	Supply	PENALTY				
Depot A	400 300	600 400	100 500	500	0 600	1800	100	100	100	100	100
Depot B	150	250 500	400	100 550	0	1050	100	50	100	-	-
Demand	300	900	500	550	600	2850					
PENALTY	250	350	300	400	0						
	250	350	300	-	0						
	400	600	100	-	0						
	400	-	100	-	0						
	-	-	100	-	0						

Table 4.18: Sixth iteration table

From the table above, since column D5 is left, we allocate the remaining resources in Deport A to the unit. Hence the total demands are met.

$x_{1D_1} = 300$

$x_{1D_2} = 400$

$x_{1D_3} = 500$

$x_{1D_5} = 600$

$x_{2D_2} = 500$

$x_{2D_4} = 550$

Now, substituting into the objective function:

$$Z = (400 * 300) + (600 * 400) + (100 * 500) + (250 * 500) + (100 * 550) + (0 * 600)$$

=120000 + 240000 + 50000 + 125000 + 55000

The Total Transportation Cost = ₦ 590,000

4.4. Computer Program Approach in attaining the minimum Transportation Cost.

A computer is an electronic system designed to manipulate and provide useful information. It does this job through a set of instructions called PROGRAM. it data processing manner is at times almost "human" A computer follows the instruction that a human would follow in solving most problem. In other word it simulates the pattern of human by not doing the thinking but it is more efficient, faster and consistent when compared to human beings. From calculating the MODI (modified distribution) method in Operation Research (Transportation Problem) in 7 full pages to get to an optimal value, to realizing, this can be built and solved in a few minutes in excel, python, MATLAB and other computer programming languages. We have seen the evolution of mathematics with help of increased computational power.

4.4.1. Choice Of Programming Language

A simple program would be developed to automate the solution repeat by one of the methods. This will be done using the python programming language which is to enhance the efficiency of the program due to its high mathematical capabilities compared to MATLAB, excel, C++ and the likes.

Python is a dynamically-typed garbage-collected programming language developed by Guido van Rossum in the late 80s to replace ABC. Much like the programming language Ruby, Python was designed to be easily read by programmers. Because of its large following and many libraries, Python can be implemented and used to do anything from webpages to scientific research.

The main aim of using python programming language is to show how you can effectively solve a transportation problem especially when you have to work with large data. With just few lines of code you can solve the problem.

PuLP

PuLP is an LP modeler written in Python. It is a Library in python that solves Linear Programming problems. The pulp package provides a modelling framework - this means that it does not solve the optimisation problems on its own, but rather provides us with the a set of commands and object types that we could use in order to represent mathematical formulations and problem instances within Python. Another popular modelling framework for Python that you might encounter is pyomo.

The actual solution of the problem instances is carried out by another type of tool called solver. PuLP will call that on our behalf, supply the structure of the model and the values of our parameters - once a solution is found, it will be send back to PuLP.

PuLP can only be used to model linear problems that fall under the LP, IP or MILP families (ie. with real or integer variables or a combination thereof). Unfortunately, it cannot be used for non-linear problems, however there are many other Python libraries that we could use for such problems.

Installation

================

Note that to install PuLP you must first have a working python software

PuLP requires Python >= 2.7 or Python >= 3.4.

The latest version of PuLP can be freely obtained from GitHub.

Pip and pypi installation

By far the easiest way to install pulp is through the use of pip.

