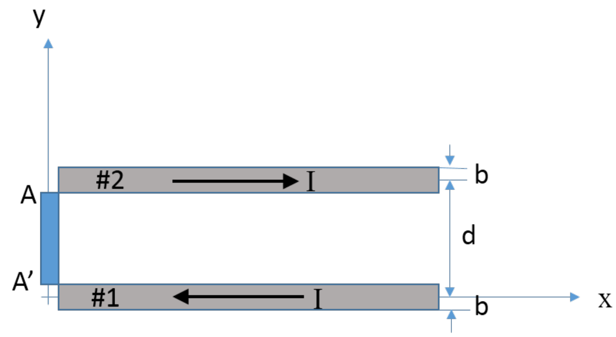
Goal: The bar AA' in the figure below serves as a conducting path (such as the blade of a circuit breaker) for the current I in two very long (semi-infinite) parallel lines. The lines have a radius b and are spaced at a distance d apart. Find the direction and the magnitude of the magnetic force on the bar.



Steps:

1. In order to compute the force, first determine the magnetic flux density $\mathbf{B}_1$ at point $(0, y)$ due to the current in line #1. Use Biot-Savart law to find $\mathbf{B}_1$.

   a) What is $\mathbf{R} - \mathbf{R}'$?

   Solution:

   $$\mathbf{R} - \mathbf{R}' = y\mathbf{a}_y - x'\mathbf{a}_x$$

   b) What is the differential current $d\mathbf{I}'$?

   Solution:

   $$d\mathbf{I}' = -I dx' \mathbf{a}_x$$

   c) What is differential magnetic flux density $d\mathbf{B}_1$?

   Solution:

   $$\begin{aligned} d\mathbf{B}_1 &= \frac{\mu_0 I}{4\pi}\left(\frac{-\mathbf{a}_x dx' \times (y\mathbf{a}_y - x'\mathbf{a}_x)}{(y^2 + (x')^2)^{3/2}}\right) \\ &= \frac{\mu_0 I}{4\pi}\frac{-y dx' \mathbf{a}_z}{(y^2 + (x')^2)^{3/2}} \end{aligned}$$

   d) What is $\mathbf{B}_1$?

   Solution:

   $$\begin{aligned} \mathbf{B}_1 &= -\mathbf{a}_z \frac{\mu_0 I}{4\pi}\int_0^\infty \frac{y}{(y^2 + (x')^2)^{3/2}} dx' \\ &= -\mathbf{a}_z \frac{\mu_0 I}{4\pi y} \end{aligned}$$

2. Use results in part (1) to find the magnetic flux density $\mathbf{B}_2$ at point $(0, y)$ due to line #2.

$$\mathbf{B}_2 = -\mathbf{a}_z \frac{\mu_0 I}{4\pi(d-y)}$$

3. What is the force $d\mathbf{F}$ acting on a short section dy of the bar AA'? Let $(0, y)$ be the position of the differential section.

   Solution:

$$\begin{aligned} d\mathbf{F} &= I dy \mathbf{a}_y \times (\mathbf{B}_1 + \mathbf{B}_2) \\ &= \frac{\mu_0 I^2 dy}{4\pi} \left(\mathbf{a}_y \times \mathbf{a}_z \left(-\frac{1}{y} - \frac{1}{d-y} \right) \right) \\ &= -\mathbf{a}_x \frac{\mu_0 I^2}{4\pi} \left(\frac{1}{y} + \frac{1}{d-y} \right) dy \end{aligned}$$

4. Integrate $d\mathbf{F}$ to find the total force on the bar.

   Solution:

$$\begin{aligned} \mathbf{F} &= -\mathbf{a}_x \frac{\mu_0 I^2}{4\pi} \int_b^{d-b} \left(\frac{1}{y} + \frac{1}{d-y} \right) dy \\ &= -\mathbf{a}_x \frac{\mu_0 I^2}{4\pi} \left[\ln\left(\frac{d-b}{b} \right) - \ln\left(\frac{b}{d-b} \right) \right] \\ &= -\mathbf{a}_x \frac{\mu_0 I^2}{2\pi} \ln\left(\frac{d}{b} - 1 \right) \end{aligned}$$

Answer:

$$\mathbf{F} = -\mathbf{a}_x \frac{\mu_0 I^2}{2\pi} \ln\left(\frac{d}{b} - 1 \right)$$