PROBABILISTIC AUTOMATA

R. G. Bukharaev

UDC 519.713.6

This paper is a survey of the most important results in the theory of probabilistic automata obtained from the start of the development of the theory through 1974. References are given to the basic papers on the topics discussed.

Introduction

This paper contains a survey of the basic results in the theory of abstract probabilistic automata contained in papers published from 1963 through 1974. It can now be stated that interest in investigations in the area of the traditional theory of probabilistic automata has diminished. This is apparently explained by the fact that many important areas of the theory have been practically completed, while further progress involves major difficulties. On the other hand, the fact that a probabilistic automaton is equivalent in certain important aspects to a general linear automaton has caused a shift of interests to the side of the latter. New tendencies have arisen. One of these is the appearance of various generalizations of the concept of a probabilistic automaton and the languages they represent; a second tendency which is more substantial and promising is the growth of interest in comparative estimates of the complexity of probabilistic and deterministic algorithms.

A number of papers have been devoted to applications of the theory of probabilistic automata: modeling of behavior in random media, problems of group behavior, teaching problems, etc. Applied aspects of probabilistic automata are not considered in the survey. However, to a certain extent we do treat the theory of linear automata and the theory of probabilistic grammars and algorithms.

Books by Bukharaev [26], Lorents [102], Pospelov [138], Paz [321], and Böhling and Dittrich [207] have been published on probabilistic automata. The book of Lorents is written from the position of a constructive direction in mathematics. A few sections of constructive probability and set theory are presented, and problems of stability (see Sec. 2) and economy of states of probabilistic automata as well as structural synthesis are investigated. The book of Pospelov tends toward the engineering point of view and is basically devoted to structure theory and questions of synthesis. The book of Paz is a type of textbook and contains a large collection of examples and exercises. In this book a great deal of attention is devoted to properties of stochastic matrices and the problem of stability of probabilistic automata. The book of Böhling and Dittrich is a reworking of lectures on stochastic automata given by the authors at the university in Bonn and treats all classical aspects of the theory (the third part of a four semester course, the first two parts of which have already been published). The monograph of Bukharaev is concerned with the abstract theory of probabilistic automata and is basically devoted to questions of the representability of multicycle channels and languages in finite probabilistic automata and to the equivalence and homomorphism of probabilistic automata. In this book, in particular, the construction of sequences of characteristic numbers of stochastic matrices is studied, and the results are applied to the investigation of properties of probabilistic automata of particular form. The book [104] is a development of the idea of the book of Lorents [102] with regard to the solution of the problem of synthesis of stable generators of random codes. Among other general material we mention the collections [42, 159], and also the book of Starke [373], which contains a chapter devoted to a systematic exposition of the basic theorems of probabilistic automata; see also [308]. The review papers on the theory of probabilistic automata and its applications include the works [22, 30, 31, 131, 207, 215, 257, 294].

1. A Mathematical Model of a Probabilistic Automaton and Its Generalizations

A probabilistic automaton (with a finite number of states n) in the sense of Carlyle [212] is a finite system of $n \times n$ matrices with nonnegative elements $\langle M(y/x), x \in \mathfrak{X}, y \in \mathfrak{Y} \rangle$ depending on two parameters, where $\mathfrak{X}$ is the set of input symbols and $\mathfrak{Y}$ is the set of output symbols of the automaton with the condition that all the

Translated from Itogi Nauki i Tekhniki. Teoriya Veroyatnostei, Matematicheskaya Statistika, Teoreticheskaya Kibernetika, Vol. 15, pp. 79-122, 1978.

matrices $A(x) = \sum_{y \in \mathfrak{Y}} M(y/x)$ are stochastic. Rabin [332] has systematically studied a finite probabilistic automaton without output in a general form consisting of a finite system of stochastic $n \times n$ matrices $\langle A(x), x \in \mathfrak{X} \rangle$. The initial state of the automaton is either given directly or in the form of a probability distribution of initial states which is equivalent to prescribing an initial stochastic state vector $\bar{\mu}(e)$ (e is the empty word). The functioning of the probabilistic automaton then consists in generating a countable vector set

$$L_M = \{\bar{\mu}(q/p),\ p \in F_{\mathfrak{X}},\ q \in F_{\mathfrak{Y}},\ |p| = |q|\}$$

in the first case and an analogous set $L_A = \{\bar{\mu}(p),\ p \in F_{\mathfrak{X}}\}$ in the second case. Here $\bar{\mu}(q/p) = \bar{\mu}(e)M(q/p)$ and $\bar{\mu}(p) = \bar{\mu}(e)A(p)$, while $M(q/p) = M(y_1/x_1) \cdot M(y_2/x_2) \ldots M(y_s/x_s)$ if $p = x_1, x_2 \ldots x_s$ and $q = y_1, y_2 \ldots y_s$ [similarly for $A(p)$]. The most important problems of the theory are the characterization of dictionary functions $\tau_{M(q/p)}^{\bar{\mu}(e)} = \bar{\mu}(e)M(q/p)\bar{e}$ ($\bar{e}$ is a column vector of ones) and

$$\chi_A^{\bar{\mu}(e),\bar{n}_F}(p) = \bar{\mu}(e)\,A(p)\,\bar{n}_F$$

($\bar{n}_F$ is the solution column vector with ones at the sites with indices from the set F and remaining coordinates zero) which effectively mean, respectively, the probability of the reaction of the automaton with output word q to input word p and the probability of finding the state of the automaton in the set F after reaction to input word p.

Special types of a probabilistic automaton – an automaton with random reactions and a Markov automaton – have been considered by Schreider [189]. Probabilistic automata of Mealy type and Moore type were introduced and studied independently by Bukharaev [17] and Starke [367]. Possibilities of various types of probabilistic automata were studied by Salomaa [344, 345]; in particular, he introduced and investigated the possibilities of the representability of languages of p-adic automata [343]. Among other works containing different definitions of a mathematical model of a probabilistic automaton we mention [61, 113, 135, 234, 235, 258, 261, 263, 350, 365]. Levin considered multichannel probabilistic automata [93]. Knast [272] and Feichtinger [239] studied a probabilistic automaton with continuous time of functioning. Starke and Thiele introduced and studied in detail asynchronous probabilistic automata [375, 377, 378]. Various generalizations of the model of a probabilistic automaton are also considered in [119, 171, 202, 228, 248, 315, 364, 393, 399]. The theory of general linear automata, which also represent a generalization of probabilistic automata, was next intensively developed. A general linear automaton is a finite system of $n \times n$ matrices $\langle M(x), x \in \mathscr{X} \rangle$ augmented by an initial row vector $\bar{\lambda}(e)$. Both the matrices $M(x)$ and the row vector $\bar{\lambda}(e)$ have arbitrary real components. The solution vector $\bar{f}$ also has arbitrary real components. Its functioning is formally defined in analogy with the functioning of the probabilistic automaton of Rabin. They were introduced by Turakainen [393, 398] (for more details see Sec. 2). Various "nonautonomous" generalizations of probabilistic automata have appeared since the beginning of the seventies. Models of a probabilistic automaton over trees introduced by Magidor and Moran [291] and later by Ellis [231], the model of a probabilistic automaton with a storing memory considered by Komiya Noriaki [275], and the two-sided probabilistic automata suggested by Kuklin [80] have an intermediate character. The last are natural generalizations of corresponding deterministic analog.

A probabilistic automaton over trees is defined as follows. Pairs (V, Σ) of finite dichotomous trees E and weight functions $V: E \to \Sigma$ define the input of the automaton. Displacement of the automaton along the tree (e.g., "upward") is determined by means of a function $r: E \to S$ and a probability measure $M(r(x), \sigma, r(x0), r(x1)) \to [0, 1]$, $\sum_{r_1, r_2} M(r, \sigma, r_1, r_2) = 1$, $\sigma \in \Sigma$, where x is a path in the tree. The automaton admits the tree if the total probability of passing along the tree and being in a set of distinguished states $F \subset S$ is greater than a constant λ.

The first definition of a probabilistic machine is found in the work of de Leeuw, Moore, Shannon, and Shapiro [95] in 1955. The definition of a probabilistic Turing machine was given by Santos [347], Fu and Li [244], and Salomaa [346], and later Santos [352] and Knast [274] introduced the definitions and systematically studied probabilistic grammars. It should be noted that both the definitions and the results of the papers mentioned on probabilistic grammars are natural and anticipated generalizations of their deterministic analogs (see Sec. 6). To conclude this section, we note the work of Trakhtenberg [160], Agafonov and Barzdin' [2, 11], and Gill [256] in which various modifications of probabilistic Turing machines are used to estimate the complexity of computations on probabilistic machines.

2. Representability of Languages and Dictionary Functions in Finite Probabilistic Automata

A language S is representable in a finite probabilistic Rabin automaton A by a set of states F (by solution vector $\bar{n}_F$), initial state vector $\bar{\mu}(e)$, and constant λ, $0 < \lambda \leq 1$, if the following condition is satisfied:

$$(p)\{p \in F_{\mathfrak{X}} \rightarrow \chi_A^{\bar{\mu}(e),\, \bar{n}_F}(p) > \lambda \sim p \in S\}.$$

A language S is representable in a finite probabilistic Mealy automaton M by an output letter y, an initial state vector $\bar{\mu}(e)$, and constant λ, $0 < \lambda \leq 1$, if the following condition is satisfied:

$$(px)\{px \in F_{\mathfrak{X}} \rightarrow \bar{\mu}(p) M(y/x) \bar{e} > \lambda \sim p \in S\}.$$

These definitions are equivalent [31] up to the representability of the empty word in the language. Rabin showed [332] that the class of languages representable in finite probabilistic automata (stochastic languages) is broader than the class of regular languages. Namely, he proved that the set of stochastic languages has the power of the continuum.

Rabin's proof has the character of an existence theorem and does not give a constructive example of a nonregular stochastic language. Such an example was constructed by Turakainen for languages in a single-letter alphabet [390] (see also [271, 342]). Salomaa [342] showed that the class of single-letter stochastic languages also has the power of the continuum. The first example of a language not representable in a finite probabilistic automaton was constructed by Bukharaev [18]. He proposed several criteria that languages be stochastic [18]. In the work [7, 87, 300, 330] whole series of examples of nonstochastic languages were constructed. In [7] a general procedure was given for constructing nonstochastic languages which encompasses most of the known examples. Another approach to constructing nonstochastic languages which is different in principle was applied by Lapin'sh [87]. A continual family of nonstochastic languages was constructed in [31].

The question arose of the existence of algebras over the set of stochastic languages, and, in particular, of the closedness of the class of stochastic languages relative to set-theoretic operations. The following results have been obtained. The class of stochastic languages is closed relative to intersection and union with a regular language [32], but, in general, it is not closed with respect to these operations [87]. The same goes for the operation of concatenation of languages even with respect to the pair of stochastic and regular languages [300]. Among other properties of the class of stochastic languages, it is known that it is closed under the operation of inverse of languages [396] but is not closed with respect to concatenation and homomorphism [395, 400].

It is of interest here to compare analogous results for stochastic dictionary functions. The interpretation of a dictionary function χ: $F_{\mathfrak{X}} \rightarrow [0, 1]$ as a fuzzy language was introduced by Zadeh [412]. Nasu and Honda clarified a number of interesting properties of the closedness of the class of stochastic dictionary functions (a probabilistic event in the language of Nasu and Honda) [299]. A numerical dictionary function $\chi(p)$ is stochastic if there is a finite probabilistic automaton, an initial state vector $\bar{\mu}(e)$, and a solution vector $\bar{n}_F$ such that $\chi(p) \equiv \chi_A^{\bar{\mu}(e),\, \bar{n}_F}(p)$, $p \in F_{\mathfrak{X}}$.

For example, the class of stochastic dictionary functions is closed under the operation of augmentation defined as $\bar{\chi}(p) = 1 - \chi(p)$, the product operation, the operation of taking **stochastic linear combinations** $\chi(p) = \sum_{i=1}^{n} \alpha_i \lambda_i(p)$, $\alpha_i \geqslant 0$, $\sum_{i=1}^{n} \alpha_i = 1$, and the operation of inversion of the argument. Nasu and Honda distinguished a number of classes of stochastic dictionary functions which are closed under the operations of union, intersection, and augmentation.

It should be noted that the stochastic condition imposed on dictionary functions is very restrictive in the construction of algebras of such functions. At the same time, for dictionary functions representable as general linear automata [398] this question can be solved in a natural way. It has been proved that the class of dictionary functions representable as finite-dimensional linear automata forms an algebra analogous to the Kleene algebra of regular languages relative to the system of operations:

1. Multiplication by a constant: $(\alpha f)(p) = \alpha f(p)$.
2. Addition: $(f + g)(p) = f(p) + g(p)$.
3. **Multiplication:** $(f \cdot g)(p) = \sum_{p_1 p_2 = p} f(p_1) g(p_2)$.

4. Iteration: If $f(e) = 0$, then $f^* = 1 + f + f^2 + \ldots$ and the elementary functions

$$f_0 : (p)\{p \in F_{\mathfrak{X}} \to f_0(p) = 0\},$$

$$f_1 : (p)\{p \in F_{\mathfrak{X}} \to f_1(p) = 1\},$$

$$f_x : f_x(p) = \begin{cases} 1, & \text{if } p = x, \\ 0, & \text{if } p \neq x, \end{cases} \quad x \in \mathfrak{X}.$$

The closure of the set of elementary functions under operations 1-4 also forms the family of all functions representable as finite-dimensional linear automata (Schützenberger, see [126]). At the same time, there is a simple connection between these two classes. Let $f(p)$ be represented as an n-dimensional linear automaton. Then there exists a probabilistic automaton with $n + 5$ states such that

$$\chi(p) = \alpha^{|p|} f(p) + \frac{1}{n+5}, \quad p \in F_{\mathfrak{X}},$$

where α is a positive number and $|p|$ is the length of the word p. This theorem, which was proved by Al'pin [31], is a modification of the result of Turakainen, who proved that the class of stochastic languages coincides with classes of languages representable as finite-dimensional general linear automata [398]. The representability of languages in general linear automata is determined in a manner similar to the way this is done for probabilistic automata with the difference that a cut point of η can be any real number.

Regarding the relation of the class of stochastic languages to certain other known classes of languages the following is known. The example of Bukharaev of a nonstochastic language is a primitively recursive language [21, 31]. On the other hand, Rabin has constructed a probabilistic automaton which represents both generally recursive and not generally recursive languages depending on the computability and noncomputability of the constant λ [31]. There is an example of a nonstochastic context-free language and at the same time an example of a stochastic context-independent language [330]. Lorents constructed an example of a finite probabilistic automaton and a cut point λ such that the language representable in this automaton is not effectively transferable [101]. Fu and Li showed that if representability of languages is defined by the condition $p \in S \sim \chi(p) > \varphi(p)$, where $\varphi(p)$ is an arbitrary dictionary function with values in (0, 1), then the class of languages representable in finite probabilistic automata in this sense coincides with the class of stochastic languages [244]. These same authors considered the representability of languages based on the principal of maximal verisimilitude. A word p is MV-representable in a probabilistic automaton M if and only if the state a is such that $\overline{\chi}_M^{\mu, a}(p) = \max_{b \in \mathfrak{A}} \chi_M^{\overline{\mu}, b}$ belongs to F. It is proved that the class of MV-representable languages occupies an intermediate position between the classes of regular and stochastic languages. Conditions were studied under which a finite probabilistic automaton represents a regular language. Rabin [332] posed the problem, and he proved the following reduction theorem. A cut point λ, $0 < \lambda \leq 1$, is called isolated for a given probabilistic automaton A if there is a positive constant δ, $\delta > 0$, such that for any word p, $p \in F_{\mathfrak{X}}$, the condition $|\chi(p) - \lambda| > \delta$ is satisfied. If a finite probabilistic automaton with n states A represents a language S with isolated cut point λ, then this language is regular, and the number of states l of a minimal deterministic automaton representing this language satisfies the inequality

$$l \leq \left(1 + \frac{1}{\delta}\right)^{n-1}.$$

This result occasioned attempts to describe necessary and sufficient conditions for the representability by a finite probabilistic automaton of regular languages. A geometric interpretation of a necessary and sufficient condition for regularity of a stochastic language was obtained in [26]. A sufficient condition that a cut point for regular (a natural generalization of regular Markov chains) probabilistic automata be isolated is also formulated there. Not long ago Bertoni [203] proved that there does not exist an algorithm which for an arbitrary given rational probabilistic automaton and an arbitrary rational cut point makes it possible to determine if the cut point is rational. The work [134, 285] is devoted to conditions that the set $\{\chi(p),\ p \in F\mathscr{X}\}$ be dense in (0, 1).

