Chanasorn Howattanakulphong 65011277

Central Limit Theorem Proof

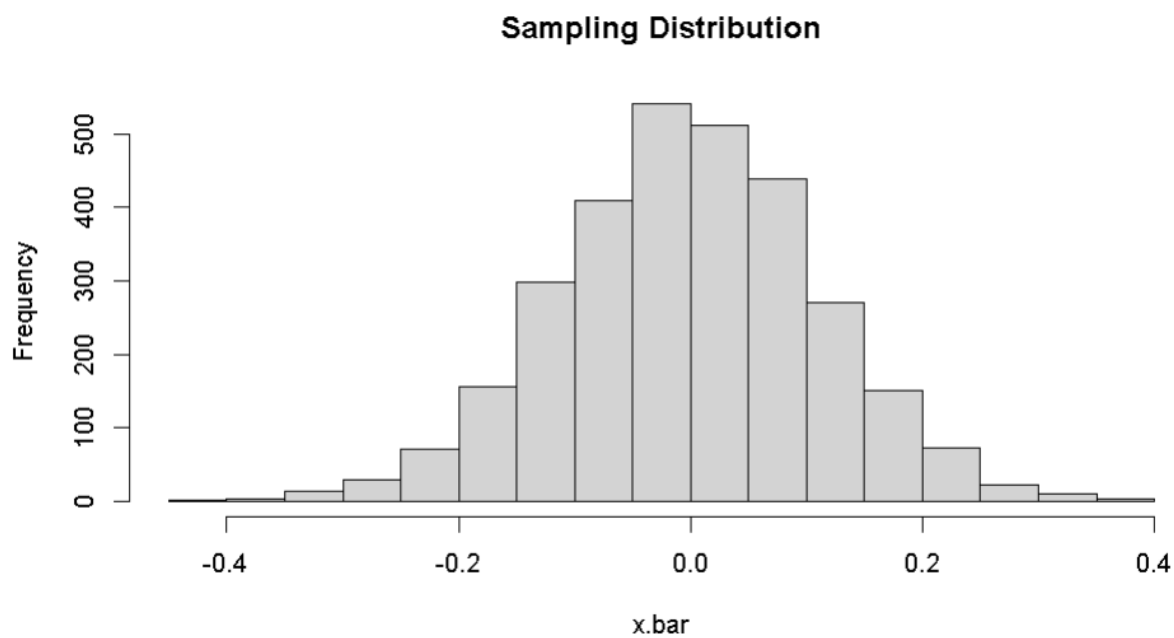
Code:

