Goal: *Magnetic-magnetic boundary conditions.* Assume that the plane $z = 0$ separates medium 1 ($z > 0$) and medium 2 ($z < 0$), with relative permeabilities $\mu_{r1} = 600$ and $\mu_{r2} = 250$, respectively. The magnetic field intensity vector in medium 1 near the boundary (for $z = 0^+$) is $\mathbf{H}_1 = (5\mathbf{a}_x - 3\mathbf{a}_y + 2\mathbf{a}_z)$ A/m. Find the magnetic field intensity in medium 2 near the boundary (for $z = 0^-$) if the surface current is either $\mathbf{J}_s = 0$ or $\mathbf{J}_s = 3\mathbf{a}_y$.

Steps:

1. State the boundary conditions for the normal and tangential magnetic fields across the boundary.
   Solution:

$$\begin{aligned} B_{1n} &= B_{2n} \\ \mu_1 H_{1n} &= \mu_2 H_{2n} \\ \mathbf{a}_{n2} \times (\mathbf{H}_1 - \mathbf{H}_2) &= \mathbf{J}_s \end{aligned}$$

2. Calculate the magnetic field intensity vector $\mathbf{H}_2$ in medium 2 near the boundary (for $z = 0^-$) if no conduction current exists at the interface ($\mathbf{J}_s = 0$).
   Solution: Since $\mathbf{J}_s$, then the tangential fields are continuous across the boundary. The normal component is scaled by the ratio of the two permeabilities.

$$H_{2n} = \frac{\mu_1}{\mu_2} H_{1n} = 4.8 \text{ A/m}$$

   Hence,

$$\mathbf{H}_2 = 5\mathbf{a}_x - 3\mathbf{a}_y + 4.8\mathbf{a}_z \quad \text{A/m for} \quad \mathbf{J_s} = 0$$

3. Calculate the magnetic field intensity vector, $\mathbf{H}_2$, in medium 2 near the boundary (for $z = 0^-$) if there is a surface current density $\mathbf{J}_s = 3\mathbf{a}_y$ at the interface.
   Solution: Since $\mathbf{J}_s$ is in the y direction, it effects the tangential H-field in the x-direction only, by a value of 3. The rest of the component values stays the same as before. Hence,

$$\mathbf{H}_2 = 2\mathbf{a}_x - 3\mathbf{a}_y + 4.8\mathbf{a}_z \quad \text{A/m}$$

Answer:

$$\mathbf{H}_2 = 5\mathbf{a}_x - 3\mathbf{a}_y + 4.8\mathbf{a}_z \quad \text{A/m for} \quad \mathbf{J_s} = 0$$

$$\mathbf{H}_2 = 2\mathbf{a}_x - 3\mathbf{a}_y + 4.8\mathbf{a}_z \quad \text{A/m for} \quad \mathbf{J_s} = 3\mathbf{a}_y$$