
[Cheng P3.40] Calculate the amount of electrostatic energy of a uniform sphere of charge with radius b and volume charge density ρ stored in the following regions:

- (a) inside the sphere,
- (b) outside the sphere.

Check your results with those of Example 3-22.

Solution:

$$\mathbf{D} = \begin{cases} \mathbf{a}_R \frac{R}{3} \rho & R < b \\ \mathbf{a}_R \frac{b^3 \rho}{3R^2} & R > b \end{cases}$$
$$\mathbf{E} = \frac{D}{\epsilon_0}$$

a)

$$W_i = \frac{1}{2} \int_V \mathbf{D} \cdot \mathbf{E} dV = \frac{1}{2} \int_0^b \frac{1}{\epsilon_0} \left(\frac{R}{3} \rho \right)^2 4\pi R^2 dR = \frac{2\pi b^5 \rho^2}{45\epsilon_0}$$

b)

$$W_0 = \frac{1}{2} \int_b^\infty \frac{1}{\epsilon_0} \left(\frac{b^3 \rho}{3R^2} \right)^2 4\pi R^2 dR = \frac{2\pi b^5 \rho^2}{9\epsilon_0}$$

Total

$$W = W_i + W_0 = \frac{4\pi b^5 \rho^2}{15\epsilon_0}$$

Answer:

(a)

$$W_i = \frac{2\pi b^5 \rho^2}{45\epsilon_0}$$

(b)

$$W_0 = \frac{2\pi b^5 \rho^2}{9\epsilon_0}$$

Total

$$W = W_i + W_0 = \frac{4\pi b^5 \rho^2}{15\epsilon_0}$$