[Cheng P.6-37] Calculate the mutual inductance per unit length between two parallel wire transmission lines $A - A'$ and $B - B'$ separated by a distance D, as shown in Fig. 6-47. Assume the wire radius is much smaller than D and the wire spacing d.

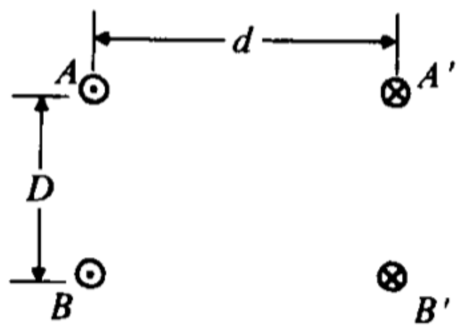
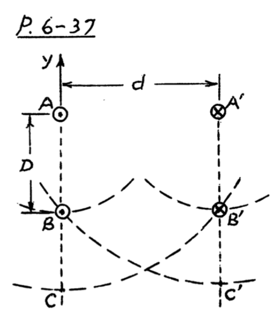


FIGURE 6–47
Coupled two-wire transmission lines (Problem P.6–37).

Solution: The magnetic flux density is a function of distance from the wire as shown in the diagram below.



$\mathbf{B}$ at a distance r from an infinitely long line carrying current I is $\mathbf{B} = \mathbf{a}_\phi \frac{\mu I}{2\pi r}$.

For a unit length flux due to I in line A that links with the second line pair $B - B'$ is

$$\Phi'_A = \frac{\mu_0 I}{2\pi} \int_{AB}^{AC} \frac{\mathrm{d}r}{r} = \frac{\mu_0 I}{2\pi} \ln \frac{AC}{AB}.$$

That unit length flux due to A' is

$$\Phi'_{A'} = \frac{\mu_0 I}{2\pi} \ln \frac{A'C'}{A'B'}.$$

The total flux linkage is therefore

$$\begin{aligned}
\Lambda'_{12} &= \Phi_{A'} + \Phi'_{A'} \\
&= \frac{\mu_0 I}{2\pi} \ln \frac{(AC)(A'C')}{(AB)(A'B')} \\
&= \frac{\mu_0 I}{2\pi} \ln \frac{(AB')(A'B)}{(AB)(A'B')} \\
&= \frac{\mu_0 I}{2\pi} \ln \frac{D^2 + d^2}{D^2}.
\end{aligned}$$

Finally we can determine the mutual inductance by simply dividing by the current

$$\begin{aligned}
M'_{12} &= \frac{\Lambda'_{12}}{I} \\
&= \frac{\mu_0}{2\pi} \ln \left(1 + \frac{d^2}{D^2}\right).
\end{aligned}$$

Answer:

$$M'_{12} = \frac{\mu_o}{2\pi} \ln \left(1 + \frac{d^2}{D^2}\right)$$