[Cheng P.6-40] Find the mutual inductance between two coplanar rectangular loops with parallel sides, as shown in Fig. 6-50. Assume that $h_1 >> h_2$ ($h_2 < w_2 < d$).

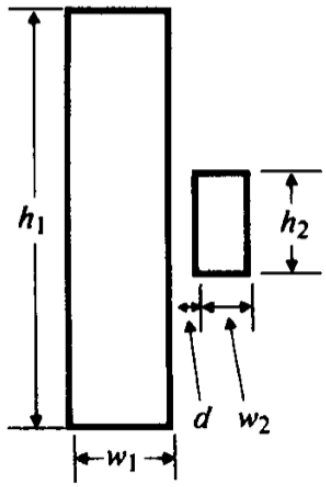

FIGURE 6–50
Two coplanar rectangular loops, $h_1 \gg h_2$ (Problem P.6–40).

Solution: We can approximate the magnetic flux from the large loop as two infinitely long wires on the long sides of the loop with lengths h_1. These infinitely long wires would carry a current I in opposite directions. We can neglect the two short sides of the large loop with length w_1.

$$\begin{aligned}\Lambda_{12} &= \frac{\mu_0 h_2 I}{2\pi}\int_0^{w_2}\left(\frac{1}{d+x}-\frac{1}{w_1+d+x}\right)\mathrm{d}x \\ &= \frac{\mu_0 h_2 I}{2\pi}\ln\left(\frac{w_2+d}{d}\cdot\frac{w_1+d}{w_1+w_2+d}\right).\end{aligned}$$

Dividing by the current we get

$$\begin{aligned}L_{12} &= \Lambda_{12}/I \\ &= \frac{\mu_0 h_2}{2\pi}\ln\left(\frac{w_2+d}{d}\cdot\frac{w_1+d}{w_1+w_2+d}\right).\end{aligned}$$

Answer:

$$L_{12} = \frac{\mu_o h_2}{2\pi}\ln\frac{(w_1+d)(w_2+d)}{d(w_1+w_2+d)}$$