Self-diffusion for Solving Inverse Problems

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Introduction

We propose self-diffusion (SDI), a novel framework for solving inverse problems without relying on pretrained generative models. Traditional diffusion-based approaches require training a model on a clean dataset to learn to reverse the forward noising process. This model is then used to sample clean solutions that are consistent with the observed data under a specific task. In contrast, self-diffusion introduces a self-contained iterative process that alternates between noising and denoising steps to progressively refine its estimate of the solution. Essentially, self-diffusion exploits the spectral bias of neural networks and modulates it through a scheduled noise process. Without relying on pretrained score functions or external denoisers, this approach still remains adaptive to arbitrary forward operators and noisy observations, making it highly flexible and broadly applicable.

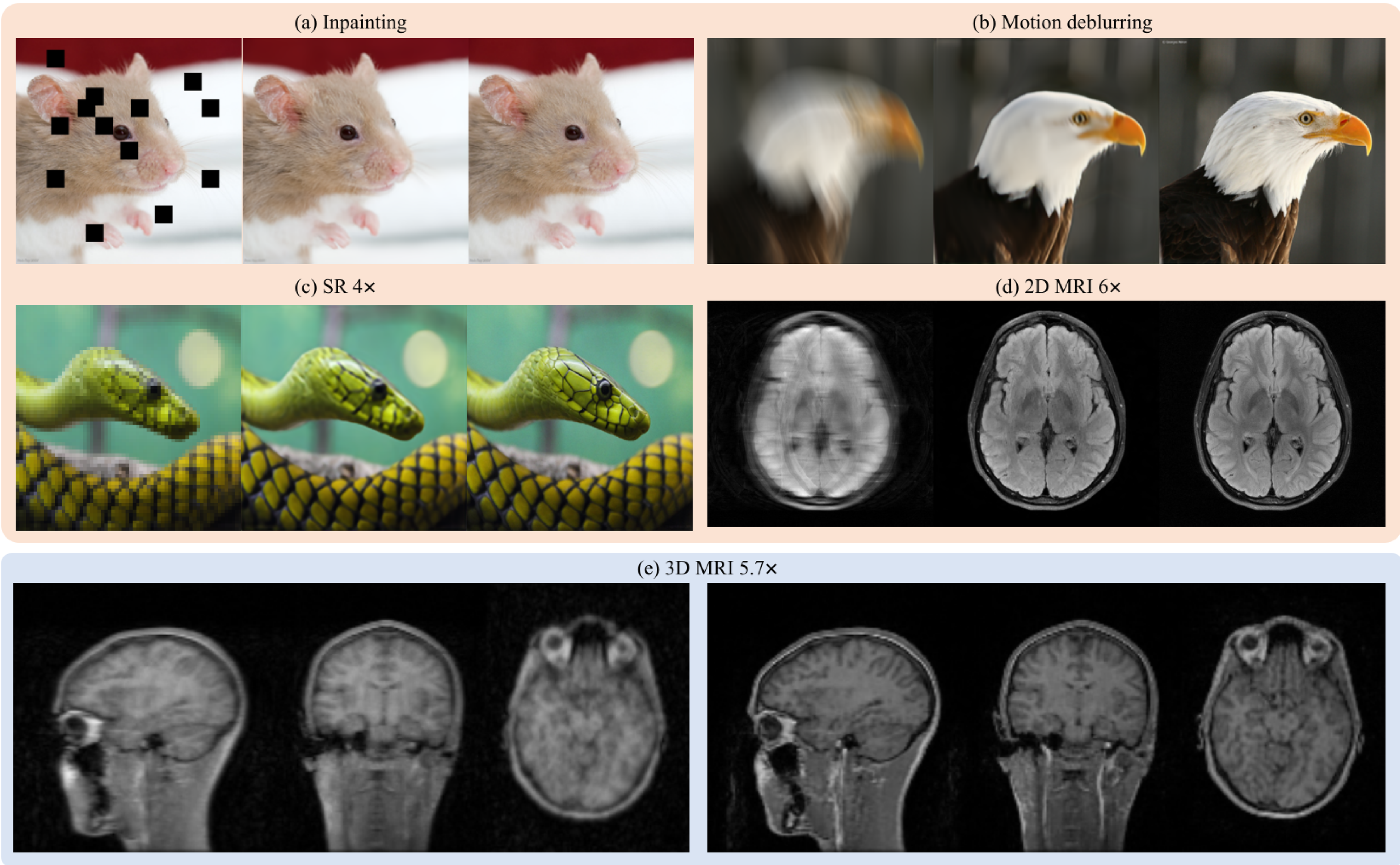


Figure 1. Demonstration of self-diffusion applied to various inverse problems across natural and medical imaging domains. (a) Image inpainting (b) Motion deblurring (c) 4× image super-resolution (SR) (d) 2D MRI reconstruction from 6× undersampled data (e) 3D MRI reconstruction from 5.7× undersampled data In each set, the leftmost image shows the degraded input, the middle shows the self-diffusion result, and the rightmost shows the ground-truth or reference.

