[Cheng P.5-1] Assuming S to be the area of the electrodes in the space-charge-limited vacuum diode in Fig. 5-2, find

(a) $V(y)$ and $\mathbf{E}(y)$ within the interelectrode region,

(b) the total amount of charge in the interelectrode region,

(c) the total surface charge on the cathode and on the anode,

(d) the transit time of an electron from the cathode to the anode with $V_0 = 200$ (V) and $d = 1$ (cm).

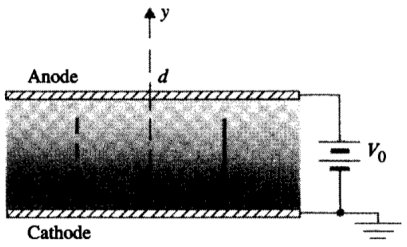

FIGURE 5–2
Space-charge-limited vacuum diode (Example 5–1).

Solution:

(a) Integrating Eq. (5-16):

$$V(y) = \left[\frac{qJ}{4\varepsilon_0}\sqrt{\frac{m}{2e}}\right]^{2/3} y^{4/3} = V_0(\frac{y}{d})^{\frac{4}{3}}$$
$$\mathbf{E}(y) = -\mathbf{a}_y\frac{dV(y)}{dy} = -\mathbf{a}_y\frac{4}{3}\frac{V_0}{d}(\frac{y}{d})^{\frac{1}{3}}$$

(b)

$$\rho(y) = \varepsilon_0\frac{dE(y)}{dy} = -\frac{4\varepsilon_0 V_0}{qd^2}\left(\frac{y}{d}\right)^{-2/3}$$
$$Q = \int_0^d \rho(y)Sdy = -\frac{4\varepsilon_0 V_0 S}{qd^{4/3}}\int_0^d y^{-2/3}dy = -\frac{4\varepsilon_o V_0}{3d}S.$$

(c) On the anode, $y = d$, $\rho_s = -\varepsilon_0 E(d) = \frac{4\varepsilon_0 V_0}{3d}$. Therefore

$$\text{Total surface charge on anode} = \rho_s S = \frac{4\varepsilon_o V_0}{3d}S$$
$$\text{Total charge on cathode} = 0.$$

(d) Substituting $V(y) = V_0 \left(\frac{y}{d}\right)^{4/3}$ in Eq. (5.12):

$$u = \frac{dy}{dt} = \left(\frac{2e}{m}V_0\right)^{1/2} \left(\frac{y}{d}\right)^{2/3}$$
$$y^{-2/3}dy = \left(\frac{2eV_0}{md^{4/3}}\right)^{1/2} dt$$
$$\int y^{-2/3}dy = \int \left(\frac{2eV_0}{md^{4/3}}\right)^{1/2} dt$$
$$y = \left(\frac{2eV_0}{9md^{4/3}}\right)^{3/2} t^3.$$

Therefore, the transit time is given by

$$T_t = 3d \left(\frac{m}{2eV_0}\right)^{1/2}.$$

For $V_0 = 200\,[\text{V}]$, $d = 10^{-2}\,[\text{m}]$, $m = 9.1 \times 10^{-31}\,[\text{kg}]$, and $e = 1.6 \times 10^{-19}\,[\text{C}]$

$$T_t = 3.58\,[\text{ns}]$$

Answer:

(a)

$$V(y) = V_0(\frac{y}{d})^{\frac{4}{3}}$$
$$\mathbf{E}(y) = -\mathbf{a}_y \frac{4}{3}\frac{V_0}{d}(\frac{y}{d})^{\frac{1}{3}}$$

(b)

$$Q = -\frac{4\varepsilon_o V_0}{3d}S$$

(c)

$$\text{Total charge on anode} = \frac{4\varepsilon_o V_0}{3d}S$$
$$\text{Total charge on cathode} = 0$$

(d)

$$\text{Transit time } T_t = 3.58\,ns$$