Worksheet 2
Friday, January 27th, 2023
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logistics

1. My office hours next week will be Monday 11am-2pm at Evans 1066. This may change for future weeks...

linear stuff

2. A 3×3 *magic square* is a 3×3 matrix with real entries such that the sums of each row, column and main diagonals are all the same. For example, $\begin{pmatrix} 2 & 7 & 6 \\ 9 & 5 & 1 \\ 4 & 3 & 8 \end{pmatrix}$ is a magic square. Show that the set $\mathcal{M}_3$ of 3×3 magic squares is a vector space.

3. Check that $\mathbb{R}$ is a vector space over $\mathbb{Q}$. Think about whether $\mathbb{R}$ has a basis, then google it.

4. Give an example of a nonempty subset of $\mathbb{R}^2$ that is closed under scalar multiplication but isn't a subspace of $\mathbb{R}^2$.

5. Let U, W be subspaces of V. Show that the intersection $U \cap W$ is also a subspace of V. Does the union $U \cup W$ have to be a subspace?

6. We say that a function $f : \mathbb{R} \to \mathbb{R}$ is *periodic* if there exists some $p \in \mathbb{R}, p > 0$ such that $f(x + p) = f(x)$ for all $x \in \mathbb{R}$. Does the set of all periodic functions form a vector space with the usual addition and scalar multiplication? What if we replace $\mathbb{R}$ with $\mathbb{Q}$?

7. Just like the complex numbers $\mathbb{C}$ are constructed by adjoining a new element i that satisfies $i^2 = -1$ to the reals, the *quaternions* $\mathbb{H}$ are constructed by also adjoining j, k satisfying $j^2 = k^2 = -1$, $ij = -ji = k, jk = -kj = i, ki = -ik = j$.

a) Show that $ijk = -1$.

b) Check that $\mathbb{H}$ satisfies every field axiom except commutativity of multiplication.

c) Meditate on what a theory of "vector spaces" over $\mathbb{H}$ might look like, and then take Math 113, 114, 250a, and 251, not necessarily in that order.[Extra credit]