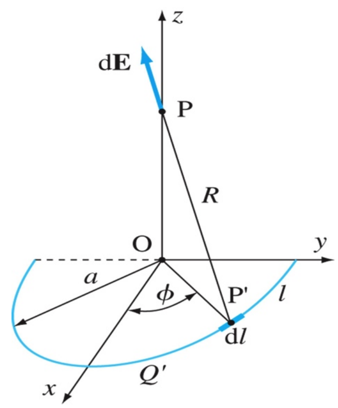
For the semi-circular line charge in the figure below, the electric field at an arbitrary point on the z-axis has an x and a z component (confirm). Find the z-component of the field from the potential $V(0,0,z)$. Can you find the x-component too using $V(0,0,z)$?



(a)

Solution:

(a) Due to symmetry about the z-axis, any $\mathbf{a}_y$ component contributions to the electric field on one side of the semi-circle will cancel with the equal but opposite in direction $\mathbf{a}_y$ components on the other side of the semi-circle. To solve for the field we approach it similar to the case of a ring. However, we just integrate ϕ over half the ring instead of the whole way around.

$$\mathbf{E} = \frac{Q' a}{a\varepsilon_0(\sqrt{z^2+a^2})^3}\Big(-\frac{a}{\pi}\mathbf{a}_x + \frac{z}{2}\mathbf{a}_z\Big)$$

(b) The potential $V(0,0,z)$ for the semi-circle can be found by integrating the contributions to the potential across small sections of the semi circle

$$V = \frac{1}{4\pi\varepsilon_0}\int_l \frac{Q' dl}{R}$$
$$V = \frac{Q'}{4\pi\varepsilon_0\sqrt{a^2+z^2}}\int_l dl$$
$$V = \frac{Q'}{4\pi\varepsilon_0\sqrt{a^2+z^2}}\pi a$$
$$V = \frac{Q' a}{4\varepsilon_0\sqrt{z^2+a^2}}.$$

To solve for the field, we simply make use of the relation between potential and electric field $\mathbf{E} = -\nabla V$.

$$E_z = -\frac{\mathrm{d}}{\mathrm{d}z}V$$
$$E_z = -\frac{\mathrm{d}}{\mathrm{d}z}\frac{Q' a}{4\varepsilon_0\sqrt{z^2+a^2}}$$
$$E_z = \frac{Q' a}{a\varepsilon_0(\sqrt{z^2+a^2})^3}\frac{z}{2}$$

(c) Unfortunately, we cannot use the potential at $V(0,0,z)$ and the gradient to solve for the electric field in the $\mathbf{a}_x$ direction as before (i.e. using $E_x = -\frac{\mathrm{d}}{\mathrm{d}x}V$). This is because we would also need the expression for the potential off of the z-axis, which is not provided.

Answer:

(a) Due to symmetry about the z-axis there will be no $\mathbf{a}_y$ component in the electric field intensity.

(b) $E_z = \frac{Q' a}{a\varepsilon_0(\sqrt{z^2+a^2})^3}\frac{z}{2}$

(c) No