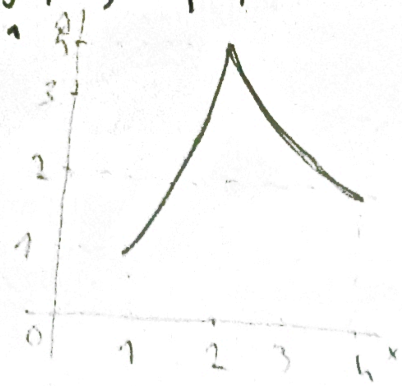
Obsahy rovinných obrazců:

př: $\int_1^4 f(x)\,dx$; $f(x) = \begin{cases} x^2; & x \in \langle 1;2\rangle \\ \frac{8}{x}; & x \in \langle 2;4\rangle \end{cases}$



$\int_1^2 x^2\,dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

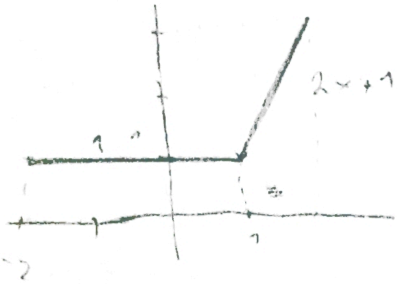
$\int_2^4 \frac{8}{x}\,dx = 8\int_2^4 \frac{1}{x}\,dx = 8\left[\ln|x|\right]_2^4 = 8\cdot(\ln 4 - \ln 2) =$

$= 8\ln 2$

$\Rightarrow \frac{7}{3} + 8\ln 2$

př: $\int_{-2}^{2} (x + |x-1|)\,dx$

$x + |x-1| = \begin{cases} x \in \langle -2;1\rangle \\ x - x + 1 = 1 \\ x \in \langle 1;2\rangle \\ x + |x-1| = 2x-1 \end{cases}$



$\int_{-2}^{1} 1\,dx = [x]_{-2}^{1} = 1-(-2) = 3$

$\int_1^2 2x-1\,dx = [x^2-x]_1^2 = 2-0 = 2$

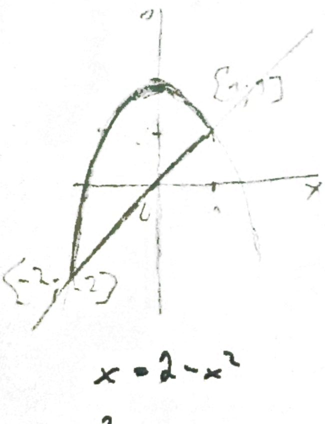
$\Rightarrow 3+2=5$

př: $y = 2-x^2$

$y = x$

$\int_{-2}^{1} x^2+x-2\,dx = \left[\frac{x^3}{3} + \frac{x^2}{2} - 2x\right]_{-2}^{1} = -\frac{7}{6} - \frac{10}{3} = \frac{-7-20}{6} =$

$= -\frac{9}{2} \Rightarrow \frac{9}{2}$



$x = 2-x^2$

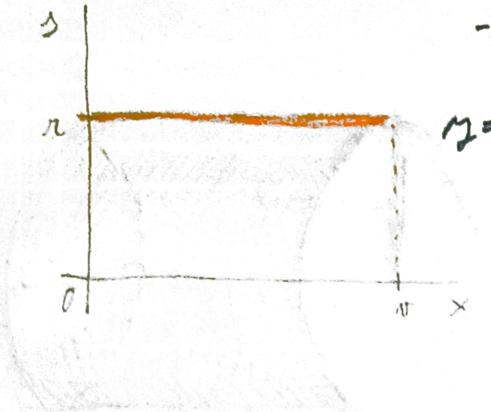
$x^2+x-2=0$

$x_1=1;\ x_2=-2$

$x \in \langle -2;1\rangle$

Objemy rotačních těles:

př:

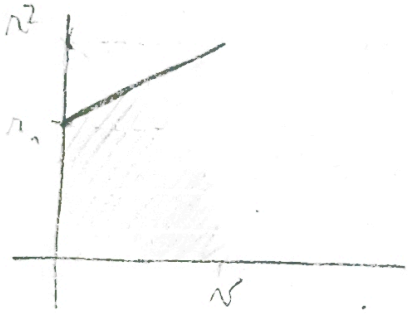


- určit objem válce o výšce v a poloměru r

$y = r$

$$V=\pi\cdot\int_0^v (r)^2\,dx = \pi\left[r^2\cdot x\right]_0^v = \pi(r^2v - r^2 0) = \pi r^2 v$$

př:



