An infinitely long cylinder of radius a in free space is charged with a volume charge density

$$\rho_u = \rho_0 \left(1 - \frac{r}{a}\right)$$

for $0 \leq r \leq a$, where ρ_0 is a constant and r is the radial coordinate in the cylindrical coordinate system. Find the charge per unit length on the cylinder.

Solution:

$$\begin{aligned}
\rho_l &= \int_0^{2\pi} \int_0^a \rho_u \, r dr d\phi \\
&= 2\pi\rho_0 \int_0^a \left(r - \frac{r^2}{a}\right) dr \\
&= 2\pi\rho_0 \left(\frac{a^2}{2} - \frac{a^3}{3a}\right) dr \\
&= \frac{\pi\rho_o a^2}{3}
\end{aligned}$$

Answer:

$$\rho_l = \frac{\pi\rho_o a^2}{3}$$