Theory of Machines and Languages

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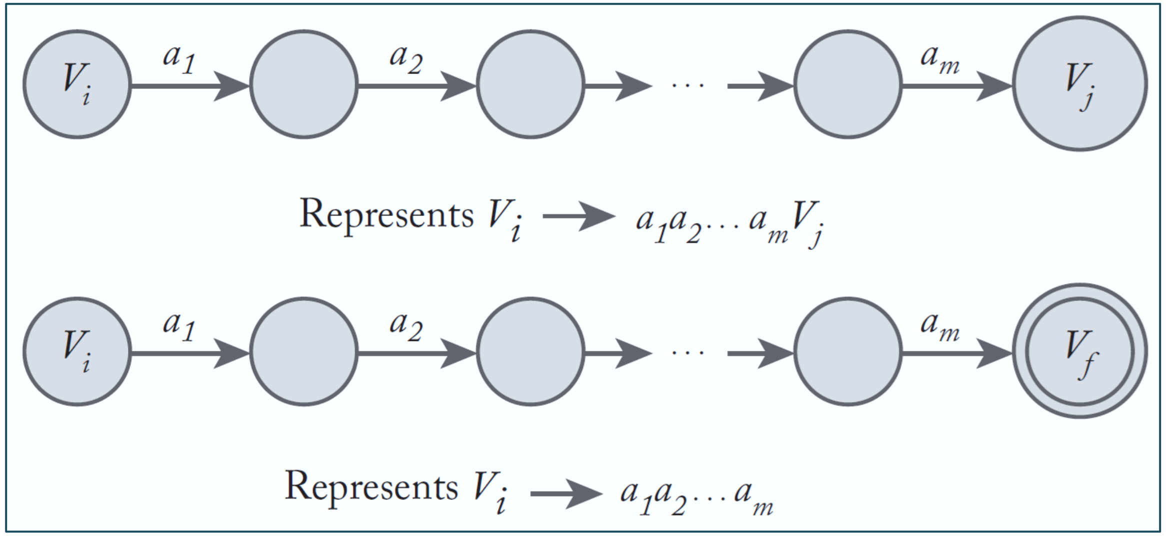
1403-1404

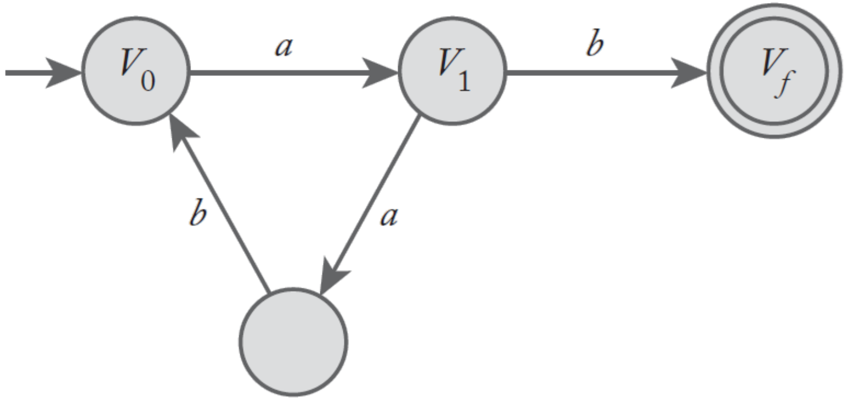
Regular Grammars

- Right-Linear Grammars Generate Regular Languages



- **Example**

$$V_0 \rightarrow aV_1,$$
$$V_1 \rightarrow abV_0|b$$



Regular Grammars

- Every regular language can be generated by some right-linear grammar
- Example

Construct a right-linear grammar for $L(aab^*a)$.

$\delta(q_0, a) = \{q_1\}$	$q_0 \longrightarrow aq_1$
$\delta(q_1, a) = \{q_2\}$	$q_1 \longrightarrow aq_2$
$\delta(q_2, b) = \{q_2\}$	$q_2 \longrightarrow bq_2$
$\delta(q_2, a) = \{q_f\}$	$q_2 \longrightarrow aq_f$
$q_f \in F$	$q_f \longrightarrow \lambda$

Regular Grammars

- A language L is regular if and only if there exists a left-linear grammar G such that $L = L(G)$

