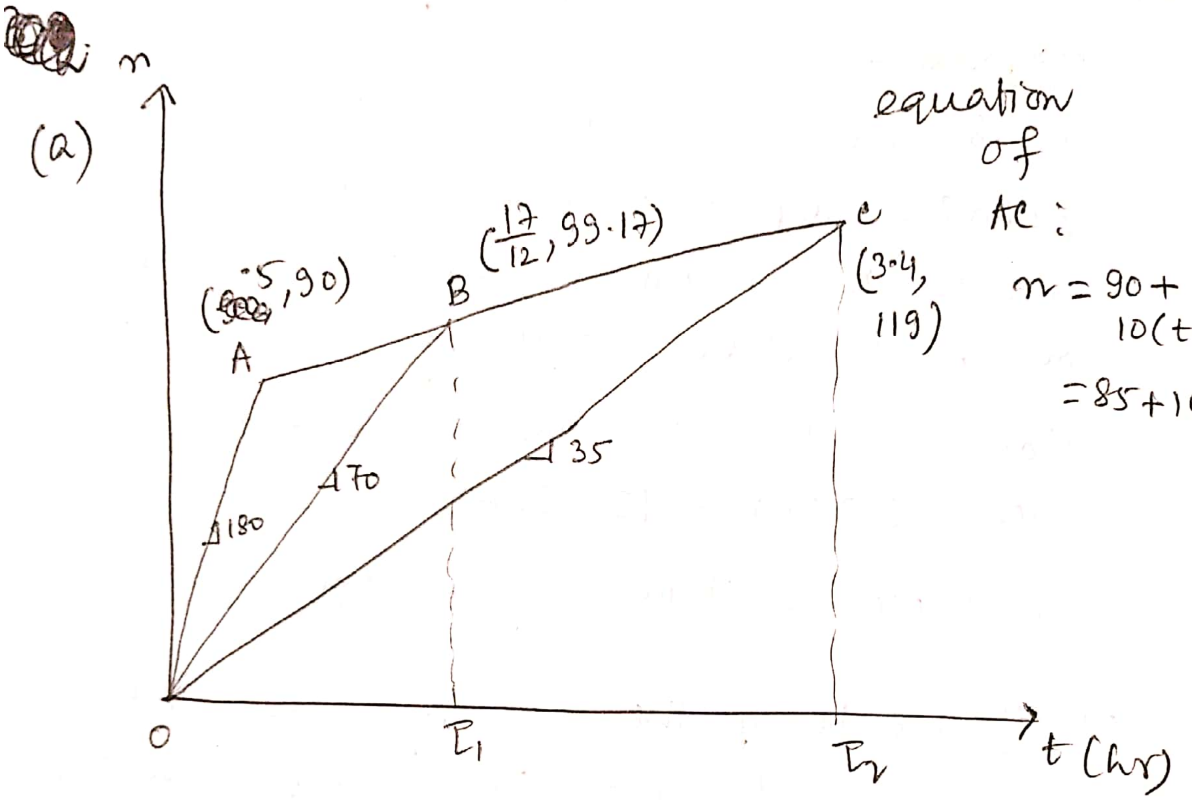
3.2

(a)



equation of AC :

$n = 90 + 10(t - .5)$

$= 85 + 10t$

arrival to airport $A(t) = \begin{cases} 180t & t < 0.5 \\ 85 + 10t & t \geq 0.5 \end{cases}$

arrival for security or departure for ticket $D_1(t) = \begin{cases} 70t & t < T_1 \\ 10t & t \geq T_1 \end{cases}$

departure from security $D_2(t) = \begin{cases} 35t & t < T_2 \\ 85 + 10t & t \geq T_2 \end{cases}$

$T_1 = \text{solve}(85 + 10t = 70t) = \frac{17}{12}$ hr

$T_2 = \text{solve}(35t = 85 + 10t) = 3.4$ hr

Delay at ticket counter = area (Δ OAB) $\approx$ 38.96 hr.

Delay b/w counter & security = area (Δ OBC) $\simeq$ 84.3 hr.

