Algorithmic Game Theory

SS 2015

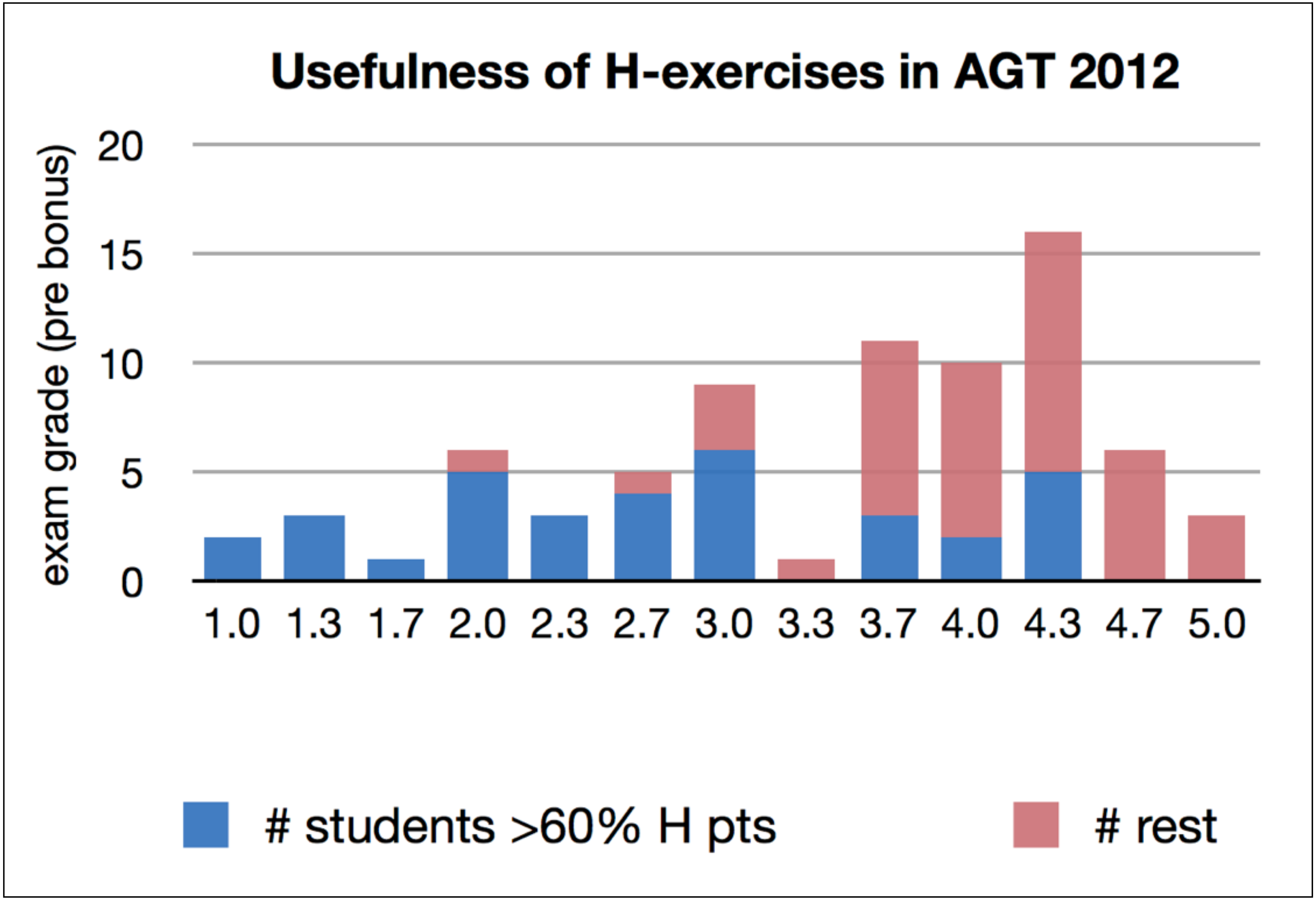
DSS
Decision Sciences & Systems

Administrative Issues

- Lectures: Felix Brandt
  - Tuesdays, 12.15 - 13.45, MI HS 2
    - No lectures on May 5th and 26th
  - *AGT course website*
  - Lecture slides, tablet notes, and hopefully video recordings will be published in Moodle after each lecture.
- Tutorials: Johannes Hofbauer & Paul Stursberg
  - Group 1: Mondays, 10.15 - 11.45, Room 01.10.011
  - Group 2: Mondays, 18.00 - 19.30, Room 01.10.011
  - Group 3: Tuesdays, 10.15 - 11.45, Room 01.10.011
  - No tutorials on May 4th, 5th, 25th, and 26th
  - Please sign up for one tutorial using TUMonline after 20.00 today.
  - Exercise sheets will be published in Moodle each Tuesday.
  - Use the Moodle AGT discussion board to discuss exercises.

Exercises

- Exercises are not mandatory, but highly recommended.
- Each exercise sheet usually contains
  - one **G** (game) exercise
    - *interactive*, you play with/against each other
    - answer has to be submitted using Moodle by midnight each Saturday
  - two **H** (homework) exercises
    - you are strongly encouraged to prepare those before the tutorial
    - will be discussed in the tutorial
    - solution hints will be published online after the tutorial
  - one **T** (tutorial) exercise
    - will be presented by the tutor in the tutorial
    - students are not expected to prepare them beforehand
    - solution hints will be published online after the tutorial
- The real meat of this course is taking place in the tutorials!
- There will be 2-3 midterm quizzes via Moodle (**Q**-exercises).


Usefulness of H-exercises in AGT 2012
exam grade (pre bonus)
20
15
10
5
0
1.0
1.3
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2.0
2.3
2.7
3.0
3.3
3.7
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4.3
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5.0
students >60% H pts
rest

Exam & Bonus

- Exam
  - Probably between July 14th and July 17th 2015
  - Grading scale on course website
  - Winter term exam in early October (if necessary)
- You can get a grade bonus by doing well in G-exercises and Q-exercises.
  - The overall grade you get for G- and Q-exercises (so-called G-grade) can be used to *improve* the grade of a *passed* exam.
  - If you pass the exam and your G-grade is better than your exam grade, then your final grade will the weighted average of your exam grade (80%) and your G-grade (20%).
  - The bonus only applies to the exam of the summer term 2015, the grades of later exams are not affected.
  - More details on course website

Motivation

- What is game theory?
  - The mathematical study of strategic behavior in interactive environments, in which the well-being of each agent not only depends on his own decisions but also on those of other agents.
  - "*Interactive decision theory*"
  - One characteristic of game theory is the lack of an indisputable notion of optimality.
  - Applications: *auctions, voting, college admission, cost-sharing, routing, file sharing, reputation systems, airport security, understanding interaction (e.g., in economics, biology, politics)*
- This course focusses on foundations rather than applications.

