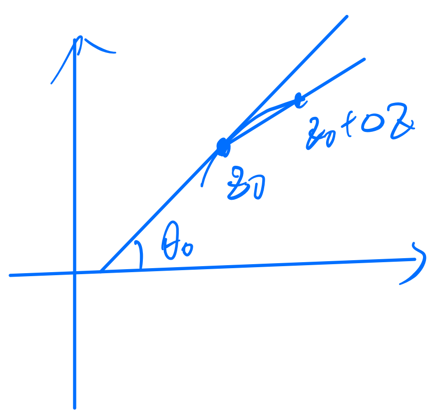
设 $f(z)$ 在 D 中解析 $z_0 \in D$ $f'(z_0) \neq 0$

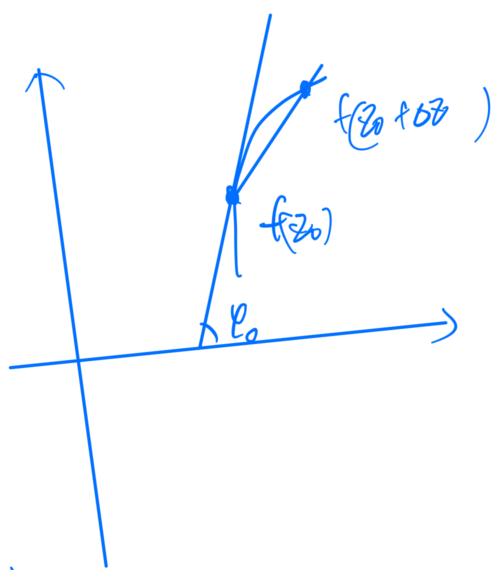
$$\Rightarrow \Delta f(z_0) = f'(z_0)\Delta z + o(|\Delta z|)$$

$$\lim_{\Delta z \to 0}$$

向量 $\Rightarrow \vec{\Delta f} = re^{i\theta} \cdot \vec{\Delta z} \Leftrightarrow \begin{pmatrix} r & \\ & r \end{pmatrix} \begin{pmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{pmatrix} \vec{\Delta z}$

即在 z_0 充分小邻域，$f(z)$ 可看作线性变换





$$\varphi_0 - \theta_0 = \arg(f'(z_0))$$

$$z_0 + \Delta z = z_0 + |\Delta z| + e^{i(\theta_0 + \Delta\theta)}$$

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{|\Delta f| e^{i(\varphi_0 + \Delta\varphi)}}{|\Delta z| e^{i(\theta_0 + \Delta\theta)}}$$

$$|f'(z_0)| = \lim_{\Delta z \to 0} \frac{|\Delta f|}{|\Delta z|} \quad (伸缩率)$$

$$\arg f'(z_0) = \varphi_0 - \theta_0$$

$f' = 0$ 可能不保角

例 $f(z) = z^2$

$$\sigma(t) = \gamma(t)^2$$

$$\sigma'(t) = 2\gamma\gamma'$$

$$\operatorname{Arg} \sigma'(0) = \lim_{t \to 0} \operatorname{Arg} \sigma'(t) = \operatorname{Arg} \gamma(t) + \operatorname{Arg} \gamma'(t)$$

$$= \operatorname{Arg} \frac{\gamma(t)}{t} + \operatorname{Arg} \gamma'(t) \quad (t \to 0)$$

$$= 2 \operatorname{Arg} \gamma'(0)$$

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