Universität Konstanz

# Minimization of Weighted Automata

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# Introduction

In the last presentation . . .

- two models for stochastic dynamical systems were considered:
  Weighted Automata (WA) and Differential Equations (DE)

- an example for modelling a CRN's dynamics in both models was given:
  - DE Solving Chemical Master Equation
  - WA Monte Carlo CTMC

# Goals specified

1. Implement minimization algorithm for weighted automata [1]. ✓
2. Implement model reduction algorithm for ODEs [2].
3. Develop reproducible benchmarks
4. Write report including

# What has been done so far

- Software Requirement Specification & Software Design Document
- Random Basis Minimal WA Construction Algorithm by Kiefer/Schützenberger [1]
- Execution of example by Matlab script and hand
- Implementation of minimization & equivalence algorithm, interfaces, TUI, CLI, tests

# The Weighted Automaton Minimization Algorithm I

Weighted Automaton $A = (n, \Sigma, \alpha, \mu, \eta)$, where

- $n$ the number of states
- $\Sigma$ the input alphabet
- $\alpha$ the initial vector with a non-zero value for all starting states
- $\mu$ the set of transition matrices, one per input character
- $\eta$ the final vector with non-zero values for all ending states

Author claims $\mathcal{O}(log^2 n)$ runtime, but this is not correct as we will see later In the following slide we use the notion of a forward reduction, but the backwards reduction is analogous besides minor variations

# The Weighted Automaton Minimization Algorithm II

- Find a basis $F$ of the prefix space using random vectors $r_i$
  - Add the vectors of all prefix words up to length n together and multiply this vector by n different factors yielding $\{v_1, \dots, v_n\}$
  - Factors are derived by random vectors and structure of prefixes
  - Base is then the maximally linear independent subset of $\{\alpha, v_1, \dots, v_n\}$

- Use basis to do Schützenberger Construction [3]: $\overrightarrow{A} = (\overrightarrow{n}, \Sigma, \overrightarrow{\alpha}, \overrightarrow{\mu}, \overrightarrow{\eta})$ With
  - $\overrightarrow{\mu} = \overrightarrow{F} \mu \overrightarrow{F}^{-1}$
  - $\overrightarrow{\alpha} = e_1$
  - $\overrightarrow{\eta} = \overrightarrow{F} \eta$
  - $\overrightarrow{n} = \text{rank}(\overrightarrow{\mu})$

# The Weighted Automaton Minimization Algorithm: Pseudo Code

b

# The Weighted Automaton Minimization Algorithm: Example

c

# Implementation Details

d

# Up Next

e

# Bibliography


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