př: Určit podle spojitosti, jestli $x^3-5x^2+3x+1=0$ má řešení na intervalu

a) $\langle -4;-2\rangle$

b) $\langle -2;2\rangle$

c) $\langle 2;5\rangle$

a) $f(a)=f(-4)=-155$
$f(b)=f(-2)=-33$ $\quad f(a)\cdot f(b)>0$ → nemusí mít kořen na tomto intervalu

b) $f(a)=f(-2)=-33$
$f(b)=f(2)=-5$ $\quad f(a)\cdot f(b)>0$ → ne

c) $f(a)=f(2)=-5$
$f(b)=f(5)=16$ $\quad f(a)\cdot f(b)<0$ → má kořen na tomto intervalu

__easy limity:__

$$\lim_{x\to 2}(x^2-3)=2^2-3=1$$

$$\lim_{x\to -2}\frac{3x+4}{x^2+1}=\frac{-6+4}{5}=-\frac{2}{5}$$

$$\lim_{x\to \frac{\pi}{6}}\sin x=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{3x^2-x}{x}=\lim_{x\to 0}\frac{x(3x-1)}{x}=\lim_{x\to 0}3x-1=-1$$

$$\lim_{x\to -1}\frac{x^2+4x+3}{x^3+1}\underset{\lim_{x\to -1}}{=}\frac{(x+1)(x+3)}{x^3+1}\Rightarrow \text{l'Hopitalovo pravidlo}\Rightarrow\frac{2x+4}{3x^2}=\frac{-2+4}{3}=\frac{2}{3}$$

$$\lim_{x\to 3}\frac{x-3}{\sqrt{x+1}-2}=\lim_{x\to 3}\frac{(x-3)(\sqrt{x+1}+2)}{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}\overset{\lim_{x\to 3}}{=}\frac{(x-3)(\sqrt{x+1}+2)}{(x+1)-4}=\frac{\cancel{(x-3)}(\sqrt{x+1}+2)}{\cancel{x-3}}=$$

$$=\sqrt{x+1}+2=4$$

another limit:

$$\lim_{x\to -2} \frac{x^2-6x+7}{x^2-5x+7} = \frac{4-6(-2)+7}{4+10+7} = \frac{23}{21}$$

$$\lim_{x\to 3} \frac{x^3+2x^2-9x-18}{x^2+2x-15} \overset{l'H.}{=} \lim_{x\to 3} \frac{3x^2+4x-9}{2x+2} = \frac{30}{8} = \frac{15}{4}$$

$$\lim_{x\to 0} \frac{\sqrt{x+3}-2}{x-1} = \frac{\sqrt{3}-2}{-1} = 2-\sqrt{3}$$

$$\lim_{x\to 6} \frac{12-2x}{\sqrt{x-3}-\sqrt{2x-9}} = \lim_{x\to 6} \frac{(12-2x)\cdot\left(\sqrt{x-3}+\sqrt{2x-9}\right)}{\left(\sqrt{x-3}-\sqrt{2x-9}\right)\left(\sqrt{x-3}+\sqrt{2x-9}\right)} \overset{\lim\limits_{x\to 6}}{=} \frac{(12-2x)\left(\sqrt{x-3}+\sqrt{2x-9}\right)}{x-3-2x+9} =$$

$$= \frac{\dots}{-x+6} = \frac{\dots}{0} \dots \text{nasrol}$$

$$\overset{l'H.:}{=} \frac{12-2x}{(x-3)^{\frac{1}{2}}-(2x-9)^{\frac{1}{2}}} = \frac{-2}{\frac{1}{2}(x-3)^{-\frac{1}{2}}\cdot 1-\frac{1}{2}(2x-9)^{-\frac{1}{2}}\cdot 2} = \frac{-2}{-0{,}288\dots} = \frac{2}{0{,}289} = \underline{\underline{4\sqrt{3}}}$$

$$\lim_{x\to 1_+} \frac{1}{x-1}+2 = \infty$$

$$\lim_{x\to 0} \frac{(x+1)\cdot\sin x}{3x^2+5x} \underset{\lim\limits_{x\to 0}}{=} \frac{(x+1)\cdot\sin x}{x(3x+5)} \underset{\lim\limits_{x\to 0}}{=} \frac{\sin x}{x}\cdot\frac{x+1}{3x+5} = 1\cdot\frac{1}{5} = \frac{1}{5}$$

$$\lim_{x\to 3_+} \frac{2}{x-3}-1 = \infty$$

$$\lim_{x\to -3} \frac{2}{x-3}-1 = -\infty$$

nevlastní limity time...

