Worksheet 7
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Problems come from a variety of sources including Axler and random problems floating online. Only a few are written by me.

linear stuff

1. Let V be a finite-dimensional vector space, and U, W subspaces of V. Recall that the annihilator U^0 of U is defined to be the space of linear functionals $f : V \to \mathbb{F}$ such that $f(u) = 0$ for all $u \in U$.

Prove that $U^0 \subset W^0$ iff $W \subset U$. When is U^0 equal to W^0?

2. Compute $(x^4 + 4x^3 + x - 10)/(x^2 + 3x - 5)$.

3. Prove that every odd degree polynomial with real coefficients has a real zero.

4. Prove the *reverse triangle inequality* for complex numbers:

$$\text{If } w, z \in \mathbb{C}, \text{ then } ||w| - |z|| \leq |w - z|$$