20/11/18

Prob. Models for Computer Scientists

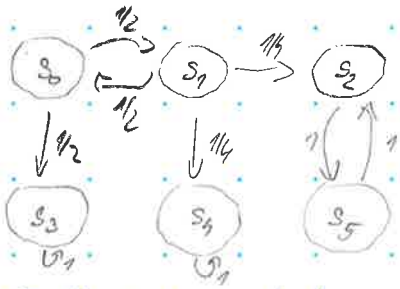
Datum

Wichtiges

Excercise / Lecture notes

(1) ITERATIVE METHOD

Ex. 1



a) $Pr(\Diamond \{s_3\})$ iterative method

b) $\Diamond\Box \{s_3, s_4, s_5\}$. $\Box\Diamond \{s_3, s_4, s_5\}$

c) $\Diamond\Box \{s_2\}$ $\Box\Diamond \{s_2\}$.

d) $\Diamond\Box \{s_2, s_5\}$ $\Box\Diamond \{s_2, s_5\}$.

III $\begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$

$x = A x + b$

$Pr(s_i \models \Diamond^{\leq n} T) =: x_n(i)$

$x_{n+1} = A x_n + b \Rightarrow \begin{bmatrix} x_{n+1}(1) \\ \vdots \\ x_{n+1}(i) \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_i \end{bmatrix} + \begin{bmatrix} A_{ij} \end{bmatrix} \cdot \begin{bmatrix} x_n(1) \\ \vdots \\ x_n(i) \end{bmatrix}$

$x_0(i) = 0$ (They do not reach T in 0 steps cause they are in $S_?$)

$(x = \vec{0})$.

Back to Example

got this by looking at P.

$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $x_1 = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$

$x_2 = \begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/4 \end{bmatrix}$

$x_3 = \begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/4 \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5/8 \\ 1/4 \end{bmatrix}$

$x_4 = \begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} 5/8 \\ 1/4 \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5/8 \\ 5/16 \end{bmatrix}$

a) I TRACE DISTRIBUTION

$Pr(Cyl(s_0 s_3)) + \sum_{k=1}^{\infty} Pr(Cyl(s_0 (s_4 s_5)^k s_3))$

$= 1/2 + 1/2 \cdot \sum_{k=1}^{\infty} (1/4)^k = 1/2 \sum_{k=0}^{\infty} (1/4)^k = 1/2 \cdot 4/3 = 2/3$

II SYSTEM OF LIN. EQ.

$S_0 = \{s_4, s_2, s_5\}$ $P(s_0, T) = 0$

$S_1 = \{s_3\}$ $P(s_1, T) = 1$

$S_2 = \{s_0, s_1\}$

$X_0 = Pr(s_0 \models \Diamond \{s_3\})$

$X_1 = Pr(s_1 \models \Diamond \{s_3\})$

$X_0 = 1/2 + 1/2 X_1$ $\left(P(s_0, s_1) + \sum_{s \in S_2} P(s_0, s) \cdot X\right)$

$X_1 = 0 + 1/2 X_0$

substitution

$X_1 = 0 + 1/2 \cdot (1/2 + 1/2 X_1) \Rightarrow 3/4 X_1 = 1/4 \Rightarrow X_1 = 1/3$

$\Rightarrow X_0 = 1/2 + 1/6 = 2/3$.

$Pr(s_i \models \Diamond^{\leq n+1} T) = \sum_{k=0}^{n+1} Pr(s_i \models \Diamond^{=k} T) = Pr(s_i \models \Diamond^{1} T) + \sum_{k=2}^{n+1} Pr(s_i \models \Diamond^{=k} T) =$

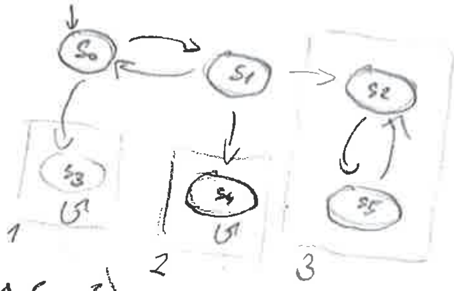
$= b_i + \sum_{k=2}^{n+1} \left(\sum_{s_j \in S_2} P(s_i, s_j) \cdot Pr(s_j \models \Diamond^{=k-1} T) \right)$

$\sum_{s_j \in S_2} \left(P(s_i, s_j) \cdot \sum_{k=2}^{n+1} Pr(s_j \models \Diamond^{=k-1} T) \right)$

$= b_i + \sum_{s_j \in S_2} P_{ij} \underbrace{Pr(s_j \models \Diamond^{\leq n} T)}_{x_n(j)}$?

b) $\Diamond\Box\{s_3, s_4, s_5\}$ / $\Box\Diamond\{s_3, s_4, s_5\}$

We now look at the BSCCs



$= Pr(\Diamond\{s_3\}) + Pr(\Diamond\{s_4\})$! not $Pr(\Diamond\{s_5\})$

there is no trace that ends with s_5 nor with $\{s_3, s_4, s_5\}$

$\rightarrow = 1$! not well explained

c) $\Diamond\Box\{s_2\} = 0$

$\Box\Diamond\{s_2\} = 1/6$

d) $\Diamond\Box\{s_2, s_5\} = 1/6$

$\Box\Diamond\{s_2, s_5\} = 1/6$

~~DEF.~~ THEOREM

$\Box\Diamond T = \sum \Diamond BSCC;\ \exists t \in T\ \exists t \in BSCC$

$\Diamond\Box T = \sum \Diamond BSCC;\ \forall t \in BSCC\ \exists t \in T$

LECTURE

$a^n b^n$ PDA

$(ab)^n$ FA

$a b a^n$

FG — safety

liveness

GF — fairness