Theory of Machines and Languages

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Identifying Nonregular Languages

A Pumping Lemma

Let L be an infinite regular language. Then there exists some positive integer m such that any $w \in L$ with $|w| \geq m$ can be decomposed as

$$w = xyz$$

with

$$|xy| \leq m,$$

and

$$|y| \geq 1,$$

such that

$$w_i = xy^i z,$$

is also in L for all $i = 0, 1, 2, \ldots$.

- We have given the pumping lemma only for infinite languages
- Finite languages are always regular

Identifying Nonregular Languages

- **Example**
  - Use the pumping lemma to show that $L = \{a^n b^n : n \geq 0\}$ is not regular

- Assume that L is regular, so that the pumping lemma must hold
- We can choose $m = n$
- Therefore, the substring y must consist entirely of a's
- Suppose $|y| = k$
- Then the string obtained by using $i = 0$ is $w_0 = a^{m-k} b^m$, which is clearly not in L
- This contradicts the pumping lemma and indicates that the assumption that L is regular must be false

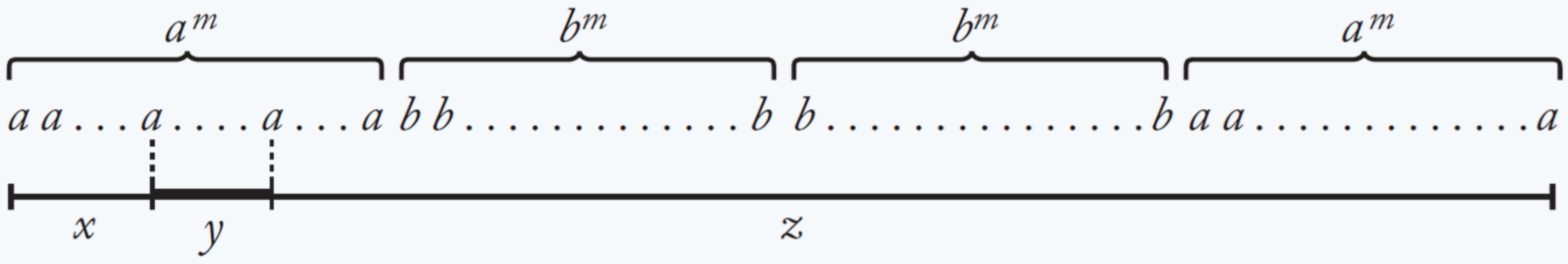
Identifying Nonregular Languages

- **Example**
  - Show that $L = \{ww^R : w \in \Sigma^*\}$ is not regular

- Given m, we pick as our string w $= a^m b^m b^m a^m$, which is in L
- Because of the constraint $|xy| \le m$, y consists entirely of a's

$$y = a^k \qquad 1 \le k \le m$$

- $i = 0 \quad \Rightarrow \quad w_2 = a^{m-k} b^m b^m a^m$ is not in L



Identifying Nonregular Languages

- **Example**
  - Let $\Sigma = \{a, b\}$. Show that $L = \{w \in \Sigma^*: n_a(w) < n_b(w)\}$ is not regular.

- Given m, we pick as our string $\mathrm{w} = (a)^m b^{m+1}$, which is in L
- Because of the constraint $|xy| \leq m$:

$$y = a^k \quad 1 \leq k \leq m$$

- $i = 2 \quad \Rightarrow \quad w_2 = a^{m+k} b^{m+1} \quad$ is not in L

Identifying Nonregular Languages

- **Example**
  - Show that $L = \{(ab)^n a^k : n > k,\ k \geq 0\}$ is not regular

- Given m, we pick as our string $w = (ab)^{m+1}a^m$, which is in L
- Because of the constraint $|xy| \leq m$, both x and y must be in the part of the string made up of ab's
- If $y = a$ $\Rightarrow$ We choose $i = 0$ and get a string not in L
- If $y = ab$ $\Rightarrow$ We choose $i = 0$ and get the string $(ab)^m a^m$
- In the same way, we can deal with any possible choice of y

Identifying Nonregular Languages

- **Example**
  - Show that $L = \{a^n : n \text{ is a perfect square}\}$ is not regular

Given the opponent's choice of m, we pick

$$w = a^{m^2}.$$

If $w = xyz$ is the decomposition, then clearly

$$y = a^k$$

with $1 \leq k \leq m$. In that case,

$$w_0 = a^{m^2-k}.$$

But $m^2 - k > (m-1)^2$, so that w_0 cannot be in L. Therefore, the language is not regular.