
A point charge $2Q$ is placed at the center of an air-filled spherical metallic (perfectly conducting) shell, charged with Q and situated in air. The inner and outer radii of the shell are a and b ($a < b$). What is the total charge on the inner and outer surface of the shell, respectively? Find the potential of the shell.

Solution: Every field line from the inner point charge is terminated at a negatively charged point on the metallic shell. We also know that there are no volumetric charges within conductors at electrostatic equilibrium. As a result, the Q of charge on the outer conductor will redistribute itself when the point charge is placed within. This will mean that a surface charge of $-2Q$ is induced on the inner part of the outer sphere, while an outer charge of $3Q$ is induced to maintain the Q of charge originally on the outer shell.

The electric field intensity outside the sphere, using Gauss' law, is

$$\mathbf{E} = \frac{3Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R \quad b \leq R \leq \infty.$$

Therefore we can solve for the potential as

$$V = - \int_{\infty}^b \frac{3Q}{4\pi\epsilon_0 R^2} dr$$
$$V = \frac{3Q}{4\pi\epsilon_0 b}.$$

Answer: Inner charge is $-2Q$, outer charge is $3Q$, potential of the shell is $3Q/(4\pi\epsilon_0 b)$.