Population languages generated by finite automata

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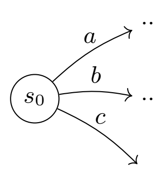
...

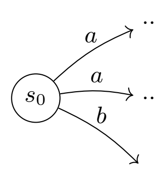
Abstract. We are interested in ...

1 Introduction

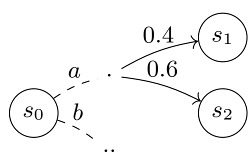
The literature offers several notions of a probabilistic automaton, arising from different interpretations of non-determinism. Let (S, Σ, δ) be the state space, alphabet and the transition function respectively. We summarise below several notions. Other interesting notions include the stochastic sequential machines [3], Markov decision process, interval Markov chains, see [6, 7] for early surveys on the topic.



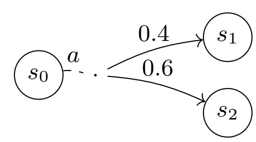
DFA,
$\delta : S \times \Sigma \to S$,
$\mathcal{L} : \Sigma^* \to S$



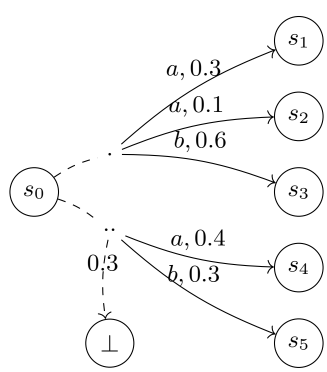
NFA,
$\delta : S \times \Sigma \to 2^S$,
$\mathcal{L} : \Sigma^* \to 2^S$



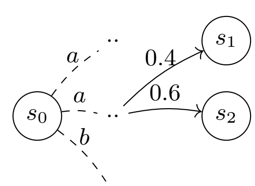
det. Rabin PA [4, 2],
$\delta : S \times \Sigma \to \mathcal{D}(S)$
$\mathcal{L} : \Sigma^* \to \mathcal{D}(S)$



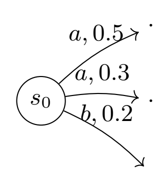
LMC, [1]
$\delta : S \to (\mathcal{D}(S) \to \Sigma)$
$\mathcal{L} : \Sigma^* \to \mathcal{D}(S)$



Segala-type automaton
$\delta \subseteq S \times \mathcal{D}(\Sigma \times S \oplus \{\bot\})$
$\mathcal{L} : \Sigma^* \to 2^{\mathcal{D}(S)}$



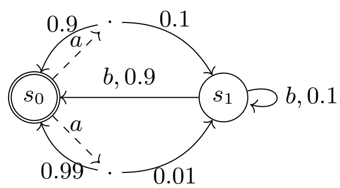
Simple Segala or non-det. Rabin [5]
$\delta : S \times \Sigma \to 2^{\mathcal{D}(S)}$,
$\mathcal{L} : \Sigma^* \to 2^{\mathcal{D}(S)}$



Fully prob. Segala [5],
$\delta : S \to \mathcal{D}(\Sigma \times S \oplus \{\bot\})$,
$\mathcal{L} : \Sigma^* \to \mathcal{D}(S)$

After processing a word $w \in \Sigma^*$, it is included in the language if the acceptance criterion is met. In DFA or NFA, the word is accepted if the automaton can end processing the word in an accepting state, where $F \subseteq S$ are accepting states. In the case of PA without nondeterminism, processing the words ends with a distribution over states, so acceptance of a word is quantified through an acceptance probability μ_F. In the case of PA with nondeterminism (simple Segala or non-det. Rabin type), processing a word can end up in a set of distributions, and, assuming randomised adversaries for resolving non-det., the resulting acceptance is any convex combination of the reachable sets of distributions Tanja: which citation is most relevant here? .

Example 1.



In this nondet. PA, if the acceptance is a Dirac distribution centered in s_0, the probability of accepting words $(ab)^n$ is $\mathcal{L}((ab)^n)(s_0) = [0.81^n, 0.871^n]$ - a convex union of the two det. PA, each resolving the non-determinism for processing a at s_0. All other words are non-accepting. We assume that actions not enabled in s_0 or s_1 halt the execution. As belief-transformer (generator of a population language), this automaton has the probability of seeing b at position $2n$ in interval $[0.81^n, 0.871^n]$. A sensible definition of a union or a complement for such a language requires finite non-combinable unions of convex combinations - e.g. in the case of complement, capturing the intervals $[0, 0.81^n]$ and $[0.871^n, 1]$.

(ab)^n: Either 0.1*0.

We are interested in language-theoretic view of population semantics of PAs - the Boolean operations over the population languages, the respective automata constructions and minimisation.

2 Population automata

3 Boolean operations

4 Minimisation, bisimulation notions

Tanja: Here I have some results on the case of LMC - decidability for population semantics and NP-hardness for standard language semantics. Here a biological example is relevant (possibly also elsewhere, but here is most straightforward in my head).

5 Examples

References

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