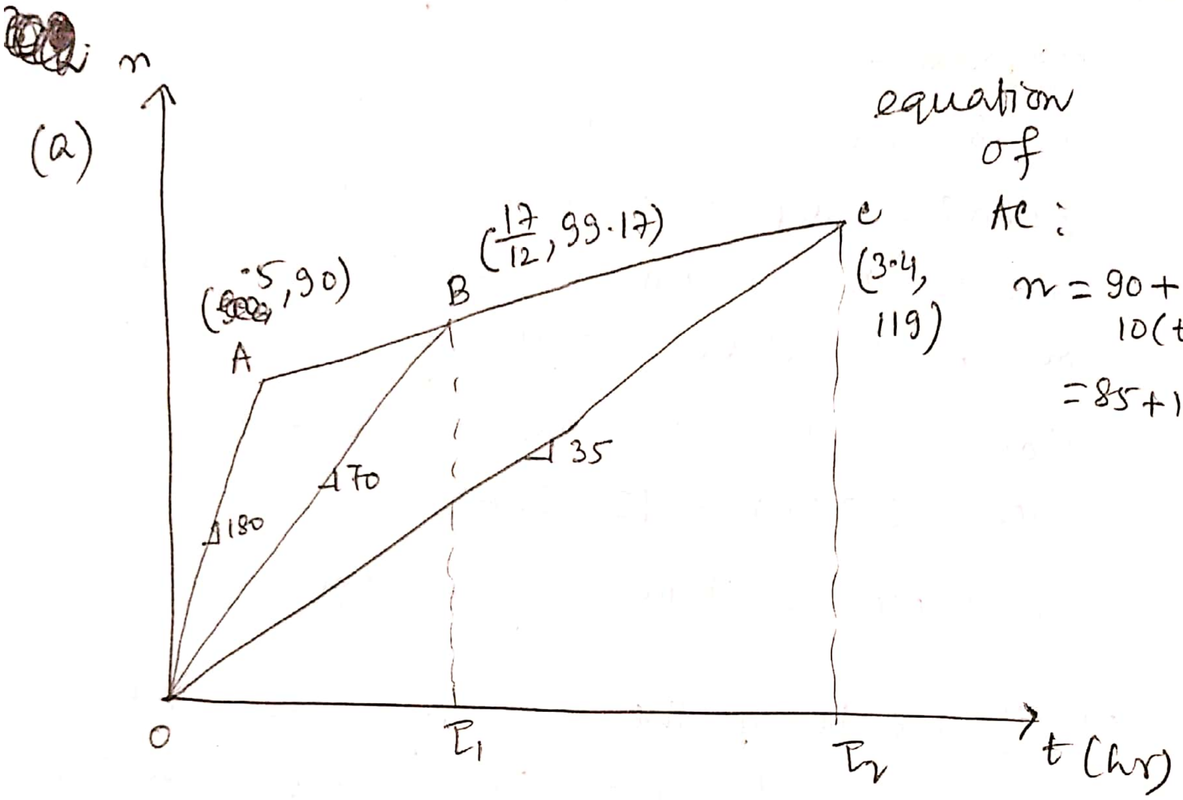
3.2

(a)



equation of AC:

$n = 90 + 10(t - .5)$
$= 85 + 10t$

arrival to airport $A(t) = \begin{cases} 180t & t < 0.5 \\ 85 + 10t & t \geq 0.5 \end{cases}$

[arrival for security or departure for ticket] $D_1(t) = \begin{cases} 70t & t < T_1 \\ 10t & t \geq T_1 \end{cases}$

[departure from security] $D_2(t) = \begin{cases} 35t & t < T_2 \\ 85 + 10t & t \geq T_2 \end{cases}$

$T_1 = \text{solve}(85 + 10t = 70t) = \frac{17}{12}$ hr

$T_2 = \text{solve}(35t = 85 + 10t) = 3.4$ hr

Delay at ticket counter = area (Δ OAB) $\approx$ 38.96 hr.

