
[Cheng P.3-30] The space between a parallel-plate capacitor of area S is filled with a dielectric whose permittivity varies linearly from ε_1 at one plate ($y = 0$) to ε_2 at the other plate ($y = d$). Neglecting fringing effects, find the capacitance.

Solution: $\varepsilon = \frac{\varepsilon_2 - \varepsilon_1}{d}y + \varepsilon_1$. Assume Q on plate at $y = d$ and a plate surface area S .

$$\begin{aligned}\mathbf{E} &= -\mathbf{a}_y \frac{\rho_s}{\varepsilon} = -\mathbf{a}_y \frac{Q}{S\varepsilon} \\ V &= -\int_{y=0}^{y=d} \mathbf{E} \cdot d\mathbf{l} = \frac{Qd \ln(\varepsilon_1/\varepsilon_2)}{S(\varepsilon_2 - \varepsilon_1)} \\ C &= \frac{Q}{V} = \frac{S(\varepsilon_2 - \varepsilon_1)}{d \ln(\varepsilon_1/\varepsilon_2)}.\end{aligned}$$

Answer:

$$C = \frac{Q}{V} = \frac{S(\varepsilon_2 - \varepsilon_1)}{d \ln(\varepsilon_1/\varepsilon_2)}.$$