Worksheet 9
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Problems come from a variety of sources including Axler and random problems floating online. Only a few are written by me.

linear stuff

1. Suppose $T \in L(V)$, where V is an n-dimensional vector space. Show that $\text{span}(v, Tv, \cdots, T^{n-1}v)$ is an invariant subspace of T.

2. Let V be a finite-dimensional vector space, and let $T \in L(V)$. Show that T and the dual map $T' \in L(V')$ share the same minimal polynomial.

3. Show that $T \in L(V)$ is invertible if and only if x does not divide the minimal polynomial of T.

4. Let $T \in L(V)$ have minimal polynomial x^k. Show that we have *strict* inclusions

$$0 \subset \ker T \subset \ker T^2 \subset \cdots \subset \ker T^k = V$$