Delay b/w counter & security = area (Δ OBC) $\approx$ 84.3 hr.

Total delay = 38.96 + 84.3 = 123.26 hr.

(b) If ticket counter capacity is increased to 180/hr, the curve $O \to B \to C$ defining departure from counter will be changed to $O \to A \to C$, as the counters will be able to serve everything.

However, the delay overall will still be area (ΔOAC) = 123.26 hr.

So, there won't be any change.

(c) If we bump the security capacity to 70/hr or beyond (like 180/hr) as the question asks, the curve $O \to C$ defining departure from security changes to

$O \to B \to C$.

Hence the net delay becomes area (ΔOAB).

$\therefore$ The delay is reduced by the entire delay b/w counter & security

i.e. by 84.3 hr.

(d) ~~To avoid complications, we keep the ticket counter and security the same, so that we essentially see them as one single server~~

(d) Just by increasing the capacity of security, we can reduce delay by

$\frac{84.3}{123.26} \times 100 \approx 68.39\%$. To get better results like 70%, we will need to improve both counter and security together

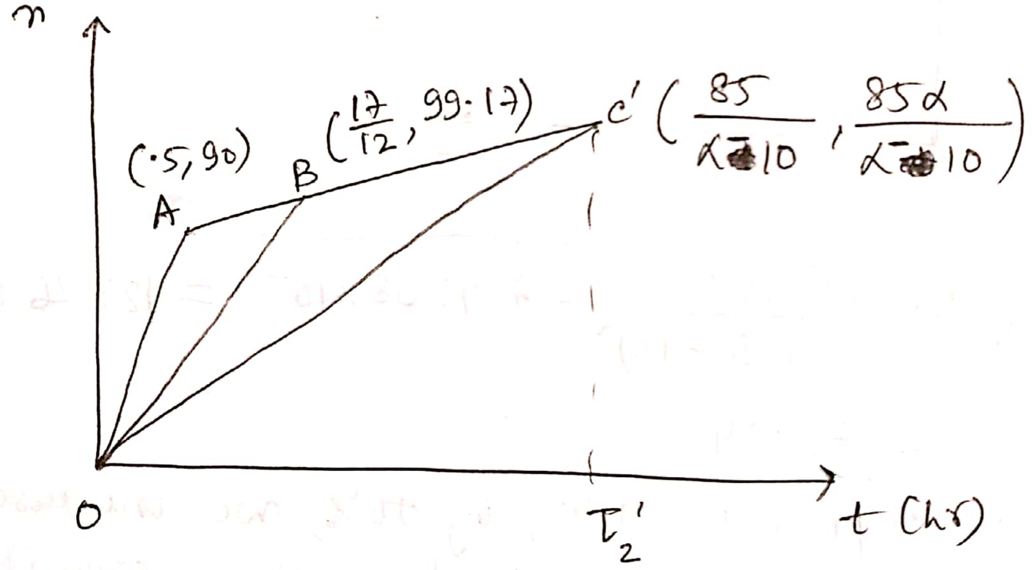
(c) The method of getting security capacity better is the cheapest method of bettering the system.

~~Increase~~

$$D_2'(t) = \begin{cases} \alpha t & t < T_2' \\ 85 + 10t & t \geq T_2' \end{cases}$$

$$T_2' = \text{solve}\left[\alpha t = 85 + 10t\right] = \frac{85}{\alpha - 10}$$

So, the net departure curve is given by c'



$$\text{area}\,(OAc') = 123.26 \times 0.5$$

$$\frac{1}{4}\sqrt{(180-\alpha)^2 \frac{85^2}{(\alpha-10)^2} - 7.4508\times10^{-9}} = 123.26 \times 0.5$$