In windows (please make sure pip is on your path):

```
c:\Python34\Scripts\> pip install pulp
```

4.1 Test for optimality of the Vogel's method

To / From	D1	D2	D3	D4	D5 Dummy	Supply	
Depot A	400 300	600 400	100 500	500	0 600	1800	$U_1 =$
Depot B	150	250 500	400	100 550	0	1050	$U_1 =$
Demand	300	900	500	550	600	2850	$U_1 =$
	$V_1 =$	$V_1 =$	$V_1 =$	$V_1 =$	$V_1 =$	$V_1 =$	

Table 4.1: First Iteration of Non-optimal Table

Checking for optimality for all occupied cell, we shall begin from cell $u_i$since it is the cell with the highest allocations. Thus, we allocate zero to cell u_i. That is u_1=0

Now, for the basic feasible solution, we use the equation $u_i + v_j = c_{ij}$

Where u_i and v_j represents the column and row values that must be occupied, and $\boldsymbol{c_{ij}}$ is the actual unit transportation cost for the cell ij

Cell (1, 1) = $u_1 + v_1 = c_{11}$= $0 + v_1 = 400$; $v_1 = 400$

Cell (1, 2) = $u_1 + v_2 = c_{12}$= $0 + v_2 = 600$; $v_2 = 600$

Cell (1, 3) = $u_1 + v_3 = c_{13}$= $0 + v_3 = 100$; $v_3 = 100$

Cell (1, 5) = $u_1 + v_5 = c_{15}$= $0 + v_5 = 0$; $v_5 = 0$

Cell (2, 2) = $u_2 + v_2 = c_{22}$= $u_2 + 600 = 250$; $u_2 = -350$

Cell (2, 4) = $u_2 + v_4 = c_{33}$= $-350 + v_4 = 100$; $v_4 = 450$

For non-basic variables $d_{ij} = c_{ij} - u_i - v_j$

Let **c_{ij}** denote – actual unit cost. and

d_{ij} denote the Unit shadow cost

Where d_{ij} equals the cost increase or decrease that would occur by allocating to a cell.

Cell (1,4) = $c_{11} - u_1 - v_4$; 500 – 0 – 450 = 50; $d_{14} = 50$

Cell (2, 1) = $c_{21} - u_2 - v_1$; 150 - (-350) – 400 = 100; $d_{21} = 100$

Cell (2, 3) = $c_{23} - u_2 - v_3$; 400 - (-350) – 100 = 650; $d_{23} = 650$

Cell (2, 5) = $c_{25} - u_2 - v_5$; 0 – 0 - (-350) = 350; $d_{25} = 350$

The difference in the actual units cost and the shadow costs in the unallocated cells are reflected in the table below.

Table 4.2: Final Iteration Table

To / From	D1	D2	D3	D4	D5 Dummy	Supply	Ui
Depot A	400 300	600 400	100 500	500	0 600	1800	$U_1 = 0$
Depot B	150	250 500	400	100 550	0	1050	U_2 =-350
Demand	300	900	500	550	600	2850	
Vj	$V_1 = 400$	$V_2 = 600$	$V_3 = 100$	$V_4 = 450$	$V_5 = 0$		

From the above calculation, there are no negative values. The results are either equal to zero or positive which obeys the optimality criterion.

Therefore, the solution is optimal.

Hence the minimum cost

$Z = (400 * 300) + (600 * 400) + (100 * 500) + (250 * 500) + (100 * 550) + (0 * 600)$

=120000 + 240000 + 50000 + 125000 + 55000

The Total Transportation Cost = ₦ 590,000

4.2. Discussion of result

Through this project it was discovered that the Vogel's Approximation Method gave the optimal solution of ₦360,000 which is not as high as the North-West Corner Rule initial solution of ₦805,000. It is also lower than the initial solution obtained from the Least Cost Method. This implies that the Vogel's Approximation Method gives the optimal transportation routes and also minimizes the total transportation cost compared to the other two methods. The modified distribution method was used to improve the solution obtained by the Least Cost Method. In other words, the MODI method was used to test for optimality of the solution.

