Goal: *Large square and small circular concentric coplanar loops.* Fig. Q6.33 shows two concentric wire loops lying in the same plane, in free space. One is a large square loop of side length a and the other is a small circular loop of radius b ($b \ll a$). The loops are oriented in the same, counter-clockwise, direction. The square loop carries a low-frequency time-harmonic current of intensity $i(t) = I_o \sin \omega t$, and the resistance of the circular loop is R. Determine the induced current in the circular loop, neglecting its own magnetic field.

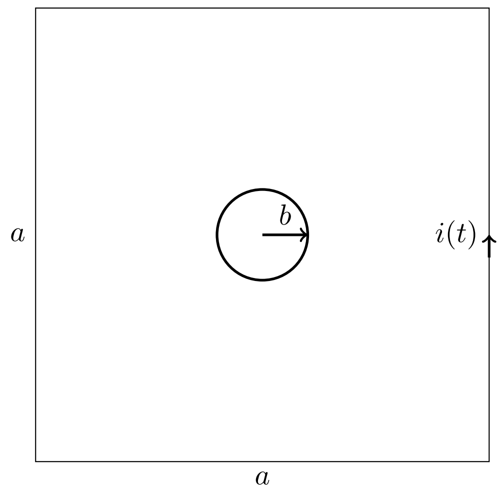

Figure 6.33 Magnetically-coupled large square and small circular loops in free space.

Steps:

1. What is the B-field at the center of the square loop due to $i(t)$?

   Solution: The B-field points out of the page at the center of the square loop.

$$B_{\text{center}} = \frac{2\sqrt{2}\mu_o i}{\pi a}$$

2. What is the magnetic flux through the circular loop?

   Solution:

$$\int \mathbf{B} \cdot d\mathbf{s} = B_{\text{center}} S = B_{\text{center}} = \frac{2\sqrt{2}\mu_o i}{a} b^2$$

3. What is the emf induced?

   Solution:

$$e_{\text{ind}}(t) = -\frac{d\Phi}{dt} = \frac{2\sqrt{2}\mu_o}{a} b^2 \frac{di}{dt}$$

4. What is the induced current in the circular loop?

   Solution:

$$i_{\text{ind}} = \frac{e_{\text{ind}}}{R} = \frac{2\sqrt{2}\mu_o b^2 I_o}{aR} \frac{di}{dt} = \frac{2\sqrt{2}\mu_o b^2 \omega}{aR} \cos \omega t$$

Answer:

$$i_{\text{ind}} = \frac{2\sqrt{2}\mu_o b^2 \omega}{aR} \cos \omega t$$