Uniform distribution:

PDF : $f(x) = \frac{1}{b-a}$ $(a<b)$

$E(x) = \frac{a+b}{2}$

$Var(x) = \frac{(b-a)^2}{12}$ SD $= \sqrt{Var}$

exponential random variable:

PDF : $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$, if $x<0$ $f(x)=0$

$E(x) = \frac{1}{\lambda}$

$Var(x) = \frac{1}{\lambda^2}$ SD $= \sqrt{Var}$

cdf: for $x \geq a$ $(P(X \geq a))$

$F(x) = 1 - e^{-a\lambda}$

Normal Random variable $X \sim N(\mu, \delta^2)$

Pdf: $f(x) = \frac{1}{\sqrt{2\pi\delta^2}} e^{\frac{-(x-\mu)^2}{2\delta^2}}$

δ = Var

μ = mean

$E(x) = \mu$

$Var(x) = \delta^2$

CDF:

$P(c \leq x \leq d) = \int_c^d f(x)\,dx$

$P(X \leq x) = \int_a^x f(t)\,dt$ for interval (a,b)

$F(a) = 0$, $F(b) = 1$

→ ex in pt Q2 p2

Example 1. *The density function of a continuous random variable, X, is given by*

$$f(x) = \begin{cases} \frac{x^2}{9} & \text{if } 0 \leq x \leq 3; \\ 0 & \text{otherwise .} \end{cases}$$

(i) What is the probability that X is between 1 and 2? $\int_1^2 \frac{x^2}{9}\,dx$

(ii) What is the probability that X is between 1 and 2, if it is known that it is between 1 and 3.

$$\frac{\int_1^2 \frac{x^2}{9}\,dx}{\int_1^3 \frac{x^2}{9}\,dx}$$

Answer: (i) $\frac{7}{27}$, *(ii)* $\frac{7}{26}$.

Expectation (crv in general)

$E(x) = \int_{\mathbb{R}} x f(x)\,dx$

if x is non negative:

$E(x) = \int_0^\infty P(X>x)\,dx = \int_0^\infty P(X>y)\,dy$

Fubini's thm (for finite)

$\int_a^b \int_c^d f(x,y)\,dx\,dy = \int_c^d \int_a^b f(x,y)\,dy\,dx$

Variance (in general)

$Var(x) = E(x^2) - (\text{mean})^2$

$= E(x^2) - (E(x)^2)$

$Var(ax+b) = a^2 Var(x)$

Joint distribution

$P((x,y)) = \iint_B f_{x,y}(x,y)\,dx\,dy$

Conditional PDF

$P(X \in B \mid X \in A) = \int_B f_{X|A}(x)\,dx$