Goal: Prove that the electric potential at an arbitrary point on the z-axis produced by the semi-circular line charge of the figure in problem 2-3 is

$$V = \frac{\rho_l a}{4\varepsilon_o \sqrt{z^2 + a^2}},$$

where ρ_l is the line charge density in C/m.

Steps:

1. Choose a coordinate system.

   Solution: Cylindrical coordinate system.

2. Find the expression of differential arc length. Then find the differential charge dQ.

   Solution: Differential arc length is

   $$dl = a\, d\phi'$$

   Hence

   $$dQ = \rho_l\, a\, d\phi'$$

3. Find the expression for $|\mathbf{R} - \mathbf{R}'|$.

   Solution:

$$\begin{aligned} \mathbf{r} &= z\mathbf{a}_z \\ \mathbf{r}' &= a\cos\phi'\mathbf{a}_x + a\sin\phi'\mathbf{a}_y \\ |\mathbf{r} - \mathbf{r}'| &= \sqrt{z^2 + a^2} \end{aligned}$$

4. Integrate.

   Solution: A direct integration yields

$$V = \frac{\rho_l\, a}{4\varepsilon_o\sqrt{z^2 + a^2}}$$

Answer: Proof problem