[Cheng P.6-17] The magnetic flux density $\mathbf{B}$ for an infinitely long cylindrical conductor has been found in Example 6-1. Determine the vector magnetic potential $\mathbf{A}$ both inside and outside the conductor from the relation $\mathbf{B} = \nabla \times \mathbf{A}$.

Solution: Suppose the conductor has radius b and carries current I. *Inside* the conductor $(0 \leq r \leq b)$,

$$\mathbf{B}_{\text{in}} = \mathbf{a}_\phi \frac{\mu_0 I}{2\pi b^2} r.$$

Outside the conductor $(r \geq b)$,

$$\mathbf{B}_{\text{out}} = \mathbf{a}_\phi \frac{\mu_0 I}{2\pi r}.$$

Then, comparing $\mathbf{a}_\phi$ components of $\mathbf{B} = \nabla \times \mathbf{A}$, we can write

$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$\Longrightarrow \mathbf{a}_\phi B = \mathbf{a}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right)$$

For an infinite conductor oriented along z, A_r is invariant along z. Therefore,

$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$\Longrightarrow B = -\frac{\partial A_z}{\partial r}$$

Substituing the expressions for $\mathbf{B}_{\text{in}}$ and $\mathbf{B}_{\text{out}}$ into the above and integrating,

$$\mathbf{A}_{\text{in}} = \mathbf{a}_z \left[-\frac{\mu_0 I}{4\pi} \frac{r^2}{b^2} + c_1 \right], \quad (0 \leq r \leq b)$$
$$\mathbf{A}_{\text{out}} = \mathbf{a}_z \left[-\frac{\mu_0 I}{2\pi} \ln(r) + c_2 \right], \quad (r \geq b)$$

At $r = b$, $\mathbf{A}_{\text{in}} = \mathbf{A}_{\text{out}}$ which allows us to write

$$c_2 = \frac{\mu_0 I}{4\pi} (2 \ln(b) - 1) + c_1$$

Answer:

$$\mathbf{A}_{\text{in}} = \mathbf{a}_z \left[-\frac{\mu_0 I}{4\pi} \frac{r^2}{b^2} + c_1 \right], \quad (0 \leq r \leq b)$$
$$\mathbf{A}_{\text{out}} = \mathbf{a}_z \left[-\frac{\mu_0 I}{2\pi} \ln(r) + c_2 \right], \quad (r \geq b)$$
$$c_2 = \frac{\mu_0 I}{4\pi} (2 \ln(b) - 1) + c_1$$