$\lim\limits_{x\to\infty} \frac{2}{x-3} - 1 = -1$

$\lim\limits_{x\to-\infty} \frac{2}{x-3} - 1 = -1$

$\lim\limits_{x\to\infty} \frac{x-3}{x} = \lim\limits_{x\to\infty} \frac{x}{x} - \frac{3}{x} = 1 - \frac{3}{\infty} = 1$

$\lim\limits_{x\to\infty} \frac{x-5x^2+3}{x^2} = \frac{1}{x} - 5 + \frac{3}{x^2} = -5$

$\lim\limits_{x\to\infty} \frac{(x-3)(x+2)}{x^2} = \frac{x^2+2x-3x-6}{x^2} = \lim\limits_{x\to\infty} 1 - \frac{1}{x} - \frac{6}{x^2} = 1$

$\lim\limits_{x\to\infty} \frac{x-5x^2+3}{x} = 1 - 5x + \frac{3}{x} = -\infty$

$\lim\limits_{x\to\infty} \frac{2x^3-4x-3}{1-2x+5x^3} \underset{\lim\limits_{x\to\infty}}{=} \frac{\cancel{x^3}\left(2-\frac{4}{x^2}-\frac{3}{x^3}\right)}{\cancel{x^3}\left(\frac{1}{x^3}-\frac{2}{x^2}+5\right)} = \frac{2}{5}$

$\lim\limits_{x\to\infty} \frac{(2x-4)(3x+7)(x-1)}{3x(x+2)(4x+5)} = \lim\limits_{x\to\infty} \frac{\left(2-\frac{4}{x}\right)\left(3+\frac{7}{x}\right)\left(1-\frac{1}{x}\right)}{3\left(1+\frac{2}{x}\right)\left(4+\frac{5}{x}\right)} = \frac{2\cdot 3\cdot 1}{3\cdot 1\cdot 4} = \frac{6}{12} = \frac{1}{2}$

$\lim\limits_{x\to\infty} \frac{(2-x)(1-3x)(4+x)}{(15x-2)(3+2x)(x-6)} = \lim\limits_{x\to\infty} \frac{\left(\frac{2}{x}-1\right)\left(\frac{1}{x}-3\right)\left(\frac{4}{x}+1\right)}{\left(15-\frac{2}{x}\right)\left(\frac{3}{x}+2\right)\left(1-\frac{6}{x}\right)} = \frac{-1\cdot(-3)\cdot 1}{15\cdot 2\cdot 1} = \frac{1}{10}$

$\lim\limits_{x\to\infty} \frac{(3-x^2)(4+x^3)(3-7x)}{(3+x-x^2)(2+5x)(3+x^3)} =$

$= \lim\limits_{x\to\infty} \frac{\left(\frac{3}{x^3}-\frac{1}{x}\right)\left(\frac{4}{x^3}+1\right)\left(\frac{3}{x^3}-\frac{7}{x^2}\right)}{\left(\frac{3}{x^3}+\frac{1}{x^2}-\frac{1}{x}\right)\left(\frac{2}{x^3}+\frac{5}{x^2}\right)\left(\frac{3}{x^3}+1\right)} =$ ok tak jinak

$= \lim\limits_{x\to\infty} \frac{\left(\frac{3-x^2}{x^2}\right)\left(\frac{4+x^3}{x^3}\right)\left(\frac{3-7x}{x}\right)}{\left(\frac{3+x-x^2}{x^2}\right)\left(\frac{2+5x}{x}\right)\left(\frac{3+x^3}{x^3}\right)} = \frac{\left(\frac{3}{x^2}-1\right)\left(\frac{4}{x^3}+1\right)\left(\frac{3}{x}-7\right)}{\left(\frac{3}{x^2}+\frac{1}{x}-1\right)\left(\frac{2}{x}+5\right)\left(\frac{3}{x^3}+1\right)} = \frac{-1\cdot 1\cdot(-7)}{-1\cdot 5\cdot 1} = -\frac{7}{5}$

x^6 x^6

$$\lim_{x\to\infty}\frac{(2-5x^2)(3+x^5)(x^3+5x-x^4)}{(3-x+x^6)(2-x^3)(4-9x^2)}=\frac{\left(\frac{2}{x^2}-5\right)\left(\frac{3}{x^5}+1\right)\left(\frac{1}{x}+\frac{5}{x^3}-1\right)}{\left(\frac{3}{x^6}-\frac{1}{x^5}+1\right)\left(\frac{2}{x^3}-1\right)\left(\frac{4}{x^2}-9\right)}=\frac{-5\cdot1\cdot(-1)}{1\cdot(-1)\cdot(-9)}=\frac{5}{9}$$

$x^2 \quad x^5 \quad x^4 \Rightarrow x^{11}$

$x^6 \quad x^3 \quad x^2 \Rightarrow x^{11}$

$$\lim_{x\to\infty}\frac{(4+3x^2)(5+4x^3)}{(6+5x^4)(7-x)}\ \begin{matrix}\Rightarrow x^5\\ \Rightarrow x^5\end{matrix}=\lim_{x\to\infty}\frac{\left(4\frac{4}{x^2}+3\right)\left(\frac{5}{x^3}+4\right)}{\left(\frac{6}{x^4}+5\right)\left(\frac{7}{x}-1\right)}=\frac{3\cdot4}{5\cdot(-1)}=-\frac{12}{5}$$

$$\lim_{x\to\infty}\frac{(4x-3)(3x+1)^5}{(7-6x^2)^3}=\lim_{x\to\infty}\frac{\left(4-\frac{3}{x}\right)\frac{(3x+1)^5}{x^5}}{\frac{(7-6x^2)^3}{(x^2)^3}}=\frac{\left(4-\frac{3}{x}\right)\left(3+\frac{1}{x}\right)^5}{\left(\frac{7}{x^2}-6\right)^3}=\frac{4\cdot3^5}{(-6)^3}=-\frac{9}{2}$$

