
Goal: A very large slab of material of thickness d lies perpendicularly to a uniform magnetic field of intensity $\mathbf{H}_0 = \mathbf{a}_z H_0$. Find the magnetic field intensity inside the dielectric slab if it has a permeability μ or if it is a permanent magnet having a magnetization vector $\mathbf{M}_i = \mathbf{a}_z M_i$.

Steps:

1. State the magnetostatic boundary conditions for tangential and normal components at the interface between the dielectric slab and air.

Solution:

$$\begin{aligned}\mathbf{a}_n \cdot \mathbf{B}_1 &= \mathbf{a}_n \cdot \mathbf{B}_2 \\ \mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2) &= \mathbf{J}_s.\end{aligned}$$

Here, \mathbf{J}_s is 0.

2. What is the magnetic field intensity \mathbf{H} inside the dielectric slab if the slab material has permeability μ ?

Solution: Tangential Component:

$$\begin{aligned}\mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2) &= \mathbf{J}_s \\ \mathbf{a}_n \times (0 - \mathbf{H}_2) &= 0 \\ \mathbf{a}_n \times \mathbf{H}_2 &= 0.\end{aligned}$$

Normal component of the field:

$$\begin{aligned}\mathbf{a}_n \cdot \mathbf{B}_1 &= \mathbf{a}_n \cdot \mathbf{B}_2 \\ \mathbf{a}_z B_{1n} &= \mathbf{a}_z B_{2n} \\ \mu_0 H_0 &= \mu H_{2n} \\ H_{2n} &= \frac{\mu_0}{\mu} H_0.\end{aligned}$$

$$\therefore \mathbf{H}_2 = \frac{\mu_0}{\mu} H_0 \mathbf{a}_z$$

3. What is the magnetic field intensity \mathbf{H} inside the magnetic slab if the slab is a permanent magnet having a magnetization vector $\mathbf{M}_i = \mathbf{a}_z M_i$?

Solution: Tangential component $\mathbf{a}_n \times \mathbf{H}_2 = 0$. Normal component:

$$\begin{aligned}\mathbf{a}_z B_{1n} &= \mathbf{a}_z B_{2n} \\ \mathbf{a}_z \mu_0 H_0 &= \mathbf{a}_z \mu_0 (H_{2n} + M_i) \\ H_{2n} &= H_0 - M_i\end{aligned}$$

\therefore The magnetic field intensity inside the slab is $\mathbf{H}_2 = (H_0 - M_i) \mathbf{a}_z$.

Answer:

$$\begin{aligned}\mathbf{H}_2 &= \frac{\mu_0}{\mu} H_0 \mathbf{a}_z \quad \text{Dielectric slab} \\ \mathbf{H}_2 &= (H_0 - M_i) \mathbf{a}_z \quad \mathbf{M}_i = a_z M_i\end{aligned}$$