Gravity and Coulumbic Forces Simulator

Documentation

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1 The Problem Statement:

To completely solve for an N-body system under an inverse square force, given each body's initial position and velocity.

1.1 Explanation:

In the physical world, this translates to creating a simulation for Newtonian gravitational force or columbic electrostatic force. For the purposes of explanation, we will continue with gravity. We are given a set of initial conditions for each body, namely the position and velocity components in Cartesian coordinates with respect to a given referent. In the case of a gravitational simulator, the referent is the heaver mass. The sun in our solar system, the Earth in an earth-moon system, etc. We are also given a time after which we need to know the final positions and velocities of each of the N-bodies. For example, if the time given is 365 days, we need to know the final conditions of each of the N-bodies after 365 days(365 days after the initial conditions). The other criteria were that the input for the initial conditions needs to be in the form of a CSV file, and each body has to have an output CSV file consisting of its 'middle' conditions, the positions, and velocities after every calculation.

2 The Package:

A program was written in C++ to analyze the N-body system. It takes in a **CSV file** with initial conditions in the following format: mass, x, y, z, Vx, Vy, Vz, q. Where (x, y, z) are the coordinates of the body, (Vx, Vy, Vz) are its velocity components, q is its charge, and mass is the mass.

```
(Mass, x, Y, z, Vx, Vy, Vz, q)
1988500e24, 0, 0, 0, 0, 0, 0, 0
3.302e23, 3.545665911686573E+10, -5.355388811959700E+10, -7.628685664591890E+09,  3.089761965787503E+04, 2.927661264869901E+04, -4.419009364366762E+02, 0.0
48.685e23, -6.678287883912063E+10, -8.525786375646330E+10, 2.683757316608179E+09, 2.732443556883071E+04, -2.175972728517292E+04, -1.875396996235638E+03, 0.0
5.97219e24, -2.679537537927854E+10, 1.446331553260101E+11, -7.008776435680687E+06, -2.976558009593398E+04, -5.532012996003417E+03, -4.594874802426041E-01, 0.0
6.4171e23, 9.288163578680943E+10, 2.057978923558269E+11, 2.034214333819464E+09, -2.116658268629578E+04, 1.202703532444972E+04, 7.712798408539241E+02, 0.0
1.89818722e27, 4.549989009412312E+11, -6.115805549564068E+11, -7.639729318131268E+09, 1.033424076603879E+04, 8.424037369862027E+03, -2.662787604069234E+02, 0.0
5.6834e26, 8.213461842100713E+11, -1.247963915440769E+12, -1.099199102208489E+10, 7.543771358776889E+03, 5.295817437955165E+03, -3.927748585301070E+02, 0.0
86.813e24, 2.296332567877831E+12,  1.864229101021707E+12, -2.282515777686954E+10, -4.332465449574797E+03, 4.977761950007296E+03, 7.430553599878942E+01, 0.0
102.409e24, 4.407232236134411E+12, -7.826696549275551E+11, -8.546307111596137E+10, 9.249547678472091E+02, 5.393739130909577E+03, -1.321862726111025E+02, 0.0
1.307e22, 2.103438886527093E+12, -4.661219489333332E+12, -1.094831727759385E+11, 5.111373207997374E+03, 1.093778504899072E+03, -1.592388441832147E+03, 0.0
```

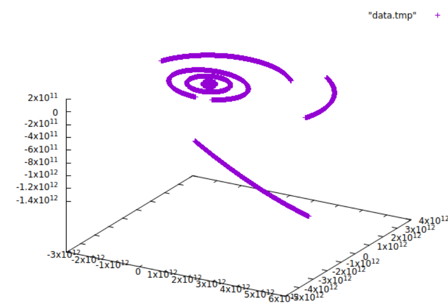
Figure 1: CSV input file for our Solar System(10 body system)

The CSV file modeling our solar system(10-body system, sun to Pluto) is given above(Figure 1). The charge for all the planets is taken to be zero. All values are given in SI, units. The data for the solar system was taken from JPL Horizon's web application time was 12:00 am January 1st, 2021. The data follow the given order line 2-Sun, line 3-Mercury, line 4-Venus, until line 10-Pluto. Note that all the values are with respect to the sun.