A generalization of the reduction theorem of Rabin to deterministic automata with a countable number of states is obtained in [32]. The significance of this result lies in the topological-metric interpretation of the hypotheses of the reduction theorem, thanks to which possibilities of broad generalizations of it arise.

Paz proved that if the set of transition matrices of an initial Mealy probabilistic automaton $\{M(q(p)),\ |p| = |q|\}$ is finite, then such an automaton represents by output letter a regular language [325].

Starke and Turakainen studied nonstandard forms of the representability of languages in finite probabilistic automata with the symbol > in the condition $\{p \in S \sim \chi(p) > \lambda\}$ replaced by the symbols $\geq, <, \leq, =$ [368, 397].

We denote the corresponding classes of languages by L(>), L(≥), etc., and by $L_{rat}(>)$ etc., the corresponding classes of languages representable in finite probabilistic automata with rationally given elements of the transition matrices, vectors, and cut point. There are the proper inclusions: {Class of regular languages} $\subset L_{rat}(=) \subset L(>)$. Further,

$$L_{rat}(\geqslant) = L_{rat}(>) = L_{rat}(<) = L_{rat}(\leqslant).$$

It is not known if these relations are true in the general case.

Definite languages are a subclass of the class of stochastic languages. A language S is called definite if for some integer k whether a word p, $|p| \geq k$, belongs to the language S is determined by whether the word p', where $p = \tilde{p}p'$ and $|p'| = k$, belongs to this language. Rabin proved that probabilistic automata with strictly positive elements of the transition matrices (actual automata) and with isolated cut point are definite languages [332]. A number of papers are devoted to the generalization and refinement of conditions that a stochastic language be definite [45, 78, 97, 123, 321, 322, 381]. Some other special subclasses of stochastic languages have been considered by Salomaa [343, 344], Starke [367], and Bukharaev [26]. In connection with the definition of definite languages and actual automata, Rabin [332] formulated the problem of stability of probabilistic automata which was then developed in a number of investigations. Let A be an actual automaton, and let λ be an isolated point. There exists ε, ε > 0, such that for each automaton A' with transition probabilities which differ from the transition probabilities of the automaton A by less than ε and with λ as cut point the automaton A' with this λ represents the same language as the automaton A. Kochkarev and Ritter subsequently considered the stability problem. Kochkarev introduced the concept of partial stability of a probabilistic automaton for a language with respect to which the point λ remains an isolated cut point, and he proved a corresponding generalization of the stability theorem [75, 77].

An interesting fact emerges in the proof: If A is a probabilistic automaton and R is a language such that all the matrices A(p), p ∈ R, are regular stochastic matrices, then the language R is regular. On the basis of this, it is possible to construct classes of regular languages with respect to which a given probabilistic automaton is stable. Ritter introduced a measure of the strong stability of a probabilistic automaton as the maximum number of linear independent stable perturbations, where a stable perturbation is a vector $\bar{\xi}$ with coordinates having sum zero which when added to some row of one of the transition matrices of the automaton gives a probabilistic automaton equivalent to the original one. It was proved that for a probabilistic automaton without unattainable states the dimension of strong stability is not less than the number of surplus states of the automaton. For a minimal probabilistic automaton with n states (n ≥ 5) the upper bound of the dimension of strong stability is equal to n − 4, and this bound is best possible [338, 339]. Regarding the problem of stability of probabilistic automata see also [99, 105, 127, 240].

Paz [326] posed the following problem: Of what type are deterministic devices less powerful than Turing machines which are capable of separating any pair of languages which are separable by a finite probabilistic automaton with computable parameters? In particular, are automata with stored memory sufficient for this? Kosaraju gave a negative answer to the last question [276].

We return to criteria for the representability of languages and dictionary functions in finite probabilistic automata. Since the class of stochastic languages is a set with the cardinality of the continuum, this criterion must contain continual choice. We shall present some forms of a criterion obtained in [18, 21, 31]. Let S be a language, and let q be the projection S_q of this language determined by the condition $(p)\{p \in F_{\mathfrak{X}} \rightarrow qp \in S \sim p \in S_q\}$. Then in order that the language S be stochastic it is necessary and sufficient that all its q-projections for all words q of particular length (e.g., all x-projections, $x \in \mathscr{X}$) be stochastic [25].

If it is assumed that the index function of a probabilistic automaton takes not only the values 0 and 1 but any real values, then the stochastic dictionary functions which can be represented by such automata can be characterized as follows. Let E be the space of all real-valued dictionary functions with the naturally defined operations of addition and multiplication by a number. In E we define linear p translations by the condition $f \rightarrow f_p$, where f_p is a function such that $(q) f_p(q) = f(pq)$. We denote by $E_f \leq E$ the space spanned by the set $\{f_p, p \in F_{\mathfrak{X}}\}$. In order that f be representable by a probabilistic automaton it is necessary and sufficient that in E_f there exists a polygon with vertex which passes to the interior of the polygon under p translations.

The next result to a certain extent also characterizes stochastic dictionary functions. A (normalized) probabilistic language over a set T of terminal symbols is a system $\hat{L} = (L, \mu)$, where L is a language over T and μ is a (probability) measure on the set L. Ellis [231] proved that each complete (i.e.,

$$(q)(q \in Q) \rightarrow \exists b \{(b \in T) \& M(q, b) \neq \varnothing\})$$

automaton admitting trees (see Sec. 2) admits a normalized probabilistic language.

Rabin pointed out that the potentially broader possibilities of finite probabilistic automata with respect to representability of languages as compared with deterministic automata cannot be utilized in practice [332]. Nevertheless, probabilistic recognition of regular languages may be more economical than deterministic recognition with regard to the number of states of the automaton required. Rabin proved that there exists a probabilistic automaton with precisely two states and a sequence λ_n, $1 \leq n < \infty$, of isolated cut points such that for each n a deterministic automaton B_n with the fewest number of states representing a stochastic, regular language S_n has at least n states. The work [45, 100] is also devoted to the problem of economy of states in the probabilistic representation of regular languages.

3. Autonomous Properties of Multicycle Channels

A probabilistic automaton M unites a multicycle channel $\langle \mathfrak{X}, \mathfrak{Y}, \tau_M^{\bar{\mu}}(q/p) \rangle$. In this connection the question arises of the structure of multicycle channels united by finite probabilistic automata. The problem was posed by Carlyle [213], and he made considerable progress in its solution. Since

$$\tau_M^{\bar{\mu}}(qq'/pp') = \bar{\mu}(q/p)\cdot\bar{\tau}(q'/p'),$$

where $\bar{\tau}(q'/p') = M(q'/p')\bar{e}$ for the probabilistic automaton M, it follows that each $n \times n$ matrix of the form $P = (\tau^{\overline{M}}(q_i q_j'/p_i p_j'))$ for any $n \geq 1$ and equal word length $|p_i| = |q_i|$, $|p_j'| = |q_j'|$ can be represented in the form $P = GH$, where G and H are $n \times k$ and $k \times n$ rectangular matrices, and k is the number of states of the probabilistic automaton (the composite sequential matrix). The rank of a multicycle channel $r(\tau)$ is the maximal rank of the ranks of all composite sequential matrices P of the channel τ, and is ∞ if no such maximum exists.

For each multicycle channel τ with k states there is the decomposition

$$\tau(qq'/pp') = \sum_{i=1}^{r} a_i(q/p)\cdot\tau(q_i q'/p_i p'),$$

where $r \leq k$ and p_i, q_i, a_i are functions only of $\tau(q''/p'')$ for all p'', q'' of lengths not more than $2k - 1$. By applying this relation it is possible to compute recurrently the probabilities $\tau(q/p)$ for words of arbitrary length $|p| = |q|$, using only values of τ in the segment of pairs of words of length not greater than $2k - 1$. Carlyle proved also that if a multicycle channel $\tau(q/p)$ is of finite rank r, then it has a realization as a finite automaton in the class of pseudoprobabilistic automata (see the definition in Sec. 4).

A criterion that a multicycle channel be autonomous was described independently by Bukharaev [17] and Starke [367].

In order that a multicycle channel $\tau(q/p)$ be autonomous it is necessary and sufficient that the following conditions be satisfied:

1. $\tau(q/p) = 0$ if $|p| \neq |q|$;
2. $\tau(q_1/p_1) = 0$ and $|p_1| = |q_1|$ imply $\tau(q_1 q/p_1 p) = 0$;
3. The relation $\tau(q_1 q/p_1 p)/\tau(q_1/p_1) = \tau_{p_1, q_1}(q/p)$ is a conditional probability measure for fixed p_1, q_1, and $\tau(q_1/p_1) \neq 0$ defined for all words $p \in F_{\mathfrak{X}}$, $q \in F_{\mathfrak{Y}}$ [17].

Each conditional probability measure $\tau_{p_1, q_1}(q/p)$, $p_1 \in F_{\mathfrak{X}}$, $q_1 \in F_{\mathfrak{Y}}$ $|p_1| = |q_1|$ defines, in turn, an autonomous channel, the set of which constitutes the set of states of the channel $\tau(q/p)$. The following conditions are equivalent to those above:

1. $\tau(q/p) = 0$ if $|p| \neq |q|$;
2. $\sum_{y \in \mathfrak{Y}} \tau(qy/px) = \tau(q/p)$ for any words $|p| = |q|$, $x \in \mathfrak{X}$.

A necessary and sufficient condition for the representability of a multicycle channel $\tau(q/p)$ in a finite probabilistic automaton was obtained in [263] and independently in [31]. We introduce some enumeration of pairs of words (p, q) of the same length from the free semigroups $F_{\mathfrak{X}}$ and $F_{\mathfrak{Y}}$, respectively. In this case we may consider the pair (p, q) as the index of the (p, q)-th coordinate of the vector channel I in the countable linear vector space E^{∞} to be equal to $\tau(q/p)$. The concepts of a (stochastic) linear combination and matrix transformation extend naturally to vector channels I. In particular, suppose that the matrix $D(q'/p') = (d_{(p_1, q_1), (p_2, q_2)})$

of countable dimension has identity elements at the sites $p_2 = p'p_1$, $q_2 = q'q_1$ and zeros elsewhere; i.e., it realizes a "shift" (p', q') in the enumeration of the coordinates of the vector I. The set of channels Γ, $\Gamma \subset E^\infty$, we call stochastic if arbitrary (p', q')-shifts of its elements are nonnegative linear combinations of the elements of Γ. A support set of the set ω, $\omega \subset E^\infty$, is any set $\Gamma(\omega)$ such that any element of ω can be represented as a stochastic linear combination of elements of $\Gamma(\omega)$.

In order that an autonomous channel I be finitely autonomous it is necessary and sufficient that the support set of the set of states of the channel I be a finite stochastic set [27]. See also [263, 277, 405].

4. Homomorphism, Equivalence, Reduction, and Minimization of Probabilistic Automata

The definition of equivalence of probabilistic automata and the solution of the problem of minimization in the class of initially equivalent automata were presented by Carlyle [212]. We shall adhere, however, to considerations of expository convenience from the point of view of modern concepts rather than to chronology.

We identify a state vector $\bar{\mu}$ with a state a if $\bar{\mu}$ has a-coordinate equal to 1 (we even write $\bar{\mu} = a$ in this case).

Two state vectors $\bar{\mu}_1$ and $\bar{\mu}_2$ (states a_1 and a_2) of a probabilistic automaton M (or two probabilistic automata M_1 and M_2 with the same sets of input and output letters) are called equivalent if their dictionary functions coincide identically:

$$\tau_M^{\bar{\mu}_1}(q/p) \equiv \tau_M^{\bar{\mu}_2}(q/p)\,(\tau_M^{a_1}(q/p) \equiv \tau_M^{a_2}(q/p)),$$
$$\tau_{M_1}^{\bar{\mu}_1}(q/p) \equiv \tau_{M_2}^{\bar{\mu}_2}(q/p)\,(\tau_{M_1}^{a_1}(q/p) \equiv \tau_{M_2}^{a_2}(q/p)),$$
$$p \in F_{\mathfrak{X}},\quad q \in F_{\mathfrak{Y}},\quad |p| = |q|.$$

We further use the terminology of the work [31], applying the term "initial equivalence" in place of the established term "equivalence," while reserving the term "weak equivalence" for equivalence of probabilistic automata not with respect to the channel $\tau(q/p)$ but with respect to the dictionary function $\chi(p)$ (see below). A probabilistic automaton M_1 is called initially equivalently (equivalently) imbedded in M_2, $M_1 \lesssim M_2$ (respectively, $M_1 \lessapprox M_2$), if for any state a_1 (respectively, state vector $\bar{\mu}_1$) there exists a state a_2 equivalent to it (a state vector $\bar{\mu}_2$) of the probabilistic automaton M_2; M_1 and M_2 are initially equivalent (equivalent), $M_1 \sim M_2$ (respectively, $M_1 \approx M_2$), if they are both initially equivalently (equivalently) imbedded in one another. It is obvious that $\lesssim \to \lessapprox$ and $\sim \to \approx$, and the reverse implications do not always hold.

We say a probabilistic automaton M is of Mealy type if the conditional probability measure of the automaton satisfies the condition $\mu(a', y/a, x) \equiv \mu(a'/a, x) \cdot \mu(y/a, x)$ and is of Moore type if $\mu(a', y/a, x) \equiv \mu(a'/a, x) \cdot \mu(y/a')$. Starke proved that for any probabilistic automaton there exists a probabilistic automaton of Moore type initially equivalent to it which can be chosen such that the deterministic properties of the transition functions and the finiteness of the sets $\mathfrak{X}$ and $\mathfrak{A}$ are preserved or such that the property of finiteness of the sets $\mathfrak{Y}$ and $\mathfrak{A}$ is preserved, while the labeling function $\mu(y/a')$ for each state a' is a deterministic function. The corresponding theorem for a probabilistic automaton of Mealy type does not hold. Starke presented an example of a probabilistic automaton which cannot be initially equivalently imbedded in any probabilistic automaton of Mealy type [373].

Carlyle calls a probabilistic automaton observable [214] if there exists a partial function $\delta: \mathfrak{A} \times \mathfrak{X} \times \mathfrak{Y} \to \mathfrak{A}$ such that $\{\mu(a', y/a, x) > 0\} \Leftrightarrow \{a' = \delta(a, x, y)$ is defined$\}$. For any probabilistic automaton M there exists an observable probabilistic automaton M^* such that $M \lesssim M^*$ and $M \approx M^*$. Here the probabilistic automaton M^* may be infinite even if M is finite.

Carlyle carried over a well-known result from the theory of deterministic automata to probabilistic automata. Let $|\mathfrak{A}| = n$. Then $\{\bar{\mu}_1 \sim \bar{\mu}_2\} \leftrightarrow (p)\ \{p \in \mathfrak{X}^{n-1} \Leftrightarrow \tau_M^{\bar{\mu}_1}(q/p) = \tau_M^{\bar{\mu}_2}(q/p)\}$. The corresponding result generalizes to state vectors belonging to different automata. The relations $\sim$ and $\lesssim$ for finite probabilistic automata and the relation $\sim$ for state vectors are algorithmically solvable.

A probabilistic automaton is called reduced (minimal) [200] if $a_1 \sim a_2$ implies $a_1 = a_2$ ($a \sim \bar{\mu}$ implies $\bar{\mu} = a$). A probabilistic automaton M' is called the reduced form of the probabilistic automaton M (the minimal form of M) if M' is reduced (respectively, minimal) and $M \sim M'$ (respectively, $M \approx M'$). Starke calls a probabilistic automaton M strongly reduced if $\bar{\mu}_1 \sim \bar{\mu}_2$ implies $\bar{\mu}_1 = \bar{\mu}_2$;, the strongly reduced form of a probabilistic automaton is defined similarly. Even [233] showed that the minimal (strongly reduced) properties are algorithmically solvable properties for finite probabilistic automata. Carlyle [212] (see also Nawrotzki [301]) proved that there always exists a reduced form M' of a probabilistic automaton M which for a finite automaton can be constructed while preserving the property of observability, the deterministic property of the transition functions, and the property of being an automaton of Mealy type. Carlyle's method of gluing together equivalent states of M (see below) is used in the proof; namely, the set of states of the reduced automaton is constructed as the factor set $\mathfrak{A}/_\sim$ with respect to the equivalence relation ~ and one sets

$$\mu'([a'],\ y/[a],\ x)=\sum_{a_1\in[a]} P_{[a]}(a_1)\cdot\sum_{a_2\in[a']}\mu(a_2,\ y/a_1,\ x),$$

where $[a]$ is the equivalence class of $\mathfrak{A}/_\sim$ containing a and $P_{[a]}$ is an arbitrary probability distribution over the class $[a]$. All reduced forms of M have the same number of states $(|\mathfrak{A}|)$. If M is a reduced probabilistic automaton and M ~ M', where M' is minimal (strongly reduced), then M is also minimal (respectively, strongly reduced).

