Worksheet 5
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Problems come from a variety of sources including Axler and random problems floating online. Only a few are written by me.

linear stuff

1. Give an example of a function $T : \mathbb{C} \to \mathbb{C}$ which is not $\mathbb{C}$-linear such that $T(x + y) = T(x) + T(y)$. That is, find a T that does not satisfy $T(\lambda x) = \lambda T(x)$ for some $\lambda \in \mathbb{C}$.

2. Let U, V, W be vector spaces, and let $T : V \to W$ be a linear transformation. Show the following:

a) T is injective if and only if $TS_1 = TS_2$ implies $S_1 = S_2$ for all $S_1, S_2 \in \mathcal{L}(U, V)$.

b) T is surjective if and only if $S_1T = S_2T$ implies $S_1 = S_2$ for all $S_1, S_2 \in \mathcal{L}(W, U)$.

3. Let V be an n-dimensional vector space and $T \in \mathcal{L}(V, V)$, Show that the following are equivalent.

a) range T = null T

b) $T^2 = 0$, n is even, and dim(range T) $= n/2$

4. Let $T \in \mathcal{L}(\mathbb{C}^6, \mathbb{C}^3)$, then what are the possible values of $\dim(\text{range}(T))$? Find an explicit example of T for each possibility.