Theory of Computation

Lab Session 2

February 04 , 2016

Innopolis University

News

Technical Report

The technical report (essay) is 20% of the final mark and will be divided in 2 parts:

1. An essay on a topic no more than 5 pages long.
2. A live presentation on the topic.

There are extra points, 5 out the final 100 marks, (optional) for a video on the topic.

News

Essay description

- The essay is on a specific topic. We will publish a list of topics by this Friday on moodle.
- You are free to choose another topic: your TA needs to approve it.
- The essay is 5 pages long.
- The essay will be in groups of 3 or 4 students.
- You need to communicate your TA the name of the students that compose each group: by February 18th.
- The essay is to be submitted on April 04th.

News

Presentation description

- The presentation will be about the essay.
- All students in each group need to present.
- The presentation is max 20 minutes.
- The day of the presentation is to be defined (we will notify later).

Agenda

- Exercises on Finite State Automaton (FSA)

FSA

- Finite State Automaton is a model of computation.
- It is a simple computing device: it acts as a language acceptor.

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Let's see an example.

FSA - Example (intuition)

If $\Sigma = \{a, b\}$ and L_1 is defined as

$$L_1 = \{x \in \Sigma^* \mid x \text{ ends with } aa\}$$

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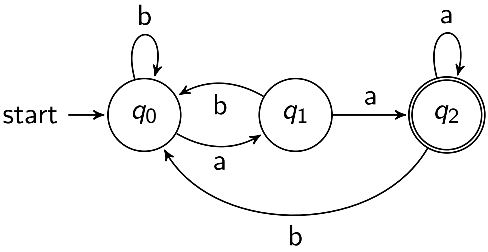
Does the following FSA accepts all strings represented by the language L_1?



Example (informally)

String $x = ababaa$ belongs to L_1. Meaning, $x \in \Sigma^*$ and it ends with aa. Let's see if the FSA accepts the string x

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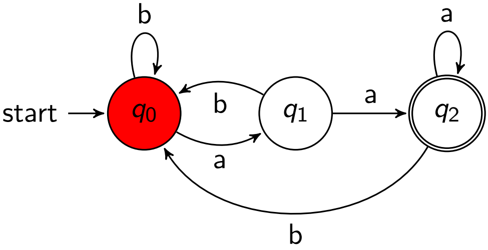
q_0 is the starting point (it is graphically denoted by *start*). So the FSA starts in state q_0

$x = ababaa$



Example (informally)

Then, we go through each character of x, following the transitions in the FSA

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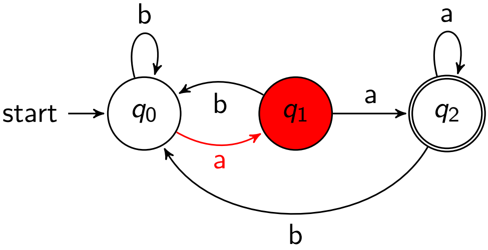
$x = \mathbf{a}babaa$

From state q_0 and label a, we reach state q_1

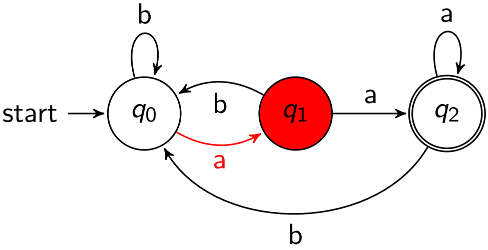


Example (informally)

Then, we go through each character of x, following the transitions in the FSA

$x = \mathbf{a}baba a$

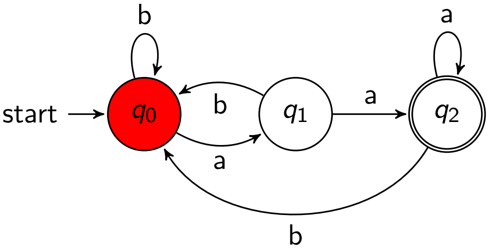
From state q_0 and label a, we reach state q_1



We repeat the process for all characters in x. If at the end we reach a final state (graphically denoted by the double circle state), we say the FSA accepts the string x.

