Goal: The circuit in the figure below is situated in a magnetic field

$$\mathbf{B} = \mathbf{a}_z 3\cos\left(5\pi 10^7 t - \frac{2}{3}\pi x\right) \quad \mu\text{T}.$$

Assuming $R = 15\,\Omega$, find the current i.

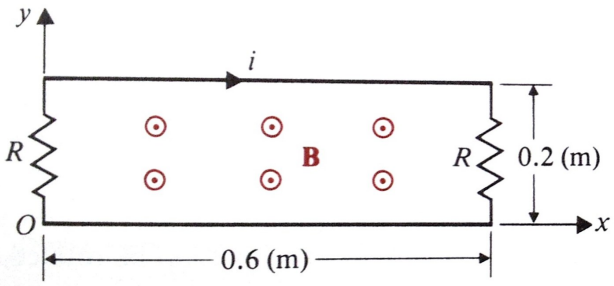

Figure 7-10. A circuit in time-varying magnetic field.

Steps:

1. State the integral form of Faraday's law.

   Solution:

   $$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

2. Calculate the magnetic flux through the loop.

   Solution:

   $$\begin{aligned}\int_S \mathbf{B}\cdot d\mathbf{s} &= \int_0^{0.6} \mathbf{a}_z 3\cos\left(5\pi 10^7 t - \frac{2}{3}\pi x\right) 10^{-6}\mathbf{a}_z 0.2dx \\ &= -\frac{0.18}{2\pi}\left[\sin\left(5\pi 10^7 - 0.4\pi\right) - \sin(5\pi 10^7 t)\right] 10^{-6} \quad \text{(V)}\end{aligned}$$

3. Calculate the electromotive force inducted in the loop.

   Solution:

   $$V = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s} = 45\left[\cos(5\pi 10^7 - 0.4\pi) - \cos(5\pi 10^7 t)\right] \quad \text{(V)}$$

4. Calculate current i.

   Solution:

   $$i = -\frac{V}{2R} = -1.5\left[\cos(5\pi 10^7 t - 0.4\pi) - \cos(5\pi 10^7 t)\right] = -1.76\sin(5\pi 10^7 - 0.2\pi) \quad \text{(A)}$$

Answer:

$$i = -1.76\sin(5\pi 10^7 - 0.2\pi) \quad \text{(A)}$$