Theory of Computation

Lab Session 3

February 11, 2016

Agenda

- A homework exercise on Finite State Automaton (FSA)
- Operations on FSA (Exercises)

Exercise

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Operations on FSA

Operations

Suppose L_1 and L_2 are both languages over the alphabet A. If $x \in A^*$, then knowing whether $x \in L_1$ and whether $x \in L_2$ is enough to determine whether $x \in L_1 \cup L_2$.

Operations

Suppose L_1 and L_2 are both languages over the alphabet A. If $x \in A^*$, then knowing whether $x \in L_1$ and whether $x \in L_2$ is enough to determine whether $x \in L_1 \cup L_2$.
If we have one algorithm to accept L_1 and another to accept L_2, how can we formulate an algorithm to accept $L_1 \cup L_2$? (similarly for $L_1 \cap L_2$ and $L_1 \backslash L_2$).

Intersection (Formally)

Suppose $M^1 = (Q^1, A, \delta^1, q_0^1, F^1)$ and $M^2 = (Q^2, A, \delta^2, q_0^2, F^2)$ are finite automata accepting L_1 and L_2, respectively. Let M be the complete FSA $M = (Q, A, \delta, q_0, F)$, where

$$Q = Q^1 \times Q^2$$
$$q_0 = (q_0^1, q_0^2)$$

the transition function δ is defined by the formula

$$\delta((q, p), a) = (\delta^1(q, a), \delta^2(p, a))$$

for every $q \in Q^1$, every $p \in Q^2$, and every $a \in A$. And the set of final states is defined as

$$F = \{(q, p) \mid q \in F^1 \wedge p \in F^2\}$$

M accepts the language $L_1 \cap L_2$.

Intersection (Example)

Let M^1 be a complete FSA defined as

$$
\begin{array}{ll}
M^1 = \langle & \\
\quad \{q_0, q_1\}, & \text{set of states} \\
\quad \{a\}, & \text{input alphabet} \\
\quad \{((q_0, a), q_1), ((q_1, a), q_0)\}, & \text{partial transition function} \\
\quad q_0, & \text{initial state} \\
\quad \{q_1\} & \text{set of final states} \\
\rangle &
\end{array}
$$

Intersection (Example)

Let M^1 be a complete FSA defined as

$$\begin{aligned} M^1 = \langle &\{q_0, q_1\}, \{a\}, \\ &\{((q_0, a), q_1), ((q_1, a), q_0)\}, \\ &q_0, \{q_1\}\rangle \end{aligned}$$

and M^2 be a complete FSA defined as

Intersection (Example)

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and M^2 be a complete FSA defined as

$$\begin{aligned} M^2 = \langle &\{p_0\}, \{a\}, \\ &\{((p_0, a), p_0)\}, \\ &p_0, \{p_0\}\rangle \end{aligned}$$

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$$\begin{aligned} M^1 = \langle & \{q_0, q_1\}, \{a\}, \\ & \{((q_0, a), q_1), ((q_1, a), q_0)\}, \\ & q_0, \{q_1\}\rangle \end{aligned}$$

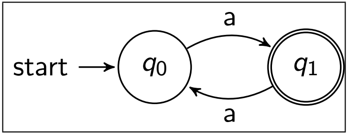
and M^2 be a complete FSA defined as

$$\begin{aligned} M^2 = \langle & \{p_0\}, \{a\}, \\ & \{((p_0, a), p_0)\}, \\ & p_0, \{p_0\}\rangle \end{aligned}$$

then

$$\begin{aligned} (M^1 \cap M^2) = \langle & \{(q_0, p_0), (q_1, p_0)\}, \{a\}, \\ & \Big\{\Big(((q_0, p_0), a), (q_1, p_0)\Big), \Big(((q_1, p_0), a), (q_0, p_0)\Big)\Big\}, \\ & (q_0, p_0), \{(q_1, p_0)\}\rangle \end{aligned}$$

Intersection (Example — Graphically)



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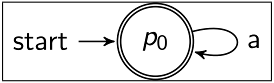

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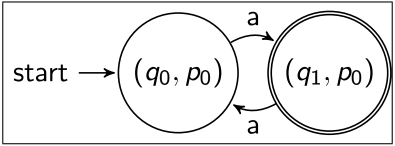

Union (Formally)

Suppose $M^1 = (Q^1, A, \delta^1, q_0^1, F^1)$ and $M^2 = (Q^2, A, \delta^2, q_0^2, F^2)$ are finite automata accepting L_1 and L_2, respectively. Let M be the complete FSA $M = (Q, A, \delta, q_0, F)$, where

$$Q = Q^1 \times Q^2$$
$$q_0 = (q_0^1, q_0^2)$$

the transition function δ is defined by the formula

$$\delta((q, p), a) = (\delta^1(q, a), \delta^2(p, a))$$

for every $q \in Q^1$, every $p \in Q^2$, and every $a \in A$. And the set of final states is defined as

$$F = \{(q, p) \mid q \in F_1 \vee p \in F_2\}$$

M accepts the language $L_1 \cup L_2$.

