[Cheng P.5-7] Refer to Problem P.5-6.

(a) Calculate the time it takes for the charge density in the sphere to diminish to 1% of its initial value.

(b) Calculate the change in the electrostatic energy stored in the sphere as the charge density diminishes from the initial value to 1% of its value. What happens to this energy?

(c) Determine the electrostatic energy stored in the space outside the sphere. Does this energy change with time?

Solution:

(a)

$$e^{-(\sigma/\varepsilon)t} = \frac{\rho}{\rho_0} = 0.01 \rightarrow t = \frac{\ln 100}{(\sigma/\varepsilon)} = 4.88 \times 10^{-12}\,[\text{s}] = 4.88\,[\text{ps}].$$

(b)

$$W_i = \frac{\varepsilon}{2}\int_{v'} E_i^2 dv' = \frac{2\pi\rho_0 b^3}{45\varepsilon} e^{-2(\sigma/\varepsilon)t} = (W_i)_0 \left[e^{-(\sigma/\varepsilon)t}\right]^2.$$

Therfore,

$$\frac{W_i}{(W_i)_0} = \left[e^{-(\sigma/\varepsilon)t}\right]^2 = 0.01^2 = 10^{-4}\,[\text{J}].$$

This energy is dissipated as heat loss.

(c) The electrostatic energy stored outside the sphere is given by

$$\begin{aligned} W_o &= \frac{\varepsilon}{2}\int_b^\infty E_o^2 4\pi R^2 dR \\ &= \frac{Q_o^2}{8\pi\varepsilon_0 b} \\ &= 45\,[\text{kJ}]. \end{aligned}$$

This energy is constant.

Answer:

(a) $4.88\,[\text{ps}]$

(b) $10^{-4}\,[\text{J}]$, dissipated as heat loss.

(c) $45\,[\text{kJ}]$, the energy is constant over time.