Therefore, the optimal transportation routes are:

Depot 1 To Destination 1 = Miakunkele To Bosso Emir Junction

Depot 1 To Destination 2 = Miakunkele To Kasuan Gwari

Depot 1 To Destination 3 = Miakunkele To Maikunkele Junction

Depot 2 To Destination 2 = Dutsen-Kura Gwari To Kasuan Gwari

Depot 2 To Destination 4 = Dutsen-Kura Gwari To Dutsen-Kura Hausa

4.2 RESULTS FROM THE PROGRAM

4.2.1 The Northwest Corner Method

This is the results gotten from the codes in appendix A. we can see that the result gotten from the program is the same as the result gotten in 4.1.

```
Run:   main

Welcome to the CBC MILP Solver
Version: 2.10.3
Build Date: Dec 15 2019

command line - C:\Users\DELL\Desktop\projects\vogel\venv\lib\site-packages\pulp\solverdir
At line 2 NAME          MODEL
At line 3 ROWS
At line 18 COLUMNS
At line 53 RHS
At line 67 BOUNDS
At line 68 ENDATA
Problem MODEL has 13 rows, 10 columns and 26 elements
Coin0008I MODEL read with 0 errors
Option for timeMode changed from cpu to elapsed
Presolve 0 (-13) rows, 0 (-10) columns and 0 (-26) elements
Empty problem - 0 rows, 0 columns and 0 elements
Optimal - objective value 805000
After Postsolve, objective 805000, infeasibilities - dual 0 (0), primal 0 (0)
Optimal objective 805000 - 0 iterations time 0.002, Presolve 0.00
Option for printingOptions changed from normal to all
Total time (CPU seconds):       0.01   (Wallclock seconds):       0.01

North-West Corner Method Results:
Optimal
L_('Depot_1',_'Destination_1') = 300.0
L_('Depot_1',_'Destination_2') = 900.0
L_('Depot_1',_'Destination_3') = 500.0
L_('Depot_1',_'Destination_4') = 100.0
L_('Depot_1',_'Dummy') = 0.0
L_('Depot_2',_'Destination_1') = 0.0
L_('Depot_2',_'Destination_2') = 0.0
L_('Depot_2',_'Destination_3') = 0.0
L_('Depot_2',_'Destination_4') = 450.0
L_('Depot_2',_'Dummy') = 600.0
Total Cost: 805000.0

Process finished with exit code 0
```

