Technische Universität München
Institut für Informatik (I18)
Prof. Dr. F. Brandt / J. Hofbauer, P. Stursberg

SS 2015
Exercise sheet 1
April 14, 2015

Algorithmic Game Theory (IN2239)

G-exercise due: April 18, 2015, 23:59 (Moodle, https://www.moodle.tum.de)

Exercise 1 *Playing the horses* (**G**)

A bookmaker offers bets on horses A, B and C for horse races. For example, if you bet €100 on B with odds of 1 : 3 and B wins, you get your stake back and three times your stake as gain, for a total of €400. If you lose, of course, you get nothing and lose your stake.
You have €120 to spend and the odds are as follows.

Horse	Odds
A	1:1
B	1:3
C	1:4

The race will be held in the tutorials. If you are not registered for any tutorial, grades will be awarded according to the results of the race in group 1. Place your bets by submitting them in Moodle. If you make a profit you get one point, if you make no profit you get none.

Exercise 2 *Transitivity* (**H**)

Let $\succsim$ be a complete preference relation.
Show that $\succsim$ is transitive if and only if $\succ$ and $\sim$ are both transitive.

Exercise 3 *Extending rational preferences* (**H**)

Let X, Y be disjoint sets and $R \subseteq X \times X$, $R' \subseteq Y \times Y$ complete and transitive relations on X and Y, respectively. Show that there is a complete and transitive extension of $R \cup R'$ on $X \cup Y$.

Exercise 4 *Representing preferences by utility functions* (**T**)

(a) Show that rational preferences over a countable set A of alternatives can be represented by a utility function $u\colon A \to [-1, 1]$.

(b) Let $A = [0, 1] \times [0, 1]$. Define lexicographic preferences $\succsim$ over A such that for all $x, y \in A$,

$$x \succsim y \text{ iff } x_1 > y_1 \text{ or both } x_1 = y_1 \text{ and } x_2 \geqslant y_2.$$

Show that this preference relation cannot be represented by a utility function.