A very (infinitely) long homogeneous dielectric cylinder, of radius a and relative dielectric permittivity ε_r, is uniformly charged with free charge density ρ throughout its volume. The cylinder is surrounded by air.

(a) Calculate the voltage between the axis and the surface of the cylinder.
(b) Find the bound charge distribution in the cylinder.

Solution:

(a) Apply Gauss' law on a cylindrical surface to find $\mathbf{D}$ and calculate $\mathbf{E} = \mathbf{D}/\varepsilon$. Then, apply the standard formula for V to find

$$V = \frac{\rho a^2}{4\varepsilon_0 \varepsilon_r}\,.$$

(b)

$$\rho_{p,v} = \frac{\rho(\varepsilon_r - 1)}{\varepsilon_r}\,, \quad 0 \le r < a$$
$$\rho_{p,s}(r = a) = \frac{\rho(\varepsilon_r - 1)a}{2\varepsilon_r}\,.$$

Answer:

(a)

$$V = \frac{\rho a^2}{4\varepsilon_0 \varepsilon_r}\,.$$

(b)

$$\rho_{p,v} = \frac{\rho(\varepsilon_r - 1)}{\varepsilon_r}\,, \quad 0 \le r < a$$
$$\rho_{p,s}(r = a) = \frac{\rho(\varepsilon_r - 1)a}{2\varepsilon_r}\,.$$