8/11/2018
Prob. Modelling for Computer Scientists
Exercise/Lecture Notes
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Notation $\Diamond \sim F$ (Finally / Eventually)
$\Box \sim G$ (Globally)

① Repetition of slides of previous lecture

PROOF
② $\Box\Diamond S' = \{ s_0 s_1 s_2 \dots \mid \forall i \geq 0 \; \exists j \geq i \;\; s_j \in S' \}$ is measurable

$\Diamond\Box S' = \{ s_0 s_1 s_2 \mid \exists i \geq 0 \; \forall j \geq i \;\; s_j \in S' \}$

$= \bigcup_{i=0}^{\infty} \left(\bigcap_{j=1}^{\infty} \{ s_0 s_1 \dots s_j \dots \mid s_j \in S' \} \right) \; \Box$

$Cyl(\underbrace{* * * *}_{\in S \setminus S'} s_j)$ $\in S'$

set G is noted by S' to not correlate with Global operator

F is noted by $\hat{S}$ to not correlate with Future operator

$S_?$?

③ Reachability computation →
→ I. Multiply Transition Matrix
II. Recursive computation $\left(\sum P(s,t) \cdot x_t + \sum P(s,u) \right)$
III. Linear eq. solution + pruning

PRUNING $S = \hat{S} \cup S' \cup S_?$

④ Constrained reachability $\overset{I.}{\to}$ III. with pruning the states from $\hat{S}$
$Pr(\hat{S} \cup S')$

with I. Transient computations $P^n(s,t) \Leftrightarrow Pr(X_{m+n} = t \mid X_m = s) = P$

$\hookrightarrow Pr^D(\Diamond^{\leq n} S') \cdot \overset{D[S]}{Pr}(\Diamond^{=n} S') = I \cdot$ something $= /$ something $<$ SOMETHING
↳ initial distr.

⑤ quiz (ask Matej if you did not know answer)
⑥ try assignment 3 Excercise 1

① Solution for original HW AS 2 exercise 1 — 13/11/2018
Show that two sets of paths are measurable ⟺ you are able to compute $\dots$

a) the set of path in state in state s_{init} and remaining forever in states from A
b) remaining forever in states A and passing through a state B after exactly 5 steps

$a) = \Box A = \{ s_0 s_1 s_2 \dots \mid \forall i \; s_i \in A \} = \Diamond\bar{A} \Rightarrow Pr(\overline{\Diamond\bar{A}}) = 1 - Pr(\Diamond\bar{A})$

b) =

Recall $\Diamond G = \{ s_0 s_1 \dots \mid \exists i \; s_i \in G \} = Cyl(s_0) \uplus Cyl(s_0 s_1) \dots$
$\bigcup_{k=0}^{\infty} \{ cylinder(s_0 \dots s_k) \mid s_0 \dots s_{k-1} \notin G, s_k \in G \}$

$Pr(\Diamond\bar{A}) = \sum_{k=0}^{\infty} Pr(Cylinder(\overset{\in A}{s_0 \dots} \overset{\notin A}{s_k})) \; \Box$

$\Diamond^{=5} B \cup \bigcup_{k=0}^{\infty} \{ Cyl(s_0 \dots s_k) \mid s_5 \in B, s_0, \dots s_{k-1} \notin A, s_k \in A \} \; \Box$? TB looked at

Assignment 2 Exer 2.
~~Example for b1~~

a) $P^{(3)}_{03} = Pr(X_3 = s_3 \mid X_0 = s_0) = P^3_{03}$ TRANSIENT

$= Pr(\Diamond^{=3}\{s_3\}) = Pr(\{Cyl(q_0, q_1, q_2, q_3) \mid q_0 = s_0, q_3 = s_3\})$

b) $\Pi_0 = [1/4, 1/4, 1/4, 1/4]$

$Pr(X_3 = s_3 \mid X_0 \sim \Pi_0) = \Pi_0 P^{(3)}$

! In PRISM add ~~a~~ dummy states s.t. $\forall s \in S \; P(dummy, s) = 1/4 \wedge$? $= Pr(X_4 = s_4 \mid X_0 = dummy)$

