[Cheng P.6-5] A current I flows in a $w \times w$ square loop as in Fig. 6-36. Find the magnetic flux density at the off-center point $P(w/4, w/2)$.

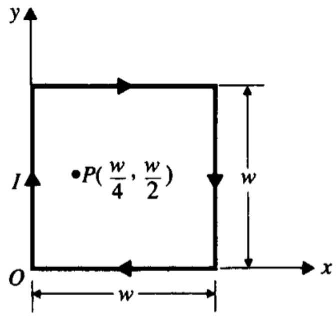
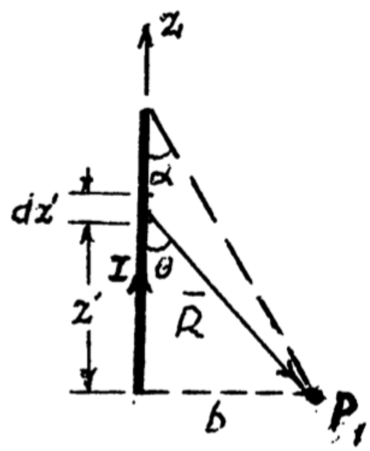


FIGURE 6–36
A square loop carrying a current I (Problem P.6–5).

Solution: We first find $\mathbf{B}_{P_1}$ at P_1 flush with one end of a wire carrying a current I and making an angle α with the other end as shown below.

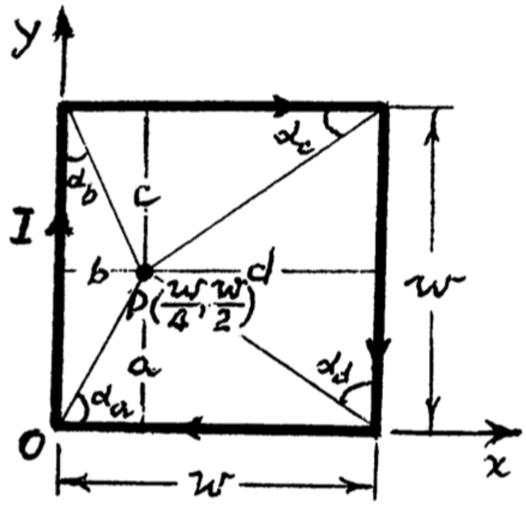


$$d\mathbf{B}_{P_1} = \frac{\mu_0 I}{4\pi R^2}\mathbf{a}_z dz' \times \mathbf{a}_R \qquad z' = \cot b,\ dz' = -b\csc^2\theta d\theta$$

$$d\mathbf{B}_{P_1} = \frac{\mu_0 I}{4\pi R^2}(\mathbf{a}_\phi \sin\theta d\theta) \qquad R = b\csc\theta,\ \mathbf{a}_z \times \mathbf{a}_R = \mathbf{a}_\phi \sin\theta$$

$$\mathbf{B}_{P_1} = -\mathbf{a}_\phi \frac{\mu_0 I}{4\pi b}\int_{\pi/2}^{\alpha} \sin\theta d\theta = \mathbf{a}_\phi \frac{\mu_0 I}{4\pi b}\cos\theta$$

Now we can apply this result to the four-sided loop below.



Here we have

$$\mathbf{B}_P = \mathbf{a}_z \frac{\mu_0 I}{4\pi}\left(\frac{1}{a}\cos\alpha_a + \frac{1}{b}\sin\alpha_a + \frac{1}{b}\cos\alpha_b + \frac{1}{c}\sin\alpha_b + \frac{1}{c}\cos\alpha_c + \frac{1}{d}\sin\alpha_c + \frac{1}{d}\cos\alpha_d + \frac{1}{a}\sin\alpha_d\right)$$

For this problem $a = c = \frac{w}{2}$, $b = \frac{w}{4}$, and $d = \frac{3}{4}w$. Likewise $\alpha_a = \tan^{-1} 2 = 63.2°$, $\alpha_b = 90° - 63.2° = 26.6°$, $\alpha_c = \tan^{-1}\frac{2}{3} = 33.7°$, and $\alpha_d = 56.3°$. Finally,

$$\mathbf{B}_P = \mathbf{a}_z\, 3.44\frac{\mu_0 I}{\pi w}.$$

Answer:

$$\mathbf{B}_P = \mathbf{a}_z\, 3.44\frac{\mu_0 I}{\pi w}$$