

Kružnice

ji. Najděte středovou rovnici kružnice $S[3, 7], r=2$

$$(x-m)^2 + (y-n)^2 = r^2$$

$$(x-3)^2 + (y-5)^2 = 4$$

+ obecnou:

$$x^2 + y^2 - 2mx - 2ny + (m^2 + n^2 - r^2) = 0$$

$$x^2 - 3 \cdot 2 \cdot x + 9 + y^2 - 2 \cdot 5 \cdot y + 25 = 4$$

$$x^2 - 6x + 9 + y^2 - 10y + 25 = 4$$

$$x^2 + y^2 - 6x - 10y + 30 = 0$$

ji: Ještě body na stejně kružnici? A[2; 1], B[2; 7], C[4; 5], D[-1; 2]

$$m \quad (2-m)^2 + (1-m)^2 = r^2$$

$$(2-m)^2 + (7-m)^2 = r^2$$

$$(4-m)^2 + (5-m)^2 = r^2$$

$$(-1-m)^2 + (2-m)^2 = r^2$$

$$(1-m)^2 = r^2 - (2-m)^2$$

$$(5-m)^2 = r^2 - (5-m)^2$$

$$(1-m)^2 = (5-m)^2$$

$$1-2m+m^2 = 25-10m+m^2$$

$$-2m+10m = 25-1$$

$$8m = 24$$

$$\underline{m=3}$$

→ druhému body:

$$(2-3)^2 + (1-3)^2 = r^2$$

$$1+4 = 5 \rightarrow \text{neplatí}$$

$$D: (-1-3)^2 + (2-3)^2 = 5$$

$$-4^2 + (-1)^2 = 5$$

platí.

~~$$(2m)^2 = r^2 \quad (7-m)^2$$~~
~~$$(2-m)^2 = r^2 + (5-m)^2$$~~

KD-Úloha

$$(2-m)^2 = r^2 - (5-m)^2$$

$$(4-m)^2 = r^2 - (5-m)^2$$

$$(2-m)^2 = (4-m)^2$$

$$2^2 - 4m + m^2 = 4^2 - 8m + m^2$$

$$-4m + 8m = 16 - 4$$

$$4m = 12$$

$$\underline{m=3}$$

pr: Zjistit vzdálenost bodu $x \{7; 6\}$ od středu kružnice $x^2 - 4x + y^2 - 2y + 10 = 0$

$$(x^2 - 4x) + (y^2 - 2y) + 10 = 0$$

$$\cancel{(x^2 - 4x)} + \cancel{(y^2 - 2y)} + 10 = 0$$

$$(x-2)^2 = x^2 - 4x + 4$$

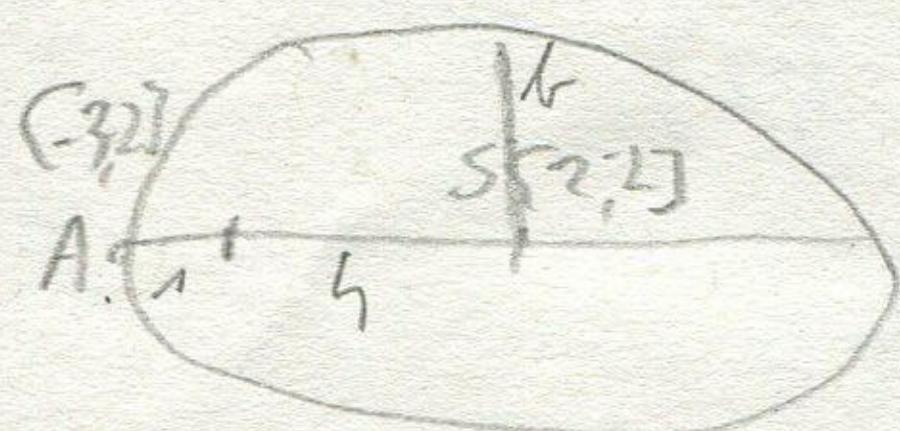
$$(y-1)^2 = y^2 - 2y + 1$$

$$(x-2)^2 + (y-1)^2 + 5 = 0$$

$$(x-2)^2 + (y-1)^2 = -5 \leq r^2 \dots \text{NR}$$

elipsa

pr: Najít stranou rovnicí elipsy, $S[2; 2]$, $e=5$, $A[-3; 2]$



$$\frac{(x-m)^2}{a^2} + \frac{(y-m)^2}{b^2} = 1$$

$$|SA| = 5 \quad b^2 = a^2 - e^2$$

$$1^2 = 5^2 - h^2$$

$$h = 3$$

$$\frac{(x-2)^2}{5^2} + \frac{(y-2)^2}{3^2} = 1$$

pr: Najít střed, ohniska a hranici polohou elipsy $25x^2 + 9y^2 + 150x - 36y + 36 = 0$