Figure 4.1: North-West Corner Method Results

4.2.2 The Least cost method

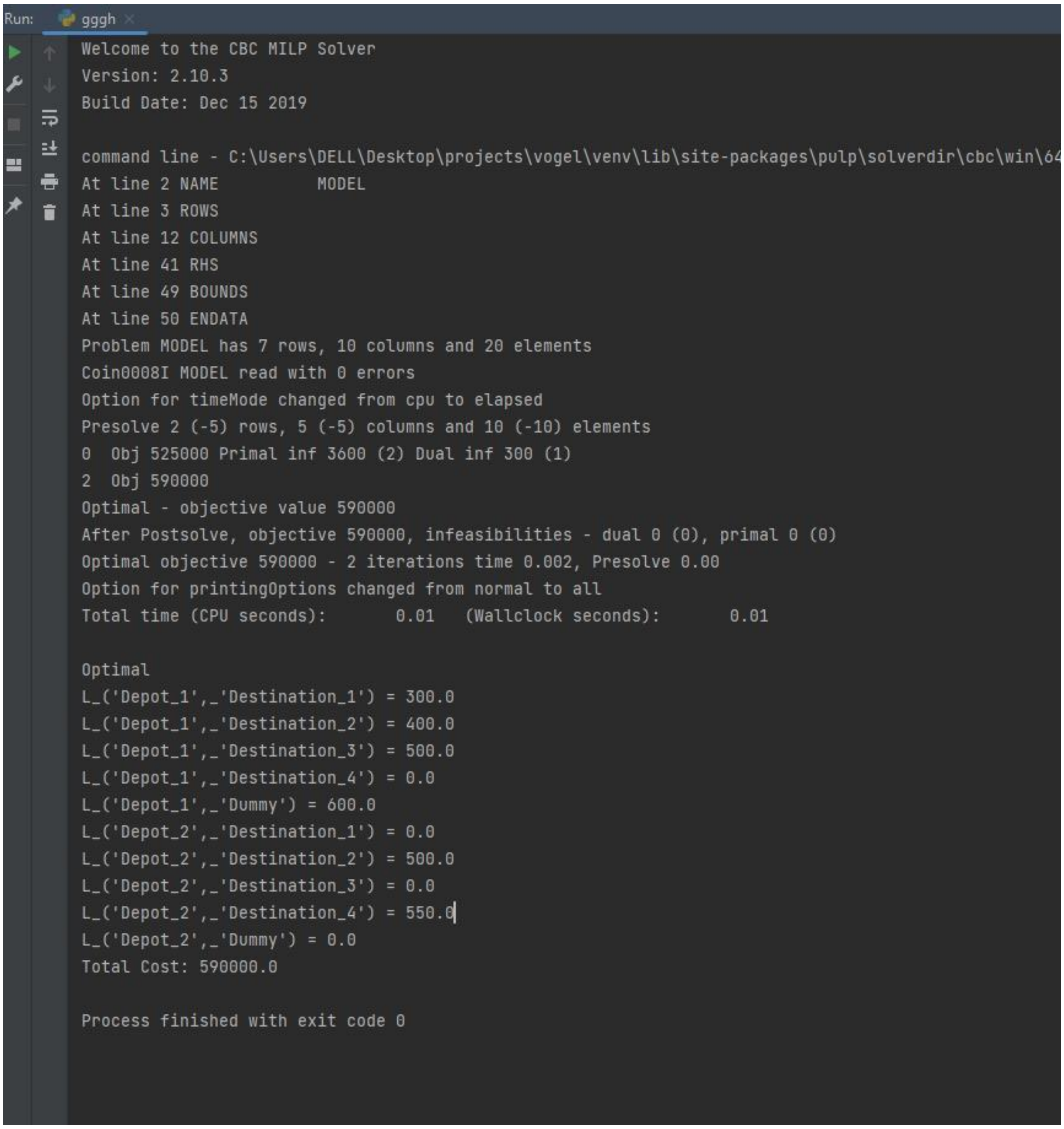
This is the results gotten from the codes in appendix B. we can see that the result gotten from the program is the same as the result gotten in 4.2.

```
Run:    main
Selected cell: ('Depot_1', 'Dummy'), Quantity: 600
Selected cell: ('Depot_1', 'Destination_3'), Quantity: 500
Selected cell: ('Depot_2', 'Destination_4'), Quantity: 550
Selected cell: ('Depot_2', 'Destination_1'), Quantity: 300
Selected cell: ('Depot_2', 'Destination_2'), Quantity: 200
Selected cell: ('Depot_1', 'Destination_2'), Quantity: 700
Welcome to the CBC MILP Solver
Version: 2.10.3
Build Date: Dec 15 2019

command line - C:\Users\DELL\Desktop\projects\face-detection\venv\lib\site-packages\pulp\solver
At line 2 NAME          MODEL
At line 3 ROWS
At line 18 COLUMNS
At line 53 RHS
At line 67 BOUNDS
At line 68 ENDATA
Problem MODEL has 13 rows, 10 columns and 26 elements
Coin0008I MODEL read with 0 errors
Option for timeMode changed from cpu to elapsed
Presolve 0 (-13) rows, 0 (-10) columns and 0 (-26) elements
Empty problem - 0 rows, 0 columns and 0 elements
Optimal - objective value 620000
After Postsolve, objective 620000, infeasibilities - dual 0 (0), primal 0 (0)
Optimal objective 620000 - 0 iterations time 0.002, Presolve 0.00
Option for printingOptions changed from normal to all
Total time (CPU seconds):       0.01   (Wallclock seconds):       0.01

Optimal
L_('Depot_1',_'Destination_1') = 0.0
L_('Depot_1',_'Destination_2') = 700.0
L_('Depot_1',_'Destination_3') = 500.0
L_('Depot_1',_'Destination_4') = 0.0
L_('Depot_1',_'Dummy') = 600.0
L_('Depot_2',_'Destination_1') = 300.0
L_('Depot_2',_'Destination_2') = 200.0
L_('Depot_2',_'Destination_3') = 0.0
L_('Depot_2',_'Destination_4') = 550.0
L_('Depot_2',_'Dummy') = 0.0
Total Cost: 620000.0
```