Difference (Formally)

Suppose $M^1 = (Q^1, A, \delta^1, q_0^1, F^1)$ and $M^2 = (Q^2, A, \delta^2, q_0^2, F^2)$ are finite automata accepting L_1 and L_2, respectively. Let M be the FA $M = (Q, A, \delta, q_0, F)$, where

$$Q = Q^1 \times Q^2$$
$$q_0 = (q_0^1, q_0^2)$$

the transition function δ is defined by the formula

$$\delta((q, p), a) = (\delta^1(q, a), \delta^2(p, a))$$

for every $q \in Q^1$, every $p \in Q^2$, and every $a \in A$. And the set of final states is defined as

$$F = \{(q, p) \mid q \in F_1 \wedge p \notin F2\}$$

M accepts the language $L_1 \setminus L_2$.

Exercise (first part)

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Let $A = \{0, 1\}$ be the alphabet.

1. Build a complete FSA M_1 that recognises the language:
   $L_1 = \{x \in A^* \mid\ x \text{ has an even number of 1's}\}$;

Exercise (first part)

Let $A = \{0, 1\}$ be the alphabet.

1. Build a complete FSA M_1 that recognises the language:
   $L_1 = \{x \in A^* \mid \ x \text{ has an even number of 1's}\}$;
2. Build a complete FSA M_2 that recognises the language:
   $L_2 = \{x \in A^* \mid \ x \text{ has an odd number of 0's}\}$;

Exercise (first part)

Let $A = \{0, 1\}$ be the alphabet.

1. Build a complete FSA M_1 that recognises the language: $L_1 = \{x \in A^* \mid \ x \text{ has an even number of 1's}\}$;
2. Build a complete FSA M_2 that recognises the language: $L_2 = \{x \in A^* \mid \ x \text{ has an odd number of 0's}\}$;
3. Build a complete FSA that accepts when either M_1 or M_2 accepts.
4. Build a complete FSA that accepts when both M_1 and M_2 accept.
5. Build a complete FSA that accepts when M_1 accepts and M_2 rejects.

Exercise (second part)

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Let $A = \{0, 1\}$ be the alphabet.

1. Build a complete FSA M_a that recognises the language:
   $L_a = \{x \in A^* \mid x$ is the binary representation of an integer, and it is divisible by 2$\}$;

Exercise (second part)

Let $A = \{0, 1\}$ be the alphabet.

1. Build a complete FSA M_a that recognises the language: $L_a = \{x \in A^* \mid x$ is the binary representation of an integer, and it is divisible by $2\}$;
2. Build a complete FSA M_b that recognises the language: $L_b = \{x \in A^* \mid x$ is the binary representation of an integer, and it is divisible by $3\}$; (this was part of the homework)

Exercise (second part)

Let $A = \{0, 1\}$ be the alphabet.

1. Build a complete FSA M_a that recognises the language: $L_a = \{x \in A^* \mid x$ is the binary representation of an integer, and it is divisible by $2\}$;
2. Build a complete FSA M_b that recognises the language: $L_b = \{x \in A^* \mid x$ is the binary representation of an integer, and it is divisible by $3\}$; (this was part of the homework)
3. Build a complete FSA that accepts when both M_a and M_b accept.
4. Build a complete FSA that accepts when either M_a or M_b accepts.
5. Build a complete FSA that accepts when M_a accepts and M_b rejects.

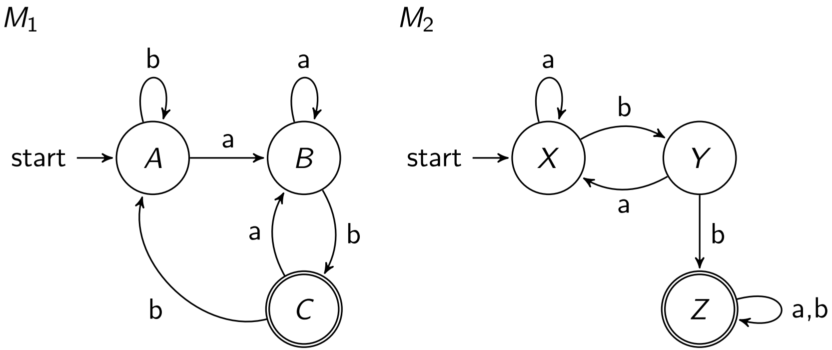
Exercise

Let M_1 and M_2 be the complete FSAs depicted below, accepting languages L_1 and L_2, respectively. Draw complete FSAs accepting the following languages.

i $L_1 \cup L_2$

ii $L_1 \cap L_2$

iii $L_1 \backslash L_2$



Exercises - Homework (The Pumping Lemma)

Theorem

The language $L = \{vv^R : v \in \Sigma^*$ where Σ is the alphabet $\Sigma = \{a, b\}$ is not regular

Proof

Provide a proof for the previous Theorem using the Pumping Lemma