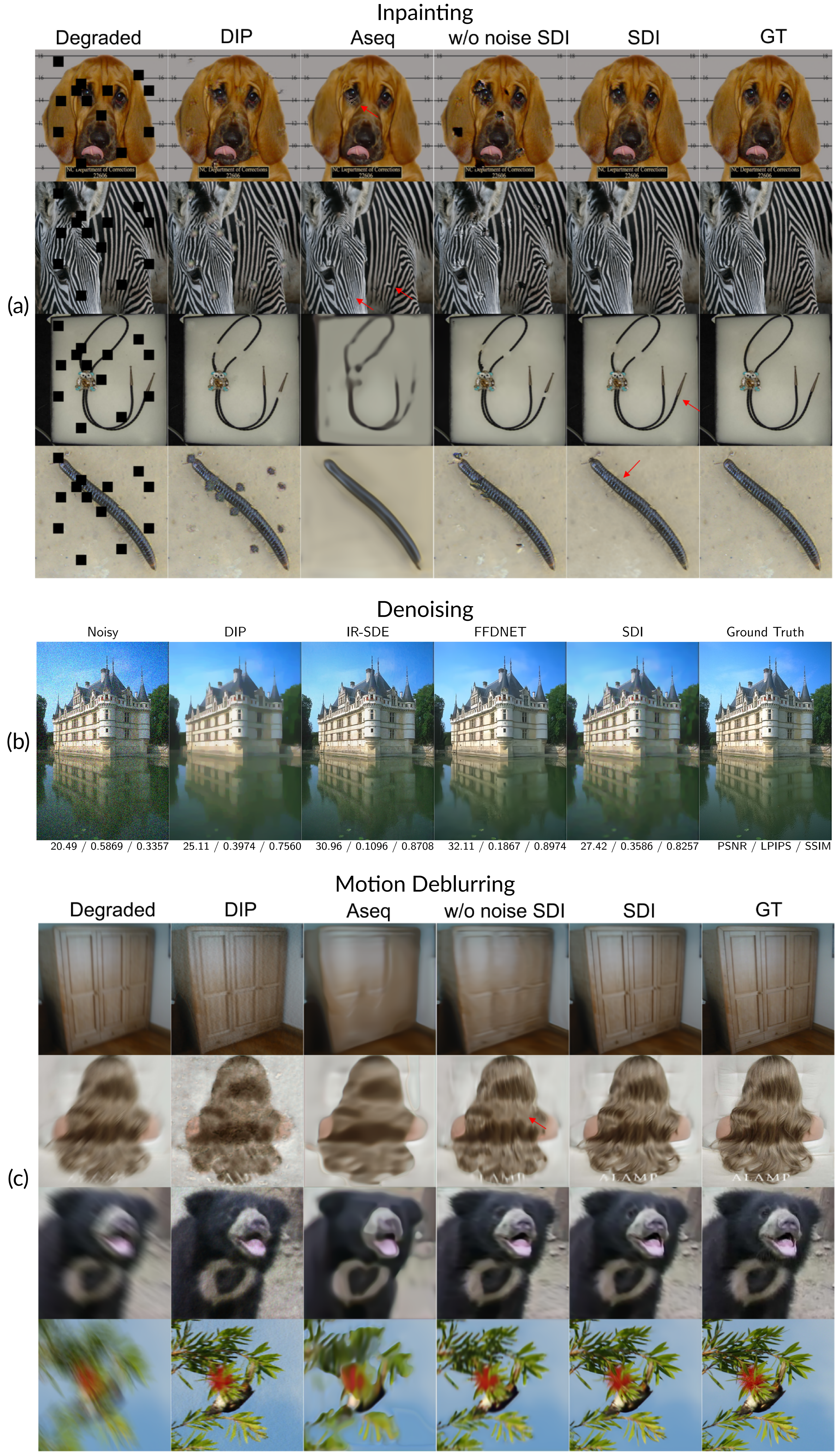
Method

To map the estimated noisy solution $\mathbf{x}_t = \mathbf{x}_t^{\text{true}} + \sigma(t)\epsilon_t$ to the true solution $\mathbf{x}^{\text{true}}$, a self-diffusion process trains a self-denoiser $D_{\theta_{t,k}}$ at each noise step $t = T-1, \dots, 0$ over $k = 0, \dots, K-1$ iterations to minimize the loss

$$L_{t,k} = \| \mathcal{A} D_{\theta_{t,k}}(\mathbf{x}_t^{\text{true}} + \sigma(t)\epsilon_t) - \mathbf{y} \|^2 , \tag{1}$$

where $\mathbf{x}_t^{\text{true}}$ is the estimate of $\mathbf{x}^{\text{true}}$ within noise step t, initialized as $\mathbf{x}_T^{\text{true}} = \epsilon_0$, with $\epsilon_0 \sim \mathcal{N}(0, I)$. The noise $\epsilon_t \sim \mathcal{N}(0, I)$ is sampled once at the start of noise timestep t and is fixed across all K iterations within that timestep. It is then resampled for the next noise step $t-1$. The noise schedule is $\sigma_t = \sqrt{1-\bar{\alpha}_t}$, where $\bar{\alpha}_t = \prod_{i=0}^{t}(1-\beta_i)$ and $\beta_t = \beta_{\text{end}} + \frac{t}{T-1}(\beta_{\text{start}} - \beta_{\text{end}})$. After K iterations at each noise step, the self-denoiser $D_{\theta_{t,k}}$ produce the estimated solution $\mathbf{x}_{t-1}^{\text{true}}$ for the next noise step $t-1$. As the noise level $\sigma(t)$ decreases to zero, the predicted solution converges to the true solution $\mathbf{x}^{\text{true}}$

