
[Cheng P.3-26] Determine the boundary conditions for the tangential and normal components of \mathbf{P} at an interface between two perfect dielectric media with dielectric constants ε_{r1} and ε_{r2} .

Solution: We know that

$$\mathbf{P}_{t/n} = \varepsilon_0(1 - \varepsilon_r)\mathbf{E}_{t/n},$$

for both the tangential and normal components. Therefore, we simply use this relation with the boundary conditions that we already know for the tangential and normal components of \mathbf{E} . This yields

$$\begin{aligned}\mathbf{E}_{t1} &= \mathbf{E}_{t2} \\ \frac{1}{\varepsilon_{r1} - 1}\mathbf{P}_{t1} &= \frac{1}{\varepsilon_{r2} - 1}\mathbf{P}_{t2} \\ \varepsilon_{r1}\mathbf{E}_{n1} &= \varepsilon_{r2}\mathbf{E}_{n2} \\ \frac{\varepsilon_{r1}}{\varepsilon_{r1} - 1}\mathbf{P}_{n1} &= \frac{\varepsilon_{r2}}{\varepsilon_{r2} - 1}\mathbf{P}_{n2}.\end{aligned}$$

Answer:

$$\begin{aligned}\frac{1}{\varepsilon_{r1} - 1}\mathbf{P}_{t1} &= \frac{1}{\varepsilon_{r2} - 1}\mathbf{P}_{t2} \\ \frac{\varepsilon_{r1}}{\varepsilon_{r1} - 1}\mathbf{P}_{n1} &= \frac{\varepsilon_{r2}}{\varepsilon_{r2} - 1}\mathbf{P}_{n2}.\end{aligned}$$