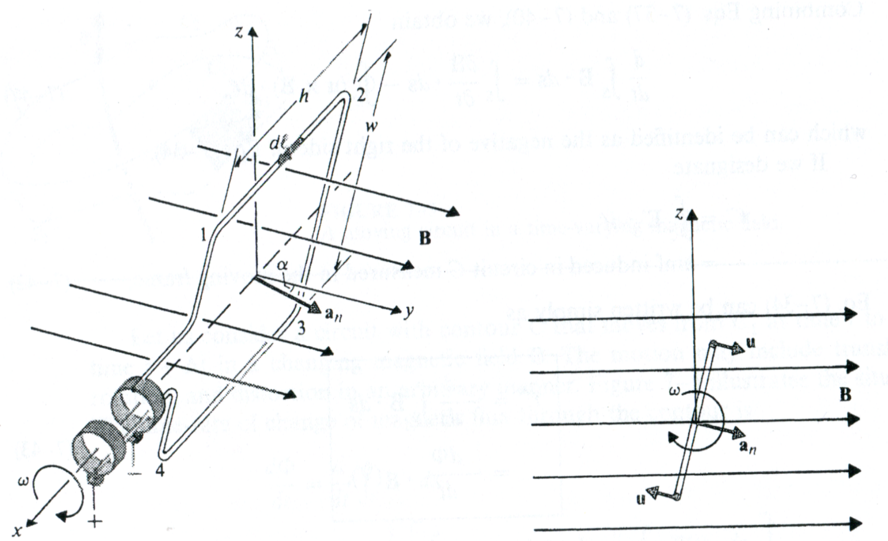
Goal: Assuming that a resistance R is connected across the slip rings of the rectangular conducting loop that rotates in a constant magnetic field $\mathbf{B} = \mathbf{a}_y B_0$, shown in Fig. 7-6, prove that the power dissipated in R is equal to the power required to rotate the loop at an angular frequency ω.



(a) Perspective view. (b) View from +x direction.

Steps:

1. Determine the emf $\mathcal{V}$ and current $\mathcal{I}$ induced in the conducting loop.

   Solution:

$$
\begin{aligned}
\mathcal{V} &= -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{S} \\
&= \omega B_0 hw \sin\omega t\,. \\
\mathcal{I} &= \frac{\omega B_0 hw \sin\omega t}{R}\,.
\end{aligned}
$$

2. Determine the power dissipated in the resistor.

   Solution:

$$
\begin{aligned}
P_d &= \frac{\mathcal{V}^2}{R} \\
&= \frac{\omega^2 B_0^2 h^2 w^2 \sin^2\omega t}{R}\,.
\end{aligned}
$$

3. What is the magnetic force exerted on side 1-2 of the loop?

   Solution:

$$
\mathbf{F}_{12} = \mathbf{a}_z i h B_0\,.
$$

4. What is the velocity of side 1-2 of the loop?

   Solution:

$$\mathbf{u}_{12} = \frac{\omega w}{2}\left(\mathbf{a}_y \cos\omega t - \mathbf{a}_z \sin\omega t\right)$$

5. What is the magnetic force exerted on side 4-3 of the loop?

   Solution:

$$\mathbf{F}_{43} = -\mathbf{a}_z ihB_0 \,.$$

6. What is the velocity of side 4-3 of the loop?

   Solution:

$$\mathbf{u}_{43} = \frac{\omega w}{2}\left(-\mathbf{a}_y \cos\omega t + \mathbf{a}_z \sin\omega t\right)$$

7. What is mechanical power required to rotate coil? (Remember that $P = \mathbf{F} \cdot \mathbf{u}$, where P is power, $\mathbf{F}$ is force, and $\mathbf{u}$ is velocity)

   Solution:

$$\begin{aligned} P_m &= -\left(\mathbf{F}_{12} \cdot \mathbf{u}_{12} + \mathbf{F}_{43} \cdot \mathbf{u}_{43}\right) \\ &= \omega B_0 hwi \sin\omega t \\ &= P_d \,. \end{aligned}$$

Answer: Proof problem