Theory of Machines and Languages

Fatemeh Deldar

1403-1404

Turing Machines

- **Exercise**

Describe the action of the Turing machine

$$\begin{aligned} \delta(q_0, a) &= (q_0, a, R), \qquad \delta(q_0, \square) = (q_1, \square, L), \\ \delta(q_1, a) &= (q_1, a, L), \qquad \delta(q_1, \square) = (q_0, \square, R) \end{aligned}$$

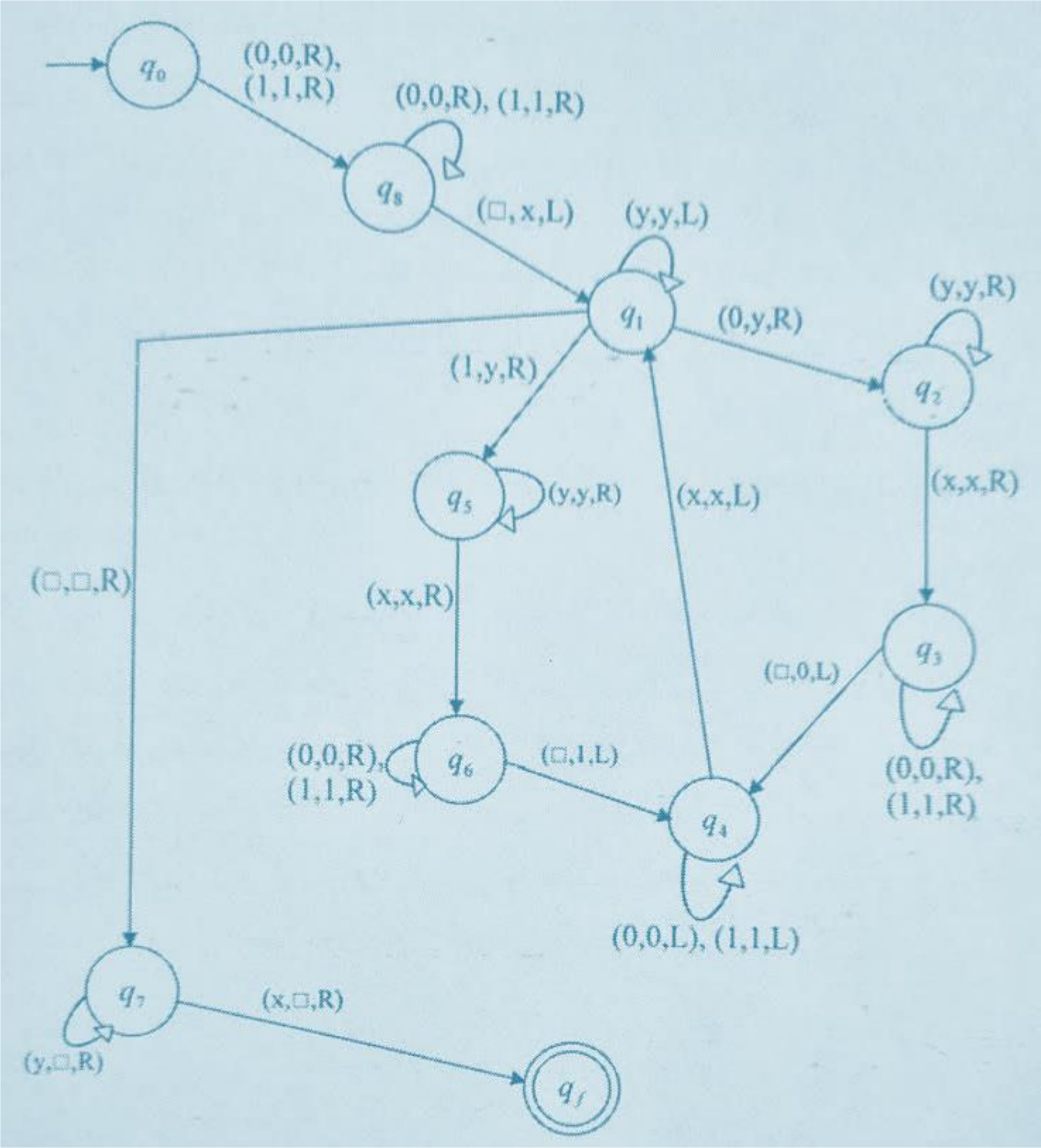
if it starts in state q_0 and input is $w = a^n$.

- **Exercise**

Design a standard Turing machine that accepts $L = \{(ab)^n, n \geq 1\}$.

Turing Machines

- **Exercise:** **Construct a Turing machine to compute the function $f(w) = w^R$, where $w \in \{0, 1\}^+$.**



Turing Thesis

- Any computation that can be carried out by mechanical means can be performed by some Turing machine
  1. Anything that can be done on any existing digital computer can also be done by a Turing machine
  2. No one has yet been able to suggest a problem, solvable by an algorithm, for which a Turing machine program cannot be written
  3. Alternative models have been proposed for mechanical computation, but none of them is more powerful than the Turing machine model
  - Turing's thesis is still an assumption

Other Models of Turing Machines

- Two automata are equivalent if they accept the same language. Consider two classes of automata C_1 and C_2. If for every automaton M_1 in C_1 there is an automaton M_2 in C_2 such that

$$L(M_1) = L(M_2),$$

we say that C_2 is at least as powerful as C_1. If the converse also holds and for every M_2 in C_2 there is an M_1 in C_1 such that $L(M_1) = L(M_2)$, we say that C_1 and C_2 are equivalent.