Figure 4.2: Least cost Method Result

4.2.3 The Vogel's method

This is the results gotten from the codes in appendix C. we can see that the result gotten from the program is the same as the result gotten in 4.3.



```
Run: gggh
Welcome to the CBC MILP Solver
Version: 2.10.3
Build Date: Dec 15 2019

command line - C:\Users\DELL\Desktop\projects\vogel\venv\lib\site-packages\pulp\solverdir\cbc\win\64
At line 2 NAME          MODEL
At line 3 ROWS
At line 12 COLUMNS
At line 41 RHS
At line 49 BOUNDS
At line 50 ENDATA
Problem MODEL has 7 rows, 10 columns and 20 elements
Coin0008I MODEL read with 0 errors
Option for timeMode changed from cpu to elapsed
Presolve 2 (-5) rows, 5 (-5) columns and 10 (-10) elements
0  Obj 525000 Primal inf 3600 (2) Dual inf 300 (1)
2  Obj 590000
Optimal - objective value 590000
After Postsolve, objective 590000, infeasibilities - dual 0 (0), primal 0 (0)
Optimal objective 590000 - 2 iterations time 0.002, Presolve 0.00
Option for printingOptions changed from normal to all
Total time (CPU seconds):       0.01   (Wallclock seconds):       0.01

Optimal
L_('Depot_1',_'Destination_1') = 300.0
L_('Depot_1',_'Destination_2') = 400.0
L_('Depot_1',_'Destination_3') = 500.0
L_('Depot_1',_'Destination_4') = 0.0
L_('Depot_1',_'Dummy') = 600.0
L_('Depot_2',_'Destination_1') = 0.0
L_('Depot_2',_'Destination_2') = 500.0
L_('Depot_2',_'Destination_3') = 0.0
L_('Depot_2',_'Destination_4') = 550.0
L_('Depot_2',_'Dummy') = 0.0
Total Cost: 590000.0

Process finished with exit code 0
```

Figure 4.3: Vogel's Method of Approximation Results

Since the sole aim of this problem is determining the best route that will minimize the cost of transportation, we can just use the program code in Appendix D which will provide us with the optimize cost and the optimal transportation routes. This is the result for Appendix D

```
command line - C:\Users\DELL\Desktop\projects\vogel\venv\lib\site-packages\pulp\solverdir\cbc\win\64
At line 2 NAME          MODEL
At line 3 ROWS
At line 12 COLUMNS
At line 41 RHS
At line 49 BOUNDS
At line 50 ENDATA
Problem MODEL has 7 rows, 10 columns and 20 elements
Coin0008I MODEL read with 0 errors
Option for timeMode changed from cpu to elapsed
Presolve 2 (-5) rows, 5 (-5) columns and 10 (-10) elements
0  Obj 525000 Primal inf 3600 (2) Dual inf 300 (1)
2  Obj 590000
Optimal - objective value 590000
After Postsolve, objective 590000, infeasibilities - dual 0 (0), primal 0 (0)
Optimal objective 590000 - 2 iterations time 0.002, Presolve 0.00
Option for printingOptions changed from normal to all
Total time (CPU seconds):       0.01   (Wallclock seconds):       0.01

Optimal
L_('Depot_1',_'Destination_1') = 300.0
L_('Depot_1',_'Destination_2') = 400.0
L_('Depot_1',_'Destination_3') = 500.0
L_('Depot_1',_'Destination_4') = 0.0
L_('Depot_1',_'Dummy') = 600.0
L_('Depot_2',_'Destination_1') = 0.0
L_('Depot_2',_'Destination_2') = 500.0
L_('Depot_2',_'Destination_3') = 0.0
L_('Depot_2',_'Destination_4') = 550.0
L_('Depot_2',_'Dummy') = 0.0
Total Cost: 590000.0
```

