[Cheng P.6-14] Find the total magnetic flux through a circular toroid with a rectangular cross section of height h. The inner and outer radii of the toroid are a and b, respectively. A current I flows in N turns of closely wound wire around the toroid. Determine the percentage of error if the flux is found by multiplying the cross-sectional area by the flux density at the mean radius.

Solution:

$$B_\phi = \frac{\mu_0 NI}{2\pi r},$$

$$\Phi = \int_S B_\phi \, ds = \frac{\mu_0 NI}{2\pi} \int_a^b \frac{h}{r} \, dr = \frac{\mu_0 NIh}{2\pi} \ln \frac{b}{a}$$

If B_ϕ at $r = \frac{a+b}{2}$ is used,

$$\Phi' = \frac{\mu_0 NIh}{\pi} \left(\frac{b-a}{b+a} \right),$$

$$\text{Percent error} = \frac{\Phi' - \Phi}{\Phi} \times 100$$

$$= \left[\frac{2(b-a)}{(b+a)\ln(b/a)} - 1 \right] \times 100\%.$$

Answer:

$$\phi = \frac{\mu_o NIh}{2\pi} \ln \frac{b}{a}$$

$$\% \text{ error} = \left[\frac{2(b-a)}{(b+a)\ln \frac{b}{a}} - 1 \right] \times 100$$