Goal: A circular rod of magnetic material with permeability μ is inserted coaxially in the long solenoid of Fig. 6-4. The radius of the rod, a, is less than the inner radius, b, of the solenoid. The solenoid has n turns per unit length and carries a current I. Find $\mathbf{H}$, $\mathbf{B}$, and $\mathbf{M}$ inside the solenoid, as well as current densities $\mathbf{J}_m$ and $\mathbf{J}_{m,s}$.

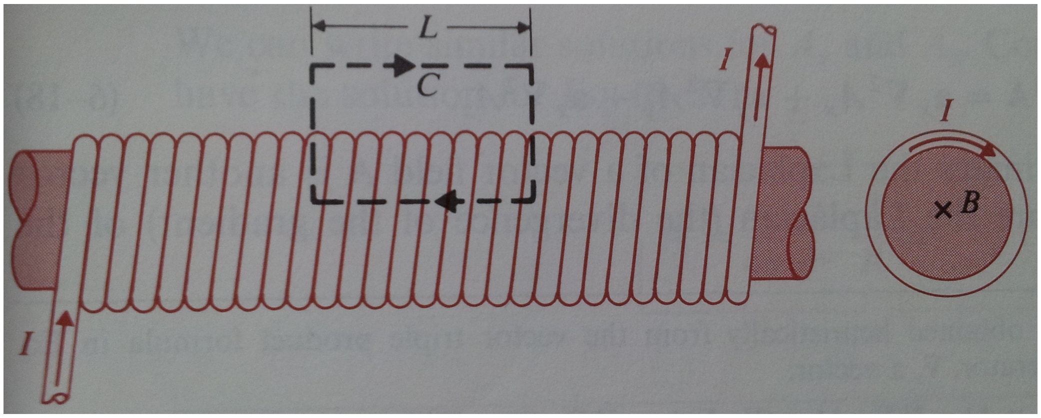

Fig. 6-4. A long solenoid with closely wound windings carrying a current I.

Steps:

1. What is the $\mathbf{H}$-field inside the solenoid?
   Solution:

$$\mathbf{H} = \mathbf{a}_z nI$$

2. What is the $\mathbf{B}$-field inside the solenoid?
   Solution:

$$\begin{aligned} \mathbf{B} &= \mathbf{a}_z \mu nI \quad \text{for} \quad r < a \\ \mathbf{B} &= \mathbf{a}_z \mu_o nI \quad \text{for} \quad a < r < b \end{aligned}$$

3. What is the $\mathbf{M}$ inside the solenoid?
   Solution:

$$\mathbf{M} = \frac{\mathbf{B}}{\mu_o} - \mathbf{H}$$

   For $r < a$,

$$\mathbf{M} = \mathbf{a}_z \left(\frac{\mu}{\mu_o} - 1 \right) nI$$

   For $a < r < b$,

$$\mathbf{M} = 0$$

4. Calculate the current densities $\mathbf{J}_m$ and $\mathbf{J}_{m,s}$ equivalent to magnetization.
Solution:

$$\mathbf{J}_m = \nabla \times \mathbf{M} = 0$$

$$\mathbf{J}_{ms} = \mathbf{M} \times \mathbf{a}_n = (\mathbf{a}_z \times \mathbf{a}_r)\left(\frac{\mu}{\mu_o} - 1\right) nI = \mathbf{a}_\phi \left(\frac{\mu}{\mu_o} - 1\right) nI$$

Answer:

$$\mathbf{H} = \mathbf{a}_z nI$$

$$\begin{aligned} \mathbf{B} &= \mathbf{a}_z \mu nI \quad \text{for} \quad r < a \\ \mathbf{B} &= \mathbf{a}_z \mu_o nI \quad \text{for} \quad a < r < b \end{aligned}$$

For $r < a$,

$$\mathbf{M} = \mathbf{a}_z \left(\frac{\mu}{\mu_o} - 1\right) nI$$

For $a < r < b$,

$$\mathbf{M} = 0$$

$$\mathbf{J}_{ms} = \mathbf{a}_\phi \left(\frac{\mu}{\mu_o} - 1\right) nI$$