1

1.1 Is $\sqrt{\frac{175}{252}}$ an irrational number. _______5/6 (F)____

1.2 There are 8 elements in the set ℘({∅, a, {a}, {{a}} }) . _____2^3 = 8 (F) _

1.3 The truth set of P is { x ∈ D | P(x) } for the predicate P within domain D. _______T_______

1.4 For all rational numbers x and y, the yth power of x is also rational. _______$2^{(1/2)}$ (F)_

1.5 $\forall w \in \mathbf{Z}, \exists x \in \mathbf{Z}, \forall y \in \mathbf{Z}, \exists z \in \mathbf{Z}, w+x = y+z$. _______T__________

1.6 For any given predicate Q, we always have $\sim(\forall x \in D, Q(x)) \equiv \exists x \in D, \sim Q(x)$. ______T_________

1.7 $f(x) = log_3(5^{x*x})$. $f(x) \in O(x^2)$ ______$c*n^2$_(T)__

1.8 x > 0 is a proposition. ___x – free var (F)_

1.9 The implication of $\exists x, \forall y, P(x, y) \Rightarrow \forall y, \exists x, P(x, y)$ is true. _______T_________

1.10 $\{\varnothing\} \subseteq \{\varnothing\}$. _______T__________

2

- 2.1. You cannot study at Innopolis University if and only if you are not healthy now
  - 2.1. $\neg a \leftrightarrow b$
- You will do physical examination every year unless you are not healthy now
  - 2.2 $\neg b \rightarrow c$ **

3

$\forall y \exists x ($

$T(x) \wedge TF(x, y)$

$\wedge$

$\forall z((T(z) \wedge TF(z, y)) \rightarrow (z = x)))$

$\forall y \exists x \forall z$

$((T(x) \wedge TF(x, y))$

$\wedge$

$((z \neq x) \rightarrow \neg(T(z) \wedge TF(z, y))))$

4.1

- P;Q;Z;P ⇒ (QVZ); (P ⇒ Q)V(P ⇒ Z)
- T T T T T
- T T F T T
- T F T T T
- **T F F F F**
- F T T T T
- F T F T T
- F F T T T
- F F F T T

4.2

- **Q is false** while **P and Z are true**

5

- **$(P \land Z) \Rightarrow (Q \lor S)$**
- $\equiv \neg(P \land Z) \lor (Q \lor S)$ (by implication)
- $\equiv (\neg P \lor \neg Z) \lor (Q \lor S)$ (De Morgan)
- $\equiv (\neg P \lor \neg Z \lor Q \lor S)$ (associativity)
- $\equiv (Q \lor \neg P \lor \neg Z \lor S)$ (commutativity)

6 (set eq def)

- $\mathbf{A \subseteq B}$ **and** $\mathbf{B \subseteq A}$

- If $\mathbf{x \in P \cup (P \cap Q)}$**,** then $\mathbf{x \in P}$ **or** $\mathbf{x \in P \cap Q}$. By intersection, we have $\mathbf{x \in P}$ **or** $\mathbf{x \in P}$ **and** $\mathbf{x \in Q}$. In both cases $\mathbf{x \in P}$. So we have $\mathbf{P \cup (P \cap Q) \subseteq P}$**.**

- If $\mathbf{y \in P}$, then $\mathbf{y \in P}$ **or** $\mathbf{y \in P \cap Q}$, so we have $\mathbf{y \in P \cup (P \cap Q)}$. Thus, $\mathbf{P \subseteq P \cup (P \cap Q)}$**.**
  Since $\mathbf{P \cup (P \cap Q) \subseteq P}$ and $\mathbf{P \subseteq P \cup (P \cap Q)}$, we have $\mathbf{P \cup (P \cap Q) = P}$ holds.

7

- $(5k + 1) + (5n + 2) + (5m + 4) = 5(k + n + m) + 7 =$

  $5(k + n + m + 1) + \mathbf{2}$

- $(5k + 1) * (5n + 2) * (5m + 4) =$

$(5*k*5*n + 5*k*2 + 5*n*1 + 2*1) * (5m+4) =\ldots=$

$(5*q + 2) * (5m+4) =$

$(5*q*5*m + 5*q*4 + 5*m*2 + 2*4) = 5*p+8 =$

$5*(p+1) + \mathbf{3}$