Figure 4.4: The Optimized Solution

Therefore, the optimal transportation routes are:

Depot 1 To Destination 1 = Miakunkele To Bosso Emir Junction

Depot 1 To Destination 2 = Miakunkele To Kasuan Gwari

Depot 1 To Destination 3 = Miakunkele To Maikunkele Junction

Depot 2 To Destination 2 = Dutsen-Kura Gwari To Kasuan Gwari

Depot 2 To Destination 4 = Dutsen-Kura Gwari To Dutsen-Kura Hausa

CHAPTER 5

5.0 CONCLUSION AND RECOMMENDATION

5.1 Conclusion

This research determined the optimal transportation routes in the distribution of Coca-cola products in Minna metropolis. The Data used was collected from Coca Cola company mini depots in Minna. The two mini depots are located in Maikunkele, and Dutsen Kura Gwari Minna, Niger State. These mini depots supply four different units which represents the various supply chain. These locations are namely; Destination 1= Bosso Emir Junction, Destination 2= Kasua Gwari, Destination 3= Maikunkele Junction and Destination 4=Dutsen-Kura Hausa. Three different approaches were used to obtain the feasible solution. It was discovered that the Vogel's Approximation Method gives the optimal transportation routes and also minimizes the total transportation cost compared to the other two methods. The modified distribution method was used to improve the solution obtained by the Least Cost Method. Also, the computer programming approach to solving transportation problem

5.2 Recommendation

The following are my recommendation based on my findings:

i. The Coca Cola Company should pay more attention to the routes that minimizes the total cost of transportation because it contributes optimally to their profit.

ii. If management desires to improve the defined maximum profit within the model's constraints, only the funds assigned to transportation along highly demanded items should be increased by decreasing those allotted to transportation along less demanded products within the range.

iii. The company should consider employing people experienced in Operations Research techniques who also have a thorough awareness of the business environment and knowledge of managerial responsibilities and functions. These individuals should be capable of efficiently interpreting the results of mathematical analysis to top managers in the company's specific context, as well as possessing the necessary competence in the use of computer skills for easy handling of more complex mathematical techniques involved in company decision making.

iv. More research should be conducted to incorporate other elements that can affect the company's optimal transportation cost.

v. More works should be considered on some linear programming computer

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APPENDIX

Appendix A

```
from pulp import *


def solve_north_west_corner_method(model, supply, demand, costs):
    # Decision Variable
    logistics = LpVariable.dicts('L', [(i, r) for i in supply for r in demand], lowBound=0)

    # Define Objective Function
    model += lpSum([logistics[i, r] * costs[(i, r)] for r in demand for i in supply])

    # Add supply constraints
    for i in supply:
        model += lpSum([logistics[(i, r)] for r in demand]) == warehouse_inventory[i]

    # Add demand constraints
    for r in demand:
        model += lpSum([logistics[(i, r)] for i in supply]) == regional_demand[r]

    # North-West Corner Method
    i_index = 0
    r_index = 0
    while i_index < len(supply) and r_index < len(demand):
        quantity = min(warehouse_inventory[supply[i_index]], regional_demand[demand[r_index]])
        model += logistics[(supply[i_index], demand[r_index])] == quantity

        warehouse_inventory[supply[i_index]] -= quantity
        regional_demand[demand[r_index]] -= quantity

        if warehouse_inventory[supply[i_index]] == 0:
            i_index += 1
        if regional_demand[demand[r_index]] == 0:
```

```
        r_index += 1

    # Solve the model
    model.solve()

    return model

# Initialize Model
qcells_model_nwc = LpProblem("Q_Cells_Distribution_Problem_NWC", LpMinimize)

region = ["Destination_1", "Destination_2", "Destination_3", "Destination_4", "Dummy"]
demand = [300, 900, 500, 550, 600]
regional_demand = dict(zip(region, demand))
warehouse = ["Depot_1", "Depot_2"]
supply = [1800, 1050]
warehouse_inventory = dict(zip(warehouse, supply))
costs = {("Depot_1", "Destination_1"): 400, ("Depot_1", "Destination_2"): 600, ("Depot_1", "Destination_3"): 100, ("Depot_1", "Destination_4"): 500, ("Depot_1", "Dummy"): 0,
         ("Depot_2", "Destination_1"): 150, ("Depot_2", "Destination_2"): 250, ("Depot_2", "Destination_3"): 400, ("Depot_2", "Destination_4"): 100, ("Depot_2", "Dummy"): 0}

# Solve the problem using North-West Corner method
solve_north_west_corner_method(qcells_model_nwc, warehouse, region, costs)

# Print the results
print("North-West Corner Method Results:")
print(LpStatus[qcells_model_nwc.status])

for v in qcells_model_nwc.variables():
    print(v.name, "=", v.varValue)

print("Total Cost:", value(qcells_model_nwc.objective))
```

