In a certain region, the magnetic vector potential is given as the following function in a cylindrical coordinate system: $\mathbf{A} = 2R^2\mathbf{a}_\phi \, \mathrm{T \cdot m}$.

(a) Find the magnetic flux density vector in this region.

(b) Obtain the magnetic flux through a circular contour $1\,\mathrm{m}$ in radius that lies in the plane $z = 0$ and is centered at the coordinate origin.

(c) Check the results by evaluating the circulation of $\mathbf{A}$ along the contour.

Solution:

(a) Using $\mathbf{B} = \nabla \times \mathbf{A}$,

$$\begin{aligned}
\mathbf{B} &= \frac{1}{R\,\sin\theta}\frac{\partial(A_\phi\,\sin\theta)}{\partial\theta}\mathbf{a}_R + \frac{1}{R}\left(-\frac{\partial(RA_\phi)}{\partial R}\right)\mathbf{a}_\theta \\
&= \frac{2R^2\,\cos\theta}{R\,\sin\theta}\mathbf{a}_R - \frac{6R^2}{R}\mathbf{a}_\theta \\
&= 2R\,\cot\theta\,\mathbf{a}_R - 6R\,\mathbf{a}_\theta\,(\mathrm{T}).
\end{aligned}$$

(b) Flux through the surface enclosed by the given contour, for which $\theta = \pi/2$ and the unit normal vector is $-\mathbf{a}_\theta$:

$$\begin{aligned}
\Phi_m &= \int_0^{2\pi}\int_0^1 R\,\sin\theta\,dR\,d\phi\,(-6R)(-\mathbf{a}_\theta\cdot\mathbf{a}_\theta) \\
&= 6\cdot 2\pi\cdot\left[\frac{R^3}{3}\right]_0^1 \\
&= 4\pi\,(\mathrm{T\cdot m^2}).
\end{aligned}$$

(c) Circulation of $\mathbf{A}$ along the contour:

$$\begin{aligned}
\Phi_m &= \int_0^{2\pi} R\,\sin\theta\,d\phi\,2R^2\,(\mathbf{a}_\phi\cdot\mathbf{a}_\phi) \\
&= 2\cdot 2\pi \\
&= 4\pi\,(\mathrm{T\cdot m^2}).
\end{aligned}$$

Answer:

(a) $\mathbf{B} = 2R\,\cot\theta\,\mathbf{a}_R - 6R\,\mathbf{a}_\theta\,(\mathrm{T})$

(b) $\Phi_m = 4\pi\,(\mathrm{T\cdot m^2})$

(c) Proof problem