$$25x^2 + 150x + 9y^2 - 36y + 36 = 0$$

$$(5x+15)^2 + (3y-6)^2 + 36 - 225 - 36 = 0$$

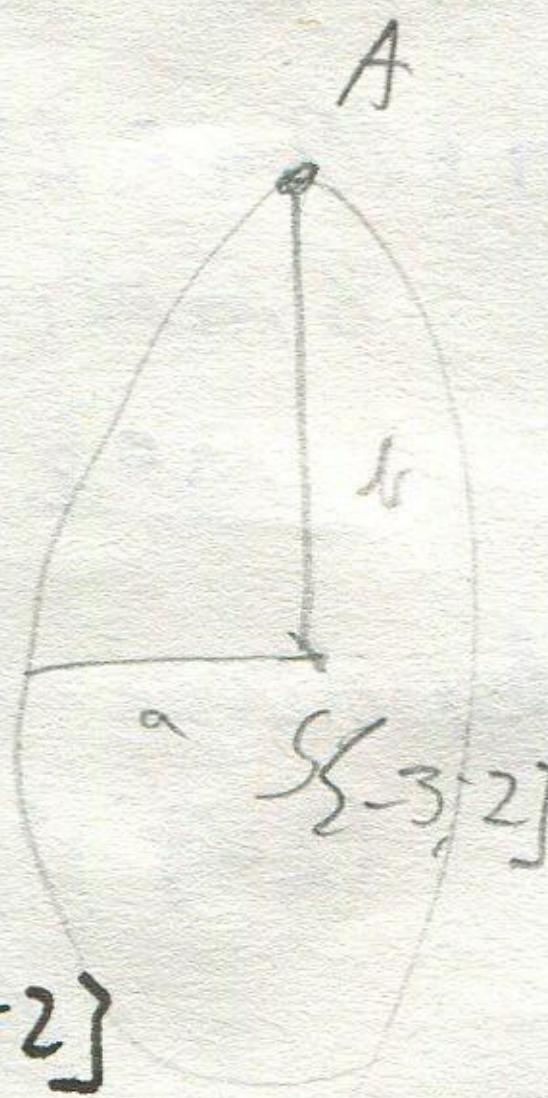
$$5(x+3)^2 + 3(y-2)^2 = 225$$

$$\cancel{\frac{5(x+3)^2}{225}} + \cancel{\frac{3(y-2)^2}{225}} = 1$$

$$S = \{-3; 2\}$$

$$a^2 = 9 \quad f^2 = 25$$

$$a = 3 \quad f = 5$$



$$A = [-3+5; 2] = [2; 2]$$

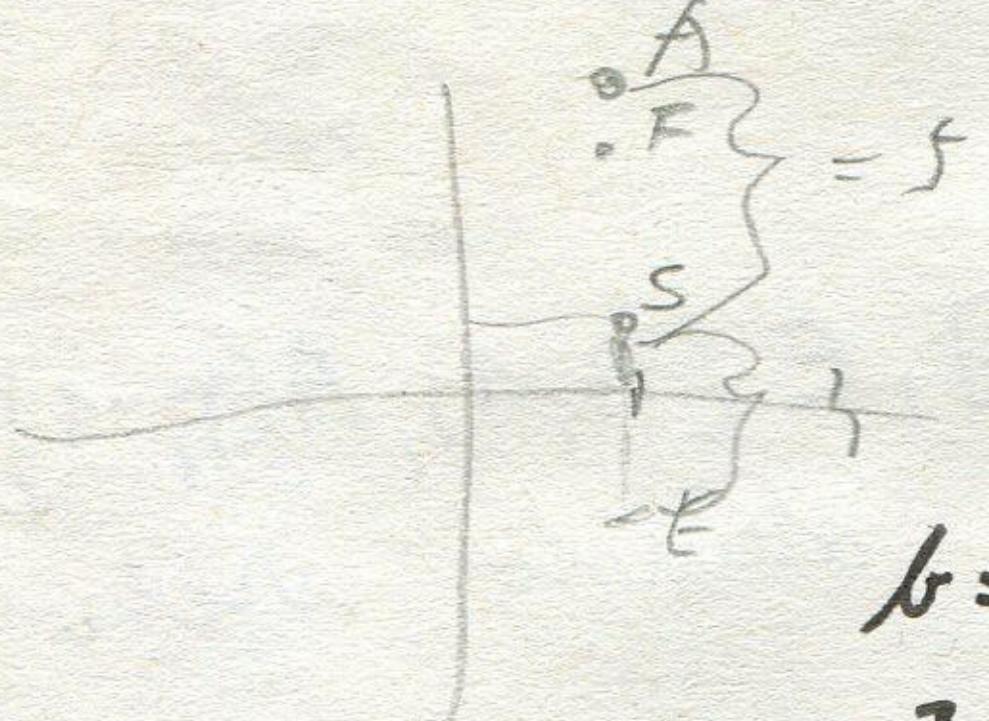
$$b^2 = a^2 - e^2$$

$$b^2 + e^2 = a^2$$

$$c^2 / a^2 - e^2 = h \rightarrow E[-3; 2+h] = [-3; 6]$$

$$F[-3; 2-h] = [-3; 2]$$

