[Cheng P.3-27] What are the boundary conditions that must be satisfied by the electric potential at an interface between two perfect dielectrics with dielectric constants ε_{r1} and ε_{r2}?

Solution: First, we know that the electrostatic potential V is continuous across a boundary. To see this, let's find the voltage difference between a and b, two points on either side of the boundary a length Δ apart, in the presence of a constant $\mathbf{E}$.

$$\begin{aligned} V_{ab} &= -\int_a^b \mathbf{E} \cdot dl \\ V_{ab} &= -E_0(b-a) \\ V_{ab} &= -E_0(\Delta). \end{aligned}$$

Now if we let $\Delta \to 0$, then $V_{ab} \to 0$. So if we have a voltage V_1 on one side of the boundary and V_2 on the other we can say

$$V_1 = V_2,$$

Futhermore, we know from our electric field boundary conditions we can say

$$\begin{aligned} \varepsilon_1 \mathbf{E}_{n1} &= \varepsilon_2 \mathbf{E}_{n2} \\ \mathbf{E} &= -\nabla V \\ \varepsilon_1 \frac{\partial V_1}{\partial n} &= \varepsilon_2 \frac{\partial V_2}{\partial n}. \end{aligned}$$

Answer:

$$\begin{aligned} \varepsilon_1 \frac{\partial V_1}{\partial n} &= \varepsilon_2 \frac{\partial V_2}{\partial n} \\ V_1 &= V_2 \end{aligned}$$