Goal: A current I flows in the inner conductor of an infinitely long coaxial line and returns via the outer conductor. The radius of the inner conductor is a, and the inner and outer radius of the outer conductor are b and c, respectively. Find the magnetic flux density $\mathbf{B}$ and plot is as a function of r for $0 < r < c$.

Steps:

1. Determine an Amperian path for this structure.
   Solution: A circle of variable radius r.

2. What is the current density inside each conductor? Assume that the current is uniformly distributed inside each conductor.
   Solution:

$$\mathbf{J}_1 = \frac{I}{\pi a^2}\mathbf{a}_z \,.$$
$$\mathbf{J}_2 = \frac{-I}{\pi\left(c^2 - b^2\right)}\mathbf{a}_z$$

3. Apply Ampere's law to find the magnetic flux density $\mathbf{B}$ inside the inner conductor ($0 < r < a$).
   Solution:

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu \int_0^{2\pi}\int_0^{r} \mathbf{J}_1 dS$$
$$2\pi r B_\phi = \mu \frac{I}{\pi a^2}\pi r^2$$
$$\mathbf{B} = \mathbf{a}_\phi \frac{\mu r I}{2\pi a^2}$$

4. Apply Ampere's law to find $\mathbf{B}$ in the region between the two conductors ($a < r < b$).
   Solution:

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu I$$
$$\mathbf{B} = \mathbf{a}_\phi \frac{\mu I}{2\pi r}$$

5. Apply Ampere's law to find $\mathbf{B}$ in the outer conductor ($b < r < c$).
   Solution:

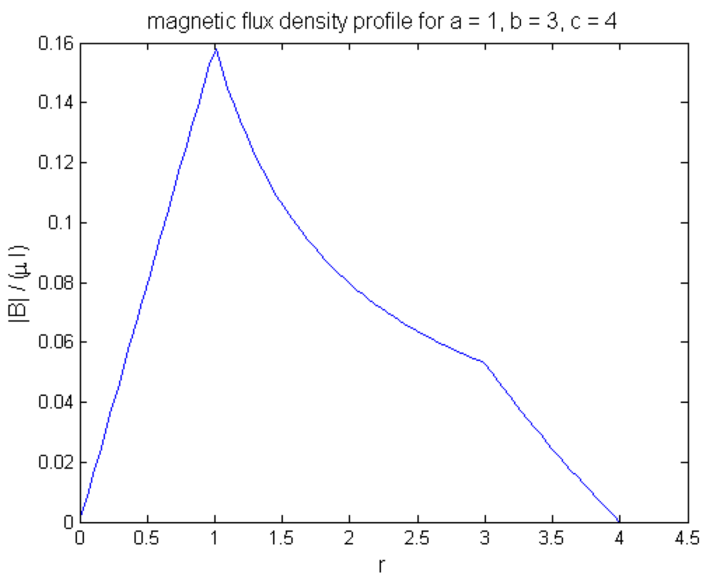
$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu\left(I + \int_0^{2\pi}\int_b^{r} \mathbf{J}_2 r dr d\theta\right)$$
$$\mathbf{B} = \mathbf{a}_\phi \frac{\mu I}{2\pi r}\frac{c^2 - r^2}{c^2 - b^2}$$

6. Use Ampere's law to compute $\mathbf{B}$ in the region outside the coaxial line ($r > c$).
Solution:

$$\oint \mathbf{B} \cdot d\mathbf{l} = 0$$
$$B_\phi = 0$$

There is no magnetic field outside the cable because there is zero net current enclosed by the Amperean loop.

7. Plot $|\mathbf{B}|$ versus r for all $0 < r < c$.



Solution:

Answer:

(a) For 0<r<a

$$\mathbf{B} = \mathbf{a}_\phi \frac{\mu r I}{2\pi a^2}$$

(b) For b<r<c

$$\mathbf{B} = \mathbf{a}_\phi \frac{\mu I}{2\pi r} \frac{c^2 - r^2}{c^2 - b^2}$$

(c) For r>c

$$B_\phi = 0$$