Goal: *Moving contour near an infinite dc line current.* Assume that the current in the straight wire conductor from Fig. Q6.12 is time-invariant, with intensity I, and that the contour moves away from the wire at a constant velocity v, as shown in Fig. Q6.17. At $t = 0$, the distance of the closer parallel side of the contour from the wire is $x = c$. Determine the emf induced in the contour.

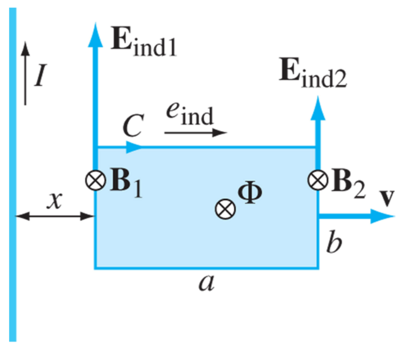

Figure 6.17 Evaluation of the emf in a rectangular contour moving in the magnetic field due to an infinitely long wire with a steady current.

Steps:

1. What is the B-field everywhere due to the line current?

   Solution:
   $$\mathbf{B} = \frac{\mu_o I}{2\pi r}\mathbf{a}_\phi$$

2. What is the total magnetic flux through the contour?

   Solution: The position as a function of time is $x(t) = c + vt$. Then we integrate over the area of the contour.
   $$\Phi(t) = \frac{\mu_o I b}{2\pi}\ln\frac{x+a}{x} = \frac{\mu_o I b}{2\pi}\ln\frac{c+a+vt}{c+vt}$$

3. What is the emf induced in the contour?

   Solution:
   $$e_{\text{ind}}(t) = -\frac{d\Phi}{dt} = -\frac{d\Phi}{dx}\frac{dx}{dt} = -\frac{d\Phi}{dx}v = \frac{\mu_o I a b v}{2\pi}\frac{1}{x(x+a)} = \frac{\mu_o I a b v}{2\pi(c+vt)(c+a+vt)}$$

Answer:
$$e_{\text{ind}}(t) = \frac{\mu_o I a b v}{2\pi(c+vt)(c+a+vt)}$$