Theory of Machines and Languages

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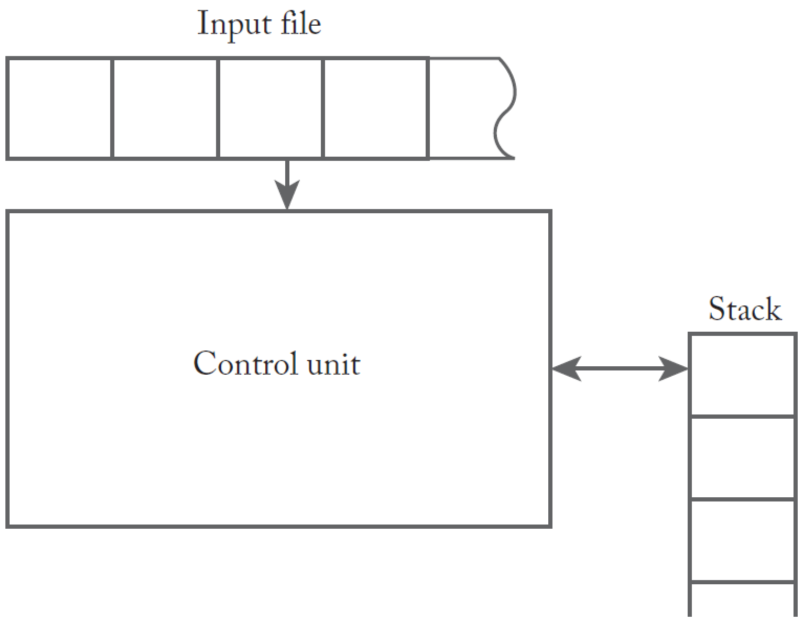
Pushdown Automata

Nondeterministic Pushdown Automata

- Each move of the control unit:
  - Reads a symbol from the input file
  - Changes the contents of the stack



Nondeterministic Pushdown Automata

- A **nondeterministic pushdown accepter (npda)** is defined by the septuple

$$\boxed{M = (Q, \Sigma, \Gamma, \delta, q_0, z, F),}$$

where

Q is a finite set of internal states of the control unit,

Σ is the input alphabet,

Γ is a finite set of symbols called the **stack alphabet**,

$\delta : Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \to$ set of finite subsets of $Q \times \Gamma^*$ is the transition function,

$q_0 \in Q$ is the initial state of the control unit,

$z \in \Gamma$ is the **stack start symbol**,

$F \subseteq Q$ is the set of final states.

Nondeterministic Pushdown Automata

- **Example**

$$\delta(q_1, a, b) = \{(q_2, cd), (q_3, \lambda)\}$$

- **At any time the control unit is in state q_1, the input symbol is a, and the symbol on top of the stack is b, then one of two things can happen:**
  1. **The control unit goes into state q_2 and the string cd replaces b on top of the stack**
  2. **The control unit goes into state q_3 with the symbol b removed from the top of the stack**
- **We assume that the insertion of a string into a stack is done symbol by symbol, *starting at the right end of the string***

Nondeterministic Pushdown Automata

- **Example**
  - Consider an npda with

$$Q = \{q_0, q_1, q_2, q_3\},$$
$$\Sigma = \{a, b\},$$
$$\Gamma = \{0, 1\},$$
$$z = 0,$$
$$F = \{q_3\},$$

  - with initial state q0 and

$$\delta(q_0, a, 0) = \{(q_1, 10), (q_3, \lambda)\},$$
$$\delta(q_0, \lambda, 0) = \{(q_3, \lambda)\},$$
$$\delta(q_1, a, 1) = \{(q_1, 11)\},$$
$$\delta(q_1, b, 1) = \{(q_2, \lambda)\},$$
$$\delta(q_2, b, 1) = \{(q_2, \lambda)\},$$
$$\delta(q_2, \lambda, 0) = \{(q_3, \lambda)\}.$$

