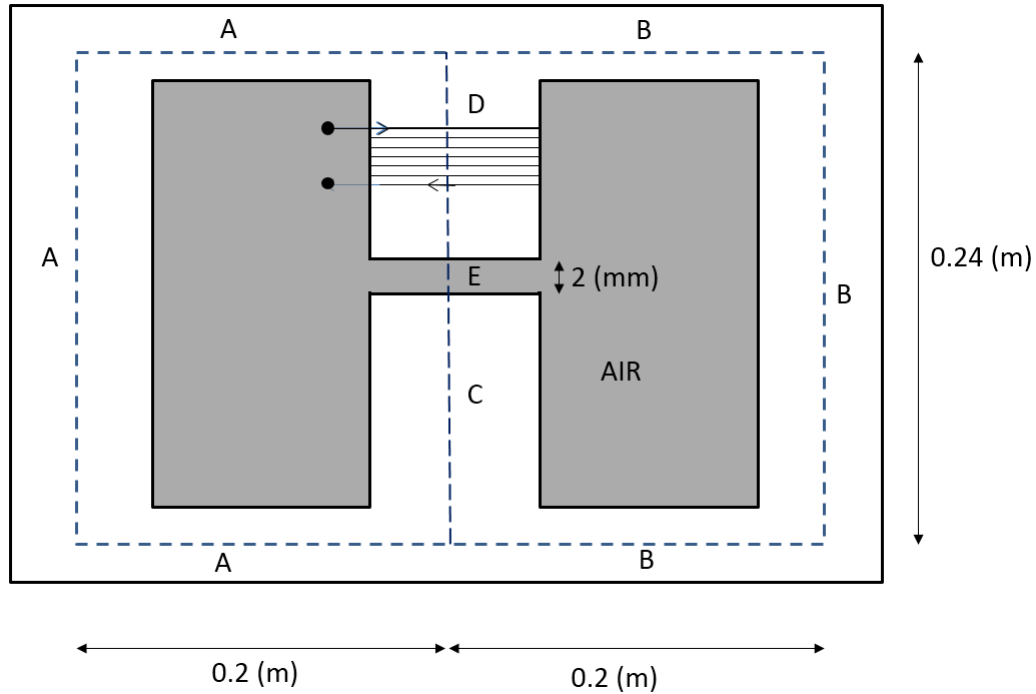**Goal:** Consider the magnetic circuit shown in the figure below. A current of 3 A flows through 200 turns of wire on the center leg. Assuming the core has constant cross-sectional area of 10^{-3} m^2 and a relative permeability of 5000, compute the magnetic flux density vector and the magnetic field intensity vector in each leg of the core and in the air gap.

Steps:

1. In order to solve this problem, first derive an equivalent circuit model for the magnetic circuit. Calculate the equivalent reluctance for each segment of the circuit (segments are labelled in the figure above).

   a) Segment A:
      Solution:
      $$R_A = \frac{l_c}{\mu_0 \mu_r S}$$
      $$= \frac{0.24 + 0.2 + 0.2}{5000 \mu_0 10^{-3}}$$
      $$= 1.019 \cdot 10^5 \ (\text{H}^{-1}).$$

   b) Segment B:
      Solution:
      $$R_B = \frac{l_c}{\mu_0 \mu_r S}$$
      $$= \frac{0.24 + 0.2 + 0.2}{5000 \mu_0 10^{-3}}$$
      $$= 1.019 \cdot 10^5 \ (\text{H}^{-1}).$$

c) Segment C:

Solution:

$$
\begin{aligned}
R_C &= \frac{l_c}{\mu_0 \mu_r S} \\
&= \frac{0.119}{5000 \mu_0 10^{-3}} \\
&= 1.894 \cdot 10^4 \ (\mathrm{H}^{-1}) .
\end{aligned}
$$

d) Segment D:

Solution:

$$
\begin{aligned}
R_D &= \frac{l_c}{\mu_0 \mu_r S} \\
&= \frac{0.119}{5000 \mu_0 10^{-3}} \\
&= 1.894 \cdot 10^4 \ (\mathrm{H}^{-1}) .
\end{aligned}
$$

e) Segment E:

Solution:

$$
\begin{aligned}
R_D &= \frac{l_c}{\mu_0 \mu_r S} \\
&= \frac{0.002}{\mu_0 10^{-3}} \\
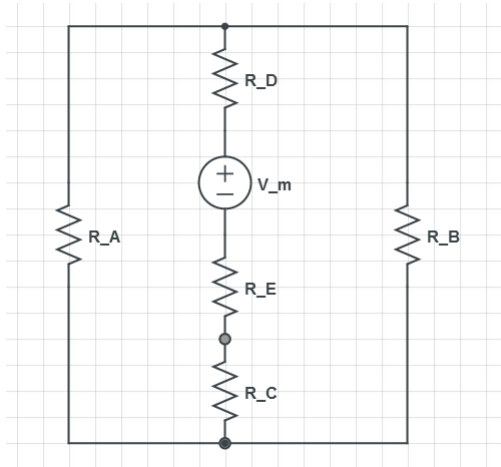&= 1.5915 \cdot 10^6 \ (\mathrm{H}^{-1}) .
\end{aligned}
$$

2. Next, compute the mmf due to the current $I = 3\,\mathrm{A}$ flowing in the 200-turn coil.

Solution:

$$
\begin{aligned}
V_m &= NI \\
&= 600 \ (\mathrm{A}) .
\end{aligned}
$$

3. Using the quantities derived above, draw an equivalent circuit model. Recall that reluctance and mmf are analogous to resistance and emf in an electric circuit, respectively.

Solution:

R_D

V_m

R_A

R_B

R_E

R_C

Note, the polarity of mmf source is obtained by applying the right hand rule.

4. Solve the circuit to determine the magnetic flux Φ in each segment. Magnetic flux Φ is analogous to current in an electric circuit.
Solution:

$$\Phi_A = \Phi_B = \frac{1}{2} \frac{V_m}{\frac{1}{2}R_A + 2R_C + R_E}$$
$$= 0.179 \text{ (mWb)}$$
$$\Phi_C = \Phi_D = \Phi_E = \frac{V_m}{\frac{1}{2}R_A + 2R_C + R_E}$$
$$= 0.357 \text{ (mWb)}.$$

5. Using the result of part 4., find the magnetic flux density in each segment of the magnetic circuit. *Solution:*

$$B = \frac{\Phi}{S}$$
$$B_A = B_B = 0.179 \text{ (T)}$$
$$B_C = B_E = B_D = 0.357 \text{ (T)}.$$

6. Using the result of part 5., find the magnetic field intensity in each segment of the magnetic circuit.

Solution:

$$H = \frac{1}{\mu_0 \mu_r} B$$

$$H_A = H_B = \frac{1}{5000\mu_0} B_A$$

$$= 28.5 \text{ (A/m)}$$

$$H_C = H_D = \frac{1}{5000\mu_0} B_C$$

$$= 57.0 \text{ (A/m)}$$

$$H_E = \frac{1}{\mu_0} B_E$$

$$= 2.85 \cdot 10^5 \text{ (A/m)} \, .$$

Answer:

$$B_A = B_B = 0.179 \text{ (T)}$$

$$B_C = B_E = B_D = 0.357 \text{ (T)}$$

$$H_A = H_B = 28.5 \text{ (A/m)}$$

$$H_C = H_D = 57.0 \text{ (A/m)}$$

$$H_E = 2.85 \times 10^5 \text{ (A/m)}$$