$\alpha \simeq 53.59$ ~~pax/hr~~

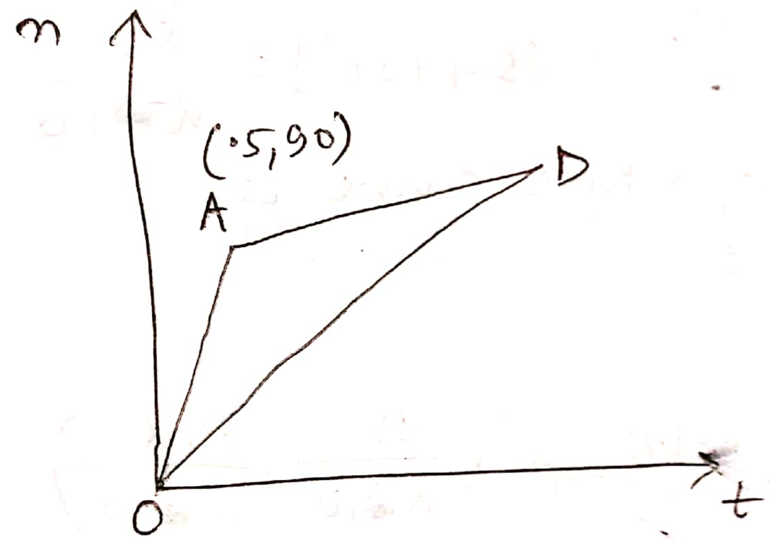
So, to reduce delay by 50%, we increase the capacity of security to 53.59 pax/hr without changes to ticket counter.

~~After~~

For 70% delay reduction,

Both ticket counter and security capacity should be increased to β.

$$D_1''(t) = D_2''(t) = \begin{cases} \beta t & t < T_3 \\ 85 + 10t & t \geqslant T_3 \end{cases}$$



$$\frac{1}{4}\sqrt{\frac{(180-\beta)^2 85^2}{(\beta-10)^2} - 7.4508\times10^{-9}} = 123.26 \times 0.3$$

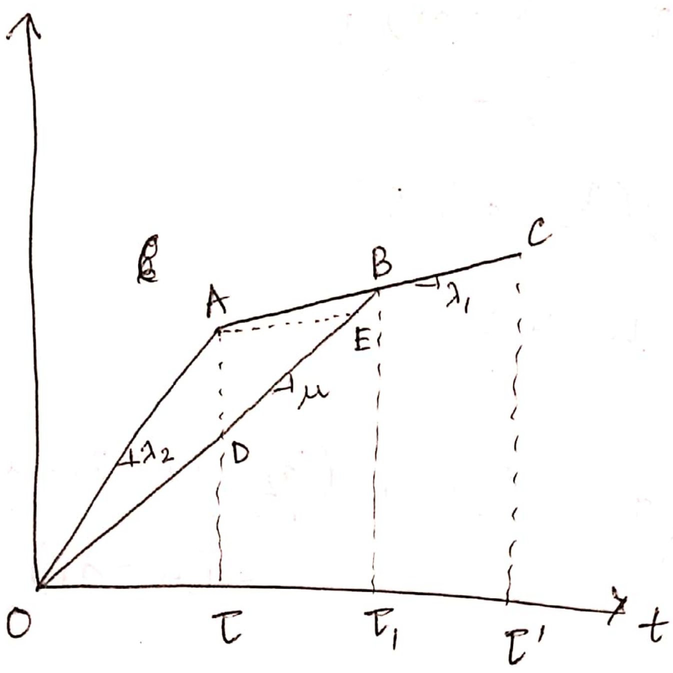
$\beta \approx 72.04.$

So, to reduce delay by 70%, we increase both ticket counter and security capacity to 72.04 pax/hr.

