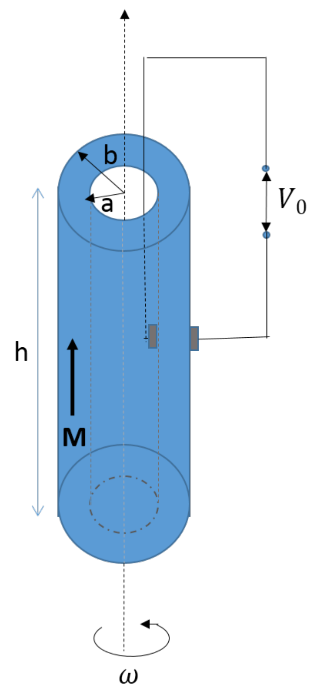
Goal: A hollow cylindrical magnet with inner radius a and outer radius b rotates about its axis at an angular frequency ω. The magnet has a uniform axial magnetization $\mathbf{M} = \mathbf{a}_z M_0$. Sliding brush contacts are provided at the inner and outer surface as shown in Fig. 7-12 (shown below). Assuming that $\mu_r = 5000$ and $\sigma = 10^7$ [S/m] for the magnet, find magnetic field intensity $\mathbf{H}$, magnetic flux density $\mathbf{B}$, open-circuit voltage V_0, and short-circuit current I.



Goal:

1. Calculate the magnetic field intensity $\mathbf{H}$ from the magnetization vector $\mathbf{M}$.

   Solution:

$$\mu_r = 1 + \chi_m$$
$$\chi_m = 4999\,.$$
$$\mathbf{H} = \frac{\mathbf{M}}{\chi_m} = \mathbf{a}_z \frac{M_0}{4999}\,.$$

2. Determine the magnetic flux density vector $\mathbf{B}$.

   Solution:

$$\mathbf{B} = \mathbf{a}_z \frac{5000}{4999} \mu_0 M_0\,.$$

3. Motion of the magnet will induce motional emf $\mathcal{V}_m$ in the loop. Find the velocity of magnet $\mathbf{u}$.

   Solution:

$$\mathbf{u} = \mathbf{a}_\phi \omega r\,.$$

4. Find the motional emf $\mathcal{V}$ induced in the loop.

   Solution:

$$\begin{aligned}\mathcal{V} &= \oint (\mathbf{u} \times \mathbf{B}) \cdot dl \\ &= \int_b^a (\mathbf{a}_\phi \omega r \times \mathbf{a}_z B) \cdot (\mathbf{a}_r\, dr) \\ &= -\frac{2500}{4999} \mu_0 M_0 \omega \left(b^2 - a^2\right)\,.\end{aligned}$$

5. What is the voltage V_0 that appears between the two terminals of the circuit if they are left open?

   Solution:

$$V_0 = \mathcal{V}\,.$$

6. Now consider the case where the terminals of the circuit are short-circuited. In this case, the induced emf $\mathcal{V}$ must equal IR where R is the resistance of the magnet for a current I flowing in the radial direction. Determine the current density J and radial electric field E in the magnet.

   Solution:

$$\begin{aligned}J &= \frac{I}{2\pi r h}\,. \\ E &= \sigma^{-1} J \\ &= \frac{I}{2\pi r h \sigma}\,.\end{aligned}$$

7. Determine I from the electric field computed in part 6.

   Solution:

$$\begin{aligned}-\int_b^a E\, dr &= \mathcal{V} \\ \frac{I}{2\pi h \sigma} \ln(b/a) &= \mathcal{V} \\ I &= \mathcal{V} \frac{2\pi h \sigma}{\ln(b/a)}\,.\end{aligned}$$

Answer:

$$\mathbf{H} = \mathbf{a}_z \frac{M_0}{4999}$$

$$\mathbf{B} = \mathbf{a}_z \frac{5000}{4999} \mu_0 M_0$$

$$V_0 = -\frac{2500}{4999} \mu_0 M_0 \omega \left(b^2 - a^2\right)$$

$$I = V_0 \frac{2\pi h \sigma}{\ln(b/a)}$$