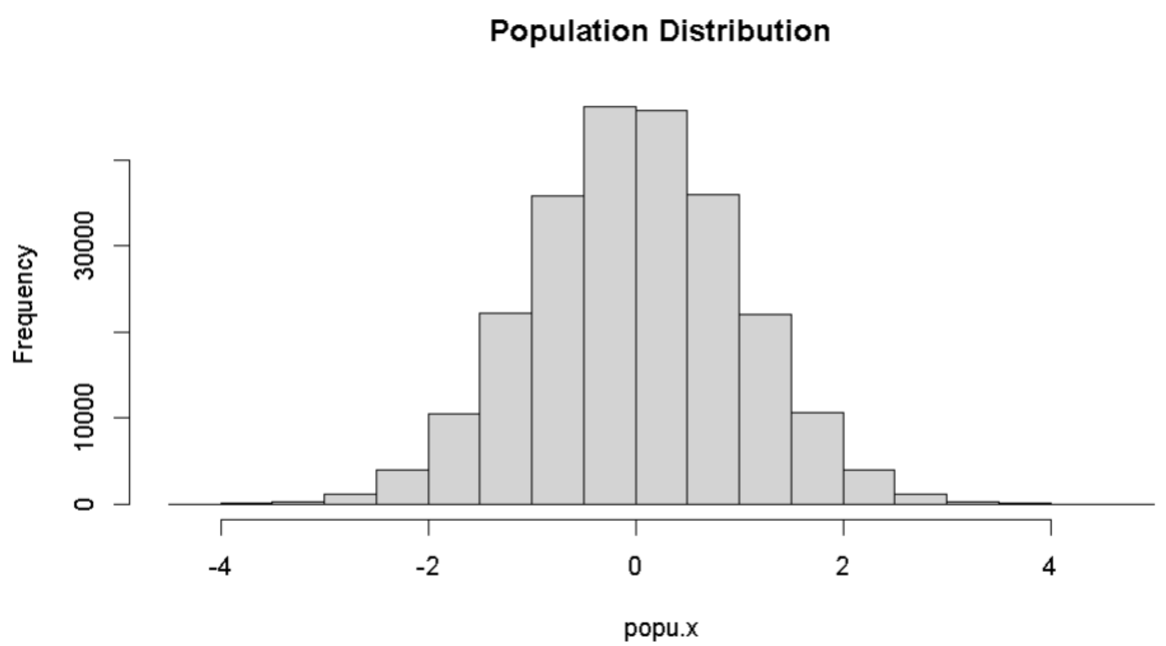
```r
# Central Limit Theorem (CLT) Proof

# Parameters
pop_mean <- 0          # Set the population mean
pop_sd <- 1            # Set the population standard deviation
sample_size <- 80      # Set the sample size
num_samples <- 3000    # Set the number of samples

# Initialize vectors to store sample means and population data
x.bar <- numeric(num_samples)  # Create an empty vector to store sample means
popu.x <- rnorm(n = sample_size * num_samples, mean = pop_mean, sd = pop_sd)  # Generate the population data

# Generate sampling distribution
for (i in 1:num_samples) {  # Loop to generate multiple samples
  samp.x <- sample(popu.x, size = sample_size)  # Randomly sample from the population data
  x.bar[i] <- mean(samp.x)  # Calculate and store the sample mean
}

# Population distribution
hist(popu.x, main = "Population Distribution")  # Create a histogram of the population data

# Sampling distribution
hist(x.bar, main = "Sampling Distribution")  # Create a histogram of the sample means

# Rule 1 proof
cat("Rule 1 - Sample Mean vs Population Mean:\n")  # Display Rule 1 title
cat("Sample Mean: ", mean(x.bar), "\n")  # Calculate and display the sample mean
cat("Population Mean: ", pop_mean, "\n")  # Display the population mean

# Rule 2 proof
cat("\nRule 2 - Sample Standard Deviation vs Population Standard Deviation / sqrt(n):\n")  # Display Rule 2 title
cat("Sample Standard Deviation: ", sd(x.bar), "\n")  # Calculate and display the sample standard deviation
cat("Expected Sample Standard Deviation: ", pop_sd / sqrt(sample_size), "\n")  # Display the expected sample standard deviation
```

Result:

```
> # Rule 1 proof
> cat("Rule 1 - Sample Mean vs Population Mean:\n")  # Display Rule 1 title
Rule 1 - Sample Mean vs Population Mean:
> cat("Sample Mean: ", mean(x.bar), "\n")  # Calculate and display the sample mean
Sample Mean:  -0.001595953
> cat("Population Mean: ", pop_mean, "\n")  # Display the population mean
Population Mean:  0
>
> # Rule 2 proof
> cat("\nRule 2 - Sample Standard Deviation vs Population Standard Deviation / sqrt(n):\n")  # Display Rule 2 title

Rule 2 - Sample Standard Deviation vs Population Standard Deviation / sqrt(n):
> cat("Sample Standard Deviation: ", sd(x.bar), "\n")  # Calculate and display the sample standard deviation
Sample Standard Deviation:  0.1117671
> cat("Expected Sample Standard Deviation: ", pop_sd / sqrt(sample_size), "\n")  # Display the expected sample standard deviation
Expected Sample Standard Deviation:  0.1118034
```

Graphs:


Sampling Distribution
Frequency
0
100
200
300
400
500
-0.4
-0.2
0.0
0.2
0.4
x.bar


Population Distribution
Frequency
0
10000
30000
-4
-2
0
2
4
popu.x

Conclusion:

From the given definition, my R code demonstrates the CLT for a chi-squared population with 4 degrees of freedom. To modify the code, I made it more general so that it can be applied to other populations and sample sizes. The "Sampling Distribution" and the "Population Distribution" histogram approximates a normal distribution. The "Sample Mean" (-0.001891103) is close to the "Population Mean," (0) confirming Rule 1 of the Central Limit Theorem. The "Sample Standard Deviation" (0.1140372) is approximately equal to the "Expected Sample Standard Deviation" (0.1118034), confirming Rule 2 of the Central Limit Theorem.