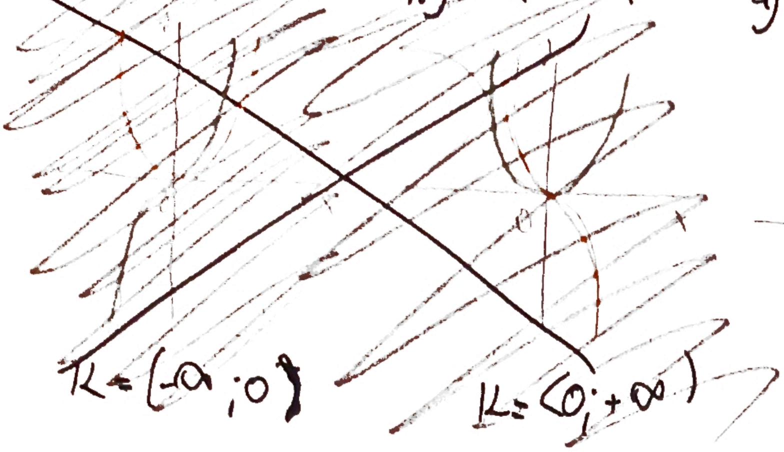
dP – 8

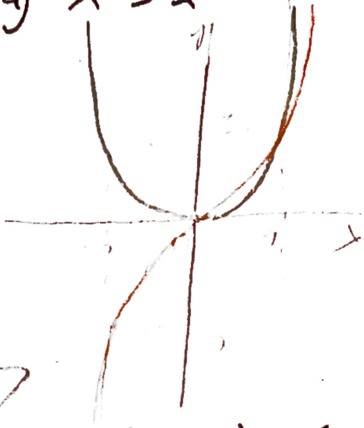
Mocninné fce

~~(8) a) $x^6 > x^5$~~ ~~b) $-x^3 \leq x^4$~~



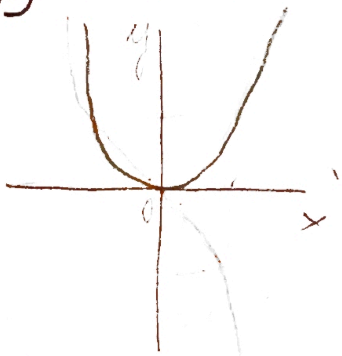
~~$K = (-\infty; 0]$~~ ~~$K = \langle 0; +\infty)$~~

a) $x^6 > x^5$



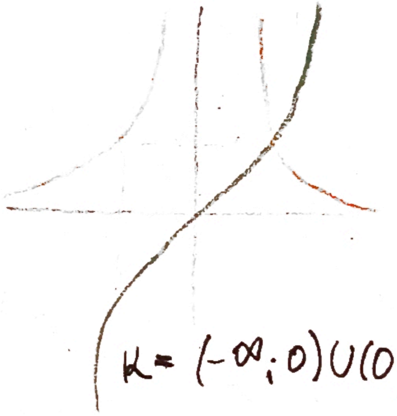
$K = (-\infty; 0) \cup (1; \infty)$

b) $-x^3 \leq x^4$



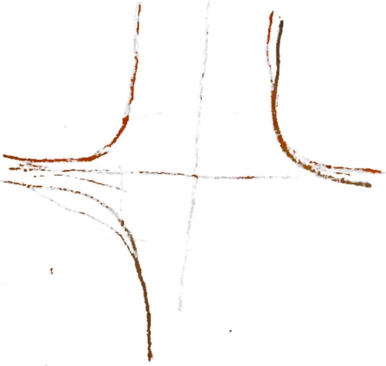
$K = (-\infty; -1\rangle \cup \langle 0; \infty)$

c) $x^{-2} > x^5 \left(= \frac{1}{x^2} > x^5\right)$



$K = (-\infty; 0) \cup (0; 1)$

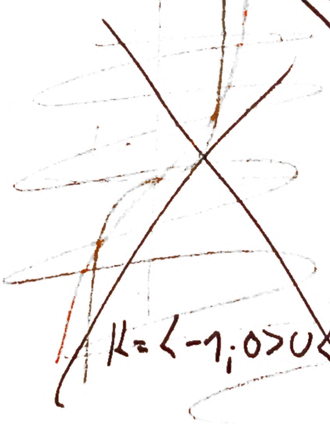
d) $x^{-4} \leq x^{-5} \left(\frac{1}{x^4} \leq \frac{1}{x^5}\right)$



