Goal: Determine the electric potential at the center of a non-uniformly charged spherical surface of radius a, with $\rho_s(\theta) = \rho_{s,o} \sin(2\theta)$, with $\rho_{s,o}$ a constant and the angle $0 \leq \theta \leq \pi$.

Steps:

1. Choose the coordinate system.

   Solution: Spherical coordinate system.

2. Find the differential surface element in the coordinate system chosen above. Then find the differential charge dQ on the spherical surface.

   Solution: Differential surface element is

   $$ds' = (R')^2 \sin\theta' d\theta' d\phi'$$

   Hence, differential charge dQ is (evaluated on the sphere surface)

   $$dQ = \rho_{s,o}\, a^2 \sin\theta' \sin(2\theta')\, d\theta' d\phi'$$

3. Find a expression for $|\mathbf{R} - \mathbf{R}'|$.

   Solution: Observation point R is at the origin. $\mathbf{R}' = R'\mathbf{a}_{R'} = \alpha\mathbf{a}_R$. Then

$$|\mathbf{R} - \mathbf{R}'| = |a|\,\mathbf{a}_{R'} = a$$

4. Integrate.

   Solution:

$$dV = \frac{\rho_{s,o}\, a^2 \sin\theta' \sin(2\theta')\, d\theta' d\phi'}{4\pi\varepsilon_o \alpha} = \frac{\rho_{s,o} a}{4\pi\varepsilon_o} \sin\theta' \sin 2\theta' d\theta' d\phi'$$

   Then

$$V = \frac{\rho_{s,o}\, a}{4\pi\varepsilon_o} \int_0^{2\pi} d\phi' \int_0^{\pi} d\theta' \sin\theta' \sin 2\theta' = 0$$

Answer:

$$V = \frac{\rho_{s,o}\, a}{4\pi\varepsilon_o} \int_0^{2\pi} d\phi' \int_0^{\pi} d\theta' \sin\theta' \sin 2\theta' = 0$$