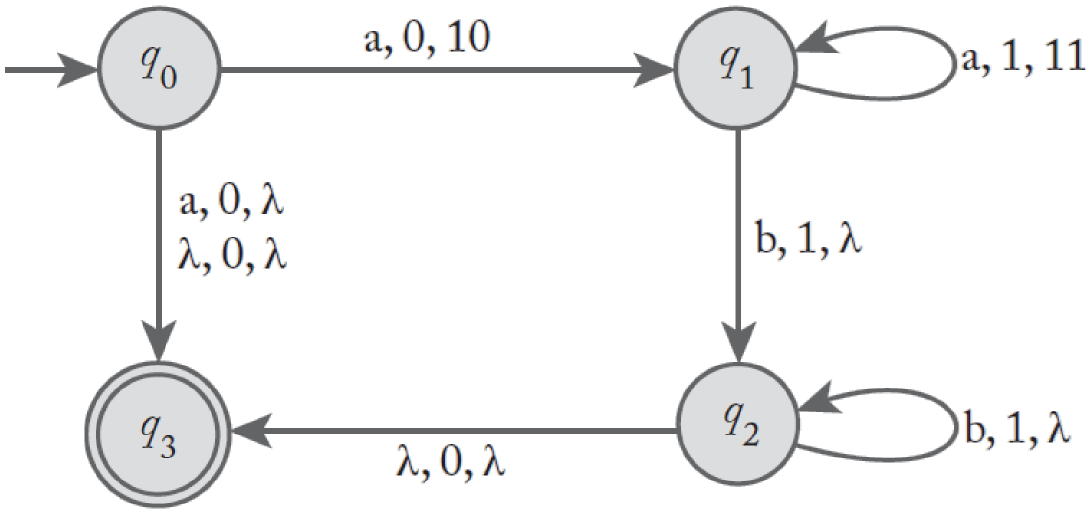
Notice that transitions are not specified for all possible combinations of input and stack symbols

Nondeterministic

$$\Rightarrow \quad L = \{a^n b^n : n \geq 0\} \cup \{a\}$$

Nondeterministic Pushdown Automata

- **Example**
  - **The transition graph**



$$\begin{aligned}
\delta(q_0, a, 0) &= \{(q_1, 10), (q_3, \lambda)\}, \\
\delta(q_0, \lambda, 0) &= \{(q_3, \lambda)\}, \\
\delta(q_1, a, 1) &= \{(q_1, 11)\}, \\
\delta(q_1, b, 1) &= \{(q_2, \lambda)\}, \\
\delta(q_2, b, 1) &= \{(q_2, \lambda)\}, \\
\delta(q_2, \lambda, 0) &= \{(q_3, \lambda)\}.
\end{aligned}$$

Nondeterministic Pushdown Automata

- **Instantaneous description**
  - **A move from one instantaneous description to another will be denoted:**

$$(q_1, aw, bx) \vdash (q_2, w, yx) \iff (q_2, y) \in \delta(q_1, a, b)$$

An arbitrary number of steps

$$(q_1, w_1, x_1) \overset{*}{\vdash} (q_2, w_2, x_2)$$

Nondeterministic Pushdown Automata

The Language Accepted by a Pushdown Automaton

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$ be a nondeterministic pushdown automaton. The language accepted by M is the set

$$L(M) = \left\{ w \in \Sigma^* : (q_0, w, z) \overset{*}{\vdash}_M (p, \lambda, u), p \in F, u \in \Gamma^* \right\}.$$

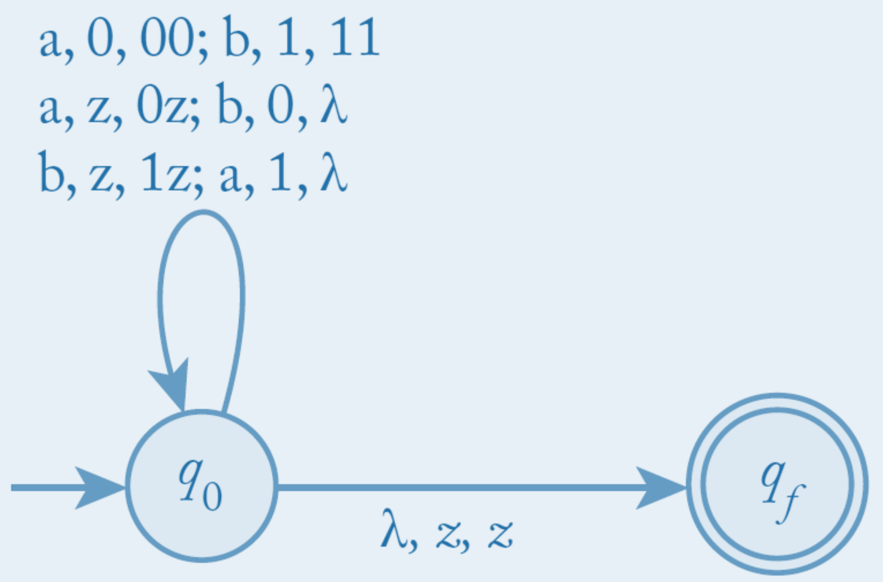
In words, $\boxed{\text{the language accepted by } M \text{ is the set of all strings that can put } M \text{ into a final state at the end of the string}}$. The final stack content u is irrelevant to this definition of acceptance.

Nondeterministic Pushdown Automata

- **Example**

Construct an npda for the language

$$L = \{w \in \{a,b\}^* : n_a(w) = n_b(w)\}$$



$$\boxed{\begin{aligned}(q_0, baab, z) &\vdash (q_0, aab, 1z) \vdash (q_0, ab, z) \\ &\vdash (q_0, b, 0z) \vdash (q_0, \lambda, z) \vdash (q_f, \lambda, z)\end{aligned}}$$