3.5

~~2.~~

(a)



line ABC equation:

$n = \lambda_2 \tau + \lambda_1 (t - \tau)$

$= (\lambda_2 - \lambda_1)\tau + \lambda_1 t$

$$A(t) = \begin{cases} \lambda_2 t & t < \tau \\ (\lambda_2 - \lambda_1)\tau + \lambda_1 t & t \geq \tau \end{cases}$$

$\tau_1 = \text{solve}\left((\lambda_2 - \lambda_1)\tau + \lambda_1 t = \mu t \right)$

$= \dfrac{\lambda_2 - \lambda_1}{\mu - \lambda_1} \tau$

~~Total~~ Maximum queue length $= AD$

$= (\lambda_2 - \mu)\tau$

Longest delay to a customer

$= AE$

$= \dfrac{\lambda_2 \tau}{\mu} - \tau = \tau \dfrac{(\lambda_2 - \mu)}{\mu}$

Duration of queue $= \tau_1 = \dfrac{\lambda_2 - \lambda_1}{\mu - \lambda_1} \tau$

Total delay $=$ area$(\Delta\, OAB) =$ area$(\Delta OAD) +$ area(ΔABD)

$= \frac{1}{2}\left[AD \times \tau + AD(\tau_1 - \tau) \right] = \dfrac{(\lambda_2 - \mu)\tau \tau_1}{2}$

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Total delay

$$= \frac{(\lambda_2-\mu)\tau}{2} \cdot \frac{(\lambda_2-\lambda_1)}{(\mu-\lambda_1)}\tau$$

$$= \frac{(\lambda_2-\mu)(\lambda_2-\lambda_1)\tau^2}{2(\mu-\lambda_1)}$$

(b) Total cost $(\bar{c}) = \beta\mu + \frac{(\lambda_2-\mu)(\lambda_2-\lambda_1)}{2(\mu-\lambda_1)}\tau^2\delta$

$$\frac{d\bar{c}}{d\mu} = 0 \Rightarrow \beta + \delta\tau^2\left[-\frac{(\lambda_2-\lambda_1)^2}{2(\mu-\lambda_1)^2}\right] = 0$$

$$\Rightarrow \frac{(\mu-\lambda_1)^2}{(\lambda_2-\lambda_1)^2} = \frac{\tau^2\delta}{2\beta}$$

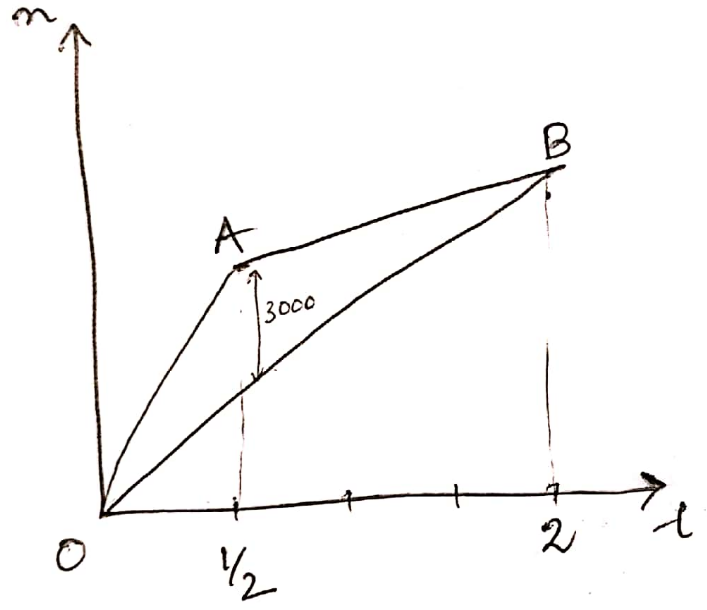
$$\Rightarrow \boxed{\mu = \lambda_1 + \frac{\tau(\lambda_2-\lambda_1)}{\sqrt{2\beta/\delta}}}$$

3.6.

(a)
$$A(t) = \begin{cases} 10000t & t < \tfrac{1}{2} \\ 2000t + 4000 & t \geq \tfrac{1}{2} \end{cases}$$

$$D(t) = \begin{cases} 4000t & t < \tau \\ 2000t + 4000 & t \geq \tau \end{cases}$$

$$\tau = \text{solve}\,(4000t = 2000t + 4000) = 2\ \text{hr.}$$



For User equilibrium, there won't be any vehicle taking the off-ramp.