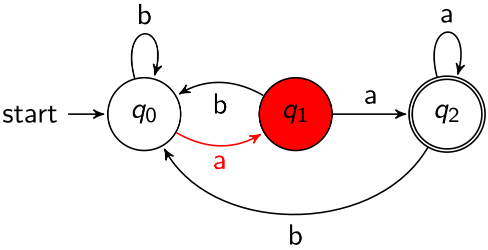
Example (informally) (1)

$x = ababaa$



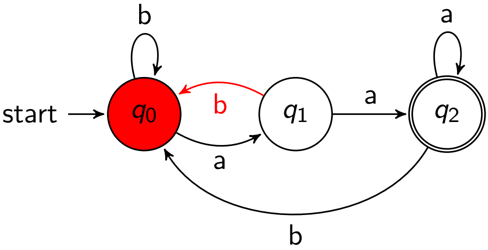
Example (informally) (2)

$x =$ **a***babaa*



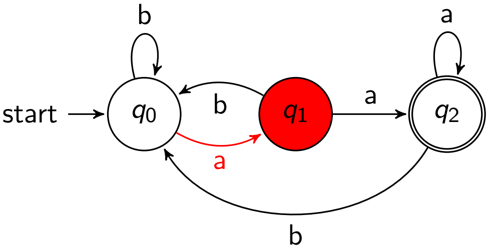
Example (informally) (3)

$x = a\mathbf{b}abaa$



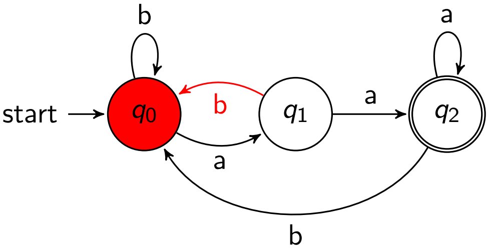
Example (informally) (4)

$x = ab\mathbf{a}baa$



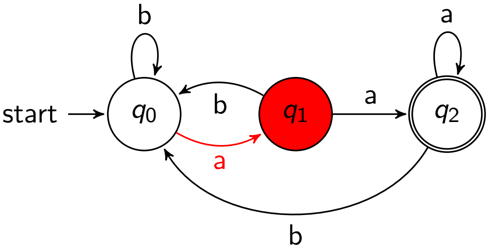
Example (informally) (5)

$x = aba\mathbf{b}aa$



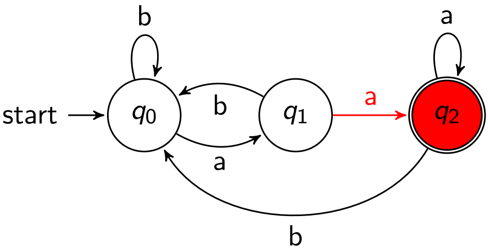
Example (informally) (6)

$x = abab\mathbf{a}a$



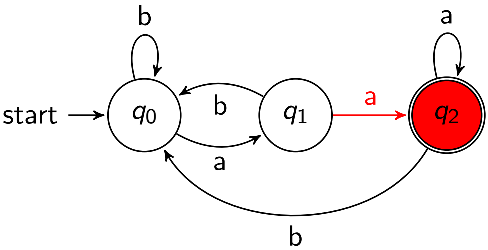
Example (informally) (7)

$x = ababa\mathbf{a}$



Example (informally) (7)

$x = ababa\mathbf{a}$



We went through all characters of x and ended up in a final state: string x belongs to language L_1.

FSA (Formal definition)

A complete Finite State Automaton

A complete Finite State Automaton is a tuple $< Q, \Sigma, q_0, A, \delta >$, where

Q is a finite set of *states*;
Σ is a finite *input alphabet*;
$q_0 \in Q$ is the *initial* state;
$A \subseteq Q$ is the set of *accepting* states;
$\delta : Q \times \Sigma \to Q$ is a total *transition* function.

For any element q of Q and any symbol $\sigma \in \Sigma$, we interpret $\delta(q, \sigma)$ as the state to which the FSA moves, if it is in state q and receives the input σ.

The extended transition δ^*

A move sequence starts from an initial state and is *accepting* if it reaches one of the final states (informally explained with the previous example).
Formally, this transition is defined recursively:

the extended transition δ^*
Let $M = < Q, \Sigma, q_0, A, \delta >$ be a complete finite state automaton.
We define the extended transition function

$$\delta^* : Q \times \Sigma^* \to Q$$

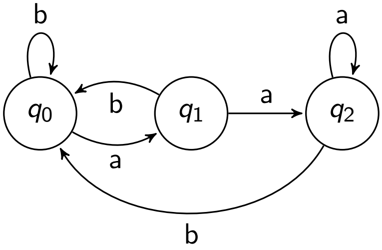
as follows:

