[Cheng P.6-12] Two identical coaxial coils, each of N turns and radius b, are separated by a distance d, as depicted in Fig. 6-39. A current I flows in each coil in the same direction.

(a) Find the magnetic flux density $\mathbf{B} = \mathbf{a}_x B_x$ at a point midway between the coils.

(b) Show that dB_x/dx vanishes at the midpoint.

(c) Find the relation between b and d such that d^2B_x/dx^2 also vanishes at the midpoint.

Such a pair of coils are used to obtain an approximately uniform magnetic field in the midpoint region. They are known as ***Helmholtz coils***.

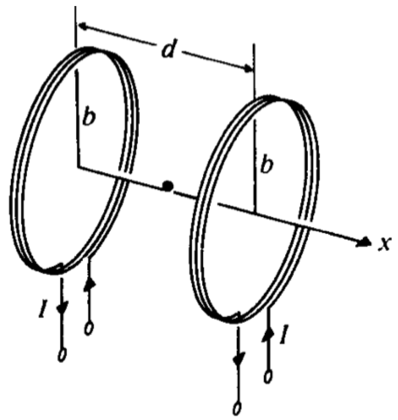
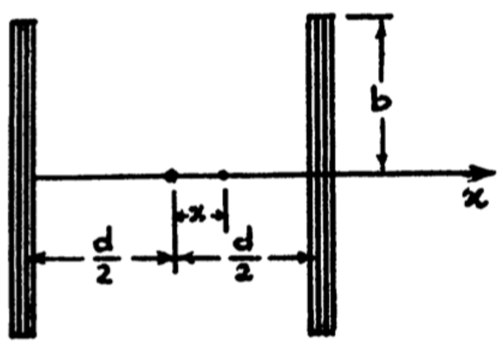


FIGURE 6–39
Helmholtz coils (Problems P.6–12).

Solution:



Using Eq. (6-38) and the diagram above

$$B_x = \frac{N\mu_0 I b^2}{2}\left\{\frac{1}{[(d/2+x)^2+b^2]^{3/2}} - \frac{1}{[(d/2-x)^2+b^2]^{3/2}}\right\}.$$

(a) At $x = 0$,

$$B_x = \frac{N\mu_0 I b^2}{[(d/2)^2+b]^{3/2}}.$$

(b)

$$\frac{dB_x}{dx} = \frac{N\mu_0 I b^2}{2}\left\{-\frac{3(d/2+x)}{[(d/2+x)^2+b^2]^{5/2}} + \frac{3(d/2-x)}{[(d/2-x)^2+b^2]^{5/2}}\right\}.$$

At the midpoint $x = 0$, $\frac{dB_x}{dx} = 0$.

(c)

$$\begin{aligned}\frac{dB_x^2}{dx^2} = -\frac{3N\mu_0 I b^2}{2}&\left\{\frac{1}{[(d/2+x)^2+b^2]^{5/2}} + \frac{1}{[(d/2-x)^2+b^2]^{5/2}}\right.\\ &\left.-\frac{5(d/2+x)^2}{[(d/2+x)^2+b^2]^{7/2}} - \frac{5(d/2-x)^2}{[(d/2-x)^2+b^2]^{7/2}}\right\}.\end{aligned}$$

At $x = 0$,

$$\frac{dB_x^2}{dx^2} = -3N\mu_0 I b^2\left\{\frac{b^2-4(d/2)^2}{[(d/2)^2+b^2]^{7/2}}\right\}.$$

This equals 0 if $b = d$.

Answer:

(a)

$$B_x = \frac{N\mu_0 I b^2}{[(d/2)^2+b]^{3/2}}$$

(b) Proof problem

(c) Proof problem, $b = d$