Two infinitely long coaxial cylindrical surfaces, $r = a$ and $r = b$ $(b > a)$, carry surface charge densities $\rho_{sa}$ and $\rho_{sb}$ respectively.

(a) Determine $\mathbf{E}$ everywhere

(b) What must be the relation between $a$ and $b$ in order that $\mathbf{E}$ vanishes for $r > b$?

*Solution:* Due to cylindrical symmetry, $\mathbf{E} = \mathbf{a}_r E_r(r)$. Applying Gauss' law with a cylinder of radius $r$:

(a)

$$\begin{aligned}
2\pi r\varepsilon_0 E_r(r < a) &= 0 \\
E_r(r < a) &= 0 \\
2\pi r\varepsilon_0 E_r(a < r < b) &= 2\pi a\rho_{s,a} \\
E_r(a < r < b) &= \frac{a\rho_{s,a}}{\varepsilon_0 r} \\
2\pi r\varepsilon_0 E_r(r < b) &= 2\pi a\rho_{s,a} + 2\pi b\rho_{s,b} \\
E_r(r > b) &= \frac{a\rho_{s,a} + b\rho_{s,b}}{\varepsilon_0 r}
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{a\rho_{s,a} + b\rho_{s,b}}{\varepsilon_0 r} &= 0 \\
\frac{b}{a} &= -\frac{\rho_{s,a}}{\rho_{s,b}}
\end{aligned}$$

*Answer:*

(a)

$$\begin{aligned}
E_r(r < a) &= 0 \\
E_r(a < r < b) &= \frac{a\rho_{s,a}}{\varepsilon_0 r} \\
E_r(r > b) &= \frac{a\rho_{s,a} + b\rho_{s,b}}{\varepsilon_0 r}
\end{aligned}$$

(b)

$$\frac{b}{a} = -\frac{\rho_{s,a}}{\rho_{s,b}}$$