pří: Najít obecnou rovnici elipsy, $S[2;1]$, $A[2;6]$, $E[2;-3]$



$$\frac{(x-m)^2}{a^2} + \frac{(y-n)^2}{b^2} = 1 \quad \frac{(x-2)^2}{25} + \frac{(y-1)^2}{\cancel{9}} = 1$$

$$b = ?$$

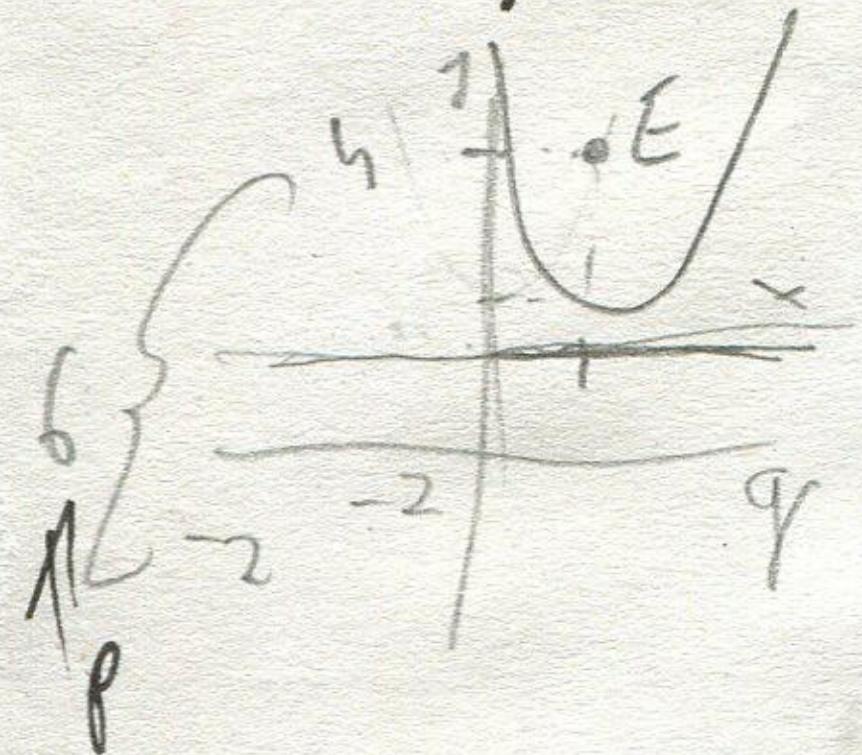
$$a^2 = 25 \quad b^2 = a^2 - c^2 = 25 - 16 \Rightarrow 3$$

$$c^2 = 16$$

$$\begin{aligned} 9. (x^2 - 4x + 4) + 25(y^2 - 2y + 1) &= 1 \\ 9x^2 - 36x + 36 + 25y^2 - 50y + 25 - 1 &= 0 \\ 9x^2 + 25y^2 - 36x - 50y + 60 &= 0 \end{aligned}$$

parabola

pří: Najít vrcholovou rovnici paraboly, $E[2;4]$, vzdálu průměru $y = -2$.



