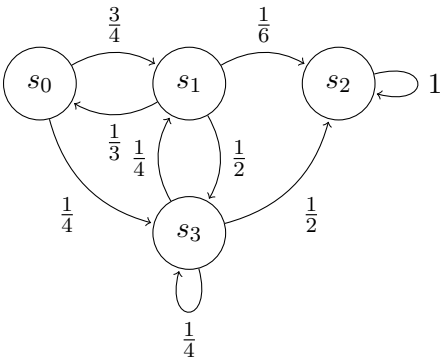
Exercise 1: Probabilities in DTMCs (4 points)

Consider the following DTMC.



a) Compute the probability of going from s_0 to s_3 in *exactly* 3 steps.

b) Compute the probability of being in state s_3 after exactly 3 steps assuming a uniform initial distribution (over all states).

c) Compute the limiting probability of being in state s_3.

d) Compute the probability of going from s_0 to s_3 in *at most* 3 steps.

e) Compute the probability of reaching (without a bound on the number of steps) s_3 when starting in s_0.

Exercise 2: Duelling Cowboys (3 points)

We consider the following scenario.

Three Cowboys: "The Good" (G), "The Bad" (B), and "The Ugly" (U) meet each other in the desert for a famous duel.

- The three may shoot as long as anyone else is still alive. Due to differences in (re)loading times, we assume that they shoot in turns. That is, The Good shoots first, then The Bad and finally The Ugly.
- The Good has a chance of a half hitting anyone. If he hits, he does so uniformly over the living contestants.

- The Bad has a chance of 0.9 of hitting anyone. If The Ugly is alive, then he aims for him with probability p. If The Ugly is already dead, then he surely aims at The Good.
- The Ugly hits The Good with a chance of q. If he does not hit The Good or The Good is already dead, he hits The Bad.

a) For certain values of p and q, this scenario is a stochastic process. For which values?

b) Depict a DTMC for this process. Please indicate for each state (i) who is alive and (ii) whose turn it is.

Exercise 3: The Gruffalo game (3 points)

A mouse enters the forest and wants to find a way out. Towards the exit, he needs to take 30 steps, distributed in nine ordered blocks of different sizes. The sizes are $4, 3, 3, 4, 3, 2, 5, 3, 3$ respectively. It can proceed if the 6-sided dice shows 1, 2 or 3 mice, by taking the corresponding number of steps forward. If the dice shows a fox (owl or snake resp.), the fox (owl or snake resp.) enters the forest. Once four foxes (owls or snakes resp.) are in the forest, the mouse goes one step back, but it does not move if the previous step is in a different block. If four foxes, four owls and four snakes are in the forest, the game is over. If the mouse manages to exit the forest, the players win.

Think of describing this game with a DTMC.

a) Depict the states of the game after the first, the second and the third roll of the die.

b) How many states does it have?

Use several paths, dont wri