$K = (0; 1\rangle$

~~e) $x^{-3} \leq x^{-7}$~~

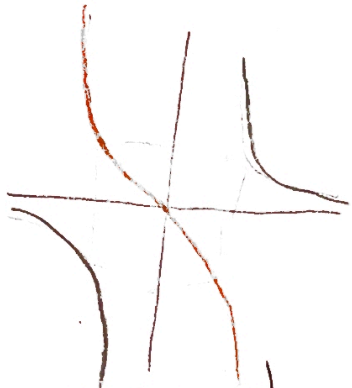
~~$\frac{1}{x^3} \leq \frac{1}{x^7}$~~ Obě sadání



$K = \langle -1; 0) \cup \langle 1; \infty)$

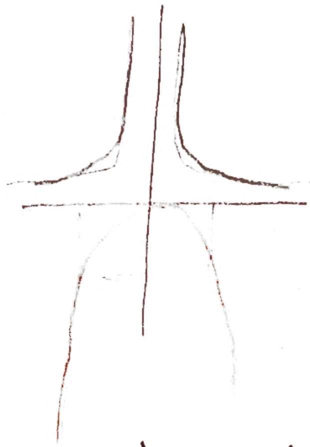
e) $x^{-3} \le -x^7$

$\frac{1}{x^3} \le (-x)^7$



$K=(-\infty;0)$

f) $x^{-6} > -x^6$



$K=(-\infty;0)\cup(0;\infty)$

g) $-x^3 \ge x^{-6}$

$K=(-\infty;-1\rangle$

h) $-x^{-2} \le x^5$

$K=\langle -1;0)\cup(0;\infty)$

39

a) $(2x)^3 = 8x^3$

b) $\left(\frac{3}{2}\right)^5 \cdot \left(\frac{2^{-1}}{3}\right)^3 = \frac{9}{256}$

c) $\frac{(12x)^2}{(3x)^3} = \frac{144x^2}{27x^3} = \frac{144}{27}\cdot\frac{x^2}{x^3} = \frac{144}{27x} = \frac{16}{3x}$

d) $\left(\frac{-x^{-1}\cdot y^2}{x^3}\right)^{-5} = \frac{-x^{-1\cdot(-5)}\cdot y^{2\cdot(-5)}}{x^{3\cdot(-5)}} = \frac{-x^5}{x^{-15}}\cdot y^{-10} =$

$= -x^{5-(-15)}\cdot y^{-10} = -x^{20}\cdot y^{10}$

e) $[(x^{-2}y)^3]^2\cdot(y^{-1})^{-6} = [x^{-6}y^3]^2\cdot(y^6) = x^{-12}\cdot y^6\cdot y^6 = x^{-12}\cdot y^{12} = \left(\frac{y}{x}\right)^{12}$

f) $\left(\frac{x}{2y^2}\right)^{-3}\cdot\left(\frac{y}{x^{-3}}\right)^5 = \frac{x^{-3}}{2^{-3}y^{-6}}\cdot\frac{y^5}{x^{-15}} = \frac{y^6}{x^3\cdot\frac{1}{8}}\cdot y^5\cdot x^{15} = \frac{8y^{11}x^{15}}{x^3} = 8y^{11}x^{12}$

43 a) $(x^2)^{-1} : 9(x\cdot 3)^2 = \frac{x^{-2}}{9x^{-6}} = \frac{x^6}{9x^2} = \frac{x^4}{9}$ $x\neq 0$

b) $\frac{(x^4y^{-3})^{-2}}{(x^3)^{-3}y^5} = \frac{x^{-8}y^6}{x^{-9}y^5} = \frac{x^{-8}}{x^{-9}}\cdot\frac{y^6}{y^5} = x^{-8-(-9)}\cdot y^{6-5} = xy$ $x;y\neq 0$

c) $0{,}8^{-1}\cdot a^2b^{-3}\cdot 1{,}5a^{-2}b^5 = \frac{5}{4}\cdot 1{,}5\cdot a^0\cdot b^2 = \frac{15}{8}b^2$

(2)

d) $\left(\frac{a^2b^{-3}}{c^{-2}d^3}\right)^{-1}\cdot\left(\frac{c^4d^{-1}}{a^{-3}b^2}\right)^2 = \frac{a^{-2}b^3}{c^2d^{-3}}\cdot\frac{c^8\cdot d^{-2}}{a^{-6}b^4} = \frac{b^3d^3}{a^2c^2}\cdot\frac{a^6\cdot c^8}{b^4d^2} =$

