[Cheng P.6-34] A very long conductor in free space carrying a current I is parallel to, and at a distance d from, an infinite plane interface with a medium.

(a) Discuss of the normal and tangential components of $\mathbf{B}$ and $\mathbf{H}$ at the interface:

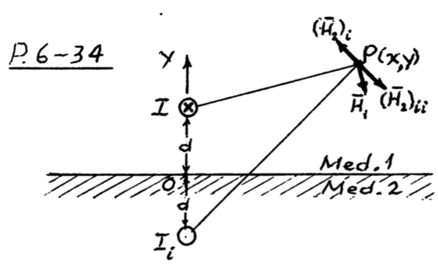
  (i) if the medium is infinitely conducting;

  (ii) if the medium is infinitely permeable.

(b) Find and compare the magnetic field intensities $\mathbf{H}$ at an arbitrary point in the free space for the two cases in part (a).

(c) Determine the surface current densities at the interface, if any, for the two cases.

Solution: We will use the diagram below to solve the question.



(a) (i) If the medium is infinitely conducting (i.e. $\sigma_2 \to \infty$) then $\mathbf{B} = \mathbf{H} = 0$. We also know from the boundary conditions that B_n is continous so $B_{1n} = B_{2n} = 0$. Likewise from the other boundary conditions, $\mathbf{a}_y \times \mathbf{H}_1 = \mathbf{J}_s \to -\mathbf{a}_z H_1 = \mathbf{J}_s$. There will also be an image current created as shown in the diagram d below the plane. It is flowing out of the page.

  (ii) If the medium is infinitely permeable (i.e. $\mu_2 \to \infty$) then $\mathbf{H}_2 = 0$, however $\mathbf{B}_2$ is finite. There is no surface current here so $H_{1t} = H_{2t} = 0$. B_n is continuous so $B_{1n} = B_{2n}$. There will also be an image current created as shown in the diagram d below the plane. However, it is flowing into the page.

(b) (i) The magnetic field intensity at point P is composed of the field from the wire $\mathbf{H}_1$ superimposed with its image $(\mathbf{H}_2)_i$. It is given by $\mathbf{H}_P = \mathbf{H}_1 + (\mathbf{H}_2)_i$, where

$$\mathbf{H}_1 = \frac{I}{2\pi}\left[\mathbf{a}_x \frac{y-d}{x^2+(y-d)^2} - \mathbf{a}_y \frac{x}{x^2+(y-d)^2}\right],$$
$$(\mathbf{H}_2)_i = \frac{I}{2\pi}\left[-\mathbf{a}_x \frac{y+d}{x^2+(y+d)^2} + \mathbf{a}_y \frac{x}{x^2+(y+d)^2}\right].$$

  (ii) The field at point P is similar to the previous part but in this case the image current direction is reversed. As a result

$$\mathbf{H}_P = \mathbf{H}_1 + (\mathbf{H}_2)_{ii}$$
$$\mathbf{H}_P = \mathbf{H}_1 - (\mathbf{H}_2)_i$$

(c) (i) $\mathbf{J}_s = -\mathbf{a}_z (\mathbf{H}_P)_x|_{y=0} = \mathbf{a}_z \left(\frac{Id}{x^2+d^2}\right)$

(ii) $\mathbf{J}_s = 0$

Answer:

(a) Using the boundary conditions and the fact that an image is created from the plane.

(b) (i) $\mathbf{H}_P = \mathbf{H}_1 + (\mathbf{H}_2)_i$, where

$$\mathbf{H}_1 = \frac{I}{2\pi}\left[\mathbf{a}_x \frac{y-d}{x^2+(y-d)^2} - \mathbf{a}_y \frac{x}{x^2+(y-d)^2}\right],$$
$$(\mathbf{H}_2)_i = \frac{I}{2\pi}\left[-\mathbf{a}_x \frac{y+d}{x^2+(y+d)^2} + \mathbf{a}_y \frac{x}{x^2+(y+d)^2}\right].$$

(ii) $\mathbf{H}_P = \mathbf{H}_1 - (\mathbf{H}_2)_i$

(c) (i) $\mathbf{J}_s = \mathbf{a}_z \left(\frac{Id}{x^2+d^2}\right)$

(ii) $\mathbf{J}_s = 0$