
Goal: Find the work done by electric forces in moving a charge $Q = 1 \text{ nC}$ from the coordinate origin to the point $(1 \text{ m}, 1 \text{ m}, 1 \text{ m})$ in the electrostatic field given by $\mathbf{E}(x, y, z) = (x\mathbf{a}_x + y^2\mathbf{a}_y - \mathbf{a}_z) \text{ V/m}$ along the straight line.

Steps:

1. Choose a path that would facilitate the computation of the work, based on the fact that $\int_C \mathbf{E} \cdot d\mathbf{l}$ is path-independent.

Solution: The parametric line describing the path from origin to point $(1 \text{ m}, 1 \text{ m}, 1 \text{ m})$ is given by:

$$\mathbf{l}(t) = t(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) \text{ [m]} \quad t \in [0, 1]$$

2. Compute scalar potential difference between point $(1 \text{ m}, 1 \text{ m}, 1 \text{ m})$ and the origin.

Solution:

$$\begin{aligned} V &= - \int_0^1 \mathbf{E}(x, y, z) \cdot d\mathbf{l}(t) \\ &= - \int_0^1 (x\mathbf{a}_x + y^2\mathbf{a}_y - \mathbf{a}_z) \cdot (\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) dt \\ &= 0.1667 \text{ [V]} \end{aligned}$$

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3. Use the potential to compute work done by electric forces, recalling the fundamental definition of electric potential difference as work per unit charge.

Solution:

$$\begin{aligned} W &= qV \\ &= 166.67 \text{ pJ} \end{aligned}$$

Therefore, the work done by the electric field is -166.67 pJ .

Answer: $W = -166.67 \text{ pJ}$