Cyclic Quadrilaterals

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§ 1   Theory

§ 2   Examples

§3 Problems

Minimum is [TBD✎]. Problems with the ⬦ symbol are required.

[3✎] **Problem 1** (AIME II 2011/10) A circle with center O has radius 25. Chord $\overline{AB}$ of length 30 and chord $\overline{CD}$ of length 14 intersect at point P. The distance between the midpoints of the two chords is 12. The quantity OP^2 can be represented as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find the remainder where $m + n$ is divided by 1000.

[4✎] **Problem 2** (NIMO Janurary 2013/8) Let $AXYZB$ be a convex pentagon inscribed in a semicircle with diameter AB. Suppose that $AZ - AX = 6$, $BX - BZ = 9$, $AY = 12$, and $BY = 5$. Find the greatest integer not exceeding the perimeter of quadrilateral $OXYZ$, where O is the midpoint of AB.

[6✎] **Problem 3** (AMC 12A 2017/24) Quadrilateral $ABCD$ is inscribed in circle O and has sides $AB = 3$, $BC = 2$, $CD = 6$, and $DA = 8$. Let X and Y be points on $\overline{BD}$ such that

$$\frac{DX}{BD} = \frac{1}{4} \quad \text{and} \quad \frac{BY}{BD} = \frac{11}{36}.$$

Let E be the intersection of intersection of line AX and the line through Y parallel to $\overline{AD}$. Let F be the intersection of line CX and the line through E parallel to $\overline{AC}$. Let G be the point on circle O other than C that lies on line CX. What is $XF \cdot XG$?