$$\lim_{x\to\infty}\frac{(4x-3)^2(3x+1)^5}{(7-6x^2)^3}=\frac{\left(\frac{4x-3}{x}\right)^2\left(\frac{3x+1}{x}\right)^5}{\left(\frac{7-6x^2}{x^2}\right)^3\cdot\frac{1}{x}}=\frac{\left(4-\frac{3}{x}\right)^2\left(3+\frac{1}{x}\right)^5}{\left(\frac{7}{x^2}-6\right)^3\cdot\frac{1}{x}}=\frac{\left(4-\frac{3}{x}\right)^2\left(3+\frac{1}{x}\right)^5\cdot x}{\left(\frac{7}{x^2}-6\right)^3}=$$

$x^7 \quad x^7$

$$=\frac{4^2\cdot3^5\cdot\infty}{(-6)^3}=\frac{\infty}{-\dots}=-\infty$$

$$\lim_{x\to\infty}\frac{\sqrt{2x^3-1}}{x^{\frac{3}{2}}}=\lim_{x\to\infty}\sqrt{\frac{2x^3-1}{x^3}}=\lim_{x\to\infty}\sqrt{2-\frac{1}{x^3}}=\sqrt{2}$$

$$\lim_{x\to\infty}\left(3x-\sqrt{9x^2-10x+1}\right)=\frac{3x-\sqrt{9x^2-10x+1}}{1}\cdot\frac{3x+\sqrt{9x^2-10x+1}}{3x+\sqrt{9x^2-10x+1}}=$$

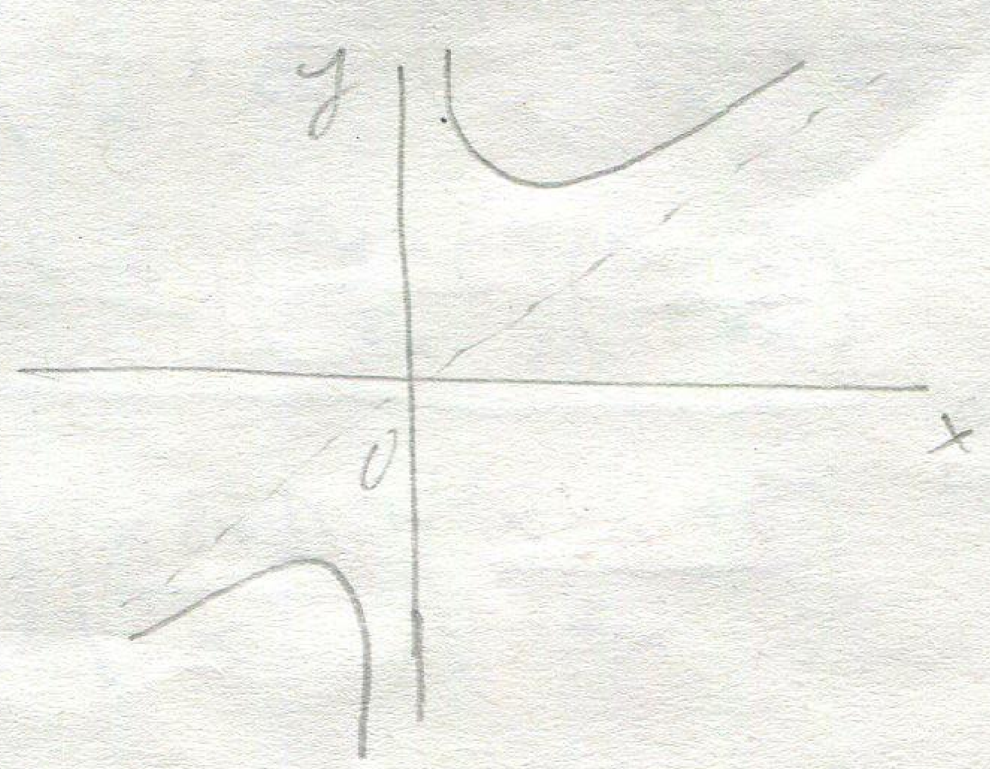
$$=\lim_{x\to\infty}\frac{9x^2-(9x^2-10x+1)}{3x+\sqrt{9x^2-10x+1}}=\frac{10x-1}{3x+\sqrt{9x^2-10x+1}}=\frac{10-\frac{1}{x}}{3+\sqrt{9-\frac{10}{x}+\frac{1}{x^2}}}=\frac{10}{3+\sqrt{9}}=\frac{10}{6}=\frac{5}{3}$$

asymptoty:

př: f: $y=x+\frac{1}{x}$

$$a=\lim_{x\to\infty}\frac{x+\frac{1}{x}}{x}=\lim_{x\to\infty}\frac{1+\frac{1}{x^2}}{1}=1$$

$$b=\lim_{x\to\infty}\left(\left(x+\frac{1}{x}\right)-1x\right)=\lim_{x\to\infty}\frac{1}{x}=0$$



