[Cheng P.6-13] A thin conducting wire is bent into the shape of a regular polygon of N sides. A current I flows in the wire. Show that the magnetic flux density at the center is

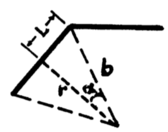
$$\mathbf{B} = \mathbf{a}_n \frac{\mu_0 N I}{2\pi b} \tan \frac{\pi}{N},$$

where b is the radius of the circle circumscribing the polygon and $\mathbf{a}_n$ is a unit vector normal to the plane of the polygon. Show also that, as N becomes very large, this result reduces to that given in Eq. (6-38) with z=0.

Solution: Use Eq. (6-35) for a wire of length 2L.

$$\mathbf{B} = \mathbf{a}_\phi \frac{\mu_0 I L}{2\pi r\sqrt{L^2 + r^2}}.$$

Using the diagram below $\alpha = \frac{\pi}{N}$, $\frac{L}{r} = \tan\alpha = \tan\frac{\pi}{N}$.



Using this result

$$\mathbf{B} = \mathbf{a}_n N \left(\frac{\mu_0 I L}{2\pi r b}\right)$$
$$\mathbf{B} = \mathbf{a}_n \frac{N \mu_0 I}{2\pi b} \tan\frac{\pi}{N}.$$

When N is very large, $\tan\frac{\pi}{N} \approx \frac{\pi}{N}$. Therefore $\mathbf{B} \to \mathbf{a}_n \frac{\mu_0 I}{2b}$. This is the same as Eq. (6-38) with $z = 0$.

Answer: Proof problem