For which right sides (find a condition on  $b_1, b_2, b_3$ ) are these systems solvable?

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix}
1 & 4 & 2 & b_1 \\
2 & 5 & 4 & b_2 \\
-1 & -4 & -2 & b_3
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 4 & 2 & b_1 \\
0 & 0 & 0 & b_2 - 2b_1 \\
0 & 0 & 0 & b_3 - b_1
\end{bmatrix}$$

$$\begin{bmatrix}
R_2 - 2R_1 \\
R_3 - R_1
\end{bmatrix}$$
free variable
$$b_2 - 2b_1 = 0$$

b2.261

Find the reduced R for each of these (block) matrices:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 2 & 4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} A & A \end{bmatrix} \qquad C = \begin{bmatrix} A & A \\ A & 0 \end{bmatrix}$$

C 
$$\cdot$$
  $\begin{bmatrix} A & A \\ A & O \end{bmatrix}$ 

A  $A \\ O - A \end{bmatrix}$ 

A  $A \\ O - A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \end{bmatrix}$ 

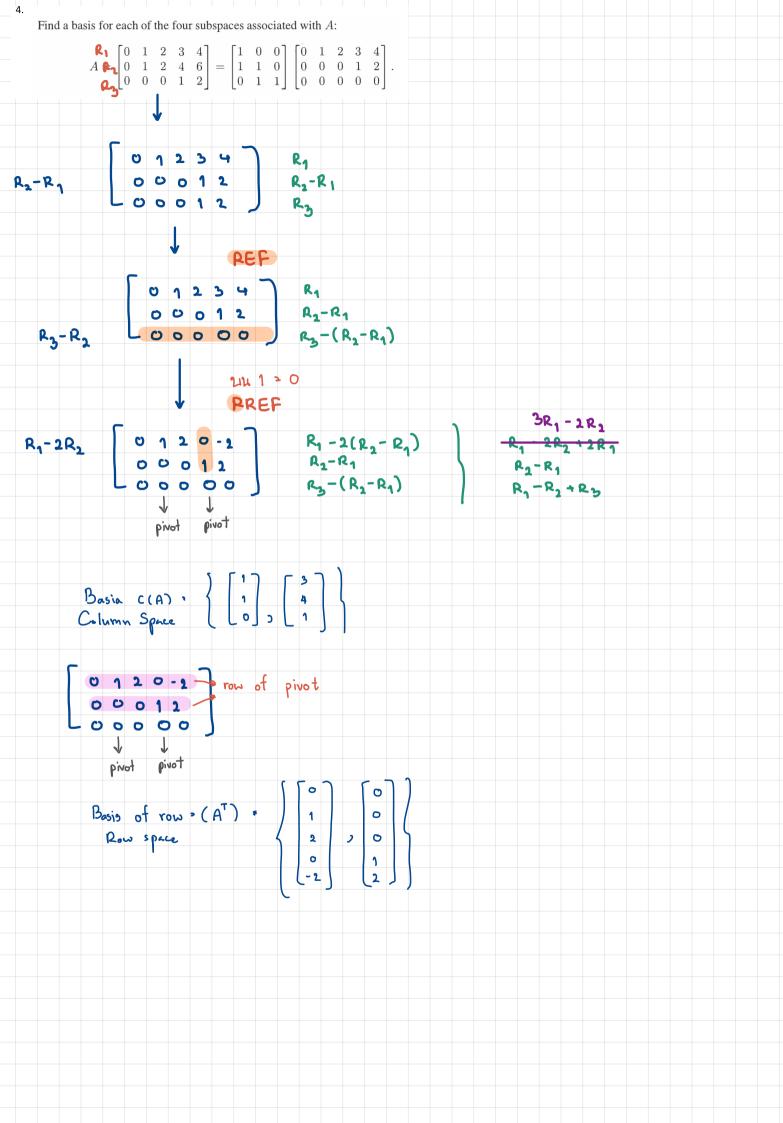
A  $A \\ O A \\ O A \end{bmatrix}$ 