1. For every $q \in Q$, $\delta^*(q, \epsilon) = q$
2. For every $q \in Q$, every $y \in \Sigma^*$, and every $\sigma \in \Sigma$,

$$\delta^*(q, y\sigma) = \delta(\delta^*(q, y), \sigma)$$

The extended transition (Example)

The complete FSA M contains the following transitions



$$
\begin{aligned}
\delta^*(q_1, baa) &= \delta(\delta^*(q_1, ba), a) \\
&= \delta(\delta(\delta^*(q_1, b), a), a) \\
&= \delta(\delta(\delta(\delta^*(q_1, \epsilon), b), a), a) \\
&= \delta(\delta(\delta(q_1, b), a), a) \\
&= \delta(\delta(q_0, a), a) \\
&= \delta(q_1, a) \\
&= q_2
\end{aligned}
$$

Acceptance by a FSA

The extended transition function is used to determine what it means for a FSA to accept (or reject) a string or a language. Formally:

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Acceptance by a FSA

Let $M =< Q, \Sigma, q_0, A, \delta >$ be a complete FSA, and let $x \in \Sigma^*$. The string x is accepted by M if

$$\delta^*(q_0, x) \in A$$

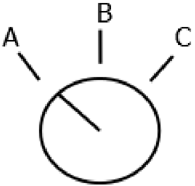
and it is rejected by M otherwise. The language accepted by M is the set

$$L(M) = \{x \in \Sigma^* \mid x \text{ is accepted by } M\}$$

If L is a language over Σ, L is accepted by M iff $L = L(M)$

Example - Three Position Switch

Consider a three-position electrical switch.The switch consists of a knob that can rotate in one of the three positions A, B, C after a shift in a clockwise or counterclockwise (shifts from A to C or C to A are not allowed). Model the operation of the switch with a complete FSA.



Example - Three Position Switch

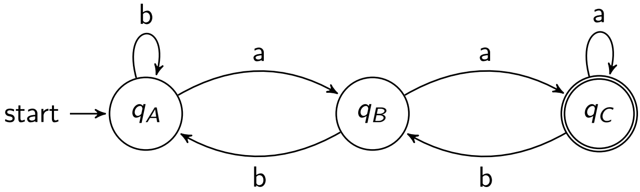
The complete Finite State Automaton

$$
\begin{tikzcd}
A \arrow[loop left, "\text{Counter}"] \arrow[r, bend left, "\text{Clockwise}"] & B \arrow[l, bend left, "\text{Counter}"] \arrow[r, bend left, "\text{Clockwise}"] & C \arrow[l, bend left, "\text{Counter}"] \arrow[loop right, "\text{Clockwise}"]
\end{tikzcd}
$$

A complete FSA models the possible configurations of a system with states, and events or external actions (inputs) that can cause a configuration change with transitions. An automaton that solves the proposed exercise is shown. It has a set of states $Q = (A, B, C)$ to represent the position and a transition function δ that models changes in position of the switch following the application of an appropriate motion to the knob.

Example - Three Position Switch

The Automaton Acceptor



An automaton acceptor of the language sequences of moves that lead from A to C.

Exercises

Exercises (first part)

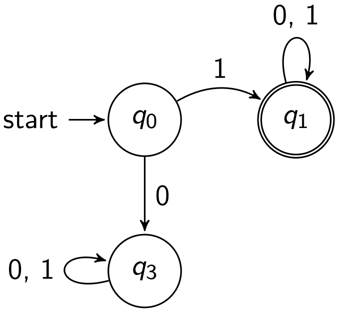
Build complete FSAs that recognise the following languages:
Let Σ be the alphabet $\Sigma = \{0, 1\}$

- $L_0 = \{x \in \Sigma^* \mid \ x \text{ starts with } 1\}$;
- $L_1 = \{x \in \Sigma^* \mid \ x \text{ does not begin with } 1\}$;
- $L_2 = \{x \in \Sigma^* \mid \text{ any } 0 \text{ in } x \text{ is followed by at least a } 1\}$.
  Strings example: 010111, 1111, 01110111011.
- $L_3 = \{x \in \Sigma^* \mid \ x \text{ ends with } 00\}$;
- $L_4 = \{x \in \Sigma^* \mid \ x \text{ contains exactly 3 zeros}\}$;

Solution (1)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

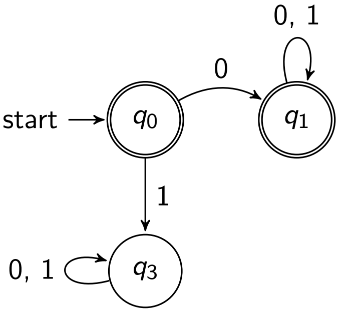
- $L_0 = \{x \in \Sigma^* \mid x \text{ starts with } 1\}$;



