# Theory of Machines and Languages

Fatemeh Deldar

1403-1404

# Turing Machines as Transducers

- A function $f$ with domain $D$ is said to be **Turing-computable** or just **computable** if there exists some Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, \Box, F)$ such that

$$q_0 w \overset{*}{\vdash}_M q_f f(w), \qquad q_f \in F,$$

for all $w \in D$.

- ***All the common mathematical functions, no matter how complicated, are Turing-computable***

# Turing Machines as Transducers

- **Example**
  - **Given two positive integers x and y, design a Turing machine that computes $x + y$**
    - **Positive integer $x$ is represented by $w(x) \in \{1\}^+$, such that $|w(x)| = x$**
    - **We want to design a Turing machine for performing the computation:**

$$q_0 w(x) 0 w(y) \overset{*}{\vdash} q_f w(x+y) 0$$

$$\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R), \\
\delta(q_0, 0) &= (q_1, 1, R), \\
\delta(q_1, 1) &= (q_1, 1, R), \\
\delta(q_1, \square) &= (q_2, \square, L), \\
\delta(q_2, 1) &= (q_3, 0, L), \\
\delta(q_3, 1) &= (q_3, 1, L), \\
\delta(q_3, \square) &= (q_4, \square, R).
\end{aligned}$$

$$\begin{aligned}
q_0 111011 &\vdash 1q_0 11011 \vdash 11q_0 1011 \vdash 111q_0 011 \\
&\vdash 1111q_1 11 \vdash 11111q_1 1 \vdash 111111q_1 \square \\
&\vdash 11111q_2 1 \vdash 1111q_3 10 \\
&\overset{*}{\vdash} q_3 \square 111110 \vdash q_4 111110.
\end{aligned}$$

# Turing Machines as Transducers

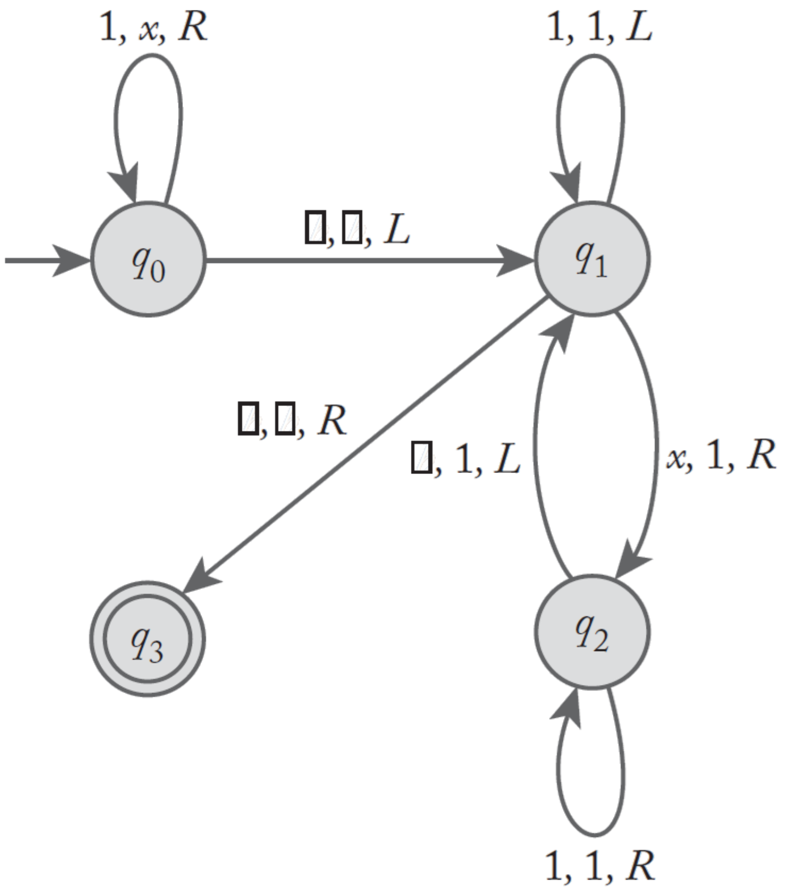
- **Example**
  - Design a Turing machine that copies strings of 1's

$$q_0 w \overset{*}{\vdash} q_f ww$$

1. Replace every 1 by an *x*
2. Find the rightmost x and replace it with 1
3. Travel to the right end of the current nonblank region and create a 1 there
4. Repeat Steps 2 and 3 until there are no more *x*'s

# Turing Machines as Transducers

- **Example (Cont.)**




$$
\begin{aligned}
q_0 11 &\vdash x q_0 1 \vdash x x q_0 \square \vdash x q_1 x \\
&\vdash x 1 q_2 \square \vdash x q_1 11 \vdash q_1 x 11 \\
&\vdash 1 q_2 11 \vdash 11 q_2 1 \vdash 111 q_2 \square \\
&\vdash 11 q_1 11 \vdash 1 q_1 111 \\
&\vdash q_1 1111 \vdash q_1 \square 1111 \vdash q_3 1111.
\end{aligned}
$$

# Turing Machines

- **Exercise:** **Construct Turing machines that will accept the following language**

$$L = \{a^n b^{2n} : n \geq 1\}$$