Theory of Machines and Languages

Fatemeh Deldar

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Turing Machines

Introduction

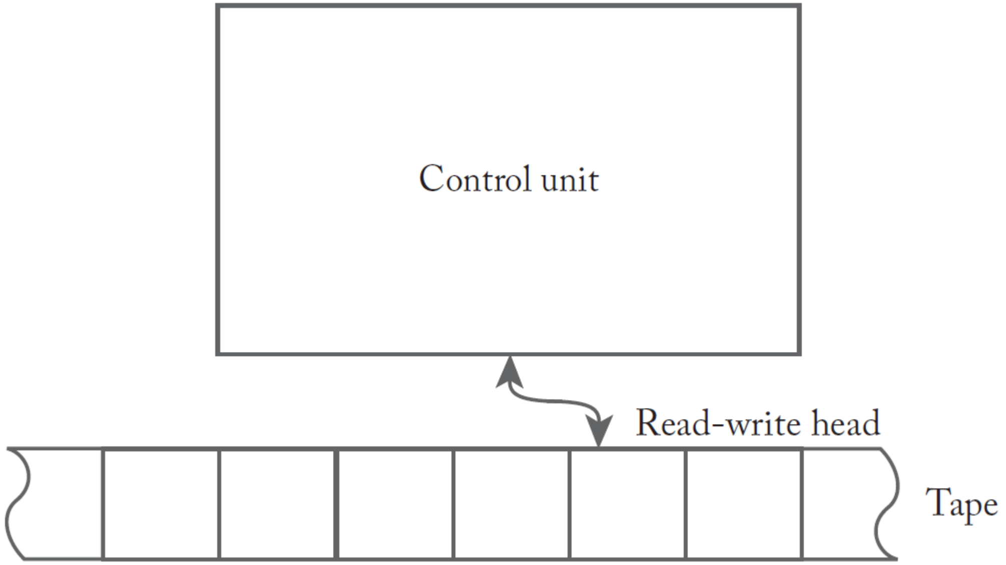
- If we compare finite automata with pushdown automata, we see that **the nature of the temporary storage creates the difference between**
  - If there is no storage, we have a finite automaton
  - If the storage is a stack, we have the more powerful pushdown automaton

> - What would happen if we used two stacks, three stacks, a queue, or some other storage device?!
> - What can we say about the most powerful of automata and the limits of computation?!

  - This leads to the fundamental concept of a ***Turing machine***

Definition of a Turing Machine

- **A Turing machine is an automaton whose:**
  - Temporary storage is a tape
  - This tape is divided into cells, each of which is capable of holding one symbol
  - Associated with the tape is a read-write head that can travel right or left on the tape and that can read and write a single symbol on each move



Definition of a Turing Machine

- A Turing machine M is defined by

$$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F),$$

where

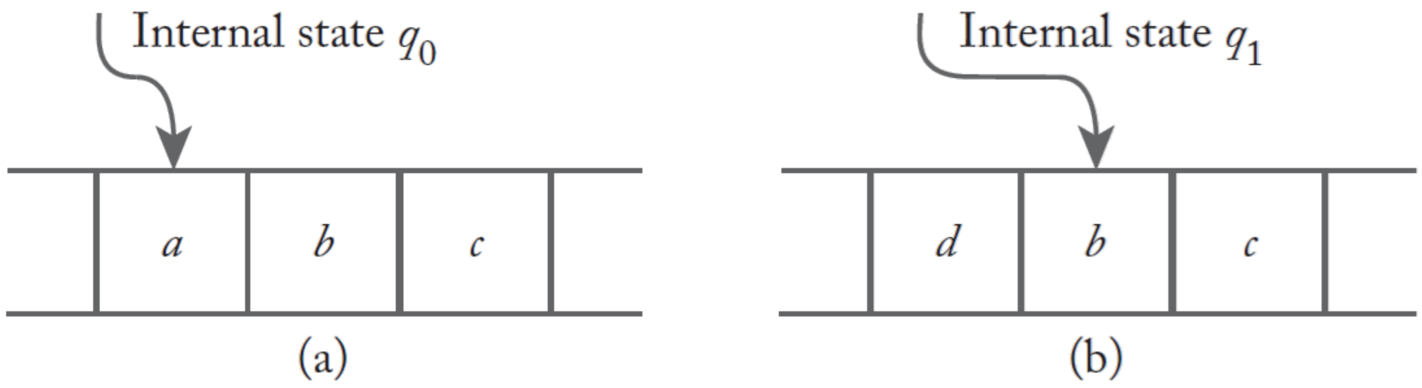
Q	is the set of internal states,
Σ	is the input alphabet,
Γ	is a finite set of symbols called the **tape alphabet**,
δ	is the transition function,
$\square \in \Gamma$	is a special symbol called the **blank**,
$q_0 \in Q$	is the initial state,
$F \subseteq Q$	is the set of final states.

