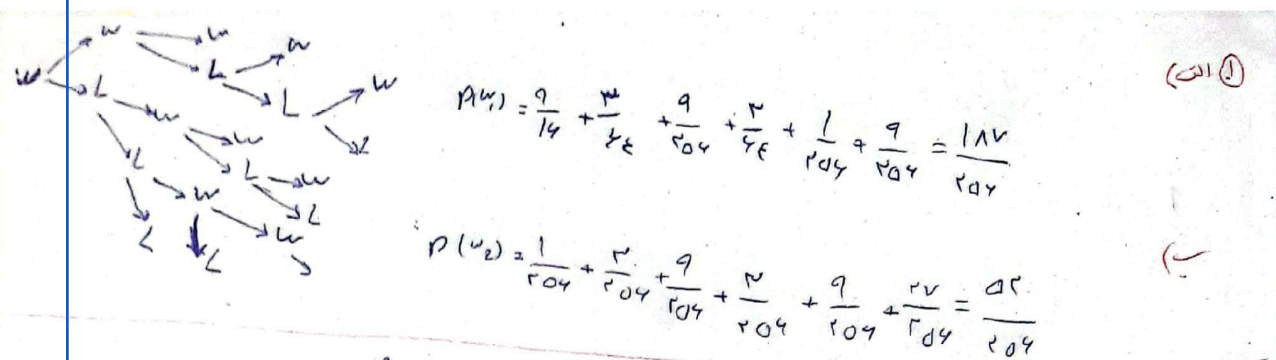


به نامه خدا

پاسخ تمرین سری چهارم

سوال (۱)



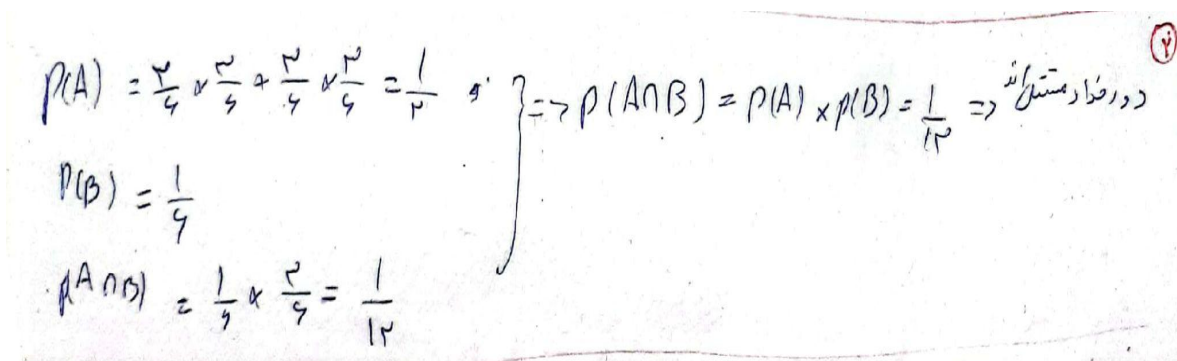
Handwritten probability tree diagram and calculations for Question 1. The tree starts with a root node branching into two main paths, each labeled 'L'. Each 'L' path further branches into two sub-paths, labeled 'W' and 'L'. The final outcomes are 'WW', 'WL', 'LW', and 'LL'.

Calculations:

$$P(W_1) = \frac{9}{14} + \frac{3}{24} + \frac{9}{204} + \frac{3}{64} + \frac{1}{204} + \frac{9}{204} = \frac{187}{204}$$

$$P(W_2) = \frac{1}{204} + \frac{3}{204} + \frac{9}{204} + \frac{3}{204} + \frac{9}{204} + \frac{27}{204} = \frac{52}{204}$$

سوال (۲)



Handwritten probability calculations for Question 2. The calculations show the probability of two independent events A and B occurring together.