$= a^{6-2}\cdot b^{3-4}\cdot c^{8-2}\cdot d^{3-2} = a^4b^{-1}c^6d \qquad a;b;c;d\neq 0$

e) $[m^{-2}\cdot 3m^3\cdot(6m)^{-1}]^{-2} : (2m^{-1})^0 = m^4\cdot 3^{-2}m^{-6}\cdot(6m)^2 : 1 =$

$= m^4\cdot\frac{1}{8}\cdot m^{-6}\cdot 36\cdot m^2 = m^{4+(-6)+2}\cdot\frac{9}{2} = m^0\cdot\frac{9}{2} = \frac{9}{2}$

f) $\left[m^4\cdot\left(\frac{2}{m}\right)^5\cdot\left(\frac{m^2}{4}\right)^{-3}\cdot\frac{1}{16}\right]^{-1} = m^{-4}\cdot\left(\frac{2}{m}\right)^{-5}\cdot\left(\frac{m^2}{4}\right)^3\cdot 16 =$

$= \frac{2^{-5}\cdot m^5}{m^4}\cdot\frac{m^6}{64}\cdot 16 = \frac{1}{32}\cdot\frac{1}{64}\cdot 16\cdot m\cdot m^6 = \frac{m^7}{128} \qquad m\neq 0$

414 a) $\frac{9}{5}$

b) $\frac{6^{3n-1}\cdot 9^{n-1}\cdot 10^{2n-1}}{30^{2n-4}\cdot 32^n\cdot 12^{1-n}} = \frac{3^{3n-1}\cdot 2^{3n-1}\cdot 3^{n-1}\cdot 3^{n-1}\cdot 5^{2n-1}\cdot 2^{2n-1}}{2^{2n-4}\cdot 5^{2n-4}\cdot 3^{2n-4}\cdot 2^n\cdot 2^n\cdot 2^n\cdot 2^n\cdot 2^n\cdot 2^{1-n}\cdot 2^{1-n}\cdot 3^{1-n}} =$

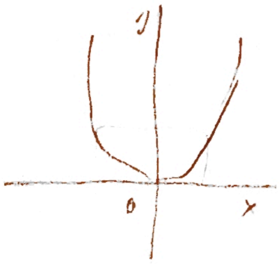
$= \frac{3^{3n-1+n-1+n-1}\cdot 2^{3n-1+2n-1}\cdot 5^{2n-1}}{2^{2n-4+5n+1-n+1-n}\cdot 3^{2n-4+1-n}\cdot 5^{2n-4}} = \frac{3^{5n-3}\cdot 2^{5n-2}\cdot 5^{2n-1}}{2^{5n-2}\cdot 3^{n-3}\cdot 5^{2n-4}} =$

$= 3^{(5n-3)-(n-3)}\cdot 5^3 = 3^{4n}\cdot 5^3 \qquad n\neq 0$

c) $\frac{21^{2n-2}\cdot 15^{n+2}\cdot 49^n}{20^{2n-5}\cdot 14^{n-2}\cdot 16^{3-2n}}$

59

$f_1(x) = \frac{(-x)^3\cdot(-x)^5}{x^2\cdot(-x)^2} = \frac{(-x)^8}{x^4} = x^4$



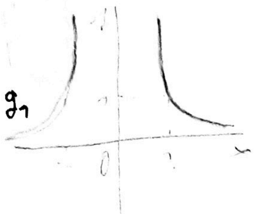
$f_2(x) = \frac{x^4\cdot(-x)^{-4}}{x\cdot(-x)^5}\cdot\frac{-x^6}{x^{-2}\cdot(-x)^{-3}} =$

$\frac{1}{x^2}\cdot\frac{-1}{x^{-3}} = -x^{-5}$

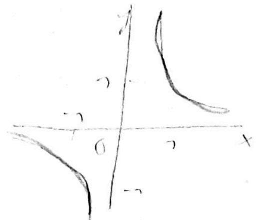
$= \frac{-x^0}{-x^6}\cdot\frac{-x^6}{-x^{4-5}} = \frac{1}{-x^{-5}} = -x^5$