$\Sigma \subseteq \Gamma - \{\square\}$

$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

Definition of a Turing Machine

- **Example** $\delta(q_0, a) = (q_1, d, R)$



(a) before the move and (b) after the move

Definition of a Turing Machine

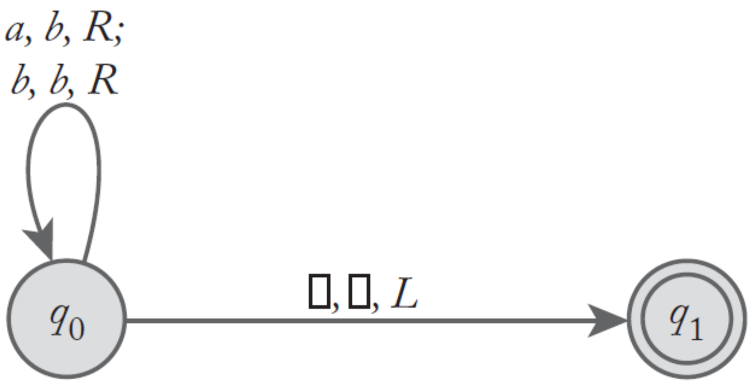
- **Example**
  - **Consider the Turing machine defined by**

$$
\begin{aligned}
Q &= \{q_o, q_1\}, & \delta(q_0, a) &= (q_0, b, R), \\
\Sigma &= \{a, b\}, & \delta(q_0, b) &= (q_0, b, R), \\
\Gamma &= \{a, b, \square\}, & \delta(q_0, \square) &= (q_1, \square, L). \\
F &= \{q_1\},
\end{aligned}
$$

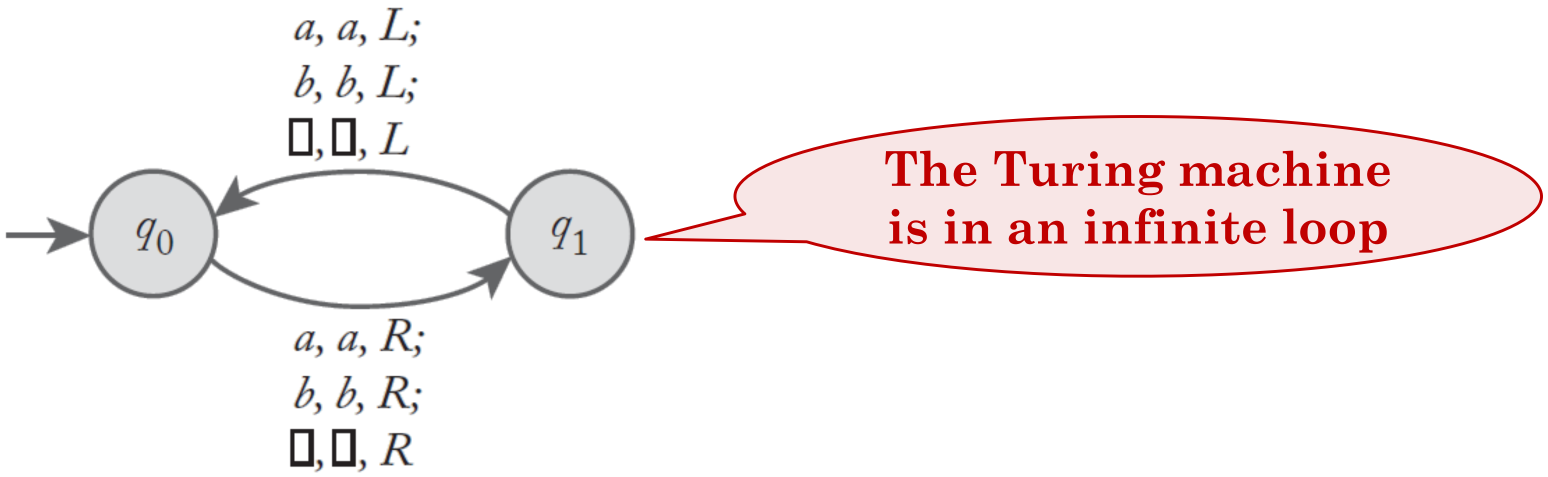
- **Any subsequent *a* will also be replaced with a *b*, but *b*'s will not be modified**
- **When the machine encounters the first blank, it will move left one cell, then halt in final state q_1**

