Consider a rectangular dielectric parallelepiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. The polarization vector in the dielectric is given by:

$$\mathbf{P} = P_o \left(\frac{x}{a}\mathbf{a}_x + \frac{y}{b}\mathbf{a}_y + \frac{z}{c}\mathbf{a}_z \right)$$

where P_o is a constant.

(a) Find the densities of volume and surface bound (polarization) charge in the parallelepiped.
(b) Show that the total bound charge in the parallelepiped is zero.

Solution:

(a)

$$\begin{aligned} \rho_p &= -\nabla \cdot \mathbf{P} \\ &= -P_0 \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \end{aligned}$$

$$\rho_{p,s} = \mathbf{a}_n \cdot \mathbf{P} = \begin{cases} P_0 & x = a, y = b, z = c \\ 0 & x = 0, y = 0, z = 0\,. \end{cases}$$

(b) Total bound charge = Volume + Surface polarization charge $= \rho_p \cdot abc + P_0 \cdot (ab + bc + ac) = 0\,.$

Answer:

(a)

$$\rho_p = -P_0 \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$$

$$\rho_{p,s} = \begin{cases} P_0 & x = a, y = b, z = c \\ 0 & x = 0, y = 0, z = 0\,. \end{cases}$$

(b) Total bound charge = $0\,.$