Expected Background

- It is expected that your are familiar with
  - standard proof techniques (e.g., proof by induction, proof by contradiction),
  - basic mathematical concepts (e.g., directed graphs, probabilities, convexity, continuity, convergence), and
  - basic concepts from theoretical computer science (e.g., polynomial-time algorithms, NP-completeness, linear programming).
- Some of the mathematical background required is nicely covered in this document by Itzhak Gilboa.
  - You can find more references on the course website.

Key Questions

- How do we formalize rational decision-making?
- Which actions should/will a decision-maker take in interactive situations?
  - *How can these situations be compactly represented?*
    - The naive representation is of exponential size.
  - *Is it possible to efficiently compute these actions?*
- Which coalitions will form in cooperative settings? How should individual contributions be valued?
  - *How can these settings be compactly represented?*
    - Again, the naive representation is of exponential size.
  - *Is it possible to efficiently compute these values and coalitions?*
- How can we design mechanisms for self-interested agents in order to obtain a desirable outcome?

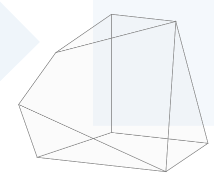
Decision Sciences and Systems

Prof. Martin Bichler and Prof. Felix Brandt

dss.in.tum.de

operations research (WI4)

Optimal planning using linear and integer linear programming: production planning, vehicle routing, inventory management, etc.



multiagent systems (sem.)

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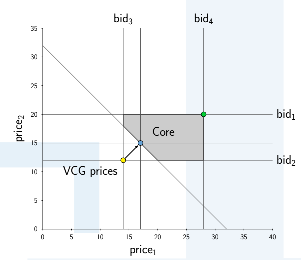
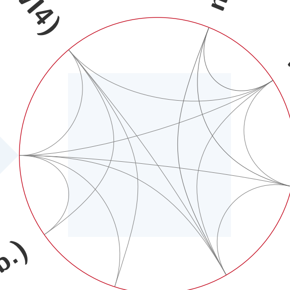
	$0, 1$	$1, 0$	$\frac{1}{2}, \frac{1}{2}$



auction theory and market design

Design of efficient auctions: combinatorial auctions, spectrum auction design, assignment markets, etc.



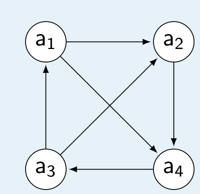
algorithmic game theory

Modeling interaction and computing optimal strategies: utility theory, normal-form games, Nash equilibrium, etc.

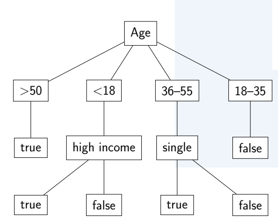
computational social choice

Rational choice and the axiomatics of voting rules: Arrow's impossibility theorem, Kemeny's rule, tournament solutions, etc.

economics and computation (sem.)



decision support systems (lab.)



business analytics

Learning human choice behavior from empirical observations: decision trees, logistic regression, Bayesian networks, etc.

Technische Universität München

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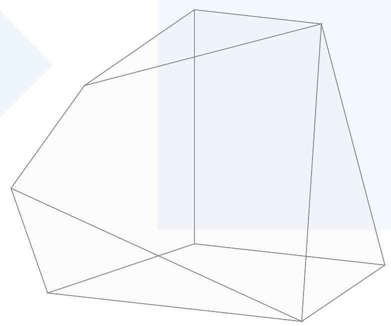

operations research (WI4)

multiagent systems (sem.)

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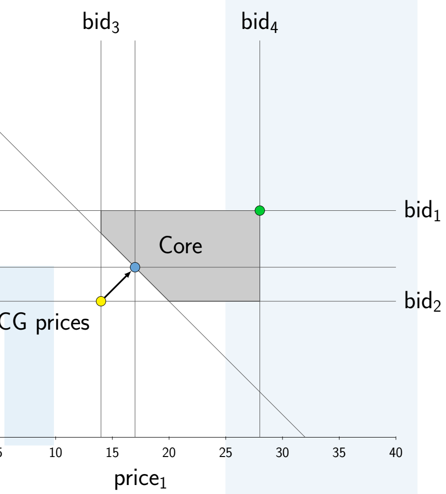
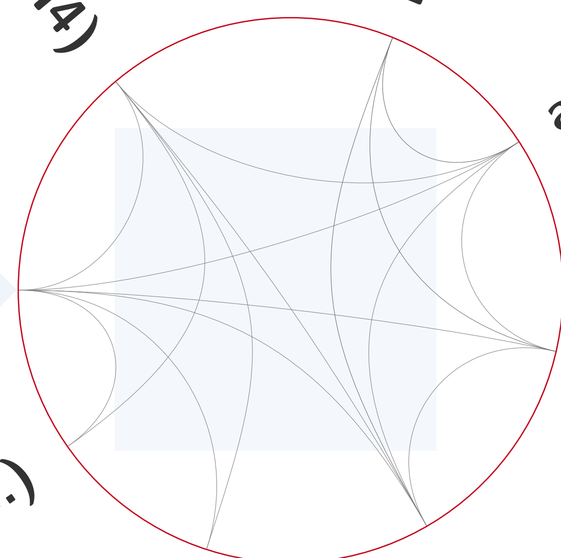


algorithmic game theory

Modeling interaction and computing optimal strategies: utility theory, normal-form games, Nash equilibrium, etc.

theory and market design



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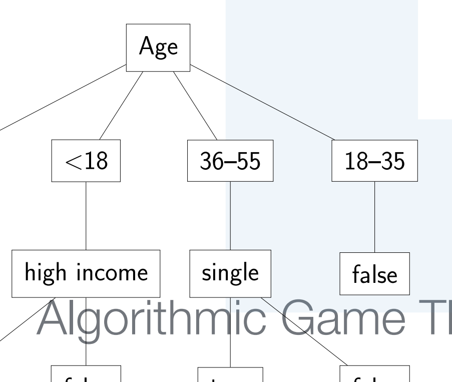
Rational choice and the axiomatics of voting rules: Arrow's impossibility theorem, Kemeny's rule, tournament solutions, etc.