In considering a probabilistic automaton up to equivalence, it is possible to further reduce its number of states. Ott [309, 310] and Paz [320] showed that there exist finite minimal probabilistic automata M for which M' has a fewer number of states and $M\lneqq M'$. For this it is necessary (but not sufficient) that M be strongly reduced.

Each minimal form of a probabilistic automaton is initially equivalently imbedded in it. If $M_1 \approx M_2$ and M_1, M_2 are both minimal, then $M_1 \sim M_2$. We note that if at least one minimal form of a probabilistic automaton M is strongly reduced, then all its minimal forms are strongly reduced.

Let $\mathfrak{A}_0\subseteq\mathfrak{A}$ be the set of all states $a\in\mathfrak{A}$ such that $a \sim \bar{\mu}$ for some undetermined state vector $\bar{\mu}$. A reduced probabilistic automaton M has a minimal form if and only if for each $a\in\mathfrak{A}_0$ there exists $\bar{\mu}_a$ such that $\bar{\mu}_a \sim a$ and $\bar{\mu}_a(\mathfrak{A}_0)=0$. Since the latter condition can always be satisfied for a finite $\mathfrak{A}_0$, in this case there always exists a minimal form of the probabilistic automaton which for finite $\mathfrak{A}$ can be effectively found (Bacon [200]). The relations $\approx$ and $\lneqq$ for a finite probabilistic automaton are algorithmically solvable.

Even [233] showed that deterministic probabilistic automata are reduced if and only if they are minimal. For probabilistic automata with deterministic transition functions this is not true in general.

Starke [373] presented an example of a reduced but not strongly reduced deterministic probabilistic automaton.

Bukharaev introduced the definition of a homomorphism of probabilistic automata and established the connection of this concept with the concept of equivalence [17, 26, 27, 31]. A probabilistic automaton M_1 with n_1 states is homomorphically mapped onto a probabilistic automaton M_2 with n_2 states, $n_1 \geq n_2$, if there exists an $n_2 \times n_1$ matrix H of rank n_2 having the sum of its row elements equal to one such that

$$M_1(y/x)H=HM_2(y/x),\quad x\in\mathfrak{X},\quad y\in\mathfrak{Y}.$$

Suppose that a probabilistic automaton M_1 is homomorphically mapped onto a probabilistic automaton M_2. Then automata M_1 and M_2 are equivalent, and the state vector $\bar{\mu}_1$ is equivalent to the state vector $\bar{\mu}_2 = \bar{\mu}_1 H$.* If here the matrix H consists only of zeros and ones, then the automata are initially equivalent.

In the theory of equivalence of probabilistic automata a major role is played by the linear vector space E_M spanned by the set of column vectors

$$L_M=\{\bar{\tau}_M(q/p),\ p\in F_{\mathfrak{X}},\ q\in F_{\mathfrak{Y}},\ |p|=|q|\}$$

and by the matrix N_M composed of the column vectors forming a basis in E_M. If probabilistic automata M_1 and M_2 are homomorphic, then the dimensions of the spaces E_{M_1} and E_{M_2} coincide. In order that two state vectors $\bar{\mu}_1$ and $\bar{\mu}_2$ of a probabilistic automaton M be equivalent, it is necessary and sufficient that the following matrix equation hold:

$$(\bar{\mu}_1-\bar{\mu}_2)N_M=\bar{0}.$$

This implies that in order that a probabilistic automaton not have noncoincident equivalent state vectors it is necessary and sufficient that this matrix equation have only the zero solution. Here the dimension of the space E_M is equal to the number of states of the automaton A [31]. On the basis of the concept of homomorphism Bukharaev formulated a necessary and sufficient condition for the equivalence of two probabilistic automata

*In the definitions and formulations of results on homomorphism and equivalence references to the admissible set of states $\mathfrak{Z}$ considered in [27, 31] are purposely omitted.

and described the class of all probabilistic automata equivalent to a given one [26, 31].* To describe the criterion it is necessary to introduce two new concepts. An object formally described and functioning like a probabilistic automaton but with the condition on the nonnegativity of the elements of the transition matrices removed we call a pseudoprobabilistic automaton. The process of minimization of a probabilistic automaton having equivalent state vectors can, in general, be reduced to a pseudoprobabilistic automaton. For each probabilistic automaton M we consider an operation of the form $\tilde{M} = DM$, where the stochastic matrix D is a solution of the matrix equation

$$DN_M = N_M.$$

The (pseudo-) probabilistic automata M and $\tilde{M}$ are equivalent. We call $\tilde{M}$ the canonical form of M.

In order that probabilistic automata M_1 and M_2 be equivalent it is necessary and sufficient that some canonical forms of these automata be homomorphically mapped onto the same, generally speaking, pseudoprobabilistic automaton [31]. Let M be a probabilistic automaton with n states, let T_1 and T_2 be two arbitrary stochastic $n \times n$ matrices, and let N_M be the basis matrix. In order that a system of $n \times n$ matrices $M'(y/x)$, $x \in \mathfrak{X}$, $y \in \mathfrak{Y}$, determine a probabilistic automaton equivalent to the given one it is necessary and sufficient that the following system of conditions be satisfied:

$$T_1 M(y/x) T_1^{-1} N = T_2 M'(y/x) T_2^{-1} N,$$
$$m'_{ij}(y/x) \geqslant 0, \; i, \; j = 1, 2, \ldots, n, \; x \in \mathfrak{X}, \; y \in \mathfrak{Y},$$
$$T_1 N_M = N.$$

Let M be a probabilistic automaton with n mutually inequivalent states. Carlyle showed that in order that a system of $n \times n$ matrices $M'(y/x)$, $x \in \mathfrak{X}$, $y \in \mathfrak{Y}$, determine a probabilistic automaton initially equivalent to a given one it is necessary and sufficient that the conditions

$$P^{-1} M'(y/x) P N_M = M(y/x) N_M,$$
$$m'_{ij}(y/x) \geqslant 0, \; i, \; j = 1, 2, \ldots, n,$$

be satisfied, where P is an arbitrary matrix of permutations [212]. Bukharaev [31] noted that the theory of equivalence and homomorphism of probabilistic automata in regard to the representability of dictionary functions $\chi(p)$ (weak equivalence, initial weak equivalence) is constructed in complete analogy to the theory with regard to the representability of channels $\tau(q/p)$. In this case two state vectors $\bar{\mu}_1$ and $\bar{\mu}_2$ are weakly equivalent if

$$\chi_A^{\bar{\mu}_1, \bar{n}_F}(p) \equiv \chi_A^{\bar{\mu}_2, \bar{n}_F}(p), \; p \in F_{\mathfrak{X}},$$

and a probabilistic automaton A_1 is weakly homomorphically mapped onto a probabilistic automaton A_2 if there exists a suitable rectangular matrix H of full rank such that

$$A_1(x) H = H A_2(x), \quad x \in \mathfrak{X}.$$

The basis matrix N_A is taken in the linear vector space E_A spanned by the vector set $L_A = \{\bar{\mu}(p), p \in F_{\mathfrak{X}}\}$. Formulations of theorems are duplicated by transformation of the corresponding concepts.

We shall present a result pertaining to the problem of weak equivalence. Any probabilistic automaton of Rabin A is the weak-homomorphic image of a free automaton D with solution vector $\bar{n}(A)$ having p-th coordinate equal to the value of the dictionary function $\chi_A(p)$ [31].

We now consider the problem of the practical realization of methods of minimizing the number of states of finite probabilistic automata. Carlyle [212] proposed a computational process making it possible to check the successive minimization of the number of states of a probabilistic automaton under the condition that at least a pair of equivalent states is known. This "gluing method" of Carlyle consists in the following. Let M be a probabilistic automaton with n states having a pair of equivalent states, for example, a_1 and a_2. Then the system of matrices $M'(y/x)$, $x \in \mathfrak{X}$, $y \in \mathfrak{Y}$, obtained from the system of matrices $M(y/x)$, $x \in \mathfrak{X}$, $y \in \mathfrak{Y}$, by depleting a_1 from a row and column and replacing the column a_2 by the sum of the columns a_1 and a_2, defines a probabilistic automaton with n − 1 states which is initially equivalent to A. The problem of finding a minimal form of a finite probabilistic automaton was solved by Even [233] as the problem of distinguishing in the basis matrix N_M a minimal collection of rows such that any of its rows is a convex linear combination of the

*The class of probabilistic automata with the same number of states which are initially equivalent to a given one was described by Carlyle [212].

distinguished rows. In the work [85, 184] methods are proposed for minimizing the number of states of a finite probabilistic automaton based on analogies with known methods of minimizing deterministic automata but which, in general, do not lead to a reduced form. Among these papers related to the problem in question we mention [86, 129, 185, 221, 232, 246, 253, 302, 320, 371, 372, 374].

5. Structure Theory of Probabilistic Automata

The problem of structural decomposition of a probabilistic automaton with a deterministic output function in loop-free form was solved by Bacon [199] following methods of Hartmanis.* A partition π of the set of states of a probabilistic automaton possesses the substitution property if and only if each transition matrix M admits enlargement corresponding to the given partition. Two partitions π and τ of the set of states are independent if and only if

$$\sum_{j\in\pi_k\cap\tau_l} a_{ij}(x)=\left[\sum_{j\in\pi_k} a_{ij}(x)\right]\cdot\left[\sum_{j\in\tau_l} a_{ij}(x)\right]$$

for all subsets π_k and τ_l of partitions of π and τ and all $i\in\mathfrak{A}$, $x\in\mathfrak{X}$. A probabilistic automaton admits sequential (quasiseries) decomposition if there exist partitions π and τ and a partition φ^{π}, determined by a function φ, such that

1. π has the substitution property and $\varphi^{\pi} \geq \pi$;
2. $\pi\cdot\tau = 0$ and π, τ are independent;
3. $(\varphi^{\pi}\cdot\tau, \tau)$ form a partition [199].

The result also admits a generalized formulation for many partitions. A probabilistic automaton M admits parallel decomposition if there exist partitions π, τ and a partition φ^{π} determined by a function φ such that

1. π possesses the substitution property and $\varphi^{\pi} \geq \pi$;
2. $\pi\cdot\tau = 0$ and π, τ are independent;
3. τ possesses the substitution property [199].

This method of loop-free decomposition is generalized in [274, 414]. See also [9, 51, 53, 54, 60, 122, 292]. Giorgadze and Burshtein obtained statistical estimates of the number of probabilistic automata with rational elements of the transition matrices which admit decomposition in the sense of Bacon [194] or in the sense of decomposition of a deterministic automaton in the Davis – Chentsov representation of a probabilistic automation in certain classes of automata [174, 225]. Let n be the number of states, let p(n) be the number of input letters, and let l(n) be the common denominator of all transition probabilities of the automaton. Then under the condition $\lim\limits_{n\to\infty}\frac{p(n)\,l(n)}{\ln n}<1$ any prescribed probabilistic automaton of class $\{n, p(n), l(n)\}$ admits sequential decomposition asymptotically with probability one. Conversely, if the condition $\lim\limits_{n\to\infty}\frac{\ln n}{p(n)\,l(n)}=0$ is satisfied, sequential decomposition is impossible asymptotically for almost every automaton of this class. Similar results for parallel decomposition are given in [50]. Rotenberg applied the idea of enlargement to the asymptotic decomposition of a homogeneous Markov chain into a family of chains describing random walks in each ergodic set of the initial chain [145].

Another direction in the structure theory of probabilistic automata is connected with the work of Davis [225] who showed that a finite, homogeneous Markov chain can be realized as a finite, deterministic automaton with random input; viz., any stochastic n × n matrix A can be represented in the form of a stochastic linear combination of simple (deterministic – stochastic) matrices: $A=\sum_{i=1}^{N}\alpha_i C_i$ (the system of components α_1, $\alpha_2, \ldots, \alpha_N$ forms an implication vector [23]). This implies the possibility of representing a finite probabilistic automaton as a sequential composition of the control generator of random codes without memory and a finite deterministic automaton as has been noted by many authors [114, 118, 174, 186, 224, 409]. In this aspect of the problem the problem of constructing a minimal implication vector is important.

*J. Hartmanis, "Loop-free structure of sequential machines," Inf. Control, 5, 25-43 (1962).

Closely related to this direction there is the method of synthesis of stochastic vectors, matrices, and probabilistic automata proposed by El-Choroury and Gupta [229]. In [23] the problem of obtaining an implication vector $\tilde{\alpha}$ of a given stochastic matrix A is considered as a "realization" problem with complexity equal to the number of nonzero components of $\bar{\alpha}$, and results are obtained which are qualitatively different from the results in the problem of obtaining minimal forms of the realization of functions of algebraic logic in the class of contact schemes.*

Problems of synthesis of probabilistic automata from basic probabilistic elements have been considered in the work of a number of Georgian mathematicians [108, 109, 110, 152, 155, 157, 158].

Skhirtladze proved that for any binary rational number p of the interval (0, 1), which in the binary representation contains n significant orders after the comma, it is possible to construct a contact scheme having conductance with probability p from contacts having conductance with probability 1/2, and for this n such contacts suffice [155]. In [158] the method was generalized to multipole schemes. Makarevich and Giorgadze proved that with special modeling of finite probabilistic automata of Rabin with rational elements of the transition matrices by means of networks of logical-probabilistic elements a basis of the form $\{\&, \vee, \sim$, delay element d, $\gamma\}$ is complete [110]. Here γ is some elementary probabilistic automaton of Moore type with two states, two input letters, and two output letters which realizes probability 1/2. In this modeling a random time lag T occurs with mathematical expectation which depends on the length of the input word and has the estimate

$$k(\log_2 k+3) < E(T) < 2n(\log_2 k+3).$$

Makarevich showed that under the assumption that the logical-probabilistic elements have a finite basis (finiteness of the set of elementary probabilities "stipulated" by the basis is essential) the latter can be complete only for a realization of rational finite probabilistic automata with random, integral time lag [108-109]. Paz proved that any probabilistic automata with n states can be realized as a network of certain, specially combined n – 1 probabilistic automata with two states (whirl, combination) [327]. Zech [415] demonstrated the completeness of the basis of elements $\{\neg$, stochastic $\&, \vee\}$

The possibility of representing a probabilistic automaton as a sequential composition of a control generator of random codes and a deterministic automaton led to the appearance of methods of minimization [85] and decomposition [81, 82] of a probabilistic automaton based on direct utilization of the corresponding methods for deterministic automata. On the other hand, interest arose in the problem of synthesis of generators of random codes. In [16] the problem was posed of synthesizing a control generator of random codes; necessary and sufficient conditions were obtained for the existence of a solution, and a general method of synthesis was proposed. See also [177, 362]. A method of approximate synthesis of an arbitrary Bernoulli distribution using the aforementioned structural realization of an autonomous probabilistic automaton (a regular, homogeneous Markov chain) and its limiting properties was suggested by Gill [254]. This direction was further developed in [88, 104, 288] on the basis of the use of the regularity property in the concept of a bistochastic circulant and its generalizations introduced by Lorents and Metra.

Regarding methods of practical synthesis of probabilistic automata see [1, 139, 156, 162, 168, 197, 247, 249, 270, 363]. An algebraic structure theory of probabilistic automata was suggested by Santos in [350]. See also [280, 308].

A structural approach to the theory of probabilistic automata is developed in the book of Pospelov [138] where known methods of synthesis and reduction of probabilistic automata are also presented.

In conclusion, we mention an approach which stands somewhat apart from our approach regarding the definition of the basic object and the methodology of investigation: the structural theory of probabilistic transformers in which probability (more correctly, frequency) plays the role of a carrier of information, while this basis consists completely of discrete deterministic elements. The first work in this direction was that of Geints [47]. See also [335]. In our country at the present time there is a rather extensive literature on this topic, an idea of which can be gained from the work of Kir'yanov [71].

