An infinite sheet of surface charge density ρ_s on the xy-plane has a hole of radius a on it, centered at the origin of the coordinate system. Find the electric field intensity at an arbitrary point on the positive z-axis.

Solution: We can find the total electric field by integrating the electric field due to each differential area on the infinite sheet, and recognizing that due to symmetry only the E_z component of the electric field will exist. Denoting r as the distance from the origin to a point on the xy-plane,

$$
\begin{aligned}
d\mathbf{E}_z &= d\mathbf{E}\frac{z}{\sqrt{r^2+z^2}} \\
&= \frac{1}{4\pi\epsilon_0}\frac{\rho_s ds}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}}
\end{aligned}
$$

The total electric field can then be found by integrating this expression over the location where the charge exists

$$
\begin{aligned}
\mathbf{E}_z &= \frac{z}{4\pi\epsilon_0}\int_0^{2\pi}\int_a^{\infty}\frac{\rho_s r dr d\phi}{(r^2+z^2)^{3/2}}\mathbf{a}_z \\
&= \frac{z\rho_s}{2\epsilon_0}\int_a^{\infty}\frac{rdr}{(r^2+z^2)^{3/2}}\mathbf{a}_z \\
&= \frac{\rho_s}{2\epsilon_0}\frac{z}{\sqrt{a^2+z^2}}\mathbf{a}_z
\end{aligned}
$$

Alternatively, the total electric field can be found using superposition of the electric field due to a positively charged infinite sheet plus a negatively charged disk. The result will be the same.

Answer:

$$
\mathbf{E}_z = \frac{\rho_s}{2\epsilon_0}\frac{z}{\sqrt{a^2+z^2}}\mathbf{a}_z
$$