An infinitely long straight conductor with a circular cross section of radius b carries a steady current I. Determine the magnetic flux density inside and outside the conductor.

Solution: First we note that this is a problem with cylindrical symmetry and that Ampère's circuital law can be used to our advantage. If we align the conductor along the z-axis, the magnetic flux density $\mathbf{B}$ will by ϕ-directed and will be constant along any circular path around the z-axis. Figure 6-2(a) shows a cross section of the conductor and the two circular paths of integration, C_1, and C_2, inside and outside, respectively, the current-carrying conductor. Note again that the directions of C_1 and C_2 and the direction of I follow the right hand rule. (When the fingers of the right hand follow the directions of C_1 and C_2, the thumb of the right hand points to the direction of I.)

(a) *Inside the conductor:*

$$\begin{aligned} \mathbf{B}_1 &= \mathbf{a}_\phi B_{\phi 1} \\ d\ell &= \mathbf{a}_\phi r_1 d\phi \\ \oint_{C_1} \mathbf{B}_1 \cdot d\ell &= \int_0^{2\pi} B_{\phi 1} r_1 d\phi = 2\pi r_1 B_{\phi 1}. \end{aligned}$$

The current through the area enclosed by C_1 is

$$I_1 = \frac{\pi r_1^2}{\pi b^2} I = \left(\frac{r_1}{b}\right)^2 I.$$

Therefore, from Ampère's circuital law,

$$\mathbf{B}_1 = \mathbf{a}_\phi B_{\phi 1} = \mathbf{a}_\phi \frac{\mu_0 r_1 I}{2\pi b^2}, \; r_1 \le b. \tag{1}$$

(b) *Outside the conductor:*

$$\begin{aligned} \mathbf{B}_2 &= \mathbf{a}_\phi B_{\phi 2} \\ d\ell &= \mathbf{a}_\phi r_2 d\phi \\ \oint_{C_2} \mathbf{B}_2 \cdot d\ell &= \int_0^{2\pi} B_{\phi 2} r_2 d\phi = 2\pi r_2 B_{\phi 2}. \end{aligned}$$

Path C_2 outside the conductor encloses the total current I. Hence

$$\mathbf{B}_2 = \mathbf{a}_\phi B_{\phi 2} = \mathbf{a}_\phi \frac{\mu_0 I}{2\pi r_2}, \quad r_2 \ge b. \tag{2}$$

Examination of (1) and (2) reveals that the magnitude of $\mathbf{B}$ increases linearly with r_1 from 0 until $r_1 = b$, after which it decreases inversely with r_2. The variation of B_ϕ versus r is sketched in Fig. 6-2(b).

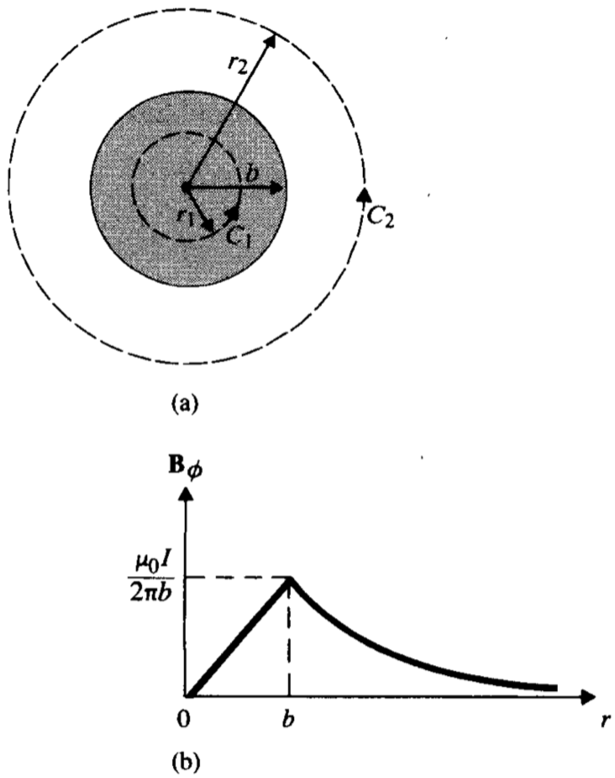

FIGURE 6–2
Magnetic flux density of an infinitely long circular conductor carrying a current I out of paper (Example 6–1).

Answer:

$$\text{For } r \leq b: \; B_1 = a_\phi \frac{\mu_o r I}{2\pi b^2}$$
$$\text{For } r \geq b: \; B_1 = a_\phi \frac{\mu_o I}{2\pi r}$$