Theory of Machines and Languages

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Deterministic Finite Accepter

- **Extended transition function $\boldsymbol{\delta^*}: \boldsymbol{Q} \times \boldsymbol{\Sigma^*} \rightarrow \boldsymbol{Q}$**
- **Example:**

$$\delta(q_0, a) = q_1$$

$$\delta(q_1, b) = q_2$$

$$\Rightarrow \quad \delta^*(q_0, ab) = q_2$$

$$\delta^*(q_0, ab) = \delta(\delta^*(q_0, a), b).$$

$$\begin{aligned} \delta^*(q_0, a) &= \delta(\delta^*(q_0, \lambda), a) \\ &= \delta(q_0, a) \\ &= q_1. \end{aligned}$$

$$\delta^*(q_0, ab) = \delta(q_1, b) = q_2,$$

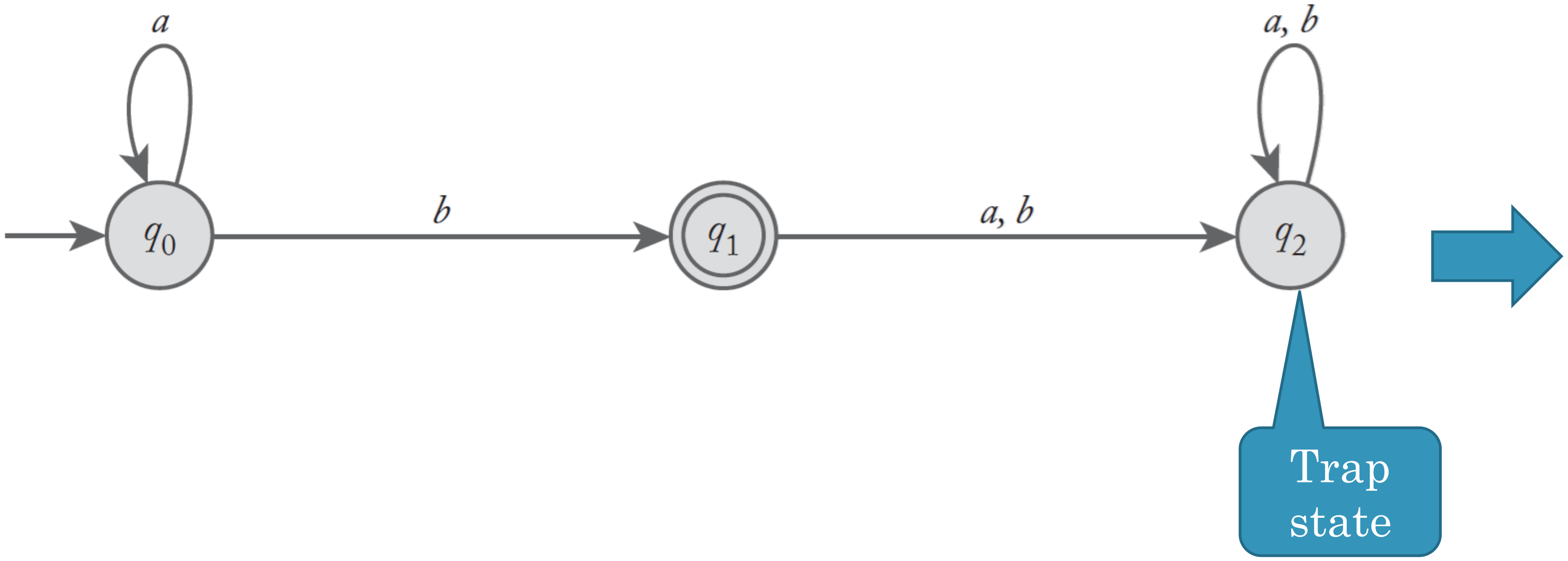
- **We can define $\boldsymbol{\delta^*}$ recursively**

$$\begin{aligned} \delta^*(q, \lambda) &= q, \\ \delta^*(q, wa) &= \delta(\delta^*(q, w), a), \end{aligned}$$

Languages and Dfa's

The language accepted by a dfa $M = (Q, \Sigma, \delta, q_0, F)$ is the set of all strings on Σ accepted by M. In formal notation,

