بسم اللّه الرّحمن الرّحیم

دانشگاه صنعتی اصفهان ــ دانشکدهٔ مهندسی برق و کامپیوتر
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نظریهٔ زبان‌ها و ماشین‌ها

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Examples of Turing Machines

Example: *Here we describe a Turing machine (TM) M_2 that decides $A = \{0^{2^n} | n \geq 0\}$, the language consisting of all strings of 0s whose length is a power of 2.*

M_2 = "On input string w:

1. Sweep left to right across the tape, crossing off every other 0.

2. If in stage 1 the tape contained a single 0, accept.

3. If in stage 1 the tape contained more than a single 0 and the number of 0s was odd, reject.

4. Return the head to the left-hand end of the tape.

5. Go to stage 1."

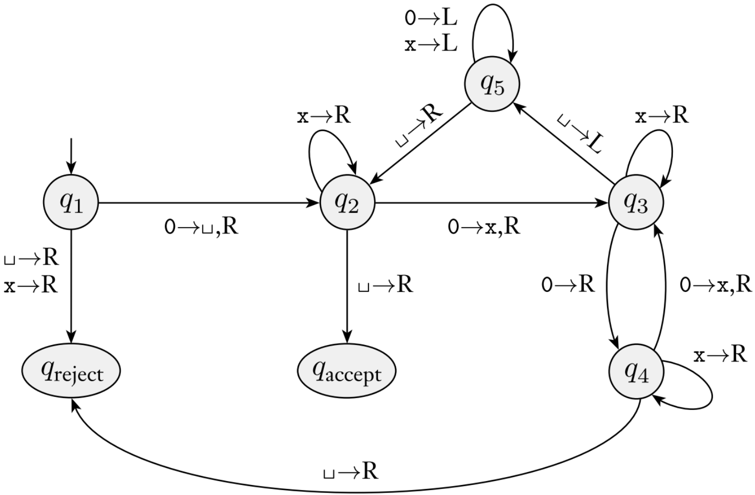
منظور از عبارت *"every other"*، «یکی‌درمیان» است.

منطق عملکرد ماشین تورینگ M_2

Each iteration of stage 1 cuts the number of 0s in half. As the machine sweeps across the tape in stage 1, it keeps track of whether the number of 0s seen is even or odd. If that number is odd and greater than 1, the original number of 0s in the input could not have been a power of 2. Therefore, the machine rejects in this instance. However, if the number of 0s seen is 1, the original number must have been a power of 2. So in this case, the machine accepts. Now we give the formal description of $M_2 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$:

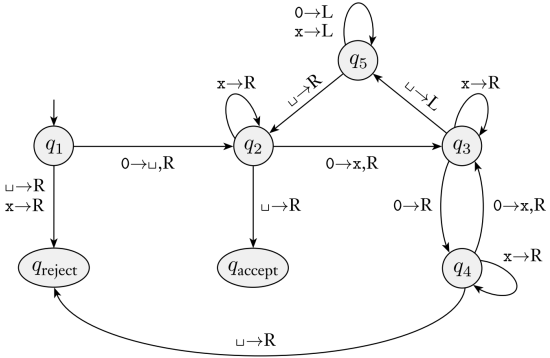
- ☞ $Q = \{q_1, q_2, q_3, q_4, q_5, q_{\text{accept}}, q_{\text{reject}}\}$,
- ☞ $\Sigma = \{0\}$, *and*
- ☞ $\Gamma = \{0, x, \sqcup\}$.
- ☞ *We describe δ with a state diagram.*
- ☞ *The start, accept, and reject states are q_1, q_{accept}, and q_{reject}, respectively.*

State diagram for Turing machine M_2



In this state diagram, the label $0 \rightarrow \sqcup, R$ appears on the transition from q_1 to q_2. This label signifies that when in state q_1 with the head reading 0, the machine goes to state q_2, writes $\sqcup$, and moves the head to the right. In other words, $\delta(q_1, 0) = (q_2, \sqcup, R)$. For clarity, we use the shorthand $0 \rightarrow R$ in the transition from q_3 to q_4, to mean that the machine moves to the right when reading 0 in state q_3 but doesn't alter the tape, so $\delta(q_3, 0) = (q_4, 0, R)$. This machine begins by writing a blank symbol over the leftmost 0 on the tape so that it can find the left-hand end of the tape in stage 4. Whereas we would normally use a more suggestive symbol such as # for the left-hand end delimiter, we use a blank here to keep the tape alphabet, and hence the state diagram, small.

$$q_1 0000 \vdash \sqcup q_2 000 \vdash \sqcup x q_3 00 \vdash \sqcup x 0 q_4 0 \vdash \sqcup x 0 x q_3 \sqcup \vdash \sqcup x 0 q_5 x \sqcup \vdash$$
$$\sqcup x q_5 0 x \sqcup \vdash \sqcup q_5 x 0 x \sqcup \vdash q_5 \sqcup x 0 x \sqcup \vdash \sqcup q_2 x 0 x \sqcup \vdash \sqcup x q_2 0 x \sqcup \vdash$$
$$\sqcup x x q_3 x \sqcup \vdash \sqcup x x x q_3 \sqcup \vdash \sqcup x x q_5 x \sqcup \vdash \sqcup x q_5 x x \sqcup \vdash \sqcup q_5 x x x \sqcup \vdash$$
$$q_5 \sqcup x x x \sqcup \vdash \sqcup q_2 x x x \sqcup \vdash \sqcup x q_2 x x \sqcup \vdash \sqcup x x q_2 x \sqcup \vdash \sqcup x x x q_2 \sqcup \vdash$$
$$\sqcup x x x \sqcup q_{\text{accept}}$$



Example: ***The following is a formal description of***

$$M_1 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}}),$$

the Turing machine that we informally described for deciding the language $B = \{w\#w \mid w \in \{0,1\}^*\}$.

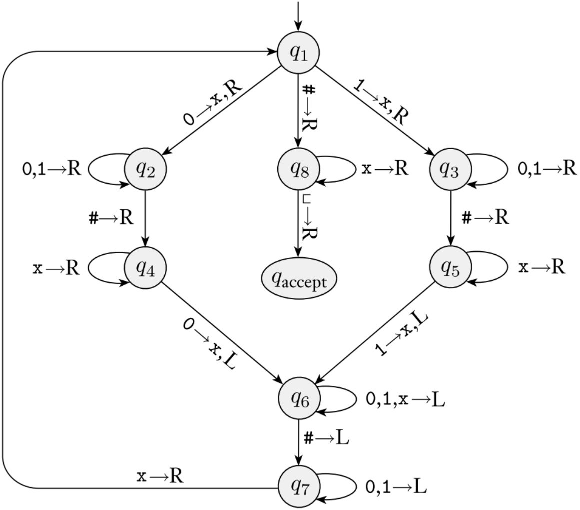
☞ $Q = \{q_1, q_2, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$,

☞ $\Sigma = \{0, 1, \#\}$, ***and*** $\Gamma = \{0, 1, \#, x, \sqcup\}$.

☞ ***We describe δ with a state diagram (see the following figure).***

☞ ***The start, accept, and reject states are q_1, q_{accept}, and q_{reject}, respectively.***

State diagram for Turing machine M_1



You will find the label $0, 1 \rightarrow R$ on the transition going from q_3 to itself. That label means that the machine stays in q_3 and moves to the right when it reads a 0 or a 1 in state q_3. It doesn't change the symbol on the tape. To simplify the figure, we don't show the reject state or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. Thus, because in state q_5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that the head moves right in each of these transitions to the reject state.

Example: $L = \{a^i b a^j | 0 \le i < j\}$