(20) $g_1(x) = \left(\frac{x^{-3}\cdot x^{\frac12}}{x^2}\right)^{\frac12} \cdot \frac{x^{-2}\cdot x^{4}}{x^{\frac13}} = \frac{x^{-\frac32}\cdot x^{\frac14}\cdot x^{-2}\cdot x^{\frac12}}{x\cdot \cancel{x^{\frac14}}} = \frac{x^{-2}\cdot x^{-2}}{x} = \frac{1}{x^4}$

g_1



g_2



$g^2(x) = \sqrt[3]{\frac{x\cdot\sqrt{x}}{x^{-2}}} \cdot \frac{x^{-4}\cdot\sqrt{x}}{\sqrt[3]{x^2}} = \cancel{\frac{x^{\frac32}\cdot x^{\frac12\cdot\frac32}}{x^{-2\cdot\frac32}} \cdot \frac{x^{-4}\cdot x^{\frac12}}{x^{\frac32}} = \frac{x^3\cdot x^{\frac34}\cdot x^{\frac12}}{x^4} =}$

$\cancel{= \frac{x^{\frac{12+3+2}{4}}}{x^4} = \frac{x^{\frac{17}{4}}}{x^4} = x^{\frac14} = \sqrt[4]{x}}$

$\cancel{= \frac{x^{\frac13}\cdot x^{\frac12\cdot\frac13}\cdot x^{-4}\cdot x^{\frac12}}{x^{-2\cdot\frac13}\cdot x^{\frac23}} = \frac{x^{\frac12}\cdot x^{\frac12}\cdot x^{\frac23}}{x^4} = \frac{x^{\frac{1+3+4}{6}}}{x^4} = \frac{x^{\frac86}}{x^4} = \cancel{x}}$

... blbé opsaný zadání...

$\sqrt[3]{\frac{x^{1+\frac12}}{x^{-2}}} \cdot \frac{x^{-4+\frac12}}{x^{\frac23}} = \frac{x^{\frac32\cdot\frac13}}{x^{-2\cdot\frac13}} \cdot \frac{x^{-\frac72}}{x^{\frac23}} = \frac{x^{\frac36}}{x^{-\frac23}} \cdot x^{-\frac72-\frac23} = \cancel{x^{\frac76}}\cdot x^{-\frac{25}{6}} =$

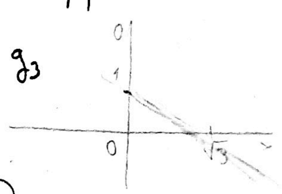
$= x^{\frac76+\left(-\frac{25}{6}\right)} = x^{-3} = \frac{1}{x^3}$

$g_3(x) = \frac{\sqrt{3x} - x\cdot\sqrt{x}}{\sqrt{3x}} = \frac{\sqrt{3}\cdot x^{\frac12} - x\cdot x^{\frac12}}{\sqrt{3}\cdot x^{\frac12}} = \frac{\sqrt{3}-x}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} - \frac{x}{\sqrt{3}} = 1 - \frac{x}{\sqrt{3}} = 1 - \frac{\sqrt{3}}{3}$

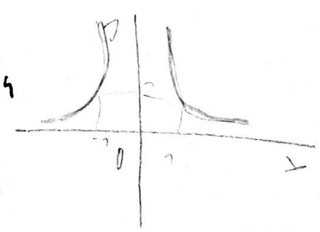
$g_4(x) = \left(1+\frac{1}{x^2}\right)^{-\frac12} \cdot (1+x^2)^{\frac12} \cdot x^{-2} = \frac{\sqrt{1+x^2}}{\sqrt{1+\frac{1}{x^2}}\cdot x^2} = \frac{\sqrt{1+x^2}}{\sqrt{\frac{x^2+1}{x^2}}} \cdot \frac{1}{x^2} =$

$= \frac{\frac{\sqrt{1+x^2}}{\frac{\sqrt{1+x^2}}{|x|}}}{x^2} = \frac{\sqrt{1+x^2}\cdot|x|}{\sqrt{1+x^2}\cdot x^2} = \frac{|x|}{x^2}$

g_3



g_4



(4)

① $f: y = \sqrt{\frac{1-x}{1+x}}$; def. obor, funkční hodnota v b. $\frac{1}{2}$

$y = \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} = \underline{\underline{1}}$ $\frac{1}{2} \cdot \frac{2}{1} = \frac{2}{2}$

