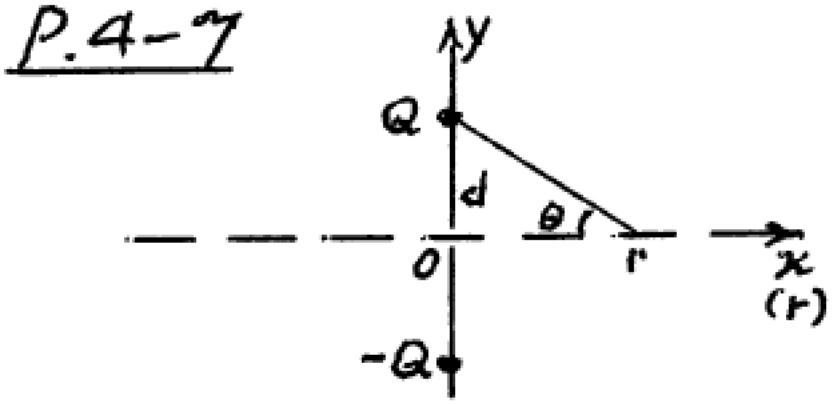
[Cheng P.4-7] A point charge Q exists at a distance d above a large grounded conducting plane. Determine

(a) the surface charge density ρ_s,

(b) the total charge induced on the conducting plane.

Solution:



With reference to the diagram above,

$$\begin{aligned}\mathbf{E}\,|_{y=0} &= -\mathbf{a}_y \frac{Q}{4\pi\varepsilon R^2}(2\sin\theta)\\ &= -\mathbf{a}_y \frac{Qd}{2\pi\varepsilon(d^2+r^2)^{3/2}}.\end{aligned}$$

Now we can find the surface charge density ρ_s and total charge Q_{tot} on the conducting plate.

(a)

$$\begin{aligned}\rho_s &= \mathbf{a}_y \cdot \varepsilon \mathbf{E}\,|_{y=0}\\ \rho_s &= -\frac{Qd}{2\pi(d^2+r^2)^{\frac{3}{2}}}\end{aligned}$$

(b)

$$\begin{aligned}Q_{\text{tot}} &= \int_0^\infty \rho_s 2\pi r dr\\ &= -Q\end{aligned}$$

Answer:

(a)

$$\rho_s = -\frac{Qd}{2\pi(d^2+r^2)^{\frac{3}{2}}}$$

(b) $Q_{\text{tot}} = -Q$