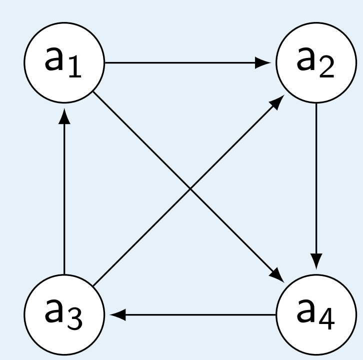
decision support systems (lab.)

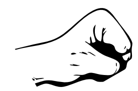
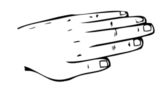
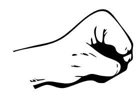
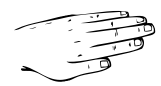
business analytics

economics and computation (sem.)



Learning human choice behavior from empirical observations: decision trees, logistic regression, Bayesian networks, etc.



			
	$\frac{1}{2}, \frac{1}{2}$	$0, 1$	$1, 0$
	$1, 0$	$\frac{1}{2}, \frac{1}{2}$	$0, 1$
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multiagent systems (sem.)

algorithmic game theory

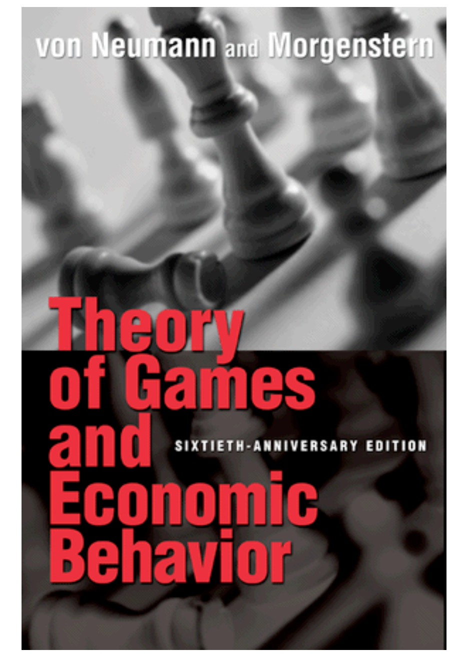
Modeling interaction and computing optimal strategies: utility theory, normal-form games, Nash equilibrium, etc.

Related Courses

- Course & Tutorial “Operations Research (WI IV)” (Bichler)
  - *decision theory, linear programming, discrete optimization*
- Course & Tutorial “Computational Social Choice” (Brandt)
  - *preference aggregation, voting rules, complexity considerations*
- Course & Tutorial “Auction Theory & Market Design” (Bichler)
  - *combinatorial auctions, spectrum license auctions, procurement*
- Course & Tutorial “Games on Graphs” (Luttenberger)
  - *software verification, parity games, stochastic games*
- Course & Tutorial “Social Computing/Social Gaming” (Groh)
  - *social network analysis, graph visualization, social games*
- Seminars “Economics and Computation” (Brandt), “Multiagent Systems” (Brandt), “Auction Theory and Market Design” (Bichler)

The Origin of Game Theory

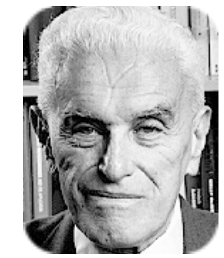
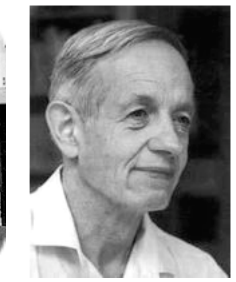
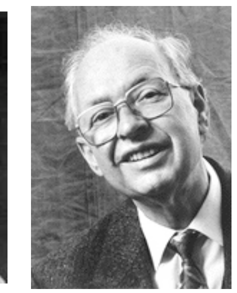
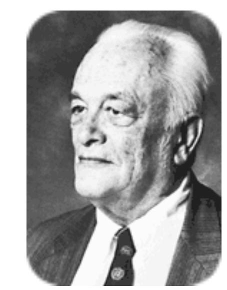
- considered a landmark publication of the 20th century (first published in 1944)
  - American Scientist's 100 books that shaped a century of science
  - Boston Public Library's 100 most influential books of the century
  - available online for free

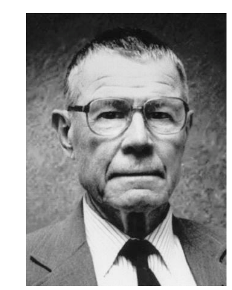
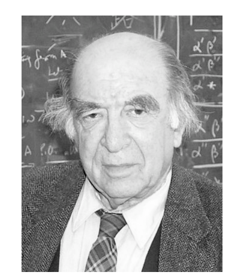


- Von Neumann was a Hungarian mathematician with groundbreaking contributions to quantum physics, calculus, set theory, topology, economics, computer science, hydrodynamics, statistics, etc.
  - E.g., von Neumann computer architecture, mergesort, cellular automata, linear programming, game theory, atomic bomb

Nobel Prize Laureates

- Harsanyi, Nash, and Selten (1994)
  - *equilibria*
- Vickrey (1996)
  - *incentives*
- Aumann and Schelling (2005)
  - *game theory*
- Hurwicz, Maskin, and Myerson (2007)
  - *mechanism design*
- Roth and Shapley (2012)
  - *stable allocations*



Synopsis

- **Introduction**
- **Utility theory** (preference relations, expected utility)
- **Normal-form games** (prisoner's dilemma, Pareto-dominance)
- **Nash equilibrium** (pure and mixed equilibria, axiomatization)
- **Computing equilibria** (algorithms, computational complexity)
- **Alternative solution concepts** (refinements, iterated dominance)
- **Cooperative games** (Shapley value, coalition formation)
- **Zero-sum games** (minimax theorem, Shapley's saddles)
- **Concise representations** (graphical games, symmetric games)
- **Extensive-form games** (Stackelberg games, subgame-perfect equilibria, parlor games)
- **Mechanism design** (GS impossibility, VCG mechanism)
- **Stable matchings** (Gale-Shapley algorithm, roommate problem)

