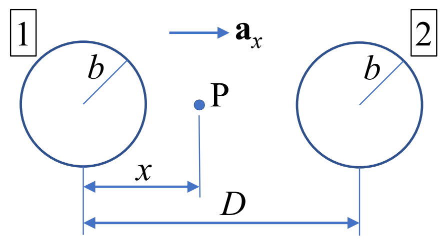
[Cheng P.3-47] The conductors of an isolated two-wire transmission line, each of radius b, are spaced at a distance D apart. Assuming $D >> b$ and a voltage V_0 between the lines, find the force per unit length on the lines.

Solution:



Assume line charge densities ρ_l and $-\rho_l$ on conductors 1 and 2, respectively. At point P,

$$
\begin{aligned}
\mathbf{E}_p &= \mathbf{E}_1 + \mathbf{E}_2 \\
&= \mathbf{a}_x \left[\frac{\rho_l}{2\pi\epsilon_0 x} + \frac{\rho_l}{2\pi\epsilon_0 (D-x)} \right] \quad (\mathrm{V/m}). \\
V_0 = V_1 - V_2 &= \int_b^{D-b} \mathbf{E}_p \cdot \mathbf{dx} \\
&= \frac{\rho_l}{2\pi\epsilon_0} \int_b^{D-b} \left(\frac{1}{x} + \frac{1}{D-x} \right) dx \\
&= \frac{\rho_l}{2\pi\epsilon_0} \left(\ln \frac{D-b}{b} - \ln \frac{b}{D-b} \right) \\
&= \frac{\rho_l}{\pi\epsilon_0} \ln \frac{D-b}{b} \\
&\simeq \frac{\rho_l}{\pi\epsilon_0} \ln \frac{D}{b} \quad (\mathrm{V}). \\
C' = \frac{\rho_l}{V_0} &= \frac{\pi\epsilon_0}{\ln(D/b)} \quad (\mathrm{F/m}). \\
F' = &= \nabla W_e = \mathbf{a}_x \frac{V_0}{2} \frac{\partial C'}{\partial D} \\
&= -\mathbf{a}_x \frac{\pi\epsilon_0 V_0^2}{2D[\ln(D/b)]^2} \quad \text{N/m (in the direction of decreasing D)}
\end{aligned}
$$

Answer:

(a)

$$V_o = V_1 - V_2 = \frac{\rho_l}{\pi\varepsilon_o} \ln \frac{D}{b} \quad \mathrm{V}.$$

(b)

$$C' = \frac{\pi\varepsilon_o}{\ln \frac{D}{b}} \quad \mathrm{F/m}.$$

(c)

$$F' = -a_x \, \frac{\pi \varepsilon_o V_o^2}{2D(\ln \frac{D}{b})^2} \quad \text{N/m.}$$