③ max. + min. fce
$h: y = -(-2+2x-x^2)$

$y = x^2 - 2x + 2$

vrchol:
$f'(x) = 2x - 2$
$0 = 2x - 2$
$2 = 2x$
$x = 1$

$y = 1 - 2 + 2 = 1$
$[1;1]$ ← vrchol, minimum
maximum nemá

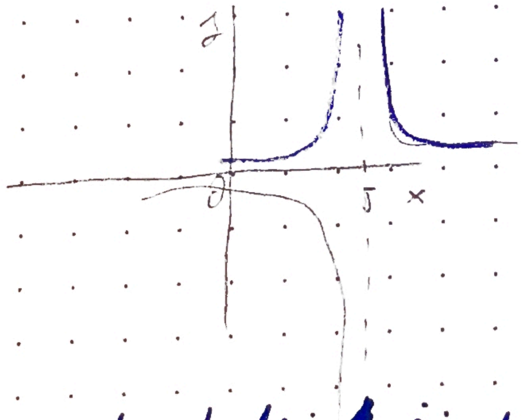
$D(f)$:

$1-x > 0$ ∧ $1+x > 0$ ∨ $1-x < 0$ ∧ $1+x < 0$

$1 > x$ | $1 < x$
$x < 1$ | $x > 1$
$x > -1$ | $x < 1$

$\underline{\underline{x \in (-1;1)}}$

② $g: y = \frac{3}{(x-5)^2}$; graf + vlastnosti



nespojitá
bez monotonie
ani sudá ani lichá
omezená zdola
neprostá

④ inverzní fce k fci k: $y = \sqrt{4-x}$

$\boxed{x \leq 4}$

$|y^2| = 4 - x$

$x = -|y^2| + 4$

$\underline{\boxed{y = -|x^2| + 4}}$ i guess

⑤ $\left\{\frac{n+3}{n}\right\}_{n=1}^{\infty}$

$4; \frac{5}{2}; \frac{6}{3}; \frac{7}{4}; \frac{8}{5}; \ldots$

geometrická posl ⇓

klesající

(27)

a) $$\frac{1+\sqrt{2}+\sqrt{3}}{1+\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{1}+\sqrt{2}+\sqrt{3})\cdot(1-\sqrt{2}-\sqrt{3})}{(1+\sqrt{2}-\sqrt{3})(1-\sqrt{2}-\sqrt{3})} = \frac{1\cancel{-\sqrt{2}}\cancel{-\sqrt{3}}\cancel{+\sqrt{2}}-2-\sqrt{2}\sqrt{3}\cancel{+\sqrt{3}}-\sqrt{2}\sqrt{3}-3}{\cancel{1\,2\,2-2\sqrt{6}}\,1\cancel{-\sqrt{2}}-\sqrt{3}\cancel{+\sqrt{2}}-2\cancel{-\sqrt{6}}-\sqrt{3}\cancel{+\sqrt{6}}+3} =$$

$$= \frac{-4-2\sqrt{6}}{2-2\sqrt{3}} = \frac{(-4-2\sqrt{6})(2+2\sqrt{3})}{\cancel{2}\,4-12} = \frac{-8-8\sqrt{3}-4\sqrt{6}-4\sqrt{18}}{-8} = \frac{8+8\sqrt{3}+4\sqrt{6}+4\sqrt{18}}{8} =$$

$$= \frac{2+2\sqrt{3}+\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2}+\sqrt{3}\cdot\sqrt{2}\cdot\sqrt{3}}{2} = \frac{2+2\sqrt{3}+4\sqrt{2}+3\sqrt{2}}{2} = \frac{2+7\sqrt{2}+2\sqrt{3}}{2}$$

b) $$\frac{\sqrt{5}+1}{\sqrt{10}-\sqrt{5}+\sqrt{2}-1} = \frac{(\sqrt{5}+1)(\sqrt{10}+\sqrt{5}+\sqrt{2}+1)}{(\sqrt{10}-\sqrt{5}+\sqrt{2}-1)(\sqrt{10}+\sqrt{5}+\sqrt{2}+1)} = \frac{\sqrt{15}+5+\sqrt{10}+\sqrt{5}+\sqrt{10}+\sqrt{5}+\sqrt{2}+1}{10+\sqrt{15}+\sqrt{20}+\sqrt{10}-\sqrt{15}-5+\sqrt{10}-\sqrt{5}+\sqrt{20}+\sqrt{10}+2+\sqrt{2}-\sqrt{10}-\sqrt{5}-\sqrt{2}-1} =$$

$$= \frac{6+\cancel{4}\sqrt{15}+2\sqrt{10}+2\sqrt{5}+\sqrt{2}}{6+2\sqrt{20}+2\sqrt{10}-2\sqrt{5}} = \cancel{\frac{6+\sqrt{3}\cdot\sqrt{5}+2\cdot\sqrt{2}\cdot\sqrt{5}+2\cdot\sqrt{5}+\sqrt{2}}{}}$$

.... jindy.