Appendix B

```
def solve_least_cost_method(model, supply, demand, costs):
    while True:
        # Check if there are still valid cells to consider
        if not costs:
            break

        # Find the cell with the lowest cost
        min_cost_cell = min(costs, key=costs.get)

        # Find the minimum of supply and demand
        quantity = min(warehouse_inventory[min_cost_cell[0]], regional_demand[min_cost_cell[1]])

        print(f"Selected cell: {min_cost_cell}, Quantity: {quantity}")

        # Add the decision variable to the model
        model += lpSum([logistics[i, r] for i, r in logistics if i == min_cost_cell[0] and r ==
min_cost_cell[1]]) == quantity

        # Update supply and demand
        warehouse_inventory[min_cost_cell[0]] -= quantity
        regional_demand[min_cost_cell[1]] -= quantity

        # Remove the row or column with exhausted supply or demand
        if warehouse_inventory[min_cost_cell[0]] == 0:
            costs = {(i, r): c for (i, r), c in costs.items() if i != min_cost_cell[0]}
        else:
            costs = {(i, r): c for (i, r), c in costs.items() if r != min_cost_cell[1]}

    # Solve the model after applying the least-cost method
    model.solve()
```

```
    return model


# Initialize Model
qcells_model = LpProblem("Q_Cells_Distribution_Problem", LpMinimize)


region = ["Destination_1", "Destination_2", "Destination_3", "Destination_4", "Dummy"]
demand = [300, 900, 500, 550, 600]
regional_demand = dict(zip(region, demand))
warehouse = ["Depot_1", "Depot_2"]
supply = [1800, 1050]
warehouse_inventory = dict(zip(warehouse, supply))
costs = {("Depot_1", "Destination_1"): 400, ("Depot_1", "Destination_2"): 600, ("Depot_1",
"Destination_3"): 100, ("Depot_1", "Destination_4"): 500, ("Depot_1", "Dummy"): 0,
        ("Depot_2", "Destination_1"): 150, ("Depot_2", "Destination_2"): 250, ("Depot_2",
"Destination_3"): 400, ("Depot_2", "Destination_4"): 100, ("Depot_2", "Dummy"): 0}


# Decision Variable
logistics = LpVariable.dicts('L', [(i, r) for i in warehouse for r in region], lowBound=0)


# Define Objective Function
qcells_model += lpSum([logistics[i, r] * costs[(i, r)] for r in region for i in warehouse])


# Add supply constraints
for i in warehouse:
    qcells_model += lpSum([logistics[(i, r)] for r in region]) == warehouse_inventory[i]


# Add demand constraints
for r in region:
    qcells_model += lpSum([logistics[(i, r)] for i in warehouse]) == regional_demand[r]


# Solve the problem using the least-cost method
solve_least_cost_method(qcells_model, warehouse, region, dict(costs))  # Create a copy of costs
dictionary
```

```
# Print the results
print(LpStatus[qcells_model.status])

for v in qcells_model.variables():
    print(v.name, "=", v.varValue)

print("Total Cost:", value(qcells_model.objective))
```

