Theory of Machines and Languages

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Parsing and Ambiguity

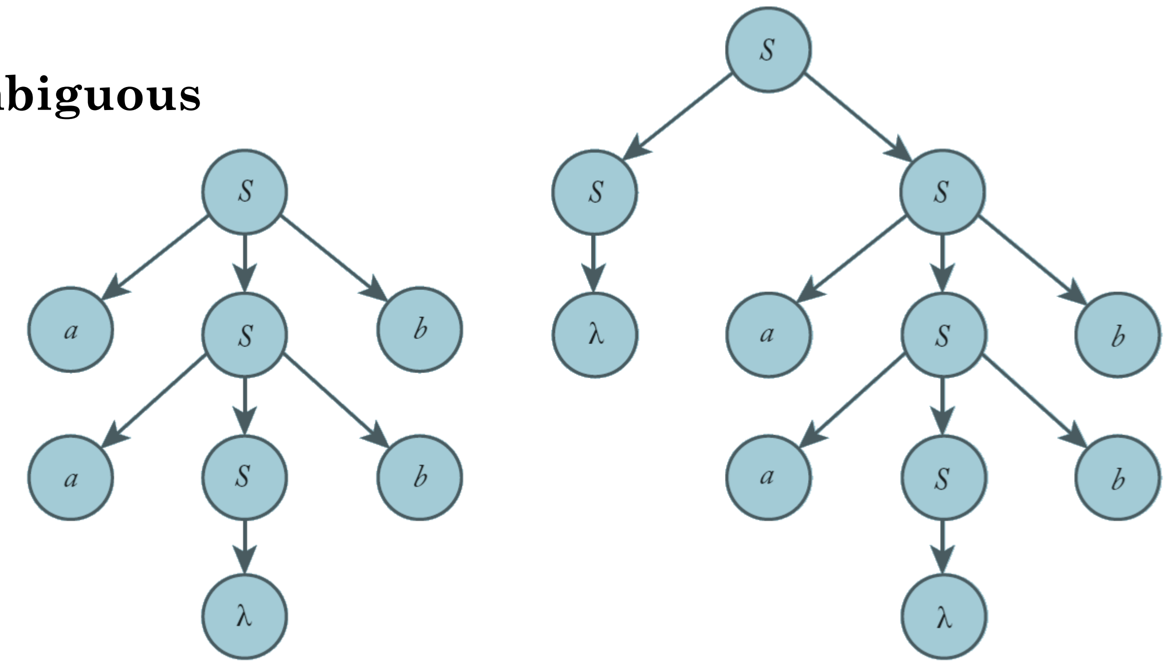
- **Example**
  - s-grammar for $L(aa^*b + b)$
    - $S \rightarrow aA \mid b$
    - $A \rightarrow aA \mid b$
  - s-grammar for $L = \{a^n b^n : n \geq 1\}$
    - $S \rightarrow aA$
    - $A \rightarrow aAB \mid b$
    - $B \rightarrow b$

Parsing and Ambiguity

- **Ambiguity in Grammars and Languages**

  A context-free grammar G is said to be **ambiguous** if there exists some $w \in L(G)$ that has at least two distinct derivation trees. Alternatively, ambiguity implies the existence of two or more leftmost or rightmost derivations.