Recommended Readings

- None of these books is required to pass the course!
- Available online and in print:
  - Osborne & Rubinstein: *A Course in Game Theory* (MIT Press, 1994)
  - Aumann: *Game Theory*, in J. Eatwell, M. Milgate, and P. Newman: The New Palgrave, A Dictionary of Economics, Vol. 2 (MacMillan, 1987)
  - Shoham & Leyton-Brown: *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations* (Cambridge UP, 2009)
  - Nisan, Roughgarden, Tardos, & Vazirani: *Algorithmic Game Theory* (Cambridge UP, 2007)
- Available in print:
  - Peters: *Game Theory - A Multi-leveled Approach* (Springer, 2008)
  - Maschler, Solan, & Zamir: *Game Theory* (Cambridge UP, 2013)
  - Myerson: *Game Theory - Analysis of Conflict* (Harvard UP, 1991)
  - Fudenberg & Tirole: *Game Theory* (MIT Press, 1991)
  - Mas-Colell, Whinston, & Green: *Microeconomic Theory* (Oxford UP, 1995)

Rational Agents

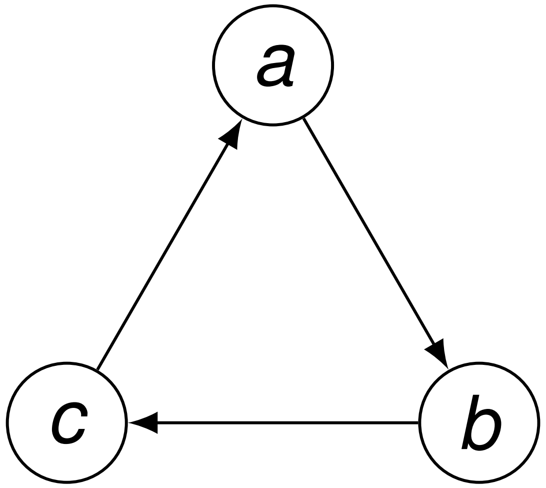
- What is an agent?
  - An agent is an autonomous entity which has the ability to interact with its environment.
    - e.g., human beings, robots, or software agents
  - It is usually assumed that agents are (unboundedly) rational.
- A prerequisite for making rational decisions are preferences over the set of alternatives A.
  - These are typically modeled as binary preference relations.
  - x is at least as good as y: $x \succsim y$
  - A preference relation can be factorized in its asymmetric part (the strict preference relation) and its symmetric part (the indifference relation).
  - $x \succ y \Leftrightarrow (x \succsim y) \wedge \neg(y \succsim x)$ and $x \sim y \Leftrightarrow (x \succsim y) \wedge (y \succsim x)$

Preference Relations

- A preference relation is called rational if it is
  - complete (i.e., $\forall x,y \in A$: $(x \succsim y) \vee (y \succsim x)$) and
  - transitive (i.e., $\forall x,y,z \in A$: $(x \succsim y) \wedge (y \succsim z) \Rightarrow (x \succsim z)$).
  - Preference relations do not capture the “intensity” of preferences.
- An agent is called rational if he chooses the most desirable among all feasible alternatives.
  - Preferences are observable through choice behavior
- Arguments for transitivity
  - Transitivity is sufficient to ensure that every finite non-empty set of alternatives admits a most desirable (or maximal) alternative, i.e., an alternative x such that there is no y with $y \succ x$.
  - *Money pump*

Transitivity

- Humans sometimes exhibit intransitive preferences.

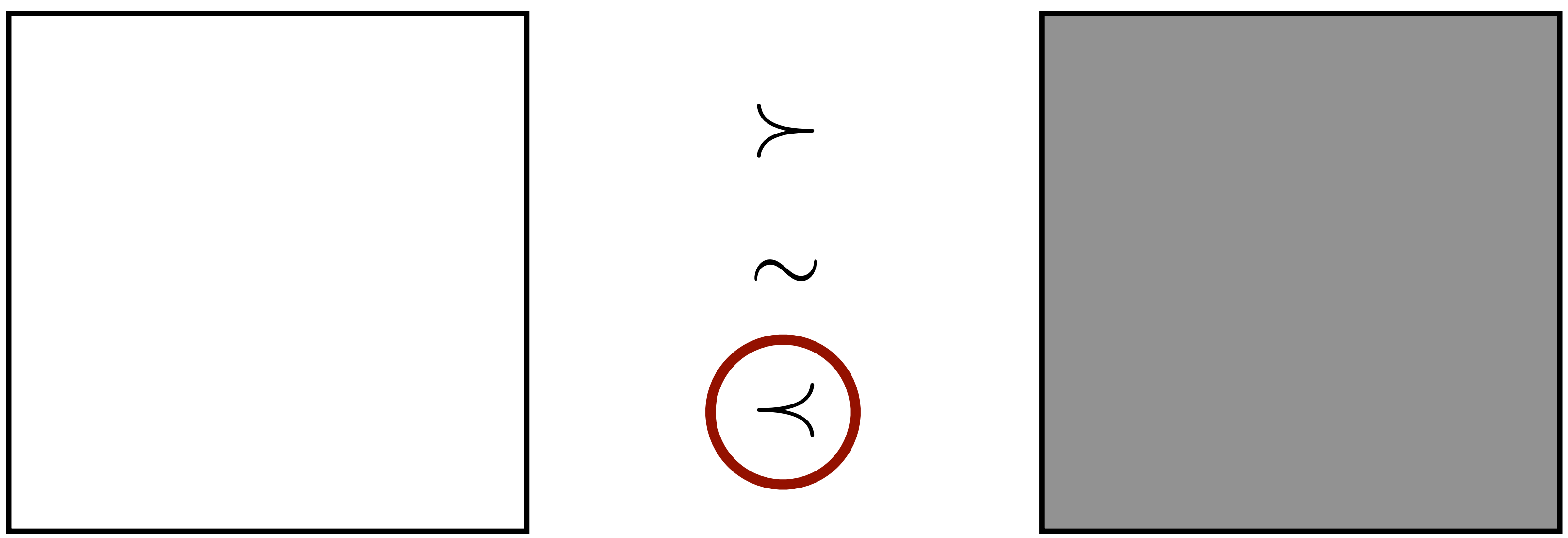


- Two examples
  - Aggregation of multiple criteria
  - Indistinguishability

Aggregation of Multiple Criteria

- Assume an agent has rational preferences when considering each criterion in isolation and consider the following aggregated preference relation:
    - $x \succsim y \;\Leftrightarrow\;$ x beats y for at least as many criteria as y beats x
- The resulting preference relation can be intransitive!
- Example
    - $A \subseteq \mathbb{R}^3$ (top speed, power, fuel consumption)
    - *sports car:* (*300,250,-20*)
    - *SUV:* (*200,300,-15*)
    - *sedan:* (*250,200,-10*)
    - “schizophrenic” preferences

Which Box is Darker?



