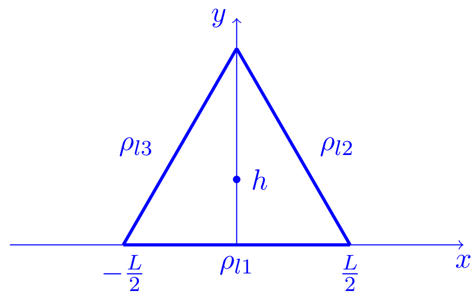
Three uniform line charges, ρ_{l1}, ρ_{l2}, and ρ_{l3}, each of length L, form an equilateral triangle. Assuming that $\rho_{l1} = 2\rho_{l2} = 2\rho_{l3}$, determine the electric field intensity at the center of the triangle.

Solution: Aligning the triangle as below, we can tell by symmetry that there will only be an electric field in the y direction.



The electric field contribution due to each line can be found by integrating over the given line

$$
\begin{aligned}
\mathbf{E}_{y,1} &= \frac{1}{4\pi\epsilon_0}\int_{-L/2}^{L/2} \frac{\rho_{l1}}{h^2+l^2}\frac{h}{\sqrt{h^2+l^2}}dl\mathbf{a}_y \\
&= \frac{\rho_{l1}}{4\pi\epsilon_0}\frac{L}{h\sqrt{h^2+L^2/4}}\mathbf{a}_y \\
\mathbf{E}_{y,2} &= \frac{1}{4\pi\epsilon_0}\int_{-L/2}^{L/2} \frac{\rho_{l2}}{h^2+l^2}\frac{h}{\sqrt{h^2+l^2}}dl\mathbf{a}_y\left(-\cos\frac{\pi}{3}\right) \\
&= -\frac{\rho_{l2}}{8\pi\epsilon_0}\frac{L}{h\sqrt{h^2+L^2/4}}\mathbf{a}_y \\
\mathbf{E}_{y,3} &= \frac{1}{4\pi\epsilon_0}\int_{-L/2}^{L/2} \frac{\rho_{l3}}{h^2+l^2}\frac{h}{\sqrt{h^2+l^2}}dl\mathbf{a}_y\left(-\cos\frac{\pi}{3}\right) \\
&= -\frac{\rho_{l3}}{8\pi\epsilon_0}\frac{L}{h\sqrt{h^2+L^2/4}}\mathbf{a}_y
\end{aligned}
$$

Using $\rho_{l1} = 2\rho_{l2} = 2\rho_{l3}$ and the centre of the triangle is at $h = L/2\sqrt{3}$, this simplifies to

$$\mathbf{E} = \mathbf{E}_{y,1} + \mathbf{E}_{y,2} + \mathbf{E}_{y,3} = \frac{3\rho_{l1}}{4\pi\epsilon_0 L}\mathbf{a}_y = \frac{3\rho_{l2}}{2\pi\epsilon_0 L}\mathbf{a}_y$$

Answer:

$$\mathbf{E} = \frac{3\rho_{l2}}{2\pi\epsilon_0 L}\mathbf{a}_y$$