Low-level Vision



MRI Reconstruction

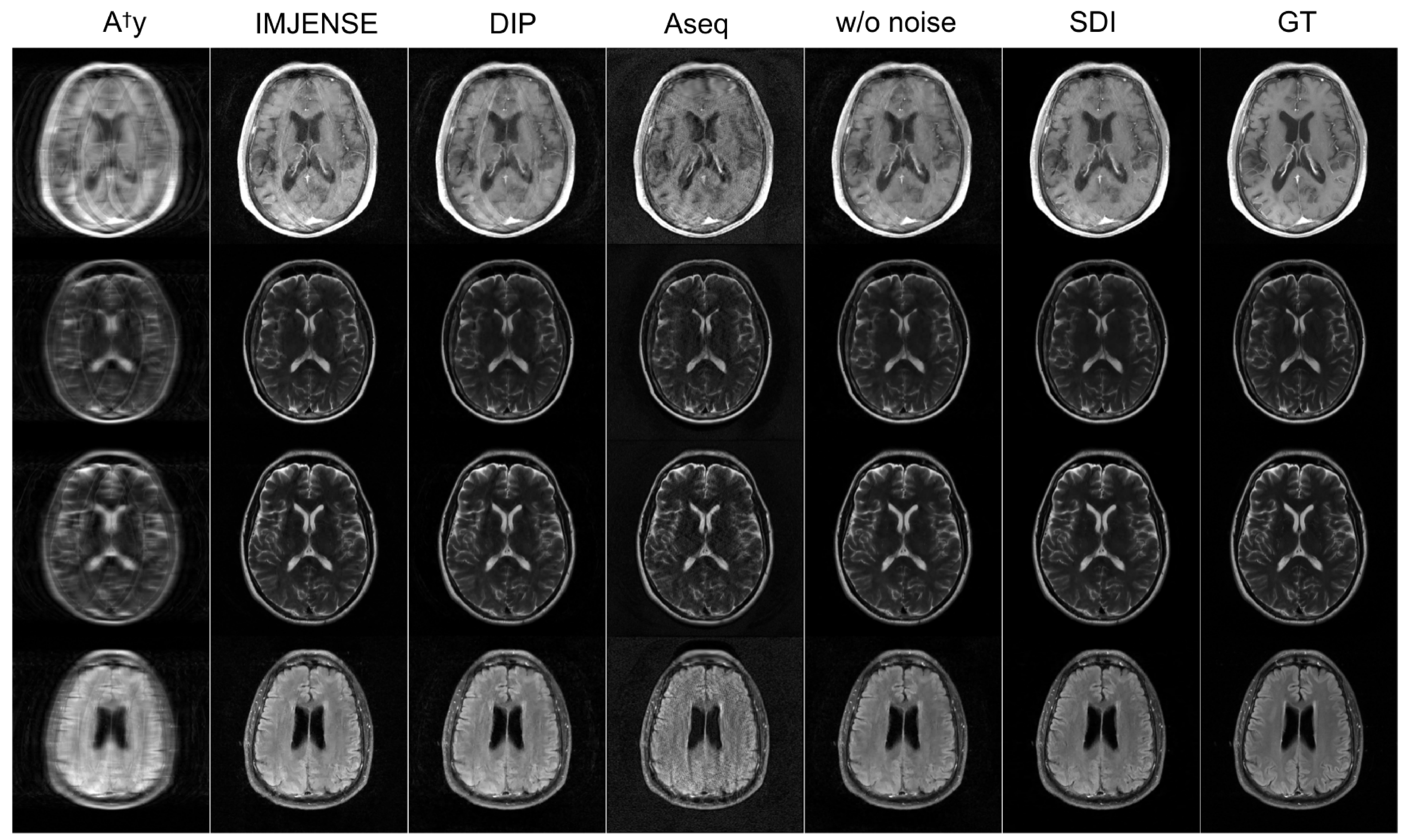


Figure 2. Reconstruction from 4× undersampled k-space with 20 ACS lines using different methods (IMJENSE, DIP, Aseq, w/o noise SDI, SDI).