$\succ$

$\sim$

$\prec$

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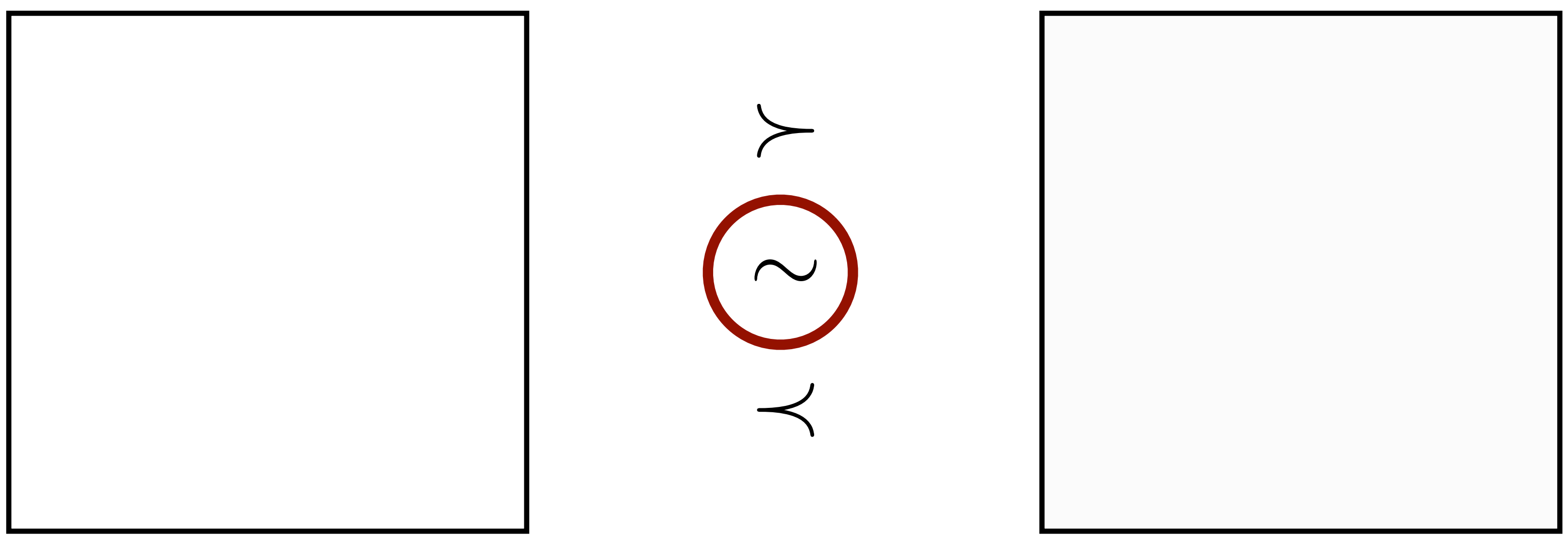

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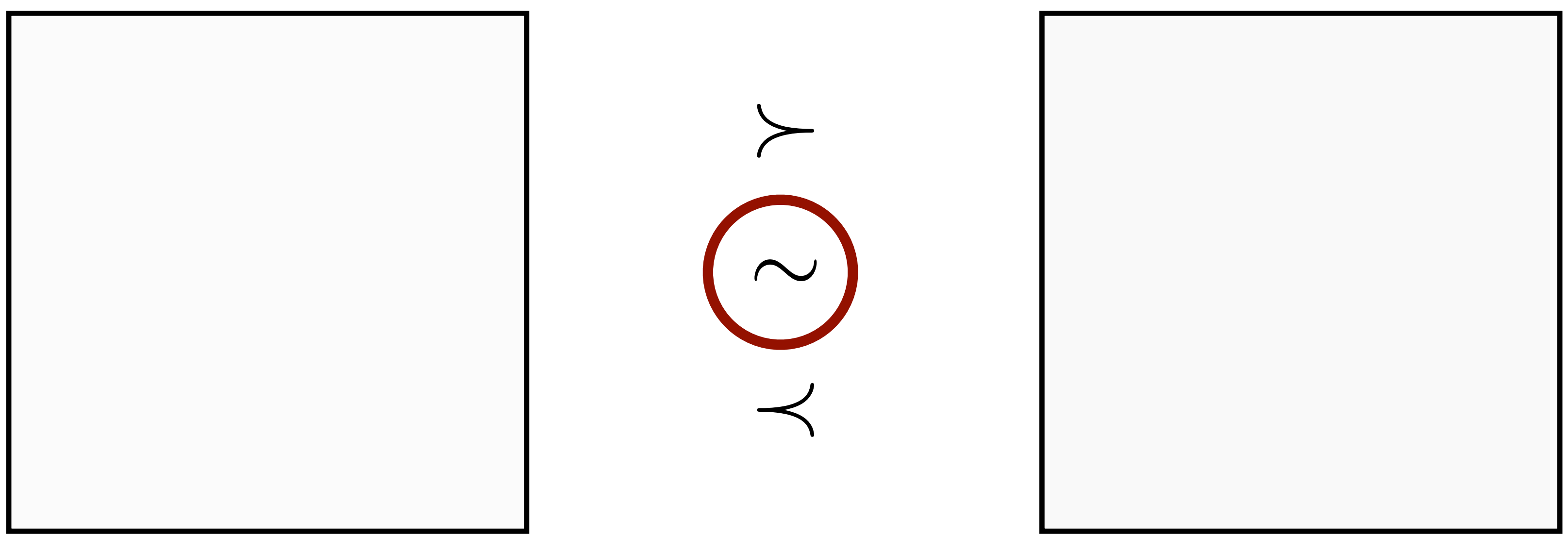

