Goal: A finite line charge of length L carrying uniform line charge density ρ_l is coincident with the x-axis. In the plane bisecting the line charge (yz-plane) determine potential V.

Steps:

1. Choose an appropriate coordinate system.

   Solution: Cartesian

2. Determine the expressions for the differential length element $d\mathbf{l}'$ and the distance between source and observation point $|\mathbf{R} - \mathbf{R}'|$.

   Solution:

   $$\left|\mathbf{R} - \mathbf{R}'\right| = (x'^{\,2} + y^2 + z^2)^{1/2}$$

3. Evaluate the potential V as a function of y and z using performing a line integration.

   Solution:

   $$\begin{aligned} V &= \int_{-L/2}^{L/2} \frac{\rho_l}{4\pi\varepsilon_0 (x'^{\,2} + y^2 + z^2)^{1/2}}\, dx' \\ &= \frac{\rho_l}{2\pi\varepsilon_0}\left[\ln\left(\sqrt{(L/2)^2 + (z^2 + y^2)} + L/2\right) - \ln\sqrt{y^2 + z^2}\right] \end{aligned}$$

4. Next, determine the electric field $\mathbf{E}(y, z)$ in the same plane. Can you use Gauss' law to compute this electric field? Explain.

   Solution: No, Gauss' law cannot be used in the case of finite length charge.

5. Based on the symmetry of the geometry, which components of the electric field are non-zero in the bisecting plane ?

   Solution: Due to symmetry, the y-component and z-component of the electric field are non-zero in the bisecting plane.

6. Use Coulumb's law to compute the electric field.

   Solution:

   $$\mathbf{E} = (\mathbf{a}_y + \mathbf{a}_z)\frac{\rho_l}{2\pi\varepsilon_0 (y^2 + z^2)}\,\frac{L/2}{\sqrt{(L/2)^2 + y^2 + z^2}}$$

7. Determine the electric field using the expression for potential V in part (3).

   Solution: Same as part (6). Use $\mathbf{E} = -\nabla V$.

Answer:

$$V = \frac{\rho_l}{2\pi\varepsilon_0}\left[\ln\left(\sqrt{(L/2)^2 + (z^2 + y^2)} + L/2\right) - \ln\sqrt{y^2 + z^2}\right]$$