Goal: Find the resistance of two conducting spheres immersed in a lossy dielectric.

Steps:

1. Assume charges $+/-Q$ on the two spheres. What are the potentials on the spheres? What approximations can be made, given that the distance between the spheres is much larger than their radii?
   Solution: Let a and b be radius' of the two spheres, and d be separation between these spheres. Let there be $+Q$ on sphere a and $-Q$ on sphere b. Scalar potential outside a uniformly charged sphere is given by $V = Q/(4\pi\epsilon R)$ (this comes from Gauss' law). From this,

$$V_a = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{d-a}\right)$$
$$V_b = \frac{-Q}{4\pi\varepsilon}\left(\frac{1}{b} - \frac{1}{d-b}\right)$$

2. Find the potential difference between the spheres and hence, the capacitance Q/V.
   Solution:

$$\begin{aligned} C &= \frac{Q}{V} \\ &= \frac{Q}{V_a - V_b} \\ &= 4\pi\varepsilon\left(\frac{1}{\frac{1}{a} - \frac{1}{d-a} + \frac{1}{b} - \frac{1}{d-b}}\right) \end{aligned}$$

3. From capacitance, find the resistance.
   Solution:

$$\begin{aligned} RC &= \varepsilon/\sigma \\ R\left(\frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{d-a} + \frac{1}{b} - \frac{1}{d-b}}\right) &= \varepsilon/\sigma \\ R &= \frac{1}{4\pi\sigma}\left(\frac{1}{a} - \frac{1}{d-a} + \frac{1}{b} - \frac{1}{d-b}\right) \\ &\approx \frac{1}{4\pi\sigma}\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right) \end{aligned}$$

Answer:

$$\begin{aligned} R &= \frac{1}{4\pi\sigma}\left(\frac{1}{a} - \frac{1}{d-a} + \frac{1}{b} - \frac{1}{d-b}\right) \\ &\approx \frac{1}{4\pi\sigma}\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right) \end{aligned}$$