Goal: A capacitor consists of two coaxial metallic cylindrical surfaces of a length $L = 30\,\text{mm}$ and radii $r_1 = 5\,\text{mm}$ and $r_2 = 7\,\text{mm}$. The dielectric material between the surfaces has a relative permittivity $\varepsilon_r = 2 + (4/r)$, where r is measured in mm. Determine the capacitance.

Steps:

1. Use Gauss' law to compute the electric field inside the dielectric due to charge $+Q$ on the inner conductor and $-Q$ on the outer conductor.

   Solution:

$$\begin{aligned}\mathbf{E} &= \mathbf{a}_r \frac{Q}{2\pi\varepsilon r L} \\ &= \mathbf{a}_r \frac{Q}{4\pi\varepsilon_0 (r+2) L}\end{aligned}$$

2. Compute potential difference between the two conductors.

   Solution:

$$\begin{aligned}V &= -\int_{r_o}^{r_i} \mathbf{E} \cdot d\mathbf{r} \\ &= \frac{Q}{4\pi\varepsilon_0 L} \ln(r+2)\Big|_5^7 \\ &= \frac{Q}{4\pi\varepsilon_0 L} \ln\left(\frac{9}{7}\right) .\end{aligned}$$

3. What is the capacitance of this cylindrical capacitor?

   Solution:

$$\begin{aligned}C &= \frac{Q}{V} \\ &= \frac{4\pi\varepsilon_0 L}{\ln\left(\frac{9}{7}\right)} \\ &= 1500\varepsilon_0 \\ &= 13.26\,\mu\text{F} .\end{aligned}$$

Answer: C=13.26 μF