

4.1.
(a) $K_0(x) = K_c(1+x)$

$$K_0(x'_B) = K_c$$

$$\Rightarrow K_c(1+x'_B) = K_c$$

$$x'_B = 0$$

$$u = 100 \text{ km/hr}$$

$$w = 20 \text{ km/hr}$$

$$K_c = 25 \text{ veh/hr}$$

$$Q_{\max} = 2500 \text{ veh/km}$$

$$x = x - 100t$$

$$x_D = x + 20t$$

$$G(x) = \int_x^5 K_0(x) dx = 25 \left[\left(x + \frac{x^2}{2} \right) + 5 + \frac{5^2}{2} \right]$$

$$= 437.5 - 25x - 12.5x^2$$

$$f(x_u) = G(x_u) = -12500t^2 + 2500tx + 2500t$$

$$-12.5x^2 - 25x + 437.5$$

$$f(x_D) = G(x_D) + K(x_D - x)$$

$$= -5000t^2 - 500tx + 2500t - 12.5x^2 - 25x + 437.5$$

$$f(x'_B) = G(x'_B) + (t-0)Q - (x-0)K_c$$

$$= 2500t - 25x + 437.5 \quad (\text{X}) \text{ not used}$$

However $\frac{dK_0}{dx} = K_c > 0$.

So, $x'_B = 0$ is not an option.

$$N(t, x) = \min \{ f(x_u), f(x_D) \}$$

eqⁿ of shock trajectory:

$$f(x_u) = f(x_D)$$

$$\Rightarrow -12500t^2 + 2500tx = -5000t^2 - 500tx$$

$$\Rightarrow 3000tx = 12000t^2$$

$$\boxed{x = 40t}$$