Calculations:

$$P(A) = \frac{2}{4} \times \frac{2}{9} + \frac{2}{4} \times \frac{2}{9} = \frac{1}{2}$$

$$P(B) = \frac{1}{9}$$

$$P(A \cap B) = \frac{1}{2} \times \frac{2}{9} = \frac{1}{9}$$

Conclusion: $P(A \cap B) = P(A) \times P(B) = \frac{1}{18} \Rightarrow$ درضا مستقلند

سوال ۳)

راحل اول)

$$P = (13/20 * 12/19 * 7/18) / (7/20)$$

راحل دوم)

$$P(\text{توزیع درستی} | \text{توزیع}) = \frac{\frac{\binom{13}{2}}{\binom{20}{2}} \times \frac{\binom{7}{1}}{\binom{18}{1}}}{\frac{\binom{7}{2}}{\binom{20}{2}} \times \frac{\binom{13}{1}}{\binom{18}{1}} + \frac{\binom{13}{2}}{\binom{20}{2}} \times \frac{7}{18} + \frac{\binom{7}{1}}{\binom{20}{2}} \times \frac{13}{18}} = \frac{24}{57}$$

سوال ۴)

$$P(\text{توزیع درستی} | \text{توزیع}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\left. \begin{aligned} P(\text{جمع دو تا ۵ بر روی ۲ تاس}) &= \frac{7}{36} \\ P(\text{هر دو تاس ۵ باشد}) &= \frac{1}{36} \end{aligned} \right\} \Rightarrow P(\text{جمع دو تا ۵ بر روی ۲ تاس} | \text{هر دو تاس ۵ باشد}) = \frac{\frac{7}{36}}{\frac{7}{36} + \frac{1}{36}} = \frac{1}{2}$$

سوال ۵)

سوال ۶)

$$\text{امید ریاضی برای جایزه} = 0.1^2 * 10^6 - 15 * 10^3 = 10^4 - 1.5 * 10^4 = -0.5 * 10^3$$

بند ب:

$$\begin{aligned} \text{امید ریاضی دریافت جایزه} &= 0.1^2 * 10^6 + (1 - 0.1^2) * 0.1^2 * 10^6 - 15 * 10^3 = \\ &= 10^4 (1 + 1 - 0.01) - 1.5 * 10^4 \quad \text{این بار سود می دهد} \end{aligned}$$

سوال ۷)

$$\begin{cases} p_n = 1.24 p_{n-1} \\ p_1 = A \end{cases}$$

سوال ۸)

$$\begin{aligned} r = 1.24 \rightarrow p(n) &= \alpha 1.24^n \\ p(1) = A \Rightarrow \alpha &= \frac{A}{1.24} \Rightarrow p(n) = \frac{A}{1.24} 1.24^n \\ A = 10^6 \Rightarrow n = 4 \Rightarrow p(10) &= \frac{10^6}{1.24} \times 1.24^{10} = 1.24^9 \times 10^6 \end{aligned}$$

سوال (٩)

$$P(n) = 2P(n-1) + P(n-2)$$

$$P(1) = 0$$

$$P(2) = 2$$

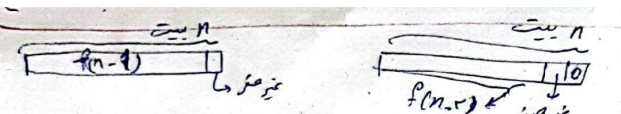
$$P(3) = 0$$

$$P(4) = 4$$

$$P(5) = 1$$

(٩)

سوال (١٠)



$$P(n) = 2P(n-1) + 2P(n-2)$$

$f(1) = 3 \Rightarrow 0, 1, 2$
 $f(2) = 1 \Rightarrow 0, 1, 2, 10, 11, 12, 20, 21, 22$

(١٠)

سوال (١١)

معادله مشخصه: $r^2 - 2r - 2 = 0 \Rightarrow r = 1 \pm \sqrt{3}$

$$f(n) = \alpha_1 (1 + \sqrt{3})^n + \alpha_2 (1 - \sqrt{3})^n$$

$$\left. \begin{aligned} f(1) &= 3 = \alpha_1 (1 + \sqrt{3}) + \alpha_2 (1 - \sqrt{3}) \\ f(2) &= 1 = \alpha_1 (1 + \sqrt{3})^2 + \alpha_2 (1 - \sqrt{3})^2 \end{aligned} \right\} \Rightarrow \begin{cases} \alpha_1 = \frac{2\sqrt{3} + 3}{9} \\ \alpha_2 = \frac{3 - 2\sqrt{3}}{9} \end{cases}$$

$$f(n) = \frac{2\sqrt{3} + 3}{9} (1 + \sqrt{3})^n + \frac{3 - 2\sqrt{3}}{9} (1 - \sqrt{3})^n$$

(١١)

