Theory of Machines and Languages

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Identifying Nonregular Languages

- **Example**

Show that the language

$$L = \{a^n b^k c^{n+k} : n \geq 0, k \geq 0\}$$

is not regular.

It is not difficult to apply the pumping lemma directly, but it is even easier to use closure under homomorphism. Take

$$h(a) = a, h(b) = a, h(c) = c,$$

then

$$\begin{aligned} h(L) &= \{a^{n+k} c^{n+k} : n + k \geq 0\} \\ &= \{a^i c^i : i \geq 0\}. \end{aligned}$$

But we know this language is not regular; therefore, L cannot be regular either.

Identifying Nonregular Languages

- **Example**
  - Show that the language $L = \{a^n b^l : n \neq l\}$ is not regular

- Choosing a string with $n = l + 1$ or $n = l + 2$ will not do
- We take $n = m!$ and $l = (m + 1)!$
- Suppose $|y| = k$
- $m! + (i - 1)k = (m + 1)!$
  - This is always possible since $i = 1 + \frac{mm!}{k}$ and $k \leq m$
  - The right side is therefore an integer

Identifying Nonregular Languages

- **Exercise**

Determine whether or not the following languages on $\Sigma = \{a\}$ are regular:

$L = \{a^n : n \geq 2 \text{ is a prime number}\}$.

Solution: Suppose L is regular and m is given. Let p be the smallest prime number such that $p \geq m$. Then we pick $w = a^p$ in L. The string y must then be a^k and the pumped strings will be

$$w_i = a^{p+(i-1)k} \in L, \text{ for } i = 0, 1, \ldots$$

However, $w_{p+1} = a^{p+pk} = a^{p(1+k)} \notin L$. Therefore L is not regular.

Identifying Nonregular Languages

- **Exercise**

Determine whether or not the following languages on $\Sigma = \{a\}$ are regular:

$L = \{a^n : n = k^3 \text{ for some } k \geq 0\}$.

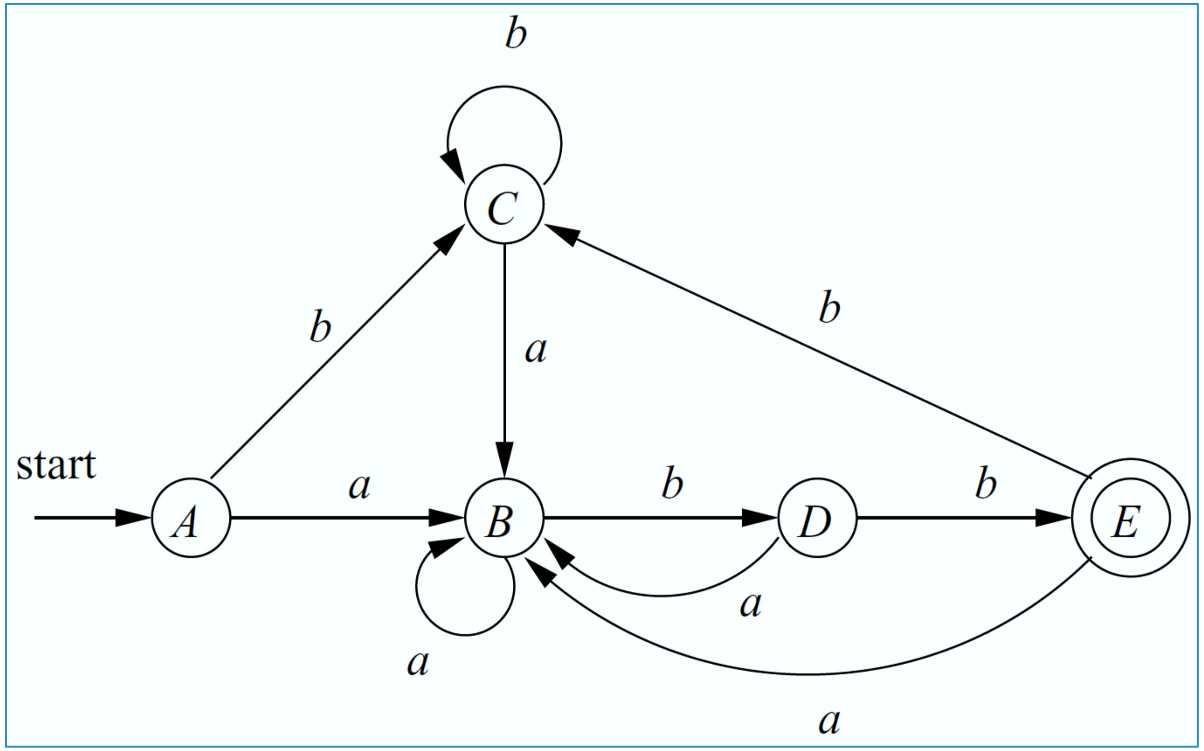
Solution: Suppose L is regular and m is given. We pick $w = a^{m^3}$ in L. The string y must then be a^k and the pumped strings will be

$$w_i = a^{m^3+(i-1)k} \in L, \text{ for } i = 0, 1, \ldots$$

However, $w_2 = a^{m^3+k} \notin L$, because that $m^3 + k < m^3 + m < (m+1)^3$. Therefore L is not regular.