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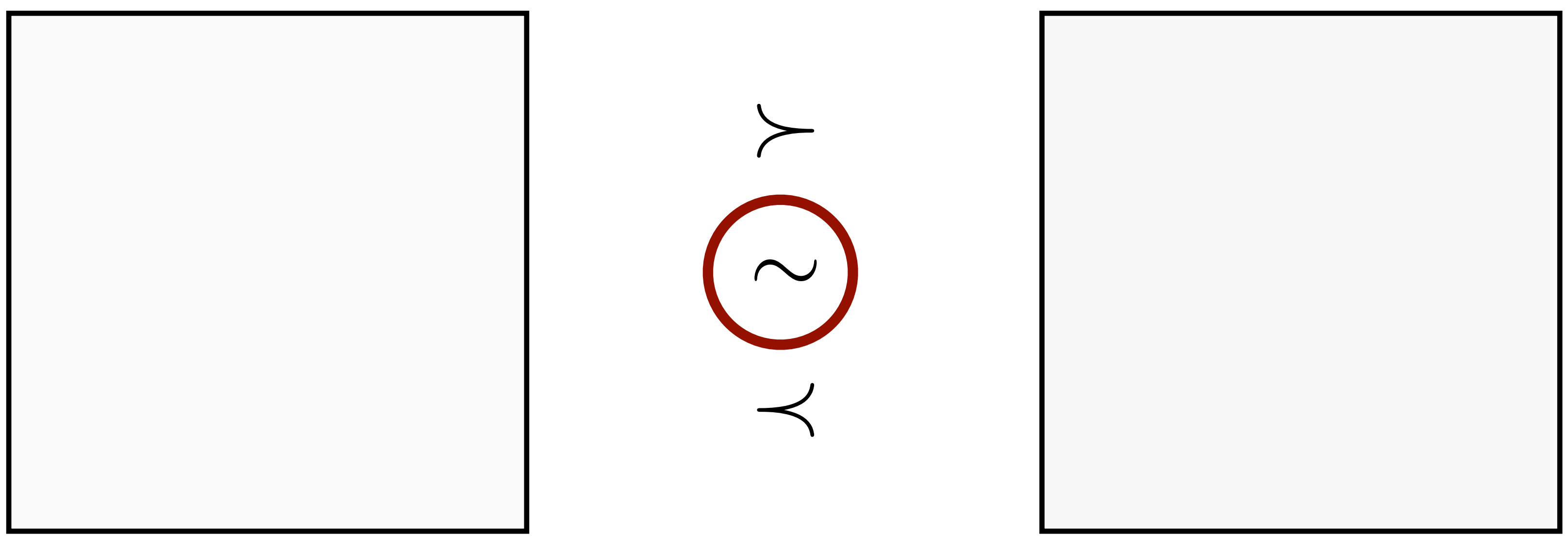

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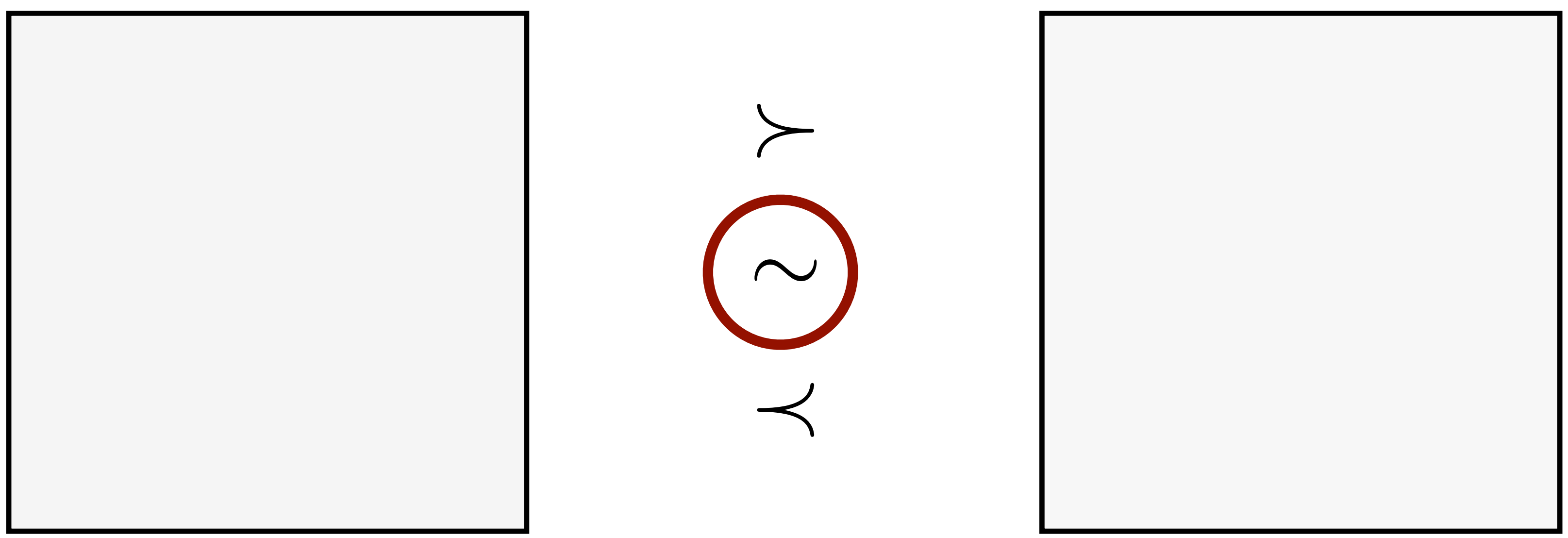

