4.1.

(a) $K_0(x) = K_c(1+x)$

$u = 100$ Km/hr

$w = 20$ Km/hr

$K_c = 25$ veh/hr

$Q_{max} = 2500$ veh/Km

$K_0(x_B') = K_c$

$\Rightarrow K_c(1+x_B') = K_c$

$x_B' = 0$

$x_U = x - 100t$

$x_D = x + 20t$

$$G(x) = \int_x^5 K_0(x)\,dx = 25\left[-\left(x+\frac{x^2}{2}\right) + 5 + \frac{5^2}{2}\right]$$

$$= 437.5 - 25x - 12.5x^2$$

$$f(x_U) = G(x_U) = -125000t^2 + 2500tx + 2500t - 12.5x^2 - 25x + 437.5$$

$$f(x_D) = G(x_D) + k(x_D - x)$$

$$= -5000t^2 - 500tx + 2500t - 12.5x^2 - 25x + 437.5$$

$$f(x_B') = G(x_B') + (t-0)Q - (x-0)K_c$$

$$= 2500t - 25x + 437.5$$ X not used

However $\frac{dK_0}{dx} = K_c > 0$.

So, $x_B' = 0$ is not an option.

$$\boxed{N(t,x) = \min\{f(x_u), f(x_D)\}}$$

eqn of shock trajectory:

$f(x_U) = f(x_D)$

$\Rightarrow -125000t^2 + 2500tx = -5000t^2 - 500tx$

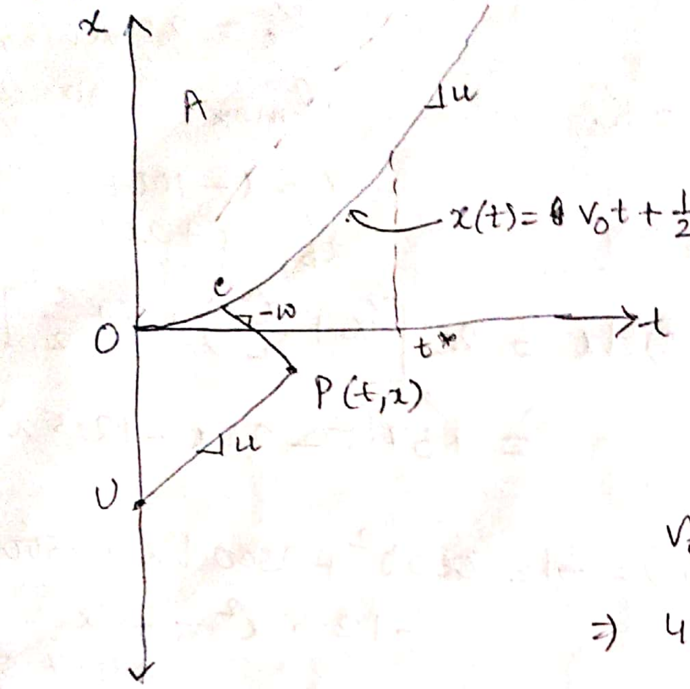
$\Rightarrow 3000tx = 120000t^2$

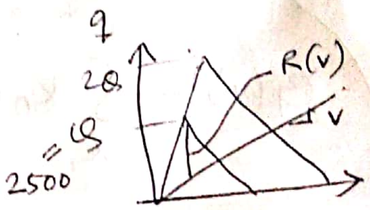
$$\boxed{x = 40t}$$

Scanned with CamScanner

$V_0 = 40$ Km/hr
$a = 2 m/s^2$
$= 25920$ Km/hr$^2$

4.1.
(b)