26

a) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

b) $\frac{12}{\sqrt{6}} = \frac{12\sqrt{6}}{6} = 2\sqrt{6}$

c) $\frac{\sqrt{3}-\sqrt{14}}{\sqrt{6}} = \frac{\sqrt{18}-\sqrt{84}}{6} = \frac{\sqrt{3}\cdot\sqrt{3}\cdot\sqrt{2} - \sqrt{2}\cdot\sqrt{2}\cdot\sqrt{21}}{6} = \frac{3\sqrt{2} - \cancel{2}4\sqrt{21}}{6}$

d) $\frac{4\sqrt{15}+5\sqrt{12}}{2\sqrt{3}} = \frac{4\sqrt{45}+5\sqrt{36}}{6} = \frac{4\sqrt{5}\cdot\sqrt{3}\cdot\sqrt{3}+30}{6} = \frac{12\sqrt{5}+30}{6} = 2\sqrt{5}+5$

e) $\frac{2}{3-\sqrt{5}} = \frac{2(3+\sqrt{5})}{3^2-\sqrt{5}^2} = \frac{6+2\sqrt{5}}{9-5} = \frac{3+\sqrt{5}}{2}$

f) $\frac{1}{1+\sqrt{7}} = \frac{1(1-\sqrt{7})}{1-7} = \frac{1-\sqrt{7}}{-6}$

g) $\frac{15}{2\sqrt{3}-3} = \frac{5(2\sqrt{3}+3)}{12-9} = 10\sqrt{3}+15$

h) $\frac{2\sqrt{3}-\sqrt{10}}{\sqrt{6}-\sqrt{5}} = \frac{(2\sqrt{3}-\sqrt{10})(\sqrt{6}+\sqrt{5})}{6-5} = 2\sqrt{3\cdot6}+2\sqrt{3\cdot5}-\cancel{10}\sqrt{6\cdot10}-\sqrt{10}\sqrt{5} =$

$= 2\cdot\sqrt{3}\cdot\sqrt{3}\cdot\sqrt{2}+2\cdot\sqrt{3}\cdot\sqrt{5}-2\cdot\sqrt{3}\cdot\sqrt{5}-5\sqrt{2} = 6\sqrt{2}-5\sqrt{2} = \sqrt{2}$

i) $\frac{2+\sqrt{8}}{6-3\sqrt{2}} = \frac{(2+\sqrt{8})(6+3\sqrt{2})}{6^2-9\cdot2} = \frac{12+\cancel{2}6\sqrt{2}+6\sqrt{8}+3\sqrt{16}}{18} = \frac{4+2\sqrt{2}+2\sqrt{8}+\sqrt{16}}{6} = \frac{8+2\sqrt{2}+2\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2}}{6} =$

$= \frac{8+2\sqrt{2}+4\sqrt{2}}{6} = \frac{8+6\sqrt{2}}{6} = \frac{4}{3}+\sqrt{2}$

j) $\frac{\sqrt{27}-1}{2+4\sqrt{3}} = \frac{(\sqrt{27}-1)(2-4\sqrt{3})}{4-48} = \frac{2\sqrt{27}-4\sqrt{81}-2+4\sqrt{3}}{-44} = \frac{2\cdot\sqrt{9}\cdot\sqrt{3}-36-2+4\sqrt{3}}{-44} = \frac{10\sqrt{3}-38}{-44} = \frac{5\sqrt{3}-19}{-22}$

k) $\frac{3\sqrt{5}-5\sqrt{3}}{3\sqrt{5}+5\sqrt{3}} = \frac{(3\sqrt{5}-5\sqrt{3})(3\sqrt{5}-5\sqrt{3})}{9\cdot5-25\cdot3} = \frac{9\cdot5-15\sqrt{15}-15\sqrt{15}+25\cdot3}{-30} = \frac{120-30\sqrt{15}}{-30} = 4+\sqrt{15}$

l) $\frac{\sqrt{6}-2\sqrt{2}}{\sqrt{15}-2\sqrt{5}} = \frac{(\sqrt{6}-2\sqrt{2})(\sqrt{15}+2\sqrt{5})}{15-4\cdot5} = \frac{6\sqrt{5}+\cancel{12\sqrt{5}}-2\sqrt{30}-4\sqrt{10}}{-5} = \cancel{6\cdot\sqrt{3}\sqrt{5}+12\cdot\sqrt{5}} =$

