Okolí a přírůstky argumentu a funkce

1. Vyjádřete množiny všech reálných x splňující podmínku jako okolí bodu – určete jejich střed a poloměr:

(a) $|x-3| < \frac{2}{5}$;

(b) $|2x+4| < 10$;

(c) $|1-2x| < \sqrt{9}$.

2. Zapište pomocí nerovnosti i pomocí intervalu δ-okolí bodu a:

(a) $a = 7,\ \delta = 0{,}45$;

(b) $a = -\frac{7}{5},\ \delta = \frac{3}{2}$;

(c) $a = \sqrt{50},\ \delta = \sqrt{18}$.

3. Vyjádřete přírůstek funkce f v bodě a:

(a) $f:\ y = 2x+3$, a obecné reálné číslo;

(b) $f:\ y = 2x+3$, $a = 5$;

(c) $f:\ y = \frac{3}{x-2}$, $a = 1$.

Spojitost v bodě a na intervalu

1. Z definice spojitosti dokažte, že:

(a) funkce $f:\ y = 3x+5$ je spojitá v bodě 7;

(b) funkce $g:\ y = -4x+7$ je spojitá v bodě 1.

2. Určete, zda funkce f je spojitá na intervalu I:

(a) $f:\ y = |x|+1$, $I = (-1;\ 3)$;

(b) $f:\ y = x^{-2}$, $I = \langle 1;\ 2\rangle$;

(c) $f:\ y = x^{-2}$, $I = \langle -1;\ 1\rangle$.

3. Pomocí spojitosti zdůvodněte, že rovnice

$$x^3 - 8x^2 + x + 38 = 0$$

má alespoň jedno řešení na intervalech $(-2;\ 0)$, $(1;\ 3)$ a $(6;\ 10)$.

4. S využitím vlastností spojitých funkcí řešte nerovnice:

(a) $x^2 - 7x + 10 < 0$;

(b) $\dfrac{x^2-7x}{x+2} \geq 0$;

(c) $\dfrac{3}{x-5} \leq 2$.

Limita funkce

1. Dokažte z definice limity, že:

(a) $\lim\limits_{x\to 3} 3x - 4 = 5$;

(b) $\lim\limits_{x\to -1} \dfrac{x^2-1}{x+1} = -2$.

2. Vypočítejte:

(a) $\lim\limits_{x\to -3} 7x + 10$;

(b) $\lim\limits_{x\to 0} \dfrac{x+\sqrt{x+4}}{1-\sqrt{9-x}}$;

(c) $\lim\limits_{x\to 0} \dfrac{x+3\sin x}{1+\cos x}$;

(d) $\lim\limits_{x\to 1} \dfrac{\log(x+9)}{\log(x+99)}$.

3. Vypočítejte:

(a) $\lim\limits_{x\to 2} \dfrac{x^2-1}{x^2-3x+2}$;

(b) $\lim\limits_{x\to 0} \dfrac{1-(x-1)^2}{x^2+4x}$;

(c) $\lim\limits_{x\to 0} \dfrac{4-x}{2-\sqrt{x}}$;

(d) $\lim\limits_{x\to 7} \dfrac{\sqrt{x-3}-2}{x-7}$.

4. Vypočítejte:

(a) $\lim\limits_{x\to 0} \dfrac{\sin x}{2x}$;

(b) $\lim\limits_{x\to 0} \dfrac{3x+\sin x}{x}$;

(c) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$;

(d) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{3x^2+x}$.

5. Vypočítejte:

(a) $\lim\limits_{x\to \infty} \dfrac{x+3}{x-2}$;

(b) $\lim\limits_{x\to \infty} \dfrac{3x^2-4x+1}{2-x^2}$;

(c) $\lim\limits_{x\to \infty} \dfrac{(1+5x^4)x^2}{(3x^2+1)^3}$.

6. Vypočítejte:

(a) $\lim\limits_{x\to 1+} \dfrac{x+3}{x-1}$;

(b) $\lim\limits_{x\to 1-} \dfrac{x+3}{x-1}$;

(c) $\lim\limits_{x\to 3+} \dfrac{1}{(x-3)^2}$;

(d) $\lim\limits_{x\to 3-} \dfrac{1}{(x-3)^2}$;

(e) $\lim\limits_{x\to 0+} \ln x - \operatorname{cotg} x$.

