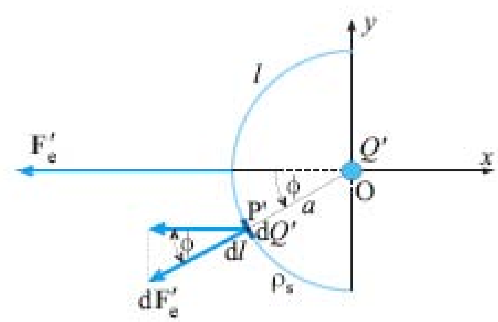
Goal: For the structure composed of an infinitely long line charge distribution ρ_l along the z-axis and a charged semi-cylinder with surface charge density ρ_s at $r = a$, $\frac{\pi}{2} \le \phi \le \frac{3\pi}{2}$, find the force per unit length on the semi-cylinder.

Steps:

1. Choose a coordinate system
   Solution: Let us define a coordinate system for the problem as follows:



   Hence, the cylinder is expressed as: $r = a$, $\pi/2 \le \phi \le 3\pi/2$, $-\infty < z < \infty$.

2. Find the electric field due to the line charge at the semi-cylinder.
   Solution: Due to symmetry, only a radial component of the electric field will exist due to the infinite line charge. The electric field a radial distance r from the line can be found by integrating the differential electric field created by a differential length of the line charge

$$dE_r = dE\frac{r}{\sqrt{z^2+r^2}}$$
$$E_r = \frac{1}{4\pi\epsilon_0}\int_{-\infty}^{\infty}\frac{\rho_l dz}{z^2+r^2}\frac{r}{\sqrt{z^2+r^2}}$$
$$E_r = \frac{\rho_l r}{4\pi\epsilon_0}\frac{2}{r^2}$$
$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0 r}\mathbf{a}_r$$

   hence, at the position of the cylinder, the field is

$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0 a}\mathbf{a}_r\,.$$

3. Find the force acting on a small element of the semi-cylinder ds.
   Solution: Consider a differential surface element on the surface of the cylinder: $ds = r\,d\phi\,dz$, carrying charge $dQ = \rho_s a\,d\phi\,dz$ at position $\mathbf{R} = a\mathbf{a}_r + z\mathbf{a}_z$. Because of the field of the line charge, this dQ receives a force:

$$d\mathbf{F} = dQ\mathbf{E} = \rho_s a\,d\phi\,dz\frac{\rho_l}{2\pi\varepsilon_0 a}\mathbf{a}_r = \frac{\rho_l\rho_s}{2\pi\varepsilon_0 a}d\phi\,dz\,(\mathbf{a}_x\cos\phi + \mathbf{a}_y\sin\phi)$$

4. Integrate over a length L of the semi-cylinder to find the force per unit length.

Solution:

$$\mathbf{F} = \int_{\text{semi-cylinder}} d\mathbf{F} = \frac{\rho_l \rho_s}{2\pi\varepsilon_0} \int_{z=0}^{z=L} dz \int_{\phi=\pi/2}^{\phi=3\pi/2} d\phi \, (\mathbf{a}_x \cos\phi + \mathbf{a}_y \sin\phi) = -\mathbf{a}_x \frac{\rho_l \rho_s}{\pi \epsilon_0} L$$

Hence, per unit length:

$$\frac{\mathbf{F}}{L} = -\mathbf{a}_x \frac{\rho_l \rho_s}{\pi \varepsilon_0}$$

Confirm that the units are Newton/m.

Answer:

$$\mathbf{F}' = -a_z \, \frac{\rho_l \rho_s}{\pi \varepsilon_o}$$