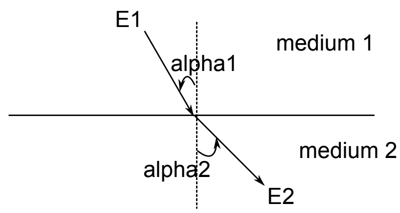
Goal: Two lossy dielectric media with permittivities and conductivities $(\varepsilon_1, \sigma_1)$ and $(\varepsilon_2, \sigma_2)$ are in contact. An electric field with a magnitude E_1 is incident from medium 1 upon the interface at an angle α_1 measured from the common normal, as in Fig. 5-10. Find the electric field in medium 2 E_2 and the surface charge density. How would these change if the media were perfect dielectrics?



Steps:

1. State the boundary conditions for the E-field, D-field and current density across the boundary.
   Solution:

$$\begin{aligned} E_{1t} &= E_{2t} \\ J_{1n} &= J_{2n} \\ D_{1n} - D_{2n} &= \rho_s \end{aligned}$$

2. Find the magnitude and direction of $\mathbf{E}_2$ in medium 2.
   Solution: From the boundary conditions for E-field and current density:

$$\begin{aligned} E_2 \sin\alpha_2 &= E_1 \sin\alpha_1 \\ \sigma_2 E_2 \cos\alpha_2 &= \sigma_1 E_1 \cos\alpha_1 \\ E_2 &= E_1 \sqrt{\sin^2\alpha_1 + \left(\frac{\sigma_1}{\sigma_2}\cos\alpha_1\right)^2} \qquad (A) \\ \alpha_2 &= \tan^{-1}\left(\frac{\sigma_2}{\sigma_1}\tan\alpha_1\right) \qquad (B) \end{aligned}$$

3. Find the surface charge density at the interface.
   Solution:

$$\begin{aligned} \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} &= \rho_s \\ \rho_s &= \left(\frac{\sigma_1}{\sigma_2}\varepsilon_2 - \varepsilon_1\right) E_{1n} = \left(\frac{\sigma_1}{\sigma_2}\varepsilon_2 - \varepsilon_1\right) E_1 \cos\alpha_1 \end{aligned}$$

4. Compare the results in parts (a) and (b) with the case in which both media are perfect dielectrics.
   Solution: If both media are perfect dielectrics, then $\sigma_1 = \sigma_2 = 0$ and Eq. A becomes

$$E_2 = E_1 \sqrt{\sin^2\alpha_1 + \left(\frac{\varepsilon_1}{\varepsilon_2}\cos\alpha_1\right)^2}$$

and Eq. B becomes

$$\alpha_2 = \tan^{-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\tan\alpha_1\right)$$

Answer:

$$\begin{aligned}
E_2 &= E_1\sqrt{\sin^2\alpha_1 + \left(\frac{\sigma_1}{\sigma_2}\cos\alpha_1\right)^2} \\
\alpha_2 &= \tan^{-1}\left(\frac{\sigma_2}{\sigma_1}\tan\alpha_1\right)
\end{aligned}$$

$$\rho_s = \left(\frac{\sigma_1}{\sigma_2}\varepsilon_2 - \varepsilon_1\right)E_1\cos\alpha_1$$

For perfect dielectrics:

$$E_2 = E_1\sqrt{\sin^2\alpha_1 + \left(\frac{\varepsilon_1}{\varepsilon_2}\cos\alpha_1\right)^2}$$

$$\alpha_2 = \tan^{-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\tan\alpha_1\right)$$