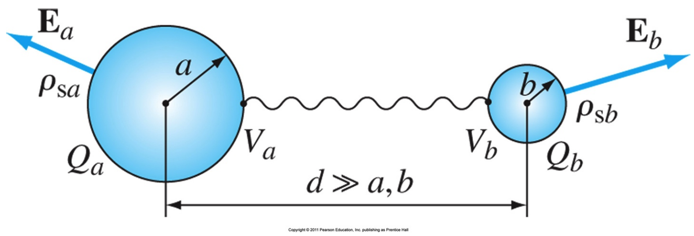
Goal: Consider the system of two metallic spheres connected by a wire as shown in the figure below. Assume that $a = 5\,\text{cm}$, $b = 1\,\text{cm}$, and $d = 1\,\text{m}$, as well as that the total charge of the two spheres is $Q = 600\,\text{pC}$. Find the potential of the spheres and the electric field intensities E_a, E_b near the surfaces of the spheres.



Steps:

1. Since the two metallic sphere are connected by a wire, the total charge distributes itself between two spheres such that the potential on each sphere is equal, i.e. $V_a = V_b$. Provide expressions for potentials V_a and V_b assuming uniform surface charge densities ρ_{sa} and ρ_{sb} on the spheres.

   Solution:

$$V_a = \frac{Q_a}{4\pi\varepsilon_0 a}$$
$$V_b = \frac{Q_b}{4\pi\varepsilon_0 b}$$

   where,

$$Q_a = \rho_{sa} 4\pi a^2$$
$$Q_b = \rho_{sb} 4\pi b^2 \,.$$

2. Compute charges Q_a and Q_b, and surface charge densities ρ_{sa} and ρ_{sb}.

   Solution:

$$Q = Q_a + Q_b \,.$$

   By equating V_a and V_b in part 1 and using the above relation, we get

$$Q_a = \frac{a}{a+b} Q = 500\,\text{pC}\,,$$
$$Q_b = \frac{b}{a+b} Q = 100\,\text{pC}\,.$$

3. What is the value of potential V_a and V_b?

   Solution:

$$V_a = V_b = 90\,\text{V}\,.$$

4. Use the electrostatic boundary condition for perfect conductors to find the electric fields $\mathbf{E}_a$ and $\mathbf{E}_b$.

   Solution:

$$
\begin{aligned}
E_a &= \frac{\rho_{sa}}{\varepsilon_0} = \frac{Q_a}{4\pi\varepsilon_0 a^2} = 1.8\,\text{kV/m} \\
E_b &= \frac{\rho_{sb}}{\varepsilon_0} = \frac{Q_b}{4\pi\varepsilon_0 b^2} = 9\,\text{kV/m}
\end{aligned}
$$

Answer:

$$
V_a = V_b = 90\ V
$$
$$
E_a = 1.8\,kV/m, \;\; E_b = 9\,kV/m
$$