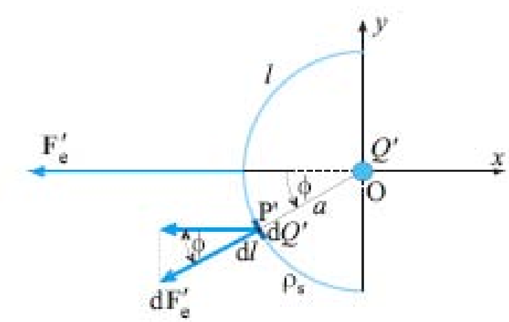
For the structure composed of an infinitely long line charge distribution ρ_l along the z-axis and a charged semi-cylinder with surface charge density ρ_s at $r = a$, $0 \leq \phi \leq \pi$, find the force per unit length on the semi-cylinder.

Solution: Let us define a coordinate system for the problem as follows:



Hence, the cylinder is expressed as: $r = a$, $\pi/2 \leq \phi \leq 3\pi/2$, $-\infty < z < \infty$. In class, we showed that the line charge distribution produces a field:

$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0 r}\mathbf{a}_r$$

Hence, at the position of the cylinder, the field is:

$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0 a}\mathbf{a}_r\,.$$

Now, consider a differential surface element on the surface of the cylinder: $ds = r\,d\phi\,dz$, carrying charge $dQ = \rho_s a\,d\phi\,dz$ at position $\mathbf{R} = a\mathbf{a}_r + z\mathbf{a}_z$. Because of the field of the line charge, this dQ receives a force:

$$d\mathbf{F} = dQ\mathbf{E} = \rho_s a\,d\phi\,dz\frac{\rho_l}{2\pi\varepsilon_0 a}\mathbf{a}_r = \frac{\rho_l\rho_s}{2\pi\varepsilon_0 a}d\phi\,dz\,(\mathbf{a}_x\cos\phi + \mathbf{a}_y\sin\phi)$$

Integrating to find the total force (assuming a length L for the semi-cylinder):

$$\mathbf{F} = \int_{\text{semi-cylinder}} d\mathbf{F} = \frac{\rho_l\rho_s}{2\pi\varepsilon_0}\int_{z=0}^{z=L} dz\int_{\phi=\pi/2}^{\phi=3\pi/2} d\phi\,(\mathbf{a}_x\cos\phi + \mathbf{a}_y\sin\phi) = -\mathbf{a}_x\frac{\rho_l\rho_s}{\pi\epsilon_0}L$$

Hence, per unit length:

$$\frac{\mathbf{F}}{L} = -\mathbf{a}_x\frac{\rho_l\rho_s}{\pi\varepsilon_0}$$

Confirm that the units are Newton/m.

Answer:

$$\frac{\mathbf{F}}{L} = -\mathbf{a}_x\frac{\rho_l\rho_s}{\pi\varepsilon_0}$$