6. Probabilistic Grammars and Algorithms

An A-machine [95] is defined as a sequential computable function defined on all initial sequences of some sequence A of zeros and ones. If the sequence is determined by a random pickup with probability p of

*S. V. Yablonskii, "On algorithmic difficulties in the synthesis of minimal contact schemes," Probl. Kibern., No. 2 (1959).

generating one, then we obtain a p-machine. A set S is strongly p-enumerable if there exists a p-machine producing (to any order) this set of symbols with positive probability. S is called p-enumerable if S is the set of those symbols which any p-machine produces at least once with probability >1/2. The main result of the paper [95] is that if A_p is a sequence defined by the binary expansion of the number p, then the following three propositions are equivalent:

1. S is A_p-enumerable;
2. S is p-enumerable;
3. S is strongly p-enumerable.

The conclusion is that if p is a computable real number, then the set S can only be recursively enumerable. On the other hand, there exist nonrecursively enumerable p and strongly p-enumerable sets for noncomputable values of p. The definition of a probabilistic Turing machine is due to Santos [347] and is a natural generalization of the definition of a deterministic Turing machine. A k-place random function is a function $\Phi(m_1, m_2, \ldots, m_k, m)$ of integral arguments mapping N^{k+1} into [0, 1] and satisfying the relation $\sum_{m=0}^{\infty} \Phi(m_1, m_2, \ldots, m_k, m) \leq 1$ for each group of values of the remaining arguments; the numbers on the tape of the machine are represented, as usual, by sequences of ones separated by a special symbol. A k-place random function Φ is considered computable on a given probabilistic Turing machine if the probability of reworking the writing on the tape of the sequence $m_1, m_2, \ldots, m_k$ into the number m is equal to the appropriate value. An integral k-place deterministic function f is probabilistically computable if there exists a probabilistic Turing machine Φ and a constant $\lambda \in (0, 1)$ such that $f(m_1, m_2, \ldots, m_k) = m$ if and only if

$$\Phi(m_1, m_2, \ldots m_k, m) = \sup\{\Phi_\delta(m_1, m_2, \ldots, m_k, m'): m' = 0, 1, 2 \ldots\} > \lambda.$$

The class of deterministically computable functions is a proper subclass of the class of probabilistically computable functions; in particular, it is uncountable [347].

Maslov showed that any function determined by a probabilistic Turing machine can be obtained from the function x + 1, 0 choice functions, and the probability distribution $\left(\frac{1}{2}, \frac{1}{2}, 0, \ldots\right)$ by means of operations of superposition, primitive recursion, and minimization (analogous to the corresponding operations in the deterministic case), and, conversely, each such function is determined by a probabilistic machine [116]. According to Salomaa, a probabilistic grammar of type i is defined as a triple $[G, \bar{\delta}, \varphi]$, where G is a deterministic grammar of type i, φ is a univalent mapping of the set of rules $\{f_1, f_2, \ldots, f_k\}$ of the grammar G into the set of all stochastic (rational) k-vectors, and $\bar{\delta}$ is a stochastic k-vector. To each outcome $D = f_{j_1} \cdot f_{j_2} \cdot \ldots \cdot f_{j_{r+1}}$ there corresponds a number $\psi(D)$, where for r = 0, $\psi(D) = [\delta]_{j_1}$ and

$$\psi(D) = \psi(D')[\varphi(f_{j_r})]_{j_{r+1}},$$

where D' is the start of an outcome of length r. A representable language is defined as the set of all words for which there exists an outcome such that $\psi(D) > \lambda \geq 0$ (pim languages) or such that $\sum_{D \in D_p} \psi(D) > \lambda \geqslant 0$ (pis languages). If the condition that φ and δ be stochastic is removed, then a weighted grammar is obtained and, correspondingly, the wim and wis languages. Each (rational) stochastic language is an (r)w3s language. Conversely, if a weighted (rational) grammar G_W does not contain rules of the form $x \to y$ (where x, y are nonterminal symbols), then a representable wis language is a (rational) stochastic language. Each p3m and p3s language is a language of type 3. Languages of type 3 are strictly contained in the set of all (r)w3m languages. The language $\{a^n b^n, n \geq 1\}$ is an (r)w3s language but is not an (r)w3m language. Moreover, each recursively enumerable set is an (r)p2m and (r)p2s language for a special interpretation of the grammar [346].

Knast [274] and Santos [352] define a probabilistic grammar as the natural generalization of a deterministic grammar of the corresponding type by introducing the concepts of probabilistic production as a conditional probability measure $\rho(\tau/\sigma)$ in place of $\sigma \to \tau$ in the deterministic case. A probabilistic grammar G_{st} is well defined and simple (unambiguous) if for each word of the language $L(G_{st}, 0)$ there is a unique sequence of productions which generates this word $pc\lambda_{G_{st}}(p) > 0$. Knast proved that a probabilistic language of type 3 is well defined if and only if it is stochastic. In the general case the class of stochastic languages L(M) is a proper subclass of the probabilistic languages of type $3L_{st}^3$. The class of probabilistic languages of type 3 forms a Boolean algebra with respect to the operations of intersection, taking sums, and complementation. If

L is a probabilistic, context-free language and $L \subset \Sigma^*$, then there exists Σ', a Dyck language $D \subset (\Sigma')^*$, and a homomorphism h: $(\Sigma')^* \to \Sigma^*$ such that for some probabilistic language $L_{st}^3 \subset (\Sigma')^*$ $L = h(D \cap L_{st}^3)$. The class of probabilistic context-free languages coincides with the class of probabilistic languages of type 3 [274]. Santos studied the connection of probabilistic grammars with asynchronous probabilistic automata, probabilistic Turing machines, and probabilistic automata with stored memory [347].

Another important direction in the theory of probabilistic machines is the comparative estimates of complexity of probabilistic and deterministic computations. Barzdin' showed that for any recursive function f there exists a recursive predicate Γ such that:

1. any deterministic computation of Γ requires for almost all values of the argument x no less than f(x) steps;
2. for any $\Delta < 1$ there exists a 1/2-machine M such that the probability of computing it at each value of the argument x is the value of the predicate $\Gamma(x)$, and for an infinitely large number of values of x the computation time $\tau_M(x)$ does not exceed $2|x|$, greater than Δ [11].

We define a time signaling relative to the predicate $\Gamma(x)$ for a probabilistic Turing machine $t_M(\Delta, x)$ as the smallest α such that

$$P\{M(x) = \Gamma(x) \& \tau_M(x) \leqslant \alpha\} > \Delta.$$

Trakhtenbrot [160] established that if a 1/2-machine computes the predicate Γ with reliability $\Delta \geq 1/2$, then there is a Turing machine N computing Γ such that

$$t_N(x) \leqslant 2^{t_M(\Delta, x)} \log^2 t_M(\Delta, x).$$

There exist languages which with reliability $\Delta > 1/2$ are separated on 1/2-machines in time $\log n$, while any Turing machine separating them works on the order of not less than n^2 steps [160].

Freivald [166] showed that for each $\varepsilon > 0$ there exists a probabilistic Turing machine which recognizes the symmetry of words in a binary alphabet with probability exceeding $1 - \varepsilon$ in time $c|p|\log^2|p|$, where c is a constant not depending on p (compare with the estimate for a deterministic Turing machine†). On the other hand, if some probabilistic Turing machine recognizes the symmetry of words in a binary alphabet with probability exceeding some $\Delta > 1/2$ in time t(p), then there exists a constant C such that for infinitely many words x

$$t(p) > C|p|\log_2|p|.$$

Gill [256] advanced the following conjecture: If f is a recursive function and it is computable on a probabilistic Turing machine with probability exceeding some $\Delta > 1/2$ in time t(p), then there exists also a deterministic Turing machine which computes f with a signaling time $\tilde{t}(p)$ such that for some constant $C > 0$ and infinitely many p, $\tilde{t}(p) \leq Ct(p)$. Freivald refuted this conjecture by presenting an example of a set, the recognition of which on a probabilistic machine with probability $1 + 0(1)$ requires time $C|p|\log\log|p|$, while any deterministic machine for all but a finite number of words requires time not less than $C|p|\log|p|$ [166]. See also [2, 72].

7. Other Questions

The problem of estimating the complexity of identification of a function of finite, homogeneous Markov chains was solved already in the work of Blackwell and Koopmans [205] and Gilbert [251].

Using similar methods, Carlyle showed that for the recognition of the difference of two state vectors of a probabilistic automaton with n states an unconditional experiment of length $n - 1$ is sufficient. The equivalence of two probabilistic automata is recognized by an unconditional experiment of length $n_1 + n_2 - 1$, where n_1 and n_2 are, respectively, the number of states of these automata [216]. Muchnik developed a powerful mathematical apparatus for estimating the complexity of experiments with general linear automata and, in particular, extended to them the results of Carlyle noted above [125]. Paz showed that to determine the connection of a probabilistic automaton with n states it is possible to construct an unconditional experiment of length $n - 1$ [321]. Santos has considered multiple experiments. He showed that for each probabilistic automaton M and some set of state vectors H it is possible to construct an unconditional diagnostic experiment of multiplicity no higher than $n - 1$ and of length no greater than $(1/2)n(n - 1)$. If the number of nonequivalent states in the automaton A is equal to k, then the estimates for an experiment recognizing these state vectors

†Ya. M. Barzdin', "The complexity of recognizing symmetry on Turing machines," Probl. Kibern., No. 15 (1965).

will be k and $(k-1)(n-1)$, respectively. Analogous results are also obtained for recognition by an automaton [353]. Makarevich and Matevosyan obtained an estimate of the length of an adjustable experiment which is equal to $l(T) < (\lambda + n)(m/e)^{\lambda+n}$. Here λ is the length of a simple input sequence, $m = |\mathfrak{X}|$, and ε is the smallest nonzero transition probability of the automaton [112]. A similar problem was considered in [314]. In the work mentioned above an experiment on a probabilistic automaton is understood in the abstract sense as the possibility of identification of objects on the basis of giving a finite system of data on the functioning of the mathematical model of the automaton (or automata). Rabin posed the problem of organizing a real experiment with a probabilistic automaton [332]. It was analyzed in detail by Böhme [208-210].

Since real experiments with probabilistic automata are of statistical nature, the actual solution of experiment problems is obtained with a certain error. To obtain estimates of the expected error Böhme used methods of information theory.

Barashko and Bogomolov considered problems of a probabilistic experiment on deterministic automata [10].

Lorents [102, 103] provided a constructive direction in the theory of probabilistic automata. The author notes that the theory of finite deterministic automata is constructive, and his objective is to preserve these constructive features in the theory of probabilistic automata. The first step in this direction is the assumption that the elements of the stochastic transition matrices are constructive real numbers. However, even with this assumption many primary procedures in the classical theory cannot be carried out. For example, it has been proved that an algorithm recognizing regular stochastic matrices is impossible. Thus, the circle of admissible means is restricted. Nevertheless, Lorents succeeded in proving constructive analogs of a number of important results after imposing the necessary restrictions. This pertains to the theorems on quasidefinite systems of matrices, results on the problem of stability of a probabilistic automaton, questions of economy of states in replacing a deterministic automaton by a probabilistic one, and the reduction theorem of Rabin.

In recent years interest has arisen in the study of the behavior of collectives of probabilistic automata, in the study of their behavior in random media, and, in particular, in the study of probabilistic automata with variable structure. Characteristic for most of the work is the experimental approach of modeling the corresponding game situations on a computer. Theoretical aspects of the problem have been studied by Valakh and Korolyuk [34, 35, 36], who investigated the optimal behavior of stochastic automata in media with various properties and by Varshavskii and Vorontsova [37, 38, 39]. Chandrasekaran and Shen [219], Chentsov [176], and Sawaragi Yoshkikazu and Baba Norio [198] investigated the learning problem for stochastic automata with variable structure. A survey on the problem can be found in [294]. See also [62, 68, 69, 119, 136, 161, 164, 165, 170, 176, 283, 284, 296, 297, 315].

LITERATURE CITED

1. G. A. Agasandyan and V. G. Sragovich, "On the structural synthesis of probabilistic automata," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 6, 121-125 (1971).
2. V. N. Agafonov and Ya. M. Barzdin', "On sets related to probabilistic machines," Z. Math. Log. Grundl. Math., 20, No. 6, 481-498 (1974).
3. G. P. Agibalov, "Recognition of operators realizable in linear autonomous automata," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 3, 99-108 (1970).
4. G. P. Agibalov and Ya. G. Yufat, "On simple experiments for linear initial automata," Avtom. Vychisl. Tekh., No. 2, 17-19 (1972).
5. Yu. A. Al'pin, "On decompositions produced by probabilistic automata," in: Probabilistic Automata and Their Applications [in Russian], Zinatne, Riga (1971), pp. 23-26.
6. Yu. A. Al'pin, "The condition of stability of probabilistic automata," in: Probabilistic Methods and Cybernetics, No. 9, Kazan. Univ. (1971), pp. 3-5.
7. Yu. A. Al'pin and R. G. Bukharaev, "On a sufficient condition for the nonrepresentability of languages in finite probabilistic automata," Dokl. Akad. Nauk SSSR, 223, No. 4 (1975).
8. G. L. Areshyan and G. B. Marandzhyan, "On some questions in the theory of probabilistic automata," Tr. Vychisl. Tsentra Akad. Nauk Arm. SSR Erevan. Univ., No. 2, 73-81 (1964).
9. Yu. M. Afanas'ev, A. M. Krysanov, and Yu. P. Letunov, "Sequential decomposition of probabilistic automata," Avtom. Telemekh., No. 3, 84-88 (1973).
10. A. S. Barashko and A. M. Bogomolov, "On experiments and automata with a source of random signals at input," Avtom. Vychisl. Tekh., No. 3, 6-14 (1969).

11. Ya. M. Barzdin', "On computability on probabilistic machines," Dokl. Akad. Nauk SSSR, 189, No. 4, 699-702 (1969).
12. O. S. Belokon', "Investigation of convergence processes in the simplest systems of probabilistic automata," Kibernetika, No. 1, 46-50 (1972).
13. O. S. Belokon', "Analysis of the structure of the companion matrix in a system of finite probabilistic automata," Akad. Nauk Ukr. SSR, Inst. Kibern. Sekts. Mat. Metody Issled. Optimiz. Sistem, Preprint 73-42, Kiev (1973).
14. A. M. Bogomolov and V. A. Tverdokhlebov, "On experiments with probabilistic automata," in: Kibernetika, Donetskoe Otd., Tr. Sem., No. 1, Kiev (1969), pp. 34-40.
15. R. G. Bukharaev, "On the imitation of probabilistic distributions," Uch. Zap. Kazan. Gos. Univ., 123, No. 6, 56-67 (1963).
16. R. G. Bukharaev, "On controllable generators of random variables," Uch. Zap. Kazan. Gos. Univ., 123, No. 6, 68-87 (1963).
17. R. G. Bukharaev, "Some equivalences in the theory of probabilistic automata," Uch. Zap. Kazan. Gos. Univ., 124, No. 2, 45-65 (1964).
18. R. G. Bukharaev, "A criterion for the representability of events in finite probabilistic automata," Dokl. Akad. Nauk SSSR, 164, No. 2, 289-291 (1965).
19. R. G. Bukharaev, "Automatic transformation of probabilistic sequences," Uch. Zap. Kazan. Gos. Univ., 125, No. 6, 24-33 (1966).
20. R. G. Bukharaev, "Two corrections to the paper 'Some equivalence in the theory of probabilistic automata,'" Uch. Zap. Kazan. Gos. Univ., 125, No. 6, 110 (1966).
21. R. G. Bukharaev, "On the representability of events in probabilistic automata," Uch. Zap. Kazan. Gos. Univ., 127, No. 3, 7-20 (1967).
22. R. G. Bukharaev, "The theory of probabilistic automata," Kibernetika, No. 2, 6-23 (1968).
23. R. G. Bukharaev, "On the problem of minimization of the input of a probabilistic automaton generating a homogeneous finite Markov chain," Uch. Zap. Kazan. Gos. Univ., 129, No. 4, 3-11 (1969).
24. R. G. Bukharaev, "Criteria for the representability of events in finite probabilistic automata," Dokl. Akad. Nauk SSSR, 164, No. 2, 289-291 (1965).
25. R. G. Bukharaev, "Criteria for the representability of events infinite probabilistic automata," Kibernetika, No. 1, 8-17 (1969).
26. R. G. Bukharaev, Probabilistic Automata [in Russian], Kazan State Univ. (1970).
27. R. G. Bukharaev, "The abstract theory of probabilistic automata," in: Probabilistic Automata and Their Applications [in Russian], Zinatne, Riga (1971), pp. 9-22.
28. R. G. Bukharaev, "Problems of synthesis of probabilistic transformers," in: Probabilistic Automata and Their Applications [in Russian], Zinatne, Riga (1971), pp. 61-75.
29. R. G. Bukharaev, "Automatic synthesis of a controllable generator of random codes," in: Probabilistic Automata and Their Applications [in Russian], Zinatne, Riga (1971), pp. 97-101.
30. R. G. Bukharaev, "Applied aspects of probabilistic automata," Avtom. Telemekh., No. 9, 76-86 (1972).
31. R. G. Bukharaev, "The theory of abstract probabilistic automata," in: Probl. Kibern., No. 30, Nauka, Moscow (1975), pp. 147-197.
32. R. G. Bukharaev and R. R. Bukharaev, "The topological method of reduction of automata," Izv. Vyssh. Uchebn. Zaved., Mat., No. 5, 31-39 (1974).
33. É. M. Vaisbrod and G. Sh. Rozenshtein, "On the 'life' time of stochastic automata," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 4, 52-59 (1965).
34. V. Ya. Valakh, "Optimization of the behavior of finite and stochastic automata in random media," in: Teor. Optimal'n. Reshenii. Tr. Sem., No. 3, Kiev (1967), pp. 3-29.
35. V. Ya. Valakh, "The time of residence of a stochastic automaton in the set of states with minimal penalty," in: Teor. Avtom. Metody Formal'nogo Sinteza Vychisl. Mashin Sistem. Tr. Sem., No. 7, Kiev (1969), pp. 62-75.
36. V. Ya. Valakh, "On the question of an optimal stochastic automaton in a composite medium," in: Teor. Optimal'n. Reshenii. Tr. Sem., No. 4, Kiev (1969), pp. 53-62.
37. V. I. Varshavskii and I. P. Vorontsova, "On the behavior of stochastic automata with variable structure," Avtom. Telemekh., 24, No. 33, 353-360 (1963).
38. V. I. Varshavskii and I. P. Vorontsova, "Stochastic automata with variable structure," in: Teor. Konechn. Veroyatn. Avtom., Nauka, Moscow (1965), pp. 301-308.
39. V. I. Varshavskii and I. P. Vorontsova, "The use of stochastic automata with variable structure for solving some problems of behavior," in: Samoobuchayushchiesya Avtomt. Sistemy, Nauka, Moscow (1966), pp. 158-164.

