Goal: Find the distribution of the electric scalar potential inside and outside a sphere $R = \alpha$, with volume charge density: $\rho = \rho_o R/\alpha$, where R is the radial coordinate of the spherical coordinate system.

Steps:

1. Choose a coordinate system.

   Solution: Spherical coordinate system.

2. Find the total charge inside the sphere.

   Solution:

$$\begin{aligned} \text{Total charge enclosed by sphere of radius } R &= \int_0^R \int_0^{2\pi} \int_0^{\pi} \frac{\rho_o R'}{\alpha} R'^2 \sin\theta d\theta' d\phi' \, dR' \\ &= \frac{\rho_o}{\alpha}(2\pi)(2)\frac{1}{4}R^4 \\ &= \rho\alpha^3\pi \quad \text{for} \quad R = \alpha \end{aligned}$$

3. Find the voltage in the region $R \geq \alpha$.

   Solution:

$$V(R) = \frac{\rho_o \alpha^3}{4\varepsilon_o R} \quad \text{for} \quad R \geq \alpha$$

4. Using Gauss' Law, find the electric inside the sphere.

Solution: Use the spherical symmetry of the problem. Use

$$
\begin{aligned}
\int \mathbf{E} \cdot \mathbf{ds} &= \frac{Q_{\text{enclosed}}}{\varepsilon_o} \\
E_R(4\pi R^2) &= \frac{1}{\varepsilon_o}\left(\frac{\rho_o}{\alpha}\pi R^4\right) \\
E_R &= \frac{\rho_o R^2}{4\varepsilon_o \alpha}
\end{aligned}
$$

5. Find the electric potential (voltage) inside the sphere, appropriately choosing the reference point.

Solution:

$$
V(R) = \frac{\rho_o \alpha^2}{3\varepsilon_o}\left(1 - \frac{R^3}{4\alpha^3}\right), \quad \text{for} \quad R < \alpha
$$

Answer:

$$V(R) = \frac{\rho_o \alpha^3}{4\varepsilon_o R} \quad \text{for} \quad R \geq \alpha$$
$$V(R) = \frac{\rho_o \alpha^2}{3\varepsilon_o}\left(1 - \frac{R^3}{4\alpha^3}\right), \quad \text{for} \quad R < \alpha$$