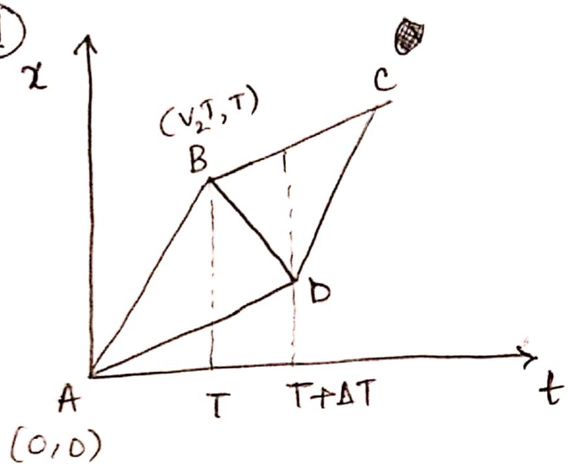
1.1.

(1)




Equating displacements to D through AD and $A \to B \to D$.

$$V_0 (T+\Delta T) = V_2 T - V_1 \Delta T$$

$$\Delta T = \frac{V_2 - V_0}{V_0 + V_1} T$$

For parallelogram ABCD, AD = BC

So, $BC = AD = V_0 (T + \Delta T)$

Traversing to C through $A \to B \to C$,

AB @ $V_2$ for time T + BC @ $V_0$ for time $(T+\Delta T)$

$$\therefore \text{average speed} = \bar{V} = \frac{V_2 T + V_0 (T + \Delta T)}{2T + \Delta T}$$

$$= \frac{V_2 + V_0 \left(1 + \frac{V_2 - V_0}{V_0 + V_1}\right) \cancel{T}}{\cancel{T} \left(2 + \frac{V_2 - V_0}{V_0 + V_1}\right)}$$

$$= \frac{2 V_2 V_0 + V_1 V_2 + V_0 V_1}{V_0 + 2V_1 + V_2}$$ [independent of T]

(2) $$\bar{V} = \frac{2.6.3 + 9.6 + 3.9}{3 + 2.9 + 6} = \frac{117}{27} = \frac{13}{3} \text{ mph} = 4.33 \text{ mph}$$

(3) With more switching, more U-turn penalty is caused. In general otherwise, the average speed will not depend on the value of T. So, the strategy should be to increase T to reduce the U-turn penalty.

1.4. $a = 1 - v - x/4$

$\ddot{x} = 1 - \dot{x} - x/4$

$4\ddot{x} + 4\dot{x} + (x-4) = 0$

$x - 4 = y$ (say)

$\Rightarrow 4\ddot{y} + 4\dot{y} + y = 0$

$y = e^{rt}$ (say)

$e^{rt}(4r^2 + 4r + 1) = 0$ [2nd order homogeneous diff. eqn.]

$\Rightarrow r = -1/2, -1/2$ (repeated roots)

$\therefore y = (c_1 t + c_2) e^{-t/2}$ (general form)

$\Rightarrow x = 4 + (c_1 t + c_2) e^{-t/2}$ (substituting $x$)

$x(0) = 0 \Rightarrow c_2 = -4$

$x(t) = 4 + (c_1 t - 4) e^{-t/2}$

$v(t) = (c_1) e^{-t/2} + (c_1 t - 4)(-1/2) e^{-t/2}$

$v(0) = 0 \Rightarrow c_1 + 2 = 0 \Rightarrow c_1 = -2$

$\therefore x(t) = (-2t - 4) e^{-t/2} + 4 = 4 - 2e^{-t/2}(t+2)$

$\therefore v(t) = \dot{x} = -2e^{-t/2} - 2(t+2)(-1/2)e^{-t/2} = t e^{-t/2}$

$\therefore a(t) = \dot{v} = e^{-t/2} + t(-\frac{1}{2})e^{-t/2} = \frac{1}{2} e^{-t/2}(2-t)$

Scanned with CamScanner

## 1.7. Eco-Driving

For the assignment, I used Carbon Monoxide (CO) as the parameter to decide on the best strategy. The python script to estimate the emission is attached.

As a continuation of example 1.2.6 of the textbook, we plot Emission vs Distance for different scenarios of β as shown in figure below.