7. Z grafu příslušné funkce odvoďte hodnotu limity:

(a) $\lim\limits_{x\to 1+} \dfrac{x+3}{x-1}$;

(b) $\lim\limits_{x\to 1-} \dfrac{x+3}{x-1}$;

(c) $\lim\limits_{x\to \infty} e^{x}$;

(d) $\lim\limits_{x\to \infty} e^{-x}$;

(e) $\lim\limits_{x\to \pi+} \operatorname{tg} x$.

Asymptoty

Určete asymptoty grafu funkce:

$$f_1:\ y=\frac{1}{x-3}$$

$$f_2:\ y=\frac{x^2-2x}{x}$$

$$f_3:\ y=\frac{1}{x^2-5}$$

$$f_4:\ y=\frac{x^2}{x+10}$$

a) $|x-3| < \frac{2}{5} \approx U(3; \frac{2}{5})$

b) $|2x+4| < 10$

$|x-(-2)| < 5 \approx U(-2;5)$

c) $|1-2x| < \sqrt{9}$

$|\frac{1}{2}-x| < \frac{\sqrt{9}}{2} \approx U(\frac{1}{2}; \frac{\sqrt{9}}{2})$

a) $U(7;0{,}45) \approx |x-7| < 0{,}45$

b) $U(-\frac{7}{5}; \frac{3}{2}) \approx |x+\frac{7}{5}| < \frac{3}{2}$

c) $U(\sqrt{50}; \sqrt{18}) \approx |x-\sqrt{50}| < \sqrt{18} = |x-5\sqrt{2}| < 3\sqrt{2}$

3. $\boxed{\Delta y = f(x) - f(a)}$

a) $\Delta y = 2x+3-[2a+3] = 2x-2a = 2\cdot(x-a)$

b) $\Delta y = 2x+3-[2\cdot5+3] = 2x+3-13 = 2x-10 = 2(x-5)$

c) $\Delta y = \frac{3}{x-2} - \frac{3}{1-2} = \frac{3}{x-2} - \frac{3}{(-1)} = \frac{3}{x-2}+3 = \frac{3}{x-2} + \frac{3(x-2)}{x-2} =$

$= \frac{3+3x-6}{x-2} = \frac{3x-3}{x-2} = \frac{3(x-1)}{x-2}$

1. a) Abychom dokázali, že je fce spojitá (v bodě), musíme najít takové $\delta > 0$, aby pro všechna x splňující $|x-a| < \delta$ platilo, že přírůstek fce v bodě a $|f(x)-f(a)| < \varepsilon$ pro libovolně malé ε.

$|f(x)-f(a)| < \varepsilon$

$|3x+5-(3\cdot7+5)| < \varepsilon$

$|3x+\cancel{5}-21-\cancel{5}| < \varepsilon$

$|x-7| < \frac{\varepsilon}{3}$

$|x-a| < \frac{\varepsilon}{3}$

Přímý důkaz:

Nechť $\varepsilon > 0$. Zvolme $\delta = \frac{\varepsilon}{2}$.

$|x-7| < \delta$

$|x-7| < \frac{\varepsilon}{3}$

$|3x-21| < \varepsilon$

$|3x+5-3\cdot7-5| < \varepsilon$

$|3x+5-(3\cdot7+5)| < \varepsilon$

JEDNODUŠEJI: Fce je v daném bodě spojitá, pokud platí: $\lim_{x\to a} f(x) = f(a)$
uvnitř definičního oboru

$\lim 3x+5 = 21+5 = 26 = f(a)$

b. $|f(x)-f(a)| < \varepsilon$

$|-4x+7+4-7| < \varepsilon$

$|-4x+4| < \varepsilon$

$|1-x| < \frac{\varepsilon}{4}$

Přímý důkaz:

Nechť $\varepsilon > 0$. Zvolme $\delta = \frac{\varepsilon}{4}$

$|1-x| < \delta$

$|1-x| < \frac{\varepsilon}{4}$

$|-4x+4| < \varepsilon$

$|-4x+7-(-4+7)| < \varepsilon$

JEDNODUŠEJI:

