Goal: Two identical coaxial coils, each of N turns and radius b, are separated by a distance d, as depicted in Fig. Q6-39. A current I flows in each coil in the same direction. Find the magnetic flux density midway between the coils, and show that $\frac{dB_x}{dx}$ vanishes at the midpoint. Find the relation between b and d such that $\frac{d^2B_x}{dx^2}$ also vanishes at the midpoint. Such a pair of coils are used to obtain an approximately uniform magnetic field in the midpoint region. They are known as Helmholtz coils.

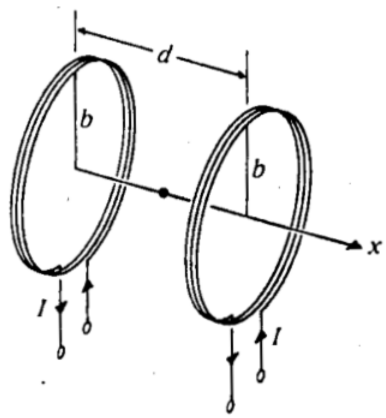

Fig. 6-39

Steps:

1. Find the magnetic flux density $\mathbf{B} = \mathbf{a}_x B_x$ at a point midway between the coils.
   Solution: Use the equation

$$B_x = \frac{N\mu_o I b^2}{2}\left(\frac{1}{((d/2+x)^2+b^2)^{3/2}} + \frac{1}{((d/2-x)^2+b^2)^{3/2}}\right)$$

   Then at the midpoint

$$B_x = \frac{N\mu_o I b^2}{((d/2)^2+b^2)^{3/2}}$$

2. Show that $\frac{dB_x}{dx}$ vanishes at the midpoint.
   Solution:

$$\frac{dB_x}{dx} = \frac{N\mu_o I b^2}{2}\left(-\frac{3(d/2+x)}{((d/2+x)^2+b^2)^{5/2}} + \frac{3(d/2-x)}{((d/2-x)^2+b^2)^{5/2}}\right)$$

   Clearly, the equation is zero for $x = 0$.

3. Find the relation between b and d such that $\frac{d^2B_x}{dx^2}$ also vanishes at the midpoint.
   Solution:

$$\frac{d^2B_x}{dx^2} = -\frac{3N\mu_o I b^2}{2}\left(\frac{1}{((d/2+x)^2+b^2)^{5/2}} - \frac{5(d/2+x)^2}{((d/2+x)^2+b^2)^{7/2}} + \frac{1}{((d/2-x)^2+b^2)^{5/2}} - \frac{5(d/2-x)^2}{((d/2-x)^2+b^2)^{7/2}}\right)$$

   Now at $x = 0$

$$\frac{d^2B_x}{dx^2} = -3N\mu_o I b^2\left(\frac{b^2-4(d/2)^2}{((d/2)^2+b^2)^{7/2}}\right)$$

   The above equation is zero if $b = d$.

Answer:

(a)

$$B_x = \frac{N\mu_o I b^2}{2}\left(\frac{1}{((d/2+x)^2+b^2)^{3/2}} + \frac{1}{((d/2-x)^2+b^2)^{3/2}}\right)$$

(b) Proof problem

(c) b = d