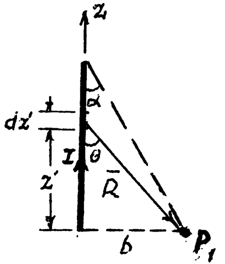
P.6-5



We first find $\bar{B}_{P_1}$ at P_1 flush with one end of a wire carrying a current I and making an angle α with the other end as shown.

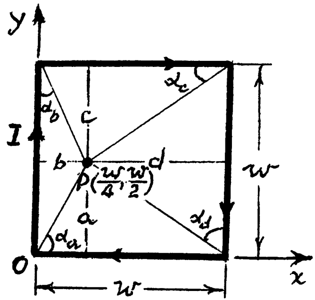
$$d\bar{B}_{P_1} = \frac{\mu_0 I}{4\pi R^2} dz' \times \bar{a}_R \qquad z' = b\cot\theta,\ dz' = -b\csc^2\theta\, d\theta,$$

$$= \frac{\mu_0 I}{4\pi b}\left(-\bar{a}_\phi \sin\theta\, d\theta\right). \qquad R = b\csc\theta,$$

$$\bar{a}_z \times \bar{a}_R = \bar{a}_\phi \sin\theta.$$

$$\bar{B}_{P_1} = -\bar{a}_\phi \frac{\mu_0 I}{4\pi b}\int_{\pi/2}^{\alpha} \sin\theta\, d\theta$$

$$= \bar{a}_\phi \frac{\mu_0 I}{4\pi b}\cos\alpha.$$



Applying the above result to the four-sided loop at left, we have

$$\bar{B}_P = \bar{a}_z \frac{\mu_0 I}{4\pi}\Big(\frac{1}{a}\cos\alpha_a + \frac{1}{b}\sin\alpha_a + \frac{1}{b}\cos\alpha_b + \frac{1}{c}\sin\alpha_b + \frac{1}{c}\cos\alpha_c + \frac{1}{d}\sin\alpha_c + \frac{1}{d}\cos\alpha_d + \frac{1}{a}\sin\alpha_d\Big).$$

For this problem, $a = c = \frac{w}{2}$, $b = \frac{w}{4}$, $d = \frac{3}{4}w$.

$\alpha_a = \tan^{-1} 2 = 63.4°$, $\alpha_b = 90° - 63.4° = 26.6°$; $\alpha_c = \tan^{-1}\frac{2}{3} = 33.7°$; $\alpha_d = 56.3°$

$$\bar{B}_P = \bar{a}_z\, 3.44\, \frac{\mu_0 I}{\pi w}.$$