[Cheng P.6-4] A current I flows lengthwise in a very long, thin conducting sheet of width w, as shown in Fig. 6-35.

(a) Assuming that the current flows into the paper, determine the magnetic flux density $\mathbf{B}_1$ at point $P_1(0, d)$.

(b) Use the result in part (a) to find the magnetic flux desnity $\mathbf{B}_2$ at point $P_2(2w/3, d)$.

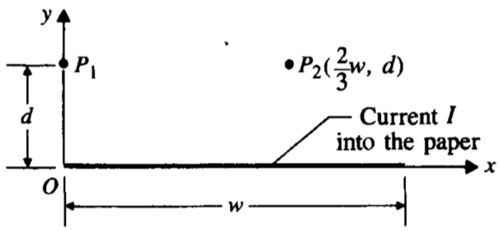
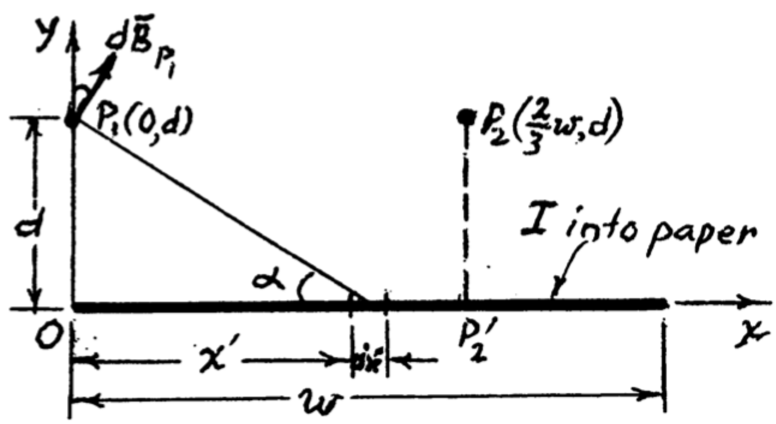


FIGURE 6–35
A thin conducting sheet carrying a current I (Problem P.6–4).

Solution:



(a) Using Eq. (6-33c) and the diagram above

$$d\mathbf{B}_{P1} = \mathbf{a}_x dB_x + \mathbf{a}_y dB_y$$
$$= \mathbf{a}_x dB_{P1} \sin\alpha + \mathbf{a}_y dB_{P1} \cos\alpha$$
$$dB_{P1} = \frac{\mu_0 (I/w) dx'}{2\pi(x'^2 + d^2)^{3/2}}$$
$$\sin\alpha = \frac{d}{(x'^2 + d^2)^{1/2}} \qquad \cos\alpha = \frac{x'}{(x'^2 + d^2)^{1/2}}$$

Therefore, $\mathbf{B}_{P1} = \mathbf{a}_x B_x + \mathbf{a}_y B_y$ where

$$B_x = \frac{\mu_0 I d}{2\pi w} \int_0^w \frac{dx'}{x'^2 + d^2} = \frac{\mu_o I}{2\pi w} \tan^{-1}(\frac{w}{d})$$
$$B_y = \frac{\mu_0 I}{2\pi w} \int_0^w \frac{x' dx'}{x'^2 + d^2} = \frac{\mu_o I}{4\pi w} \ln(1 + \frac{w}{d})$$

(b) To find $\mathbf{B}$ at $P_2(\frac{2}{3}w, d)$, we add vectorially the contributions of the current strips to the right and to the left of point P_2' using the result in part (a).

$$\mathbf{B}_{P_2} = \mathbf{B}_{2R} + \mathbf{B}_{2L}$$

$$\mathbf{B}_{2R} = \frac{\mu_0 I}{2\pi w}\left[\mathbf{a}_x \tan^{-1}\left(\frac{w}{3d}\right) + \mathbf{a}_y \frac{1}{2}\ln\left(1 + \frac{w^2}{9d^2}\right)\right]$$

$$\mathbf{B}_{2L} = \frac{\mu_0 I}{2\pi w}\left[\mathbf{a}_x \tan^{-1}\left(\frac{2w}{3d}\right) - \mathbf{a}_y \frac{1}{2}\ln\left(1 + \frac{4w^2}{9d^2}\right)\right]$$

$$\mathbf{B}_{P_2} = \frac{\mu_o I}{2\pi w}\left[\mathbf{a}_x \left(\tan^{-1}\frac{w}{3d} + \tan^{-1}\frac{2w}{3d}\right) - \mathbf{a}_y \ln\sqrt{\frac{1+(\frac{2w}{3d})^2}{1+(\frac{w}{3d})^2}}\right]$$

Answer:

(a)

$$\mathbf{B}_{P_1} = \mathbf{a}_x B_x + \mathbf{a}_y B_y \quad where$$
$$B_x = \frac{\mu_o I}{2\pi w}\tan^{-1}(\frac{w}{d}),$$
$$B_y = \frac{\mu_o I}{4\pi w}\ln(1 + \frac{w}{d})$$

(b)

$$\mathbf{B}_{P_2} = \frac{\mu_o I}{2\pi w}\left[a_x(\tan^{-1}\frac{w}{3d} + \tan^{-1}\frac{2w}{3d}) - a_y \ln\sqrt{\frac{1+(\frac{2w}{3d})^2}{1+(\frac{w}{3d})^2}}\right]$$