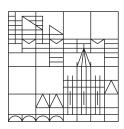
Probabilistic Automata: Semantics, Equivalence & Minimization

Master Thesis

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Some New Abstract Text

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1 Introduction

2 Background

2.1 Algebraic Preliminaries

Definition 2.1 (Equivalence Relation).

Definition 2.2 (Monoid).

Definition 2.3 (Ring, Semiring).

Theorem 2.3.1 (The Ring of Formal Power Series).

2.2 Category Theory

2.3 Probability Theory

Definition 2.4 (Sample space, events, σ -algebra, measurable space). Let Ω be a set, namely the set of possible outcomes of a chance experiment, called **sample space**.

Let $\mathcal{F} \subseteq \mathcal{P}(\Omega)$ with \mathcal{P} the powerset, a set of subsets of the sample space, whose elements are called **events**.

Let Ω be a sample space, \mathcal{F} a set of events. \mathcal{F} is called a σ -algebra over Ω , if and only if

• the sample space is contained in the set of events,

$$\Omega\in\mathcal{F}$$

• the set of events is closed under complementation,

$$A \in \mathcal{F} \Rightarrow \Omega \setminus A \in \mathcal{F}$$

• and the set of events is closed under countable union:

$$\forall i \geq 0 : A_i \in F \Rightarrow (\bigcup_{n \in \mathbb{N}A_n) \in \mathcal{F}}$$

The pair (Ω, \mathcal{F}) is called a **measurable space**.

Let $(\Omega_1, \mathcal{F}_1)$, $(\Omega_2, \mathcal{F}_2)$ measurable spaces. A funtion $f: \Omega_1 \to \Omega_2$ is called a **measurable function** if and only if for every $A \in \mathcal{F}_2$ the preimage of A under f is in \mathcal{F}_1 .

$$\forall A \in \mathcal{F}_2: f^{-1}(A) \in \mathcal{F}_1$$

Definition 2.5 (Probability Space, Probability Measure, Discrete and Continuous Probability Space and Measure).

Definition 2.6 (Random Variable, Probability Distribution, Distribution Funtion and Cumulative Density Function).

Definition 2.7 (Mean, Median, Mode, Expectation, Variance).

Definition 2.8 (Joint Probability Distribution, Conditional Probability, Bayes Rule, Independence, Conditional Independence).

Definition 2.9 (Stochastic Process, Bernouilli & Binomial Process).

Definition 2.10 (Geometric Distribution).

Definition 2.11 ((Negative) Exponential Distr.).

Theorem 2.11.1 (Memoryless property).

Definition 2.12 (Markov Property, Markov Process, Time Homogeneity).

2.4 Automata Theory

Definition 2.13 (Transition System).

Definition 2.14 (Labelled Transition System).

Definition 2.15 (Path, Trace, Cylinder Sets, Prefix, Postfix).

Definition 2.16 (Determinism and Non-Determinism, Internal vs. External, Adversaries/Policies).

Definition 2.17 (Weighted Automata).

Definition 2.18 (Probabilistic Automata, Initial Distribution, transition probability function, stochastic matrix, transition probability matrix).

Remark (Probabilistic vs. Non-Deterministic Choice).

Remark (Sokolova's System Types Paper and focus on MC-like models).

Definition 2.19 (Markov Chains, Discrete-Time, Continuous-Time, Labelled, (MDP?, Generalized Stochastic Petri Nets?)).

Definition 2.20 (Bisimulation Relations, Strong, weak, exact, ordinary, Prob., Buchholz).

3 Semantics, Equivalence & Minimization

The general case, may help with some things, e.g. if proven independent of the semiring used. So for each subsection here starting with WAs independent of the Semiring that is used, continue with PA results that are as independent as possible from the concrete transition structure. Finally apply the aforementioned results to LMCs/MCs. Also discriminate between DTMC, MDP, PA model(s) I.e. per subsection apply the folling structure

WA - General Case for arbitrary Semirings

PA - Results for other transition structures

MC-like models - Application of above and other literature on specific example of LMC

3.1 Semantics

3.1.1 Parametrization and Initialization

3.1.2 Trace Semantics

Execution as Sparse Matrix Multiplication, (constrained) reachability (pr.), path/trace distributions, ergodicity, state residency time, Uniformization

3.1.3 Transient Semantics & Cylinder Sets of Executions (incl. Reward/weighted semantics)

wrt. probabilities?!

Definition 3.1 (Transient probability distributiin).

3.1.4 Threshold semantics

3.1.5 Funtion Transient Semantics (?)

word functions instead of probability functions?

3.2 Equivalence & Tanja's Draft Equivalences

3.2.1 Trace semantics is decidable in P for all

paz p.36, Kiefer , Tzeng, Schützenberger, Bollig & Zeitoun, Kiefer WA should hold for PA when using sufficient eps (theoretically unclean), Doyen,

3.2.2 Transient semantics is lower NP, upper EXP

As we need to construct the postfix space which is a cylinder set, transfers also to reward/weighted transient

3.2.3 Word Function-based

Same as above, just that lower bound is less; stop on first false; maybe eq of kiefer helps here

3.3 Minimization + wrt. Draft Eqs.

Definition 3.2 (Lumping/lumpable).

3.3.1 Approaches

3.3.1.1 Partition Refinement - Coalgebraic Approach

Proof: PA minimization is in P for uninitialized.

Deiffel, WiSSmann, Paz p.24ff (IntroToProbabilisticAutomata) for all in O(n log n) as partition refinement is minimal (see sources above) for uninitialized Automata.

3.3.1.2 Schützenberger's Construction \pm Arnoldi Iteration with Housholder Reflectors &

Show either that is also in P oder correct Kiefers runtime analysis. For uninitialized see kiefer; TODO figure out if kiefers reduction is flawed or if the runtime analysis of his algo is flawed.

3.3.2 Wrt. different semantics

3.3.2.1 Decidability

3.3.2.2 Complexity

4 Conclusions

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