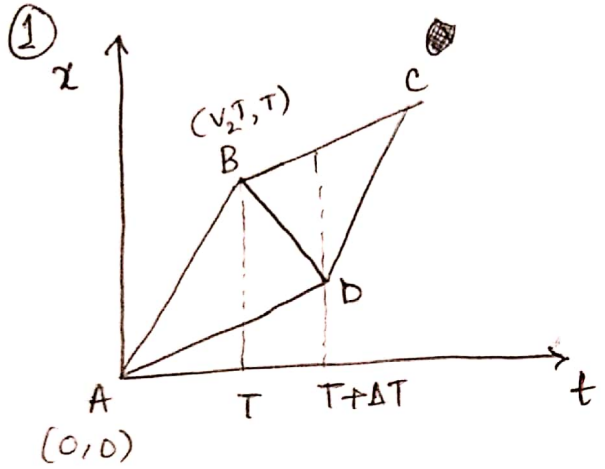
1.1.

① 

Equating displacements to D through AD and A→B→D.

$$V_0(T+\Delta T) = V_2 T - V_1 \Delta T$$

$$\Delta T = \frac{V_2 - V_0}{V_0 + V_1} T$$

For parallelogram ABCD, AD = BC

So, $BC = AD = V_0(T+\Delta T)$

Traversing to C through A→B→C,

AB @ V_2 for time T + BC @ V_0 for time $(T+\Delta T)$

$$\therefore \text{average speed} = \bar{V} = \frac{V_2 T + V_0(T+\Delta T)}{2T+\Delta T}$$

$$= \frac{V_2 + V_0\left(1 + \frac{V_2 - V_0}{V_0+V_1}\right)}{\left(2 + \frac{V_2-V_0}{V_0+V_1}\right)}$$

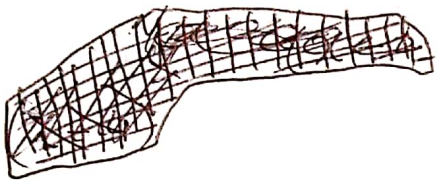
$$= \frac{2V_2V_0 + V_1V_2 + V_0V_1}{V_0 + 2V_1 + V_2}$$ [independent of T]

② $$\bar{V} = \frac{2.6.3 + 9.6 + 3.9}{3 + 2.9 + 6} = \frac{117}{27} = \frac{13}{3} \text{ mph} = 4.33 \text{ mph}$$

③ With more switching, more U-turn penalty is caused. In general otherwise, the average speed will not depend on the value of T. So, the strategy should be to increase T to reduce the U-turn penalty.

1.4. $a = 1 - v - x/4$

$\ddot{x} = 1 - \dot{x} - x/4$



$4\ddot{x} + 4\dot{x} + (x-4) = 0$

$x - 4 = y$ (say)

$\Rightarrow 4\ddot{y} + 4\dot{y} + y = 0$

$y = e^{rt}$ (say)

$e^{rt}(4r^2 + 4r + 1) = 0$ [2nd order homogeneous diff. eqn.]

$\Rightarrow r = -1/2, -1/2$ (repeated roots)

$\therefore y = (c_1 t + c_2) e^{-t/2}$ (general form)

$\Rightarrow x = 4 + (c_1 t + c_2) e^{-t/2}$ (substituting x)

$x(0) = 0 \Rightarrow c_2 = -4$

$x(t) = 4 + (c_1 t - 4) e^{-t/2}$

$v(t) = (c_1) e^{-t/2} + (c_1 t - 4)(-1/2) e^{-t/2}$

$v(0) = 0 \Rightarrow c_1 + 2 = 0 \Rightarrow c_1 = -2$

$\therefore x(t) = (-2t - 4) e^{-t/2} + 4 = 4 - 2e^{-t/2}(t+2)$

$\therefore v(t) = \dot{x} = -2e^{-t/2} - 2(t+2)(-1/2) e^{-t/2} = te^{-t/2}$

$\therefore a(t) = \dot{v} = e^{-t/2} + t(\tfrac{1}{2}) e^{-t/2} = \frac{1}{2} e^{-t/2}(2 - t)$