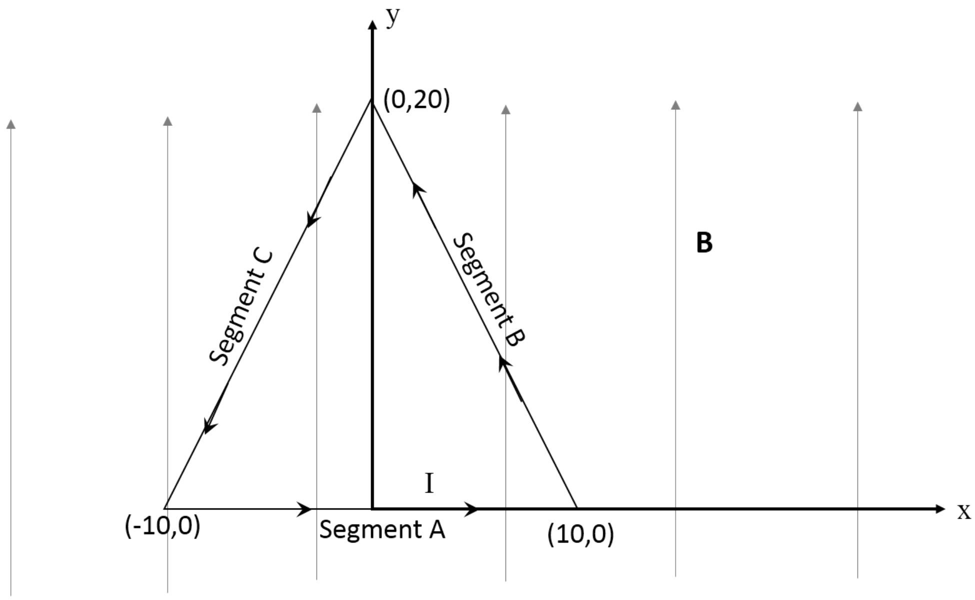
Goal: A d-c current $I = 10\,A$ flows in a triangular loop in the xy-plane as in the figure below. Assuming a uniform magnetic flux density $\mathbf{B} = \mathbf{a}_y 0.5\,T$ in the region, find the forces and torque on the loop. All dimensions are in cm.



Steps:

1. For Segment A in the figure above, what is the differential length vector $d\mathbf{l}_A$?

   Solution:

$$d\mathbf{l}_A = \mathbf{a}_x dx \qquad -0.1\text{ (m)} < x < -0.1\text{ (m)}$$

2. Determine the force $\mathbf{F}_A$ exerted on Segment A by $\mathbf{B}$.

   Solution:

$$d\mathbf{F}_A = I d\mathbf{l}_A \times \mathbf{B}$$
$$\mathbf{F}_A = I \int_{-0.1}^{0.1} \mathbf{a}_x \times 0.5\mathbf{a}_y dx$$
$$\mathbf{F}_A = 1\mathbf{a}_z \text{ (N)}$$

3. For Segment B, what is the differential length vector $d\mathbf{l}_B$?

   Solution:

$$d\mathbf{l}_B = \frac{-\mathbf{a}_x + 2\mathbf{a}_y}{\sqrt{5}} dl \qquad 0 < l < \sqrt{0.05}$$

4. Determine the force $\mathbf{F}_B$ exerted on Segment B by $\mathbf{B}$?

Solution:

$$\begin{aligned}\mathbf{F}_B &= I\frac{-\mathbf{a}_x + 2\mathbf{a}_y}{\sqrt{5}} \times 0.5\mathbf{a}_y \int_0^{\sqrt{0.05}} dl \\ &= -0.5\mathbf{a}_z\end{aligned}$$

5. For Segment C, what is the differential length vector $d\mathbf{l}_C$?

Solution:

$$d\mathbf{l}_C = \frac{-\mathbf{a}_x - 2\mathbf{a}_y}{\sqrt{5}} dl \qquad 0 < l < \sqrt{0.05}$$

6. Determine the force $\mathbf{F}_C$ exerted on Segment C by $\mathbf{B}$?

Solution:

$$\begin{aligned}\mathbf{F}_C &= I\frac{-\mathbf{a}_x - 2\mathbf{a}_y}{\sqrt{5}} \times 0.5\mathbf{a}_y \int_0^{\sqrt{0.05}} dl \\ &= -0.5\mathbf{a}_z\end{aligned}$$

7. What is total force on the triangular loop?

Solution:

$$\mathbf{F} = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C\,.$$

8. Determine the torque $\mathbf{T}$ on the loop due to $\mathbf{B}$.

Solution:

$$\begin{aligned}\mathbf{T} &= \mathbf{m} \times \mathbf{B} \\ &= I\frac{1}{2}(0.2)(0.2)\mathbf{a}_z \times 0.5\mathbf{a}_y \\ &= -0.1\mathbf{a}_x \text{ (Nm)}\,.\end{aligned}$$

Answer:

$$\begin{aligned}\mathbf{F} &= 0 \\ \mathbf{T} &= -0.1\mathbf{a}_x \text{ (Nm)}\,.\end{aligned}$$