40. V. I. Varshavskii, I. P. Vorontsova, and M. L. Tsetlin, "Teaching stochastic automata," in: Biol. Aspekty Kibern., Akad. Nauk SSSR, Moscow (1962), pp. 192-197.
41. L. N. Vasershtein, "Markov processes on a countable product of spaces describing systems of automata," Probl. Peredachi Inf., 5, No. 3, 64-72 (1969).
42. Probabilistic Automata and Their Applications, Inst. Elektron. Vychisl. Tekh. Akad. Nauk Latv. SSR, Zinatne, Riga (1971).
43. G. A. Volovnik, "Construction of matrices of transition probabilities of finite automata with disturbances," in: Nadezhn. Effektivn. Diskretn. Sistem, Zinatne, Riga (1968), pp. 147-159.
44. I. P. Vorontsova, "Algorithms for altering the transition probabilities of stochastic automata," Probl. Peredachi Inf., 1, No. 3, 122-126 (1965).
45. N. Z. Gabbasov, "On the characteristics of events representable by finite probabilistic automata," Uch. Zap. Kazan. Gos. Univ., 130, No. 3, 18-27 (1970).
46. G. G. Galustov, G. M. Pozdnyakov, and V. A. Zimovnov, "Some results on modeling discerning probabilistic automata," in: Vopr. Tekhn. Diagnostiki, Taganrog (1975), pp. 42-54.
47. B. R. Geints, "The stochastic computing machine," Elektronika, No. 14 (1967).
48. M. Gessel and Kh. D. Mondrov, "On the question of processing random sequences by abstract automata," in: Diskretn. Sistemy, Vol. 4, Zinatne, Riga (1975), pp. 7-12.
49. A. Kh. Giorgadze, "A method of constructing transition matrices for an automaton with stochastic stop elements," Soobshch. Akad. Nauk Gruz. SSR, 54, No. 1, 49-52 (1969).
50. A. Kh. Giorgadze and L. V. Burshtein, "Stochastic estimates of the decomposition of probabilistic automata," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 1, 138-145 (1974).
51. A. Kh. Giorgadze and T. L. Dzhebashvili, "On the question of decomposition of probabilistic automata," Soobshch. Akad. Nauk Gruz. SSR, 76, No. 2, 321-323 (1974).
52. A. Kh. Giorgadze and V. P. Zelentsov, "Formulation of the problem of control of random processes," in: Metody Predstavleniya Apparaturn. Analiz Sluchainykh Protsessov Polei, Vol. 1, Novosibirsk (1969), pp. 75-78.
53. A. Kh. Giorgadze and A. G. Safiulina, "On iterative decomposition of finite probabilistic automata," Avtom. Telemekh., No. 9, 81-85 (1974).
54. A. Kh. Giorgadze and A. G. Safiulina, "Methods of decomposition of probabilistic automata," Avtom. Vychisl. Tekh., No. 5, 1-5 (1974).
55. A. Kh. Giorgadze and A. G. Safiulina, "On decomposition of probabilistic automata," Kibernetika, No. 2, 6-11 (1975).
56. V. S. Gladkii, "On the inversion of matrices on probabilistic automata," in: Veroyatn. Avtom. Primen., Zinatne, Riga (1971), pp. 131-141.
57. G. Glinskii, "Theoretical informational problems of the theory of reliable automata," in: Teor. Konechn. Veroyatn. Avtom., Nauka, Moscow (1965), pp. 280-300.
58. V. B. Golovchenko, "Self-organization of a collection of probabilistic automata with the two simplest behavioral 'motives,'" Avtom. Telemekh., No. 4, 151-156 (1974).
59. V. B. Golovchenko, "On the dynamics of the interacting probabilistic automata of Moore," in: Avtomatizir. Sistemy Upr. (ASUP) – Teoriya, Metodol. Modelirov., Tekhn. Sredstva, Irkutsk (1974), pp. 138-143.
60. V. A. Gorbatov, A. M. Krysanov, and Yu. P. Letunov, "Parallel decomposition of probabilistic automata," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 5, 112-120 (1972).
61. A. P. Goryashko, "The 'diffusion' model of the functioning of a probabilistic automaton," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 4, 133-136 (1972).
62. V. B. Grigorenko, A. N. Rapoport, and E. I. Ronin, "The investigation of learning systems realized in the form of probabilistic automata," Izv. Vyssh. Uchebn. Zaved., Radiofiz., 14, No. 7, 1026-1034 (1971).
63. Ya. A. Dubrov, "On the theory of noninitial probabilistic automata," in: Teor. Avtom. Metody Formalizovan. Sinteza Vychisl. Mashin Sistem. Tr. Sem., No. 5, Kiev (1969), pp. 33-39.
64. G. E. Zhuravlev and V. N. Veselov, "Investigation of a forgetting automaton," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 4, 118-126 (1970).
65. V. P. Zarovnyi, "On the theory of infinite linear and quasilinear automata," Kibernetika, No. 4, 5-17 (1971).
66. Yasuesi Inagava, "Probabilistic automata," Suri Kacheku, Math. Sci., 9, No. 8, 39-40, 42-47 (1971).
67. Yasuesi Inagava, "Probabilistic automata," Bull. Electrotech. Lab., 29, No. 6 (1965).
68. N. P. Kandelaki and G. N. Tsertsvadze, "On the behavior of certain classes of stochastic automata in random media," Avtom. Telemekh., 24, No. 6, 115-119 (1966).

69. N. P. Kandelaki and G. N. Tsertsvadze, "Solution of the problem of localization of the characteristic numbers of matrices of automata possessing asymptotically optimal behavior in stationary random media," Tr. Vychisl. Tsentra Akad. Nauk Gruz. SSR, 9, No. 1, 144-149 (1969).
70. A. K. Kel'mans, "On the coherence of probabilistic schemes," Avtom. Telemekh., No. 3, 98-116 (1967).
71. B. F. Kir'yanov, "Equivalence of systems realizing stochastic principles of computation," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 5, 121-128 (1972).
72. I. N. Kovalenko, "A remark on the complexity of representing events in probabilistic and deterministic automata," Kibernetika, No. 2, 35-36 (1965).
73. G. Ya. Kostromin, "On finding the impulse vector for a stochastic machine," Redkollegya Zh. Avtom. Vychisl. Tekh. Akad. Nauk Latv. SSR, Riga (1972).
74. B. S. Kochkarev, "On the question of stability of probabilistic automata," Uch. Zap. Kazan. Gos. Univ., 127, No. 3, 82-87 (1967).
75. B. S. Kochkarev, "On the stability of probabilistic automata," Kibernetika, No. 2, 24-30 (1968).
76. B. S. Kochkarev, "On verifying the feasibility of a sufficient condition for the stability of probabilistic automata," in: Teor. Avtom. Sem., No. 4, Akad. Nauk Ukr. SSR, Kiev (1967), pp. 79-90; also Kibernetika, No. 4 (1969).
77. B. S. Kochkarev, "On the partial stability of probabilistic automata," Dokl. Akad. Nauk SSSR, 182, No. 5, 1022-1025 (1968).
78. B. S. Kochkarev, "On a sufficient condition for the definiteness of an event represented by a probabilistic automaton," Uch. Zap. Kazan. Gos. Univ., 129, No. 4, 12-20 (1969).
79. A. I. Krysanov, "Algorithms for parallel decomposition of probabilistic automata," in: Ekon.-Mat. Metody Programmir. Plan. Ekon. Zadach, Moscow (1972), pp. 126-134.
80. Yu. I. Kuklin, "Two-sided probabilistic automata," Avtom. Vychisl. Tekh., No. 5, 35-36 (1973).
81. A. É. Kéévalik and G. É. Yakobson, "On a means of decomposing autonomous probabilistic automata," Tr. Tallin. Politekh. Inst., No. 350, 53-59 (1973).
82. A. É. Kéévalik and G. É. Yakobson, "A method of decomposing probabilistic automata," in: Diskretn. Sistemy, Vol. 4, Zinatne, Riga (1974), pp. 13-21.
83. V. G. Lazarev and V. M. Chentsov, "On the question of obtaining a reduced form of a stochastic automaton," in: Sintez Diskretn. Avtom. Upr. Ustroistv, Nauka, Moscow (1968).
84. V. G. Lazarev and V. M. Chentsov, "Use of stochastic automata for distributing information," in: Avtom., Gibridn. Upr. Mashiny, Nauka, Moscow (1972), pp. 66-72.
85. V. G. Lazarev and V. M. Chentsov, "On minimizing the number of internal states of a stochastic automaton," in: Sintez Diskretn. Avton. Upr. Ustroistv, Nauka, Moscow (1968), pp. 150-159.
86. Ya. K. Lapin'sh, "Minimization of probabilistic automata representing finite information media," Avtom. Vychisl. Tekh., No. 1, 7-9 (1973).
87. Ya. K. Lapin'sh, "On nonstochastic languages obtained as the union or intersection of stochastic languages," Avtom. Vychisl. Tekh., No. 4, 6-13 (1974).
88. Ya. K. Lapin'sh and I. A. Metra, "On a method of synthesis of transformers of probability distributions," Avtom. Vychisl. Tekh., No. 4, 32-37 (1973).
89. A. A. Larin, "Basic concepts of the theory of ciphered automata," in: Kibern. Avtom. Upr., Tr. Sem., No. 2, Kiev (1968), pp. 3-8.
90. A. A. Larin, "Basic concepts of the theory of probabilistic ciphered automata," in: Avtom., Gibridn. Upr. Mashiny, Nauka, Moscow (1972), pp. 59-65.
91. V. I. Levin, "Determination of the characteristics of probabilistic automata with reverse channels," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 3, 107-110 (1966).
92. V. I. Levin, "An operational method of studying probabilistic automata," Avtom. Vychisl. Tekh., No. 1, 18-25 (1967).
93. V. I. Levin, "Multichannel probabilistic automata," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 6, 63-68 (1968).
94. V. I. Levin, "Analysis of the reliability of inhomogeneous Markov automata," in: Nadezhn. Effektivn. Diskretn. Sistem, Zinatne, Riga (1968), pp. 141-145.
95. K. de Leeuw, E. F. Moore, K. E. Shannon, and N. Shapiro, "Computability on probabilistic machines," in: Avtomaty [Russian translation], IL, Moscow (1956), pp. 281-305.
96. A. A. Lorents, "Some questions in the constructive theory of finite probabilistic automata," Avtom. Vychisl. Tekh., No. 5, 57-80 (1967).

97. A. A. Lorents, "Generalized quasidefinite finite probabilistic automata and some algorithmic problems," Avtom. Vychisl. Tekh., No. 5, 1-8 (1968).
98. A. A. Lorents, "Questions of reducibility of finite probabilistic automata," Avtom. Vychisl. Tekh., No. 7, 4-13 (1969).
99. A. A. Lorents, "Synthesis of stable probabilistic automata," Avtom. Vychisl. Tekh., No. 4, 90-91 (1969).
100. A. A. Lorents, "Economy of states of finite probabilistic automata," Avtom. Vychisl. Tekh., No. 2, 1-9 (1969).
101. A. A. Lorents, "On the character of events representable in finite probabilistic automata," Avtom. Vychisl. Tekh., No. 3, 91-93 (1971).
102. A. A. Lorents, Elements of the Constructive Theory of Probabilistic Automata [in Russian], Zinatne, Riga (1972).
103. A. A. Lorents, "Problems in the constructive theory of probabilistic automata," in: Veroyatn. Avtom. Primen., Zinatne, Riga (1971), pp. 37-53.
104. A. A. Lorents, Synthesis of Reliable Probabilistic Automata [in Russian], Zinatne, Riga (1975).
105. A. A. Lorents, "On the stability of finite probabilistic automata," in: Teor. Konechn. Avtom. Ee Prilozhen., No. 3, Zinatne, Riga (1974), pp. 52-60.
106. L. V. Makarevich, "On attainability in probabilistic automata," Soobshch. Akad. Nauk Gruz. SSR, 53, No. 2, 293-296 (1969).
107. A. A. Lorents, "On the realizability of probability operators in logical networks," in: Diskretn. Analiz, No. 15, Novosibirsk (1969), pp. 35-36.
108. A. A. Lorents, "The problem of completeness in the structure theory of probabilistic automata," Kibernetika, No. 1, 17-30 (1971).
109. A. A. Lorents, "On the general approach to the structure theory of probabilistic automata," in: Veroyatn. Avtom. Primen., Zinatne, Riga (1971).
110. A. A. Lorents and A. Kh. Giorgadze, "On the question of the structure theory of probabilistic automata," Soobshch. Akad. Nauk Gruz. SSR, 50, No. 1, 37-42 (1968).
111. A. A. Lorents and A. A. Matevosyan, "The transformation of random sequences in automata," Avtom. Vychisl. Tekh., No. 5, 8-13 (1970).
112. A. A. Lorents and A. A. Matevosyan, "Adjustable experiments with finite probabilistic automata," Avtom. Telemekh., No. 8, 88-92 (1972).
113. A. A. Lorents and A. A. Matevosyan, "Ergodic automata," Soobshch. Akad. Nauk Gruz. SSR, 78, No. 2, 313-315 (1975).
114. S. V. Makarov, "On the realization of stochastic matrices by finite automata," in: Vychisl. Sistemy, No. 9, Novosibirsk (1963), pp. 65-70.
115. G. B. Marandzhyan, "On distinguishing the most probable trajectories of Markov chains," Sb. Nauchn. Tr. Erevan. Politekh. Inst., 24, 231-238 (1968).
116. A. N. Maslov, "Probabilistic Turing machines and recursive functions," Dokl. Akad. Nauk SSSR, 205, No. 5, 1018-1020 (1972).
117. A. A. Matevosyan, "On the universal source of P-ary random sequences," Inst. Kibern. Akad. Nauk Gruz. SSR, Tbilisi (1974).
118. L. P. Matyushkov, "On the realization of autonomous stochastic automata," Tr. I Resp. Konferentsii Matematikov Belorussii, 1964, Vysshaya Shkola, Minsk (1965), pp. 166-170.
119. Boyan Metev, "Some probabilistic automata with variable structure," Tr. Mezhdunar. Sem. Prikl. Aspektam Teor. Avtom., Vol. 2, Varna (1971), pp. 442-449.
120. I. A. Metra, "Comparison of the number of states of probabilistic and deterministic automata representing given events," Avtom. Vychisl. Tekh., No. 5, 94-96 (1971).
121. I. A. Metra, "A stochastic calculator," in: Veroyatn. Avtom. Primen., Zinatne, Riga (1971), pp. 33-36.
122. I. A. Metra, "On the enlargement of products of stochastic matrices," Avtom. Vychisl. Tekh., No. 3, 20 (1972).
123. I. A. Metra and A. A. Smilgais, "On the definiteness and regularity of events represented by probabilistic automata," Avtom. Vychisl. Tekh., No. 4, 1-7 (1968).
124. I. A. Metra and A. A. Smilgais, "On some possibilities of representing nonregular events by probabilistic automata," Latv. Mat. Ezhegodnik, No. 3, Zinatne, Riga (1968), pp. 253-261.
125. A. A. Muchnik, "General linear automata," Probl. Kibern., No. 23, 171-208 (1970).
126. A. A. Muchnik and A. N. Maslov, "Regular, linear, and probabilistic events," Tr. Mat. Inst. Akad. Nauk SSSR, 133, 149-168 (1973).