Solution (2)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

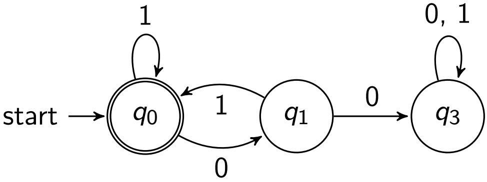
- $L_1 = \{x \in \Sigma^* \mid \ x \text{ does not begin with } 1\}$;



Solution (3)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

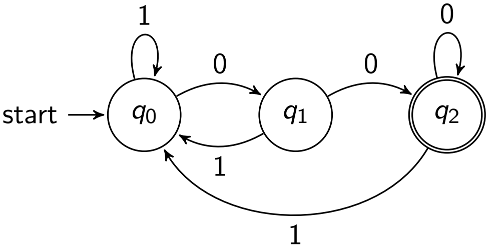
- $L_2 = \{x \in \Sigma^* \mid \text{ any 0 in } x \text{ is followed by at least a 1}\}$.
  Strings example: 010111, 1111, 01110111011.



Solution (4)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

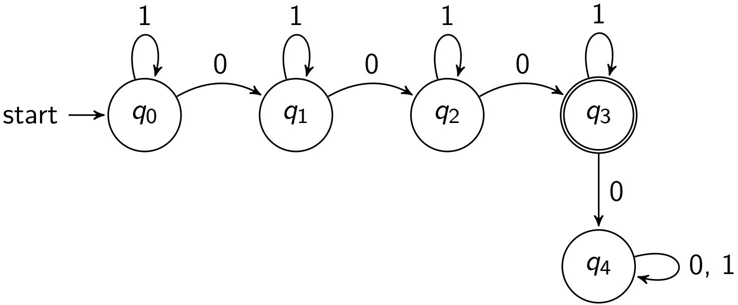
- $L_3 = \{x \in \Sigma^* \mid \ x \text{ ends with } 00\}$;



Solution (5)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

- $L_4 = \{x \in \Sigma^* \mid x \text{ contains exactly 3 zeros}\}$;



Exercises (second part)

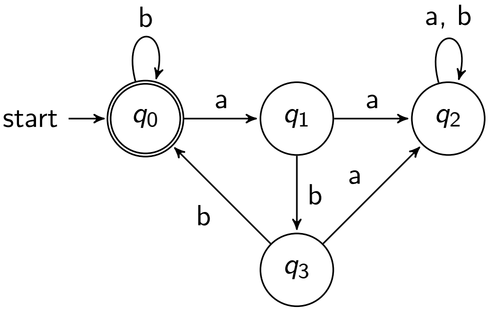
Build complete FSAs that recognise the following languages:
Let Σ be the alphabet $\Sigma = \{a, b\}$

- $L_5 = \{x \in \Sigma^* \mid$ every a in x (if there are any) is followed immediately by $bb\}$.
- $L_6 = \{x \in \Sigma^* \mid$ x ends with b and does not contain the substring $aa\}$.
- $L_7 = \{x \in \Sigma^* \mid$ x contains the substring $abbaab\}$;
- $L_8 = \{x \in \Sigma^* \mid$ x has an even number of 0's and an even number of 1's$\}$;

Solution (6)

Let Σ be the alphabet $\Sigma = \{a, b\}$

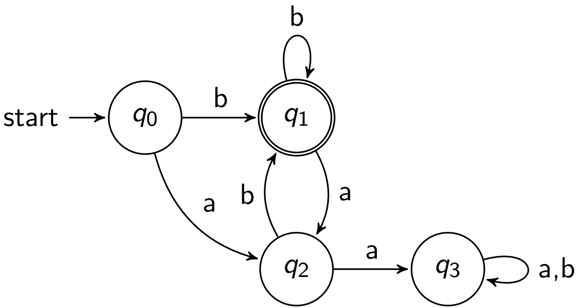
- $L_5 = \{x \in \Sigma^* \mid$
  every a in x (if there are any) is followed immediately by $bb\}$.