Other Models of Turing Machines

- **Turing Machines with a Stay-Option**

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

$$\delta\left(q_i, a\right) = \left(q_j, b, S\right)$$

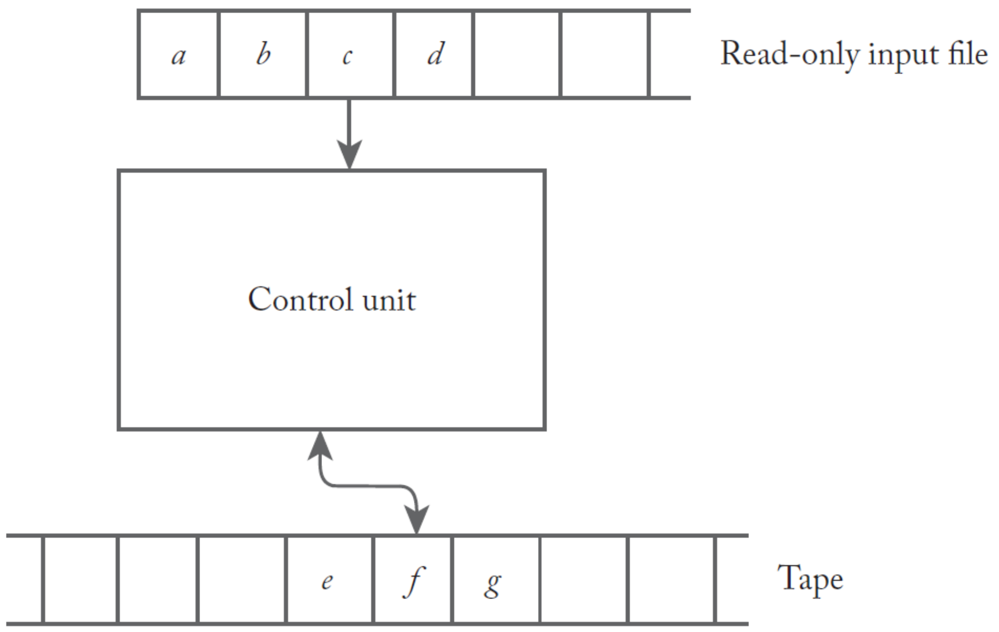
$$\widehat{\delta}\left(\widehat{q}_i, a\right) = \left(\widehat{q}_{jS}, b, R\right)$$

$$\widehat{\delta}\left(\widehat{q}_{jS}, c\right) = \left(\widehat{q}_j, c, L\right)$$

  - **The class of Turing machines with a stay-option is equivalent to the class of standard Turing machines.**

Other Models of Turing Machines

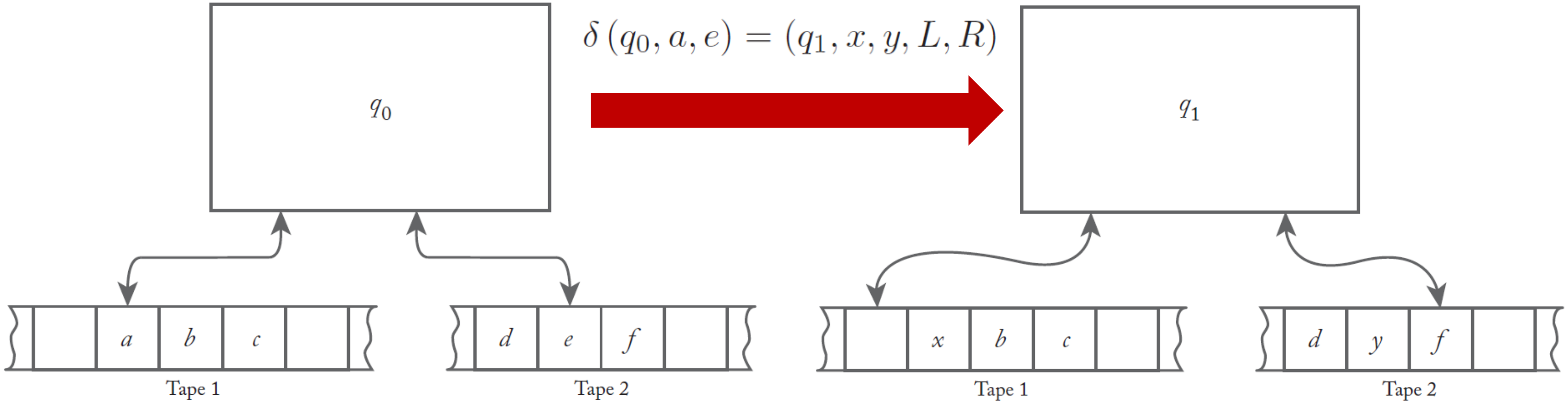
- **The Off-Line Turing Machine**



- **The class of off-line Turing machines is equivalent to the class of standard Turing machines.**

Other Models of Turing Machines

- **Multitape Turing Machines**



- **The class of multitape Turing machines is equivalent to the class of standard Turing machines.**

Other Models of Turing Machines

- **Multidimensional Turing Machines**
  - **The formal definition of a two-dimensional Turing machine**

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, U, D\}$$

  - **The class of multidimensional Turing machines is equivalent to the class of standard Turing machines.**