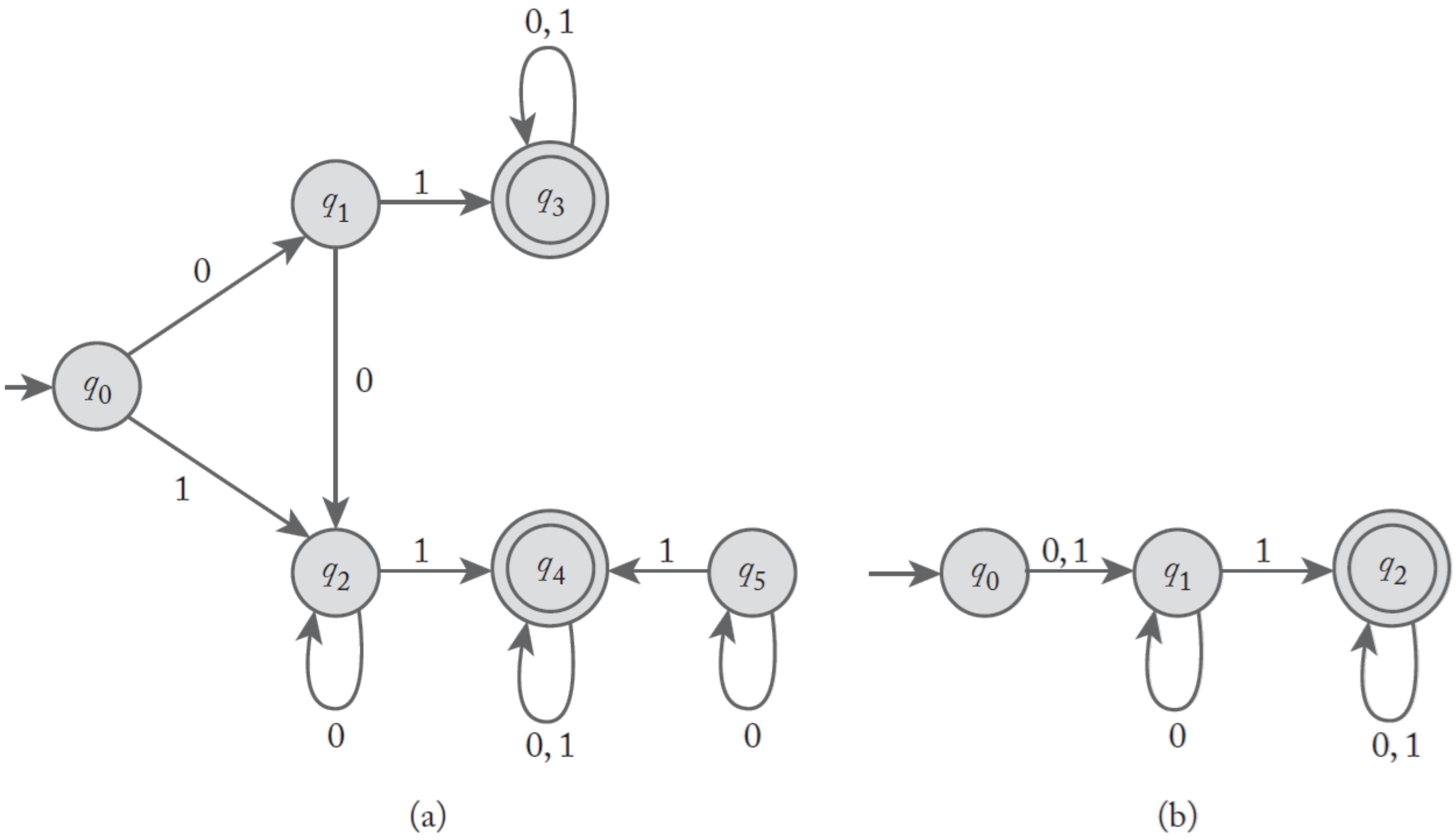
Minimizing the Number of States of a DFA

- **Example**
  - $\{A, B, C, D\}\{E\}$
  - $\{A, B, C\}\{D\}\{E\}$
  - $\{A, C\}\{B\}\{D\}\{E\}$



Minimizing the Number of States of a DFA

- **Example**



Minimizing the Number of States of a DFA

- **Example**