The other inputs taken by the program are choosing between the gravity or electric simulators(g/e) and then inputting the time of analysis(ex. 400 days in the gravity simulator). Both of these are terminal inputs. Upon completion, the program outputs N+1 CSV files. N CSV files contain the position and velocity of each body after every calculation. The last CSV file contains the position of *every body* after each calculation set. This last file, called "data.tmp" can then be used to graph the path taken by each body, see Figure 2a. The program also outputs the time taken to run the complete analysis and the final positions and velocities of each body(terminal outputs). This is in the following format: (x, y, z) | (Vx, Vy, Vz). The terminal output for the solar system after 185 days, from January 1st (initial conditions) to July 5th(185 days after initial conditions), is given below.

```
(0, 0, 0) | (0, 0, 0)
(5.24987e+10, -2.34306e+10, -6.73036e+09) | (10339.8, 46684.3, 2866.43)
(-1.0508e+11, 2.2335e+10, 6.37019e+09) | (-7462.67, -34413.8, -41.6595)
(3.44717e+10, -1.47876e+11, 7.39051e+06) | (28562.1, 6663.32, 0.365716)
(-2.2129e+11, 1.14247e+11, 7.82247e+09) | (-10208.2, -19463.4, -157.478)
(6.00519e+11, -4.55122e+11, -1.15465e+10) | (7740.66, 11040.5, -219.109)
(9.37502e+11, -1.15705e+12, -1.72028e+10) | (6977, 6070.82, -383.681)
(2.22557e+12, 1.94257e+12, -2.16223e+10) | (-4520.87, 4823.89, 76.1843)
(4.42116e+12, -6.96291e+11, -8.75592e+10) | (817.837, 5413.94, -130.084)
(2.18485e+12, -4.64312e+12, -1.34921e+11) | (5074.96, 1169.99, -1590.41)
DONE
Time taken by simulation : 79.000000 sec
```

(a) Terminal Output for given conditions



(b) Complete Solar System

Figure 2: 3D Orbital Paths of Planets

The outputs are given in the same order as the inputs.

2.1 Working:

The program is based on the basic principles of Newton's Laws. At a given instant, the program calculates the net force on one body due to every other body. Using that force, it calculates the acceleration that the body faces, due to which the body moves for some set time interval. Smaller and smaller time intervals are required for more exact calculations, but this results in larger running times. Currently, the program is set to perform the calculations at every 25s, or 3456 times per day. It was found after much trial and error that this time interval works best.

3 Results and Verification:

Below is a plot of the path of the inner planets in 400 days, given the above initial conditions(5-body system, including the sun), Figure 3. We can see that the axis are not completely planar, which is correct. The plane of rotation of the other planets is at a slight angle as compared to earth's, which can be seen here as well.

