Worksheet 4
Friday, February 10th, 2023
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linear stuff

1. Let U, W be subspaces of a vector space V. Recall the definition of direct sum: We say that $V = U \oplus W$ if every vector $v \in V$ can be expressed as a sum $v = u + w$ of vectors $u \in U, w \in W$ in a unique way. Prove that the following are equivalent.

a) $V = U \oplus W$

b) $V = U + W$ and $U \cap W = \{0\}$ (this is often used as an alternate definition of the direct sum).

c) $\dim V = \dim U + \dim W$ and $V = U + W$

d) $\dim V = \dim U + \dim W$ and $U \cap W = \{0\}$

e) If $u_1, \cdots, u_m$ is a basis for U and $w_1, \cdots, w_n$ is a basis for W, then $u_1, \cdots, u_m, w_1, \cdots, w_n$ is a basis for V.

2. Suppose that V is finite-dimensional, and $T \in \mathcal{L}(V)$. Show that T is a scalar multiple of the identity if and only if $TS = ST$ for every $S \in \mathcal{L}(V)$.