1. CSE 6220: HW4

Note: This assignment is submitted by: **Somdut Roy (GTID: sroy86)**

1. In both cases, many-to-many communication is used, each processor has $\frac{n}{p}$ tree nodes and D is the number of children per tree node (say).
   (a) Sending size $\leq \frac{n}{p}$ and receiving size $\leq \frac{Dn}{p}$.
   (b) Sending size $\leq \frac{Dn}{p}$ and receiving size $\leq \frac{n}{p}$.
   Runtime: In each case, $O(\tau log(p) + \mu \frac{n}{p} log(p))$ or $O(\tau p + \mu \frac{n}{p})$.

2. Like sorting n elements, this problem is analogous to one of size $m * p$ being arranged into p processors.
   That should take a computation and communication time of $T_{comp}(mp, p)$ and $T_{comm}(mp, p)$ respectively.

3. For rank i of a processor, define the mapping function $f(i)$ as follows:

$$f(i) = \begin{cases} 0 & i = 0 \\ (f(i-1) + i(-1)^{i-1} + p) \mod p & \text{otherwise} \end{cases}$$

   Here, 0 and 1 get mapped to 0 and 1 as we can see. For $i > 2$,
   $f(i) = (f(i-1) + i(-1)^{i-1} + p) \mod p$.
   $f(i-1) = (f(i-2) + (i-1)(-1)^{i-2} + p) \mod p$.
   Adding the two above we get,
   $f(i) = (f(i-2) + (-1)^{i-1} + p) \mod p$, which ensures the criteria mentioned in the problem (that is adjacent ranks are separated by ≤ 2 places). For example, let $p = 8$, then the f-sequence will look like 0, 1, 7, 2, 6, 3, 5, 4. This satisfies the condition mentioned in the problem.

4. The 3 dimensions will have 4, 4 and 3 bits respectively as $2^4 * 2^4 * 2^3 = 2^{11} = 2048$.
   (a) $binary(7, 15, 3) = (0111, 1111, 011)$.
   $btog(0111, 1111, 011) = (0100, 1000, 010)$.
   $decimal(01001000010) = 512 + 64 + 2 = 578$.
   (b) $binary(877) = 1101101101$ which can be split into dimensions as $(0110, 1101, 101)$.
   $gtob(0110, 1101, 101) = (0100, 1001, 110)$.
   $decimal(0100, 1001, 110) = (4, 9, 6)$.
   Torus rank of processor 877 is $(4, 9, 6)$.

5. We aim to find two processors that are connected in the tree topology may be assigned to processors in the target topology whose ranks differ in more than one bit position.