Goal: *Rectangular contour near an infinite line current.* An infinitely long straight wire carries a slowly time-varying current density of intensity $i(t)$. A rectangular contour of side lengths a and b lies in the same plane with the wire, with two sides parallel to it, as shown in Fig. 6.12. The distance between the wire and the closer parallel side of the contour is c. Determine the emf induced in the contour.

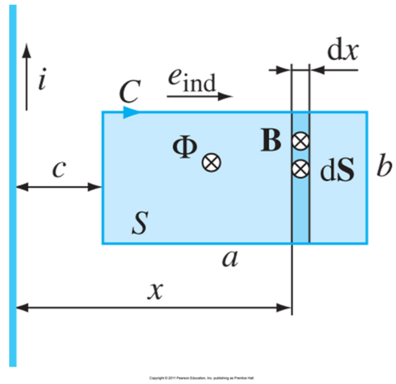

Figure 6.12 Evaluation of the emf in a rectangular contour in the vicinity of an infinitely long wire with a slowly time-varying current.

Steps:

1. State the integral form of Faraday's law.

   Solution:
   $$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial \Phi}{\partial t},$$
   where Φ is the total magnetic flux through a surface with contour C.

2. What is the $\mathbf{B}$-field everywhere due to $i(t)$?

   Solution: The B-field is into the page.
   $$B(x,t) = \frac{\mu_o i(t)}{2\pi x}$$

3. What is $\Phi(t)$, the magnetic flux through the loop?

   Solution: We integrate over the area of the loop.
   $$\Phi(t) = \int_c^{c+a} B(x,t) b \, dx = \frac{\mu_o i(t) b}{2\pi} \int_c^{c+a} \frac{dx}{x} = \frac{\mu_o i(t) b}{2\pi} \ln \frac{c+a}{c}$$

4. What is the emf induced in the loop?

   Solution:
   $$e_{ind}(t) = -\frac{d\Phi}{dt} = -\frac{\mu_o b}{2\pi} \ln \frac{c+a}{c} \frac{di}{dt}$$

Answer:

$$e_{ind}(t) = -\frac{\mu_o b}{2\pi} \ln \frac{c+a}{c} \frac{di}{dt}$$