A spherical distribution of charge $\rho = \rho_0[1 - (R^2/b^2)]$ exists in the region $0 \le R \le b$. This charge distribution is concentrically surrounded by a conducting shell with inner radius R_i $(> b)$ and outer radius R_o. Determine $\mathbf{E}$ everywhere.

Solution: Due to spherical symmetry of the charge distribution (i.e. no variation of charge density ρ in $\mathbf{a}_\theta$ or $\mathbf{a}_\phi$), $\mathbf{E} = \mathbf{a}_R E_R(R)$. Applying Gauss' law with a sphere of radius R as the Gaussian surface.

(a) for $0 \le R \le b$

$$4\pi R^2 \varepsilon_0 E_R(R) = \int_0^\pi \int_0^{2\pi} \int_0^R \rho_0 \left(1 - \frac{R^2}{b^2}\right) R^2 \sin\theta dR d\phi d\theta$$

$$4\pi R^2 E_R(R) = \frac{\rho_0}{\varepsilon_0} \int_0^R \left(1 - \frac{R^2}{b^2}\right) 4\pi R^2 dR$$

$$E_R(R) = \frac{\rho_0}{\varepsilon_0} R \left(\frac{1}{3} - \frac{R^2}{5b^2}\right)$$

(b) for $b \le R \le R_i$

$$4\pi R^2 E_R(R) = \frac{\rho_0}{\varepsilon_0} \int_0^b \left(1 - \frac{R^2}{b^2}\right) 4\pi R^2 dR$$

$$E_R(R) = \frac{2\rho_0 b^3}{15\varepsilon_0 R^2}$$

(c) for $R_i < R < R_o$

$$E_R(R) = 0$$

(d) for $R > R_o$

$$E_R(R) = \frac{2\rho_0 b^3}{15\varepsilon_0 R^2}$$

Answer:

(a) for $0 \le R \le b$

$$E_R(R) = \frac{\rho_0}{\varepsilon_0} R \left(\frac{1}{3} - \frac{R^2}{5b^2}\right)$$

(b) for $b \le R \le R_i$

$$E_R(R) = \frac{2\rho_0 b^3}{15\varepsilon_0 R^2}$$

(c) for $R_i < R < R_o$

$$E_R(R) = 0$$

(d) for $R > R_o$

$$E_R(R) = \frac{2\rho_0 b^3}{15\varepsilon_0 R^2}$$