Three point charges $Q = -1\,\text{nC}$ are placed at three vertices $(a, 0, 0)$, $(0, a, 0)$ and $(0, 0, a)$ of a cube with $a = 1\,\text{m}$. Find the electric field intensity vector at (a) the coordinate origin $(0, 0, 0)$ and (b) the point on the z-axis $(0, 0, 100\,\text{m})$.

Solution:

(a) $\mathbf{E} = \dfrac{1 \times 10^{-9}}{4\pi\varepsilon_0}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) = 9(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)\,\text{N/C} = 9\sqrt{3}\mathbf{a}_r\,\text{N/C}$,
where $\mathbf{a}_r = \dfrac{1}{\sqrt{3}}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$

(b) From the large distance (100 m), the three charges can be treated as a single charge $Q_{\text{tot}} = -3\,\text{nC}$, leading to:

$$\mathbf{E} = \frac{3 \times 10^{-9}}{4\pi\varepsilon_0 100^2} = 2.7\,\mathbf{a}_z\,\text{mN/C}$$

Answer:

(a) $\mathbf{E} = 9\sqrt{3}\mathbf{a}_r\,\text{N/C}$,
where $\mathbf{a}_r = \dfrac{1}{\sqrt{3}}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$

(b) $\mathbf{E} = 2.7\,\mathbf{a}_z\,\text{mN/C}$