Goal: Determine the capacitance of an isolated conducting sphere of radius b that is coated with a dielectric layer of uniform thickness d. The dielectric has an electric susceptibility χ_e.

Steps:

1. What is the electric field inside the dielectric due to charge $+Q$ on the surface of the conducting sphere?

   Solution: For $b < R < b + d$:

   $$\mathbf{E}_1 = \mathbf{a}_r \frac{Q}{4\pi\varepsilon_0(1+\chi_e)R^2}\,.$$

2. What is the electric field in air due to charge $+Q$ on the conductor?

   Solution: For $R > b + d$:

   $$\mathbf{E}_2 = \mathbf{a}_r \frac{Q}{4\pi\varepsilon_0 R^2}\,.$$

3. What is the potential on the outer surface of the dielectric ($r = b + d$)? What is the potential on the conductor?

   Solution: Potential on the outer surface of the dielectric is

   $$\begin{aligned} V_2 &= -\int_{\infty}^{b+d} E_2 dR \\ &= \frac{Q}{4\pi\varepsilon_0(b+d)}\,. \end{aligned}$$

   Potential on the inner surface of the dielectric is

   $$\begin{aligned} V_1 &= -\int_{b+d}^{b} E_1 dR + V_2 \\ &= \frac{Q}{4\pi\epsilon_0(1+\chi_e)}\left(\frac{1}{b} + \frac{\chi_e}{b+d}\right)\,. \end{aligned}$$

4. Compute the capacitance of the spherical conductor.

   Solution:

   $$\begin{aligned} C &= \frac{Q}{V} \\ &= \frac{4\pi\epsilon_0(1+\chi_e)b(b+d)}{b(1+\chi_e)+d} \end{aligned}$$

Answer:

$$C = \frac{4\pi\epsilon_0(1+\chi_e)b(b+d)}{b(1+\chi_e)+d}$$