Goal: A current I flows in a $w \times w$ square loop as shown in Fig. a) below. Find the magnetic flux density at the off-center point $P(w/4, w/2)$.

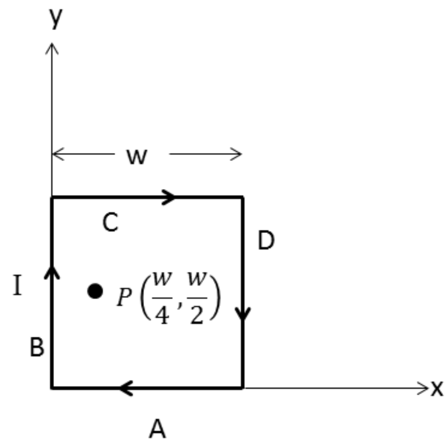

Fig a)

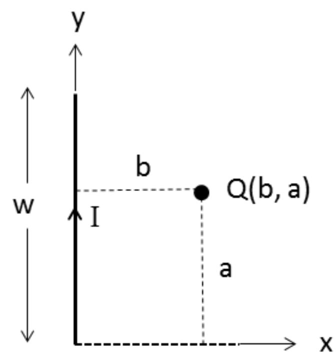

Fig b)

Steps:

1. The total magnetic flux density at point $P(w/4, w/2)$ is the sum of the contributions from each segment of the loop. Firstly, we consider the magnetic flux density at an arbitrary point $Q(b, a)$ due to a line current source of length w, as shown in Fig b). What is the source position vector $\mathbf{R}'$ for this configuration?
Solution:

$$\mathbf{R}' = y'\mathbf{a}_y$$

2. Determine the observation vector $\mathbf{R}$.
Solution:

$$\mathbf{R} = b\mathbf{a}_x + a\mathbf{a}_y$$

3. Determine the differential length vector $d\mathbf{l}'$.
Solution:

$$d\mathbf{l}' = dy'\mathbf{a}_y$$

4. Determine the differential magnetic flux density.
Solution:

$$\begin{aligned}
\mathbf{R} - \mathbf{R}' &= b\mathbf{a}_x + a\mathbf{a}_y \\
d\mathbf{B} &= \frac{\mu_0 I}{4\pi}\frac{d\mathbf{l}' \times (\mathbf{R} - \mathbf{R}')}{|\mathbf{R} - \mathbf{R}'|^3} \\
&= \frac{\mu_0 I}{4\pi}\frac{dy'\mathbf{a}_y \times (b\mathbf{a}_x + (a - y')\mathbf{a}_y}{(b^2 + (a - y')^2)^{3/2}} \\
&= -\mathbf{a}_z\frac{\mu_0 I}{4\pi}\frac{bdy'}{(b^2 + (a - y')^2)^{3/2}}
\end{aligned}$$

5. Integrate.
Solution:

$$\begin{aligned}
\mathbf{B} &= -\mathbf{a}_z\frac{\mu_0 I}{4\pi}\int_0^w \frac{b}{(b^2 + (a - y')^2)^{3/2}}dy' \\
&= -\mathbf{a}_z\frac{\mu_0 I}{4\pi b}\left[\frac{w - a}{((a - w)^2 + b^2)^{1/2}} + \frac{a}{(a^2 + b^2)^{1/2}}\right]
\end{aligned}$$

6. Use the result in part 6) to compute magnetic flux density at point P due to:
Solution:

- segment A (as marked in Fig. a): For $a = 3/4w$, and $b = w/2$

$$\mathbf{B}_A = -\mathbf{a}_z\frac{\mu_0 I}{\pi w}[0.6396]$$

- segment B : For $a = w/2$ and $b = w/4$

$$\mathbf{B}_B = -\mathbf{a}_z\frac{\mu_0 I}{\pi w}[1.7889]$$

- segment C: For $a = w/4$ and $b = w/2$

$$\mathbf{B}_C = -\mathbf{a}_z\frac{\mu_0 I}{\pi w}[0.6397]$$

- segment D: For $a = w/2$ and $b = 3w/4$

$$\mathbf{B}_D = -\mathbf{a}_z\frac{\mu_0 I}{\pi w}[0.3698]$$

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7. What is the total magnetic flux density at point $P(w/4, w/2)$?
Solution:

$$\begin{aligned}\mathbf{B} &= \mathbf{B}_A + \mathbf{B}_B + \mathbf{B}_C + \mathbf{B}_D \\ &= -\mathbf{a}_z \frac{\mu_0 I}{\pi w} [3.44]\end{aligned}$$

Answer:

$$\mathbf{B} = -\mathbf{a}_z \frac{\mu_0 I}{\pi w} [3.44]$$