So, the delay will be given by the area of $\Delta\,OAB$

$$= \frac{1}{2} \times 3000 \times 2 = 3000\ \text{hr}$$

3.6. (b) Assumption: Freeway speed = 80 Kph

Arterial speed = 30Kph.

(Plot attached)

So, taking each off-ramp causes extra time of $\frac{1\ \text{Km}}{30\ \text{Kph}} - \frac{1\ \text{Km}}{80\ \text{Kph}} = \frac{1}{48}\ \text{hr}$

(c) The attached plot shows overall reduction in delay significantly with SO.

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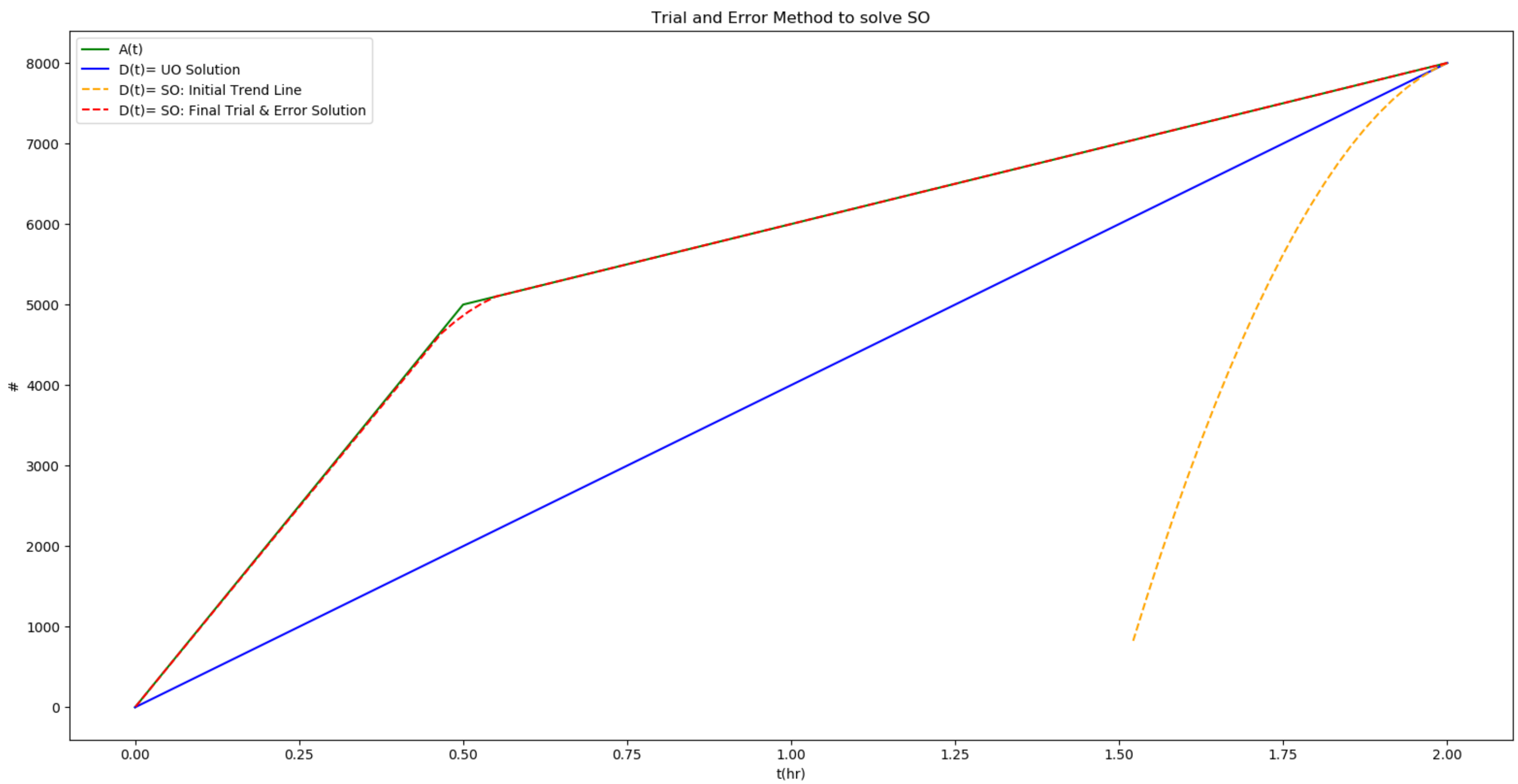