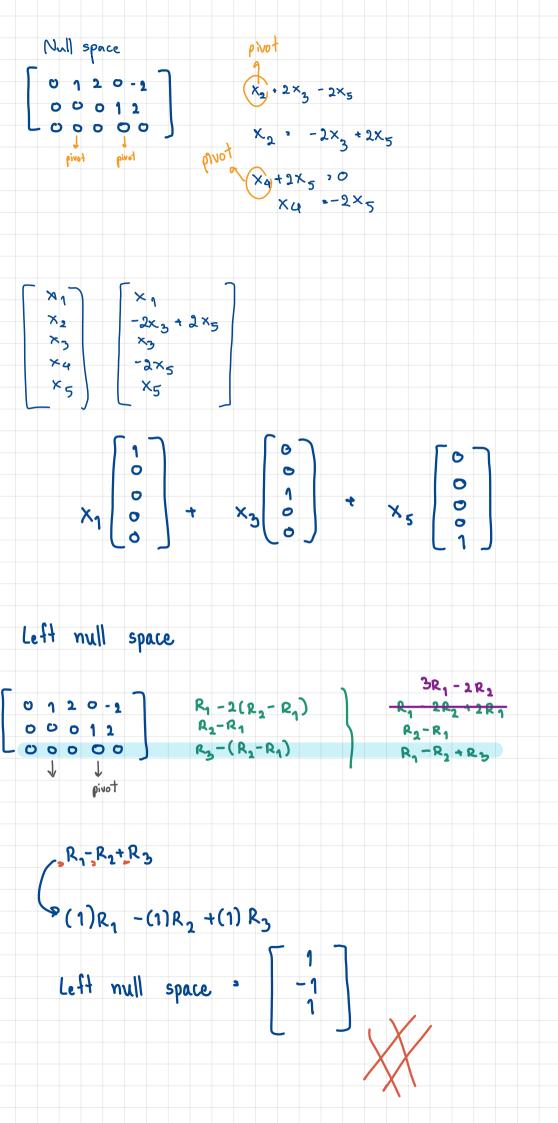
A  $A \\ O A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A \\ O A \\ O A \end{bmatrix}$ 

A  $A \\ O A \\ O A$ 

Find the complete solution (also called the <i>general solution</i> ) to $\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$		
1 3 1 2 1 1 2 1 1 3 1 2 1 1 3 1 2 1 1 3 1 2 1 1 3 1 1 1 1		
1 3 1 2 1 1 0 0 0 2 4 3-1 R <sub>2</sub> -2 R <sub>1</sub>		
[ 1 3 1 2   1 ] 0 0 2 4   1 ] 0 0 0 0   1-1 ] R <sub>2</sub> -R <sub>3</sub>		
1 3 0 0 1-0.5 R <sub>1</sub> -R <sub>2</sub> 0 0 1 2 0.5 0 0 0 0 0		
x y 2 +		
free free variable  pivot * x, z  free y, t		
Partial Sol Ax.b	Special Sol.	
free variable = 0	free variable =1  y = 1  + = 0	+ z 1 Y 2 0
X+3y+02+0t70.5 X:0.5 2+2t = 0.5 2:0.5	X+3y+02+0t70.5 X+3 20.5 X 2-2.5 2+2+20.5 2 20.5	X+3y = 0.5 X = 0.5 2+2+ = 0.5 2 - 2.5 %





## Check

$$A = \begin{bmatrix}
0 & 1 & 2 & 3 & 4 \\
0 & 1 & 2 & 4 & 6 \\
0 & 0 & 0 & 1 & 2
\end{bmatrix} \times 1$$