$$V_0 + a t^* = u$$

$$\Rightarrow 40 + 25920 t^* = 100$$

$$t^* = \frac{1}{432} \text{ hr.}$$

$c(t_c, x_c)$

$$t_c = \frac{\sqrt{5}\sqrt{1440t + 72x + 5} - 5}{2160}$$

$$= \frac{\sqrt{1 + 14.4x + 288t} - 1}{432}$$

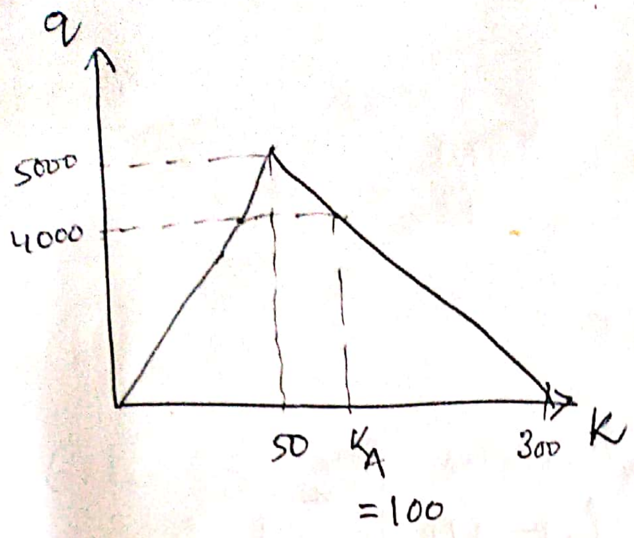
$$x_c = \frac{5}{108}\left(-\sqrt{1 + 14.4x + 288t} + 432t + 21.6x + 1\right)$$

cost P–C–O
= cost P–C + cost C–O

$$= K(x_c - x_p) + \int_0^{t_c} R(v(t))\,dt$$

$$= 300\left(\frac{5}{108}\right)\left(-\sqrt{1 + 14.4x + 288t} + 432t + 1\right) + \int_0^{t_c} Q\left(1 - \frac{V_0 + at}{u}\right)dt$$

$$= \frac{-25}{18}\left(\sqrt{5}\sqrt{1440t + 72x + 5} - 3960t + 18x - 5\right)$$




$$f(x_u) = K_A |x_u|$$
$$= 100\,(ut - x) = 100\,(100t - x)$$

$$N(t,x) = \min\{\text{cost } P \to C \to 0,\ f(x_u)\}$$

$$N(t,x) = \min\left\{ -\frac{25}{18}\left(\sqrt{5}\sqrt{1440t + 72x + 5} - 3960t + 18x - 5\right),\ 100\,(100t - x)\right\}$$

Trajectory of shock is given by:

$$\text{cost } P \to C \to 0 = f_u$$

$$-\frac{25}{18}\left(\sqrt{5}\sqrt{1440t + 72x + 5} - 3960t + 18x - 5\right) = 100\,(100t - x)$$