$$y=1x+0\Rightarrow y=x$$

pr: $f: y = \frac{x^2+1}{x+3}$

$$\frac{\frac{a}{b}}{c} = \frac{a}{b} : \frac{c}{1} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{b \cdot c}$$

$$a = \lim_{x\to\infty} \frac{\frac{x^2+1}{x+3}}{x} = \lim_{x\to\infty} \frac{x^2+1}{x^2+3x} = \lim_{x\to\infty} \frac{1+\frac{1}{x^2}}{1+\frac{3}{x}} = 1$$

$$b = \lim_{x\to\infty}\left(f(x) - ax\right) = \lim_{x\to\infty}\left(\frac{x^2+1}{x+3} - x\right) = \lim_{x\to\infty} \frac{(x^2+1)-x(x+3)}{x+3} =$$

$$= \lim_{x\to\infty} \frac{x^2+1-x^2-3x}{x+3} = \cancel{\lim_{x\to\infty} \frac{-2}{x+3} = \lim_{x\to\infty} \frac{-\frac{2}{x}}{1+\frac{3}{x}} = \frac{0}{1} \ldots \to 0\,?}$$

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$$= \lim_{x\to\infty} \frac{1-3x}{x+3} = \lim_{x\to\infty} \frac{\frac{1}{x}-3}{1+\frac{3}{x}} = -\frac{3}{1} = -3 \quad \Rightarrow \underline{y = x-3}$$

limity

plst:

i: $y=\sqrt{4-x}$ $\quad x<4$

$y^2=4-x$

$y^2-4=-x$

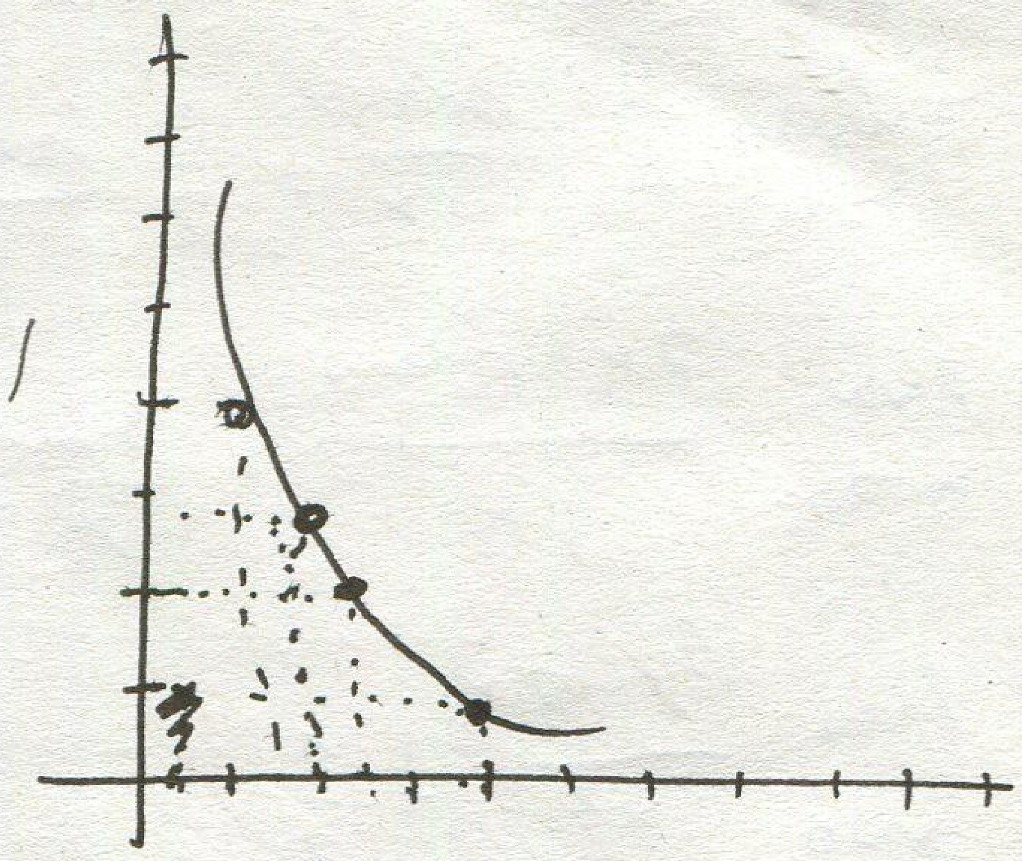
$-y^2+4=x$

$x=4-y^2=(2+y)(2-y)$

$\left\{\frac{n+3}{n}\right\}_{n=1}^{\infty}$ $\quad \frac{4}{1};\ \frac{5}{2};\ \frac{6}{3};\ \frac{7}{4}$

