A cylindrical bar magnet of radius b and length L has a uniform magnetization $\mathbf{M} = \mathbf{a}_z M_o$ along its axis. Determine the magnetic flux density at an arbitrary distant point.

Solution: A cylindrical bar magnet having a uniform magnetization $\mathbf{M} = \mathbf{a}_z M_0$ is equivalent to a magnetic volume current density

$$\mathbf{J}_{\mathrm{m}} = \nabla \times \mathbf{M} = 0$$

and a magnetic surface current density

$$\begin{aligned}\mathbf{J}_{\mathrm{ms}} &= \mathbf{M} \times \mathbf{a}_n \\ &= (\mathbf{a}_z M_0) \times \mathbf{a}_r \\ &= \mathbf{a}_\phi M_0\end{aligned}$$

on the cylinder wall. At a distant point, the magnetic flux density $\mathbf{B}$ due to $\mathbf{J}_{\mathrm{ms}}$ flowing on a cylindrical wall of length L and radius b is the same as that due to a circular loop of radius b carrying a current $I = M_0 L$. It is given by Eq. 6-44, which is the same as Eq. 6-73 obtained in Example 6-9 where the total dipole moment of the cylindrical magnet is $M_T = I\pi b^2 = M_0 L \pi b^2$.

Answer:

$$\begin{aligned}\mathbf{B} = \frac{\mu_o M_T}{4\pi R^3}(a_R \cos\theta + a_\theta \sin\theta),\ \text{where} \\ M_T = M_o L \pi b^2\end{aligned}$$