Definition of a Turing Machine

- The transition graph of the previous example



- Example



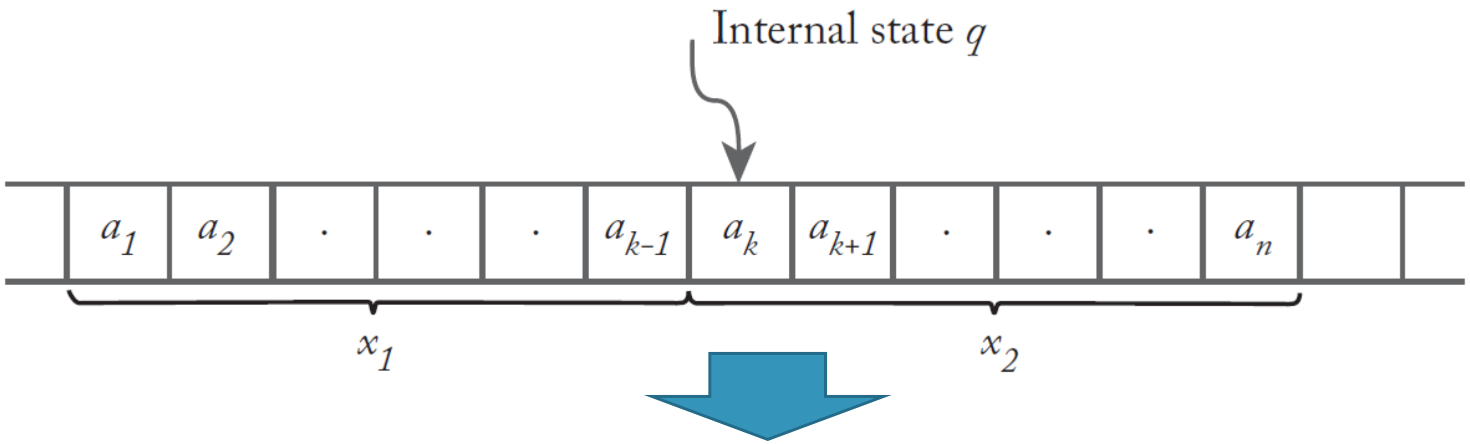
Definition of a Turing Machine

- **Standard Turing machine:**

1. **The Turing machine has *a tape that is unbounded in both directions*, allowing any number of left and right moves**
2. **The Turing machine is *deterministic* in the sense that δ defines at most one move for each configuration**
3. **There is no special input file**
   - **We assume that at the initial time the tape has some specified content**

Definition of a Turing Machine

- **Example**



$$x_1 q x_2$$

or

$$a_1 a_2 \cdots a_{k-1} q a_k a_{k+1} \cdots a_n$$

Definition of a Turing Machine

- **Example**
  - **Instantaneous descriptions**

q_0aa $\quad$ bq_0a $\quad$ $bbq_0\square$ $\quad$ bq_1b

$$q_0aa \vdash bq_0a \vdash bbq_0\square \vdash bq_1b \quad \textbf{or} \quad q_0aa \overset{*}{\vdash} bq_1b$$

Definition of a Turing Machine

$$a_1 \cdots a_{k-1} q_1 a_k a_{k+1} \cdots a_n \vdash a_1 \cdots a_{k-1} b q_2 a_{k+1} \cdots a_n \quad \Leftrightarrow \quad \delta(q_1, a_k) = (q_2, b, R)$$

$$a_1 \cdots a_{k-1} q_1 a_k a_{k+1} \cdots a_n \vdash a_1 \cdots q_2 a_{k-1} b a_{k+1} \cdots a_n \quad \Leftrightarrow \quad \delta(q_1, a_k) = (q_2, b, L)$$

$$x_1 q_i x_2 \overset{*}{\vdash} y_1 q_j a y_2$$

M **is said to halt starting from some initial configuration $x_1 q_i x_2$ if $\delta(q_j, a)$ is undefined**

$$x_1 q x_2 \overset{*}{\vdash} \infty$$

A loop and never halts

- **The sequence of configurations leading to a halt state will be called a *computation***

Turing Machines as Language Accepters

- Let $M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$ be a Turing machine. Then the language accepted by M is

$$L(M) = \left\{ w \in \Sigma^+ : q_0 w \overset{*}{\vdash} x_1 q_f x_2 \text{ for some } q_f \in F, x_1, x_2 \in \Gamma^* \right\}.$$

- **When w is not in $L(M)$, one of two things can happen:**
  1. **The machine can halt in a nonfinal state**
  2. **The machine can enter an infinite loop and never halt**

- **Example**

$$\delta(q_0, 0) = (q_0, 0, R),$$
$$\delta(q_0, \square) = (q_1, \square, R).$$

  - **If at any time a 1 is read, the machine will halt in the nonfinal state q_0**