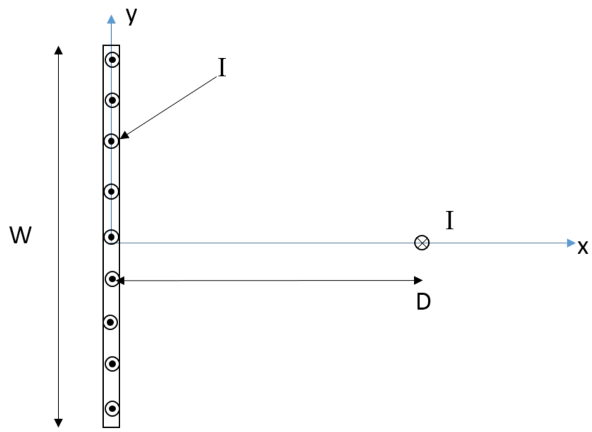
Goal: The cross section of a long thin metal plate and a parallel wire is shown in the figure below. Equal and opposite current I flow in the conductors. Find the force per unit length acting on both conductors.



Steps:

1. First determine the magnetic flux density $\mathbf{B}_{12}$ at an arbitrary point $(0, y)$ on the metal plate due the current I which flows in the wire.

   Solution: The magnetic field due to a single wire at point $(0, y)$ on the thin metal plate is

   $$\mathbf{B}_{12} = \frac{\mu_0 I}{2\pi r}\mathbf{a}_\phi \,,$$
   $$\mathbf{B}_{12} = \frac{\mu_0 I}{2\pi\sqrt{D^2+y^2}}\left(\frac{y}{\sqrt{D^2+y^2}}\mathbf{a}_x + \frac{D}{\sqrt{D^2+y^2}}\mathbf{a}_y\right)$$

2. Next, divide the plate into small strips of differential width dy. Determine the current dI which flows in the strip.

   Solution:

   $$dI = \frac{I}{w}dy\,\mathbf{a}_z\,.$$

3. Let $\mathbf{F}_{12}$ be the force per unit length exerted by the wire on the metal plate. Using the result in part (1) and (2), determine the contribution $\mathbf{dF}_{12}$ to the force due to each differential strip.

   Solution:

   $$d\mathbf{F}_{12} = \frac{\mu_0 I^2}{2\pi w}\left[dy\mathbf{a}_z \times \left(\frac{y}{D^2+y^2}\mathbf{a}_x + \frac{D}{D^2+y^2}\mathbf{a}_y\right)\right]$$
   $$= \frac{\mu_0 I^2}{2\pi w}\left(\frac{y}{D^2+y^2}\mathbf{a}_y - \frac{D}{D^2+y^2}\mathbf{a}_x\right)dy$$

4. Integrate $d\mathbf{F}_{12}$, to compute the force per unit length exerted by the current carrying wire on the metal plate.

   Solution:

$$\mathbf{F}_{12} = \mathbf{a}_y \frac{\mu_0 I^2}{2\pi w} \int_{-w/2}^{w/2} \frac{y}{D^2 + y^2} dy - \mathbf{a}_x \frac{\mu_0 I^2}{2\pi w} \int_{-w/2}^{w/2} \frac{D}{D^2 + y^2} dy = -\mathbf{a}_x \frac{\mu_0 I^2}{\pi w} \left[\tan^{-1} \left(\frac{w}{2D} \right) \right]$$

5. What is the force per unit length exerted by the metal plate on the current carrying wire?

   Solution:

$$\begin{aligned} \mathbf{F}_{21} &= -\mathbf{F}_{12} \\ &= \mathbf{a}_x \frac{\mu_0 I^2}{\pi w} \left[\tan^{-1} \left(\frac{w}{2D} \right) \right] \end{aligned}$$

Answer:

$$\begin{aligned} \mathbf{F}_{21} &= -\mathbf{F}_{12} \\ &= \mathbf{a}_x \frac{\mu_0 I^2}{\pi w} \left[\tan^{-1} \left(\frac{w}{2D} \right) \right] \end{aligned}$$