Solution (7)

Let Σ be the alphabet $\Sigma = \{a, b\}$

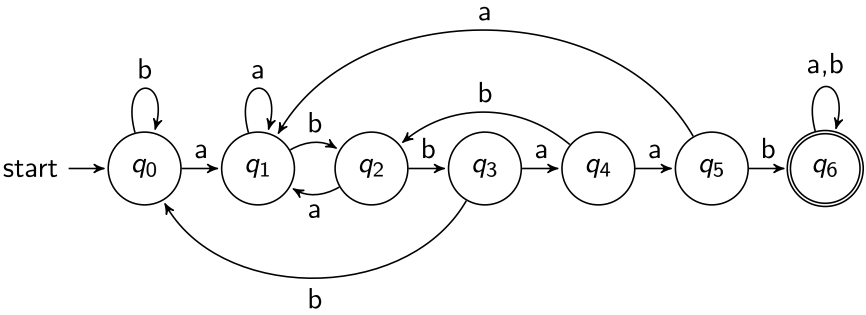
- $L_6 = \{x \in \Sigma^* \mid$
  x ends with b and does not contain the substring $aa\}$.



Solution (8)

Let Σ be the alphabet $\Sigma = \{a, b\}$

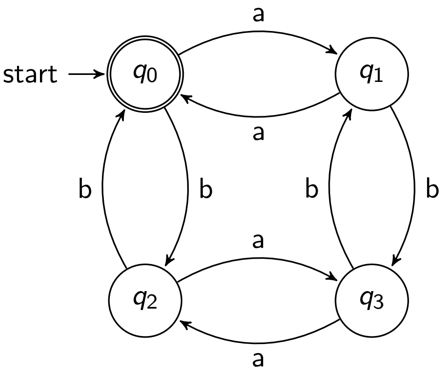
- $L_7 = \{x \in \Sigma^* \mid \ x \text{ contains the substring } abbaab\}$;



Solution (9)

Let Σ be the alphabet $\Sigma = \{a, b\}$

- $L_8 = \{x \in \Sigma^* \mid$
  x has an even number of a's and an even number of b's$\}$;



Exercises - Homework (1)

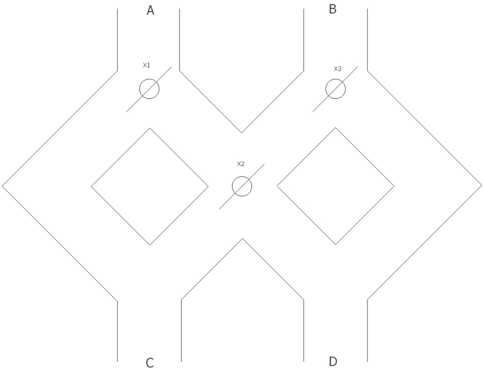
Build complete FSAs accepting the following languages over the alphabet $\Sigma = \{0, 1\}$

- $L_a = \{x \in \Sigma^* \mid \ x$ is the a binary representation of an integer, and it is divisible by $3\}$;
- $L_b = \{x \in \Sigma^* \mid$ x begins with a 1 that, when interpreted as a binary integer, is multiple of $5\}$;
- $L_c = \{x \in \Sigma^* \mid |x| \geq 2 \wedge$ whose final two symbols are the same$\}$;

Build a complete FSA accepting the following languages over the alphabet $\Sigma = \{a, b, c\}$

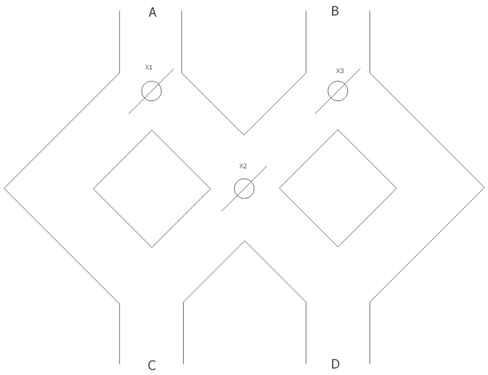
- $L_d = \{x \in \Sigma^* \mid$ the substring *abc* in x occurs an odd number of times$\}$;

Exercises - Homework (2a)



The figure is a marble toy. A marble is dropped at A or B. Levers $x1$, $x2$, and $x3$ cause the marble to fall either to the left of to the right. Whenever a marble encounters a lever, it causes the lever to reverse after the marble passes, so the next marble will take the opposite branch.

Exercises - Homework (2b)



Model this toy by a complete FSA. Let the inputs A and B represents the input into which the marble is dropped. Let acceptance corresponds to the marble existing at D; nonacceptance represents a marble exiting at C.

Exercises - Homework (3)

Implement, in the programming language of your choice, the FSAs of the previous examples and exercises (give an elegant solution to them!).