$$(x-m)^2 = \pm 2p(y-n)$$

$$|E_Q| = 6 \Rightarrow |EV| = 3 \Rightarrow E[2;1]$$

$$(x-2)^2 = \pm 2p(y-1)$$

$$(x-2)^2 = \pm 12(y-1)$$

pří: Najít ohnisko, vrchol a vzdálu průměru paraboly $x^2 - 4x - 4y + 12 = 0$

$$(x-2)^2 - 4 = 4y - 12$$

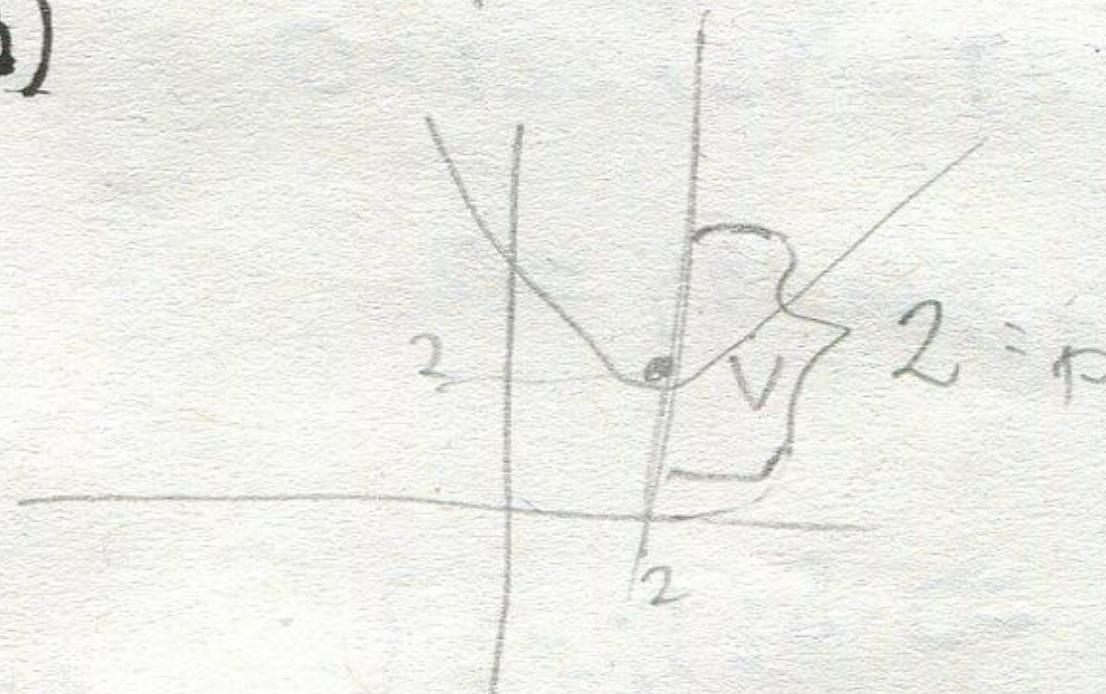
$$(x-2)^2 = 4y - \cancel{8}$$

$$(x-2)^2 = 4(y - 3)$$

$$(x-2)^2 = 4(y-2)$$

$$V[2;2]$$

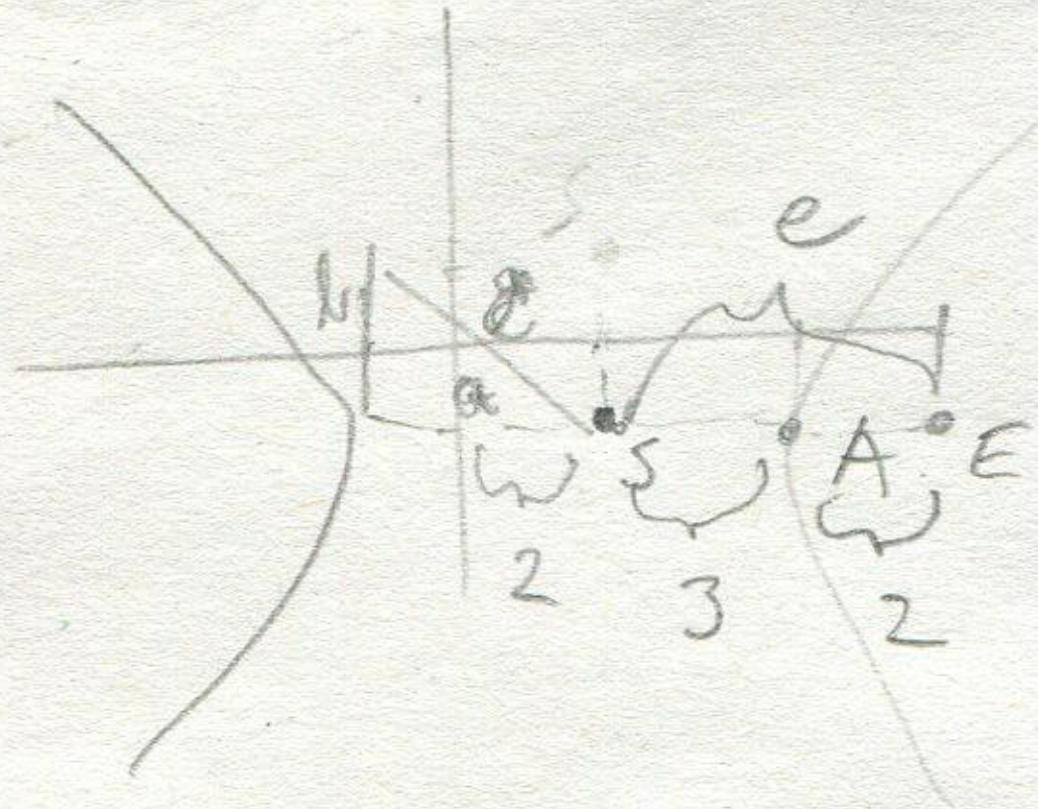
$$p = 2$$



$$\Rightarrow q \neq \gamma = 1 \\ E = [2;3]$$

Hyperbola

pr: Najít střed, ohniska, klouz. vrchol a asympt. hyperbolu, $S[2; 1]$, $\in [7; -1]$, $A(7; -1)$
 střed ohniska vrchol



$$\frac{(x-2)^2}{a^2} - \frac{(y-1)^2}{b^2} = 1$$

$$a^2 + b^2 = e^2 \quad a = 4 \\ b^2 = e^2 - a^2 \quad e = 5$$

$$\frac{(x-2)^2}{16} - \frac{(y-1)^2}{9} = 1 \quad b = 3$$

$$\rightarrow \text{asymptoly: } \frac{x-2}{a} = \pm \frac{y-1}{b}$$

