Complexity categories : linear , quadratics

↳ any linear will eventually be more efficient than quadratic

$\Theta(n^2)$ → pure quadratic aka order of n^2 ex. $3n^2$

can also be $\Theta(n^3)$: pure cubic function

same as big O

Big O notation → $O(f(n))$

$\Omega(f(n))$

$\Theta(f(n))$

$g(n) \in O(f(n))$ Proof

$g(n) \le f(n)$

$g(n) \in \Omega(f(n))$ Proof

$g(n) \ge f(n)$

Show that $5n^2 \in O(n^2)$

is to show that

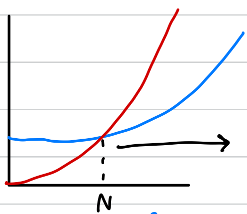
$5n^2 \le cn^2$ for $\forall n \ge N$

ex. let c be 7

$5n^2 \le 7n^2 \quad \forall n \ge N$

$0 \le 2n^2$

$0 \le n^2$



$\boxed{0 \le n}$ → we conclude that $5n^2 \le 7n^2$ for all $n \ge 0$

$\therefore 5n^2 \in O(n^2)$

ex. $n^3 \notin O(n^2)$

Assume $n^3 \in O(n^2)$

Proof there exist $c > 0$ and $N \ge 0$

$n^3 \le cn^2 \quad \forall n \ge N \ge 0$

n^2 is always positive $n \le c \quad \forall n \ge N \ge 0$

↳ n increase will defie $n \le c$

so its false

so $n^3 \notin O(n^2)$

for Ω

ex. $n^3 - n^2 \in \Omega(n^2)$

Proof find $c > 0$ and $N \ge 0$ so that

$n^3 - n^2 \ge cn^2 \quad \forall n \ge N$

let c be 2

$n^3 - n^2 \ge 2n^2$

$n \ge 3 \quad \forall n \ge N$

$\therefore n^3 - n^2 \in \Omega(n^2)$

ex $2n^2+9 \in \Theta(n^2)$

Proof: Find $c>0$ and $d>0$ and $N \geq 0$ such that

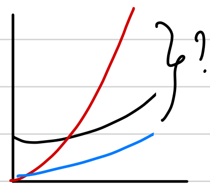
$$cn^2 \leq 2n^2+9 \leq dn^2$$

let $c = 2$ $\quad d = 3$

$$2n^2 \leq 2n^2+9 \leq 3n^2$$

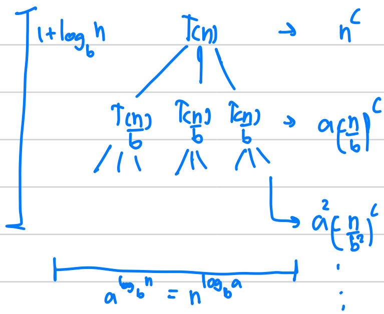
$$0 \leq 9 \leq n^2$$

$$\boxed{n \geq 3 \qquad \forall n \geq N}$$



$\therefore 2n^2+9 \in \Theta(n^2)$

Recursion tree



Master theorem

$$T(n) = aT(\frac{n}{b}) + f(n) \quad \text{for } T(0)=0 \quad T(1)=\Theta(1)$$

$f(n) \rightarrow \Theta(n^c)$

and $a \geq 1$, $b > 0$, $f(n) \geq 0$

$$T(n) = aT(\frac{n}{b}) + \Theta(n^c)$$

Case 1 if $c > \log_b a$ Then $T(n) = \Theta(n^c)$

Case 2 if $c = \log_b a$ Then $T(n) = \Theta(n^c \log n)$

Case 3 if $c < \log_b a$ Then $T(n) = \Theta(n^{\log_b a})$

$r = \frac{a}{b^c}$

Past exam

1.1 $3n+10 \in O(n)$

Proof that theres $c>0$ for $n \geq 0$ such that

$3n+10 \leq cn \quad \forall n \geq N$

Let $c=4$

$3n+10 \leq 4n$

$10 \leq n \quad \forall n \geq N$

Therefore $3n+10 \leq 4n$ for $\forall n \geq 10$

$\therefore 3n+10 \in O(n)$

1.2 $3n+10 \in O(n^2)$

Proof

$3n+10 \leq cn^2 \quad \forall n \geq N$

let $c=1$

$3n+10 \leq n^2$

$0 \leq n^2-3n-10$

$0 \leq (n-5)(n+2)$

$5 \leq n$ and $\cancel{n \leq -2}$

Therefore $3n+10 \leq n^2$ for all $n \geq 5$

$\therefore 3n+10 \in O(n^2)$

1.3 $\log_3(n^2) \in \Theta(\log_2(n^3))$

Proof

$c\log_2(n^3) \leq \log_3(n^2) \leq d\log_2(n^3) \quad \forall n \geq N$

1.4 $n\log^2 n \in O(n^2)$

Proof

$n\log^2 n \leq cn^2 \quad \forall n \geq N$

let $c=1$

$n\log^2 n \leq n^2$

$n\log^2 n - n^2 \leq 0$

$(\sqrt{n}\log n)^2-(n)^2$

$\underbrace{(\sqrt{n}\log n - n)(\sqrt{n}\log n+n)}\leq 0$

$\sqrt{n}\log n - n \leq 0 \rightarrow \sqrt{n}\log n \leq n \rightarrow$ always true

$\sqrt{n}\log n + n \geq 0 \rightarrow \sqrt{n}\log n \geq -n \Rightarrow$ always true

for $n \geq 0$