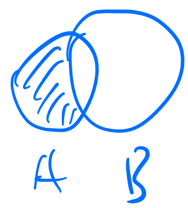
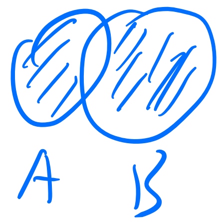
交 $A \cup B$

并 $A \cap B$

差 $A - B = \{x, x \in A$ 且 $x \notin B\}$



对称差 $A \triangle B = (A-B) \cup (B-A)$



$$\bigcup_{\alpha \in I} A_\alpha = \{x, \exists \alpha \in I \text{ s.t. } x \in A_\alpha\}$$

$$\bigcap_{\alpha \in I} A_\alpha = \{x, \forall \alpha \in I \text{ s.t. } x \in A_\alpha\}$$

若 $B \subset A$ $\quad A - B = \complement_A B \ (\complement B)$

① 交换律

$$A \cap B = B \cap A$$

$$A\cup B=B\cup A$$

② 结合律

$$A\cup(B\cup C)=(A\cup B)\cup C$$

$$A\cap(B\cap C)=(A\cap B)\cap C$$

(可任意交换)

③ 分配律

$$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$$

$$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$$

证 $A=B$ $\Leftrightarrow$ 证 $\begin{cases} A\subset B \\ B\subset A \end{cases}$

X是基本集(其他所有集合都是X子集)

$$X-A=C_X A$$

De Morgan:

设X是一基本集及$\{A_\alpha\}$

① $\complement\left(\bigcup_{\alpha\in I}A_\alpha\right)=\bigcap_{\alpha\in I}\left(\complement A_\alpha\right)$

② $\complement\left(\bigcap_{\alpha\in I}A_\alpha\right)=\bigcup_{\alpha\in I}\left(\complement A_\alpha\right)$

① Pr: 对 $\forall x\in\complement\left(\bigcup_{\alpha\in I}A_\alpha\right)$

$\Rightarrow x\notin\bigcup_{\alpha\in I}A_\alpha$

$\Rightarrow \forall\alpha\in I\quad x\notin A_\alpha$

$\Rightarrow \forall\alpha\in I\quad x\in\complement A_\alpha$

$\Rightarrow x\in\bigcap_{\alpha\in I}\left(\complement A_\alpha\right)$

对 $\forall x\in\bigcap_{\alpha\in I}\left(\complement A_\alpha\right)$

$\Rightarrow \forall\alpha\in I\quad x\in\complement A_\alpha$

$\Rightarrow \forall\alpha\in I\quad x\notin A_\alpha$

$\Rightarrow x\notin\bigcup_{\alpha\in I}A_\alpha$

$\Rightarrow x\in\complement\left(\bigcup_{\alpha\in I}A_\alpha\right)$

② Pr: 对 $\forall x\in\complement\left(\bigcap_{\alpha\in I}A_\alpha\right)$

$\Rightarrow x\notin\bigcap_{\alpha\in I}A_\alpha$

$\Rightarrow \exists\alpha\in I\quad x\notin A_\alpha$

$\Rightarrow \exists\alpha\in I\quad x\in\complement A_\alpha$

对 $\forall x\in\bigcup_{\alpha\in I}\left(\complement A_\alpha\right)$

$\Rightarrow \exists\alpha\in I\quad x\in\complement A_\alpha$

$\Rightarrow \exists\alpha\in I\quad x\notin A_\alpha$

$\Rightarrow x\notin\bigcap_{\alpha\in I}A_\alpha$

$\Rightarrow x \in \bigcup_{\alpha \in I} (\mathscr{C} A_\alpha)$ $\Rightarrow x \in \mathscr{C}\left(\bigcap_{\alpha \in I} A_\alpha\right)$

或由①：$\mathscr{C}\left(\bigcup_{\alpha \in I} \mathscr{C} A_\alpha\right) = \bigcap_{\alpha \in I} \left(\mathscr{C}\mathscr{C} A_\alpha\right) = \bigcap_{\alpha \in I} A_\alpha$

$\Rightarrow \bigcup_{\alpha \in I} (\mathscr{C} A_\alpha) = \mathscr{C}\left(\bigcap_{\alpha \in I} A_\alpha\right)$

补：

若 $A_n = \{x.\ 0 \le x < 1 + \frac{1}{n}\}$ $(n = 1, 2, 3 \cdots)$

证 $\bigcap_{i=1}^{n} A_i = \{x\ \ 0 \le x < 1 + \frac{1}{n}\}$

$\bigcap_{i=1}^{\infty} A_i = \{x\ \ 0 \le x \le 1\}$

Pr: 有 $A_1 \supset A_2 \supset A_3 \cdots$

$\Rightarrow \bigcap_{i=1}^{n} A_i = A_n$

对 $\forall n$ $A_n \supset \{x,\ 0 \le x \le 1\}$

$\Rightarrow \bigcap_{i=1}^{\infty} A_i \supset \{x,\ 0 \le x \le 1\}$

$n=1$

对 $\forall x \in \bigcap_{n=1}^{\infty} A_n$

$\Rightarrow 0 \le x < 1+\frac{1}{n} \quad (n \to \infty)$

$\Rightarrow x \in [0,1]$

$\Rightarrow \{x,\ 0 \le x \le 1\} \supset \bigcap_{n=1}^{\infty} A_n$

补：若 $A_n = \{x \quad n \le x \le n+\frac{3}{2}\} \quad n=1、2\ldots\ldots$

则 $\bigcap_{n=1}^{\infty} A_n = \emptyset$

Pr：若 $\bigcap_{n=1}^{\infty} A_n \ne \emptyset$

取 $x \in \bigcap_{n=1}^{\infty} A_n$

$\Rightarrow$ 对 $\forall n \quad n \le x \le n+\frac{3}{2}$ 不成立

（令 $n_0 = [x]+1$，$n_0 \le x \le n_0+\frac{3}{2}$ ⊗）

补：

$A_n = \{x,\ -\frac{1}{n} < x < \frac{1}{n}\} \quad (n=1、2\ldots\ldots)$

则 $\bigcap_{i=1}^{n} A_i = A_n$， $\bigcap_{n=1}^{\infty} A_n = \{0\}$

补： $A_n = \{x : n-1 < x \le n\}$ $(n=1,2,\cdots)$

证： $\bigcup_{n=1}^{\infty} A_n = (0,+\infty)$

Pr： ① 显然 $(0,+\infty) \supset A_n$

$\Rightarrow (0,+\infty) \supset \bigcup_{n=1}^{\infty} A_n$

② 对 $\forall x \in (0,+\infty)$

$\exists n_0 = I \times [\,([x]+1)$

$x \in A_{n_0}$

$\Rightarrow x \in \bigcup_{n=1}^{\infty} A_n$

$\Rightarrow (0,+\infty) \subset \bigcup_{n=1}^{\infty} A_n$

$\Rightarrow (0,+\infty) = \bigcup_{n=1}^{\infty} A_n$