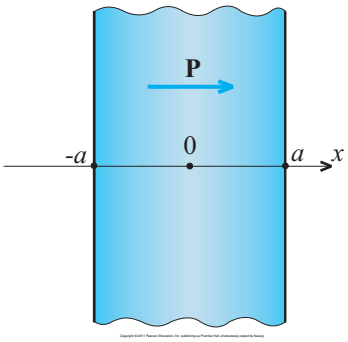
Goal: An infinitely large dielectric slab of thickness $d = 2a$, shown below, is polarized so that the polarization vector is $\mathbf{P} = P_o \frac{x^2}{a^2} \mathbf{a}_x$, where P_o is constant. The medium outside the slab is air. Find the voltage between the boundary surfaces of the slab.



Steps:

1. Compute the distribution of volume and surface bound charge of the slab. *Solution:*

$$\rho_p = -\nabla \cdot \mathbf{P} = -\frac{2P_o x}{a^2}, \rho_{p,s1} = \mathbf{a}_n \cdot \mathbf{P} = P_o = -\rho_{p,s2}$$

2. Compute the electric field intensity vector everywhere.

   Solution:

$$\mathbf{E} = -\frac{P_o x^2}{\varepsilon_o a^2} \mathbf{a}_x \,, \quad |x| < a \quad \text{and zero elsewhere.}$$

3. Compute the voltage between the boundary surfaces of the slab.

   Solution: A straight forward integration along the x direction yields

$$V = \frac{2}{3} \frac{P_o a}{\varepsilon_o}$$

Answer:

$$V = \frac{2}{3} \frac{P_o a}{\varepsilon_o}$$