[Cheng P.6-44] Determine the force per unit length between two parallel, long, thin conducting strips of equal width w. The strips are at a distance d apart and carry currents I_1 and I_2 in opposite directions as in Fig. 6-52.

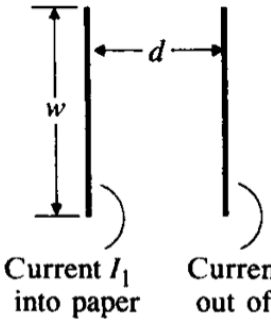

FIGURE 6–52
Cross section of two parallel strips carrying opposite currents (Problem P.6–44).

Solution: From problem P.6-4 we have the y-component of the magnetic flux density at an arbitrary point $P(d, y)$ on the right-hand strip due to I_1 in the left-hand strip

$$B_{Py} = -\frac{\mu_0 I_1}{2\pi w}\left[\tan^{-1}\left(\frac{y}{d}\right) + \tan^{-1}\left(\frac{w-y}{d}\right)\right].$$

The x-component of the force on a strip of width dy due to I_2 in the right-hand conductor is

$$\begin{aligned}
\mathrm{d}F'_{2x} &= \left(\frac{I_2}{w}\mathrm{d}y\right) B_{Py} \text{ (in the +}x\text{-direction, a repulsive force)} \\
\mathbf{F}'_2 &= \mathbf{a}_x \int \mathrm{d}F_{2x} \\
&= \mathbf{a}_x \frac{\mu_0 I_1 I_2}{2\pi w^2} \int_0^w \left[\tan^{-1}\left(\frac{y}{d}\right) + \tan^{-1}\left(\frac{w-y}{d}\right)\right] \mathrm{d}y \\
&= \mathbf{a}_x \frac{\mu_0 I_1 I_2}{2\pi w^2}\left[2w\tan^{-1}\left(\frac{w}{d}\right) - d\ln\left(1+\frac{w^2}{d^2}\right)\right] \text{ per unit length}
\end{aligned}$$

There are no net forces in the y-direction.

Answer:

$$\mathbf{F}'_{\mathbf{2}} = a_x \frac{\mu_o I_1 I_2}{2\pi w 62}\left[2w\tan^{-1}\left(\frac{w}{d}\right) - d\ln\left(1+\frac{w^2}{d^2}\right)\right]$$