$$\frac{(x-2)}{4} = \pm \frac{(y-1)}{3}$$

$$3x - 6 = 4y - 4 \rightarrow 3x - 4y + 5 = 0 \\ 3x - 6 = -4y + 4 \rightarrow 3x + 4y - 10 = 0$$

pr: Najít střed, ohniska, klouz. vrchol a asympt. hyperbolu

$$9x^2 - 90x - 16y^2 - 96y - 227 = 0$$

$$9(x^2 - 10x) - 16(y^2 + 6y) - 227 = 0$$

$$(3x - 15)^2 - (4y + 6)^2 - 227 + 81 + 144 = 0$$

$$3^2(x-5)^2 - 4^2(y+3)^2 + 11 = 0$$

$$-\frac{(x-5)^2}{16} + \frac{(y+3)^2}{9} = 1 \Rightarrow S[5; -3]$$

$$a = 4 \\ b = 3$$

$$c = \sqrt{4^2 + 3^2} = 5$$

$$A[5; 0] \quad E[5; 2] \\ B[5; -6] \quad F[-5]$$

řešení: elipsa: $x^2 + 9y^2 - 6x - 27 = 0$

$$\cancel{(x-3)^2 + 9y^2 - 36 = 0}$$

$$\frac{(x-3)^2}{36} + \frac{9y^2}{36} = 1$$

$$\frac{(x-3)^2}{36} + \frac{y^2}{\frac{36}{9}} = 1$$

povídáme to převrát, abu to je elipsa

$$S = [3; 0]$$

$$a = \sqrt{36} = 6$$

$$b = \sqrt{4} = 2$$

prímka: $2x - 3y + 6 = 0$

vezemme poloha elipsy a prímky?

