Theory of Machines and Languages

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1403-1404

Nondeterministic Finite Accepter

A **nondeterministic finite accepter** or **nfa** is defined by the quintuple

$$M = (Q, \Sigma, \delta, q_0, F),$$

where Q, Σ, q_0, F are defined as for deterministic finite accepters, but

$$\delta : Q \times (\Sigma \cup \{\lambda\}) \to 2^Q.$$

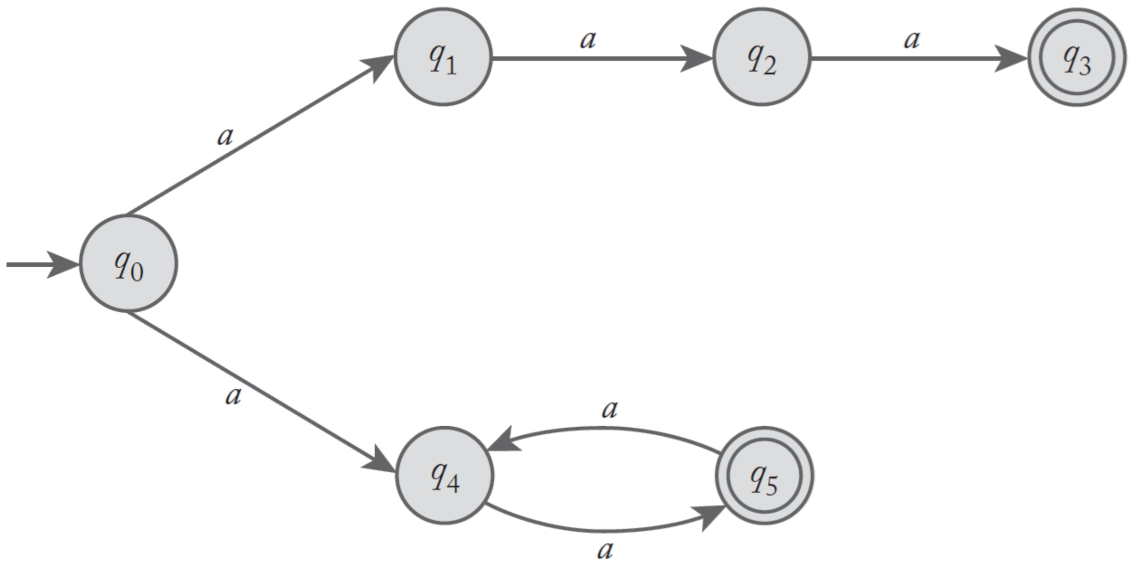
Three major differences between this definition and the definition of a dfa:

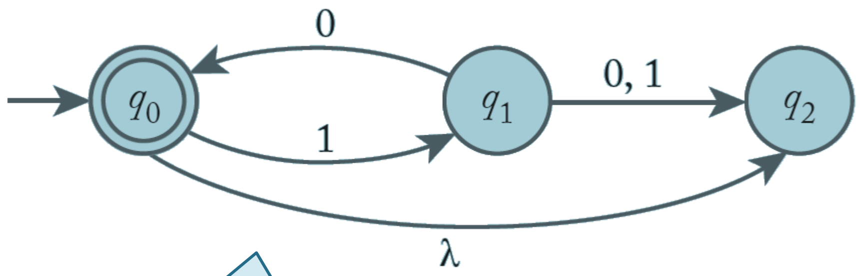
- **The range of δ is in the powerset 2^Q**
- **We allow λ as the second argument of δ**
- **The set $\delta(q_i, a)$ may be empty**

- **A string is accepted by an nfa if there is some sequence of possible moves that will put the machine in a final state at the end of the string**

Nondeterministic Finite Accepter

- **Example**





- **For 10 there are two walks, one leading to q_0, the other to q_2**
- **Even though q_2 is not a final state, the string is accepted because one walk leads to a final state**

