Chap 1 Combination Analysis
"counting"

Ex 

$0 \le m \le n$

* The system fails when there are 2 consecutive antennas fail.

Q. For given n and m (m: # defectives), what is the probability (total) that the system fails?

Ex $n = 4$, $m = 2$

Configurations

Working: $0 = 0$

Defective: $\bullet = 1$

All configurations ($n=4, m=2$)

Fail	0 0 1 1	$1\times2+1\times1=3$
Work	0 1 0 1	$1\times2^2+1=5$
Fail	0 1 1 0	$1\times2^2+1\times2=6$
work	1 0 0 1	$1\times2^3+1\times2^0=9$
work	1 0 1 0	$1\times2^3+1\times2=10$
Fail	1 1 0 0	$1\times2^3+1\times2^2=12$

Configurations = 6

Failed configurations = 3

$P(\text{Failed system}) = \frac{\#\text{Failed confi.}}{\#\text{Total confi.}} = \frac{3}{6} = \frac{1}{2}$

Q What about the general m and n

$n = 7$, $m = 3$

Work 

Fail 

Basic principle of counting

Thm 1 Suppose 2 experiments to be performed and

① For experiment 1, we have m possible outcomes.

② For each outcome of experiment 1, we have n outcomes for experiment 2.

Then # possible outcomes is $m \times n$.

① ② 1st Ex

A B C D E

[m choices] × [n choices]

out comes $= mn$

Proof Define without loss of generality (WLOG),

A = set of all possible outcomes of the 1st experiment.

$= \{1, 2, 3, \dots, m\}$

And

B = set of all possible outcomes of the 2nd exp.

Thus, the set of all possible outcomes is

$$A\times B = \begin{Bmatrix} (1,1),(1,2),\dots,(1,n) \\ (2,1),(2,2),\dots,(2,n) \\ \vdots \\ (m,1),(m,2),\dots,(m,n) \end{Bmatrix}.$$

We have the number of outcomes is $|A\times B| = |A|\times|B| = m\times n$

Thm 2 Suppose r experiments to be perfored and

① For experiment i, we have n_i possible outcomes for $1 \le i \le r$

② For each outcome of experiment i, we have n_{i+1} outcomes for experiment $i+1$ for $1 \le i \le r-1$

Then total numbers of possible outcomes is $\prod_{i=1}^{r} n_i = n_1 \times n_2 \times \dots \times n_r$.

Ex Find # possible 7 place license plates if 26 * First 3 places are letters
10 * Final 4 places are numbers

Solution

For each place ("experiment") of the first 3 places, we have 26 outcomes.

By the 2nd basic principle of counting the total number of outcomes is $26\times26\times26\times10\times10\times10\times10 = 175{,}760{,}000$