127. M. B. Nevel'son and R. Z. Khas'minskii, "On the stability of stochastic systems," Probl. Peredachi Inf., 2, No. 3, 76-91 (1966).
128. Yu. I. Neimark, V. P. Grigorenko, and A. N. Rapoport, "On optimization by independent deterministic and stochastic automata," in: Prikl. Mat. Kibernet. Mater. Vses. Mezhvuz. Simp. Prikl. Mat. Kibernt., Gorkii (1967), pp. 148-166.
129. N. Ya. Parshenkov and V. M. Chentsov, "On the question of minimization of a stochastic automaton," Probl. Peredachi Inf., 5, No. 4, 81-83 (1969).
130. N. Ya. Parshenkov and V. M. Chentsov, "The stability of internal states of a probabilistic automaton," Kibernetika, No. 6, 47-52 (1970).
131. N. Ya. Parshenkov and V. M. Chentsov, "On the theory of stochastic automata," in: Diskretn. Avtom. Seti Svyazi, Nauka, Moscow (1970, pp. 141-184.
132. N. Ya. Parshenkov and V. M. Chentsov, "Some questions in the theory of probabilistic automata," Tr. Mezhdunar. Simp. Prikl. Aspektam Teor. Avtom., Vol. 2,Varna (1971), pp. 454-463.
133. N. Ya. Parshenkov and V. M. Chentsov, "Questions in the theory of probabilistic automata," in: Avtom. Upr. Setyami Svyazi, Nauka, Moscow (1971), pp. 180-202.
134. K. M. Podnik, "On profile points of some probabilistic automata," Avtom. Vychisl. Tekh., No. 5, 90-91 (1970).
135. A. S. Pozdnyak, "Adaptive probabilistic automata," in: VI Vses. Soveshch. po Probl. Upr., 1974, Ref. Dokl. Ch. I., Nauka, Moscow (1974), pp. 57-61.
136. A. S. Pozdnyak, "Investigation of the convergence of algorithms of self-teaching stochastic automata," Avtom. Telemekh., No. 1, 88-103 (1975).
137. D. A. Pospelov, "On some problems of probabilistic logic," Tr. Mosk. Energ. Inst., No. 42, 153-159 (1962).
138. D. A. Pospelov, Probabilistic Automata [in Russian], Énergiya, Moscow (1970).
139. Val. P. Pyatkin, Vyach. P. Pyatkin, and A. K. Romanov, "On a problem of synthesis of probabilistic automata," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 4, 130-132 (1972).
140. L. A. Rastrigin and K. K. Ripa, "Statistical search as a probabilistic automaton," Avtom. Vychisl. Tekh., No. 1, 50-55 (1971).
141. K. K. Ripa, "Some stochastic properties of optimizing automata and random search," Avtom. Vychisl. Tekh., No. 3, 28-32 (1970).
142. K. K. Ripa, "Properties of a system of optimization by a collection of independent automata with random outputs," in: Probl. Sluchain. Poiska, No. 3, Zinatne, Riga (1974), pp. 27-41.
143. K. K. Ripa, "Algorithms of self-instruction in random search as probabilistic automata," in: Probl. Sluchain. Poiska, No. 2, Zinatne, Riga (1973), pp. 99-126.
144. V. N. Roginskii, "On a type of probabilistic discrete automata," Probl. Peredachi Inf., No. 17, 85-90 (1964).
145. A. R. Rotenberg, "Asymptotic enlargement of states of some stochastic automata," Probl. Peredachi Inf., 9, No. 4 (1974).
146. L. L. Rotkop, "Methods of investigating statistical automata of relay action under stochastic perturbations," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 4, 107-114 (1963).
147. I. V. Safonov, "On the question of determining transition probabilities," in: Teor. Avtom. Tr. Sem., No. 1, Kiev (1969), pp. 107-112.
148. É. M. Sil'vestrova, "Finite Markov stochastic automata with discrete time. I," Kibernetika, No. 3, 122-133 (1972).
149. É. M. Sil'vestrova, "Finite Markov stochastic automata with discrete time. II," Kibernetika, No. 4, 26-30 (1972).
150. É. M. Sil'vestrova, "On the optimal construction of probabilistic automata with alternative memory," in: Mat. Modeli Slozhn. Sistem, Kiev (1973), pp. 169-173.
151. V. G. Sragovich and Yu. A. Flerov, "On a class of stochastic automata," Izv. Akad. Nauk SSSR, Tekh. Kibern., No. 2, 66-73 (1965).
152. R. L. Skhirtladze, "On the synthesis of P-schemes from contacts with random discrete states," Soobshch. Akad. Nauk Gruz. SSSR, 26, No. 2, 181-186 (1961).
153. R. L. Skhirtladze, "Equalization of distributions of double random sequences by functions of the logic algebra," Soobshch. Akad. Nauk Gruz. SSR, 37, No. 1, 37-44 (1965).
154. R. L. Skhirtladze, "On the optimal equalization of distributions of Boolean random quantities," Soobshch. Akad. Nauk Gruz. SSR, 40, No. 3, 559-566 (1965).
155. R. L. Skhirtladze, "On a method of constructing Boolean quantities with a given probability distribution," in: Diskretn. Analiz, No. 7, Novosibirsk (1966), pp. 71-80.

156. R. L. Skhirtladze, "On a method of synthesis of a Markov automaton," Soobshch. Akad. Nauk Gruz. SSR, 55, No. 3, 549-552 (1969).
157. R. L. Skhirtladze, "Synthesis of probabilistic transformers in a code of 'diagonal' vectors," in: Issled. Nekotor. Vopr. Mat. Kibernt., Tbilisi Gos. Univ., Tbilisi (1973), pp. 81-86.
158. R. L. Skhirtladze and V. V. Chavchanidze, "On the question of synthesis of discrete stochastic devices," Soobshch. Akad. Nauk Gruz. SSR, 27, No. 5 (1961).
159. Theory of Finite and Probabilistic Automata, Tr. Mezhdunar. Simposiuma po Teorii Relein. Ustroistv Konechn. Avtomatov (IFAK), Nauka, Moscow (1965), p. 403.
160. B. A. Trakhtenbrot, "Remarks on the complexity of computations on probabilistic machines," in: Teor. Algoritmov Mat. Logiki, Vychisl. Tsentr Akad. Nauk SSSR, Moscow (1974), pp. 159-176.
161. E. S. Usachev, "A stochastic model of instruction and its properties," in: Issled. po Teorii Samonastraivayushch. Sistem, Vychisl. Tsentr Akad. Nauk SSSR, Moscow (1967), pp. 8-26.
162. E. S. Usachev, "On the realization of a stochastic model of an automaton," in: Issled. po Teorii Samonastraivayushch. Sistem, Vychisl. Tsentr Akad. Nauk SSSR, Moscow (1971), pp. 207-222.
163. T. Farago, "On a formulation of the prediction problem by means of a probabilistic automaton," in: Vychisl. Tekh. Vopr. Kibern., No. 10, Leningrad. Univ. (1974), pp. 46-61.
164. Yu. A. Flerov, "On results of stochastic automata," in: Issled. po Teorii Samonastraivayushch. Sistem, Vychisl. Tsentr Akad. Nauk SSSR, Moscow (1967), pp. 97-114.
165. Yu. A. Flerov, "The limiting behavior of a class of stochastic automata with variable structure," in: Vopr. Kibern., Adaptiv. Sistemy, Moscow (1974), pp. 140-145.
166. R. V. Freivald, "On comparison of the possibilities of probabilistic and frequency algorithms," in: Diskretn. Sistemy, Vol. 4, Zinatne, Riga (1974), pp. 280-287.
167. Fudzimato Sinti and Fukao Takesi, "Analysis of a probabilistic automaton," Byull. Elektr. Tekh. Lab., 30, No. 8 (1966).
168. G. N. Tsertsvadze, "Some properties and methods of synthesis of stochastic automata," Avtom. Telemekh., 24, No. 3, 341-352 (1963).
169. G. N. Tsertsvadze, "Stochastic automata and the problem of constructing reliable automata and their unreliable elements," Avtom. Telemekh., 25, No. 2, 213-226 (1964).
170. G. N. Tsertsvadze, "On stochastic automata asymptotically optimal in a random medium," Soobshch. Akad. Nauk Gruz. SSR, 43, No. 2, 433-438 (1966).
171. G. N. Tsertsvadze, "A stochastic automaton with a hysteresis tactic," Tbilisi Univ. Shromebi, Tr. Tbilisi Univ., 135, 57-61 (1970).
172. M. L. Tsetlin and S. L. Ginzburg, "On a construction of stochastic automata," Probl. Kibern., No. 20, 19-26 (1968).
173. V. M. Chentsov, "Synthesis of a stochastic automaton," in: Probl. Sinteza Tsifrovykh Avtomatov, No. 13, Nauka, Moscow (1967), pp. 135-144.
174. V. M. Chentsov, "On a method of synthesis of an autonomous stochastic automaton," Kibernetika, No. 3, 32-35 (1968).
175. V. M. Chentsov, "Synthesis of a stochastic automaton," in: Probl. Sinteza Tsifrovykh Avtomatov, Nauka, Moscow (1967), pp. 135-144.
176. V. M. Chentsov, "Investigation of the behavior or stochastic automata with variable structure," in: Inf. Seti Kommutatsiya, Nauka, Moscow (1968).
177. V. N. Chetverikov, E. A. Bakanovich, and A. V. Men'kov, "Investigations of controllable probabilistic elements and devices," in: Diskretn. Sistemy, Vol. 4, Zinatne, Riga (1974), pp. 57-66.
178. M. K. Chirkov, "On the analysis of probabilistic automata," in: Vychisl. Tekh. Vopr. Programmir., No. 4, Leningrad State Univ. (1965), pp. 100-103.
179. M. K. Chirkov, "Composition of probabilistic automata," in: Vychisl. Tekh. Vopr. Kibern., No. 5, Leningrad State Univ. (1968), pp. 31-59.
180. M. K. Chirkov, "Probabilistic automata and probabilistic mappings," in: Diskretn. Analiz, No. 7, Novosibirsk (1966), pp. 61-70.
181. M. K. Chirkov, "Equivalence of probabilistic finite automata," in: Vychisl. Tekh. Vopr. Kibern., No. 5, Leningrad State Univ. (1968), pp. 3-30.
182. M. K. Chirkov, "On probabilistic finite automata," in: Vychisl. Tekh. Vopr. Programmir. No. 3, Leningrad State Univ. (1964), pp. 44-57.
183. M. K. Chirkov, "Probabilistic problems of completely defining partial automata without memory," in: Vychisl. Tekh. Vopr. Kibern., No. 8, Leningrad State Univ. (1971), pp. 66-81.
184. M. K. Chirkov, "On minimization of probabilistic automata," in: Vychisl. Tekh. Vopr. Kibern., No. 9, Mosk. Gos. Univ., Moscow (1972), pp. 88-99.

185. M. K. Chirkov and Bui-Min Chi, "Reduced forms of partial probabilistic automata," in: Vychisl. Tekh. Vopr. Kibern., No. 11, Mosk. Gos. Univ., Moscow, pp. 80-93.
186. M. K. Chirkov and T. P. Shilkevich, "On the realizability of probabilistic automata by automata with random inputs," in: Metody Vychisl., No. 6, Leningrad State Univ. (1970), pp. 127-136.
187. Yu. I. Shmukler, "Theoretical informational estimates of the instruction process," in: Tekh. Kibern., Nauka, Moscow (1965), pp. 318-325.
188. Yu. I. Shmukler, "On the search for a conditional extremum by a probabilistic automaton," in: Issled. po Teorii Samonastraivayushch. Sistem, Vychisl. Tsentr Akad. Nauk SSSR, Moscow (1967), pp. 115-137.
189. Yu. A. Shreider, "Instruction models and control systems," in: Bush and Mosteller, Stochastic Models of Instructability [Russian translation], IL, Moscow (1962).
190. V. V. Yakovlev, "Stochastic functional transformers," Avtom. Vychisl. Tekh., No. 6, 25-28 (1973).
191. N. V. Yarovitskii, "Limiting behavior of a closed system of automata with random input," Kibernetika, No. 1, 57-61 (1965).
192. N. V. Yarovitskii, "Probabilistic autonomous modeling of discrete systems," Kibernetika, No. 5, 35-43 (1966).
193. N. V. Yarovitskii, "An existence theorem for ergodic distributions for a particular system of automata," in: Teor. Avtomatov, Sem., No. 1, Naukova Dumka, Kiev (1966), pp. 22-23.
194. A. Adam, "On stochastic truth functions," Coloq. Inf. Theory, Debrecen (1967), Abstracts, Budapest, pp. 1-2.
195. G. Adomian, "Linear stochastic operators," Rev. Mod. Phys., 35, No. 1, 185-207 (1963).
196. Tihomir Z. Aleksic, "On near optimal decomposition of stochastic matrices," Publ. Elektrotekh. Fak. Univ. Beogradu, Ser. Math. Fiz., No. 274-301, 135-138 (1969).
197. M. Arbib, "Realization of stochastic systems," Ann. Math. Statist., 38, No. 3, 927-933 (1967).
198. Baba Norio and Sawaragi Yoshikazu, "Consideration on the learning behaviors of stochastic automata," Trans. Soc. Instrum. Control Eng., 10, No. 1, 78-85 (1974).
199. G. C. Bacon, "The decomposition of stochastic automata," Inf. Control, 7, No. 3, 320-339 (1964).
200. G. C. Bacon, "Minimal-state stochastic finite-state systems," IEEE Trans. Circuit Theory, CT-11 (1964).
201. F. Bancilhon, "A geometric model for stochastic automata," IEEE Trans. Comput., 23, No. 12, 1290-1299 (1974).
202. R. Baztoszynski, "Some remarks on extension of stochastic automata," Bull. Acad. Pol. Sci. Ser., Sci. Math., Astron., Phys., 18, No. 9, 551-556 (1970).
203. A. T. Bertoni, "The solution of problems relative to probabilistic automata in the frame of the formal language theory," Lect. Notes Comput. Sci., 26, 107-112 (1975).
204. A. T. Bertoni, "Mathematical methods of the theory of stochastic automata," Lect. Notes Comput. Sci., 28, 9-22 (1975).
205. D. Blackwell and L. Koopmans, "On the identifiability problem for functions of finite Markov states," Ann. Math. Statist., 28, No. 4, 1011-1015 (1957).
206. Taylor L. Booth, "Probabilistic automata and system models. An overview," Fifth Asilomar Conf. Circuits and Syst., Pacific Grove Calif. (1971); Conf. Rec., North Hollywood, Calif. (1972), pp. 1-4.
207. K. H. Böhling and G. Dittrich, "Endliche stochastische Automaten, Vol. 1, Hochschulskripten, No. 6, 766a, Bibliographisches Institut, Mannheim – Vienna – Zurich (1972).
208. J. F. Böhme, "Einfache diagnostische Vorgabeexperimente mit stochastischen Automaten," Ber. Math. Forschungsinst., Oberwolfach, No. 3, 117-127 (1970).
209. J. F. Böhme, "Diagnostische Vorgabeexperimente mit stochastischen Automaten," Computing, 8, No. 2, 2 (1971).
210. J. F. Böhme, "Experimente mit stochastischen Automaten," Diss. Doct., Ing. Tech. Fak. Univ. Erlangen – Nürnberg (1970).
211. R. G. Bukharaev (Bukharajev), "Applied aspects of probabilistic automata," Prog. IFAC 5-th World Congr., Part 4, S. 1, s.a. 39-2/1-39-12/8 (1972).
212. J. W. Carlyle, "Reduced forms for stochastic sequential machines," J. Math. Anal. Appl., 7, No. 2, 167-175 (1963).
213. J. W. Carlyle, "On the external probability structure of finite-state channels," Inf. Control, 7, No. 3, 385-397 (1964).
214. J. W. Carlyle, "State-calculable stochastic sequential machines, equivalences, and events," IEEE Conf. Rec. Switch. Circuit Theory and Logic Design, Ann Arbor, Mich. (1965); Inst. Electron. Eng. Inc., New York (1965), pp. 258-263.

