Worksheet 3
Friday, February 3rd, 2023
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logistics

1. My office hours will be Monday 11am-12pm, 1pm-3pm at Evans 1066. Hopefully this is permanent.

2. You can now find these worksheets at https://www.chan.coffee/p/s23-math-110.html

linear stuff

3. a) Show that if we think of $\mathbb{C}$ as a vector space over $\mathbb{R}$, the list $1+i, 1-i$ is linearly independent. b) Show that $1+i, 1-i$ is linearly dependent in $\mathbb{C}$ as a vector space over $\mathbb{C}$.

4. Let $M_{m\times n}$ be the vector space of all $m \times n$ matrices (with the usual addition and scalar multiplication). Check if the following subsets of $M_{m\times n}$ are subspaces, and find the dimension of each subspace by finding a basis.

- $V_1 = \{X \in M_{n\times 1} : \sum_i X_{i1} = 0\}$
- $V_2 = \{X \in M_{n\times n} : X \text{ is not invertible}\}$
- $V_3 = \{X \in M_{n\times n} : X_{ij} = X_{ji} \text{ for all } 1 \leq i, j \leq n\}$
- $V_4 = \{X \in M_{n\times n} : X_{ij} = -X_{ji} \text{ for all } 1 \leq i, j \leq n\}$

5. Let $V = U \oplus W$.

a) Show that if $B_U = \{u_1, \cdots, u_m\}$ is a basis of U, and $B_W = \{w_1, \cdots, w_n\}$ is a basis of W, then $B_U \cup B_W$ is a basis of V.

b) Find a counterexample to the following statement: "If $V = U \oplus W$, every basis of V is the union of a basis of U and a basis of W".

6. Let P_3 be the vector space of polynomials of degree at most three. Does there exist a basis of P_3 *not* containing any polynomials of degree 2?