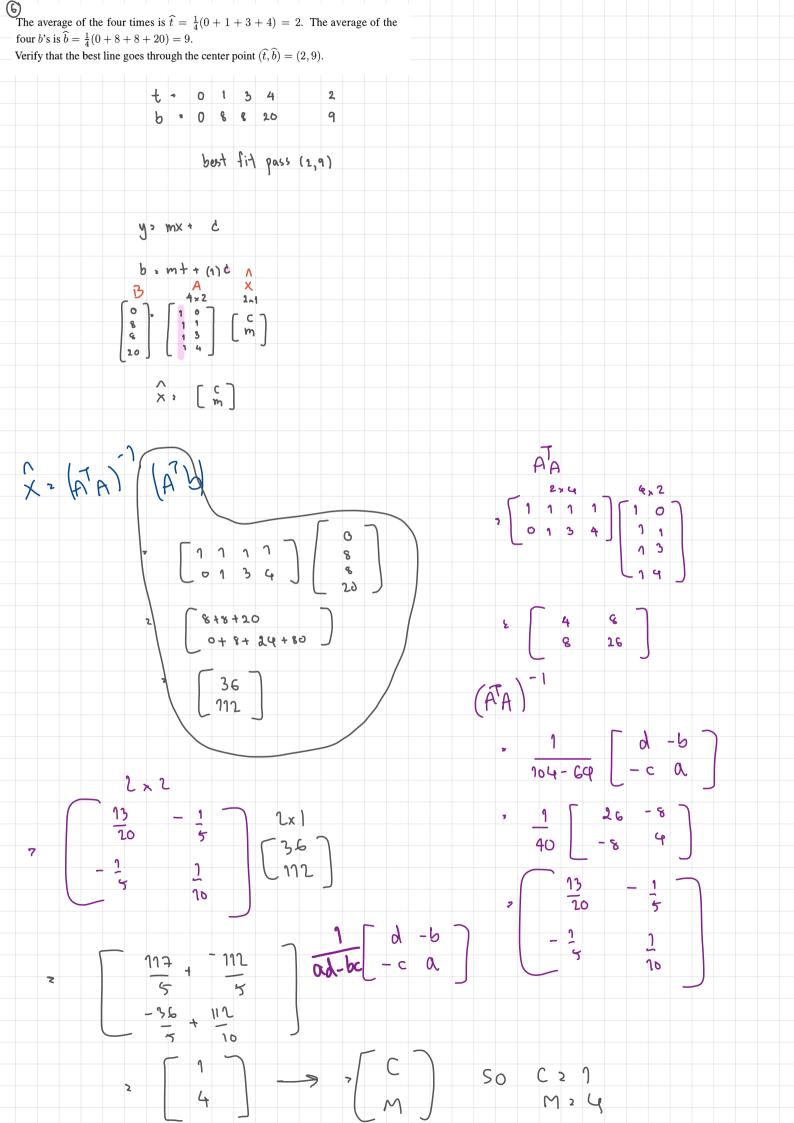
Project b onto the column space of A by solving  $A^{T}A\widehat{x} = A^{T}b$  and  $p = A\widehat{x}$ : A 2 2 x 1 (ATA)(AT)  $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}.$ ATA (X) 2 AB