215. J. W. Carlyle, "Stochastic finite-state system theory," in: System Theory, McGraw-Hill, No. 4 (1969), pp. 387-424.
216. J. W. Carlyle and A. Paz, "Realizations by stochastic finite automata," J. Comput. Syst. Sci., 5, No. 1, 26-40 (1971).
217. J. Cerny, "Note on stochastic transformers," Mat. Cas., 20, No. 2, 101-108 (1970).
218. J. Cerny and J. Vinaz, "On simple stochastic models," Mat. Cas., 20, No. 4, 293-303 (1970).
219. B. Chandrasekaran and D. W. Shen, "Adaptation of stochastic automata in nonstationary environments," Proc. Nat. Electron. Conf., Chicago, Ill. 1967, Vol. 23, Chicago, Ill. (1967), pp. 39-44.
220. B. Chandrasekaran and D. W. Shen, "Stochastic automata games," IEEE Trans. Syst. Sci. Cybern., 5, No. 2, 145-149 (1969).
221. Chen Chi-Tsong, "Minimization of linear sequential machines," IEEE Trans. Comput., 23, No. 1, 93-95 (1974).
222. Chen I-Ngo and C. L. Sheng, "The decision problems of definite stochastic automata," SIAM J. Control, 8, No. 1, 124-134 (1970).
223. V. Claus, "Ein Reduktionssatz für stochastische Automaten," Z. Angew Math. Mech., 48, No. 8, Sonderh., 115-117 (1968).
224. J. P. Cleave, "The synthesis of finite homogeneous Markov chains," Cybernetica, 15 (1962).
225. A. S. Davis, "Markov chains as random input automata," Am. Math. Monthly, 68, No. 3, 264-267 (1961).
226. K. Ecker and H. Ratschek, "Eigenschaften der von linearen Automaten erkennbaren Worte," Acta Inf., 3, No. 4, 365-383 (1974).
227. L. Eichner, "Homomorphe Darstellung endlicher Automaten in linearen Automaten," Elektron. Inf. Kybern., 9, No. 10, 587-613 (1973).
228. H. N. El-Choroury and S. C. Gupta, "Convex stochastic sequential machines," Int. J. Sci. Syst., 2, No. 1, 97-112 (1971).
229. H. N. El-Choroury and S. C. Gupta, "Realization of stochastic automata," IEEE Trans. Comput., 20, No. 8, 889-893 (1971).
230. C. A. Ellis, "Probabilistic languages and automata," PhD Diss. Univ. Ill. (1969).
231. C. A. Ellis, "Probabilistic tree automata," Inf. Control, 19, No. 5, 401-416 (1971).
232. H. J. Engelbert, "Zur Reduktion stochastischer Automaten," Elektron. Inf. Kybernk., 4, No. 2, 81-92 (1968).
233. S. Even, "Comments on the minimization of stochastic machines," IEEE Trans. Electron. Comput., 14, No. 4, 634-637 (1965).
234. G. Feichtinger, "Zur Theorie abstrakter stochastischen Automaten," Z. Wahrscheinlichkeitstheorie Verw. Geb., 9, No. 4, 341-356 (1968).
235. G. Feichtinger, "Stochastische Automaten als Grundlage linearen Lernmodelle," Statistisches Heft, 10, No. 1 (1969).
236. G. Feichtinger, "Ein Markoffsches Lernmodell für Zwei-Personen-Spiele," Elektron. Datenverarb., 11, No. 7, 322-325 (1969).
237. G. Feichtinger, "Lernprozesse in stochastischen Automaten," Lect. Notes Oper. Res. Math. Syst., 24, 66 (1970).
238. G. Feichtinger, "Gekoppelte stochastische Automaten und sequentielle Zwei-Personen-Spiele," Unternehmenforsch., 14, No. 4, 249-258.
239. G. Feichtinger, "Stochastische Automaten mit stetigem Zeitparameter," Angew. Inf., 13, No. 4, 156-164 (1971).
240. K. Fischer, R. Lindner, and H. Thiele, "Stabile stochastische Automaten," 3, No. 4, 201-213 (1967).
241. M. Fox, "Conditions under which a given process is a function of a Markov chain," Ann. Math. Statist., 33, No. 3 (1962).
242. R. V. Freivald, "Functions computable in the limit by probabilistic machines," Lect. Notes Comput. Sci., 28, 77-87 (1975).
243. S. Fujimoto, "On the partition pair and the decomposition by partition pair for stochastic automata," Trans. Inst. Electron. Commun. Eng., Jpn., 56, No. 11, 615-622 (1973).
244. Fu King-Sun and T. J. Li, "On stochastic automata and languages," Inf. Sci., 1, No. 4, 403-419 (1969).
245. B. R. Gaines, "Memory minimization in control with stochastic automata," Electron. Lett., 7, No. 24, 710-711 (1971).
246. H. Gallaire, "On the isomorphism of linear automata," Int. Comput. Symp., Venice, Proc., No. 1, 473-484 (1972).

247. S. E. Gelenbe, "On the loop-free decomposition of stochastic finite state systems," Inf. Control, 10, No. 5, 474-484 (1970).
248. S. E. Gelenbe, "On probabilistic automata with structural restrictions," IEEE Conf. Rec. 10th Ann. Sympos. Switch and Automata Theory, Waterloo, 1969, New York (1969), pp. 90-99.
249. S. E. Gelenbe, "A realizable model for stochastic sequential machines," IEEE Trans. Comput., 20, No. 2, 199-204.
250. S. E. Gelenbe, "On languages defined by probabilistic automata," Inf. Control, 16, No. 5, 487-501 (1970).
251. E. J. Gilbert, "On the identifiability problem for functions of finite Markov chains," Am. Math. Statist., 30 (1959).
252. A. Gill, "On a weight distribution problem with applications to the design of a statistic generator," J. Assoc. Comput. Mach., 10, No. 1, 110-121 (1963).
253. A. Gill, "The reduced form of a linear automaton," in: Automata Theory, Academic Press, New York – London (1966), pp. 164-175.
254. A. Gill, "Synthesis of probability transformers," J. Franklin Inst., 274, No. 1, 1-19 (1962).
255. A. Gill, "Analysis and synthesis of stable linear sequential circuits," J. Assoc. Comput, Mach., 12, No. 1, 141-149 (1965).
256. J. T. Gill, "Computational complexity of probabilistic Turing machines," Proc. 6th Ann. ACM Symp. on Theory of Comp. (1974), pp. 91-96.
257. R. M. Glorioso, "Learning in stochastic automata," 5th Asilomer Conf. Circuits and Syst., Pacific Grove, Calif. (1971); Conf. Rec., North Hollywood, Calif. (1972), pp. 11-15.
258. A. Goscinski and R. Jakubowski, "Automat stochastyczny jako model programowania dynamicznego," Podst. Sterow, 2, No. 2, 147-162 (1972).
259. T. V. Griffiths, "The unsolvability of the equivalence problem for free nondeterministic generalized machines," J. Assoc. Comput. Math., 15, No. 3, 409-413 (1968).
260. S. Guiasu, "On codification in abstract random automata," Inf. Control, 13, No. 4, 277-283 (1968).
261. T. Havranek, "An application of logical probabilistic expressions to the realization of stochastic automata," Kybernetika, 10, No. 3, 241-257 (1974).
262. A. Heller, "Probabilistic automata and stochastic transformations," Math. Syst. Theor., 1, No. 3, 197-208 (1967).
263. H. H. Homuth, "A type of a stochastic automaton applicable to the communication channel," Angew. Inf., No. 8, 362-372 (1971).
264. W. J. Horvath, "Stochastic models of behavior," Manag. Sci., 12, No. 12, B513-B518 (1966).
265. Y. Inagaki, T. Fukumura, and H. Matuura, "Some aspects of linear space automata," Inf. Control, 20, No. 5, 439-479 (1972).
266. R. A. Jarvis, "Adaptive global search in time-variant environment using a probabilistic automaton with pattern recognition supervision," IEEE Trans. Syst. Sci. Cybern., 6, No. 3, 209-217 (1970).
267. R. L. Kashyap, "Optimization of stochastic finite-state systems," IEEE Trans., Autom. Control, 11, No. 4, 685-692 (1966).
268. D. I. Kfoury, "Synchronizing sequence for probabilistic automata," Stud. Appl. Math., 49, No. 1, 101-103 (1970).
269. D. I. Kfoury and Chung L. Liu, "Definite stochastic sequential machines and definite stochastic matrices," IEEE Conf. Rec. 10th Ann. Sympos. Switch and Automata Theory, Waterloo (1969), pp. 100-105.
270. R. Knast, "On a certain possibility of structural synthesis of probabilistic automata," Place Komisji Budowy Maszyn i Elektrotechniki (Posnanskie Towarz. Przyjaciol Nauk), 1, No. 5, 57-67 (1967).
271. R. Knast, "Representability of nonregular languages in finite probabilistic automata," Inf. Control, 16, No. 3, 285-302 (1970).
272. R. Knast, "Continuous-time probabilistic automata," Inf. Control, 15, No. 4, 335-352 (1969).
273. R. Knast, "Linear probabilistic sequential machines," Inf. Control, 15, No. 2, 111-129 (1969).
274. R. Knast, "Finite-state probabilistic languages," Inf. Control, 21, No. 2, 148-170 (1972).
275. Komiya Noriaki, "Some properties of probabilistic automata with pushdown tape," Rev. Radio Res. Lab., 17, No. 90, 236-243 (1971).
276. S. Rao Kosaraju, "Probabilistic automata – a problem of Paz," Inf. Control, 23, No. 1, 97-104 (1973).
277. S. Rao Kosaraju, "A note on probabilistic input – output relations," Inf. Control, 26, No. 2, 194-197 (1974).
278. W. Kuich and K. Walk, "Block-stochastic matrices and associated finite-state languages," Computing, 1, No. 1, 50-61 (1966).

279. Eaves B. Kurtis, "Polymatrix games with joint constraints," SIAM J. Appl. Math., 24, No. 3, 418-423 (1973).
280. H. Künstler, "Algebren endlicher stochastischer Automaten und ihrer Verhaltensfunktionen. I," Elektron. Inf. Kybern., 11, No. 1-2, 61-116 (1975).
281. Toshiro Kutsuwa, Hideo Kosako, and Yoshiaki Kojima, "Some questions in the analysis of stochastic sequential machines," Trans. Inst. Electron. Commun. Eng. Jpn., A56, No. 1, 1-8 (1973).
282. S. Lakshmivarahan and M. A. L. Thatchachar, "Optimal nonlinear reinforcement schemes for stochastic automata," Inf. Sci. (USA), 4, No. 2, 121-128 (1972).
283. S. Lakshmivarahan, "Absolutely expedient learning algorithms for stochastic automata," Trans. Syst., Man Cybern., 3, No. 3, 281-286 (1973).
284. S. Lakshmivarahan, "Bayesian learning and reinforcement schemes for stochastic automata," Proc. Int. Conf. Cybern. Soc. Washington, D.C. 1972, New York (1972), pp. 369-372.
285. E. Latikka, "On density of output probabilities in ergodic probabilistic automata," Turun Yliopiston Julk. Sar. A1, No. 158 (1973).
286. W. E. Lewis, "Stochastic sequential machines; theory and applications," Doct. Diss., Northwestern Univ. (1966); Dissert. Abstr., B27, No. 8, 2782-2783 (1967).
287. C. L. Liu, "A note on definite stochastic sequential machines," Inf. Control, 14, No. 4, 407-421 (1969).
288. A. A. Lorents (Lorenc), "On a synthesis of generation of stable probability distributions," Inf. Control 24, No. 3, 212-230 (1974).
289. B. W. Lovell, "The incompletely specified finite-state stochastic sequential machine: equivalence and reduction," IEEE Conf. Rec. 10th Ann. Sympos. Switch and Automata Theory, Waterloo, 1969, New York (1969), pp. 82-89.
290. R. W. Maclaren, "A stochastic model for the synthesis of learning systems," IEEE Trans. Syst. Cybern., 2 (1966).
291. M. Magidor and G. Moran, "Probabilistic tree automata and context-free languages," Isr. J. Math., 8, No. 4, 340-348 (1970).
292. M. M. Matluk and A. Gill, "Decomposition of linear sequential circuits: residue and class rings," J. Franklin Inst., 294, No. 3, 167-180 (1972).
293. Gr. C. Moisil, "Fenomene de indeterminism la automatele cu relee temporizate," An. Univ. Timisoara, Ser. Sti. Mat., 7, No. 1, 91-94 (1969).
294. Kumpati S. Narendra and M. A. L. Thatchachar, "Learning automata – a survey," IEEE Trans. Syst., Man Cybern., 4, No. 4, 323-324 (1974).
295. Kumpati S. Narendra and R. Viswanathan, "A two-level system of stochastic automata for periodic random environments," IEEE Trans. Syst., Man Cybern., 2, No. 2, 285-289 (1972).
296. Kumpati S. Narendra and R. Viswanathan, "A note on the linear reinforcement scheme for variable structure stochastic automata," IEEE Trans. Syst., Man Cybern., 2, No. 2, 292-294 (1972).
297. Kumpati S. Narendra and R. Viswanathan, "Stochastic automata model with applications to learning systems," IEEE Trans. Syst., Man Cybern., 3, No. 1, 107-111 (1973).
298. Kumpati S. Narendra and R. Viswanathan, "On variable-structure stochastic automata models and optimal convergence," Proc. 5th Ann. Princeton Conf. Inf. Sci. Syst., Princeton, New Jersey (1971), pp. 410-414.
299. Nasu Masakazu and Honda Namio, "Fuzzy events realized by finite probabilistic automata," Inf. Control, 12, No. 4, 284-303 (1968).
300. Nasu Masakazu and Honda Namio, "A context-free language which is not accepted by a probabilistic automata (manuscript), Inf. Control, 18, No. 3, 233-236 (1971).
301. K. Nawrotzki, "Eine Bermerkung zur Reduktion stochastischer Automaten," Elektron. Inf. Kybern., 2, No. 3 (1966).
302. K. Nawrotzki, "Zur Minimalisierung stochastischer Automaten," Elektron. Inf. Kybern., 8, No. 10, 623-631 (1972).
303. K. Nawrotzki and D. Richter, "Eine Bemerkung zum allgemeinen Reduktionsproblem von P. H. Starke," Elektron. Inf. Kybern., 10, No. 8-9, 481-487 (1974).
304. T. T. Nieh, "Stochastic sequential machines with prescribed performance criteria," Inf. Control, 13, No. 2, 99-113 (1968).
305. T. T. Nieh and J. W. Carlyle, "On the deterministic realization of stochastic finite-state machines," Proc. 2nd Ann. Princeton Conf. Inform. Sci. Syst. (1960).
306. C. J. Nihoul, "La transformée stochastique et l'étude des systemes non linéaires," Bull. Scient. A.I.M., 76, No. 8-9, 803-817 (1963).