$\frac{5}{2}=4\cdot q$

rostá
omezená zdola (0)
neurčitá nebližá
spojitá



$q=\frac{5}{2}:\frac{4}{1}=\frac{5}{2}\cdot\frac{1}{4}=\frac{5}{8}$

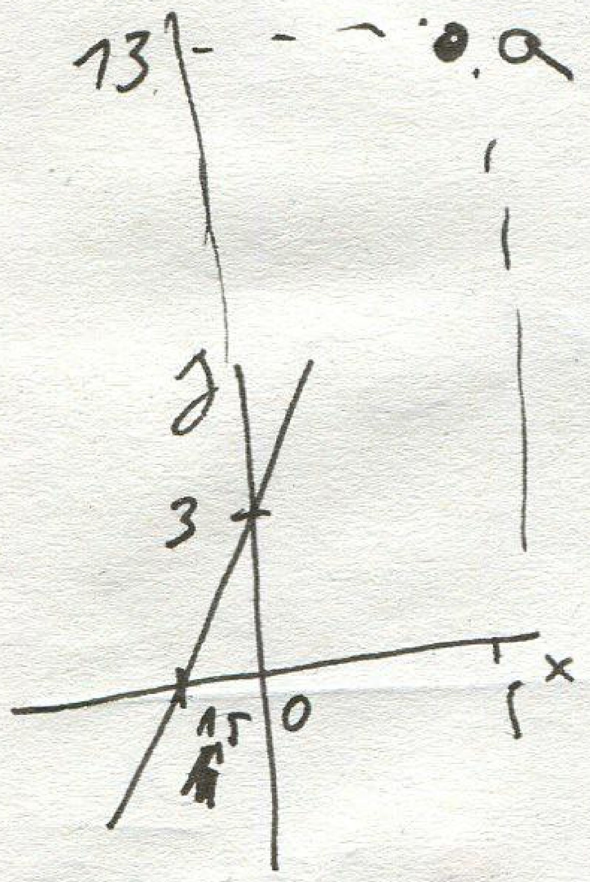
$\frac{5}{8}\cdot\frac{5}{2}=\frac{25}{16}$

není geom ani aritm

$\frac{6}{3}\cdot\frac{5}{8}=\frac{30}{24}=\frac{5}{4}$

$y=2x+3$

$a=5$



$\lim_{x\to 2}\frac{x^2-1}{x^2-3x+2}=\frac{(x+1)\cancel{(x-1)}}{(x-2)\cancel{(x-1)}}=$

$=\frac{(x+1)\frac{1}{x}}{(x-2)\frac{1}{x}}=\frac{1+\frac{1}{x}}{1-\frac{2}{x}}=\frac{\frac{3}{2}}{\text{fuck}}$

$\frac{(1+\frac{1}{x})\cdot\frac{1}{x}}{(1-\frac{2}{x})\cdot\frac{1}{x}}=\frac{\frac{1}{x}+\frac{1}{x^2}}{\frac{1}{x}-\frac{2}{x^2}}=\frac{\frac{1}{2}+\frac{1}{4}}{\frac{1}{2}-\frac{2}{4}}$ … here we go again

$\lim_{x\to 2^+}: \frac{3}{0{,}0001}=\infty$

$\lim_{x\to 2^-}: \frac{3}{-0{,}00001}=-\infty$ } neexistuje

i guess

~~lim x→3~~

$\lim_{x\to -3}(7x+10)=-21+10=-11$

$\lim_{x\to 0}\frac{x+\sqrt{x+4}}{1-\sqrt{9-x}}=\frac{0+2}{-2}=-1$

$\lim_{x\to 0}\frac{0+3\sin 0}{1+\cos 0}=\frac{0}{2}=0$

$\lim_{x\to 1}\frac{\log(x+9)}{\log(x+99)}=\frac{\log 10}{\log 100}=\frac{\log 10}{\log 10+\log 10}=\frac{\log 10}{2\log 10}=\frac{1}{2}$

$\lim_{x\to 0}\frac{1-(x-1)^2}{x^2+4x}=\frac{x(1-(x-1))\cdot(1+(x-1))}{x(x+4)}=\frac{(1-x+1)(1+x-1)}{x(x+4)}=\frac{\cancel{x}(2-x)}{\cancel{x}(x+4)}=\frac{2}{4}=\frac{1}{2}$

$\lim_{x\to 0}\frac{4-x}{2-\sqrt{x}}=\frac{4}{2}=2$

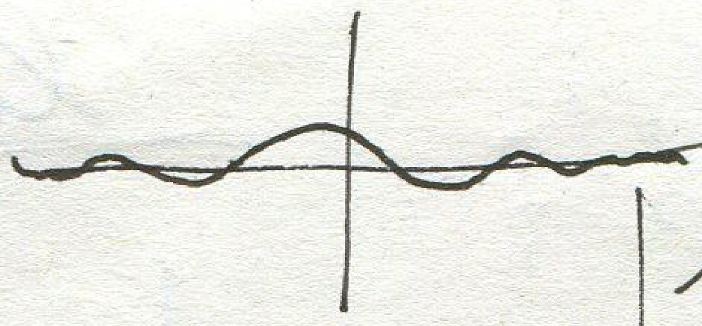
$\lim_{x\to 7}\frac{\sqrt{x-3}-2}{x-7}=\frac{(\sqrt{x-3}-2)\cdot\frac{1}{x}}{(x-7)\cdot\frac{1}{x}}=\frac{\frac{\sqrt{x-3}}{x}-\frac{2}{x}}{1-\frac{7}{x}}=\frac{\frac{2}{7}-\frac{2}{7}}{0}$ *