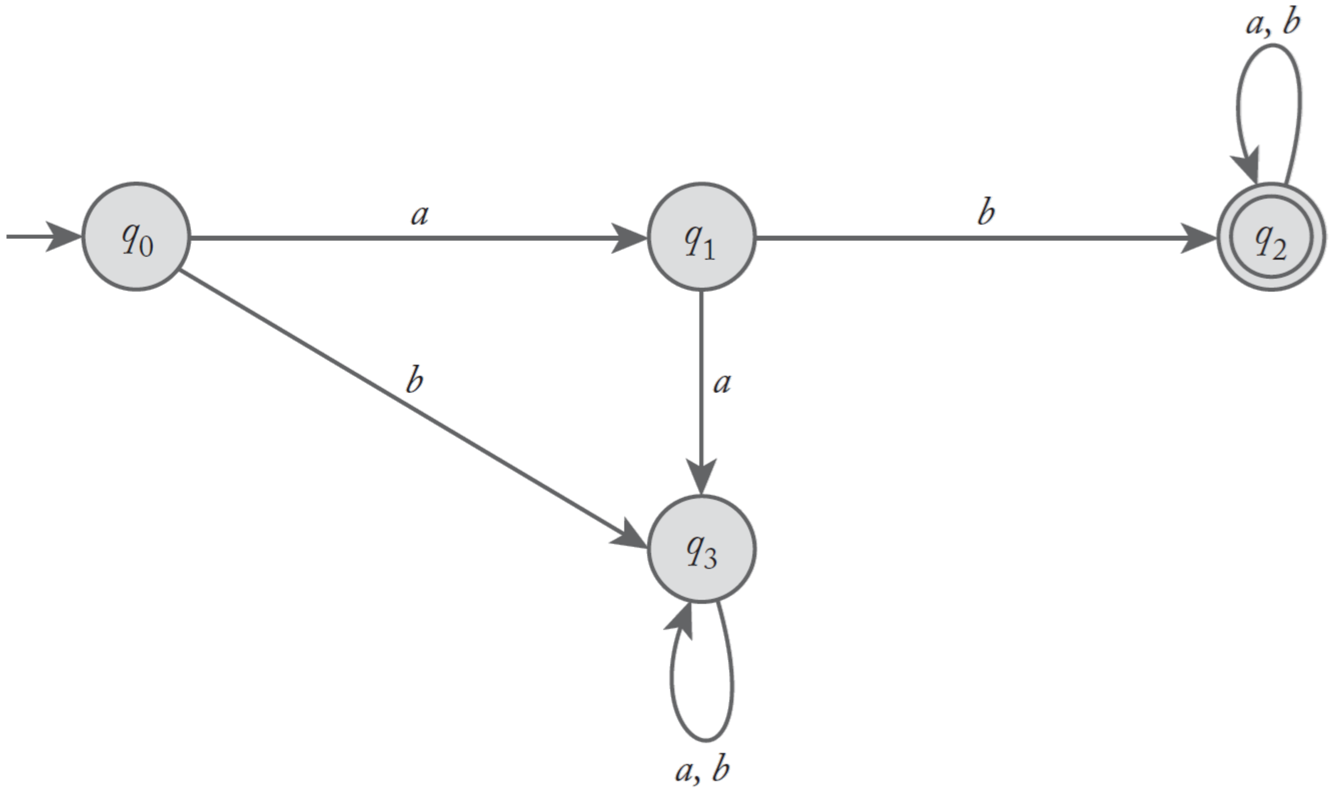
$$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}.$$



