Practical: Investigating the t-test with simulated data

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20 octobre 2023

The aim of this practical is to investigate the properties of the two-sample t-test – in particular, its false-positive rate (under H0) and power (under H1) – using simulated data that may violate some of its assumptions.

1. As a starting point, use the function `rnorm()` to draw two datasets of size $n_1 = n_2 = 10$ from a standard normal distribution, and use the function `t.test()` to test the null-hypothesis of equal means ($\mu_1 = \mu_2$).
2. Now repeat this procedure $r = 10000$ times and plot a histogram of the resulting p-values. What do you observe ?
3. Plot the same kind of histogram when the two samples are drawn from distributions with different means ($\mu_1 = -\delta/2$, $\mu_2 = \delta/2$), while retaining equal variances ($\sigma_1^2 = \sigma_2^2 = 1$). It might be a good idea to write a `function` that does the iteration and takes the means and variances as parameters.
4. Using the same kind of data, plot the power (probability of true positives) of the test as a function of δ. *Bonus* : Compare the results to the theoretical power curve derived from the t-distribution.
5. Assuming equal variances, compare the power of the two versions of the t-test (for equal and unequal variances, respectively). What happens to the difference as sample sizes increase ?
6. Assuming unequal variances, compare the false-positive rates of the two versions of the t-test (e.g., plot the false-positive rate as a function of σ_2 while keeping σ_1 constant).
7. Returning to equal variances, compare the power and false-positive rates of the t-test and its non-parametric alternative, the Wilcoxon rank-sum (or Mann-Whitney U) test (`wilcox.test()`).