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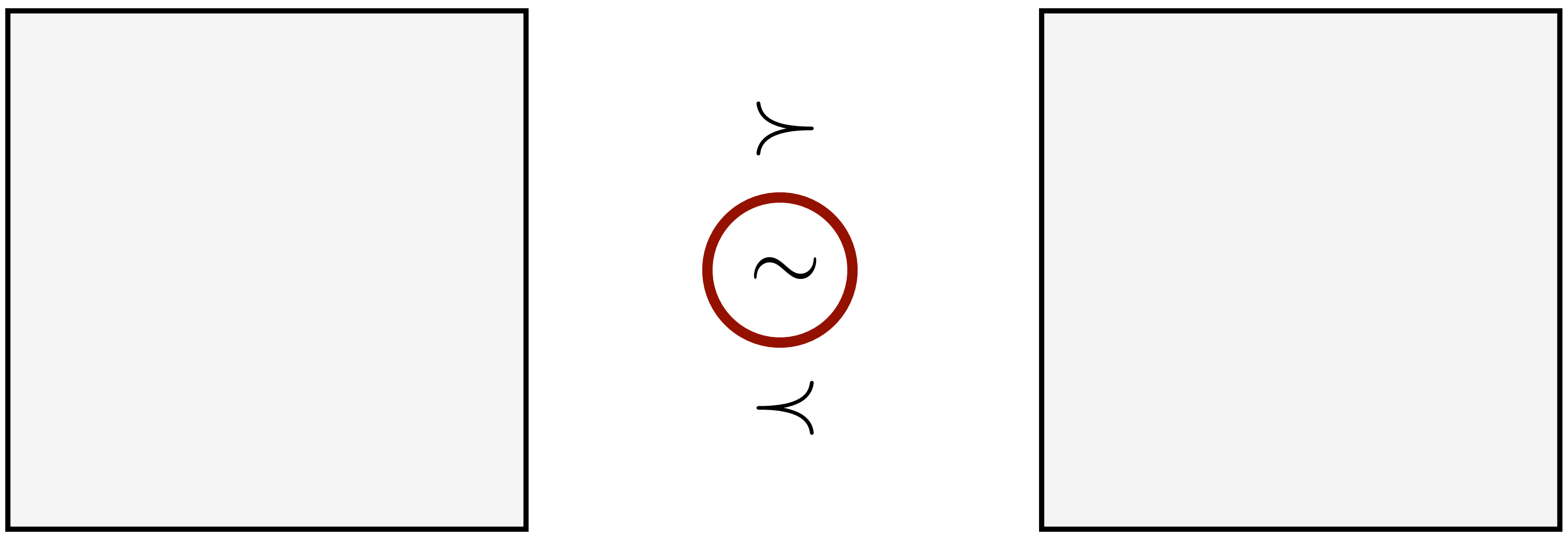

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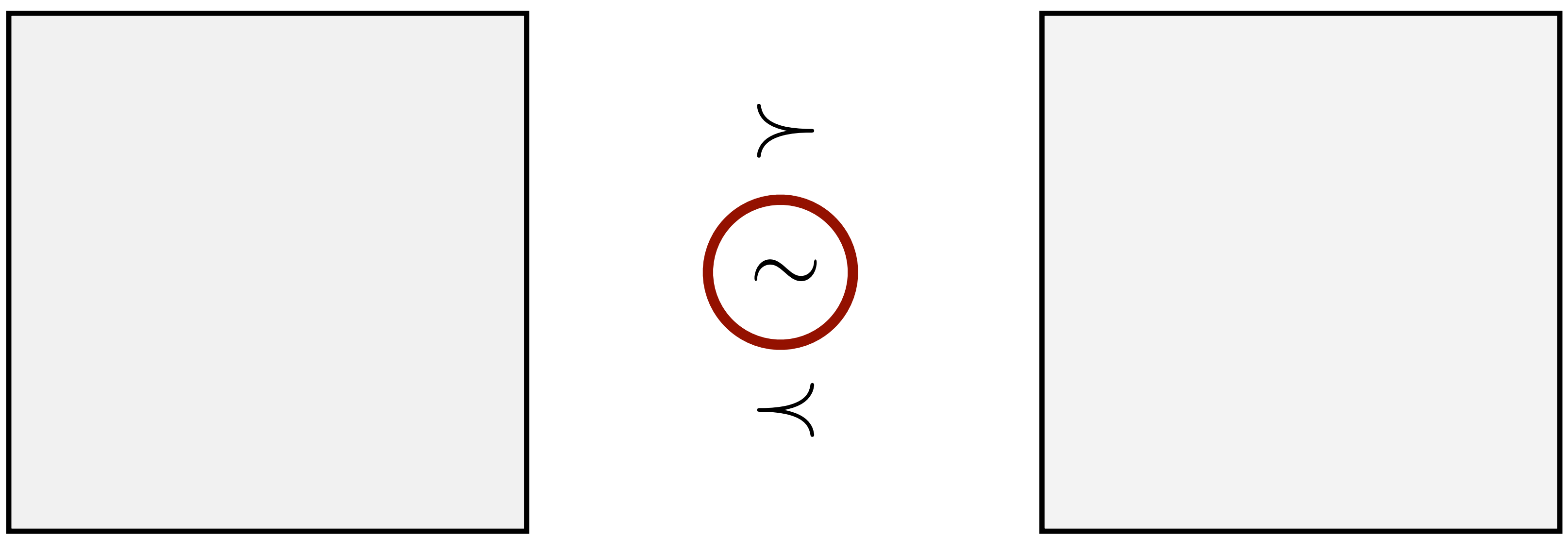

