[Cheng P.3-48] A parallel-plate capacitor of width w, length L, and a separation d has a solid dielectric slab of permittivity ε in the space between the plates. The capacitor is charged to a voltage V_0 by a battery, as indicated in Fig. 3-45. Assuming that the dielectric slab is withdrawn to the position shown, determine the force acting on the slab

(a) with the switch closed,

(b) after the switch is first opened.

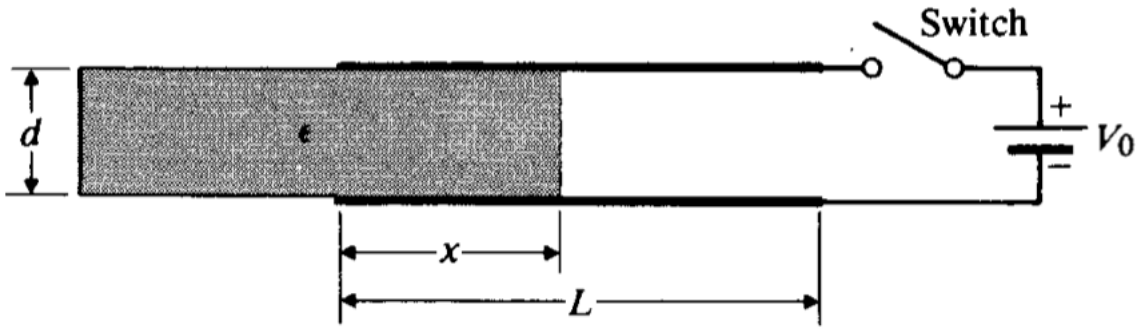

FIGURE 3–45
A partially filled parallel-plate capacitor (Problem P.3–48).

Solution:

(a) Switch closed $\rightarrow V = V_0 =$ constant.

$$
\begin{aligned}
W_e &= \frac{1}{2} C V^2, \\
C &= \frac{w}{d}\left[\varepsilon x + \varepsilon_0 (L - x)\right], \\
\mathbf{F}_V &= \nabla W_e \\
&= \mathbf{a}_x \frac{V_0^2}{2} \frac{\partial C}{\partial x} \\
&= \mathbf{a}_x \frac{V_0 w}{2d} (\varepsilon - \varepsilon_0)
\end{aligned}
$$

(b) Switch open $\rightarrow Q =$ constant $= CV_0$.

$$
\begin{aligned}
W_e &= \frac{Q^2}{2C}, \\
\mathbf{F}_Q &= -\nabla W_e = -\mathbf{a}_x \frac{Q^2}{2} \frac{\partial}{\partial x}\left(\frac{1}{C}\right) \\
&= \mathbf{a}_x \frac{Q^2 d}{2w} \frac{\varepsilon - \varepsilon_0}{\left[\varepsilon x + \varepsilon_0 (L - x)\right]^2} \\
&= \mathbf{a}_x \frac{V_0 w}{2d} (\varepsilon - \varepsilon_0)
\end{aligned}
$$

Answer:

(a)

$$\mathbf{F}_V = \mathbf{a}_x \frac{V_0 w}{2d}(\varepsilon - \varepsilon_0)$$

(b)

$$\mathbf{F}_Q = \mathbf{a}_x \frac{V_0 w}{2d}(\varepsilon - \varepsilon_0)$$