

$$3. f = (1+e^y) \cos x - ye^y$$

$$\frac{\partial f}{\partial x} = -(1+e^y) \sin x = 0 \Rightarrow x = k\pi$$

$$\frac{\partial f}{\partial y} = e^y \cos x - (y+1)e^y = 0$$

$$y+1 = \cos x$$

$$\begin{cases} x = 2k\pi \\ y = 0 \end{cases} \quad \begin{cases} x = (2k+1)\pi \\ y = -2 \end{cases}$$

$$A = \frac{\partial^2 f}{\partial x^2} = -(1+e^y) \cos x$$

$$B = \frac{\partial^2 f}{\partial x \partial y} = -e^y \sin x$$

$$C = \frac{\partial^2 f}{\partial y^2} = e^y \cos x - (y+2)e^y$$

$$\textcircled{1} \begin{cases} x = 2k\pi \\ y = 0 \end{cases}$$

$$A = -2 < 0$$

$$B = 0$$

$$C = 1 - 2 = -1$$

$$AC - B^2 = 2 > 0$$

极大值

$$\textcircled{2} x = (2k+1)\pi$$

$$A = 1 + e^{-2} > 0$$

$$\textcircled{2} \quad \begin{cases} y = -2 \end{cases}$$

$$B = 0$$

$$C = -e^{-2} < 0$$

$$AC - B^2 < 0$$

不是极值

$$8. \quad f = x^2 + 2xy + 2y^2 - 1 = 0$$

$$y^2 = 1$$

$$\frac{dy}{dx} = - \frac{2x + 2y}{2x + 4y} = 0$$

$$x + y = 0$$

$$y = \pm 1$$

$$= - \frac{F_x}{F_y}$$

$$F_x = 2x + 2y = 0$$

$$F_y = 2x + 4y = 2y$$

$$F_{xx} = 2$$

$$y'' = - \frac{F_{xx}F_y - F_xF_{xy}}{F_y^2}$$

$$= - \frac{4y}{4y^2} = - \frac{1}{y}$$

$$y = 1 \text{ 极大}$$

$$y = -1 \text{ 极小}$$

9.

$$4z^2 - 16z^2 - z + 8 = 0$$

$$7z^2 + z - 8 = 0$$

$$f = 2x^2 + 2y^2 + z^2 + 8yz - z + 8 = 0$$

$$\begin{matrix} 1 & -1 \\ 1 & 8 \end{matrix}$$

$$\begin{cases} -\frac{F_x}{F_z} = 0 \\ -\frac{F_y}{F_z} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 4x = 0 \\ 4y + 8z = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = 0 \\ y + 2z = 0 \\ z = 1 \end{cases}$$

$$A = - \frac{f_{xx}f_z - f_x f_{zx}}{f_z^2}$$

$$B = - \frac{f_{xy}f_z - f_y f_{zy}}{f_z^2}$$

$$C = - \frac{f_{yy}f_z - f_y f_{zy}}{f_z^2}$$

$$f_{xx} = 4$$

$$f_{xy} = 0$$

$$f_{yy} = 4$$

$$z_1 = -\frac{8}{7}$$

$$f_z = 28 + 8y - 1 = -148 - 1$$

$$AC - B^2 = \frac{1}{f_z^4} \left[\begin{array}{c} 0 \\ 11 \end{array} f_{xx}f_{yy}f_z^2 - \begin{array}{c} 0 \\ 11 \end{array} f_x f_z f_{yy}f_{zx} - \begin{array}{c} 0 \\ 11 \end{array} f_y f_z f_{xx}f_{zy} + \begin{array}{c} 0 \\ 11 \end{array} f_x f_y f_{zx}f_{zy} \right] - \left(\begin{array}{c} 1 \\ 0 \end{array} f_{xy}^2 f_z^2 + \begin{array}{c} 1 \\ 0 \end{array} f_x^2 f_{zy}^2 - 2 \begin{array}{c} 1 \\ 0 \end{array} f_z f_x f_{xy} f_{zy} \right)$$

$$= \frac{f_{xx}f_{yy} - f_{xy}^2}{f_z^2} = \frac{16}{f_z^2} > 0$$

$$z_1 = 1 \quad \text{极大}$$

$$z_2 = -\frac{8}{7} \quad \text{极大}$$

$$A \Leftrightarrow f_x f_{zx} - f_{xx} f_z$$