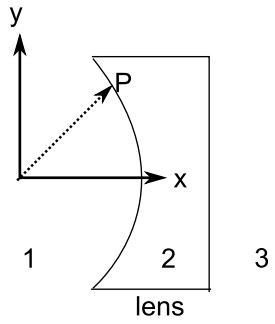
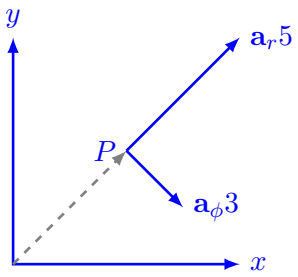
Goal: Dielectric lenses can be used to collimate electromagnetic fields. In the diagram below, the left surface of the lens is that of a circular cylinder, and the right surface is a plane. If $\mathbf{E}_1$ at point $P(r_o, 45°, z)$ is region 1 is $5\mathbf{a}_r - 3\mathbf{a}_\phi$, what must be the dielectric constant of the lens in order that $\mathbf{E}_3$ in region 3 is parallel to the x-axis?



Steps:

1. Draw the electric fields vectors at the point P.

   Solution:



2. What are the boundary conditions of the E-fields across a dielectric interface?

   Solution: The tangential electric fields is continuous across the boundary. The normal electric fields are scaled by the ratio of the dielectric constant on either side.

$$\frac{E_{\mathrm{n}2}}{E_{\mathrm{n}1}} = \frac{\varepsilon_1}{\varepsilon_2}$$

3. Compute the dielectric constant of the lens.

   Solution: We simply scale $5\mathbf{a}_r$ (the normal vector) such that its y-component is the same as the y-component of $3\mathbf{a}_\phi$. Hence dielectric constant needs to be $5/3$.

Answer: $\varepsilon_r = 5/3$