Endsem: MA 504 (Computational Differential Equation), 2018 **IIT Patna**

Instructor: P. Das Total Marks: 50, Time: 3 Hours

1. Assume that the difference scheme $\sum_{k=1}^{5} a_k u_{i-k+1}$ converges to the smooth function $u^{(iv)}(t_i)$. Then $\sum_{k=1}^{5} a_k =$ ..... since .... (fill up by only two words). [1+1]

2. Write down the central difference scheme for $u''(x)$ at $x = x_i$ by using forward and backward scheme. [2]

3. True/False: Upwind schemes are always unconditionally stable. Justify your answer. [1+2]

4. True/False: Central difference scheme in space (with space step size Δx) with forward scheme in time (with time step Δt) will give second order convergent solution in space and time for $u_t = u_{xx}$ if $\Delta t = (\Delta x)^2$. Justify your answer. [1+2]

5. True/False: For the problem $u_t + cu_x = 0, \quad 0 < x < l, c > 0, u(x, 0) = f(x)$, one can not prescribe the solution at $u(l, t) = g(t)$. Justify your answer. [1+2]

6. What is Lax equivalence theorem. Discretize the problem $u'' + a(x)u' + bu = f, \quad x \in (0, 1)$, where $a(x) = 2, x > .6$ and $a(x) = -3, x \geq .6$, suitably so that it produces first order convergent stable solution. [2+3]

7. What does the CFL condition of $u_t + cu_x = 0$ say, physically, in terms of analytical and numerical speed of a wave? Write down the CFL condition for $u_t + c(x, t)u_x = f(x, t)$, $u(x, 0) = g(x)$ where $c(x, t) > 0$. Write down the Crank Nicolson scheme for this problem. [2+2+2]

8. Find the Lax Friedrichs scheme for $u_{tt} = c^2 u_{xx}$ on $0 < x < 1$, $0 < t < 1$.(Hint:Think about converting it into a first order system of advection equations before applying scheme) [3]

9. Write down the advantages and disadvantages of first order Upwind Scheme, CTCS, Lax Friedrichs scheme, Lax Wendroff scheme for $u_t + cu_x = 0$, $u(x, 0) = f(x)$. No need to write down these schemes. [$4 \times 2 = 8$]

10. Consider the PDE $u_t = u_{xx} + u_{yy} + f(x, y, t)$,
    (a) Write down the Alternating Direction Implicit scheme for the above PDE.
    (b) Discuss the benefits of this scheme over classical central difference scheme.
    (c) Mention the order of convergence and stability condition of the Alternating Direction Implicit scheme for the above PDE. [3+2+2]

11. Consider the PDE $u_t = u_{xx} + u_{yy} + f(x, y, t)$. Discretize the above PDE by finite volume scheme by explaining all the procedures and write the time dependent reduced ODE after using finite volume method. [4]

12. Consider the problem $u_{tt} = u_{xx}, \quad 0 < x < l, t \geq 0, u(t, 0) = \alpha(t), \quad u(t, l) = \beta(t), \quad u(0, x) = f(x), \quad u_x(0, x) = g(x)$. Explain the procedure to obtain **second order convergence** in time and space by using central differrence scheme for space and time discretization. Note that the solution at 1st time level Δt is not known and forward discretization in time will only produce first order convergence. [4]