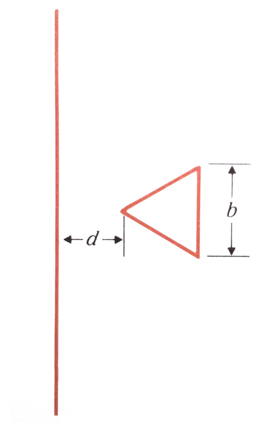
Goal: Determine the mutual inductance between a very long, straight wire and a conducting equilateral triangle loop, as shown in the figure below.



Steps:

1. Define the mutual inductance L_{12}.

   Solution:
   $$L_{12} = \frac{\Phi_{12}}{i_1}$$
   where Φ_{12} is the total magnetic flux through the second loop due to the B-field generated by i_1 in the first loop.

2. What is the B-field generated by the line current?

   Solution:
   $$\mathbf{B} = \mathbf{a}_\phi \frac{\mu_o I}{2\pi r}$$

3. Find the magnetic flux through the triangle due to the B-field generated by the line current.

   Solution:
   $$\begin{aligned}\Phi_{12} &= \int_S \mathbf{B}\cdot d\mathbf{s} = \int_S B_\phi \frac{2}{\sqrt{3}}(r-d)\,dr = \frac{\mu_o I}{\pi\sqrt{3}}\int_d^{d+\frac{\sqrt{3}}{2}b}\left(\frac{r-d}{r}\right)dr \\ &= \frac{\mu_o I}{\pi\sqrt{3}}\left[\frac{\sqrt{3}}{2}b - d\ln\left(1+\frac{\sqrt{3}b}{2d}\right)\right]\end{aligned}$$

4. What is the mutual inductance?

   Solution:
   $$L_{12} = \frac{\mu_o}{\pi\sqrt{3}}\left[\frac{\sqrt{3}}{2}b - d\ln\left(1+\frac{\sqrt{3}b}{2d}\right)\right]$$

Answer:

$$L_{12} = \frac{\mu_o}{\pi\sqrt{3}}\left[\frac{\sqrt{3}}{2}b - d\ln\left(1 + \frac{\sqrt{3}b}{2d}\right)\right]$$