[Cheng P6-39] Determine the mutual inductance between a very long, straight wire and a conducting circular loop, as shown in Fig. 6-49.

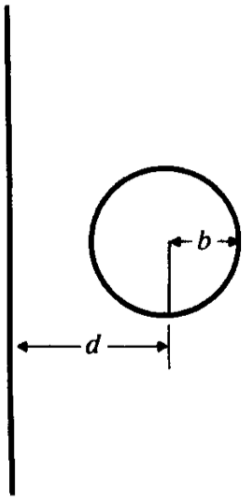
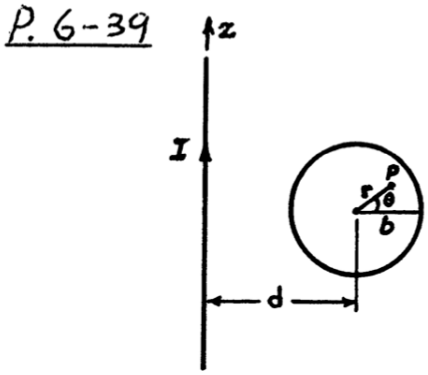


FIGURE 6–49
A long, straight wire and a conducting circular loop (Problem P.6–39).

Solution: Using the diagram below we can find the mutual inductance.



Assuming a current I, $\mathbf{B}$ from the wire at a point $P(r,\theta)$ is

$$\mathbf{B}_P = \mathbf{a}_\phi \frac{\mu_0 I}{2\pi(d + r\cos\theta)}.$$

Therefore

$$\begin{aligned}\Lambda_{12} &= \frac{\mu_0 I}{2\pi}\int_0^b \int_0^{2\pi} \frac{r\mathrm{d}r\mathrm{d}\theta}{d + r\cos\theta} \\ &= \frac{\mu_0 I}{2\pi}\int_0^b \frac{2\pi r\mathrm{d}r}{\sqrt{d^2 - r^2}} \\ &= \mu_0 I(d - \sqrt{d^2 - b^2}).\end{aligned}$$

The mutual inductance can now be found by simply dividing by the current I

$$\begin{aligned} L_{12} &= \Lambda_{12}/I \\ &= \mu_0(d - \sqrt{d^2 - b^2}). \end{aligned}$$

Answer:

$$L_{12} = \mu_o(d - \sqrt{(d^2 - b^2)})$$