Endsem: MA 504 (Computational Differential Equation), 2017 **IIT Patna**

Instructors: P. Das and P. K. Srivastava Total Marks: 50, Time: 3 Hours

****** *Question 1-11 (Post-Midsem) should be answered in a single place ONLY. Otherwise their answers will NOT be evaluated******

Post-Midsem:–

1. True/False: Upwind schemes are always unconditionally stable. Justify your answer. [3]

2. Write down a first order convergent upwind scheme for Burger's equation $u_t + \left(\frac{u^2}{2}\right)_x = 0$ by keeping the continuous problem as it is (i.e., do not assume $(u^2/2)_x = uu_x$). [3]

3. Find out the stability condition of the BTCS scheme for $u_t = u_{xx}$. [3]

4. What does CFL condition of a general PDE say, physically? Find out the CFL condition for advection equation $u_t + cu_x = 0$, $u(x, 0) = f(x)$ where $c > 0$. [2+3]

5. Statement: "It is possible that a discretized problem (obtained by difference schemes) produces first order convergence for a first order PDE. But the same discrete problem (without alternating or adding any part/term) produces second order convergence for a second order PDE." Justify this statement by producing an example. [3]

6. Find the Lax Friedrichs scheme for $u_{tt} = c^2 u_{xx}$ on $0 < x < 1$, $0 < t < 1$.(Hint:Think about converting it into a first order system of advection equations before applying scheme) [3]

7. Write down the advantages and disadvantages of first order Upwind Scheme, CTCS, Lax Friedrichs scheme, Lax Wendroff scheme for $u_t + cu_x = 0$, $u(x, 0) = f(x)$. No need to write down these schemes. [$4 \times 2 = 8$]

8. Discuss the advantage of Lax Wendroff scheme over Leapfrog scheme for Advection equation. [1]

9. Write down the Alternating Direction Implicit (ADI) method for $u_t = u_{xx} + u_{yy} + f(x, y, t)$. What can you say about the stability and consistency of this method. Is there any benefit of this method over the standard Crank Nicolson Scheme for the above two dimensional Heat equation. [3+2+1]

10. Discuss the demerits of Von Neumann Stability analysis. [2]

11. What is Methods of Lines for $u_t(x, t) = Lu(x, t) + f(x, t)$. Discuss the disadvantages of this method. [2+1]

Pre-Midsem:–

12. Solve the BVP: $y'' = \frac{3}{2}y^2$, $y(0) = 4$, $y(1) = 1$ on interval $[0, 1]$ choosing $h = 1/3$. (For starting the iterations, you may use initial guess for unknown as $(2, 1.5)$.) [4]

13. Use AB-3 ($x_{n+1} = x_n + \frac{h}{12}[23f_n - 16f_{n-1} + 5f_{n-2}]$) to solve the IVP: $x' = t^2 - x$, $x(0) = 1$ for $x \in [0, 0.3]$ choosing $h = 0.1$. For starting values use method of same order. [3]

14. Use AM-3 ($x_{n+1} = x_n + \frac{h}{12}[5f_{n+1} + 8f_n - f_{n-1}]$) to solve the IVP: $x' = t + x$, $x(0) = 1$ for $x \in [0, 3]$ choosing $h = 1$. For starting values use method of same order. [3]