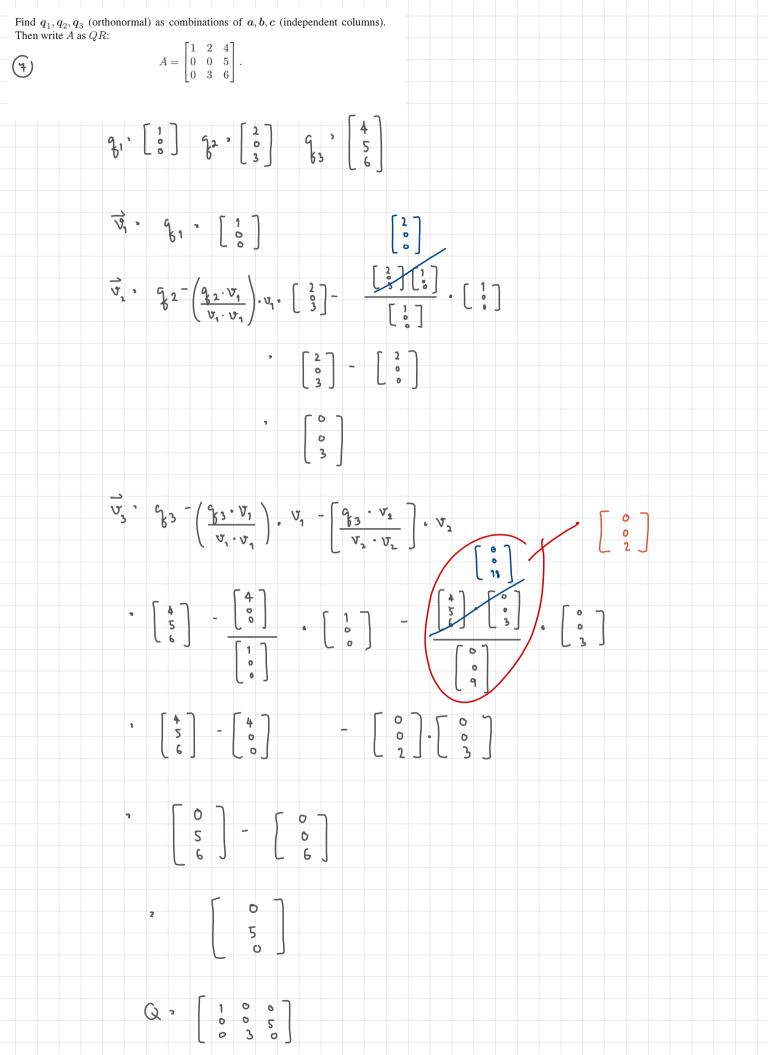
2×1 + 1×1 2 5

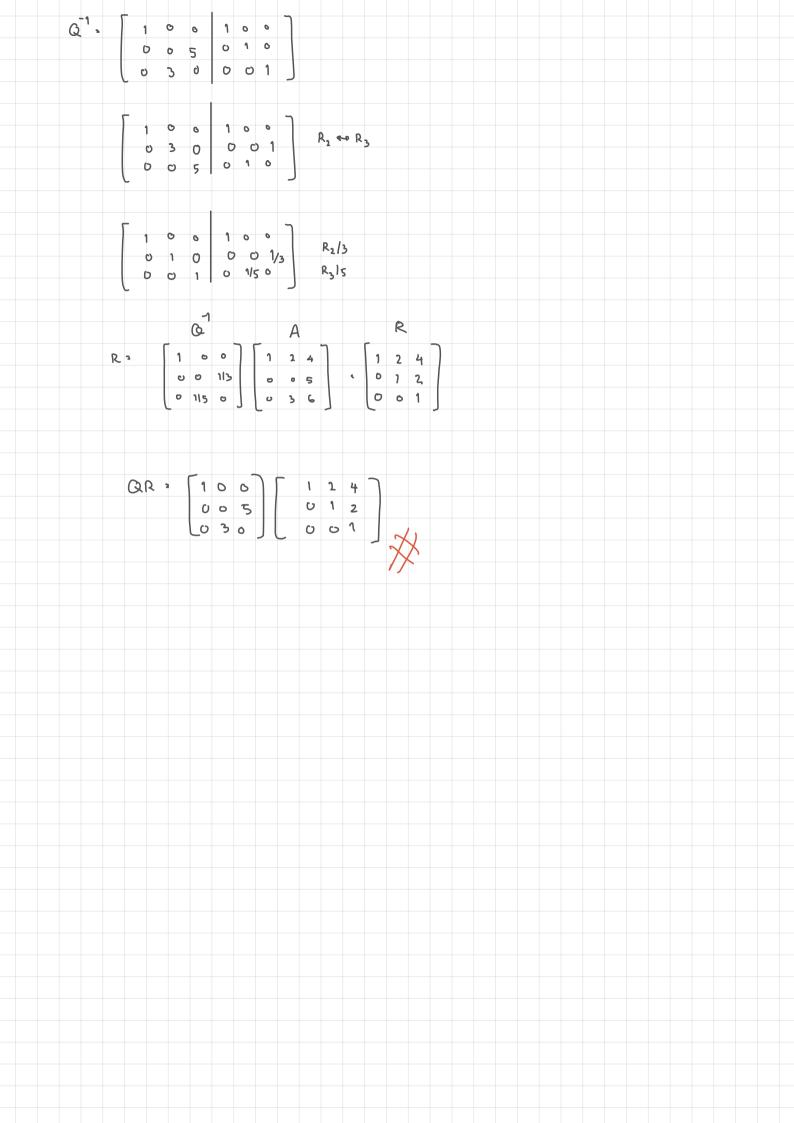
2×1 + 3×1 , Ve

2×1 + 3×1 , Ve

ATA ABJ Find e = b - p. It should be perpendicular to the columns of A. p - A (ATA) -1 AT. b 1 1 (1+1+0 1+1+0) (4+4+0) ~ 2 (A7 b) (A7 A)  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 14 \end{bmatrix}$  $\begin{array}{c|c}
 & \begin{array}{c|c}
 & 1 & 1 \\
 & 1 & 1 \\
 & 0 & 1
\end{array}
 \end{array}
 \left(\begin{array}{c}
 & 1 & 1 \\
 & 6-4 \\
 & 1
\end{array}\right)
 \left(\begin{array}{c}
 & 3 & 2 \\
 & 2 & 2
\end{array}\right)
 \left[\begin{array}{c}
 & 8 \\
 & 14
\end{array}\right]$ AA = [ 110 ] [ 11 ] = [ 2 2 2 2 3 ]  $\begin{array}{c|c}
 & \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} \frac{1}{2} \\ 2 \end{bmatrix} & \begin{bmatrix} 3 & -2 \\ -2 & 2 \end{bmatrix} & \begin{bmatrix} 8 \\ 14 \end{bmatrix}
\end{array}$  $\begin{array}{c|cccc}
1 \times 1 & 2 \times 1 & 2 \times 1 \\
2 & 2 & \times 1 & \times 1 \\
2 & 3 & \times 2 & 1
\end{array}$ 1 1 1 12-14 7  $\begin{bmatrix} 3 \times 2 & 2 \times 1 \\ 1 & 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 6 \end{bmatrix}$ 2×1+2×2 2 8 2x1 + 3×2 = 14 -x2 : 6 72+6 X 2 6 X1 2 -2 e= b-p e = b - p , [4] - [4] P 0







$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Find the reduced R for each of these (block) matrices:

2.

4.

5.

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 2 & 4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} A & A \end{bmatrix} \qquad C = \begin{bmatrix} A & A \\ A & 0 \end{bmatrix}$$

3. Find the complete solution (also called the *general solution*) to

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

Find a basis for each of the four subspaces associated with A:

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Project **b** onto the column space of A by solving  $A^{T}A\hat{x} = A^{T}b$  and  $p = A\hat{x}$ :

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$
 and  $b = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$ .

Find e = b - p. It should be perpendicular to the columns of A.

6. The average of the four times is  $\hat{t} = \frac{1}{4}(0+1+3+4) = 2$ . The average of the four b's is  $\hat{b} = \frac{1}{4}(0+8+8+20) = 9$ .

Verify that the best line goes through the center point  $(\widehat{t}, \widehat{b}) = (2, 9)$ .

7. Find  $q_1,q_2,q_3$  (orthonormal) as combinations of a,b,c (independent columns). Then write A as QR:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{bmatrix}.$$