[Cheng P.3-29] Refer to Example 3-16. Assuming the same r_i and r_o and requiring the maximum electric field intensities in the insulating materials not to exceed 25% of their dielectric strengths, determine the voltage rating of the coaxial cable

a if $r_p = 1.75r_i$;

b if $r_p = 1.35r_i$.

c Plot the variations of E_r and V versus r for both part (a) and part (b).

Solution: 25% of the dielectric strength of rubber is $0.25 \times 25 \times 10^6 = 6.25 \times 10^6 [V/m]$. 25% of the dielectric strength of polystyrene is $0.25 \times 20 \times 10^6 = 5 \times 10^6 [V/m]$.

(a)

$$
\begin{aligned}
r_p &= 1.75 r_i \\
r_i &= 1.189 r_o \\
\text{Max.} E_r &= \frac{\rho_l}{2\pi\varepsilon_0}\frac{1}{3.2 r_i} \rightarrow \frac{\rho_l}{2\pi\varepsilon_0} = 3.2 r_i 6.25 \times 10^6 = r_i 20 \times 10^6 \\
\text{Max.} E_p &= \frac{\rho_l}{2\pi\varepsilon_0}\frac{1}{4.55 r_i} \rightarrow \frac{\rho_l}{2\pi\varepsilon_0} = 4.55 r_i 5 \times 10^6 = r_i 22.75 \times 10^6 \\
\frac{\text{Max.} E_r}{\text{Max.} E_p} &= \frac{4.55}{3.22} > \frac{6.25}{5}.
\end{aligned}
$$

From this we can see that the rubber will determine the breakdown voltage.

$$
\begin{aligned}
\frac{\rho_l}{2\pi\varepsilon_0} &= r_i 20 \times 10^6 \\
\frac{\rho_l}{2\pi\varepsilon_0} &= 0.004 \times 20 \times 10^6 = 8 \times 10^4.
\end{aligned}
$$

We can determine the breakdown voltage as

$$
\begin{aligned}
V_{\text{max}} &= \frac{\rho_l}{2\pi\varepsilon_0}\left(\frac{1}{2.6} ln \frac{r_o}{r_p} + \frac{1}{3.2} ln \frac{r_p}{r_i}\right) \\
V_{\text{max}} &= 8 \times 10^4 \left(\frac{1}{2.6} ln 1.189 + \frac{1}{3.2} ln 1.35\right) \\
V_{\text{max}} &= 19.3 kV
\end{aligned}
$$

(b) Similar to part (a), $V_{\text{max}} = 1.82 kV$

Answer:

(a) 19.3 kV

(b) 1.82 kV