Total delay = 38.96 + 84.3 = 123.26 hr.

(b) If ticket counter capacity is increased to 180/hr, the curve $O \rightarrow B \rightarrow C$ defining departure from counter will be changed to $O \rightarrow A \rightarrow C$, as the counters will be able to serve everything.

However, the delay overall will still be area (ΔOAC) = 123·26 hr.

So, there won't be any change.

(c) If we bump the security capacity to 70/hr or beyond (like 180/hr) as the question asks, the curve $O \rightarrow C$ defining departure from security changes to $O \rightarrow B \rightarrow C$.

Hence the net delay becomes area (ΔOAB).

∴ The delay is reduced by the entire delay b/w counter & security i.e. by 84·3 hr.

~~(d) To avoid complications, we keep the ticket counter and security the same, so that we essentially see them as one single server~~

(d) Just by increasing the capacity of security, we can reduce delay by $\frac{84 \cdot 3}{123 \cdot 26} \times 100 \approx 68 \cdot 39\%$. To get better results like 70%, we will need to improve both counter and security together

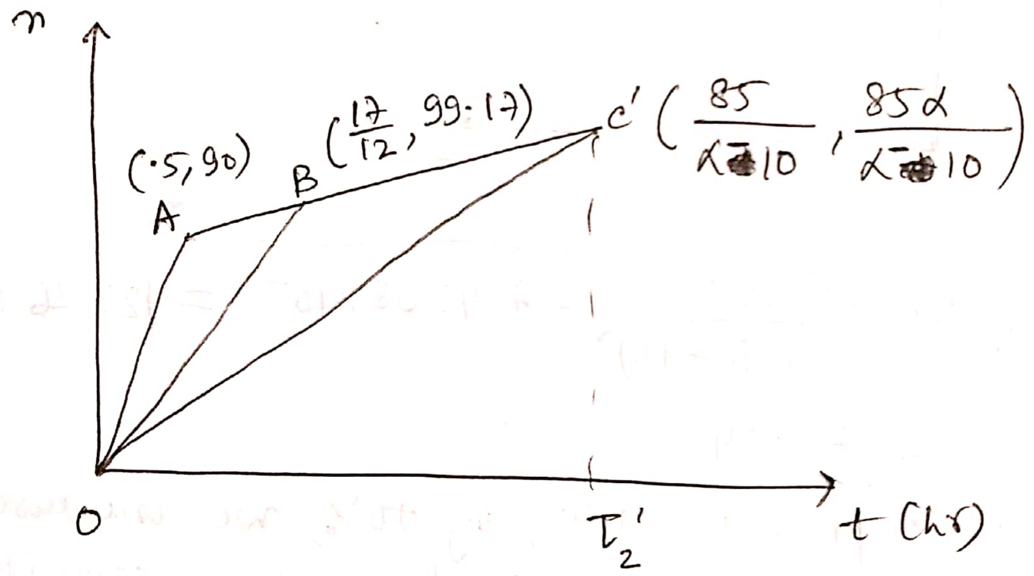
(c) The method of getting security capacity better is the cheapest method of bettering the system.

~~Therefore~~

$$D_2'(t) = \begin{cases} \alpha t & t < T_2' \\ 85 + 10t & t \geq T_2' \end{cases}$$

$$T_2' = \text{solve}\left[\alpha t = 85 + 10t\right] = \frac{85}{\alpha - 10}$$

So, the net departure curve is given by c'



$$\text{area}(OAc') = 123.26 \times 0.5$$

$$\frac{1}{4}\sqrt{(180-\alpha)^2 \frac{85^2}{(\alpha-10)^2} - 7.4508 \times 10^{-9}} = 123.26 \times 0.5$$

