ASSIGNMENT - I

DUAL NATURE OF MATTER

We know that any substance that occupies volume and has mass is called matter. We can classify matter in to particle and wave. If we have to study about a particle, we dicuss about displacement, velocity, momentum and energy. Displacement is discussed when a body moves from one place to another place. Velocity is studied when a body actually moves from rest. Actually everything depends on energy.

If we move from a place that has lower altitude and to a place that has higher altitude, it is said that we actually gained potential energy. Similarly, when a body moves from '0' velocity to a velocity 'v' it is said to gain kinetic energy of $\frac{1}{2}mv^2$. When a body is moving with a velocity 'v' it inturn has momentum 'mv'.

Similarly when we have to discuss about a wave, we talk about /discuss about amplitude, frequency, wavelength, time period, intensity and velocity. In order to discuss these both parameters simultaneously, Planck gave a theory. It is Planck's Quantum Theory. It states that, the energy is either absorbed (or) released in the form of small energy packets named 'quanta'. These are small units of energy. Energy is quantised according to the Quantum theory

But inturn, these packets were a form of energy which contains photons as smallest unit of energy. Photons are actually the particles that move with the speed of light. When a body moves with the speed of light it gains mass. According to the Quantum theory, the energy of the photon is directly proportional to the frequency of radiation. As we can see the frequency and velocity, mass on a single piece of matter, we can say that a body will consist of particle and wave nature simultaneously.

*** De Broglies Hypothesis :-**

Extending the idea of Planck, de-Broglie stated that, each and every substance that exists in the universe will have both particle and wave nature. He stated that the wavelength of the wave exhibited by a moving body is inversely proportional to its momentum. So, as faster the body moves, as lesser the wavelength becomes.

$$\lambda \propto \frac{1}{mv}$$

After removing the proportionality,

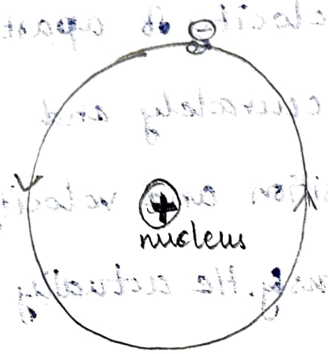
$$\lambda = \frac{h}{mv}$$

where h is plancks constant

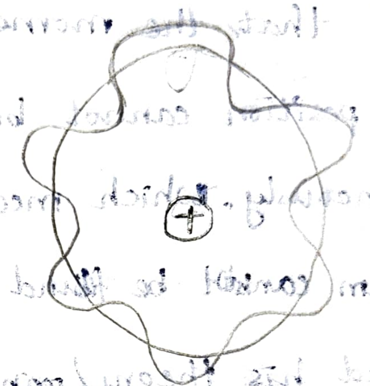
$$h = 6.636 \times 10^{-34} Js.$$

$$\lambda = \frac{h}{mv}$$

This de-Broglies hypothesis proves that the electron does not only moves in a circular path around the nucleus but also exhibits a continuous wave that is concentrated at its nucleus as centre.



Bohr's Theory



de-Broglies Hypothesis

→ derivations based on de-Broglies hypothesis.

WKT,

$$\frac{1}{2}mv^2 = KE$$

$$mv^2 = 2KE$$

$$m^2v^2 = 2mKE$$

$$(mv)^2 = 2mKE$$

from the hypothesis,

$$\lambda = \frac{h}{mv}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mKE}}}$$ → relation b/w wave length and kinetic energy of the particle.

* Heisenberg's Uncertainity Principle :

If we need to completely study a particle we need to find its existence and we should be able to track its movements. So, in this direction, Heisenberg expressed that the momentum (or) velocity of a particle and its position cannot be found accurately and simultaneously. Which means, the position and velocity/momentum cannot be found simultaneously. He actually explained his theory/principle based on the equation

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}.$$

> where Δx speaks about the uncertainity in position
> where Δp and Δv speaks about the uncertainity in momentum and velocity respectively.

> If the position of particle is found accurately Δx becomes zero.

if $\Delta x = 0$;

from the equation,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$(0)\Delta p \geq \frac{h}{4\pi}$$

$$\Delta p \geq \frac{h}{4\pi} \times \frac{1}{0}.$$

which makes Δp infinity, means the inaccuracy in calculating momentum is very high.

