

Worksheet 8

Friday, March 17th, 2023

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Problems come from a variety of sources including Axler and random problems floating online. Only a few are written by me.

linear stuff

1. Let V be a vector space and $T \in L(V)$. Show that if $W_1, W_2 \subset V$ are invariant subspaces of T , then $W_1 + W_2$ is also an invariant subspace.

2. Define $T : P(\mathbb{R}) \rightarrow P(\mathbb{R})$ by $T(p) = p'$. Find all eigenvalues and eigenvectors of T .

3. Let V be a finite-dimensional vector space. Suppose $T \in L(V)$ and $\lambda \in \mathbb{F}$. Show that there exists $\alpha \in \mathbb{F}$ such that $|\alpha - \lambda| < 0.0001$ and $T - \alpha I$ is invertible.

Bonus. Let V be finite dimensional. A hyperplane in V is defined as the kernel of a linear functional. Show that every subspace of V is an intersection of hyperplanes.