

1.

For which right sides (find a condition on b_1, b_2, b_3) are these systems solvable?

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

2.

Find the reduced R for each of these (block) matrices:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 2 & 4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} A & A \end{bmatrix} \quad C = \begin{bmatrix} A & A \\ A & 0 \end{bmatrix}$$

3.

Find the complete solution (also called the *general solution*) to

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

4.

Find a basis for each of the four subspaces associated with A :

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

5.

Project \mathbf{b} onto the column space of A by solving $A^T A \hat{\mathbf{x}} = A^T \mathbf{b}$ and $\mathbf{p} = A \hat{\mathbf{x}}$:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}.$$

Find $\mathbf{e} = \mathbf{b} - \mathbf{p}$. It should be perpendicular to the columns of A .

6.

The average of the four times is $\hat{t} = \frac{1}{4}(0 + 1 + 3 + 4) = 2$. The average of the four b 's is $\hat{b} = \frac{1}{4}(0 + 8 + 8 + 20) = 9$.

Verify that the best line goes through the center point $(\hat{t}, \hat{b}) = (2, 9)$.

7.

Find q_1, q_2, q_3 (orthonormal) as combinations of a, b, c (independent columns). Then write A as QR :

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{bmatrix}.$$