Goal: Three point charges $Q = -1$ nC are placed at three vertices $(a, 0, 0)$, $(0, a, 0)$ and $(0, 0, a)$ of a cube with $a = 1$ m. Find the electric field intensity vector at (a) the coordinate origin $(0, 0, 0)$ and (b) the point on the z-axis $(0, 0, 100\text{ m})$.

Steps:

1. Choose a coordinate system
   Solution: Cartesian

2. Find the electric field at the origin due to a point charge at each of the three vertices.
   Solution:

$$\mathbf{E}_{(a,0,0)} = \frac{-1 \times 10^{-9}}{4\pi\varepsilon_0}(-\mathbf{a}_x) = 9\mathbf{a}_x \text{N/C}$$
$$\mathbf{E}_{(0,a,0)} = \frac{-1 \times 10^{-9}}{4\pi\varepsilon_0}(-\mathbf{a}_y) = 9\mathbf{a}_y \text{N/C}$$
$$\mathbf{E}_{(0,0,a)} = \frac{-1 \times 10^{-9}}{4\pi\varepsilon_0}(-\mathbf{a}_z) = 9\mathbf{a}_z \text{N/C}$$

3. Use superposition to find the total electric field at the origin.
   Solution:

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_{(a,0,0)} + \mathbf{E}_{(0,a,0)} + \mathbf{E}_{(0,0,a)} \\ &= 9(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)\text{N/C} \\ &= 9\sqrt{3}\mathbf{a}_r \text{N/C}\end{aligned}$$

   where $\mathbf{a}_r = \frac{1}{\sqrt{3}}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$.

4. Replace the three separate charges with one equivalent charge because they are so far away.
   Solution:

$$Q_{tot} = -3 \text{ nC}$$

5. Find the electric field due to the equivalent charge *Solution:*

$$\begin{aligned}\mathbf{E} &= \frac{-3 \times 10^{-9}}{4\pi\varepsilon_0(100)^2}\mathbf{a}_x \\ &= 2.7\mathbf{a}_z \text{mN/C}\end{aligned}$$

Answer:

(a)

$$\mathbf{E} = 9\sqrt{3}\mathbf{a}_r \text{N/C}$$

where $\mathbf{a}_r = \frac{1}{\sqrt{3}}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$

(b)

$$\mathbf{E} = \frac{-3 \times 10^{-9}}{4\pi\varepsilon_0(100)^2}\mathbf{a}_x$$
$$= 2.7\mathbf{a}_z\text{mN/C}$$