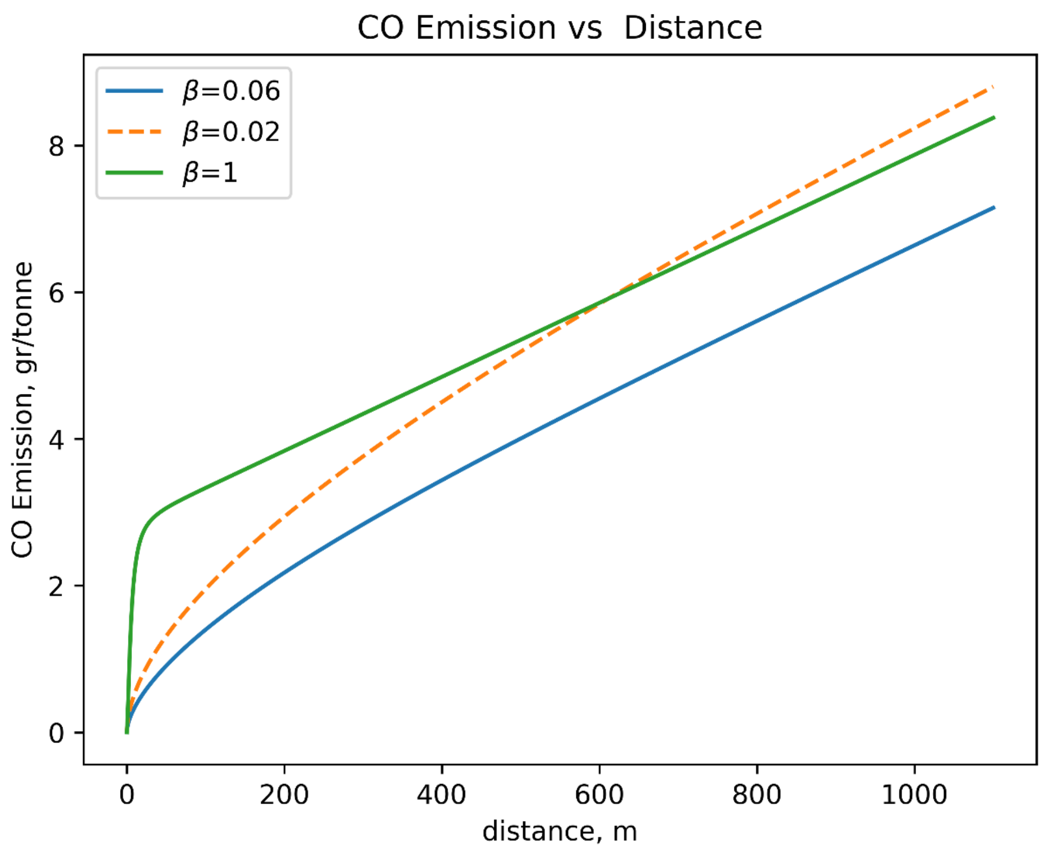
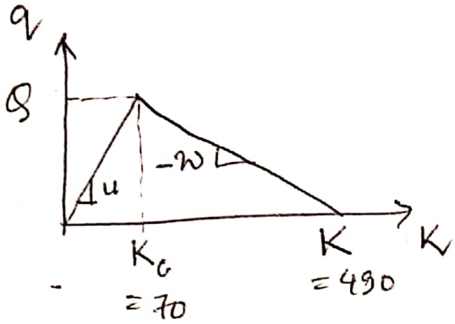



**Figure 1**: CO-Emission vs Distance based on approximate MOVES formula

Based on this, being an average driver (i.e. having β=0.06) is consistently the best strategy for minimize CO emissions. Going by the results for the others, within the limit of about 650m, being timid (β=0.02) is the second best strategy. However, beyond that, being an aggressive driver at β=1 is the second best strategy.