$= \cancel{\frac{6\cdot\sqrt{3}\cdot\sqrt{5}+12\sqrt{5}-2\sqrt{5}\cdot\sqrt{3}\sqrt{2}-4\sqrt{2}\sqrt{5}}{-5}} = \cancel{\frac{\sqrt{5}(6\sqrt{3}+12-2\sqrt{6}-4\sqrt{2})}{-5}}$

$= \frac{\sqrt{30}+2\sqrt{30}-2\sqrt{30}-4\sqrt{10}}{-5} = \frac{\sqrt{9}\cdot\sqrt{10}+2\sqrt{3}\sqrt{10}-2\sqrt{3}\sqrt{10}-4\sqrt{10}}{-5} = \frac{-1\sqrt{10}}{-5} = \frac{\sqrt{10}}{5}$

21

a) $\sqrt[4]{2}\cdot\sqrt[4]{8}=\sqrt[4]{16}=2$

b) $\sqrt[3]{16}:\sqrt[3]{2}=\sqrt[3]{\frac{16}{2}}=\sqrt[3]{8}=2$

c) $(3\sqrt[3]{16}+2\sqrt[3]{54})\cdot\sqrt[3]{4}=(3\cdot\sqrt[3]{8}\cdot\sqrt[3]{2}+2\cdot\sqrt[3]{27}\cdot\sqrt[3]{2})\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}=(6\sqrt[3]{2}+6\sqrt[3]{2})\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}=$

$=12\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}=12\cdot2=24$

d) $(\sqrt[4]{3}+\sqrt[4]{2})\cdot(\sqrt[4]{27}-\sqrt[4]{8})=\sqrt[4]{81}-\sqrt[4]{24}+\sqrt[4]{54}-\sqrt[4]{16}=3-2-\sqrt[4]{2}\cdot\sqrt[4]{2}\cdot\sqrt[4]{2}\cdot\sqrt[4]{3}+\sqrt[4]{3}\cdot\sqrt[4]{3}\cdot\sqrt[4]{3}\cdot\sqrt[4]{2}=$

$=1-\sqrt[4]{2}^{3}\cdot\sqrt[4]{3}+\sqrt[4]{3}^{2}\cdot\sqrt[4]{2}$

e) $\frac{(2\cdot\sqrt[3]{18}+3\sqrt[3]{12})}{\sqrt[3]{6}}=\frac{2\cdot\cancel{\sqrt[3]{6}}\cdot\sqrt[3]{3}+3\cdot\cancel{\sqrt[3]{6}}\cdot\sqrt[3]{2}}{\cancel{\sqrt[3]{6}}}=2\sqrt[3]{3}+3\sqrt[3]{2}$

f) $(\sqrt[4]{20}+\sqrt[4]{4}):\sqrt[4]{4}=\frac{\cancel{\sqrt[4]{4}}\cdot\sqrt[4]{5}+\cancel{\sqrt[4]{4}}\cdot1}{\cancel{\sqrt[4]{4}}}=\sqrt[4]{5}+1$

22

a) $\sqrt{72}-\sqrt{48}+2\sqrt{75}=\cancel{24}\sqrt{3}\cdot\sqrt{4}-\sqrt{4}\cdot\sqrt{4}\cdot\sqrt{3}+2\cdot\sqrt{3}\cdot\sqrt{25}=2\sqrt{3}-4\sqrt{3}+10\sqrt{3}=8\sqrt{3}$

b) $2\sqrt{8}-\sqrt{8}+11\sqrt{72}=\cancel{2}\sqrt{8}+11\sqrt{9}\sqrt{8}=34\sqrt{8}=34\cdot\sqrt{4}\sqrt{2}=68\sqrt{2}$

c) $8\sqrt{50}+4\sqrt{32}-6\sqrt{162}=8\cdot\sqrt{2}\cdot\sqrt{25}+4\cdot\sqrt{2}\cdot\sqrt{4}\cdot\sqrt{4}-6\cdot\sqrt{2}\cdot\sqrt{3}\cdot\sqrt{3}\cdot\sqrt{3}\cdot\sqrt{3}=40\sqrt{2}+8\sqrt{2}-54\sqrt{2}=-6\sqrt{2}$

d) $\sqrt{98}+\sqrt{200}+\sqrt{128}=\sqrt{2}\cdot\sqrt{7}\cdot\sqrt{7}+\sqrt{8}\cdot\sqrt{25}+\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{32}=7\sqrt{2}+5\sqrt{8}+(\sqrt{2})^{10}=7\sqrt{2}+5\cdot\sqrt{4}\cdot\sqrt{2}+$