$\alpha \simeq 53.59$ ~~pax/hr~~

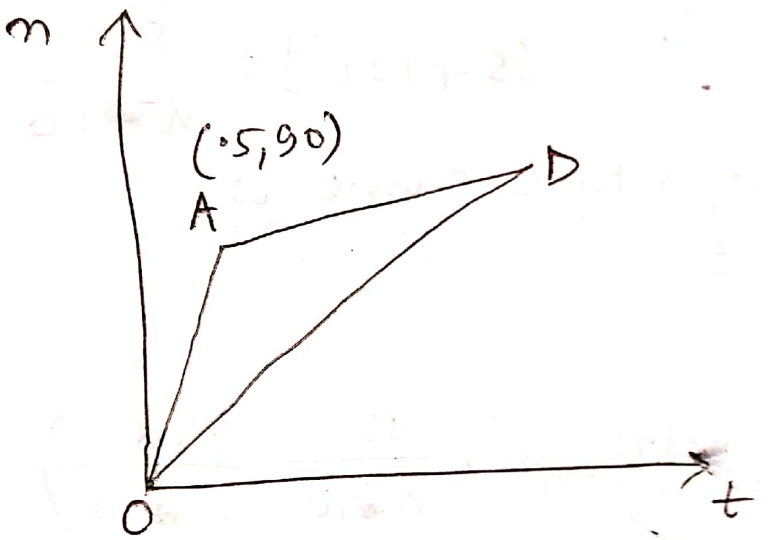
So, to reduce delay by 50%, we increase the capacity of security to 53.59 pax/hr without changes to ticket counter.

~~After~~

For 70% delay reduction,

Both ticket counter and security capacity should be increased to β.

$$D_1''(t) = D_2''(t) = \begin{cases} \beta t & t < t_3 \\ 85+10t & t \geq t_3 \end{cases}$$



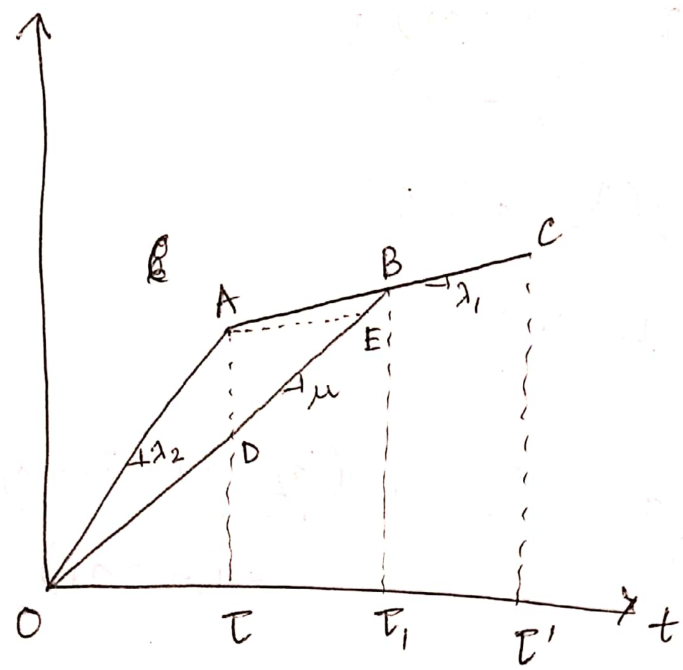
$$\frac{1}{4}\sqrt{\frac{(180-\beta)^2 85^2}{(\beta-10)^2} - 7.4508\times10^{-9}} = 123.26 \times 0.3$$

$\beta \approx 72.04.$

So, to reduce delay by 70%, we increase both ticket counter and security capacity to 72.04 pax/hr.

3.5

(a)



line ABC equation:

$$n = \lambda_2\tau + \lambda_1(t-\tau)$$
$$= (\lambda_2-\lambda_1)\tau + \lambda_1 t$$

$$A(t) = \begin{cases} \lambda_2 t & t < \tau \\ (\lambda_2-\lambda_1)\tau + \lambda_1 t & t \geq \tau \end{cases}$$

$$\tau_1 = \text{solve}\left((\lambda_2-\lambda_1)\tau + \lambda_1 t = \mu t \right)$$
$$= \frac{\lambda_2-\lambda_1}{\mu-\lambda_1}\tau$$

