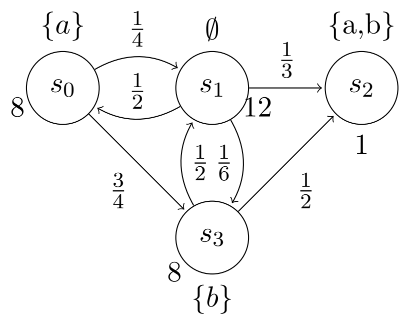
Assignment 5

Exercise 1 **6 points**

Exercise 2 **6 points**



$\mathcal{C} = (S, P, r, i_0, AP, L)$

$S = (s_0, s_1, s_2, s_3)$

$r = \{s_0 \mapsto 8, s_1 \mapsto 12, s_2 \mapsto 1, s_3 \mapsto 8\}$

$i_0 = \{s_1 \mapsto 1, s_0 \mapsto 0, s_2 \mapsto 0, s_3 \mapsto 0\}$

$P = \{(s_0, s_1) \mapsto \frac{1}{4}, (s_0, s_3) \mapsto \frac{3}{4}, \ldots\}$

$AP = \{a, b\}$

$L = \{s_0 \mapsto \{a\}, s_1 \mapsto \emptyset, s_2 \mapsto \{a, b\}, s_3 \mapsto s_3\}$

For $s, s' \in S$, use notation $R(s, s') := r(s) \cdot P(s, s')$,

a) Draw the embedded DTMC.

b) Draw the uniformized CTMC $unif(\mathcal{C}, \hat{r})$ for $\hat{r} = 10$.

c) Draw the uniformized CTMC $unif(\mathcal{C}, \hat{r})$ for $\hat{r} = 12$.

d) Write the generator matrix $\mathbf{Q} := \mathbf{R} - \mathbf{r}$

$$r := \begin{bmatrix} r(s_1) & 0 & . & . & 0 \\ 0 & r(s_2) & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & 0 & r(s_n) \end{bmatrix} \tag{1}$$

e) Compute $P_r(X_{10} = s_2)$ and $P_r(X_{100} = s_2)$ by using

- the flux-balance equations ($\frac{d}{dt}\mathbf{p}(t) = \mathbf{p}(t) \cdot \mathbf{Q}$)

- uniformization method, up to error $\epsilon = 0.001$.

f) Compute $P(\Diamond a), P(\Diamond^{\leq 100} a), P(\bar{b} U a), P(\bar{b} U^{\leq 100} a)$ (where U stands for "UNTIL", introduced in the lecture notes)

g) Compute $\lim_{t\to\infty} P_r(X_t = s_2)$. (Hint: look at the uniformized chain)

h) Use PRISM to do the computations above. Do you get the same results?

Exercise 2 (CTMC Modelling) **6 points**

We consider a server system consisting of 2 servers and a queue of capacity 1. Jobs arrive with a rate λ, and are scheduled to any available server. If no server is available, they are put in the queue. The servers handle the jobs with rate μ. If done, a job from the queue is taken, if there is one. Otherwise, the server is idle.

a) Model this system as a CTMC and compute the expected number of jobs in 100 time units for $\lambda = 1, \mu = 2$. How does this change if you add one more state to the queue? Use can use PRISM for computation.

b) In three server systems with jobs arriving at rates λ_1, λ_2, λ_3, the first two are damaged, hence all jobs are redistributed to the third server. What is the rate of jobs arrival to the third server after the damage? What is the expected waiting time to start handling a job before and after the damage?

Exercise 3 (CTMC Modelling of biochemical reaction networks) **6 points**

In the model in figure below (also available on slides 26/27 from lecture notes under the name $'mbps2015\_lec13.pdf'$), of the chain for for enzyme-catalysed substrate conversion (also known as the Michaelis-Menten scheme),

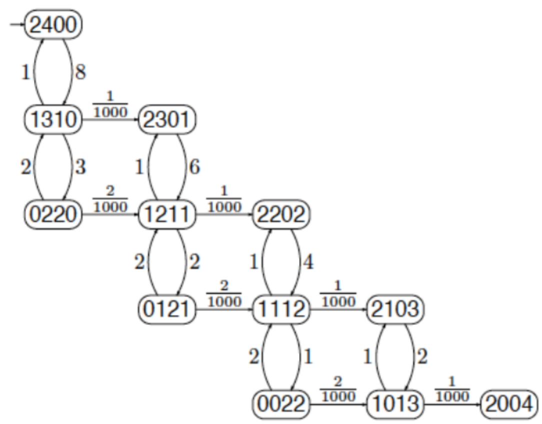
a) Compute the probability of having more than 2 products after $t = 10s$.

b) Compute the probability of having more than 20 products, for initially 20 enzymes and 40 substrates.

c) Compute the expected product number, for initially 20 enzymes and 40 substrates.

Use PRISM wherever convenient.

Enzyme-catalyzed substrate conversion as a CTMC



States:	*init*	*goal*
enzymes	2	2
substrates	4	0
complex	0	0
products	0	4

Transitions: $E + S \underset{1}{\overset{1}{\rightleftharpoons}} C \xrightarrow{0.001} E + P$

e.g., $(x_E, x_S, x_C, x_P) \xrightarrow{0.001 \cdot x_C} (x_E + 1, x_S, x_C - 1, x_P + 1)$ for $x_C > 0$

(Note on the course: Adapted from Prof. Joost-Pieter Katoen's course titled "Modelling and Verification of Probabilistic Systems", available at http://moves.rwth-aachen.de/teachings/ws-1516/movep15/.)