Goal: A suggested scheme for reducing eddy-current power loss in transformer cores with a circular cross section is to divide the cores into a large number of small insulated filamentary parts. As illustrated in Fig. 7-12, the section shown in part (a) is replaced by that in part (b). Assuming that $B(t) = B_0 \sin \omega t$ and that N filamentary areas fill 95% of the original cross-sectional area, find average eddy-current power loss in the section of core of height h in Fig. 7-12 (a), and Fig. 7-12 (b).

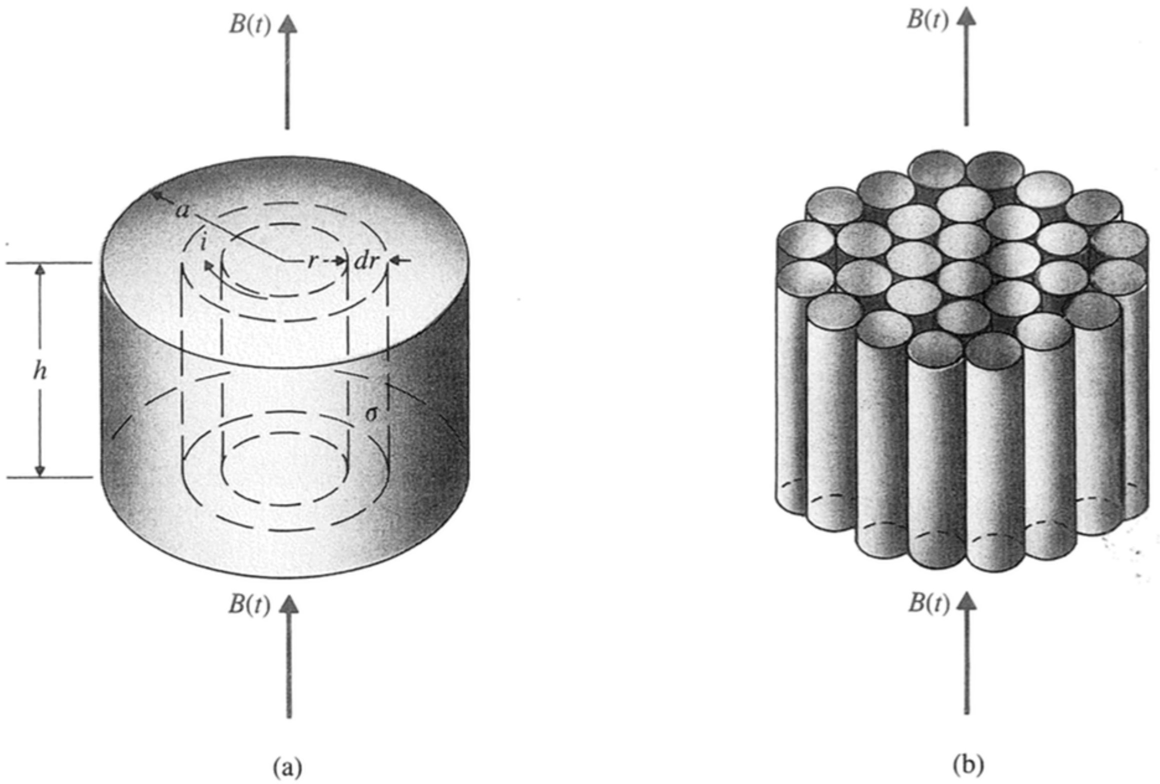

Figure 7-12

Steps:

1. We decompose the cylinder in Figure 7-12 (a) into thin cylindrical rings of of thickness dr and radius r as shown. For the ring, determine:

   a) the magnetic flux $\Phi(t)$ through the ring's cross section

   Solution:

$$\Phi = \pi r^2 B(t)$$

   b) the emf $\mathcal{V}$ induced on the ring (make sure that the direction of $\mathcal{V}$ is consistent with the direction of current (clockwise) shown in Fig. 7-12 (a))

   Solution:

$$\mathcal{V} = iR_r = \frac{d\Phi}{dt} = \pi r^2 \frac{dB(t)}{d(t)} .$$

c) the resistance R_r of the circular ring,

Solution:

$$R_r = \frac{2\pi r}{\sigma h dr}$$

d) the current i which is induced in the ring (note that $\mathcal{V} = iR_r$)

Solution:

$$\begin{aligned} i &= \mathcal{V}/R_r \\ &= \frac{\sigma h}{2} r dr \left(\frac{dB}{dt}\right) \end{aligned}$$

e) the power dP_r dissipated in the ring

Solution:

$$\begin{aligned} dP_r &= i^2 R_r \\ &= \frac{\pi\sigma h}{2} r^3 dr \left(\frac{dB}{dt}\right)^2 . \end{aligned}$$

2. Integrate dP_r to find the instantaneous and average power dissipated in the transformer core illustrated in Fig 7-12 (a).

Solution:

$$\begin{aligned} P &= \int_0^a dP_r \\ &= \frac{\pi\sigma h}{8} a^4 \omega^2 B_0^2 \cos^2 \omega t . \\ P_{av} &= \frac{\pi\sigma h}{16} a^4 \omega^2 B_0^2 . \end{aligned}$$

3. What is the radius b of each filament in the transformer core shown in Fig. 7-12(b)?

Solution:

$$\begin{aligned} N\pi b^2 &= 0.95\pi a^2 \\ b &= \sqrt{\frac{0.95}{N}} a . \end{aligned}$$

4. Using the result of part 2., determine the total power loss in the transformer core shown in 7-12 (b).

Solution: Substitute $a \to \sqrt{\frac{0.95}{N}}a$ in (2) to get power loss in single filament.

$$P' = N\left(\frac{\pi\sigma h}{8}\right)\left(a\sqrt{\frac{0.95}{N}}\right)^4 \omega^2 B_0^2 \cos^2 \omega t$$
$$= \frac{0.95^2}{N}P.$$
$$P'_{av} = \frac{0.95^2}{N}P_{av}\,.$$

Answer:

$$P'_{av} = \frac{0.95^2}{N}\frac{\pi\sigma h}{16}a^4\omega^2 B_0^2\,.$$