$x\to 7^+: \frac{0}{\text{něco}}$ $\quad x\to 7^-: \frac{0}{\text{něco}}\to 0 \Rightarrow \underline{0}$

$*\ \frac{(\sqrt{x-3}-2)(\sqrt{x-3}+2)}{(\sqrt{x-3}+2)\cancel{(x-7)}}=\frac{x-3-4}{\dots}=\cancel{x-7}$

$=\frac{1}{\sqrt{x-3}+2}=$

$=\frac{1}{4}$

4. a) $\lim\limits_{x\to 0} \frac{\sin x}{2x} =$ 

$\boxed{\lim \frac{\sin x}{x} = 1}$ $\boxed{\lim \frac{1-\cos x}{x^2} = \frac{1}{2}}$

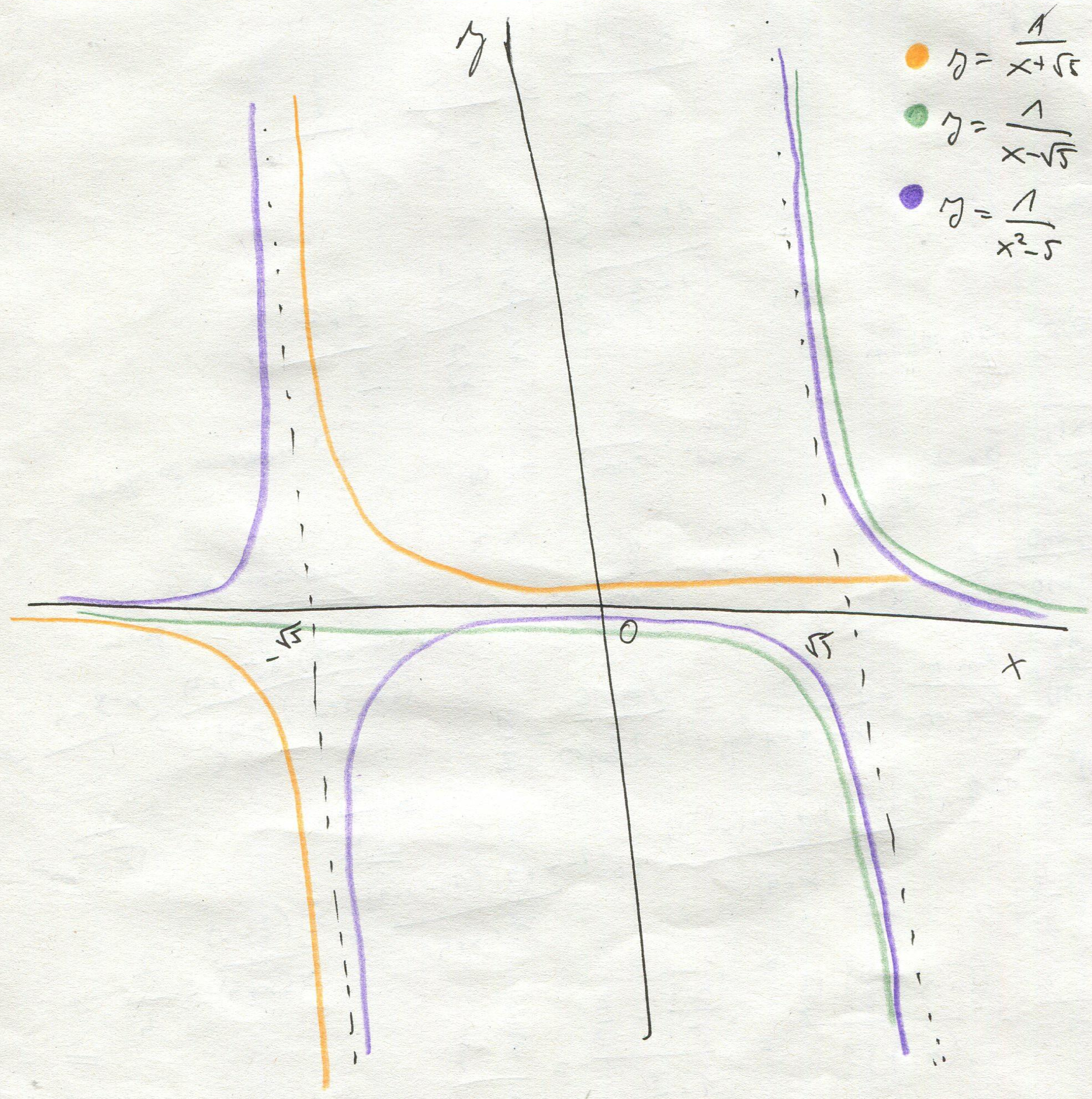
$= \lim\limits_{x\to 0} \frac{1}{2} \cdot \frac{\sin x}{x} = \frac{1}{2} \frac{\sin x}{x} = \frac{1}{2} \cdot 1 = \underline{\underline{\frac{1}{2}}}$

b) $\lim\limits_{x\to 0} \frac{3x+\sin x}{x} = \frac{3\cancel{x}}{\cancel{x}} + 1 = \cancel{3x}\ 3+1 = \underline{\underline{4}}$

c) $\lim\limits_{x\to 0} \frac{1-\cos x}{x^2} = \underline{\underline{\frac{1}{2}}}$

d) $\lim\limits_{x\to 0} \frac{\sin 2x}{3x^2+x} = \cancel{\ldots} =$ l'Hopitalovo pravidlo (= 0 nahoře, 0 dole) ($\neq \infty$) $\Rightarrow$