c) $\lim_{n \to \infty} Pr(X_n = s_3) = 0$

$\hat{\pi} P = \hat{\pi}$

$P^T \hat{\pi}^T = \hat{\pi}^T$ transposed

d) $Pr(\{\Diamond^{\leq 3} s_3\}) = Pr(\{s_0 s_3, s_0 s_1 s_3, s_0 s_1 s_0 s_3\}) = \frac{3}{4} + \frac{1}{4}\cdot\frac{1}{6} + \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{3}{4} = \frac{85}{96} \doteq 0{,}8854$

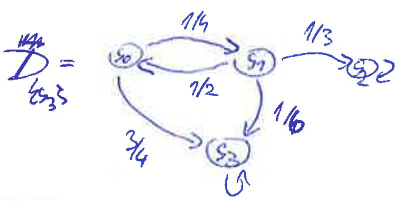
in 1 step, in 2 steps, in 3 steps

Recall D = ORIGINAL

Theorem by Example TRANSFORMATION

$Pr(\Diamond^{\leq 3}_D \{s_3\}) = Pr(\Diamond^{=3}_{D_{\{s_3\}}} \{s_3\})$

$D_{\{s_3\}} =$



e) $Pr(\Diamond\{s_3\})$

$S_0 = \{s_2\}$ states that does not reach s_3 $\quad X_{s_2} = 0$

$S_1 = \{s_3\}$ states that reach s_3 $\quad X_{s_3} = 1$

$X_{s_0} = \frac{3}{4} + \frac{1}{4} X_{s_1}$

$X_{s_1} = \frac{1}{6} + \frac{1}{2} X_{s_0}$

↓

$X_{s_1} = \frac{1}{6} + \frac{1}{2}\cdot\left(\frac{3}{4} \cdot \frac{1}{4} s_1\right)$

$X_{s_1} = \frac{1}{6} + \frac{3}{8} + \frac{1}{8} s_1$

$\frac{7}{8} s_1 = \frac{13}{24} \qquad s_1 = \frac{8}{7} \cdot \frac{13}{24} = \frac{13}{21}$ WHAT?

$X_{s_0} = \frac{3}{4} + \frac{1}{4} \cdot \frac{13}{21} = \frac{19}{21}$ □

also for iterative m.

$$\underbrace{\begin{bmatrix} X_{s_0} \\ X_{s_1} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 & 1/4 \\ 1/2 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} X_{s_0} \\ X_{s_1} \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 3/4 \\ 1/6 \end{bmatrix}}_{b}$$

$(A - I)x + b$

$$0 = \begin{bmatrix} -1 & 1/4 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} X_{s_0} \\ X_{s_1} \end{bmatrix} + \begin{bmatrix} 3/4 \\ 1/6 \end{bmatrix}$$

by complement

$Pr(\Box \overline{\{s_3\}}) = Pr(\Box\{s_0, s_1, s_2\}) =$

$\sum_{k=0}^{\infty} Cyl\left(s_0 (s_1 s_0)^k s_1 s_2\right) =$

$\sum_{k=0}^{\infty} \frac{1}{4}\cdot\frac{1}{3}\cdot\left(\frac{1}{2}\cdot\frac{1}{4}\right)^k = \frac{1}{12} \cdot \frac{1}{1 - 1/8} = \frac{2}{21}$ □

$X_{s_0} = \frac{3}{4} + \frac{1}{4}\cdot\left(\frac{1}{6} + \frac{1}{2} X_{s_0}\right) = \frac{3}{4} + \frac{1}{24} + \frac{1}{8} s_0$

$s_0 = \frac{8}{7} \cdot \left(\frac{3 \cdot 24 + 4}{24 \cdot 4}\right) = \frac{76}{7 \cdot 12} = \frac{19}{7 \cdot 3} = \frac{19}{21}$ □