$$\cancel{2916x^2 - 349920xy + 180x}$$

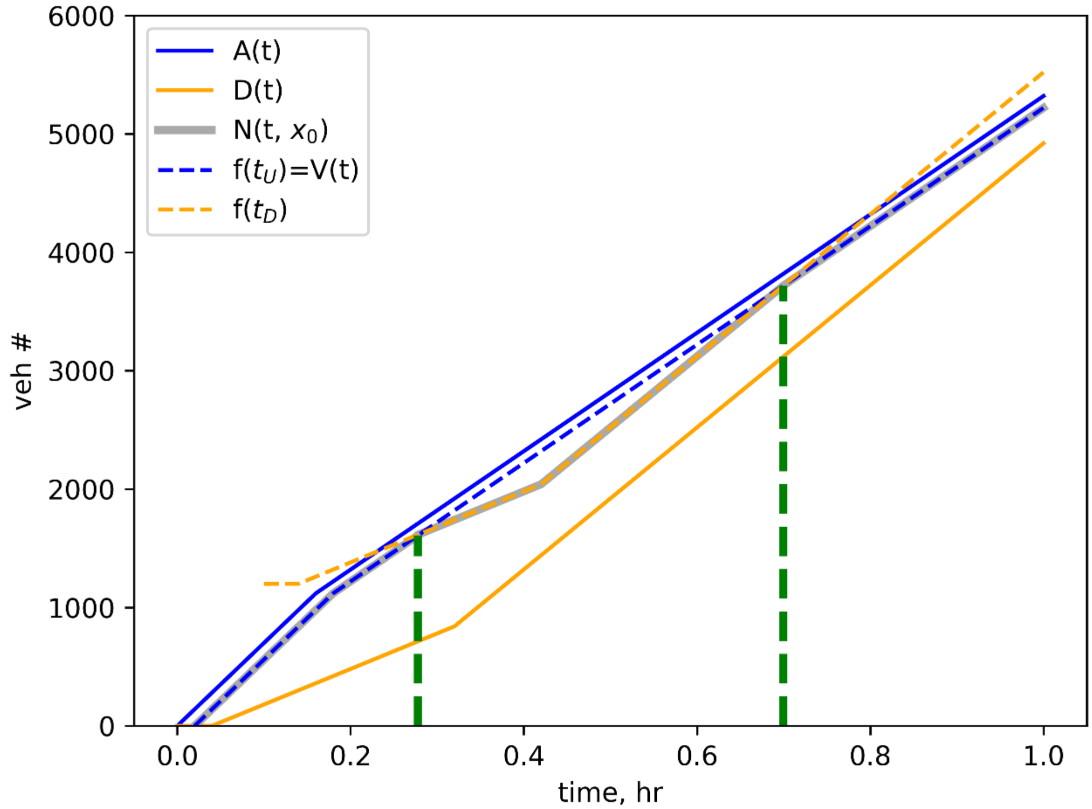
$$\boxed{2916x^2 - 349920\,xt + 180x + 10497600t^2 - 39600t = 0}$$

parabola trajectory for shock

**4.2.** The N-Curve plots are made using python with the script ***Plot_4.2.py*** attached to the submission.

For 4-lanes, the jam density K=150*4=600 veh/km. The "virtual arrival" curve is made by shifting A(t) by $(x_o-x_A)/u=(2-0)/100=0.02$ in time-axis.

The "virtual departure" is created by shifting D(t) by $(x_D-x_0)/\omega=(4-2)/20=0.1$ along time axis and by $K(x_D-x_0)=600(4-2)=1200$ along the N-axis.




(a) The shaded area represents N-curve at x=2 km.

(b) The green lines denote two intersections at t=0.2776 hr and t=0.6995 hr. in the first instance the queue starts growing and then in the second instance, the queue starts receding.

6.1. $\lambda = 500$

$u = 40\ Km/hr = \frac{2}{3}\ Km/min$

$\tau^* = \frac{l}{u} = 20\ mins \Rightarrow l = \frac{40}{3}\ Km$

$L \times (\text{Jam density}) = n_{jam}$

$\Rightarrow L = \frac{10000}{150}\ Km$

For (a) and (b),

$$\mu = \frac{100 \times L}{l} = \frac{\overset{5}{\cancel{100}} \times 1000\cancel{0} \times \cancel{3}}{\underset{5}{\cancel{150}} \times 4\cancel{0}} = 500$$

$$\rho = \frac{\lambda}{\mu} = 1$$

So, we use the graphs for $\rho = 0.99$.

(a) Based on graph, the system does not reach equilibrium by $t = 8\tau^*$ based on the extent of the plot.
It reaches equilibrium at $n = 0.45 n_{jam}$
$= 4500$.

(b) Travel time at $t = 80\ mins = 4\tau^*$
$= 1.5\tau^* = 30\ mins.$

Scanned with CamScanner

(c) After incident removal,

$$\mu = \frac{200L}{l} = 1000$$

$$\rho = \frac{500}{1000} = 0.5$$

We do not have a curve for $n(0) = \cdot 45\, n_{jam}$, but it showed closely follow $n(0) = 0.5\, n_{jam}$

It should reach equilibrium around $t = 6\tau^{*} = 120$ mins

and around $K_1^{*} = 0.15\, n_{jam}$

$= 1500$

(d) At $t = 40$ mins $= 2\tau^{*}$,

we have travel time approximately

$1.2\tau^{*} = 24$ mins.

Scanned with CamScanner