$L = \{a^n b : n \geq 0\}$

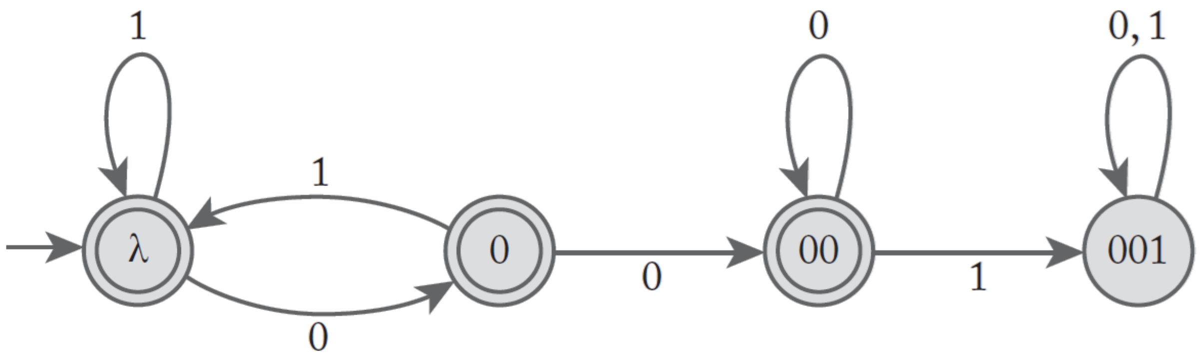
Languages and Dfa's

- **Example**
  - **Find a deterministic finite accepter that recognizes the set of all strings on $\Sigma = \{a, b\}$ starting with the prefix ab.**



Languages and Dfa's

- **Example**
  - **Find a dfa that accepts all the strings on $\{0, 1\}$, except those containing the substring 001**



Regular Languages

- A language L is called **regular** if and only if there exists some deterministic finite accepter M such that

