Goal: Figure 6-37 shows an infinitely long solenoid with air core having a radius b and n closely wound turns per unit length. The windings are slanted at an angle α and carry a current I. Determine the magnetic flux density both inside and outside the solenoid.

Fig. 6-37. A long solenoid with closely wound windings carrying a current I.

Steps:

1. Decompose the current I into a component along $\mathbf{a}_z$ and a component along $\mathbf{a}_\phi$.
   Solution: In the $\mathbf{a}_\phi$ direction
   $$I_\phi = nI\cos\alpha$$
   In the $\mathbf{a}_z$ direction
   $$I_z = 2\pi bnI\sin\alpha$$

2. What is the contribution of the $\mathbf{a}_z$ component of the current to the B-field inside the solenoid?
   Solution: zero.

3. What is the contribution of the $\mathbf{a}_z$ component of the current to the B-field outside the solenoid?
   Solution:
   $$\mathbf{B}_1 = \mathbf{a}_\phi \mu_o \frac{bnI}{r}\sin\alpha$$

4. What is the contribution of the $\mathbf{a}_\phi$ component of the current to the B-field inside the solenoid?
   Solution:
   $$\mathbf{B}_2 = \mathbf{a}_z \mu_o nI\cos\alpha$$

5. What is the contribution of the $\mathbf{a}_\phi$ component of the current to the B-field outside the solenoid?
   Solution: zero.

6. What is the total B-field inside and outside the solenoid?
   Solution: Using superposition, add up all the fields above.

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$$

Answer:

$$\text{Inside:} \quad \mathbf{B} = a_z\,\mu_o\,nI\cos\alpha$$
$$\text{Outside:} \quad \mathbf{B} = a_z\,\mu_o\frac{bnI}{r}\sin\alpha$$