$f: y = \frac{-r_1-r_2}{v}\cdot x + r_1$

$V = \frac{\pi\cdot v}{3}(r_1^2 + r_1r_2 + r_2^2)$

$$V=\pi\cdot\int_0^v\left(-\frac{r_1-r_2}{v}\cdot x - r_1\right)^2 dx =$$

$$=\pi\cdot\int_0^v \frac{(r_1-r_2)^2}{v^2}x^2 - 2\cdot\frac{(r_1-r_2)r_1}{v}x + r_1^2\,dx =$$

$$=\pi\left[\frac{(r_1-r_2)^2}{v^2}\cdot\frac{x^3}{3} - \frac{(r_1-r_2)r_1}{v}\cdot x^2 + r_1^2 x\right]_0^v =$$

$$=\pi\cdot\left(\frac{(r_1-r_2)^2}{v^2}\cdot\frac{v^3}{3} - \frac{(r_1-r_2)r_1}{v}\cdot v^2 + r_1^2\cancel{x}\right) - 0 =$$

$$=\pi\left(\frac{(r_1-r_2)\cdot v}{3} - (r_1-r_2)r_1\cdot v + r_1 v\right) =$$

$$=\pi\cdot v\left(\frac{r_1-r_2}{3} - (r_1-r_2)r_1 + r_1\right) =$$

$$=\pi\cdot v\left(\frac{r_1-r_2}{3} - \frac{(3r_1-3r_2)\cdot r_1}{3} + \frac{3r_1}{3}\right) =$$

$$=\pi\cdot v\left(\frac{r_1-r_2-3r_1^2+3r_1r_2+3r_1}{3}\right) =$$

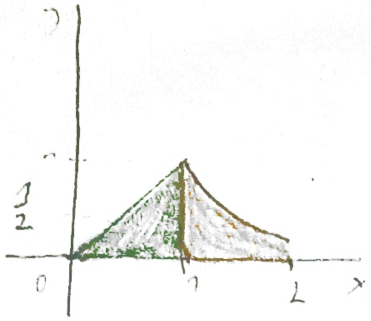
$$=\cancel{\frac{\pi v}{3}}\ \ldots$$

... je to moc dlouhý ...

$$=\frac{\pi v}{3}\left(r_1^2 + r_1r_2 + r_2^2\right)$$

př: $y = x;\ x \in \langle 0;1\rangle$

$y = \frac{1}{x};\ x \in \langle 1;2\rangle$



$V = \frac{1}{3} S_p \cdot v = \frac{1}{3}\pi \cdot 1 \cdot 1 = \frac{1}{3}\pi$

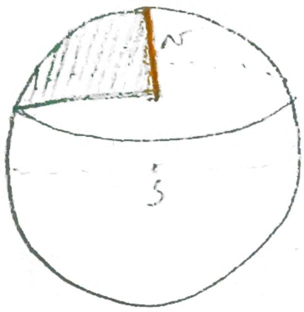
$V = \pi \cdot \int_1^2 \left(\frac{1}{x}\right)^2 dx = \pi \cdot \int_1^2 x^{-2} dx =$

$= \pi \cdot \left[-(x^{-1})\right]_1^2 = -\frac{1}{2}\pi - 1\pi = -\frac{3}{2}\pi$

$\Rightarrow \frac{3}{2}\pi$

$\frac{1}{3}\pi + \frac{2}{3}\pi = \pi$

př:



$x^2 + y^2 = r^2$

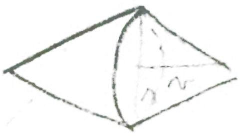
$|y| = \sqrt{r^2 - x^2}$

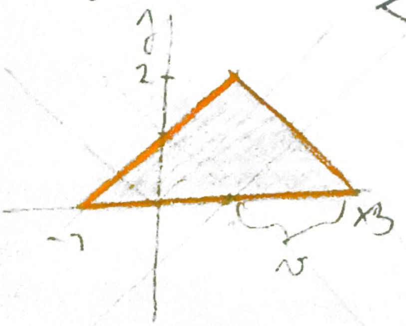
$V = \pi \cdot \int_{r-v}^{r} \left(\sqrt{r^2 - x^2}\right)^2 dx = \pi \cdot \left[\frac{r^2 x}{1} - \frac{x^3}{3}\right]_{r-v}^{r} =$

$= \pi \cdot \left(\left(\frac{r^3}{1} - \frac{r^3}{3}\right) - \left(\frac{r^2(r-v)}{1} - \frac{(r-v)^3}{3}\right)\right)$ ... už jenom úprava výrazu...

př: $y = 2 - |x-1|$

$y = 0$





$V = 2 \cdot \frac{1}{3} S_p \cdot v = 2 \cdot \frac{1}{3}\pi \cdot 2^2 \cdot 2 = \frac{16}{3}\pi$

$V = \pi \cdot \int_{-1}^{1} (x+1)^2 dx + \pi \cdot \int_1^3 (-x+3)^2 dx =$ ofiko řešení, vyjde to stejně