$\Rightarrow \frac{\cos 2x \cdot 2}{6x+1} = \frac{1 \cdot 2}{0+1} = \underline{\underline{2}}$



⑤ a) $\lim\limits_{x\to\infty} \frac{x+3}{x-2} = \lim\limits_{x\to\infty} \frac{(x+3)\cdot\frac{1}{x}}{(x-2)\cdot\frac{1}{x}} = \frac{1+\frac{3}{x}}{1-\frac{2}{x}} = \frac{1+0,\dots}{1-0,\dots} = \underline{\underline{1}}$

b) $\lim\limits_{x\to\infty} \frac{3x^2-4x+1}{2-x^2} = \lim\limits_{x\to\infty} \frac{(3x^2-4x+1)\frac{1}{x}}{(2-x^2)\frac{1}{x}} = \frac{\frac{3x^2}{x}-\frac{4x}{x}+\frac{1}{x}}{\frac{2}{x}-\frac{x^2}{x}} = \frac{3x-4+\frac{1}{x}}{\frac{2}{x}-x} = \frac{\infty}{-\infty}$

$\frac{(6x-4)\frac{1}{x}}{(-2x)\frac{1}{x}} = \frac{\frac{6x}{x}-\frac{4}{x}}{\frac{-2x}{x}} = \frac{6-\frac{4}{x}}{-2} = \frac{6-0,\dots}{-2} = \underline{-3}$

c) $\lim\limits_{x\to\infty} \frac{(1+5x^4)x^2}{(3x^2+1)^3} =$

$= \frac{x^2+5x^6}{(3x^2+1)^2(3x+1)} = \frac{x^2+5x^6}{(9x^4+2\cdot 3x^2\cdot 1+1)(3x+1)} = \frac{x^2+5x^6}{27x^6+27x^4+9x^2+1} =$

$\cancel{= \frac{2x+30x^5}{5\cdot 27x^4+36x^3+54x^2+12x+3} = \frac{30x^5}{x+2}}$

$= \frac{\cancel{x^6}\cdot\left(\frac{1}{x^4}+5\right)}{\cancel{x^6}\cdot\left(27+\frac{27}{x^2}+\frac{9}{x^4}+\frac{1}{x^6}\right)} = \frac{5}{27+0+0+0} = \underline{\underline{\frac{5}{27}}}$ Eu.

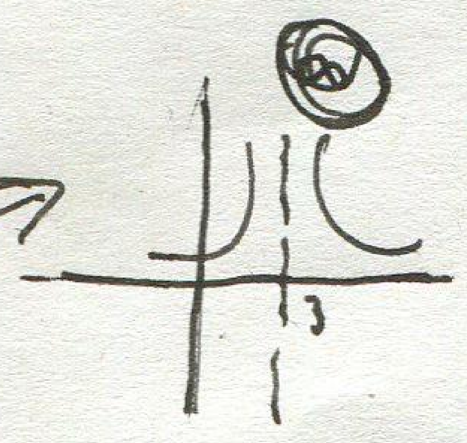
⑥

a) $\lim\limits_{x\to 1+} \frac{x+3}{x-1} = \frac{4}{0,0001} = \infty$

b) $\lim\limits_{x\to 1-} \frac{x+3}{x-1} = \frac{4}{-0,0001} = -\infty$

c) $\lim\limits_{x\to 3+} \frac{1}{(x-3)^2} = \frac{1}{\infty} = 0$

... ale je to ∞



d) $\lim\limits_{x\to 3-} \frac{1}{(x-3)^2}$ .... totéž

e) $\lim\limits_{x\to 0^+} (\ln x - \operatorname{cotg} x) = -\infty - \infty = \underline{\underline{-\infty}}$ $\quad \lim\limits_{x\to 0^+} \ln x = -\infty$