$$2x = 3y - 6$$
$$x = \frac{3y - 6}{2}$$

$$\left(\frac{3y-6}{2}\right)^2 + 9y^2 - 6\left(\frac{3y-6}{2}\right) - 27 = 0$$

$$\frac{9y^2 - 2 \cdot 6 \cdot 3y + 36}{4} + 9y^2 - \frac{18y + 36}{2} - 27 = 0 \quad | \cdot 4$$

$$(9y^2 - 36y + 36) + (36y^2 - 36y + 72) - 108 = 0$$

$$45y^2 - 72y = 0$$

~~$$y_1 = \frac{24}{45}$$~~
~~$$y_2 = 0$$~~

dva společné body \Rightarrow sečna

pr: Najděte ohnisko, vrchol a řídící přímky paraboly, která je dáná rovnicí $x^2 - 4x - 4y + 12 = 0$.

- obecná rovna \rightarrow vrcholová rovna

$$x^2 - 4x - 4y + 12 = 0$$

$$(x-2)^2 - 4 - 4y + 12 = 0$$

$$(x-2)^2 = 4y - 8$$

$$(x-2)^2 = 4(y-2)$$

parabola

- z vrcholové rovnice vym. vrchol: $V[2; 2]$, orientaci paraboly a vzdálenost řídící přímky a ohniska: (2)
- $E[2; 3]$... ohnisko
- $q = -1$... druhá řídící přímka

pr: Napište obecnou rovinu hyperboly s asymptotami $a_1: 3x+2y-9=0$;
 $a_2: 3x-2y-9=0$ a vrcholem $A[3; 3]$

- rovnice asymptot

$$3x-9 = \pm 2y$$

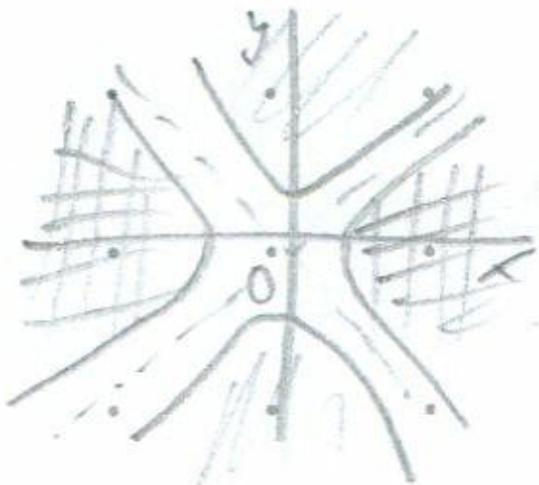
$$\frac{x-m}{a} = \pm \frac{y-n}{b}$$

$$\frac{3x-9}{6} = \pm \frac{2y}{6} \quad ; \quad \boxed{\frac{x-3}{2} = \pm \frac{y}{3}}$$

$$\frac{(x-3)^2}{4} - \frac{y^2}{9} = \pm 1$$

||

více možnosti:



- dosaďme souřadnice vrcholu A

$$-\frac{(x-3)^2}{4} + \frac{y^2}{9} = 1$$

- obecná rovna:

$$-9(x-3)^2 + 4y^2 = 36$$

$$-9x^2 + 54x - 81 + 4y^2 - 36 = 0$$

$$9x^2 - 4y^2 - 54x + 117 = 0$$

príklad: Najdiť obecnú rovnicu elipsy, ktorá má súradnice súradničného systému $S[2; 1]$, klasický orčol $[2, 6]$ a ohnisko $E(2, -3)$.

• Klasické položky a + výškodnosť e:

$$a = |AS| = \sqrt{(2-2)^2 + (1-6)^2} = \sqrt{5^2} = 5$$

$$e = |ES| = \sqrt{(2-2)^2 + (1-(-3))^2} = \sqrt{4^2} = 4$$

• medzijedinečné položky:

$$b^2 = a^2 - e^2$$

$$b^2 = 25 - 16 = 9$$

$$b = \pm 3 \rightarrow |b| = 3$$

$$\frac{(x-2)^2}{3^2} + \frac{(y-1)^2}{5^2} = 1$$

• obecná rovnica:

$$25(x-2)^2 + 9(y-1)^2 = 25 \cdot 9$$

$$25x^2 + 9y^2 - 100x - 18y + 100 + 9 - 225 = 0$$

$$25x^2 + 9y^2 - 100x - 18y - 116 = 0$$