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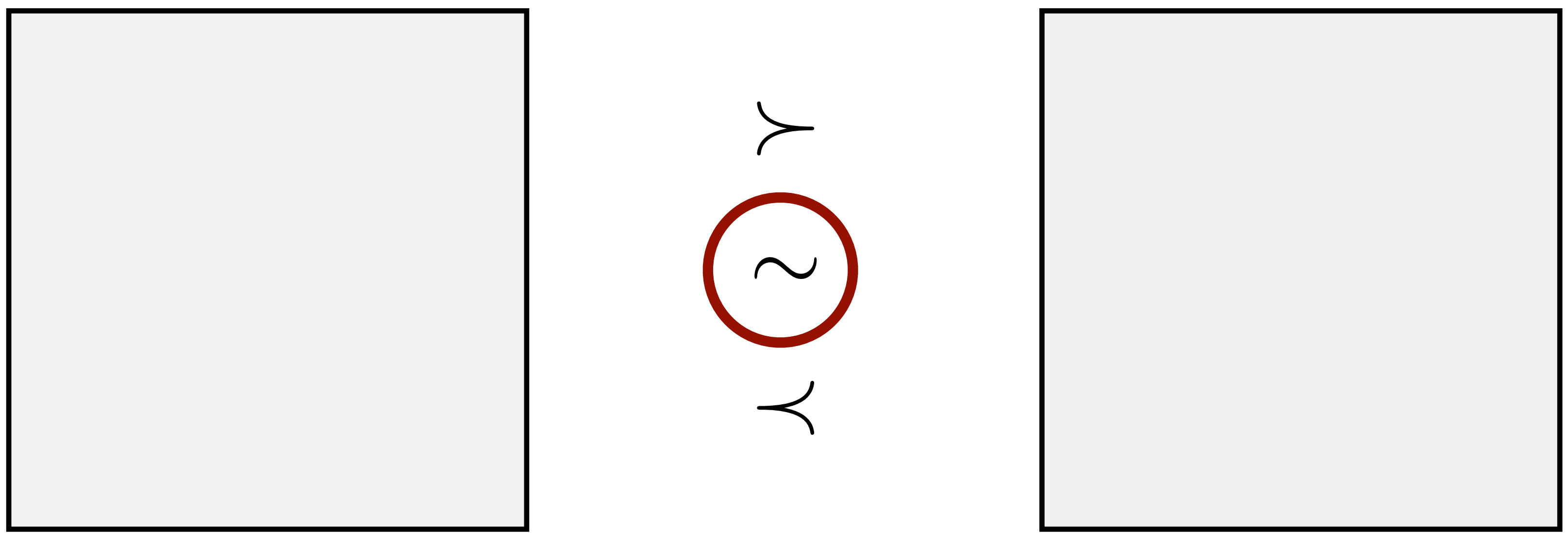

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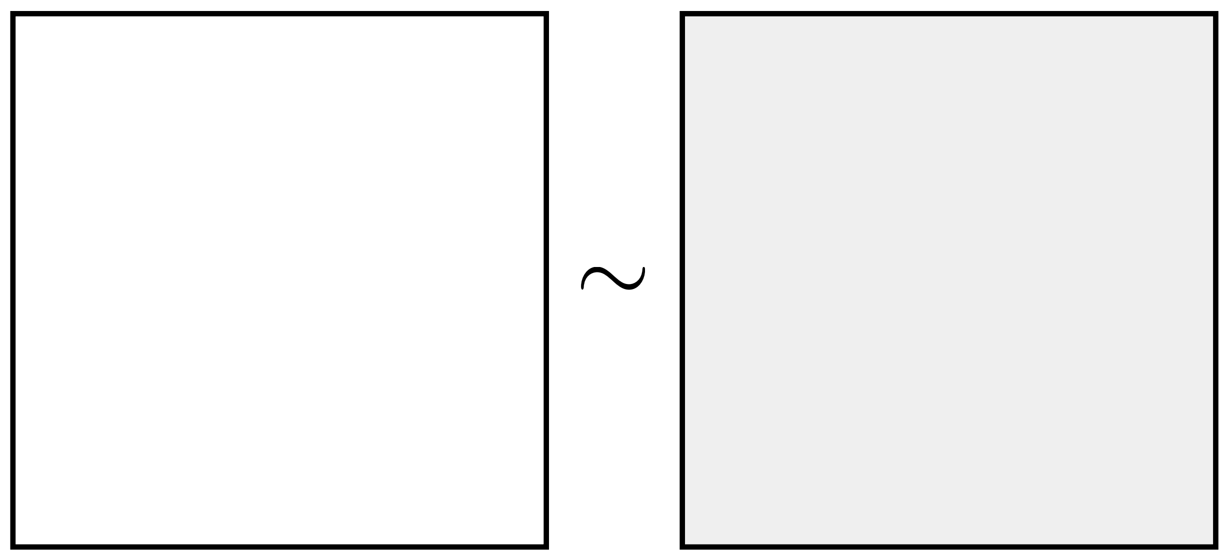
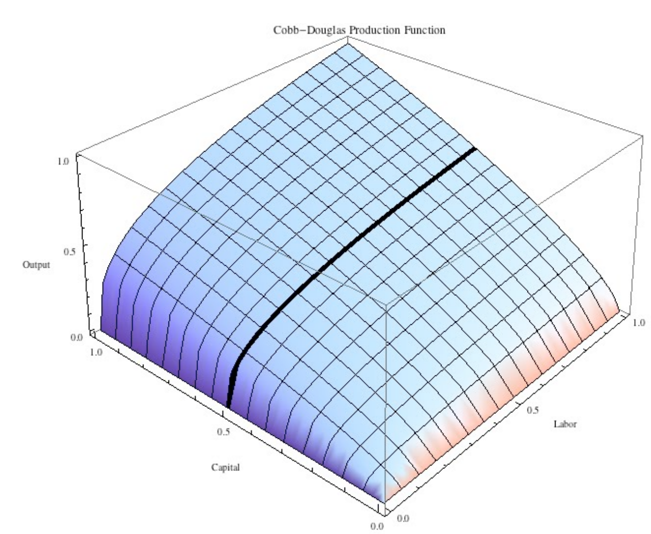
- Transitivity of the indifference relation implies the following:



Utility Functions

- A utility function $u : A \rightarrow \mathbb{R}$ represents the preference relation $\succsim$ if for all $x,y \in A$, $x \succsim y \Leftrightarrow u(x) \geq u(y)$.
  - For every strictly increasing function f, $f(u(\cdot))$ is a new utility function which represents the same preference relation.
  - In general, utility functions are purely ordinal.
- Why utility?
  - naturalness
    - increasing/decreasing utility
    - utility maximization
  - analytical convenience
    - differential calculus
    - constrained optimization (linear programming, integer programming, etc.)



Preferences and Utility

- Proposition: For a countable number of alternatives, a preference relation can be represented by a utility function iff it is rational.
  - Direction from left to right: ≥-relation (on the reals) is transitive and complete.
    Direction from right to left: Tutorial.
- When the number of alternatives is uncountable, some form of continuity is required for the above result to hold.
  - Lexicographic preferences: consider criteria in some fixed order until one criterion can distinguish the alternatives
    - E.g., safety first, altruism/spitefulness
    - Example: $A \subseteq \mathbb{R}^+ \times \mathbb{R}^+$

      $$x \succsim y \Leftrightarrow (x_1 > y_1) \vee ((x_1 = y_1) \wedge (x_2 \geq y_2))$$

    - This preference relation cannot be represented by a utility function!