
Consider the electric field $\mathbf{E} = \mathbf{a}_x E_x(x)$ with

$$E_x(x) = \begin{cases} \frac{\rho_0 x}{\epsilon_0} \left(1 - \frac{x^2}{3a^2}\right), & |x| < a \\ \frac{2\rho_0 a}{3\epsilon_0}, & x > a \\ -\frac{2\rho_0 a}{3\epsilon_0}, & x < -a \end{cases}$$

Find the corresponding charge distribution in free space.

Solution:

$$\begin{aligned} \rho_v &= \nabla \cdot \mathbf{D} \\ \rho_v &= \epsilon_0 \frac{dE_x}{dx} \\ \rho_v &= \begin{cases} \rho_0 \left(1 - \frac{x^2}{a^2}\right), & |x| < a \\ 0, & x > a \\ 0, & x < -a \end{cases} \end{aligned}$$

Answer:

$$\rho_v = \begin{cases} \rho_0 \left(1 - \frac{x^2}{a^2}\right), & |x| < a \\ 0, & x > a \\ 0, & x < -a \end{cases}$$