[Cheng P.5-14] Refer to the flat conducting quarter-circular washer in Example 5-6 and Fig 5-8. Find the resistance between the curved sides.

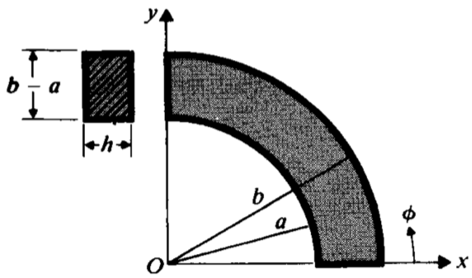

FIGURE 5–8
A quarter of a flat conducting circular washer (Example 5–6).

Solution: Starting with Poisson's equation

$$\nabla^2 V = 0 \rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0.$$

This has the solution

$$V(r) = c_1 \ln r + c_2.$$

To solve for c_1 and c_2, we use the boundary conditions $V(a) = V_0$ and $V(b) = 0$. This yields

$$V(r) = V_0 \frac{\ln(b/r)}{\ln(b/a)}.$$

From this we can solve for the current I

$$\begin{aligned}
\mathbf{E}(r) &= -\mathbf{a}_r \frac{\partial V}{\partial r} = \mathbf{a}_r \frac{V_0}{r \ln(b/a)}. \\
\mathbf{J}(r) &= \sigma \mathbf{E}_r. \\
I &= \int_S \mathbf{J} \cdot dS = \int_0^{\pi/2} \mathbf{J} \cdot (\mathbf{a}_r h r d\phi) \\
&= \frac{\pi \sigma h V_0}{2 \ln(b/a)}.
\end{aligned}$$

Now with Ohm's law we can solve for the resistance R

$$R = \frac{V_0}{I} = \frac{2\ln(b/a)}{\pi \sigma h}.$$

Answer:

$$R = \frac{2\ln(b/a)}{\pi \sigma h}$$