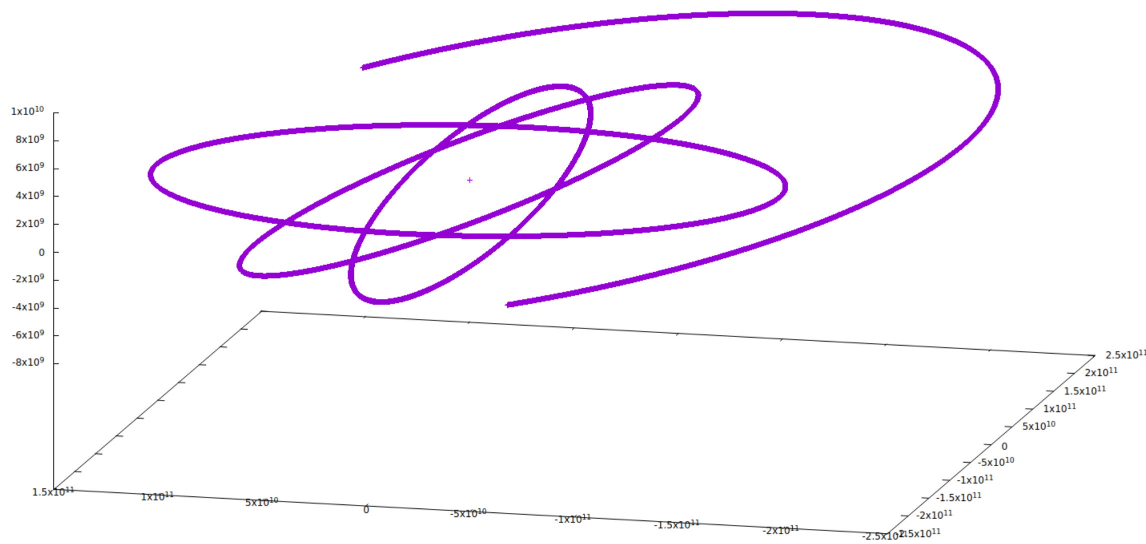


Figure 3: 3D Orbital Path of the inner Planets after 400 days

3.1 Comparison with JPL Horizon's Data

I compared the final positions and velocities outputted by my program to the JPL Horizons's data. 185 days after January 1st, 2021 was July 5th, 2022. The results are given below:

Planatary Body	My Data	
	Position(m)	Velocity(m/s)
Sun	(0, 0, 0)	(0, 0, 0)
Mercury	(5.24987e+10, -2.34306e+10, -6.73036e+09)	(10339.8, 46684.3, 2866.43)
Venus	(-1.0508e+11, 2.2335e+10, 6.37019e+09)	(-7462.67, -34413.8, -41.6595)
Earth	(3.44717e+10, -1.47876e+11, 7.39051e+06)	(28562.1, 6663.32, 0.365716)
Mars	(-2.2129e+11, 1.14247e+11, 7.82247e+09)	(-10208.2, -19463.4, -157.478)
Jupiter	(6.00519e+11, -4.55122e+11, -1.15465e+10)	(7740.66, 11040.5, -219.109)
Saturn	(9.37502e+11, -1.15705e+12, -1.72028e+10)	(6977, 6070.82, -383.681)
Uranus	(2.22557e+12, 1.94257e+12, -2.16223e+10)	(-4520.87, 4823.89, 76.1843)
Neptune	(4.42116e+12, -6.96291e+11, -8.75592e+10)	(817.837, 5413.94, -130.084)
Pluto	(2.18485e+12, -4.64312e+12, -1.34921e+11)	(5074.96, 1169.99, -1590.41)

Figure 4: Position and Velocity according to my program

Figure 4 gives the positions and velocities of each of the bodies after 185 days, as calculated by my program. This was the day when Earth reached its Aphelion. Figure 5(given below) shows the position and velocities of each of the bodies on the same day, July 5th, 2021, given by JPL Horizon. We can see that the error between the two sets is marginal, around 0.1%.

An important note is that all of our observations are heliocentric, which is why the position and velocity of the

Planatary Body	JPL Horizon's Data	
	Position	Velocity
Sun	(0, 0, 0)	(0, 0, 0)
Mercury	(5.251522710294981E+10, -2.328218766643114E+10, -6.719747562357445E+09)	(1.023464789222711E+04, 4.673803757157367E+04, 2.880469067361510E+03)
Venus	(-1.050685130627065E+11, 2.237691984335082E+10, 6.370092025256057E+09)	(-7.474941793501393E+03, -3.441129276165364E+04, -4.091682944110886E+01)
Earth	(3.400031500886266E+10, -1.482512631446154E+11, 7.101377223409712E+06)	(2.855833721383995E+04, 6.540364017659209E+03, -1.383205986789537)
Mars	(-2.212941078589625E+11, 1.142635020319666E+11, 7.822918386723340E+09)	(-1.020973168868518E+04, -1.946138643518960E+04, -1.573990882353860E+02)
Jupiter	(6.005558130057800E+11, -4.551248288881654E+11, -1.154607474886295E+10)	(.742744257421989E+03, 1.104116210178912E+04, -2.190535155834610E+02)
Saturn	(9.374931165678781E+11, -1.157069593775156E+12, -1.719328031497777E+10)	(6.976786753158771E+03, 6.068548822090276E+03, -3.825061648332497E+02)
Uranus	(2.225566048998931E+011, 1.942575686164649E+11, -2.162146268256795E+10)	(-4.521397503831477E+03, 4.824025123542581E+03, 7.626719798263992E+02)
Neptune	(4.421167618858603E+12, -6.962875070054131E+11, -8.756083211510113E+10)	(8.189224433377918E+02, 5.414130719932170E+03, -1.307816595479676E+02)
Pluto	(2.184565610593886E+12, -4.643358014319886E+12, -1.347982739267957E+11)	(5.074312189772455E+03, 1.172320199054426E+03, -1.585520864239336E+03)

