
[Cheng P.3-13] Determine the work done in carrying a $-2 (\mu\text{C})$ charge from $P_1(2, 1, -1)$ to $P_2(8, 2, -1)$ in the field $\mathbf{E} = \mathbf{a}_x y + \mathbf{a}_y x$

- (a) along the parabola $x = 2y^2$,
- (b) along straight line joining P_1 and P_2 .

Solution: For both paths we will use the formula

$$W = -q \int \mathbf{E} \cdot d\mathbf{l} = -q \int x dx + y dy.$$

- (a) Along the parabola $x = 2y^2$:

$$dx = 4y dy$$

$$W = -q \int_1^2 6y^2 dy$$

$$W = 28 \mu\text{J}.$$

- (b) Along straight line $x = 6y - 4$:

$$dx = 6 dy$$

$$W = -q \int_1^2 12y - 4 dy$$

$$W = 28 \mu\text{J}.$$

As we can see, this confirms that the static electric field is conservative.

Answer:

- (a) Along the parabola: $W = 28 \mu\text{J}$.
- (b) Along straight line: $W = 28 \mu\text{J}$.