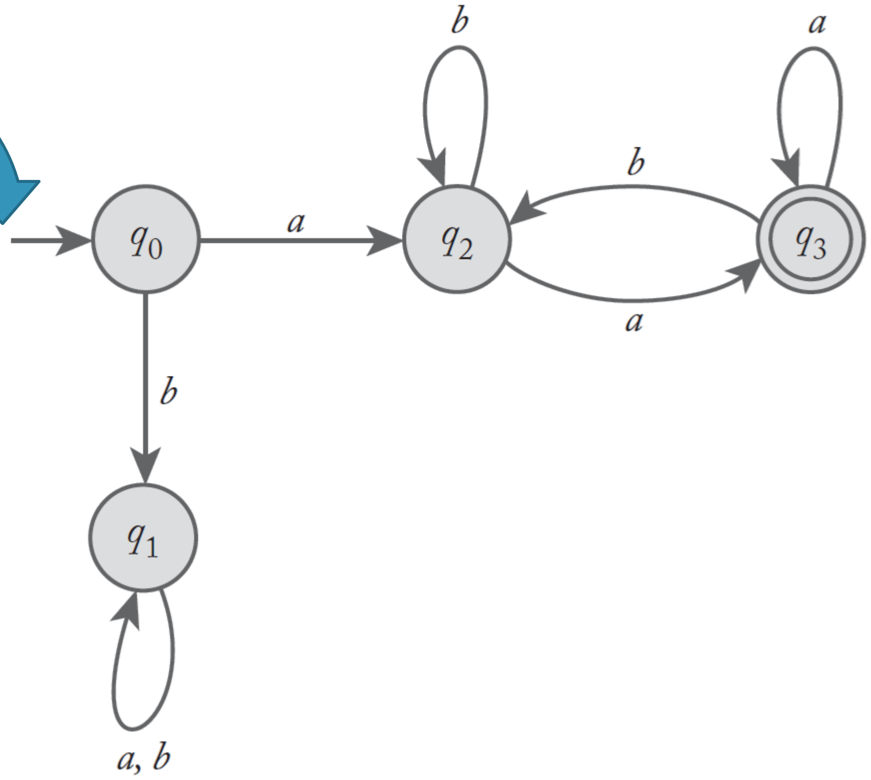
$$L = L(M).$$

- **Example**

  Show that the language

$$L = \{awa : w \in \{a,b\}^*\}$$

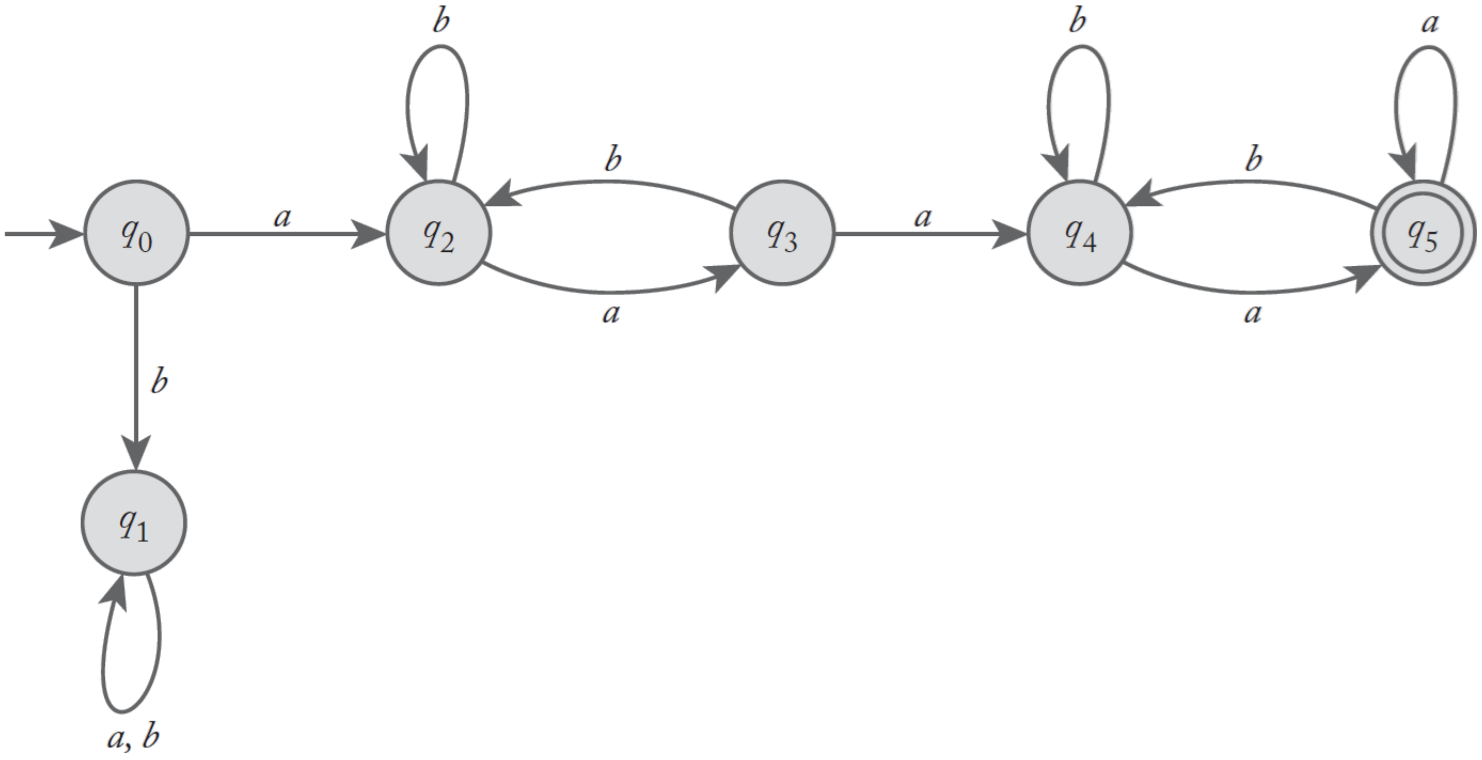
  is regular.



Regular Languages

- **Example**
  - **Let L be the language in the previous example, show that L^2 is regular**

$$L^2 = \{aw_1aaw_2a : w_1, w_2 \in \{a, b\}^*\}$$



Regular Languages

Give dfa's for the languages

(a) $L = \left\{ab^4wb^2 : w \in \{a,b\}^*\right\}$.

(b) $L = \{ab^n a^m : n \geq 3, m \geq 2\}$.

Find dfa's for the following languages on $\Sigma = \{a, b\}$.

(a) $L = \{w : |w| \mod 3 \neq 0\}$.

(b) $L = \{w : |w| \mod 5 = 0\}$.

(c) $L = \{w : n_a(w) \mod 3 < 1\}$.

(d) $L = \{w : n_a(w) \mod 3 < n_b(w) \mod 3\}$.

(e) $L = \{w : (n_a(w) - n_b(w)) \mod 3 = 0\}$.