~~Total~~ Maximum queue length = AD

$$= (\lambda_2-\mu)\tau$$

Longest delay to a customer

= AE

$$= \frac{\lambda_2\tau}{\mu} - \tau = \frac{\tau(\lambda_2-\mu)}{\mu}$$

Duration of queue $= \tau_1 = \dfrac{\lambda_2-\lambda_1}{\mu-\lambda_1}\tau$

Total delay = area(Δ OAB) = area(ΔOAD) + area(Δ ABD)

$$= \frac{1}{2}\left[AD \times \tau + AD(\tau_1-\tau)\right] = \frac{(\lambda_2-\mu)\tau\tau_1}{2}$$

Total delay

$$= \frac{(\lambda_2-\mu)\tau}{2} \cdot \frac{(\lambda_2-\lambda_1)}{(\mu-\lambda_1)}\tau$$

$$= \frac{(\lambda_2-\mu)(\lambda_2-\lambda_1)\tau^2}{2(\mu-\lambda_1)}$$

(b) Total cost $(\bar{c}) = \beta\mu + \frac{(\lambda_2-\mu)(\lambda_2-\lambda_1)}{2(\mu-\lambda_1)}\tau^2\delta$

$$\frac{d\bar{c}}{d\mu} = 0 \Rightarrow \beta + \delta\tau^2\left[-\frac{(\lambda_2-\lambda_1)^2}{2(\mu-\lambda_1)^2}\right] = 0$$

$$\Rightarrow \frac{(\mu-\lambda_1)^2}{(\lambda_2-\lambda_1)^2} = \frac{\tau^2\delta}{2\beta}$$

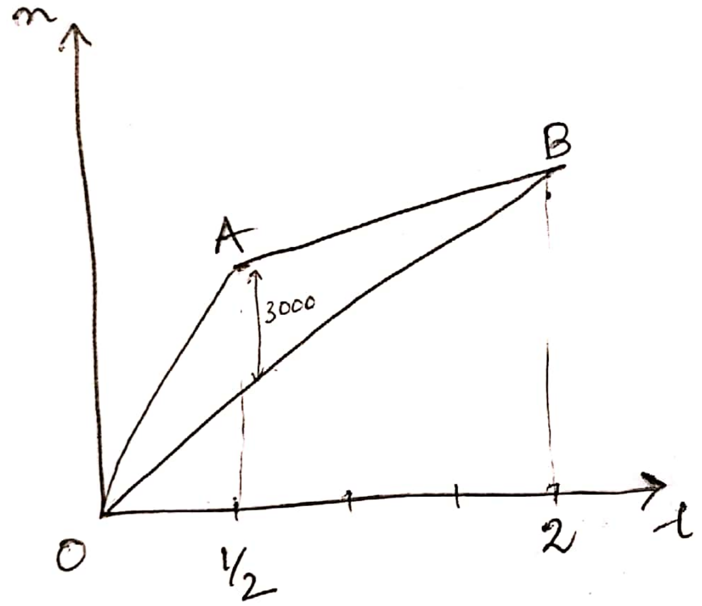
$$\Rightarrow \boxed{\mu = \lambda_1 + \frac{\tau(\lambda_2-\lambda_1)}{\sqrt{2\beta/\delta}}}$$

3.6.

(a) $A(t) = \begin{cases} 10000t & t < 1/2 \\ 2000t + 4000 & t \geq 1/2 \end{cases}$

$D(t) = \begin{cases} 4000t & t < \tau \\ 2000t + 4000 & t \geq \tau \end{cases}$

$\tau = \text{solve}(4000t = 2000t + 4000) = 2$ hr.



For User equilibrium, there won't be any vehicle taking the off-ramp.

So, the delay will be given by the area of Δ OAB

$= \frac{1}{2} \times 3000 \times 2 = 3000$ hr

3.6. (b) Assumption: Freeway speed = 80 Kph
Arterial speed = 30 Kph.

(Plot attached)

So, taking each off-ramp causes extra time of $\frac{1\text{ Km}}{30\text{ Kph}} - \frac{1\text{ Km}}{80\text{ Kph}} = \frac{1}{48}$ hr

(c) The attached plot shows overall reduction in delay significantly with SO.

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