Exponents in modular arithmetic

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NQT

1 Problems

1.1 Period of a Repeating Decimal

[2 ♟] **Problem 1** The expansion of $\frac{1}{7}$ is $0.\overline{142857}$, which is a repeating decimal with a 6 digit long sequence. How many digits long is the expansion of $\frac{1}{13}$?

[2 ♟] **Problem 2** We define the cycle of a repeating fraction $\frac{m}{n}$ as the minimum number i such that $\frac{m}{n} = 0.\overline{a_1 a_2 a_3 \ldots a_i}$. Find the cycle of $\frac{1}{23}$.

[3 ♟] **Problem 3 (AMC 10A 2019/18)** For some positive integer k, the repeating base-k representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.232323\ldots_k$. What is k?

[4 ♞] **Problem 4 (e-dchen Mock MATHCOUNTS)** What is the sum of all odd n such that $\frac{1}{n}$ expressed in base 8 is a repeating decimal with period 4?

[6 ♟] **Problem 5 (AMC 12A 2014/23)** The fraction

$$\frac{1}{99^2} = 0.\overline{b_{n-1}b_{n-2}\ldots b_2 b_1 b_0},$$

where n is the length of the period of the repeating decimal expansion. What is the sum $b_0 + b_1 + \cdots + b_{n-1}$?

[6 ♟] **Problem 6 (AMC 12B 2016/22)** For a certain positive integer n less than 1000, the decimal equivalent of $\frac{1}{n}$ is $0.\overline{abcdef}$, a repeating decimal of period 6, and the decimal equivalent of $\frac{1}{n+6}$ is $0.\overline{wxyz}$, a repeating decimal of period 4. Find n.