Figure 5: Position and Velocity from JPL Horizon's database

sun remain 0. In reality, the sun also moves with respect to its initial frame(the initial conditions), but the results that are important to us are relative to the sun, as that is what we can observe.

We can see that aside from Mercury's positions, this result is a good beginning to show that the program works correctly. Mercury's orbit can not be completely explained by Newtonian gravity alone, which is why the final coordinates of mercury in my program are slightly skewed compared to the real-world data.

The time taken to run this test was 80 seconds.

3.2 Comparison with Initial Conditions

Another method I used to verify the validity of my program was to check the position and velocities after one revolution around the sun for each of them. I have only been able to do this for the inner planets, as the time required to compute for one revolution of the outer planets becomes immense. The following are my results:

Planatary Body	Initital Conditions	
	Position(m)	Velocity(m/s)
Sun	(0, 0, 0)	(0, 0, 0)
Mercury	(3.545665911686573E+10, -5.355388811959700E+10, -7.628685664591890E+09)	(3.089761965787503E+04, 2.927661264869901E+04, -4.419009364366762E+02)
Venus	(-6.678287883912063E+10, -8.525786375646330E+10, 2.683757316608179E+09)	(2.732443556883071E+04, -2.175972728517292E+04, -4.594874802426041E-01)
Earth	(-2.679537537927854E+10,1.446331553260101E+11, -7.008776435680687E+06)	(-2.976558009593398E+04, -5.532012996003417E+03, 0.365716)
Mars	(9.288163578680943E+10, 2.057978923558269E+11, 2.034214333819464E+09)	(-2.116658268629578E+04, 1.202703532444972E+04, 7.712798408539241E+02

Figure 6: Initial Conditions

Figure 6 shows the initial conditions given, taken from JPL Horizon's database and inputted into my code. These are then compared with one revolution of each body around the sun. The final positions should be similar if the program works correctly.

Planatary Body	Final Output	
	Position(m)	Velocity(m/s)
Sun	(0, 0, 0)	(0, 0, 0)
Mercury	(3.55292e+10, -5.34982e+10, -7.63081e+09)	(30856.6, 29332.2, -433.568)
Venus	(-6.60484e+10, -8.58435e+10, 2.63321e+09)	(27510.3, -21519.2, -1882.82)
Earth	(-2.73617e+10, 1.44536e+11, -7.20255e+06)	(-29742.5, -5646.92, -0.419434)
Mars	(9.27232e+10, 2.05881e+11, 2.04045e+09)	(-21173.3, 12012.6, 771.134)

Figure 7: Positions and Velocities after one revolution

Figure 7 shows the positions and velocities after one revolution. They are very close to each other, proving that the calculations done by the program are correct. It is important to note that since the sun is also affected, it also moves, and hence one revolution would not bring the body back to the same place it started, but instead to a place closet. Another important note is that the revolutions for each planet vary. For Mercury, it is 88 days; for Venus, it is 225; for Earth, it is 365; for Mars, it is 687.

The time taken to run these tests is given below:

1. Mercury: 40 seconds
2. Venus: 98 seconds
3. Earth: 152 seconds
4. Mars: 291 seconds

4 Future Work:

5 References:

1. JPL Horizon's Web Application
2. JPL Horizon's Orbit Viewer