Trial and Error Method to solve SO
A(t)
D(t)= UO Solution
D(t)= SO: Initial Trend Line
D(t)= SO: Final Trial & Error Solution
#
t(hr)
0
1000
2000
3000
4000
5000
6000
7000
8000
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

3.11.

(a) For OR,

$$A_2(t) = (6500 \times 15\% / hr)\, t$$

$$D_2(t) = \begin{cases} (500 \times 50\%)/hr \; t & t < 20 \text{ mins} \\ \left(\frac{6500}{3} \times 50\%\right) \text{ slope} & 20 \text{ mins} < t < T_0 \\ (6500 \times 15\% / hr)\, t & t > T_0 \end{cases}$$

where T_0 is the time it takes to clear the OR Bottleneck.

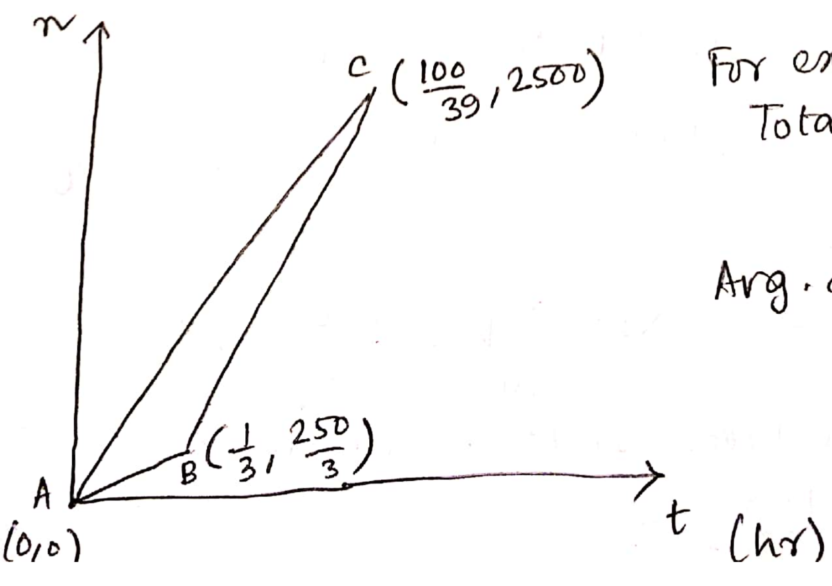
Queue formed in OR in the 20 mins

$$= (6500 \times 15\% - 500 \times 50\%) \times \frac{20}{60} = \frac{725}{3} = q_0 \text{ (say)}$$

Queue clearing rate

$$= \left(\frac{6500}{3} \times 50\% - 6500 \times .15\right)/hr = \frac{325}{3\,hr} = r \text{ (say)}$$

$$T_0 = 20 \text{ mins} + q_0/r = \frac{100}{39} hr$$



For exiting vehicles

Total delay = area (Δ ABC)

$$= \boxed{309.829 \text{ hr}}$$

$$\text{Avg. delay} = \frac{309.829}{2500 - 0} \times 60 \text{ mins}$$

$$= \boxed{7.44 \text{ mins}}$$

For Freeway, initially it stays unaffected as the 200 m of OR gets filled up first at timestamp 'τ' (say). Then one lane gets blocked reducing the capacity of the Freeway to 2/3rd. So, 2 lanes carry the load of non-existing vehicles till the time it takes for the off-ramp queue to be limited only to the 200m of OR, which is denoted by timestamp 'τ_1' (say). As the third lane gets cleared, all the queues and new non-exiting vehicles get cleared as the freeway is served at full capacity. The timestamp for that clearance is denoted by 'τ_2' (say).

