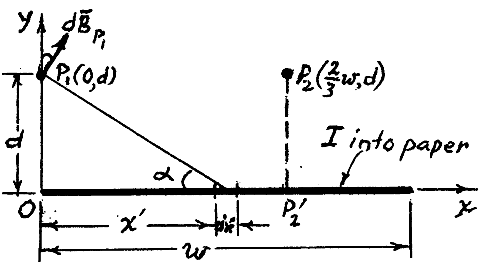
P. 6-4



a) Using Eq. (6-33c):

$$d\bar{B}_{P_1} = \bar{a}_x\, dB_x + \bar{a}_y\, dB_y$$

$$= \bar{a}_x (dB_{P_1}) \sin\alpha + \bar{a}_y (dB_{P_1}) \cos\alpha,$$

$$dB_{P_1} = \frac{\mu_0 (I/w)\, dx'}{2\pi (x'^2 + d^2)^{3/2}},$$

$$\sin\alpha = \frac{d}{(x'^2+d^2)^{1/2}}, \quad \cos\alpha = \frac{x'}{(x'^2+d^2)^{1/2}}.$$

$$\therefore\ \bar{B}_{P_1} = \bar{a}_x B_x + \bar{a}_y B_y,$$

where

$$B_x = \frac{\mu_0 I d}{2\pi w}\int_0^w \frac{dx'}{x'^2+d^2} = \frac{\mu_0 I}{2\pi w}\tan^{-1}\left(\frac{w}{d}\right),$$

$$B_y = \frac{\mu_0 I}{2\pi w}\int_0^w \frac{x'\,dx'}{x'^2+d^2} = \frac{\mu_0 I}{4\pi w}\ln\left(1+\frac{w^2}{d^2}\right).$$

b) To find B at $P_2(\frac{2}{3}w, d)$, we add vectorially the contributions of the current strips to the right and to the left of point P_2' using the result in part (a).

$$\bar{B}_{P_2} = \bar{B}_{2R} + \bar{B}_{2L}.$$

$$\bar{B}_{2R} = \frac{\mu_0 I}{2\pi w}\left[\bar{a}_x \tan^{-1}\left(\frac{w}{3d}\right) + \bar{a}_y \frac{1}{2}\ln\left(1+\frac{w^2}{9d^2}\right)\right].$$

$$\bar{B}_{2L} = \frac{\mu_0 I}{2\pi w}\left[\bar{a}_x \tan^{-1}\left(\frac{2w}{3d}\right) - \bar{a}_y \frac{1}{2}\ln\left(1+\frac{4w^2}{9d^2}\right)\right].$$

$$\therefore\ \bar{B}_{P_2} = \frac{\mu_0 I}{2\pi w}\left[\bar{a}_x\left(\tan^{-1}\frac{w}{3d} + \tan^{-1}\frac{2w}{3d}\right) - \bar{a}_y \ln\sqrt{\frac{1+(2w/3d)^2}{1+(w/3d)^2}}\right].$$