1.13.
(a)



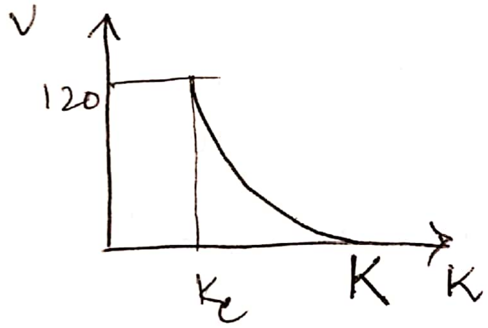

$$w = \frac{u \times K_c}{K - K_c} = 20 \text{ Km}$$

$$Q = uK_c = 8400 \text{ veh/hr}$$

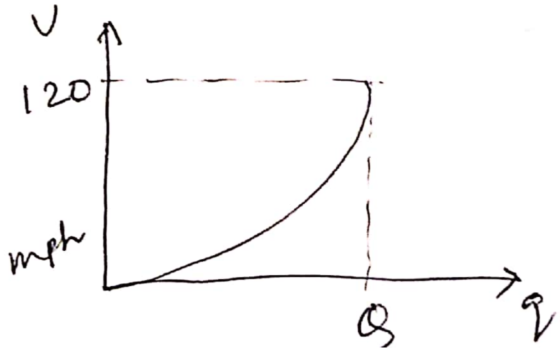
$$q = \begin{cases} 120K & K \le K_c \\ Q - 20(K - K_c) & K > K_c \end{cases}$$

$$= \begin{cases} 120K & K \le 70 \\ 9800 - 20K & K > 70 \end{cases}$$

$$v = q/K = \begin{cases} 120 & K \le 70 \\ \frac{9800}{K} - 20 & K > 70 \end{cases}$$



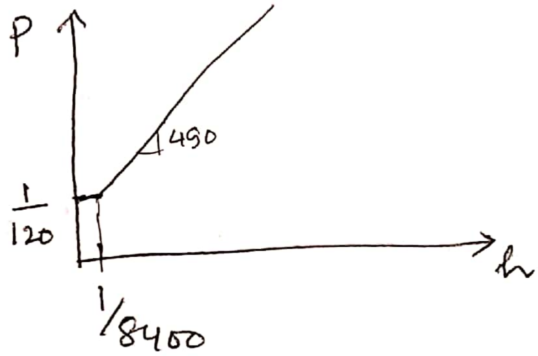

$$v = \frac{9800}{q/v} - 20 \Rightarrow v = \frac{20q}{9800 - q}$$



\# $h = \frac{1}{q}$ ; $p = 1/v$

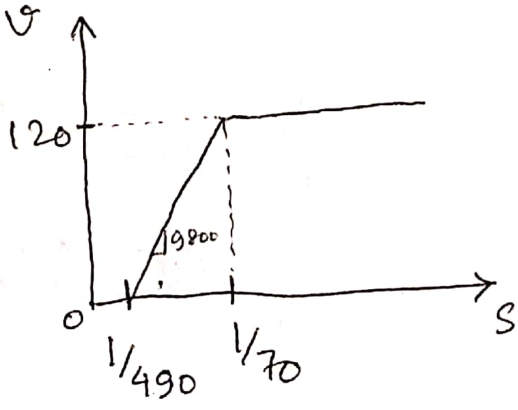
$$1/p = \frac{20/h}{9800 - 1/h} = \frac{20}{9800h - 1}$$

$$p = \begin{cases} 490h - 0.05 & h > \frac{1}{8400} \\ \frac{1}{120} & \text{otherwise} \end{cases}$$




\# $s = 1/k$

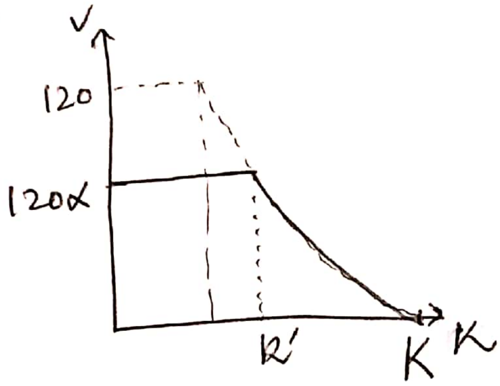
$$v = \begin{cases} 120 & s \geq \frac{1}{70} \\ 9800s - 20 & s < \frac{1}{70} \end{cases}$$



1.13.

