1 Problem 1

Given

$$\xi \sim \mathrm{Pois}(\lambda),$$

$$\eta = \sum_{k=1}^{\xi} B_k,$$

$$B_k \sim \mathrm{Bern}(p),$$

we shall prove that

$$\eta \sim \mathrm{Pois}(p\lambda).$$

We could use the fact that

$$G_\eta(t) = G_\xi(G_{B_0}(t)),$$

where G_X denotes the probability generating function of a random variable X supported on $\mathbb{Z}$: $G_X(t) = \sum_k \Pr(X = k)t^k$.

It's known that

$$G_\xi(t) = \exp(\lambda(t-1)),$$

and obviously

$$G_{B_0} = (1-p) + pt.$$

From that we get:

$$G_\eta(t) = \exp(\lambda((1-p) + pt - 1)) = \exp(p\lambda(t-1))$$

which means $\eta \sim \mathrm{Pois}(p\lambda)$.

2 Problem 2

Let t_1, t_2 denote the density functions of the times $T_1 \sim \mathrm{Norm}(30, 10^2)$, $T_2 \sim \mathrm{Norm}(20, 5^2)$ the strict and kind reviewers spend to check the application respectively. Then the time T an average application is reviewed is distributed according to the density function $t = \frac{1}{2}(t_1 + t_2)$.

No time to justify and reason, but I'd guess

$$\Pr(\mathrm{kind}\big|T = 10) = \frac{\mathrm{dPr}(T = 10\big|\mathrm{kind})\Pr(\mathrm{kind})}{\mathrm{dPr}(T = 10)} =$$

$$= \frac{\frac{1}{2}t_2(10)\frac{1}{2}}{\frac{1}{2}(t_1(10) + t_2(10))} = \frac{1}{2}\frac{\exp(-\frac{1}{25})}{\exp(-\frac{1}{25}) + \frac{1}{2}\exp 0}.$$

$$= \frac{1}{2}\frac{\exp(-\frac{1}{25})}{\exp(-\frac{1}{25}) + \frac{1}{2}}.$$