Appendix C

```
from pulp import *


def solve_vogels_method(model, supply, demand, costs):
    # Decision Variable
    logistics = LpVariable.dicts('L', [(i, r) for i in supply for r in demand], lowBound=0)

    # Define Objective Function
    model += lpSum([logistics[i, r] * costs[(i, r)] for r in demand for i in supply])

    # Add supply constraints
    for i in supply:
        model += lpSum([logistics[(i, r)] for r in demand]) == warehouse_inventory[i]

    # Add demand constraints
    for r in demand:
        model += lpSum([logistics[(i, r)] for i in supply]) == regional_demand[r]

    # Solve the model
    model.solve()

    return model

# Initialize Model
qcells_model = LpProblem("Q_Cells_Distribution_Problem", LpMinimize)

region = ["Destination_1", "Destination_2", "Destination_3", "Destination_4", "Dummy"]
demand = [300, 900, 500, 550, 600]
regional_demand = dict(zip(region, demand))
warehouse = ["Depot_1", "Depot_2"]
supply = [1800, 1050]
warehouse_inventory = dict(zip(warehouse, supply))
```

```
costs = {("Depot_1", "Destination_1"): 400, ("Depot_1", "Destination_2"): 600, ("Depot_1",
"Destination_3"): 100, ("Depot_1", "Destination_4"): 500, ("Depot_1", "Dummy"): 0,
        ("Depot_2", "Destination_1"): 150, ("Depot_2", "Destination_2"): 250, ("Depot_2",
"Destination_3"): 400, ("Depot_2", "Destination_4"): 100, ("Depot_2", "Dummy"): 0}


# Solve the problem using Vogel's approximation method
solve_vogels_method(qcells_model, warehouse, region, costs)


# Print the results
print(LpStatus[qcells_model.status])


for v in qcells_model.variables():
    print(v.name, "=", v.varValue)


print("Total Cost:", value(qcells_model.objective))
```

Appendix D

```
from pulp import *


def solve_problem(model, supply, demand, costs):
    # Decision Variable
    logistics = LpVariable.dicts('L', [(i, r) for i in supply for r in demand], lowBound=0)

    # Define Objective Function
    model += lpSum([logistics[i, r] * costs[(i, r)] for r in demand for i in supply])

    # Add supply constraints
    for i in supply:
        model += lpSum([logistics[(i, r)] for r in demand]) == warehouse_inventory[i]

    # Add demand constraints
    for r in demand:
        model += lpSum([logistics[(i, r)] for i in supply]) == regional_demand[r]

    # Solve the model
    model.solve()

    return model

# Initialize Model
qcells_model = LpProblem("Q_Cells_Distribution_Problem", LpMinimize)

region = ["Destination_1", "Destination_2", "Destination_3", "Destination_4", "Dummy"]
demand = [300, 900, 500, 550, 600]
regional_demand = dict(zip(region, demand))
warehouse = ["Depot_1", "Depot_2"]
supply = [1800, 1050]
```

```
warehouse_inventory = dict(zip(warehouse, supply))

costs = {("Depot_1", "Destination_1"): 400, ("Depot_1", "Destination_2"): 600, ("Depot_1", "Destination_3"): 100, ("Depot_1", "Destination_4"): 500, ("Depot_1", "Dummy"): 0,
        ("Depot_2", "Destination_1"): 150, ("Depot_2", "Destination_2"): 250, ("Depot_2", "Destination_3"): 400, ("Depot_2", "Destination_4"): 100, ("Depot_2", "Dummy"): 0}


solve_problem(qcells_model, warehouse, region, costs)


# Print the results
print(LpStatus[qcells_model.status])


for v in qcells_model.variables():
    print(v.name, "=", v.varValue)


print("Total Cost:", value(qcells_model.objective)).
```