[Cheng P.6-45] Refer to Problem 6-39 and Fig. 6-49. Find the force on the circular loop that is exerted by the magnetic field due to an upward current I_1 in the long straight wire. The circular loop carries a current I_2 in the counterclockwise direction.

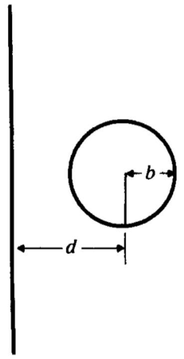
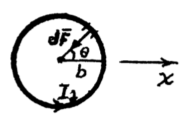


FIGURE 6–49
A long, straight wire and a conducting circular loop (Problem P.6–39).

Solution: The magnetic flux density $\mathbf{B}$ due to I_1 in the straight wire in the z-direction at an elemental arc $bd\theta$ on the circular loop is given by

$$\mathbf{B} = \mathbf{a}_\phi \frac{\mu_0 I_1}{2\pi(d + b\cos\theta)}$$

where θ,b, and d are shown in the sketch below.



$\bar{F}$ has no net y-component.

As hinted by the diagram, there will be no y-component to the force $\mathbf{F}_{\text{on loop}}$. This is because the force is given by $\mathbf{F}_{\text{on loop}} = I_2 \mathbf{d}\ell \times \mathbf{B}$. Using the right hand rule, the force in the $+y$-direction on the bottom half of the loop will cancel with the force in the $-y$-direction on the top half of the loop as the points are equidistant from the current I_1 generating $\mathbf{B}$. However, the force in the $+x$-direction on the left half of the loop is larger in than the force in the $-x$-direction on the right half of the loop as the points on the left half of the loop are closer to the current I_1 generating $\mathbf{B}$. As a result, we should expect a repulsive force. With that established, we can now perform the integration to find the force on the loop by finding the force on half the loop and doubling it.

$$\begin{aligned}
\mathbf{F}_{\text{on loop}} &= 2\left[-\mathbf{a}_x \int_0^\pi (I_2 b d\theta)\cos\theta \frac{\mu_0 I_1}{2\pi(d + b\cos\theta)}\right] \\
&= -\mathbf{a}_x \frac{\mu_0 I_1 I_2 b}{\pi} \int_0^\pi \frac{cos\theta}{d + b\cos\theta} d\theta \\
&= \mathbf{a}_x \mu_0 I_1 I_2 \left[\frac{1}{\sqrt{1 - (b/d)^2}} - 1\right] \text{ (Repulsive force).}
\end{aligned}$$

Answer:

$$\mathbf{F}_{\text{on loop}} = \mathbf{a}_x \mu_0 I_1 I_2 \left[\frac{1}{\sqrt{1-(b/d)^2}} - 1 \right] \text{ (Repulsive force).}$$