DISCRETE MATHEMATICS
ASSIGNMENT 11

PROBLEM 1

Mary is conducting experiments on the effectiveness of some pesticides on certain plants. Note that in the experiments described below, pesticide is to be administered in whole doses only (thus, no half dose or 0.2 dose).

1.1) In the first experiment, there are 7 plants of different types. The experiment involves administering between 0 to 9 doses of pesticide ZX on each plant. How many ways can this experiment be conducted?

1.2) In the second experiment, there are 7 plants of different types. The experiment involves administering between 0 to 9 doses of pesticide ZX on each plant so that no two plants receive the same number of doses. How many ways can this experiment be conducted?

1.3) In the third experiment, there are 7 identical plants. The experiment involves administering between 0 to 9 doses of pesticide ZX on each plant. How many ways can this experiment be conducted?

1.4) In the fourth experiment, there are 7 identical plants. The experiment involves administering between 0 to 9 doses of pesticide ZX on each plant so that no two plants receive the same number of doses. How many ways can this experiment be conducted?

1.5) In the fifth experiment, there are 7 plants of different types and in total 12 doses of pesticide ZX. All the 12 doses of pesticide ZX are to be administered to the plants so that each plant receives at least one dose. How many ways can this experiment be conducted?

1.6) In the sixth experiment, there are 7 identical plants and in total 12 doses of pesticide ZX. All the 12 doses of pesticide ZX are to be administered to the plants so that each plant receives at least one dose. How many ways can this experiment be conducted?

1.7) In the seventh experiment, there are 7 plants of different types and 5 different kinds of pesticides. The experiment involves administering only one dose of pesticide on each plant such that each kind of pesticide is used on at least one plant. How many ways can this experiment be conducted?

1.8) In the eighth experiment, there are 7 identical plants and 5 different kinds of pesticides. The experiment involves administering only one dose of pesticide on each plant such that each kind of pesticide is used on at least one plant. How many ways can this experiment be conducted?

1.9) In the ninth experiment, there are 7 identical plants and 10 different kinds of pesticides. Suppose we have only one dose available for each kind of pesticide. The experiment involves administering a mixture of one or more kinds of pesticide on each plant. Every kind of pesticide must be used in this experiment. How many ways can this experiment be conducted?

1.10) The tenth experiment is similar to the ninth experiment. The only difference is that, in the tenth experiment, it is not required that every kind of pesticide has to be used. How many ways can this experiment be conducted?

PROBLEM 2

A group of 7 students is assigned seats for each of two classes in the same classroom. How many ways can these seats be assigned if no student is assigned the same seat for both classes?

PROBLEM 3

How many ways can a class of 30 students be divided into 5 groups, each of which contains 6 students?

PROBLEM 4

A (0,1)-matrix is a matrix in which each entry is either '0' or '1'. How many (0,1)-matrices of dimension 5 x 5 with no zero rows are there?

Note: A zero row in a matrix is a row in which all entries are '0'.

PROBLEM 5

A (0,1)-matrix is a matrix in which each entry is either '0' or '1'. How many (0,1)-matrices of dimension 5 x 5 with no zero rows and no zero columns are there?

Note: A zero row in a matrix is a row in which all entries are '0'. A zero column in a matrix is a column in which all entries are '0'.

PROBLEM 6

A function f on a set of integers is said to be **increasing** if and only if $x \leq y$ implies $f(x) \leq f(y)$ for all x, y in the domain.

Suppose $[n]$, where n is a positive integer, denotes the set of positive integers no larger than n, i.e.

$$[n] = \{1, 2, 3, \dots, n\}$$

How many different increasing functions on the set $[10]$ are there?

PROBLEM 7

Suppose n is a positive integer. Let

- $P(n)$ be the set of all permutations of all the numbers in the set $[n] = \{1,2,3, \dots, n\}$.
- $D(n)$ be the set of all derangements in $P(n)$.
- $d(n) = |D(n)|$, the number of derangements in $P(n)$.

7.1) Prove that $d(n)$ satisfies the following recurrence equations.

$$d(1) = 0$$
$$d(n) = n \cdot d(n-1) + (-1)^n \quad \text{where } n > 1$$

7.2) Prove that $d(n)$ also satisfies the following recurrence equations.

$$d(1) = 0$$
$$d(2) = 1$$
$$d(n) = (n-1)(d(n-1) + d(n-2)) \quad \text{where } n > 2$$