ICMA 350 Probability Quiz I (30 points=10%)

Name.. Student I.D. ....

1. In how many ways can 10 people be seated in a row if **(2 points each)**

   (a) there are no restrictions on the seating arrangement?

   Answer________________________

   (b) persons A and B must NOT sit next to each other?

   Answer________________________

   (c) there are 5 men and 5 women and no 2 men or 2 women can sit next to each other?

   Answer________________________

   (d) there are 5 men and they must sit next to each other?

   Answer________________________

   (e) there are 5 married couples and each couple must sit together?

   Answer________________________

2. If $P(E) = .9$ and $P(E) = .8$, show that $P(EF) \geq .7$. In general, prove Bonferroni's inequality, namely,

$$P(EF) \geq P(E) + P(F) - 1.$$

(10 points)

3. Suppose that 6 percent of men and .36 percent of women are color blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. What if the population consisted of twice as many males as females? **(10 points)**

4. **BONUS!** A property insurer with a large number of policies identifies the level of damage when a claim occurs as low, medium or high. 30% of claims received are low damage claims, 60% are medium and 10% are high. Out of three independently occurring claims on the insurer, what is the probability at least one is a high damage claim? **(5 points)**

Solutions to ICMA 350 Probability Quiz I

1. **Seating Arrangements:**

   (a) **No restrictions:**

   - Explanation: Any of the 10 people can sit in the first seat, any of the remaining 9 in the second, and so on.
   - Solution: $10! = 10 \times 9 \times 8 \times \cdots \times 1$.

   (b) **A and B not together:**

   - Explanation: Calculate total arrangements (10!) and subtract arrangements where A and B are together. Treat A and B as a single unit, leading to 9! arrangements for the units, and 2 arrangements within the AB pair.
   - Solution: $10! - 9! \times 2$.

   (c) **Alternating men and women:**

   - Explanation: Arrange either all men or all women first (5! ways), then arrange the other group in alternate seats (5! ways). Double the total for either group starting.
   - Solution: $2 \times 5! \times 5!$.

   (d) **5 men together:**

   - Explanation: Treat the 5 men as a single unit, leading to 6! arrangements for the units, and 5! arrangements within the men group.
   - Solution: $6! \times 5!$.

   (e) **5 married couples together:**

   - Explanation: Treat each couple as a single unit (5! arrangements), with 2 ways to arrange individuals within each couple.
   - Solution: $5! \times 2^5$.

2. Bonferroni's Inequality:

   - Given $P(E) = 0.9$ and $P(F) = 0.8$, we have $P(EF) \geq P(E) + P(F) - 1 = 0.9 + 0.8 - 1 = 0.7$.
   - Proof of Bonferroni's inequality: $P(EF) = P(E) + P(F) - P(E \cup F) \geq P(E) + P(F) - 1$.

3. Probability of Color Blindness: Given:

    - Probability of a man being color-blind: $P(\text{Color Blind}|\text{Male}) = 0.06$.
    - Probability of a woman being color-blind: $P(\text{Color Blind}|\text{Female}) = 0.0036$.

    Part 1: Equal Number of Males and Females

    - Assume $P(\text{Male}) = P(\text{Female}) = 0.5$.
    - Find $P(\text{Male}|\text{Color Blind})$ using Bayes' Theorem:

    $$P(\text{Male}|\text{Color Blind}) = \frac{P(\text{Color Blind}|\text{Male}) \times P(\text{Male})}{P(\text{Color Blind})}$$

    - Calculate $P(\text{Color Blind})$ using the Law of Total Probability:

      $P(\text{Color Blind}) = P(\text{Color Blind}|\text{Male}) \times P(\text{Male}) + P(\text{Color Blind}|\text{Female}) \times P(\text{Female})$

    - Substituting values:

    $$P(\text{Color Blind}) = 0.06 \times 0.5 + 0.0036 \times 0.5 = 0.0318$$

    - Therefore:

    $$P(\text{Male}|\text{Color Blind}) = \frac{0.06 \times 0.5}{0.0318} \approx 0.9434$$

    Part 2: Twice as Many Males as Females

    - Assume $P(\text{Male}) = \frac{2}{3}$ and $P(\text{Female}) = \frac{1}{3}$.
    - Repeat the Bayes' Theorem calculation with the new probabilities:

    $$P(\text{Color Blind}) = 0.06 \times \frac{2}{3} + 0.0036 \times \frac{1}{3} \approx 0.0412$$

    - Therefore:

    $$P(\text{Male}|\text{Color Blind}) = \frac{0.06 \times \frac{2}{3}}{0.042} \approx 0.9709$$

4. Bonus - High Damage Claim: Given probabilities:

    - Low damage claim: $P(\text{Low}) = 0.30$.
    - Medium damage claim: $P(\text{Medium}) = 0.60$.
    - High damage claim: $P(\text{High}) = 0.10$.

    Objective:

    - Find the probability that out of three independently occurring claims, at least one is a high damage claim.

    Solution:

    - Calculate the probability that none of the claims are high damage: $(1 - P(\text{High}))^3$.
    - Therefore, the probability of at least one high damage claim is: $1 - (1 - P(\text{High}))^3$.
    - Substituting $P(\text{High}) = 0.10$, we get: $1 - (1 - 0.10)^3 = 0.271$.