For a sphere of radius $R = a$, with a volume charge density: $\rho_v = \rho_0 R/a$ (where R is the radial coordinate of the spherical coordinate system), use Gauss' law in the differential form to compute the electric field intensity everywhere.

Solution: Due to symmetry, the only component of the electric field will be E_R

$$\begin{aligned}
\nabla \cdot E &= \frac{\rho_v}{\epsilon_0} \\
\frac{1}{R^2}\frac{d(R^2 E_R)}{dR} &= \frac{\rho_0 R}{\epsilon_0 a} \\
R^2 E_R(R) &= \begin{cases} \int_0^R \frac{\rho_0 R'^3}{\epsilon_0 a} dR', & R < a \\ \int_0^a \frac{\rho_0 R'^3}{\epsilon_0 a} dR', & R > a \end{cases} \\
E_R(R) &= \begin{cases} \frac{\rho_0 R^2}{4\varepsilon_0 a}, & R < a \\ \frac{\rho_0 a^3}{4\varepsilon_0 R^2}, & R > a \end{cases}
\end{aligned}$$

Answer:

$$E_R(R) = \begin{cases} \frac{\rho_0 R^2}{4\varepsilon_0 a}, & R < a \\ \frac{\rho_0 a^3}{4\varepsilon_0 R^2}, & R > a \end{cases}$$