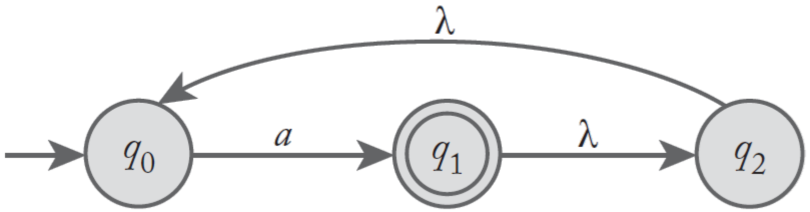
Nondeterministic Finite Accepter

- The transition function can be extended so its second argument is a string

$$\delta^*(q_i, w) = Q_j$$

Q_i is the set of all possible states the automaton may be in, having started in state q_i and having read w

- Example



$$\delta^*(q_1, a) = \{q_0, q_1, q_2\}$$
$$\delta^*(q_2, \lambda) = \{q_0, q_2\}$$
$$\delta^*(q_2, aa) = \{q_0, q_1, q_2\}$$

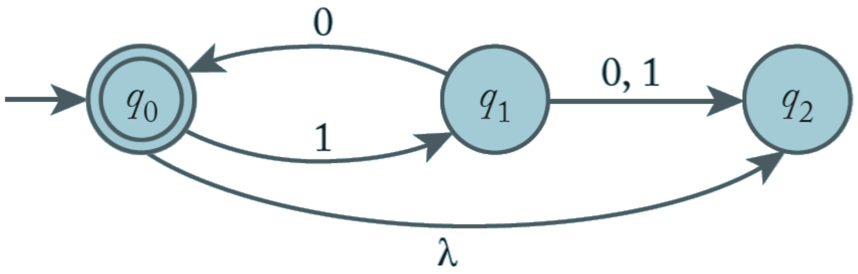
Nondeterministic Finite Accepter

The language L accepted by an nfa $M = (Q, \Sigma, \delta, q_0, F)$ is defined as the set of all strings accepted in the above sense. Formally,

$$\boxed{L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \cap F \neq \varnothing\}.}$$

In words, the language consists of all strings w for which there is a walk labeled w from the initial vertex of the transition graph to some final vertex.

- **Example**



$$L = \{(10)^n : n \geq 0\}$$

- **What happens when this automaton is presented with the string w = 110**

Dead configuration

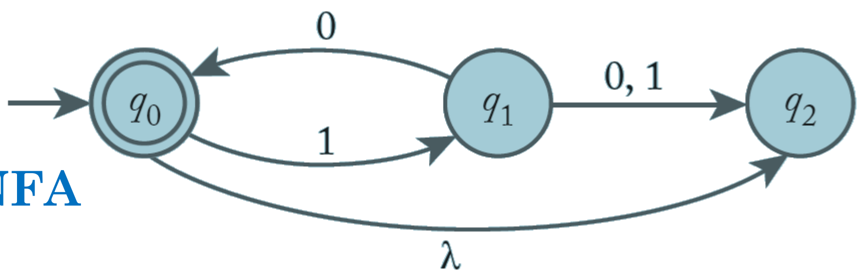
Equivalence of Dfa's and Nfa's

Two finite accepters, M_1 and M_2, are said to be equivalent if

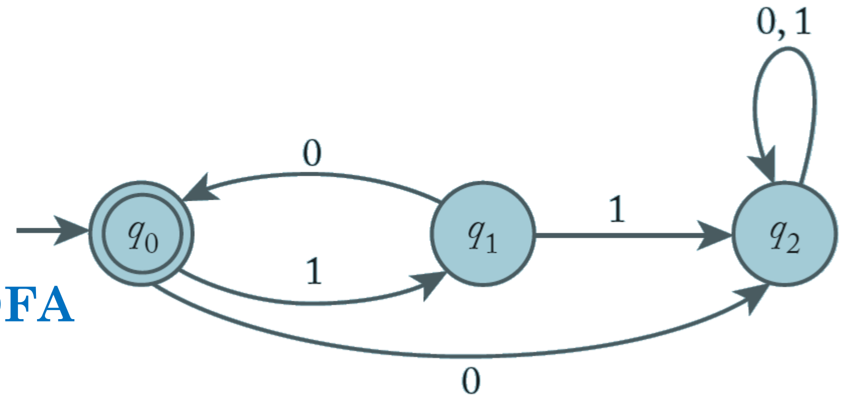
$$L(M_1) = L(M_2),$$

that is, if they both accept the same language.

- **Example**



NFA



DFA