Find the force between a charged circular loop of radius $b$ and uniform charge density $\rho_l$ and a point charge $Q$ located on the loop axis at a distance $h$ from the plane of the loop. What is the force when $h \gg b$ and when $h = 0$? Plot the force as a function of $h$. (Hint: force = electric field × charge)

*Solution:* Aligning the loop axis with the $z$ axis, the electric field created by the loop at a point $h$ on the $z$ axis of the loop can be found by recognizing that due to symmetry only the $E_z$ component of the electric field will exist.

$$\begin{aligned} d\mathbf{E}_z &= d\mathbf{E}\frac{h}{\sqrt{h^2+b^2}} \\ \mathbf{E} &= \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\frac{\rho_l b d\phi}{h^2+b^2}\frac{h}{\sqrt{h^2+b^2}}\mathbf{a}_z \\ &= \frac{\rho_l b h}{2\epsilon_0(h^2+b^2)^{3/2}}\mathbf{a}_z \end{aligned}$$

The force is then given by $\mathbf{F} = q\mathbf{E}$

$$\mathbf{F} = \frac{Q\rho_l b h}{2\epsilon_0(h^2+b^2)^{3/2}}\mathbf{a}_z$$

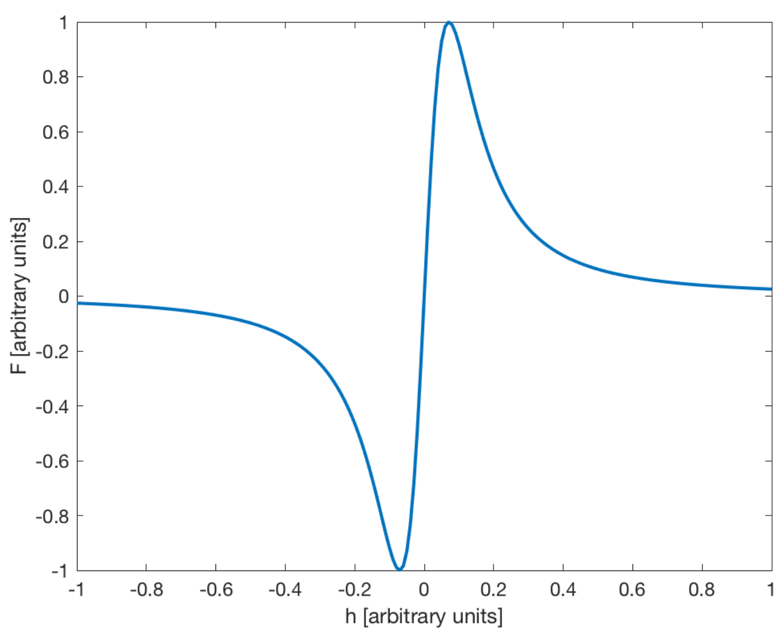
If $h \gg b$, then $(h^2+b^2)^{3/2} \to h^3$, so

$$\mathbf{F} = \frac{Q\rho_l b}{2\epsilon_0 h^2}\mathbf{a}_z$$

If $h = 0$, then

$$\mathbf{F} = \frac{Q\rho_l h}{2\epsilon_0 b^2}\mathbf{a}_z$$

The below is a plot of the force as a function of $h$



*Answer:* For $h \gg b$,

$$\mathbf{F} = \frac{Q\rho_l b}{2\epsilon_0 h^2}\mathbf{a}_z$$

For $h = 0$, then

$$\mathbf{F} = \frac{Q\rho_l h}{2\epsilon_0 b^2}\mathbf{a}_z$$