Worksheet 6
Friday, February 24th, 2023
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Problems come from a variety of sources including Axler and random problems floating online. Only a few are written by me.

linear stuff

1. Let S, T be elements of $\mathcal{L}(V)$. Show that if ST is invertible, then S and T are also invertible.

2. Let V be a vector space over $\mathbb{F}$. Let V' be the dual space of V. Show that the dual map of the identity operator on V is the identity operator on V'.

3. Let V be a vector space over $\mathbb{F}$. Let V' be the dual space of V. Furthermore, let V'' be the dual space of V'.

Define the map $\phi : V \to V''$ by

$$\phi(v)(f) = f(v) \text{ for all } f \in V'.$$

a) Show that ϕ is a linear map.

b) Show that ϕ is an isomorphism.