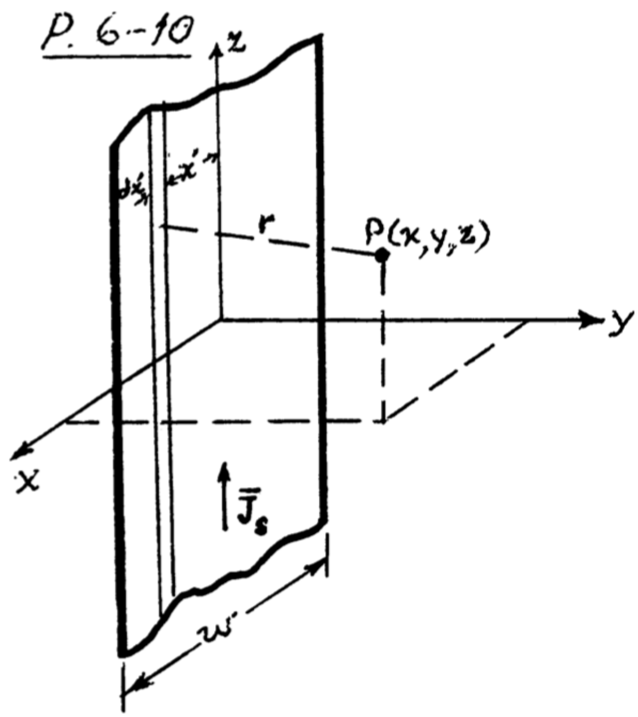
[Cheng P.6-10] A very long, thin conducting strip of width w lies in the xz-plane between $x = \pm w/2$. A surface current $\mathbf{J}_s = \mathbf{a}_z J_{s0}$ flows in the strip. Find the magnetic flux density at an arbitrary point outside the strip.

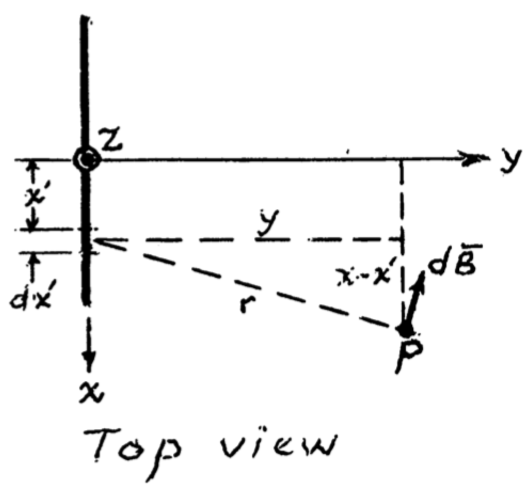
Solution: The surface current $\mathbf{J} = \mathbf{a}_z J_{s0}$ extending from $x = -w/2$ to $x = w/2$ is shown in the diagram below.



At $P(x, y, z)$ the magnetic flux density due to an infinitely long strip of current of length dx' is

$$d\mathbf{B} = \frac{\mu_0 J_{s0} dx'}{2\pi r} \left(-\mathbf{a}_x \frac{y}{r} + \mathbf{a}_y \frac{x - x'}{r} \right)$$

The distance $r = \sqrt{(x - x')^2 + y^2}$ is shown in the diagram below.



Therefore the flux density is given by

$$\mathbf{B} = \int d\mathbf{B} = \mathbf{a}_x B_x + \mathbf{a}_y B_y.$$

The vector components are given by

$$\begin{aligned}
B_x &= -\frac{\mu_0 J_{s0} y}{2\pi} \int_{-w/2}^{w/2} \frac{dx'}{(x-x')^2 + y^2} \\
&= -\frac{\mu_0 J_{s0}}{2\pi} \left[\tan^{-1} \frac{x - w/2}{y} - \tan^{-1} \frac{x + w/2}{y} \right] \\
B_y &= \frac{\mu_0 J_{s0} y}{2\pi} \int_{-w/2}^{w/2} \frac{(x-x')dx'}{(x-x')^2 + y^2} \\
&= \frac{\mu_0 J_{s0}}{4\pi} \ln \left(\frac{(x + w/2)^2 + y^2}{(x - w/2)^2 + y^2} \right).
\end{aligned}$$

Answer:

$$\begin{aligned}
&\mathbf{B} = \mathbf{a}_x B_x + \mathbf{a}_y B_y, \quad \textit{where} \\
&B_x = -\frac{\mu_0 J_{s0}}{2\pi} \left[\tan^{-1} \frac{x - w/2}{y} + \tan^{-1} \frac{x + w/2}{y} \right], \\
&B_y = \frac{\mu_0 J_{s0}}{4\pi} \ln \left(\frac{(x + w/2)^2 + y^2}{(x - w/2)^2 + y^2} \right)
\end{aligned}$$