Theory of Machines and Languages

Fatemeh Deldar

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Context-Free Languages

Context-Free Grammars

- A grammar $G = (V, T, S, P)$ is said to be **context-free** if all productions in P have the form

$$A \to x,$$

where $A \in V$ and $x \in (V \cup T)^*$.

> A language L is said to be context-free if and only if there is a context-free grammar G such that $L = L(G)$.

- **Every regular grammar is context-free, so a regular language is also a context-free one**
- **The family of regular languages is a proper subset of the family of context-free languages**

Context-Free Grammars

- **Example**

$$S \to aSa,$$
$$S \to bSb,$$
$$S \to \lambda,$$

$$L(G) = \{ww^R : w \in \{a,b\}^*\}$$

A context-free **grammar**

A context-free **language**

- **Example**

$$S \to abB,$$
$$A \to aaBb,$$
$$B \to bbAa,$$
$$A \to \lambda,$$

$$L(G) = \{ab\,(bbaa)^n\, bba\,(ba)^n : n \geq 0\}$$

Context-Free Grammars

- **Example**

$$L = \{a^n b^m : n \neq m\}$$

Grammar

$$\begin{aligned} S &\rightarrow AS_1 | S_1 B, \\ S_1 &\rightarrow aS_1 b | \lambda, \\ A &\rightarrow aA | a, \\ B &\rightarrow bB | b. \end{aligned}$$

- **Example**

$$S \rightarrow aSb | SS | \lambda.$$

Language

$$L = \{w \in \{a,b\}^* : n_a(w) = n_b(w) \text{ and } n_a(v) \geq n_b(v), \text{ where } v \text{ is any prefix of } w\}.$$

Context-Free Grammars

- Find context-free grammars for the following languages

$$L = \{a^n b^m : 2n \leq m \leq 3n\}$$

$$S \to aSbb|aSbbb|\lambda$$

$$L = \{a^n b^m c^k : n = m \text{ or } m \leq k\}$$

$$S \to AC|DBC$$
$$A \to aAb|\lambda$$
$$B \to bBc|\lambda$$
$$C \to cC|\lambda$$
$$D \to aA|\lambda$$

$$L = \left\{uvwv^R : u, v, w \in \{a, b\}^+, |u| = |w| = 2\right\}$$

$$S \to AB$$
$$A \to aa|ab|ba|bb$$
$$B \to aBa|bBb|aAa|bAb$$

Leftmost and Rightmost Derivations

- In a grammar that is not linear, a derivation may involve sentential forms with more than one variable
  - In such cases, we have a choice in the order in which variables are replaced

- **Example**

1. $S \to AB.$
2. $A \to aaA.$
3. $A \to \lambda.$
4. $B \to Bb.$
5. $B \to \lambda.$

Language

$$L(G) = \{a^{2n}b^m : n \geq 0, m \geq 0\}$$

Two derivations for aab

$$S \overset{1}{\Rightarrow} AB \overset{2}{\Rightarrow} aaAB \overset{3}{\Rightarrow} aaB \overset{4}{\Rightarrow} aaBb \overset{5}{\Rightarrow} aab$$

$$S \overset{1}{\Rightarrow} AB \overset{4}{\Rightarrow} ABb \overset{2}{\Rightarrow} aaABb \overset{5}{\Rightarrow} aaAb \overset{3}{\Rightarrow} aab.$$

Leftmost and Rightmost Derivations

- **Leftmost derivation**
  - **In each step the leftmost variable in the sentential form is replaced**
- **Rightmost derivation**
  - **In each step the rightmost variable in the sentential form is replaced**
- **Example**

$$S \to aAB,$$
$$A \to bBb,$$
$$B \to A|\lambda.$$

Leftmost derivation of the string *abbbb*

$$S \Rightarrow aAB \Rightarrow abBbB \Rightarrow abAbB \Rightarrow abbBbbB \Rightarrow abbbbB \Rightarrow abbbb$$

$$S \Rightarrow aAB \Rightarrow aA \Rightarrow abBb \Rightarrow abAb \Rightarrow abbBbb \Rightarrow abbbb$$

Rightmost derivation of the string *abbbb*