Compressed Sensing

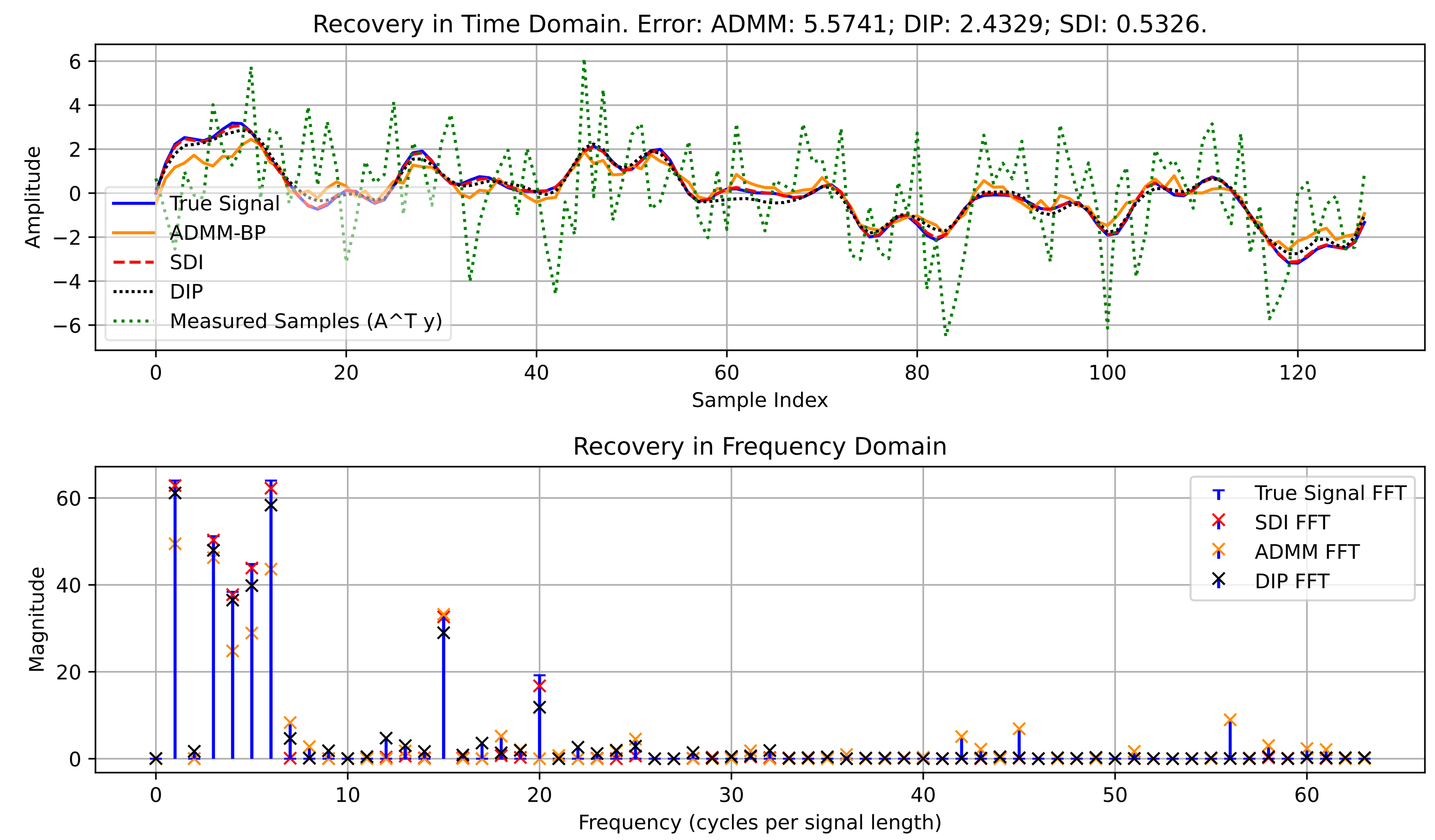


Figure 3. We apply the self-diffusion inference to recover a sparse 1D signal from compressed sensing measurements. The original signal is generated as a sum of sine wave with varying frequencies and amplitudes. The signal $\mathbf{x}$ is constructed as $\mathbf{x} = \sum_{(A,f)\in\mathcal{S}} A \cdot \sin\left(2\pi f \frac{t}{N}\right)$. The predefined set of amplitude-frequency pairs $\mathcal{S}$ is $\mathcal{S} = \{(1.0,1), (0.5,15), (0.3,20), (1.0,6), (0.8,3), (0.6,4), (0.7,5)\}$, and N is the total signal length, with $t \in \{0, 1, ..., N-1\}$. The signal is measured using a random Gaussian sampling matrix $\mathbf{A} \in \mathbb{R}^{m\times n}$, where $m \ll n$. In this experiment, $N = 128$, $m = 35$

Conclusion

Self-diffusion introduces a simple yet powerful framework for solving inverse problems without external data or pretrained models. By leveraging the intrinsic spectral bias of neural networks and regulating it through a structured noise schedule, it enables progressive, coarse-to-fine reconstruction.

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