307. Octav Onicescu and Silviu Guiasu, "Finite abstract random automata," Z. Wahrscheinlichkeitstheorie Geb., 3, No. 4, 279-285 (1965).
308. Branko Ostojic, "Teorija stochastickoga automata zasnovanana neformalnim neuronskim mrezama," Dokt. Dis. Sveuciliste u Rijeci. Then. Fak. Rijeka (1974), p. 157.
309. E. H. Ott, "Theory and application of stochastic sequential machines," Sperry Rand Res. Center, Research Paper, Sudburg, Mass. (1966).
310. E. H. Ott, "Reconsideration of the state minimization problem for stochastic finite-state systems," IEEE Conf. Rec. 7th Ann. Sympos. of Switch. Circuit and Automata Theory (1966).
311. C. V. Page, "Equivalences between probabilistic and deterministic sequential machines," Inf. Control, 9, No. 5, 469-520 (1966).
312. C. V. Page, "Strong stability problems for probabilistic sequential machines," Inf. Control, 15, No. 6, 487-509 (1969).
313. C. V. Page, "The search for a definition of partition pair for stochastic automata," IEEE Trans. Comput., 19, No. 2, 1222-1223 (1970).
314. A. C. Pan, "State identification and homing experiments for stochastic sequential machines," Proc. 4th Haw. Int. Conf. Syst. Sci., Honolulu, Hawaii, 1971, North Hollywood Calif. (1971), pp. 498-500.
315. J. J. Paredaens, "Finite stochastic automata with variable transition probabilities," Computing, 11, No. 1, 1-20 (1973).
316. J. J. Paredaens, "A general definition of stochastic automata," Computing, 13, No. 2, 93-105 (1974).
317. Behrooz Parhami, "Stochastic automata and the problems of reliability of sequential machines," IEEE Trans. Comput., 21, No. 4, 388-391 (1972).
318. A. Paz, "Some aspects of probabilistic automata," Inf. Control, 9, No. 1, 26-60 (1966).
319. A. Paz, "Minimization theorems and techniques for sequential stochastic machines," Inf. Control, 11, No. 1-2, 155-166 (1967).
320. A. Paz, "Homomorphism between stochastic sequential machines and related problems," Math. Syst. Theory, 2, No. 3, 223-245 (1968).
321. A. Paz, Introduction to Probabilistic Automata, Academic Press, New York (1971).
322. A. Paz, "Definite and quasidefinite sets of stochastic matrices," Proc. Am. Math. Soc., 16, 634-641 (1965).
323. A. Paz, "Fuzzy star functions, probabilistic automata, and their approximation by nonprobabilistic automata," IEEE Conf. Rec. 8th Ann. Sympos. Switch. and Automata Theory, Austin, Texas 1967, New York (1967), p. 2.
324. A. Paz, "A finite set of $n \times n$ stochastic matrices generating all n-dimensional probability vectors whose coordinates have finite expansion," SIAM J. Control, 5, No. 4, 545-554 (1969).
325. A. Paz, "Regular events in stochastic sequential machines," IEEE Trans. Comput., 19, No. 5, 456-457 (1970).
326. A. Paz, "Formal series, finiteness properties, and decision problems," Ann. Acad. Scient. Fennicae Suomalais. Tiedeakat. Toimituks, A-1, No. 493, 16 (1971).
327. A. Paz, "Whirl decomposition of stochastic systems," IEEE Trans. Comput., 20, No. 10, 1208-1211 (1971).
328. A. Paz and M. Rabinovitz, "Linear automata approximation problem," IEEE Trans. Comput., C-23, No. 3, 249-255 (1974).
329. A. Paz and M. Reichard, "Ergodic theorem for sequences of infinite stochastic matrices," Proc. Cambr. Phil. Soc., 63 (1967).
330. Phan Dinh Dieu, "On a necessary condition for stochastic languages," Elektron. Inf. Kybern., 8, No. 10 (1972).
331. Phan Dinh Dieu, "On a class of stochastic languages," Z. Math. Log. Grundl., 17, 421-425 (1971).
332. M. O. Rabin, "Probabilistic automata," Inf. Control, 6, No. 3, 230-245 (1963).
333. M. O. Rabin, Lectures on Classical and Probabilistic Automata. Automata Theory, Academic Press, New York – London (1966), pp. 304-313.
334. C. de Rennae Souza, "A theorem on the state reduction of synthesized stochastic machines," IEEE Trans. Comput., 18, No. 5, 473-474 (1969).
335. S. T. Riberia, "Random-pulse machines," IEEE Trans. Electron. Comput., 16, No. 3, 216-276 (1967).
336. I. Richardt, "Zur Analyse und Synthese asynchroner stochastischer Automaten," Elektron. Inf. Kybern., 10, No. 2-3, 123-132 (1974).
337. J. S. Riordan, "Optimal feedback characteristics from stochastic automaton models," IEEE Trans. Autom. Control, 14, No. 1, 89-92 (1969).

338. W. Rytter, "The strong stability problem for stochastic automata," Bull. Acad. Pol. Sci. Ser. Sci. Math., Astron. Phys., 21, No. 3, 271-275 (1973).
339. W. Rytter, "The dimension of strong stability of minimal-state stochastic automata," Bull. Acad. Pol. Sci. Ser. Sci. Math., Astron. Phys., 21, No. 3, 277-279 (1973).
340. W. Rytter, "Zagadnienie stabilnosci skonczonych stochastycznych," Pr. CO PAN, No. 6, 38 (1972).
341. W. Rytter, "The dimension of stability of stochastic automata," Inf. Control, 24, No. 3, 201-211 (1974).
342. A. Salomaa, "On probabilistic automata with one input letter," Turun Yliopiston Julk., Sar. AI, No. 85 (1965).
343. A. Salomaa, "On m-adic probabilistic automata," Inf. Control, 10, No. 2, 215-219 (1967).
344. A. Salomaa, "On events represented by probabilistic automata of different types," Can. J. Math., 20, No. 1, 242-251 (1968).
345. A. Salomaa, "On languages accepted by probabilistic and time-variant automata," Proc. 2nd Ann. Princeton Conf. Inf. Sci. Systems (1968).
346. A. Salomaa, "Probabilistic and weighted grammars," Inf. Control, 15, No. 6, 529-544 (1970).
347. E. S. Santos, "Probabilistic Turing machines and computability," Proc. Am. Math. Soc., 22, No. 3, 704-710 (1969).
348. E. S. Santos, "Fuzzy algorithms," Inf. Control, 17, No. 4, 326-339 (1970).
349. E. S. Santos, "Computability by probabilistic Turing machines," Trans. Am. Math. Soc., 159, 165-184 (1971).
350. E. S. Santos, "Algebraic structure theory of stochastic machines," Conf. Rec. 3rd Ann. ACM Symp. Theory Comput., Shaker Heights, Ohio, 1971, New York (1971), pp. 219-243.
351. E. S. Santos, "First and second covering problems of quasistochastic systems," Inf. Control, 20, No. 1, 20-37 (1972).
352. E. S. Santos, "Probabilistic grammars and automata," Inf. Control, 21, No. 1 (1972).
353. E. S. Santos, "Identification of stochastic finite-state systems," Inf. Control, 21, No. 1 (1972).
354. E. S. Santos, "A note on probabilistic grammars," Inf. Control, 25, No. 4, 393-394 (1974).
355. E. S. Santos, "Realizations of fuzzy languages by probabilistic, max-product, and maximin automata," Inf. Sci. (USA), 8, No. 1, 39-53 (1975).
356. E. S. Santos, "State-splitting for stochastic machines," SIAM J. Comput., 4, No. 1, 85-96 (1975).
357. Sawaragi Yoshikazu and Baba Norio, "Two E-optimal nonlinear reinforcement schemes for stochastic automata," IEEE Trans. Syst., Man Cybern., 4, No. 1, 126-131 (1974).
358. Sawaragi Yoshikazu and Baba Norio, "A note on the learning behavior of variable-structure stochastic automata," IEEE Trans. Syst., Man Cybern., 3, No. 6, 644-647 (1973).
359. A. Schmitt, "Vorhersage der Ausgabe stochastischer Automaten," Mitt. Ges. Matt. Datenverarb., No. 8, 36-38 (1970).
360. A. Schmitt, "Optimale Vorhersage der Ausgabe stochastischer Automaten über lange Zeiten Arbeitis-ber," Inst. Math. Masch. Datenverarb., 3, No. 6 (1971).
361. J. Shapiro and Kumpati S. Narendra, "Use of stochastic automata for parameter self-optimization with multimodal performance criteria," IEEE Trans. Syst. Sci. Cybern., 5, No. 4 (1969).
362. C. L. Sheng, "Threshold logic elements used as a probability transformer," J. Assoc. Comput. Math., 12, No. 2, 262-276 (1965).
363. C. B. Silio, Jr., "Synthesis of simplicial covering machines for stochastic finite state systems," 6th Asilomar Conf. Circuits Syst., Pacific Grove, Calif., 1972, North Hollywood, Calif. (1973), pp. 202-208.
364. C. de Rennae Souza, "Probabilistic automata with monitored final state sets," IEEE Trans. Comput., 20, No. 4, 448-450 (1971).
365. F. Stanciulescu and F. A. Oprescu, "A mathematical model of finite random sequential automata," IEEE Trans. Comput., C-17, No. 1, 28-31 (1968).
366. F. Stanciulescu and F. A. Oprescu, "Probabilisation de l'algebre sequentielle et simulation mathematique des automates sequentiels aleatoires," Bull. Math. Soc. Sci. Rsr. Mat. RSF, 12, No. 1, 123-132 (1968).
367. P. H. Starke, "Theorie stochastischen Automaten. I. Theorie stochastischen Automaten. II," Elektron Inf. Kybern., 1, No. 2 (1965).
368. P. H. Starke, "Stochastische Ereignisse und Wortmengen," Z. Math. Log. Grundl. Math., 12, No. 1-2, 61-68 (1966).
369. P. H. Starke, "Stochastische Ereignisse und stochastische Operatoren," Elektron. Inf. Kybern., 2, No. 3 (1966).
370. P. H. Starke, "Theory of stochastic automata," Kybernetika, 2, 475-482 (1966).

371. P. H. Starke, "Die Reduktion von stochastischen Automaten," Elektron. Inf. Kybern., 4, No. 2, 93-99 (1968).
372. P. H. Starke, "Über die Minimalisierung von stochastischen Rabin-Automaten," Elektron. Inf. Kybern., 5, No. 3, 153-170 (1969).
373. P. H. Starke, Abstrakte Automaten, VEB Deutsch. Verl. Wiss., Berlin (1969).
374. P. H. Starke, "Schwache Homomorphismen fur stochastische Automaten," Z. Math. Log. Grundl. Math., 15, No. 5, 421-429 (1969).
375. P. H. Starke, "Einige Bemerkungen über asynchrone stochastische Automaten," Suomalais, Tiedeakat, Toimituks, Ser. AI, No. 491, 21 (1971).
376. P. H. Starke and H. Thiele, "Zufällige Zustände in stochastischen Automaten," Elektron. Inf. Kybern., 3, No. 1, 25-37 (1967).
377. P. H. Starke and H. Thiele, "On asynchronous stochastic automata," Inf. Control, 17, No. 3, 265-293 (1970).
378. P. H. Starke and H. Thiele, "Über asynchrone stochastische Automaten," Ber. Math. Forschungsinst. Oberwolfach, 3, No. 3, 129-142 (1970).
379. K. Sugino, Y. Inagaki, and T. Fukumura, "On analysis of a probabilistic automaton by its state characteristic equation," Trans. Inst. Electr. Commun. Eng. Jpn., 51-C, No. 1, 29-36 (1968).
380. J. Sustal, "The degree of distinguishability of stochastic sequential machines and related problems," Elektron. Inf. Kybern., 10, No. 4, 203-214 (1974).
381. J. Sustal, "On the size of the set of all reducible stochastic sequential machines," Inf. Control, 26, 301-311 (1974).
382. P. Szynal and S. Janick, "Some remarks on extension of finite sets of stochastic automata," Bull. Acad. Pol. Sci. Ser.Sci.Math., Astron. Phys., 23, No. 2, 183-187 (1975).
383. Akihiro Takeuschi and Tadahiro Kitahashi, "Interaction between random media and stochastic automata," Trans. Inst. Electron. Commun. Eng. Jpn., A55, No. 10, 569-570 (1972).
384. Akihiro Takeuschi and Tadahiro Kitahashi, "On the behavior of stochastic automata in environments reacting to those outputs in random fashion," Trans. Inst. Electron. Commun. Eng. Jpn., 55D, No. 9, 587-593 (1972).
385. Akihiro Takeuschi, Tadahiro Kitahashi, and Kohkichi Tanaka, "Random environments and automata," Inf. Sci. (USA), 8, No. 2, 141-154 (1975).
386. Akihiro Takeuschi, Tadahiro Kitahashi, and Kohkichi Tanaka, "The behavior of a stochastic automaton in a random medium," Trans. Inst. Electron Commun. Eng. Jpn., C54, No. 10, 949-950 (1971).
387. Tan Choon Peng, "On two-state isolated probabilistic automata," Inf. Control, 21, No. 5, 483-495 (1972).
388. N. Tandareanu, "Functions associated to a partition on the state set of a probabilistic automaton," Inf. Control, 28, No. 4, 304-313 (1975).
389. A. J. Thomasian, "A finite criterion for indecomposable channels," Ann. Math. Statist., 34, No. 1, 337-338 (1963).
390. P. Turakainen, "On nonregular events representable in probabilistic automata with one input letter," Turun Yliopiston Julk., Sar. AI, No. 90, 14 (1966).
391. P. Turakainen, "Äärellisistä ja stokastisista automaateista," Arkhimedes, No. 2, 16-20 (1967).
392. P. Turakainen, "On m-adic stochastic languages," Inf. Control, 17, No. 4, 410-415 (1976).
393. P. Turakainen, "On probabilistic automata and their generalizations," Soumalais, Tiedeakat. Toimituks, 53 (1968).
394. P. Turakainen, "On time-variant probabilistic automata with monitors," Turun Yliopiston Julk., Sar. A, No. 130 (1969).
395. P. Turakainen, "The family of stochastic languages is closed neither under catenation nor under homomorphism," Turun Yliopiston Julk., Sar. AI, No. 133 (1970).
396. P. Turakainen, "On stochastic languages," Inf. Control, 12, No. 4, 304-313 (1968).
397. P. Turakainen, "On languages representable in rational probabilistic automata," Suomalais, Tiedeakat. Toimituks, Sar. AI, 10, No. 439 (1969).
398. P. Turakainen, "Generalized automata and stochastic languages," Proc. Am. Math. Soc., 21, No. 2, 303-309 (1969).
399. P. Turakainen, "On multistochastic automata," Inf. Control, 23, No. 2, 183-203 (1973).
400. P. Turakainen, "Some closure properties of the family of stochastic languages," Inf. Control, 18, No. 3, 253-256 (1971).
401. P. Turakainen, "Some remarks on multistochastic automata," Inf. Control, 27, No. 1, 75-86 (1975).

402. S. G. Tzafestas, "State estimation algorithms for nonlinear stochastic sequential machines," Comput. J., 16, No. 3, 245-253 (1973).
403. R. Viswanathan and Kumpati S. Narendra, "Convergence rates of optimal variable structure stochastic automata," Syst. Seventies Proc. IEEE Syst. Sci. and Cybern. Conf., Pittsburgh, Pa. (1970), pp. 147-153.
404. R. Viswanathan and Kumpati S. Narendra, "Games of stochastic automata," IEEE Trans. Syst., Man Cybern., 4, No. 1, 131-135 (1974).
405. J. N. Warfield, "Synthesis of switching circuits to yield prescribed probability relations," IEEE Conf. Rec. Switch. Circuit Theory and Logic. Design, Ann Arbor, Mich. (1965).
406. W. Wes and K. S. Fu, "A formulation of fuzzy automata and its application as a model of a learning system," Syst. Sci. Cybern., 5, No. 3, 215-223 (1969).
407. G. M. White, "Penny matching machines," Inf. Control, 2, No. 4, 349-363 (1959).
408. D. G. Willis, "Computational complexity and probability constructions," J. Assoc. Comput. Mach., 17, No. 2, 241-259 (1970).
409. J. Winkowski, "A method of realization of Markov chains," Algorythmy, No. 9 (1968).
410. T. Yasui and S. Yajima, "Two-state two-symbol probabilistic automata," Inf. Control, No. 3, 203-224 (1970).
411. T. Yasui and S. Yajima, "Some algebraic properties of sets of stochastic matrices," Inf. Control, 14, No. 4 (1969).
412. A. L. Zadeh, "Fuzzy sets," Inf. Control, 8, 338-353 (1965).
413. A. L. Zadeh, "Communicaton: fuzzy algorithms," Inf. Control, 12, No. 2, 94-102 (1968).
414. K. A. Zech, "Homomorphe decomposition stochastischer und nicht deterministischer Automaten," Elektron Inf. Kybern., 7, No. 5-6, 297-315 (1971).
415. K. A. Zech, "Eine Bemerkung über stochastische Wahrheitsfunktionen und ihre Anwendung in der Strukturtheorie stochastischer Automaten," Elektron. Inf. Kybern., 7, No. 8, 505-512 (1971).
416. H. Zunlke, "Ersetzbarkeit von stochastischen Automaten," Wiss. Z. Friederich Schiller Univ., Jena, Math. Naturwiss. Reihe, 18, No. 2, 279-283 (1969).

MULTIPLE-USER COMMUNICATION

S. I. Gel'fand and V. V. Prelov

UDC 519.72

The survey is devoted to the theory of multiple-user communication (coding of dependent sources, one-way multicomponent channels, and multiway channels).

1. Introduction

1.1. The present survey covers work on the theory of multiple-user communication reviewed in Ref. Zh. Mat. from 1961 through 1976.

Multiple-user communication as an area of information theory was established at the beginning of the 1960s after the work of Shannon [73] on two-way channels. Thereafter, during the course of 10 years, only a few papers appeared which mainly developed and refined the results of Shannon. At the beginning of the 1970s the interest in this area increased abruptly due to its possible relation to questions of constructing complex modern networks of data transmission. The papers of Cover [33] and Slepian and Wolf [76] mark the beginning of an intense investigation of multiple-user systems. This process of intense investigation still continues, and the number of papers devoted to multicomponent communication systems has increased rapidly. It may evidently be said that at the present time this area of information theory is in full bloom.

There are very few papers which may properly be considered as surveys of the theory of multiple-user communication. There is a survey of Wyner [93] in which Sec. 6 is devoted to the topic of interest here, a brief survey of Wolf [87], and the recent very detailed and substantial survey of van der Meulen [62] (and its extended version [63]) which, however, is devoted only to multicomponent channels and does not treat questions

Translated from Itogi Nauki i Tekhniki. Teoriya Veroyatnostei, Matematicheskaya Statistika, Teoreticheskaya Kibernetika, Vol. 15, pp. 123-162, 1978.

 0090-4104/80/1303-0386$07.50