G:

$$A \rightarrow Bv,$$
$$A \rightarrow v,$$

$\widehat{G}$:

$$A \rightarrow v^R B,$$
$$A \rightarrow v^R,$$

$$L(G) = \left(L(\widehat{G})\right)^R$$

$\widehat{G}$ is right-linear

↓

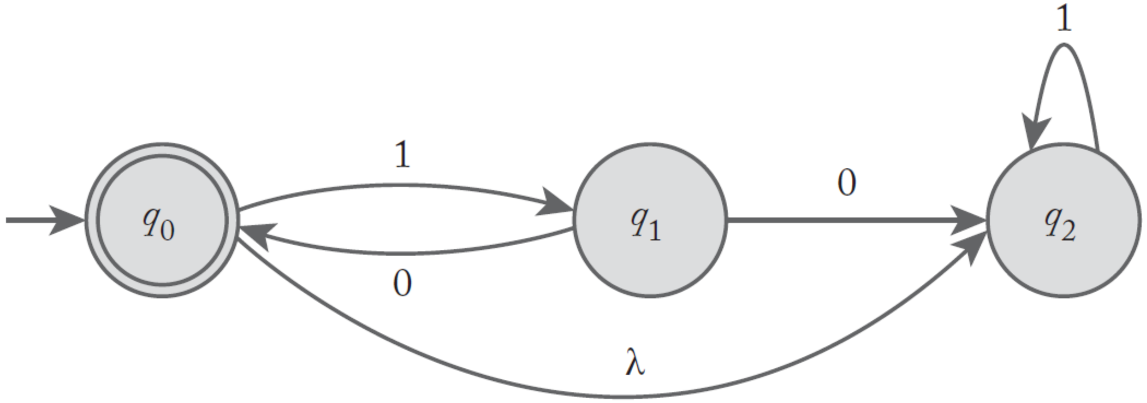
$L(\widehat{G})$ is regular

↓ The reverse of any regular language is also regular

$\left(L(\widehat{G})\right)^R = L(G)$ is regular

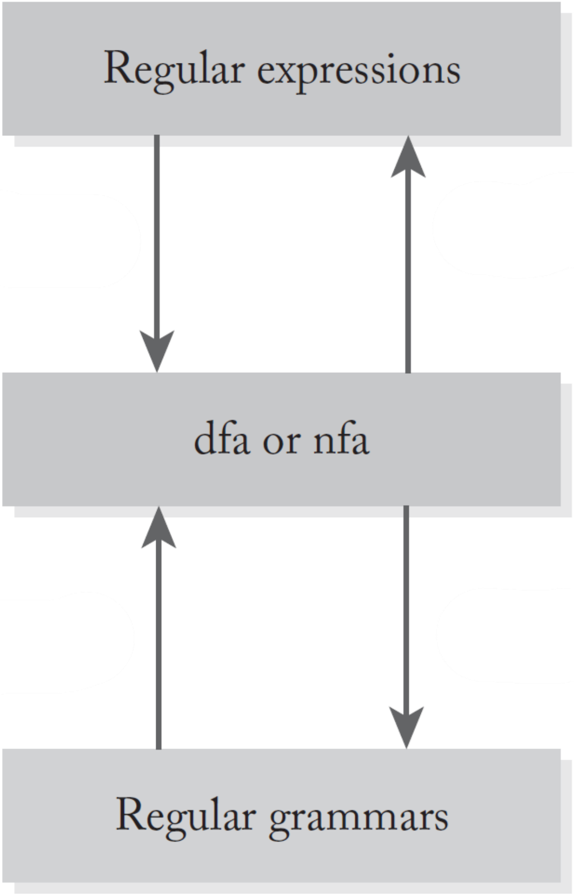
Regular Grammars

- **Exercise**
  - Construct a left-linear grammar for the following nfa



  - Find a regular grammar for the language $L = \{a^n b^m \colon n + m \text{ is odd}\}$

Regular Grammars



Properties of Regular Languages

Closure Properties of Regular Languages

- If L_1 and L_2 are regular languages, then so are $L_1 \cup L_2$, $L_1 \cap L_2$, $L_1 L_2$, $\overline{L_1}$, and L_1^*. We say that the family of regular languages is closed under union, intersection, concatenation, complementation, and star-closure.
- **Proof:** If L_1 and L_2 are regular, then there exist regular expressions r_1 and r_2 such that $L_1 = L(r_1)$ and $L_2 = L(r_2)$. By definition, $r_1 + r_2$, $r_1 r_2$, and r_1^* are regular expressions denoting the languages $L_1 \cup L_2$, $L_1 L_2$, and L_1^*
- **If $M = (Q, \Sigma, \delta, q_0, F)$ be a dfa that accepts L_1** ⇒ **$\widehat{M} = (Q, \Sigma, \delta, q_0, Q - F)$ accepts $\overline{L_1}$**

Closure Properties of Regular Languages

- **Proof (Cont.)**
  - **Intersection**

$$M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$$

$$M_2 = (P, \Sigma, \delta_2, p_0, F_2)$$

$$\widehat{M} = \left(\widehat{Q}, \Sigma, \widehat{\delta}, (q_0, p_0), \widehat{F}\right)$$

$$\widehat{Q} = Q \times P$$

$$\widehat{\delta}((q_i, p_j), a) = (q_k, p_l),$$

$$\delta_1(q_i, a) = q_k$$

$$\delta_2(p_j, a) = p_l.$$

$\widehat{F}$ is defined as the set of all (q_i, p_j), such that $q_i \in F_1$ and $p_j \in F_2$