

Theory of Machines and Languages

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Pushdown Automata for Context-Free Languages

Example

 $S \rightarrow aSbb|a$.

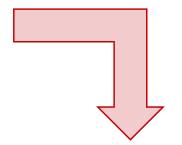
We first transform the grammar into Greibach normal form

$$S \to aSA|a,$$

$$A \rightarrow bB$$
,

$$B \to b$$

The start symbol S is put on the stack



 $\delta(q_0, \lambda, z) = \{(q_1, Sz)\}\$

$$\delta(q_1, a, S) = \{(q_1, SA), (q_1, \lambda)\}\$$

$$\delta(q_1, b, A) = \{(q_1, B)\}\$$

$$\delta(q_1, b, B) = \{(q_1, \lambda)\}\$$

 $v(q_1, v, D) = \{(q_1, \lambda)\}$

By appearing the start symbol on top of the stack the pda is put into its final state



- Remove S from the stack and replace it with SA, while reading a from the input
- For the production $S \rightarrow a$
 - Read a from the input while removing S from the stack
- •••

$$\delta(q_1, \lambda, z) = \{(q_2, \lambda)\}\$$

Pushdown Automata for Context-Free Languages

Example

$$S \rightarrow aA,$$

 $A \rightarrow aABC |bB| a,$
 $B \rightarrow b,$
 $C \rightarrow c.$

$$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}\$$

$$\delta(q_1, \lambda, z) = \{(q_f, z)\},\$$

$$\delta(q_1, a, S) = \{(q_1, A)\},\$$

$$\delta(q_1, a, A) = \{(q_1, ABC), (q_1, \lambda)\},\$$

$$\delta(q_1, b, A) = \{(q_1, B)\},\$$

$$\delta(q_1, b, B) = \{(q_1, \lambda)\},\$$

$$\delta(q_1, c, C) = \{(q_1, \lambda)\}.$$

The sequence of moves made by M in processing aaabc is

$$(q_0, aaabc, z) \vdash (q_1, aaabc, Sz)$$

$$\vdash (q_1, aabc, Az)$$

$$\vdash (q_1, abc, ABCz)$$

$$\vdash (q_1, bc, BCz)$$

$$\vdash (q_1, c, Cz)$$

$$\vdash (q_1, \lambda, z)$$

$$\vdash (q_f, \lambda, z).$$

Deterministic Pushdown Automata and Deterministic Context-Free Languages

■ A deterministic pushdown accepter (dpda) is a pushdown automaton with the following restrictions:

for every $q \in Q, a \in \Sigma \cup \{\lambda\}$ and $b \in \Gamma$

1. $\delta(q, a, b)$ contains at most one element,

2. if $\delta(q, \lambda, b)$ is not empty, then $\delta(q, c, b)$ must be empty for every $c \in \Sigma$.

For any given input symbol and any stack top, at most one move can be made

when a λ-move is possible for some configuration, no input-consuming alternative is available

Deterministic Pushdown Automata and Deterministic Context-Free Languages

A language L is said to be a **deterministic context-free language** if and only if there exists a dpda M such that L = L(M).

Example

The language

$$L = \{a^n b^n : n \ge 0\}$$

is a deterministic context-free language. The pda $M = (\{q_0, q_1, q_2\}, \{a, b\}, \{0, 1\}, \delta, q_0, 0, \{q_0\})$ with

$$\delta(q_0, a, 0) = \{(q_1, 10)\},\$$

$$\delta(q_1, a, 1) = \{(q_1, 11)\},\$$

$$\delta(q_1, b, 1) = \{(q_2, \lambda)\},\$$

$$\delta(q_2, b, 1) = \{(q_2, \lambda)\},\$$

$$\delta(q_2, \lambda, 0) = \{(q_0, \lambda)\}$$

accepts the given language.