[Cheng P.5-9] Two lossy dielectric media with permittivities and conductivities $(\varepsilon_1, \sigma_1)$ and $(\varepsilon_2, \sigma_2)$ are in contact. An electric field with a magnitude E_1 is incident from medium 1 upon the interface at an angle α_1 measured from the common normal, as in Fig. 5-10.

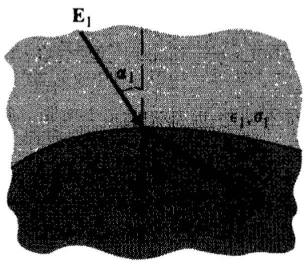

FIGURE 5–10
Boundary between two lossy dielectric media (Problem P.5–9).

(a) Find the magnitude and direction of $\mathbf{E}_2$ in medium 2.

(b) Find the surface charge density at the interface.

(c) Compare the results in parts (a) and (b) with the case in which both media are perfect dielectrics.

Solution:

(a)

$$\text{Eq. (3-118): } E_{1t} = E_{2t} \rightarrow E_2 \sin\alpha_2 = E_1 \sin\alpha_1.$$
$$\text{Eq. (3-118): } J_{1n} = J_{2n} \rightarrow \sigma_1 E_{1n} = \sigma_2 E_{2n} \rightarrow \sigma_2 E_2 \cos\alpha_2 = \sigma_1 E_1 \cos\alpha_1.$$

Therefore,

$$E_2 = E_1 \sqrt{\sin^2\alpha_1 + (\frac{\sigma_1}{\sigma_2}\cos\alpha_1)^2}.$$
$$\tan\alpha_2 = \frac{\sigma_2}{\sigma_1}\tan\alpha_1 \rightarrow \alpha_2 = \tan^{-1}(\frac{\sigma_2}{\sigma_1}\tan\alpha_1).$$

(b)

$$\text{Eq. (3-121b): } D_{2n} - D_{1n} = \rho_s \rightarrow \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \rho_s$$
$$\rho_s = (\frac{\sigma_1}{\sigma_2}\varepsilon_2 - \varepsilon_1)E_{1n} = (\frac{\sigma_1}{\sigma_2}\varepsilon_2 - \varepsilon_1)E_1\cos\alpha_1.$$

(c) If both media are perfect dielectrics, $\sigma_1 = \sigma_2 = 0$. $\alpha_2 = \tan^{-1}(\frac{\sigma_2}{\sigma_1}\tan\alpha_1)$ reverts to Eq (1-129), while $E_2 = E_1\sqrt{\sin^2\alpha_1 + (\frac{\sigma_1}{\sigma_2}\cos\alpha_1)^2}$ reverts to Eq. (3-130) and $\rho_s = 0$.

Answer:

(a)

$$\alpha_2 = \tan^{-1}(\frac{\sigma_2}{\sigma_1}\tan\alpha_1)$$

(b)

$$\rho_s = (\frac{\sigma_1}{\sigma_2}\varepsilon_2 - \varepsilon_1)E_1 \cos\alpha_1$$

(c)

$$\sigma_1 = \sigma_2 \ and \ \rho_s = 0$$