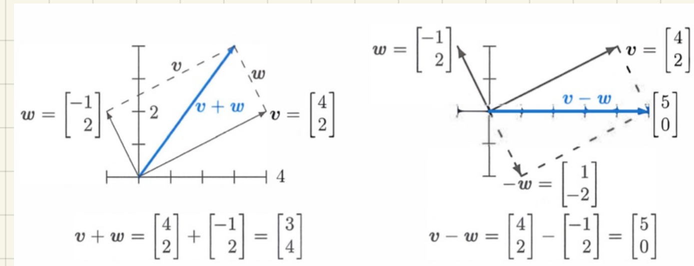
Vector & Linear Combination

- $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ $\rightarrow$ $v + w = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \end{bmatrix}$ $\rightarrow$ zero vector = Space or 0

Add

$\rightarrow$ $2v = \begin{bmatrix} 2v_1 \\ 2v_2 \end{bmatrix}$



vector addition (head to tail)

- $v \cdot w =$ $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = v_1 w_1 + v_2 w_2$

Dot product

if $= 0$ the angle is $90°$
v & w are perpendicular

length of $v \cdot w = \sqrt{v \cdot w}$

Angle between 2 vector

- Angle is $< 90°$ when $v \cdot w$ is *positive*
  Angle is $> 90°$ when $v \cdot w$ is *negative*

- ❖ Perpendicular vectors: $\|v\|^2 + \|w\|^2 = \|v - w\|^2$
  ❖ Pythagoras $(v_1^2 + v_2^2) + (w_1^2 + w_2^2) = (v_1 - w_1)^2 + (v_2 - w_2)^2$

$$0 = -2v_1 w_1 - 2v_2 w_2$$

$$v_1 w_1 + v_2 w_2 = 0$$

- **Cosine Formula**
  If v and w are nonzero vectors, then $\frac{v \cdot w}{\|v\| \, \|w\|} = \cos\theta$

Matrices

1st variable column = row of 2nd to do multiplication

ex. (3 by 2) × (2 by 1)
result (2 by 1)

$$Ax = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_1 + x_2 \\ -x_2 + x_3 \end{bmatrix} \begin{matrix} : x_1 - 0 \\ : x_2 - x_1 \\ : x_3 - x_2 \end{matrix}$$

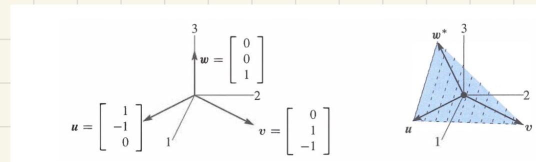
❖ A is called the difference matrix because b contains differences of x

$$Cx = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_3 \\ x_2 - x_1 \\ x_3 - x_2 \end{bmatrix}$$

x_1 x_2 x_3 $x_3 - x_2$

❖ C is called the cyclic difference matrix

- Dependence
  &
- Independence



❖ Independence w is not in the plane of u and v
❖ Dependence w^* is in the plane of u and v
Why? if we can find the combination from u & v
❖ w^* is a linear combination of u and v

$$w^* = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = -u - v$$

Identify Matrix

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ คูณ matrix จะ ค่าเท่าเดิม

$$\begin{matrix} 2x & + & 4y & - & 2z & = & 2 & (E1) \\ 4x & + & 9y & - & 3z & = & 8 & (E2) \\ -2x & - & 3y & + & 7z & = & 10 & (E3) \end{matrix} \Rightarrow \begin{matrix} 2x & + & 4y & - & 2z & = & 2 \\ & & 1y & + & 1z & = & 4 \\ & & & & 4z & = & 8 \end{matrix}$$

Step 1 $(E2) - 2 \times (E1) \rightarrow y + z = 4$
Step 2 $(E3) - (-1) \times (E1) \rightarrow y + 5z = 12$

x is eliminated $\begin{matrix} 1y & + & 1z & = & 4 & (E2') \\ 1y & + & 5z & = & 12 & (E3') \end{matrix}$

Step 2 $(E3') - (1) \times (E2') \rightarrow 4z = 8$

Elimination

Inverse only if $ad - bc \neq 0$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\boxed{\begin{matrix} * \ (AB)^{-1} = B^{-1} A^{-1} \\ * \ (ABC)^{-1} = C^{-1} B^{-1} A^{-1} \end{matrix}}$$

- Gauss-Jordan Elimination

Step 1 $K : [K \ I]$ ex. $K : \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$, $[K I] = \begin{bmatrix} -2 & 1 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix}$

Step 2 eliminate until $KI = IK^{-1}$

ex. $[K \ I] = \begin{bmatrix} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix}$

$[I \ K^{-1}] = \begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}$

Transpose

row $\rightarrow$ column

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 0 \end{bmatrix}$

$$\boxed{\begin{matrix} * \ (A+B)^T = A^T + B^T \\ (AB)^T = B^T A^T \\ (A^{-1})^T = (A^T)^{-1} \end{matrix}}$$