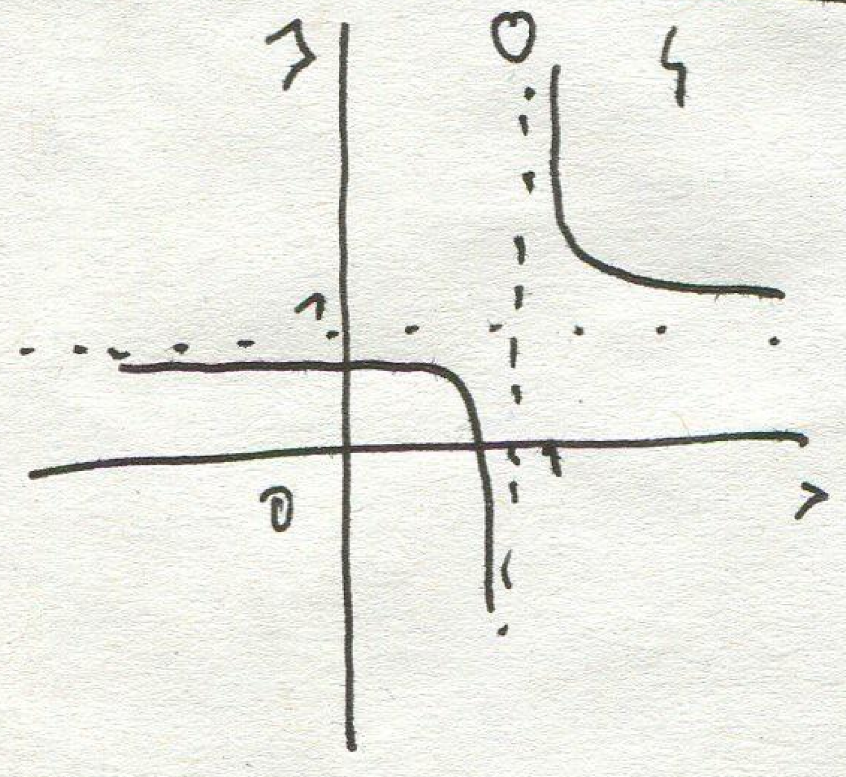
$\lim\limits_{x\to 0^+} \operatorname{cotg} x = +\infty$

⑦

a) $\lim\limits_{x\to 1+} \frac{x+3}{x-1}$ $\quad (x+3):(x-1) = 1 + \frac{4}{x-1}$

$-(x-1)$

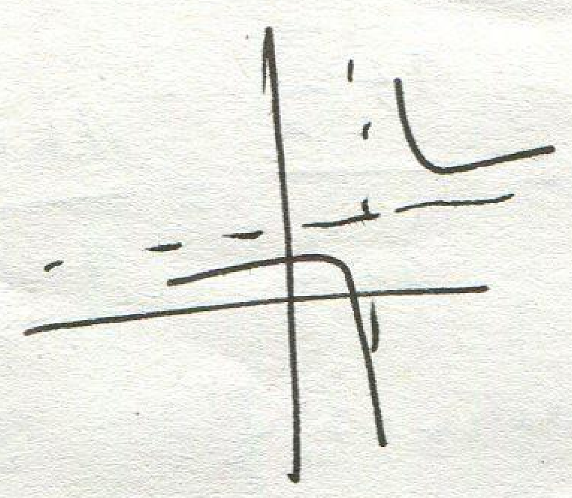
0 4



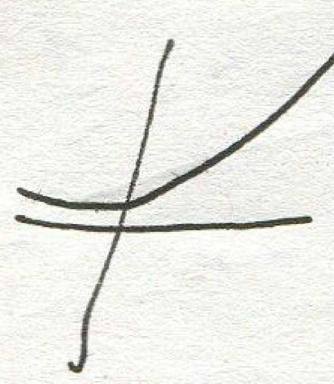
$= \infty$

b) $\lim_{x \to 1^-} \frac{x+3}{x-1} = 1 + \frac{4}{x-1}$

$= -\infty$

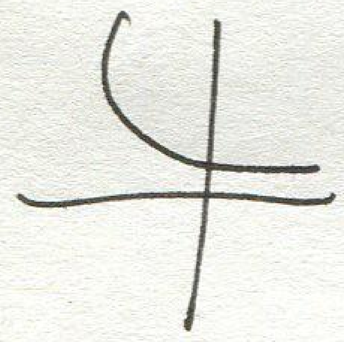


c) $\lim_{x \to \infty} e^x$



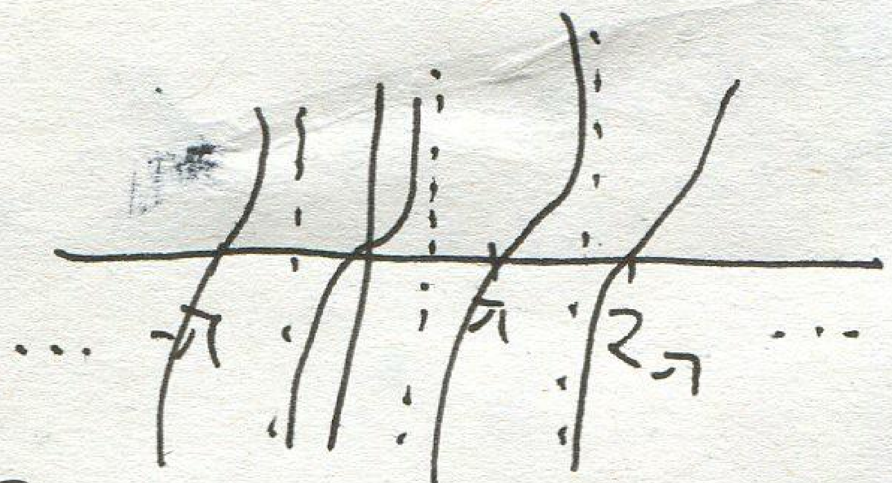
$= \infty$

d) $\lim_{x \to \infty} e^{-x}$



$= 0$

e) $\lim_{x \to \pi^+} \operatorname{tg} x$



$= 0$