(b) Speed limit $= \alpha u = 120\alpha$

# V vs K



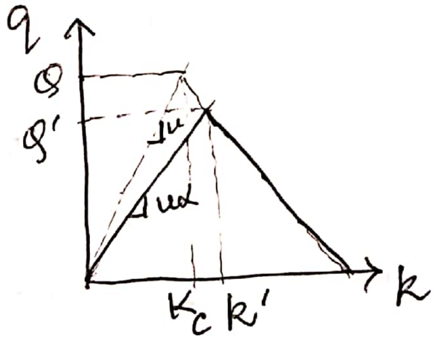

$$\frac{9800}{k'} - 20 = 120\alpha$$

$$k' = \frac{9800}{120\alpha + 20}$$

$$V = \begin{cases} 120\alpha & k \leq \dfrac{9800}{120\alpha+20} \\ \dfrac{9800}{K} - 20 & k > \dfrac{9800}{120\alpha+20} \end{cases}$$

# q vs K

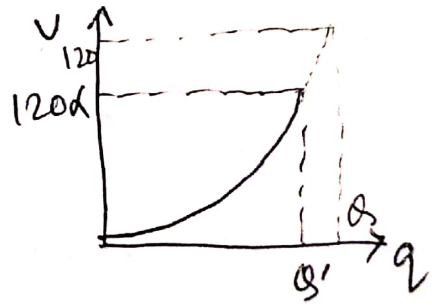
$$q = VK = \begin{cases} 120\alpha K & k \leq \dfrac{9800}{120\alpha+20} \\ 9800 - 20K & k > \dfrac{9800}{120\alpha+20} \end{cases}$$




$$Q' = \frac{9800 \times 120\alpha}{120\alpha + 20}$$

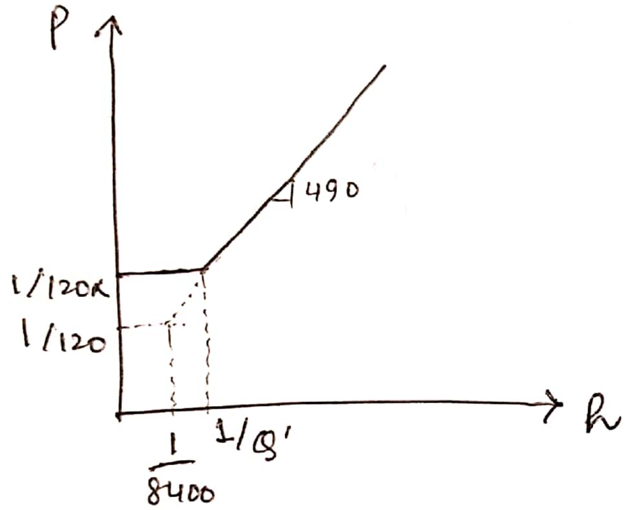
$$= \frac{58800\alpha}{6\alpha + 1}$$

# $v = \dfrac{20q}{9800-q}$ ; $q < Q'$

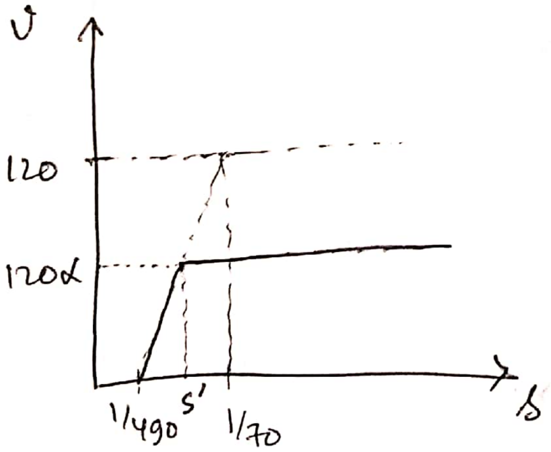


# p vs h

$$p = \begin{cases} 490h - 0.05 & h > \frac{1}{Q'} \\ \frac{1}{120\alpha} & \text{otherwise} \end{cases}$$




# v vs s




$$\frac{120\alpha}{\left(s' - \frac{1}{490}\right)} = 9800$$

$$\Rightarrow s' = \frac{3\alpha}{245} + \frac{1}{490} = \frac{6\alpha+1}{490}$$

$$V = \begin{cases} 120\alpha & s \geq s' \\ 9800s - 20 & s < s' \end{cases}$$

1.13.(c) $v = \frac{20q}{9800-q} \Rightarrow q = \frac{9800v}{v+20}$

Based on a sample of 10 million data points

mean = 7297 veh/hr, sd = 371 veh/hr.