# 第一章 The Integers 整数

## 1.1Numbers and Sequencens（数、序列）

整数集： $Z=\{0,\pm1,\pm2,...\}$ The set of integers

正整数集： $Z^{+}=\{1,2,3...\}$ The set of positive integers

有理数集： $Q$ The set of rational numbers

实数集： $R$ The set of real number

自然数集： N The set of natural numbers

$N=\{1,2,3...\}=Z^{+}$ $N=\{0,1,2,3...\}$

**The Well-Ordering Property**

**Every nonempty set of positive integers has a least element.**

【良序性质】/【良序公理】每个非空的正整数集合都有最小元。

**Definition. The real number $r$ is *rational* if there are integers $p$ and $q$, with $q\neq0$, such that $r=p/q$. If $r$ is not rational , it is said to be *irrational*.**

〖定义〗设$r$是实数，若存在整数$p$，$q$（$\neq0$），使得$r=p/q$，则称$r$为有理数，否则称$r$为无理数。

**Theorem 1.2.** $\sqrt{2}$ is irrational.

代数数/超越数 Algebraic/Transcendental

**The gretest Integer function(最大整函数)**

*Floor function*：取整函数

上取整（*ceiling function*）

Diophantine Approximation 丢番图逼近

Theorem 1.2. *The Pigeonhole Principle*

If $k$+1 or more objects are placed into $k$ boxes, then at least one box contains two or more of the objects.

〖定理 1.2〗鸽巢原则/抽屉原则

假设 $k$+1 或更多的物体装入 $k$ 个盒子(boxes),那么至少一个盒子有两个或更多的物体.

〖定理 1.3〗*Dirichlet's approximation theorem*(逼近原理).

Sequnces 序列

〖定义〗序列$\{a_n\}$($n$=0,1,2,3,...或$n$=1,2,3,...)

Bijection 双射

also called one-to-one correspondence(一一映射).

Definition:

等比数列（几何数列）: Geometric Progression

等差数列（算术数列）: Arithmetic Progression

可数集/不可数集 Countable/uncuntable

Definition. A set is *countable* if it is finite or it is infinite and there exists a one-to-one correspondence between the set of positive integers and the set. A set that is not countable is called *uncountable*

〖定义〗有限集或无限集且它可与自然数集建立 1-1 对应关系的集称为可数集；不是可数集的集合称为不可数集。

1.2 Sume and Products 和与积

Products 求积：$\prod_{j=1}^{n} a_j = a_1 a_2 \ldots a_n$

〖定义〗阶乘

***Definition.*** Let $n$ be a positive integer. The $n!$( read as "n factorial ") is the product of the integers 1, 2, ...,$n$. we also specify that $0! = 1$ .In terms of product notation, we have