Determine the magnetic flux density at a point on the axis of a solenoid with radius b and length L, and with a current I in its N turns of closely wound coil. Show that the result reduces to the following equation as $L \to \infty$.

$$B = \mu_0 \frac{N}{L} I$$

Solution: The differential magnetic field dB created by a differential length dz' at a point z along the axis of the solenoid is given by

$$dB = \frac{\mu_0 I b^2}{2[(z'-z)^2 + b^2]^{3/2}} \frac{N}{L} dz'$$

This can be integrated along the length of the solenoid to give

$$\begin{aligned} B &= \frac{\mu_0 N I b^2}{2L} \int_0^L \frac{dz'}{[(z'-z)^2 + b^2]^{3/2}} \\ &= \frac{\mu_0 N I}{2L} \left[\frac{L-z}{\sqrt{(L-z)^2 + b^2}} + \frac{z}{\sqrt{z^2 + b^2}} \right] \end{aligned}$$

In the limit as $L \to \infty$, the magnetic field in the centre of the solenoid $(z = L/2)$ will be

$$\begin{aligned} B &= \frac{\mu_0 N I}{2L} \left[\frac{L - L/2}{\sqrt{(L - L/2)^2 + b^2}} + \frac{L/2}{\sqrt{(L/2)^2 + b^2}} \right] \\ &= \mu_0 \frac{N}{L} I \end{aligned}$$

Note that because the solenoid is infinitely long, every point can be described as being in the centre. *Answer:* Proof problem