Goal: A charge Q is distributed over the wall of a circular tube of radius b and height h. The tube sits on xy-plane with its axis coinciding with z-axis. Determine V and $\mathbf{E}$ along the axis of the tube.

Steps:

1. Choose a coordinate system for this problem.

   Solution: Cylindrical (or Cartesian)

2. Determine the charge density.

   Solution:

$$\rho_s = \frac{Q}{2\pi b h}$$

3. Which components of $\mathbf{E}$ are non-zero?

   Solution: The z-component.

4. What is the distance between source and observation point $|\mathbf{R} - \mathbf{R}'|$? What is the differential surface element dS'?

   Solution:

$$\begin{aligned} \left|\mathbf{R} - \mathbf{R}'\right| &= \left((z - z')^2 + (x')^2 + (y')^2\right)^{1/2} \\ &= \left((z - z')^2 + b^2\right)^{1/2} . \\ dS' &= b\, dz\, d\theta'. \end{aligned}$$

5. Compute the potential V.

   Solution:

$$V = \frac{b\rho_s}{2\varepsilon_0} \ln \frac{\sqrt{b^2 + z^2} + z}{\sqrt{b^2 + (z-h)^2} + z - h}$$

6. Using the potential computed in part (5) determine the electric field $\mathbf{E}$. *Solution:*

$$E = -\nabla V = \mathbf{a}_z \frac{b\rho_s}{2\varepsilon_0} \left[\frac{1}{\sqrt{b^2 + (z-h)^2}} - \frac{1}{\sqrt{b^2 + z^2}} \right]$$

Answer:

(a)

$$V = \frac{b\rho_s}{2\varepsilon_0} \ln \frac{\sqrt{b^2 + z^2} + z}{\sqrt{b^2 + (z-h)^2} + z - h}$$

(b)

$$E = \mathbf{a}_z \frac{b\rho_s}{2\varepsilon_0} \left[\frac{1}{\sqrt{b^2 + (z-h)^2}} - \frac{1}{\sqrt{b^2 + z^2}} \right]$$