# Constructing Auxillary Figures

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GPU

The whole premise of this handout is that you just construct something else, and the problem instantly becomes clear.

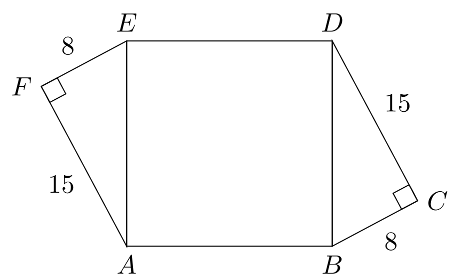
# § 1 Problems

> “Memories aren’t something you can go out of your way to create. It’s what’s left over!”
>
> Yugami-kun

[2] **Problem 1** (AIME II 2007/3) Square $ABCD$ has side length 13, and points $E$ and $F$ are exterior to the square such that $BE = DF = 5$ and $AE = CF = 12$. Find $EF^2$.

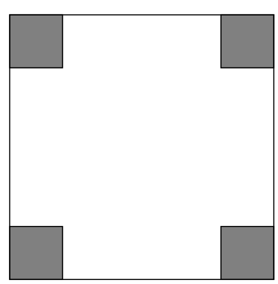
[2] **Problem 2** Let $ABCD$ be a square and $P$ be a point outside of $ABCD$ such that $\angle APB = 90°$. Prove that the bisector of $\angle APB$ bisects $ABCD$ into two polygons of equal area.

[2] **Problem 3** (PUMaC 2015) Find the distance $\overline{CF}$ in the diagram below where $ABDE$ is a square and angles and lengths are as given:




The length $\overline{CF}$ is of the form $a\sqrt{b}$ for integers $a, b$ such that no integer square greater than 1 divides $b$. What is $a + b$?

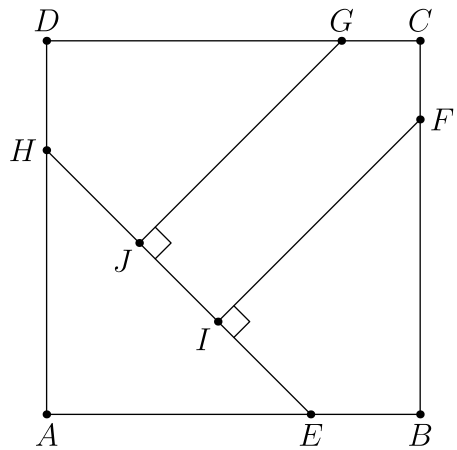
[3] **Problem 4** (AMC 8 2017/25) One-inch squares are cut from the corners of this 5 inch square. What is the area in square inches of the largest square that can be fitted into the remaining space?



[3] **Problem 5** (AIME II 2008/5) In trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$, let $BC = 1000$ and $AD = 2008$. Let $\angle A = 37°$, $\angle D = 53°$, and $M$ and $N$ be the midpoints of $\overline{BC}$ and $\overline{AD}$, respectively. Find the length $MN$.

[4] **Problem 6** (Kyiv City Math Olympiad 2014/7.4) Consider simple convex quadrilateral $ABCD$ where $AD = AB + CD$. If the angle bisectors of $\angle BAD$ and $\angle CAD$ intersect at $P$, prove that $BP = CP$.

[9] **Problem 7 (AMC 10B 2020/21)** In square $ABCD$, points $E$ and $H$ lie on $\overline{AB}$ and $\overline{DA}$, respectively, so that $AE = AH$. Points $F$ and $G$ lie on $\overline{BC}$ and $\overline{CD}$, respectively, and points $I$ and $J$ lie on $\overline{EH}$ so that $\overline{FI} \perp \overline{EH}$ and $\overline{GJ} \perp \overline{EH}$. See the figure below. Triangle $AEH$, quadrilateral $BFIE$, quadrilateral $DHJG$, and pentagon $FCGJI$ each has area 1. What is $FI^2$?




[13 ✎] **Problem 8** (ISL 2001/G1) Let $A_1$ be the center of the square inscribed in acute triangle $ABC$ with two vertices of the square on side $BC$. Thus one of the two remaining vertices of the square is on side $AB$ and the other is on $AC$. Points $B_1,\ C_1$ are defined in a similar way for inscribed squares with two vertices on sides $AC$ and $AB$, respectively. Prove that lines $AA_1,\ BB_1,\ CC_1$ are concurrent.