

3.11.

(a) For OR,

$$A_2(t) = (6500 \times 15\% / \text{hr}) t$$

$$D_2(t) = \begin{cases} (500 \times 50\% / \text{hr}) t & t < 20 \text{ mins} \\ \left(\frac{6500}{3} \times 50\% \right) & 20 \text{ mins} < t < T_0 \\ \text{slope} \\ \text{dope} & \\ (6500 \times 15\% / \text{hr}) t & t > T_0 \\ \text{slope} & \end{cases}$$

where T_0 is the time it takes to clear the OR Bottleneck.

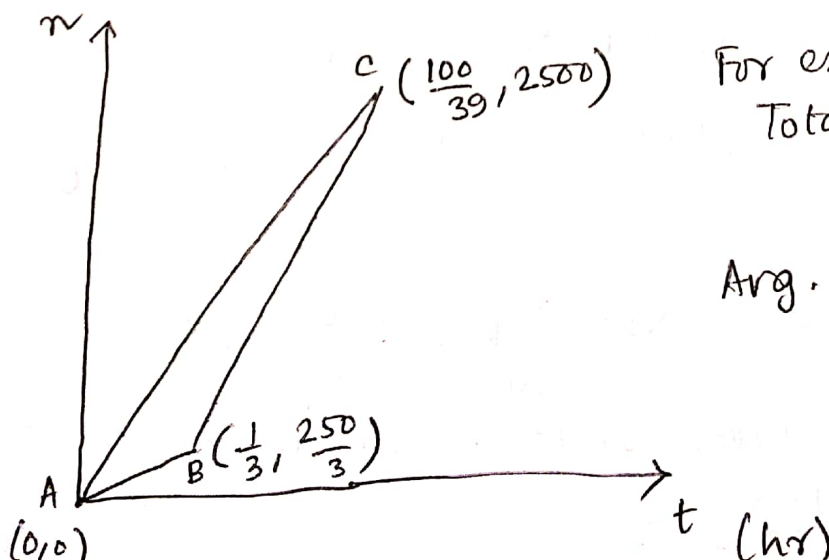
Queue formed in OR in the 20 mins

$$= (6500 \times 15\% - 500 \times 50\%) \times \frac{20}{60} = \frac{725}{3} = Q_0 \quad (\text{say})$$

Queue clearing rate

$$= \left(\frac{6500}{3} \times 50\% - 6500 \times 15\% \right) / \text{hr} = \frac{325}{3 \text{ hr}} = r \quad (\text{say})$$

$$T_0 = 20 \text{ mins} + Q_0 / r = \frac{100}{39} \text{ hr}$$



For exiting vehicles

Total delay = area (ΔABC)

$$= \boxed{309.829 \text{ hr}}$$

$$\text{Avg. delay} = \frac{309.829}{2500 - 0} \times 60 \text{ mins}$$

$$= \boxed{7.44 \text{ mins.}}$$

For Freeway, initially it stays unaffected as the 200m of OR gets filled up first at timestamp ' τ ' (say). Then one lane gets blocked reducing the capacity of the Freeway to $\frac{2}{3}$ rd. So, 2 lanes carry the load of non-existing vehicles till the time it takes for the off-ramp queue to be limited only to the 200m of OR, which is denoted by timestamp ' τ_1 ' (say). As the third lane gets cleared, all the queues and new non-existing vehicles get cleared as the freeway is served at full capacity. The timestamp for that clearance is denoted by ' τ_2 ' (say).

$$A_1(t) = (6500 \times 85\% / \text{hr}) \times t$$

$$D_1(t) = \begin{cases} (6500 \times 85\% / \text{hr}) t & t < \tau \\ (6500 \times \frac{2}{3}) / \text{hr} \text{ slope} & \tau \leq t \leq \tau_1 \\ 6500 / \text{hr} \text{ slope} & \tau_1 \leq t < \tau_2 \\ (6500 \times 85\% / \text{hr}) t & t \geq \tau_2 \end{cases}$$

Assumption: A car and allowable space behind it a queue altogether be assumed to 6m (say). So, the OR holds $\frac{200}{6} = \frac{100}{3}$ vehicles before affecting freeway lane at $t = \tau$

$$(6500 \times 15\% - 500 \times 50\%) \frac{\tau}{\text{hr}} = \frac{100}{3} \Rightarrow \tau = 165.52 \text{ s}$$

Queue formed in Freeway lane due to OR

$$Q_1 = Q_0 - \frac{100}{3} = \frac{625}{3}$$

Time stamp at ~~48~~ which that gets cleared

$$T_1 = 20 \text{ mins} + \frac{Q_1}{r} = \frac{88}{39} \text{ hr.}$$

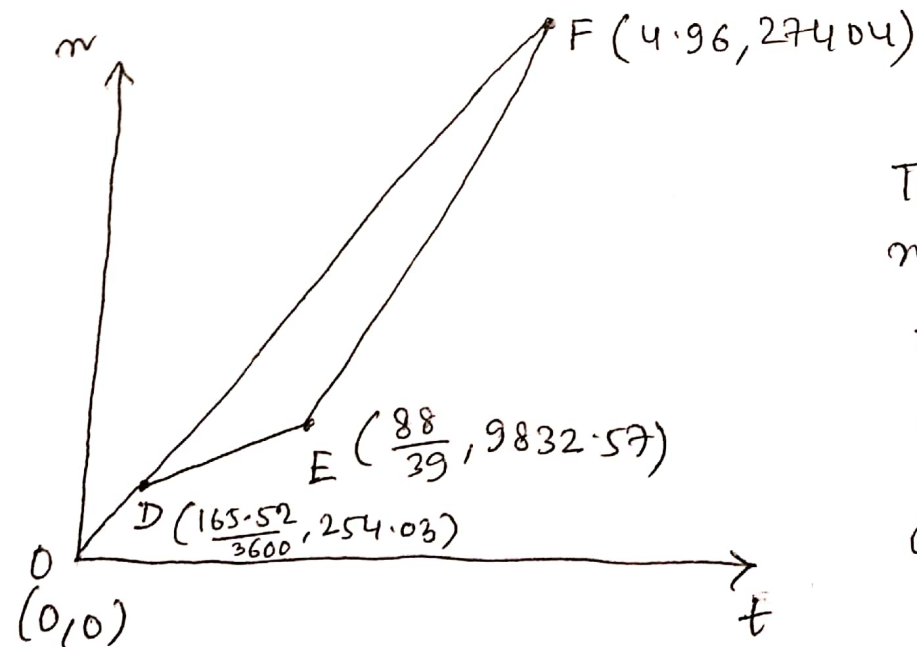
Total queue formed on the other 2-lanes from $t = T$ to $t = T_1$ is given by

$$Q_2 = (T_1 - T) \times \left[\frac{6500 \times 85\% - 6500 \times \frac{2}{3}}{\text{hr}} \right] \approx 2634.1$$

Rate of queue clearance at the freeway after $t = T_1$ is given by

$$r_2 = [6500 - 6500 \times 85\%] / \text{hr}$$

$$\therefore T_2 = T_1 + \frac{Q_2}{r_2} \approx 4.96 \text{ hr.}$$



Total delay for non-exiting vehicles

= area ($\triangle DEF$)

$$= \boxed{6472.02 \text{ hr}}$$

avg. delay

$$= \frac{6472.02 \text{ hr}}{27404 - 254.03}$$

$$\approx \boxed{14.3 \text{ mins}}$$