Assuming the electric field intensity is $\mathbf{E}_x = 100x\mathbf{a}_x\ (V/m)$, find the total electric charge contained inside

(a) a cubical volume $100\ mm$ on a side centered symmetrically about the origin.

(b) a cylindrical volume around the z-axis having a radius $50\ mm$ and a height $100\ mm$ centered at the origin.

Solution: The charge density in the region of space where the electric field exists can be found by Gauss' Law

$$\begin{aligned}\rho_v &= \varepsilon_0 \nabla \cdot E \\ &= \varepsilon_0 \frac{dE_x}{dx} \\ &= 100\varepsilon_0\end{aligned}$$

The total charge in contained within an area of space can be found by integrating this charge density over the volume of space

(a)

$$\begin{aligned}Q &= \int_{-.05}^{.05}\int_{-.05}^{.05}\int_{-.05}^{.05} 100\varepsilon_0 dxdydz \\ &= 8.85 \times 10^{-13}\, C\end{aligned}$$

(b)

$$\begin{aligned}Q &= \int_{-.05}^{.05}\int_{0}^{2\pi}\int_{0}^{.05} 100\varepsilon_0 r dr d\phi dz \\ &= 6.95 \times 10^{-13}\, C\end{aligned}$$

Answer:

(a)

$$Q = 8.85 \times 10^{-13}\, C$$

(b)

$$Q = 6.95 \times 10^{-13}\, C$$