$=17\sqrt{2}+$

$8\sqrt{2}=$ $\underline{25\sqrt{2}}$

e) $\sqrt{75}-\sqrt{300}+\sqrt{243}=\sqrt{3}\cdot5-\sqrt{6}\cdot5+\sqrt{3}\cdot9=5\sqrt{3}-5\sqrt{3}\cdot\sqrt{2}+9\sqrt{3}=\sqrt{3}\cdot(5-5\sqrt{2}+9)=\sqrt{3}\,(14-5\sqrt{2})$

f) $2\sqrt{108}-2\sqrt{27}+12\sqrt{72}=2\sqrt{2}\sqrt{2}\sqrt{27}-2\sqrt{27}+12\sqrt{2}\sqrt{6}=2\sqrt{2}\sqrt{2}\sqrt{3}\sqrt{3}\sqrt{3}-2\sqrt{3}\sqrt{3}\sqrt{3}+12\sqrt{2}\sqrt{3}\sqrt{2}=$

$=2\cdot2\cdot3\cdot\sqrt{3}-2\cdot3\cdot\sqrt{3}+12\cdot2\cdot\sqrt{3}=12\sqrt{3}-6\sqrt{3}+24\sqrt{3}=30\sqrt{3}$

23

a) $\sqrt[3]{16}-\sqrt[3]{54}+2\sqrt[3]{250}=\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}-\sqrt[3]{3}\cdot\sqrt[3]{2}\cdot\sqrt[3]{3}\cdot\sqrt[3]{3}+2\sqrt[3]{125}\sqrt[3]{2}=$

$=2\sqrt[3]{2}-3\sqrt[3]{2}+10\sqrt[3]{2}=9\sqrt[3]{2}$

b) $5\sqrt[3]{625}-\sqrt[3]{5}-10\sqrt[3]{40}=5\cdot\sqrt[3]{5}\cdot\sqrt[3]{125}-\sqrt[3]{5}-10\sqrt[3]{5}\cdot\sqrt[3]{8}=25\sqrt[3]{5}-\sqrt[3]{5}-20\sqrt[3]{5}=4\sqrt[3]{5}$

c) $\sqrt[3]{128}+2\cdot\sqrt[3]{16}-\sqrt[3]{1024}=\sqrt[3]{8}\cdot\sqrt[3]{8}\cdot\sqrt[3]{2}+2\cdot\sqrt[3]{2}\cdot\sqrt[3]{8}-\sqrt[3]{8}\cdot\sqrt[3]{8}\cdot\sqrt[3]{8}\cdot\sqrt[3]{2}=4\sqrt[3]{2}+4\sqrt[3]{2}-8\sqrt[3]{2}=0$

d) $\sqrt[3]{32}+2\sqrt[3]{4}-\sqrt[3]{500}=\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{8}+2\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}-\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{125}=2\sqrt[3]{2}\sqrt[3]{2}+2\sqrt[3]{2}\sqrt[3]{2}-5\sqrt[3]{2}\sqrt[3]{2}=$

$=-\sqrt[3]{2}\cdot\sqrt[3]{2}=-\sqrt[3]{4}$

24

a) $\sqrt[4]{48}-\sqrt[4]{3}+2\sqrt[4]{243}=\sqrt[4]{3}\cdot\sqrt[4]{16}-\sqrt[4]{3}+2\sqrt[4]{[illegible]3}\cdot\sqrt[4]{81}=(2-1+6)\sqrt[4]{3}=7\sqrt[4]{3}$

b) $\sqrt[4]{405}+3\sqrt[4]{80}-5\sqrt[4]{5}=\sqrt[4]{5}\cdot\sqrt[4]{81}+3\cdot\sqrt[4]{5}\cdot\sqrt[4]{2}\cdot\sqrt[4]{2}\cdot\sqrt[4]{2}\cdot\sqrt[4]{2}-\sqrt[4]{5}\cdot5=3\cdot\sqrt[4]{5}+6\cdot\sqrt[4]{5}-5\sqrt[4]{5}=4\sqrt[4]{5}$

25

a) $2\sqrt{7}=\sqrt{28}$

b) $5\sqrt{3}=\sqrt{75}$

c) $4\sqrt[3]{2}=\sqrt[3]{128}$

d) $3\sqrt[4]{4}=\sqrt[4]{324}$