[Cheng P.5-6] Lightning strikes a lossy dielectric sphere — $\varepsilon = \varepsilon_0$, $\sigma = 10$ (S/m) — of radius 0.1 (m) at time $t = 0$, depositing uniformly in the sphere a total charge 1 (mC). Determine, for all t,

(a) the electric field intensity both inside and outside the sphere,

(b) the current density in the sphere.

Solution:

$$\rho_o = \frac{Q_o}{(4\pi/3)b^3} = \frac{10^{-3}}{(4\pi/3)(0.1)^3} = 0.239\,[\mathrm{C/m^3}]$$
$$\rho = \rho_o e^{-(\sigma/\varepsilon)t}$$

(a)

$$R < b : \mathbf{E}_i = \mathbf{a}_R \frac{(4\pi/3)R^3\rho}{4\pi\varepsilon R^2} = \mathbf{a}_R \frac{\rho_o R}{3\varepsilon} e^{-(\sigma/\varepsilon)t} = \mathbf{a}_R\, 7.5 \times 10^9 R\, e^{-9.42\times 10^{11} t}\,[\mathrm{V/m}]$$
$$R > b : \mathbf{E}_o = \mathbf{a}_R \frac{Q_o}{4\pi\varepsilon_0 R^2} = \mathbf{a}_R \frac{9}{R^2} \times 10^6\,[\mathrm{V/m}]$$

(b)

$$R < b : \mathbf{J}_i = \sigma \mathbf{E}_i = \mathbf{a}_R\, 7.5 \times 10^{10} R\, e^{-9.42\times 10^{11} t}\,[\mathrm{A/m^2}]$$
$$R > b : \mathbf{J}_o = 0\,[\mathrm{A/m^2}]$$

Answer:

(a)

$$R < b : \mathbf{E}_i = \mathbf{a}_R\, 7.5 \times 10^9 R\, e^{-9.42\times 10^{11} t}\,[\mathrm{V/m}]$$
$$R > b : \mathbf{E}_o = \mathbf{a}_R \frac{9}{R^2} \times 10^6\,[\mathrm{V/m}]$$

(b)

$$R < b : \mathbf{J}_i = \mathbf{a}_R\, 7.5 \times 10^{10} R\, e^{-9.42\times 10^{11} t}\,[\mathrm{A/m^2}]$$
$$R > b : \mathbf{J}_o = 0\,[\mathrm{A/m^2}]$$