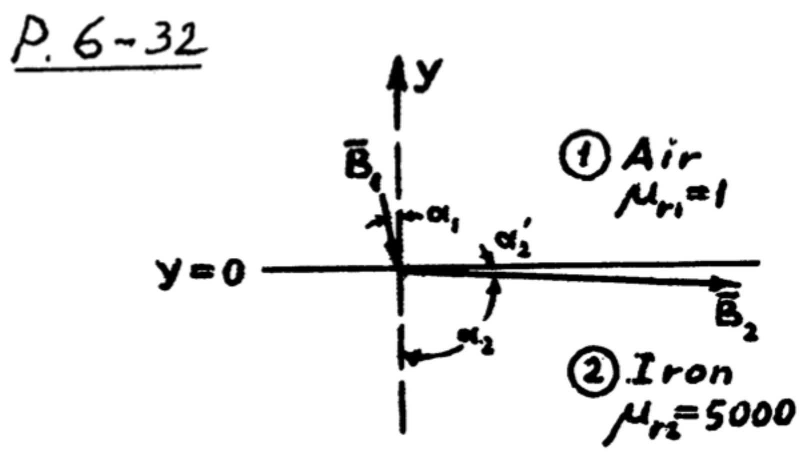
[Cheng P.6-32] Consider a plane boundary ($y = 0$) between air (region 1, $\mu_{r1} = 1$) and iron (region 2, $\mu_{r2} = 5000$).

(a) Assuming $\mathbf{B}_1 = \mathbf{a}_x 0.5 - \mathbf{a}_y 10$(mT), find $\mathbf{B}_2$ and the angle $\mathbf{B}_2$ makes with the interface.

(b) Assuming $\mathbf{B}_2 = \mathbf{a}_x 10 + \mathbf{a}_y 0.5$(mT), find $\mathbf{B}_1$ and the angle $\mathbf{B}_1$ makes with the normal to the interface.

Solution: The problem has the structure shown in the diagram below.



(a) The magnetic flux densities in both regions are given by

$$\mathbf{B}_1 = \mathbf{a}_x 0.5 - \mathbf{a}_y 10 \text{ (mT)}$$
$$\mathbf{B}_2 = \mathbf{a}_x B_{2x} - \mathbf{a}_y B_{2y}.$$

From the boundary conditions we have

$$H_{1x} = H_{2x} \to \frac{0.5}{\mu_0} = \frac{B_{2x}}{5000\mu_0}$$
$$B_{2x} = 2500 \text{ (mT)}$$
$$B_{1y} = B_{2y}$$
$$B_{2y} = -10 \text{ (mT)}$$

Putting them together we get

$$\mathbf{B}_2 = \mathbf{a}_x 2500 - \mathbf{a}_y 10 \text{ (mT)}$$

To solve for the angle we use the first boundary condition $H_{1x} = H_{2x} \to \mu_2 B_{1x} = \mu_1 B_{2x}$. Now if we divide both sides by their y-components (which are equal from the other set of boundary conditions) we get $\frac{B_{1x}}{B_{1y}} = \frac{\mu_1}{\mu_2}\frac{B_{2x}}{B_{2y}}$. Finally using $\tan\alpha = \frac{B_x}{B_y}$ we get

$$\tan\alpha_2 = \frac{\mu_2}{\mu_1}\tan\alpha_1 = 5000\frac{B_{1x}}{B_{1y}} = 250$$
$$\alpha_2 = 89.8^\circ,\ \alpha_2' = 0.2^\circ.$$

(b) The magnetic flux densities in both regions are given by

$$\mathbf{B}_2 = \mathbf{a}_x 10 + \mathbf{a}_y 0.5 \text{ (mT)}$$
$$\mathbf{B}_1 = \mathbf{a}_x B_{1x} + \mathbf{a}_y B_{1y}.$$

As before, from the boundary conditions we have

$$H_{1x} = H_{2x} \rightarrow \frac{B_{1x}}{\mu_0} = \frac{10}{5000\mu_0}$$
$$B_{1x} = 0.002 \text{ (mT)}$$
$$B_{1y} = B_{2y}$$
$$B_{2y} = 0.5 \text{ (mT)}$$

Putting them together we get

$$\mathbf{B}_2 = \mathbf{a}0.002 + \mathbf{a}_y 0.5 \text{ (mT)}$$

To find the angle we use the same process as part (a)

$$\alpha_1 = \tan^{-1}\frac{B_{1x}}{B_{1y}} \approx \frac{0.002}{0.5} = 0.23^\circ$$

Answer:

(a)

$$\mathbf{B_2} = a_x 2500 - a_y 10 \text{ (mT)}$$
$$\alpha_2 = 89.8^o$$

(b)

$$\mathbf{B_1} = a_x 0.002 + a_y 0.5 \text{ (mT)}$$
$$\alpha_1 = 0.23^o$$