1. Write the first four terms of sequences defined by the formulas in a-c ([ ] – integer part):

   a) $c_i = \dfrac{(-1)^i}{3^i}$, for all integers $i \geq 0$

   b) $e_n = \left[\dfrac{n}{2}\right] \cdot 2$, for all integers $n \geq 0$

   c) $f_n = \left[\dfrac{n}{4}\right] \cdot 4$, for all integers $n \geq 1$

   Solution:

   a) $1, -\dfrac{1}{3}, \dfrac{1}{9}, -\dfrac{1}{27}$

   b) $0, 0, 2, 2$

   c) $0, 0, 0, 4$

2. Compute the summations:

   a) $\sum_{k=1}^{5} (k+1)$

   b) $\sum_{k=-1}^{1} (k^2+3)$

   c) $\sum_{m=0}^{3} \dfrac{1}{2^m}$

   d) $\sum_{n=1}^{10} \left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$

   Solution:

   a) 20

   b) 11

   c) 1.875 or 15/8

   d) 10/11

3. Compute the products

   a) $\prod_{k=2}^{4} k^2$

   b) $\prod_{j=0}^{4} (-1)^j$

   c) $\prod_{k=2}^{2} \left(1 - \dfrac{1}{k}\right)$

   d) $\prod_{i=2}^{5} \dfrac{i(i+2)}{(i-1)(i+1)}$

   Solution:

   a) 576

   b) 1

   c) ½

   d) 35/3

4. Write each a)-c) as single summation

a) $\sum_{i=1}^{k} i^3 + (k+1)^3$

b) $\sum_{k=1}^{m} \frac{k}{k+1} + \frac{m+1}{m+2}$

c) $\sum_{m=0}^{n} (m+1)2^m + (n+2)2^{n+1}$

d) $2 \cdot \sum_{k=1}^{n} (3k^2+4) + 5 \cdot \sum_{k=1}^{n} (2k^2-1)$

e) $\left(\prod_{k=1}^{n} \frac{k}{k+1}\right) \cdot \left(\prod_{k=1}^{n} \frac{k+1}{k+2}\right)$

Solution:

a) $\sum_{i=1}^{k+1} i^3$

b) $\sum_{k=1}^{m+1} \frac{k}{k+1}$

c) $\sum_{m=0}^{n+1} (m+1)2^m$

d) $\sum_{k=1}^{n} (16k^2-3)$

e) $\prod_{k=1}^{n-1} \frac{k}{k+2}$

5. Write, using summation or product:

a) $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2$

b) $(2^2-1)(3^2-1)(4^2-1)$

c) $\frac{2}{3 \cdot 4} - \frac{3}{4 \cdot 5} + \frac{4}{5 \cdot 6} - \frac{5}{6 \cdot 7} + \frac{6}{7 \cdot 8}$

d) $(1-t)(1-t^2)(1-t^3)(1-t^4)$

Solution:

a) $\sum_{k=1}^{7} (-1)^{k+1} k^2$

b) $\prod_{i=1}^{3} \left((i+1)^2 - 1\right)$ or $\prod_{i=2}^{4} (i^2-1)$

c) $\sum_{j=2}^{6} \frac{(-1)^j j}{(j+1)(j+2)}$

d) $\prod_{n=1}^{4} (1-t^n)$

6. Transform by making the change of variable $j = i - 1$

a) $\sum_{i=1}^{n-1} \frac{i}{(n-i)^2}$

b) $\prod_{i=n}^{2n} \frac{n-i+1}{n+i}$

Solution:

a) $\sum_{j=0}^{n-2} \frac{j+1}{(n-j-1)^2}$

b) $\prod_{i=n-1}^{2n-1} \frac{n-j}{n+j+1}$

7. Prove using mathematical induction

$$4^3 + 4^4 + 4^4 + ... + 4^n = \frac{4(4^n - 16)}{3} \text{ for all } n \ge 3,\ n \in Z$$

Solution:

For given statement, the property P(n) is given equation

Lets first show that P(3) is true:

The left hand side of P(3) is $4^3 = 64$, the right side is $\frac{4(64-16)}{3} = \frac{4 \cdot 48}{3} = 64$. Thus, P(3) is true.

Now, lets show that for all integers $k \ge 3$, if P(k) is true, then P(k+1) is true:

Let k be any integer with $k \ge 3$, and suppose P(k) is true (inductive hypothsis):

$$4^3 + 4^4 + 4^5 + ... + 4^k = \frac{4(4^k - 16)}{3}$$

We must show that P(k+1) is true. That is we must show that

$$4^3 + 4^4 + 4^5 + ... + 4^{k+1} = \frac{4(4^{k+1} - 16)}{3}$$

Left size of this equation is:

$$4^3 + 4^4 + 4^5 + ... + 4^{k+1} = 4^3 + 4^4 + 4^5 + ...4^k + 4^{k+1} = \frac{4(4^k - 16)}{3} + 4^{k+1} =$$

$$= \frac{4(4^k - 16) + 3 \cdot 4^{k+1}}{3} = \frac{4 \cdot 4^k - 4 \cdot 16 + 3 \cdot 4^{k+1}}{3} = \frac{4^{k+1} - 4 \cdot 16 + 3 \cdot 4^{k+1}}{3} = \frac{4 \cdot 4^{k+1} - 4 \cdot 16}{3} = \frac{4(4^{k+1} - 16)}{3}$$

And this is the right side of P(k+1). Hence, the property is true for $n = k+1$

8. Prove using mathematical induction

$$\prod_{i=0}^{n} \left(\frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{(2n+2)!} \text{, for all integers } n \ge 1$$

Solution:

$P(0)$:

$\frac{1}{1} \cdot \frac{1}{2} = \frac{1}{2!}$ - True

Let $P(k)$ be true:

$$\prod_{i=0}^{k} \left(\frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{(2k+2)!}$$

Lets prove, that $P(k+1)$ is true

$P(k+1)$:

$$\prod_{i=0}^{k+1} \left(\frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{(2(k+1)+2)!}$$

$$\prod_{i=0}^{k} \left(\frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) \cdot \frac{1}{2(k+1)+1} \cdot \frac{1}{2(k+1)+2} = \frac{1}{(2(k+1)+2)!}$$

$$\frac{1}{(2k+2)!} \cdot \frac{1}{2k+3} \cdot \frac{1}{2k+4} = \frac{1}{(2k+4)!} \text{ - True}$$