Goal: The radius of the core and the inner radius of the outer conductor of a very long coaxial transmission line are r_i and r_o, respectively. The space between the conductors is filled with two coaxial layers of dielectrics. The dielectric constants of the dielectrics are ε_{r1} for $r_i < r < b$ and ε_{r2} for $b < r < r_o$. Determine its capacitance per unit length.

Steps:

1. Using Gauss's Law, determine an expression for the electric fields in both dielectrics.

   Solution:

$$\begin{aligned} \mathbf{E}_1 &= \mathbf{a}_r \frac{\rho_l}{2\pi\varepsilon_o\varepsilon_{r1} r}, \quad \text{for} \quad r_i < r < b \\ \mathbf{E}_2 &= \mathbf{a}_r \frac{\rho_l}{2\pi\varepsilon_o\varepsilon_{r2} r}, \quad \text{for} \quad b < r < r_o \end{aligned}$$

2. Determine an expression of the voltage between the two conductors.

   Solution: We integrate along the electric fields, yielding

$$V = -\int_{r_o}^{r_i} \mathbf{E} \cdot d\mathbf{l} = \frac{\rho_l}{2\pi\varepsilon_o} \left(\frac{1}{\varepsilon_{r1}} \ln \frac{b}{r_i} + \frac{1}{\varepsilon_{r2}} \ln \frac{r_o}{b} \right)$$

3. Determine an expression for the capacitance per unit length.

   Solution:

$$C' = \frac{\rho_l}{V} = \frac{2\pi\varepsilon_o}{\frac{1}{\varepsilon_{r1}} \ln(b/r_i) + \frac{1}{\varepsilon_{r2}} \ln(r_o/b)} \text{ (F/m)}$$

Answer:

$$C' = \frac{2\pi\varepsilon_o}{\frac{1}{\varepsilon_{r1}} \ln(b/r_i) + \frac{1}{\varepsilon_{r2}} \ln(r_o/b)} \text{ (F/m)}$$