> If the velocity /momentum of the body /particle is found accurately, $\Delta v / \Delta p$ becomes zero.

If $\Delta p = 0$,

from the equation,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}.$$

$$\Delta x (0) \geq \frac{h}{4\pi}.$$

$$\Delta x \geq \frac{h}{4\pi} \times \frac{1}{0}.$$

which makes Δx infinity, means, the inaccuracy in finding the position of body /particle is more/ high.

PLANCKS' QUANTUM THEORY

* Postulates: There are 5 postulates of the Plancks' Quantum Theory. They are,

1. Energy is relased due to the vibration of charged particles.

2. The energy is either emitted (or) absorbed in the form of small packets of energy called 'Quanta'.

3. The propogation of Quanta is in the form of a wave.

4. The energy that is absorbed (or) released is always quantised. Energy is the integral multiples of $h\nu$.

   $E = nh\nu$

   where $n = 1, 2, 3, \ldots n$ (integral values only).

5. The energy of a photon is directly proportional to the frequency of that particular particle/photon.

   $E \propto \nu$

   $\boxed{E = h\nu} \rightarrow$ for 1 photon

   $\boxed{E = nh\nu} \rightarrow$ for n photons

   where, $n = 1, 2, 3, 4, 5, \ldots n$ (integers only).

Units of plancks' constant $\Rightarrow$ $E = h\nu$

$J = h s^{-1}$

$\boxed{[h] = J s^{+1}}$

'Js' is the unit of the constant.

* Black body is actually a body with surface that absorbs and emits all type of radiations.

* PHYSICAL SIGNIFICANCE OF ψ and ψ^2

ψ is the amplitude (maximum displacement) of the wave and is called the wave function. It takes a positive value above the axis and negative value below the axis and zero when intercepting the axes. ψ is a state function but doesnot have any physical significance it only represents the amplitude of electron wave. But, the square of ψ at a point gives the probability function, which describes the probability of finding an electron around the nucleus. The point at which ψ^2 has maximum value is known as an orbital. The value of ψ^2 is always +ve. From the value of ψ^2 at different points with in the atom, it is possible to find out the orbital of the electron. Evidently ψ^2 can be interpreted as probability density. If 'dv' is the volume of small region, then $\psi^2 dv$ gives the probability of finding the electron in the region having a volume 'dv'.

* **Schrodinger's Wave Equation:**

In 1926, Erwin Schrodinger developed a new model of the atom. He incorporated the idea of quantisation and the conclusion of de-Broglies and Heisenberg's uncertainty principle in this model. In the Schrodinger wave equation, the electrons are treated as a wave motion in three dimensional space around the nucleus having nodes and quantised energies.

From the classical wave mechanics, if ψ (psi) is the amplitude or wave function of a wave moving in a three dimensional space with velocity of 'v' and frequency 'ν', the wave equation is,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + 4\pi^2 v^2 \psi^2 = 0 \quad \text{.....} (1)$$

According to de-Broglies' hypothesis,

$$\lambda = \frac{h}{mv} \qquad \text{where } v = \nu\lambda$$

$$\frac{1}{\lambda} = \frac{\nu}{v}$$

$$\frac{mv}{h} = \frac{1}{\lambda}$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + 4\pi^2 \left(\frac{mv}{h}\right)^2 \psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + 4\pi^2 \left(\frac{m^2 v^2}{h^2}\right) \psi = 0.$$

WKT,

$$KE = \frac{1}{2}mv^2$$

$$mv^2 = 2KE$$

$$m^2v^2 = 2mKE$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{4\pi^2(2mKE)}{h^2}\psi = 0.$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{8\pi^2 mKE}{h^2}\psi = 0.$$

WKT,

$$TE = KE + PE.$$

$$E = KE + V$$

$$\boxed{KE = E - V}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + 8\pi^2 m(E-V)\psi = 0.$$

Or, simply $\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0.$

This equation is known as schrodingers wave equation.

where,

$\nabla^2 = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$ is called the Laplace operator.

Here, m = mass of the electron

V = potential energy (PE)

E = total energy

h = Plancks' constant

x, y, z are the cartesian co-ordinates in the three dimensional space.