[Cheng P.3-43] Prove that Eqs (3-180) for stored electrostatic energy hold true for any two-conductor capacitor.

Solution: Two conductors at potentials V_1 and V_2 carrying charges Q and -Q:

$$W_e = \frac{1}{2}V_1 \int_{S_1} \rho_{si} dS + \frac{1}{2}V_2 \int_{S_2} \rho_{s2} dS = \frac{1}{2}Q(V_1 - V_2) = \frac{1}{2}CV^2, \qquad V = V_1 - V_2.$$

Answer:

$$W_e = \frac{1}{2}Q(V_1 - V_2)$$