- **Example**
  - **The grammar $S \to aSb|SS|\lambda$, is ambiguous**



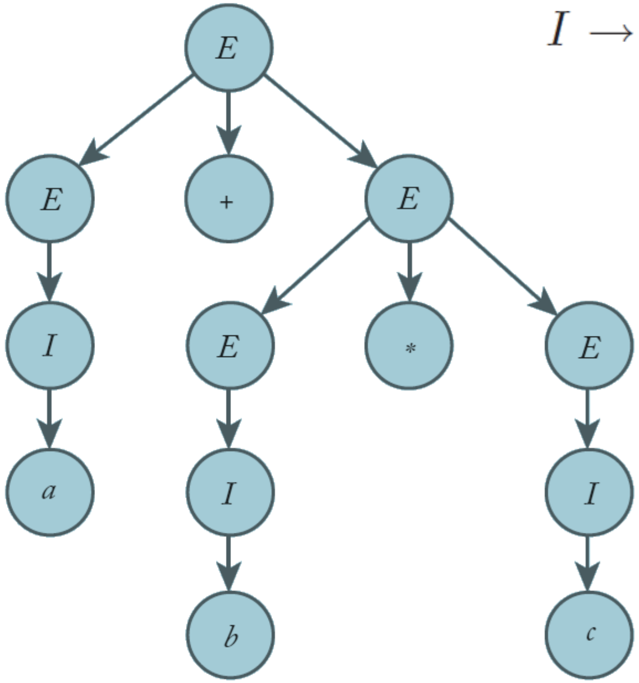
Parsing and Ambiguity

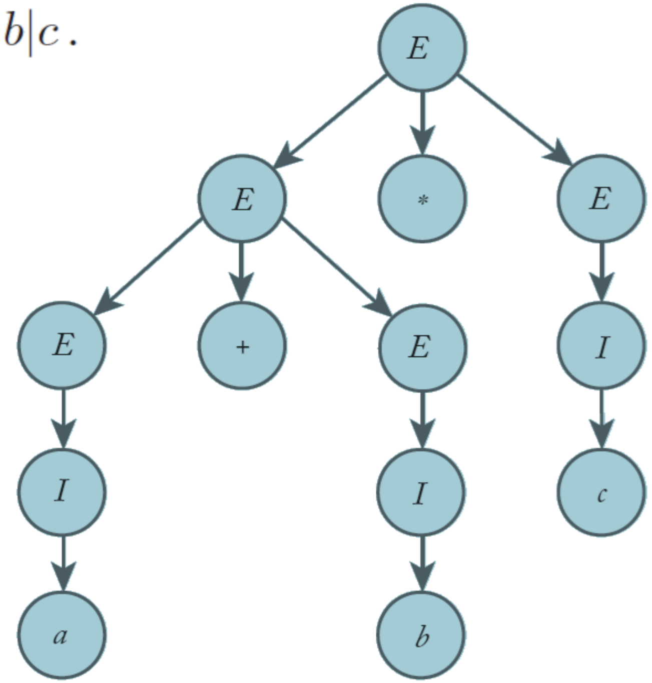
- **Example** Consider the grammar

$$
\begin{aligned}
E &\rightarrow I, \\
E &\rightarrow E + E, \\
E &\rightarrow E * E, \\
E &\rightarrow (E)\,, \\
I &\rightarrow a\,|b|c\,.
\end{aligned}
$$

- **The grammar is ambiguous**
- **One way to resolve the ambiguity is to associate precedence rules with the operators + and ∗**

Two derivation trees for $a + b * c$

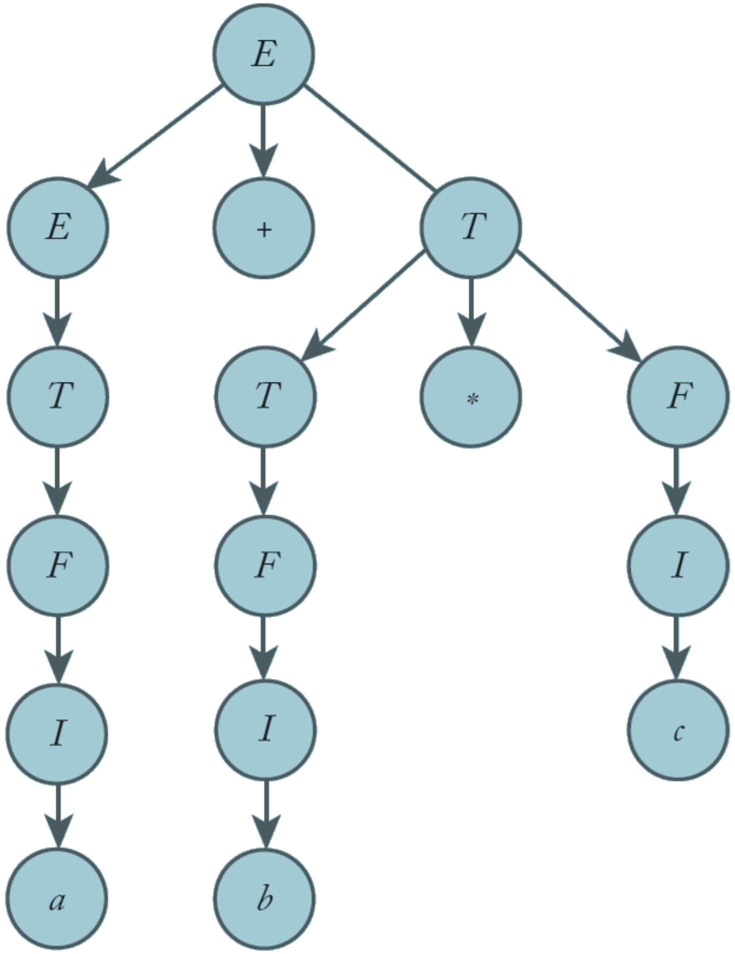




Parsing and Ambiguity

- **Example**

$$E \rightarrow T,$$
$$T \rightarrow F,$$
$$F \rightarrow I,$$
$$E \rightarrow E + T,$$
$$T \rightarrow T * F,$$
$$F \rightarrow (E),$$
$$I \rightarrow a \,|b|\, c.$$



- **The grammar is unambiguous**

Parsing and Ambiguity

- If L is a context-free language for which there exists an unambiguous grammar, then L is said to be ***unambiguous***

- If every grammar that generates L is ambiguous, then the language is called ***inherently ambiguous***

- **Example**
  - The language $L = \{a^n b^n c^m\} \cup \{a^n b^m c^m\}$ is an inherently ambiguous context-free language

$$S \rightarrow S_1 | S_2$$
$$S_1 \rightarrow S_1 c | A$$
$$A \rightarrow aAb | \lambda$$
$$S_2 \rightarrow aS_2 | B$$
$$B \rightarrow bBc | \lambda$$

The grammar is ambiguous since the string $a^n b^n c^n$ has two distinct derivations

Greibach Normal Form

- A context-free grammar is said to be in Greibach normal form if all productions have the form

$$A \to ax,$$

where $a \in T$ and $x \in V^*$.

- **Example**

$$\begin{aligned} S &\to AB, \\ A &\to aA\,|bB|\,b, \\ B &\to b \end{aligned} \quad \Rightarrow \quad \begin{aligned} S &\to aAB\,|bBB|\,bB, \\ A &\to aA\,|bB|\,b, \\ B &\to b, \end{aligned}$$

- **Example**

$$S \to abSb|aa \quad \Rightarrow \quad \begin{aligned} S &\to aBSB|aA, \\ A &\to a, \\ B &\to b, \end{aligned}$$