Goal: Assume that the $z = 0$ plane separates two lossless dielectric regions with $\varepsilon_{r1} = 2$ and $\varepsilon_{r2} = 3$. Let the electric field $\mathbf{E}_1 = 2y\mathbf{a}_x - 3x\mathbf{a}_y + (5 + z)\mathbf{a}_z$ in region 1. Find the electric field $\mathbf{E}_2$ and the electric flux density $\mathbf{D}_2$ in region 2.

Steps:

1. What is the polarization vector $\mathbf{P}_1$ in region 1?

   Solution:

$$\begin{aligned}\mathbf{P}_1 &= \varepsilon_0 \left(\varepsilon_{r1} - 1\right) \mathbf{E}_1 \\ &= \epsilon_0 \mathbf{E}_1 \,.\end{aligned}$$

2. What is the electric flux density $\mathbf{D}_1$ in region 1?

   Solution:

$$\mathbf{D}_1 = 2\varepsilon_0 \left(2y\mathbf{a}_x - 3x\mathbf{a}_y + (5 + z)\mathbf{a}_z\right)$$

3. Using the boundary conditions for electrostatic fields we can compute the electric field $\mathbf{E}_2$ and the electric flux density $\mathbf{D}_2$ in region 2 *near* the interface. Use an appropriate boundary condition to find the normal component of $\mathbf{D}_{2\mathrm{n}}$ in region 2 at the interface.

   Solution:

$$\begin{aligned}\mathbf{D}_{2\mathrm{n}}(z = 0) &= \mathbf{D}_{1\mathrm{n}}(z = 0) \\ &= 10\varepsilon_0 \mathbf{a}_z \,.\end{aligned}$$

4. Use an appropriate boundary condition to find the tangential component of the electric field $\mathbf{E}_{2\mathrm{t}}$ in region 2 at the interface.

   Solution:

$$\begin{aligned}\mathbf{E}_{2\mathrm{t}} &= \mathbf{E}_{1\mathrm{t}} \\ &= \mathbf{a}_x 2y - \mathbf{a}_y 3x \,.\end{aligned}$$

5. What is the electric field $\mathbf{E}_2$ and the electric flux density $\mathbf{D}_2$ in region 2?

   Solution:

$$\begin{aligned}\mathbf{D}_2 &= 3\epsilon_0 \left(\mathbf{a}_x 2y - \mathbf{a}_y 3x + \mathbf{a}_z \frac{10}{3}\right) \\ \mathbf{E}_2 &= \mathbf{a}_x 2y - \mathbf{a}_y 3x + \mathbf{a}_z \frac{10}{3} \,.\end{aligned}$$

Answer:

$$\mathbf{D}_2 = 3\epsilon_0 \left(\mathbf{a}_x 2y - \mathbf{a}_y 3x + \mathbf{a}_z \frac{10}{3} \right)$$

$$\mathbf{E}_2 = \mathbf{a}_x 2y - \mathbf{a}_y 3x + \mathbf{a}_z \frac{10}{3} \, .$$