[Cheng P.6-36] Refer to Example 6-16. Determine the inductance per unit length of air coaxial transmission line assuming that its outer conductor is not very thing but is of a thickness d.
(Note: Find inductance from magnetic energy)

Solution: For $b \le r \le (b+d)$,

$$\begin{aligned}\mathbf{B}_3 &= \mathbf{a}_\phi B_{3\phi} \\ &= \mathbf{a}_\phi \frac{\mu_0 I}{2\pi r}\left[1 - \frac{\pi(r^2-b^2)}{\pi(b+d)^2 - \pi b^2}\right] \\ &= \mathbf{a}_\phi \frac{\mu_0 I}{2\pi r}\left[\frac{(b+d)^2 - r^2}{(b+d)^2 - b^2}\right].\end{aligned}$$

Magnetic energy per unit length stored in the outer conductor is given by

$$\begin{aligned}W'_m &= \frac{1}{2\mu_0}\int_b^{b+d} B_{3\phi}^2 2\pi r \mathrm{d}r \\ &= \frac{\mu_0 I^2}{4\pi}\left\{\frac{(b+d)^4}{[(b+d)^2-b^2]^2}\ln\left(1+\frac{d}{b}\right) + \frac{b^2-3(b+d)^2}{4[(b+d)^2-b^2]}\right\}.\end{aligned}$$

We can determine the inductance from the stored magnetic energy per unit length using

$$\begin{aligned}L' &= \frac{2W_{m,tot}}{I^2} \\ &= \frac{2}{I^2}\left(W'_{m1} + W'_{m2} + W'_{m3}\right) \\ &= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{b}{a} + \frac{(b+d)^4}{[(b+d)^2-b^2]^2}\ln\left(1+\frac{d}{b}\right) + \frac{b^2-3(b+d)^2}{4[(b+d)^2-b^2]}\right] \text{(H/m)}.\end{aligned}$$

Answer:

$$L' = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{b}{a} + \frac{(b+d)^4}{[(b+d)^2-b^2]^2}\ln\left(1+\frac{d}{b}\right) + \frac{b^2-3(b+d)^2}{4[(b+d)^2-b^2]}\right] \text{(H/m)}.$$