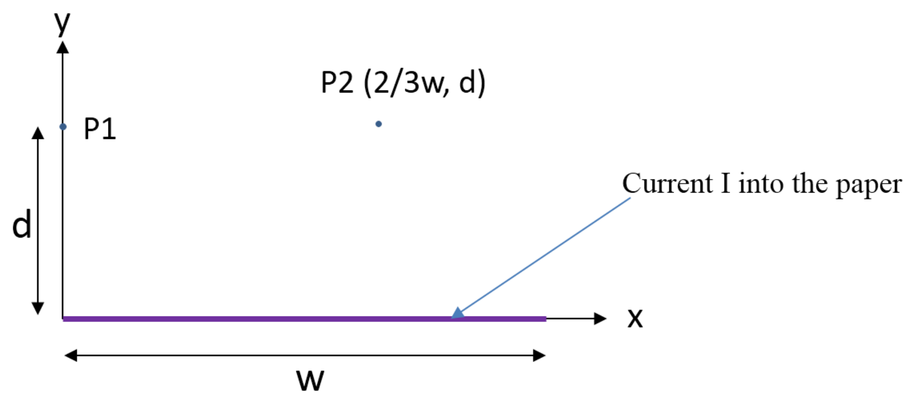
Goal: A current I flows lengthwise in a very long, thin conducting sheet of width w, as shown below. Assuming that the current flows into the paper, determine the magnetic flux density $\mathbf{B}$ at points $P_1(0, d)$ and $P_2(2w/3, d)$.



Steps:

1. The problem can be solved with superposition. What is the differential current element?
   Solution:
$$d\mathbf{I}' = \frac{I}{w} d\mathbf{l}' = -\frac{I}{w} dx' dz' \mathbf{a}_z$$

2. Determine the observation vector $\mathbf{R}$
   Solution:
$$\mathbf{R} = d\mathbf{a}_y$$

3. Determine the source vector $\mathbf{R}'$
   Solution:
$$\mathbf{R}' = x'\mathbf{a}_x + z'\mathbf{a}_z$$

4. What is the differential magnetic flux density $d\mathbf{B}$
   Solution:
$$\begin{aligned} d\mathbf{B} &= \frac{\mu_0 I}{4\pi w} d\mathbf{l}' \times (\mathbf{R} - \mathbf{R}') \\ &= \frac{\mu_0 I}{4\pi w} \frac{d\, dz' dx' \mathbf{a}_x + x' dz' dx' \mathbf{a}_y}{(d^2 + x'^2 + z'^2)^{3/2}} \end{aligned}$$

5. Solve the superposition integral.

Solution:

$$
\begin{aligned}
\mathbf{B} &= \frac{\mu_0 I}{4\pi w}\int_0^w \int_{-\infty}^{\infty} \frac{d\,dz'dx'\mathbf{a}_x + x'dz'dx'\mathbf{a}_y}{(d^2+x'^2+z'^2)^{3/2}} \\
&= \frac{\mu_0 I}{4\pi w}\int_0^w (d\mathbf{a}_x + x'\mathbf{a}_y)\left(\int_{-\infty}^{\infty} \frac{dz}{(d^2+x'^2+z'^2)^{3/2}}\right) dx' \\
&= \frac{\mu_0 I}{2\pi w}\int_0^w (d\mathbf{a}_x + x'\mathbf{a}_y)\left(\frac{1}{d^2+x'^2}\right) dx' \\
&= \frac{\mu_0 I}{2\pi w}\tan^{-1}\left(\frac{w}{d}\right)\mathbf{a}_x + \frac{\mu_0 I}{4\pi w}\ln\left(1+\frac{w^2}{d^2}\right)\mathbf{a}_y
\end{aligned}
$$

6. Use the answer of part (5) to find magnetic flux density $\mathbf{B}_2$ at point $P_2(2w/3, d)$.

Solution: Magnetic flux density due to current strip to the right (R) and to left are given by (L)

$$
\begin{aligned}
\mathbf{B}_L &= \frac{\mu_0 I}{2\pi w}\tan^{-1}\left(\frac{2w}{3d}\right)\mathbf{a}_x - \frac{\mu_0 I}{4\pi w}\ln\left(1+\frac{4w^2}{9d^2}\right)\mathbf{a}_y \\
\mathbf{B}_R &= \frac{\mu_0 I}{2\pi w}\tan^{-1}\left(\frac{w}{3d}\right)\mathbf{a}_x + \frac{\mu_0 I}{4\pi w}\ln\left(1+\frac{w^2}{9d^2}\right)\mathbf{a}_y \\
\mathbf{B} &= \mathbf{B}_L + \mathbf{B}_R
\end{aligned}
$$

Answer:

$$
\mathbf{B_1} = \frac{\mu_0 I}{2\pi w}\tan^{-1}\left(\frac{w}{d}\right)\mathbf{a}_x + \frac{\mu_0 I}{4\pi w}\ln\left(1+\frac{w^2}{d^2}\right)\mathbf{a}_y
$$

$$
\begin{aligned}
\mathbf{B}_L &= \frac{\mu_0 I}{2\pi w}\tan^{-1}\left(\frac{2w}{3d}\right)\mathbf{a}_x - \frac{\mu_0 I}{4\pi w}\ln\left(1+\frac{4w^2}{9d^2}\right)\mathbf{a}_y \\
\mathbf{B}_R &= \frac{\mu_0 I}{2\pi w}\tan^{-1}\left(\frac{w}{3d}\right)\mathbf{a}_x + \frac{\mu_0 I}{4\pi w}\ln\left(1+\frac{w^2}{9d^2}\right)\mathbf{a}_y \\
\mathbf{B_2} &= \mathbf{B}_L + \mathbf{B}_R
\end{aligned}
$$