[Cheng P.7-4] A conducting equilateral triangular loop is placed near a very long straight wire, shown in Fig. 6-48, with $d = b/2$. A current $i(t) = I \sin \omega t$ flows in the straight wire.

(a) Determine the voltage registered by a high-impedance rms voltmeter inserted in the loop.

(b) Determine the voltmeter reading when the triangular loop is rotated by $60°$ about a perpendicular axis through its center.

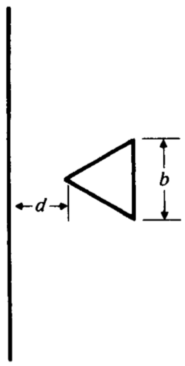
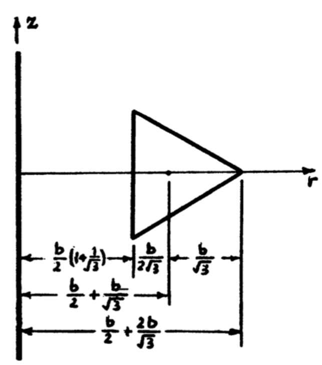


FIGURE 6–48
A long, straight wire and a conducting equilateral triangular loop (Problem P.6–38).

Solution: The magnetic flux density from the wire is given by

$$\mathbf{B} = \mathbf{a}_\phi \frac{\mu_0 I \sin \omega t}{2\pi r}.$$

The flux through the loop is $\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$, where $d\mathbf{S} = \mathbf{a}_\phi 2z dr$ and $z = \frac{\sqrt{3}}{3}(r - d)$. We will use the sketch below to solve the problem.



(a)

$$\Phi = \frac{\sqrt{3}}{3} \frac{\mu_0 I \sin \omega t}{\pi} \int_d^{\frac{\sqrt{3}}{2}(b+d)} \left(1 - \frac{d}{r}\right) dr$$
$$= \frac{\sqrt{3}\mu_0 I \sin \omega t}{3\pi} \left[\frac{\sqrt{3}}{2} b - d \ln\left(\frac{\frac{\sqrt{3}}{2} b + d}{d}\right)\right].\ d = b/2.$$

The voltage can now be calculated.

$$
\begin{aligned}
V &= -\frac{d\Phi}{dt} \\
&= -\frac{\sqrt{3}\mu_0 I\omega b}{3\pi}\left[\frac{\sqrt{3}}{2}-\frac{1}{2}\ln(\sqrt{3}+1)\right]\cos\omega t \\
&= V_m \cos\omega t. \\
V_{\text{rms}} &= \frac{\sqrt{2}}{2}|V_m| \\
&= \frac{\sqrt{6}\mu_0 I\omega b}{12\pi}\left[\sqrt{3}-\ln(\sqrt{3}+1)\right] \\
&= 0.0472\mu_0 I\omega b \text{ (V)}.
\end{aligned}
$$

(b)

$$
\begin{aligned}
z &= \frac{1}{\sqrt{3}}\left[\frac{b}{2}\left(1+\frac{4}{3}\right)-r\right]; \\
\int \mathbf{B}\cdot d\mathbf{S} &= -\frac{\mu_0 I\sin\omega t}{\sqrt{3}\pi}\int_{\frac{b}{2}\left(1+\frac{1}{\sqrt{3}}\right)}^{\frac{b}{2}\left(1+\frac{4}{\sqrt{3}}\right)}\left[\frac{b}{2}\left(1+\frac{4}{\sqrt{3}}\right)\frac{1}{r}-1\right]dr \\
&= -\frac{\mu_0 I\sin\omega t}{\sqrt{3}\pi}\left[\frac{b}{2}\left(1+\frac{4}{\sqrt{3}}\right)\ln\left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right)-\frac{\sqrt{3}}{2}b\right]. \\
V_{\text{rms}} &= \frac{1}{\sqrt{2}}\frac{\mu_0 I\omega}{\sqrt{3}\pi}\frac{b}{2}\left|\left(1+\frac{4}{\sqrt{3}}\right)\ln\left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right)-\sqrt{3}\right| \qquad = 0.0469\mu_0 I\omega b \text{ (V)}.
\end{aligned}
$$

Answer:

(a) $V_{\text{rms}} = 0.0472\mu_0 I\omega b$ (V).

(b) $V_{\text{rms}} = 0.0469\mu_0 I\omega b$ (V).