Goal: A cylindrical capacitor of length L consist of coaxial conducting surfaces of radii r_i and r_o. Two dielectric media of different dielectric constants ε_{r1} and ε_{r2} fill the space between the conducting surfaces as shown in Fig. 3-42. Determine its capacitance.

Steps:

1. Is the electric fields in both of the dielectric same or different? Think about the boundary conditions.

   Solution: The boundary condition tangential E-fields are continuous across dielectric-dielectric boundaries. The E-fields are the same on both dielectric regions.

2. Using Gauss's Law, obtain an expression for the electric field.

   Solution:

   $$\pi r L \varepsilon_o \varepsilon_{r1} E_r + \pi r L \varepsilon_o \varepsilon_{r2} E_r = \rho_l L$$

   Isolating for E_r to obtain

   $$E_r = \frac{\rho_l}{\pi r \varepsilon_o (\varepsilon_{r1} + \varepsilon_{r2})}$$

3. Obtain an expression for the voltage between the two conductors.

   Solution:

   $$V = -\int_{r_o}^{r_i} E_r \, dr = \frac{\rho_l}{\pi \varepsilon_o (\varepsilon_{r1} + \varepsilon_{r2})} \ln \frac{r_o}{r_i}$$

4. Obtain an expression for the capacitance.

   Solution:

   $$C = \frac{\rho_l L}{V} = \frac{\pi \varepsilon_o (\varepsilon_{r1} + \varepsilon_{r2}) L}{\ln(r_o / r_i)}$$

Answer:

$$C = \frac{\pi \varepsilon_o (\varepsilon_{r1} + \varepsilon_{r2}) L}{\ln(r_o / r_i)}$$