$$A_1(t) = (6500 \times 85\% / hr) \times t$$

$$D_1(t) = \begin{cases} (6500 \times 85\% / hr)\, t & t < \tau \\ (6500 \times \frac{2}{3}) / hr \text{ slope} & \tau \le t \le \tau_1 \\ 6500/hr \text{ slope} & \tau_1 \le t < \tau_2 \\ (6500 \times 85\% / hr)\, t & t \geq \tau_2 \end{cases}$$

Assumption: A car and allowable space behind it a queue altogether be assumed to 6m (say). So, the OR holds $\frac{200}{6} = \frac{100}{3}$ vehicles before affecting freeway lane at $t = \tau$

$$(6500 \times 15\% - 500 \times 50\%) / hr \;\tau = \frac{100}{3} \Rightarrow \tau = 165.52\,s$$

Queue formed in Freeway lane due to OR

$$Q_1 = Q_0 - \frac{100}{3} = \frac{625}{3}$$

Time stamp at ~~for~~ which that gets cleared

$$T_1 = 20 \text{ mins} + \frac{Q_1}{r} = \frac{88}{39} \text{ hr.}$$

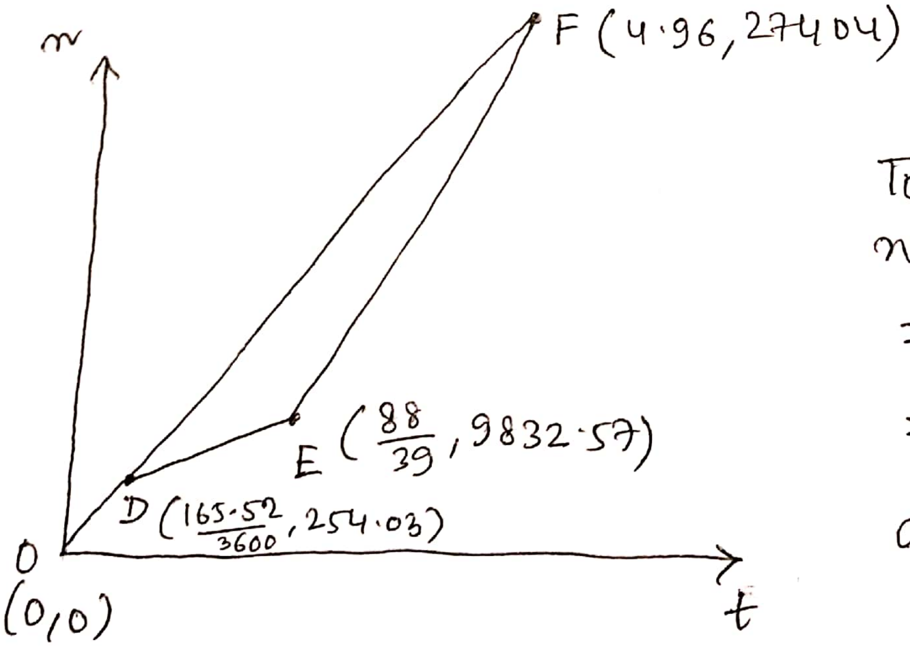
Total queue formed on the other 2-lanes from $t = T$ to $t = T_1$ is given by

$$Q_2 = (T_1 - T) \times \left[\frac{6500 \times 85\% - 6500 \times \frac{2}{3}}{hr}\right] \approx 2634.1$$

Rate of queue clearance at the freeway after $t = T_1$ is given by

$$r_2 = [6500 - 6500 \times 85\%]/hr$$

$$\therefore T_2 = T_1 + \frac{Q_2}{r_2} \approx 4.96 \text{ hr.}$$



Total delay for non-exiting vehicles

$= \text{area } (\Delta DEF)$

$= \boxed{6472.02 \text{ hr}}$

avg. delay

$$= \frac{6472.02 \text{ hr}}{27404 - 254.03}$$

$$\approx \boxed{14.3 \text{ mins}}$$