$\lim_{x\to 1} -4x+7 = -4+7 = 3 = f(a)$

2. a) $f_1: y = x$ je spojitá $\Rightarrow f_2: y = |x|$ je spojitá

$f_2: y = |x|$ je spojitá $\wedge$ $f_3: y = 1$ je spojitá $\Rightarrow f: y = |x|+1$ <u>je spojitá</u>

b) $f: y = \frac{1}{x^2}$ - podíl spojitých funkcí je spojitý ve všech bodech kromě nulových bodů jmenovatele

$x^2 = 0$

$x = 0$

$0 \notin \langle 1; 2 \rangle \Rightarrow$ <u>je spojitá</u>

c) -||-

$0 \in \langle -1; 1 \rangle \Rightarrow$ na daném intervalu <u>není spojitá</u>

3. Všechny polynomické funkce jsou spojité, tedy i naše funkce $\Rightarrow$ můžeme využít Bolzanovy - Weierstrassovy věty:

„Je-li funkce spojitá v $\langle a; b \rangle$ a $f(a) \neq f(b)$, potom ke každému číslu K, které leží mezi čísly $f(a)$ a $f(b)$, existuje jeden takový bod $c \in (a; b)$, že $f(c) = K$."

Zde $K = 0 \rightarrow$ vypočteme funkční hodnoty v krajních bodech intervalu a pokud $K = 0$ bude ležet mezi nimi, musí existovat i odpovídající c, kde $f(c) = K$, který bude

Interval $(-2;0)$: $f(-2) = -8 - 8.4 - 2 + 38 = -8 - 32 - 2 + 38 = -4$

$f(0) = 0 - 0 + 0 + 38 = 38$

$f(a) < 0 < f(b)$

$-4 < 0 < 38 \rightarrow$ řešení existuje

Interval $(1;3)$: $f(1) = 1 - 8 + 1 + 38 = 32$

$f(3) = 27 - 8.9 + 3 + 38 = 68 - 72 = -4$

$f(a) > 0 > f(b)$

$32 > 0 > -4 \rightarrow$ řešení existuje

Interval $(6;10)$: $f(6) = 216 - 288 + 6 + 38 = 260 - 288 = -28$

$f(10) = 1000 - 8.100 + 10 + 38 = 248$

$f(a) < 0 < f(b)$

$-28 < 0 < 248 \rightarrow$ řešení existuje

Pozn.: $K = 0$, takže vlastně využíváme Darbouxovu vlastnost, což je vlastně speciální případ B-W věty.

i. Využití vlastností spojitých funkcí při řešení nerovnic:
Když si rozdělíme D_f na intervaly podle nulových bodů, pak víme, že všechna x z daného intervalu mají stejné znaménko (pokud je funkce na daném intervalu spojitá).

a) $x^2 - 7x + 10 < 0$

$(x-2).(x-5) < 0$

$K = (2;5)$

	$(\infty;2)$	$(2;5)$	$(5;\infty)$
$(x-2)$	$-$	$+$	$+$
$(x-5)$	$-$	$-$	$+$
$(x-2).(x-5)$	$+$	$-$	$+$

b) $\dfrac{x^2 - 7x}{x+2} \geq 0 \qquad x \neq -2$

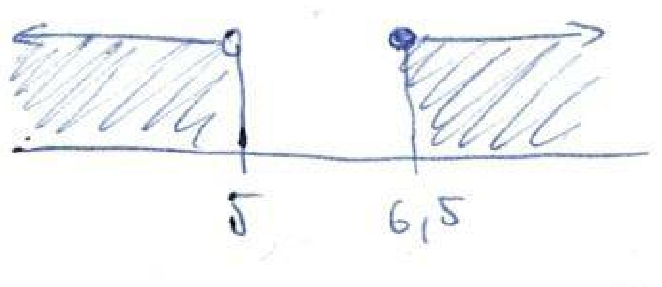
$\dfrac{x(x-7)}{x+2} \geq 0$

	$(-\infty;-2)$	$(-2;0\rangle$	$\langle 0;7\rangle$	$\langle 7;\infty)$
$(x+2)$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
$(x-7)$	$-$	$-$	$-$	$+$
$\dfrac{x.(x-7)}{x+2}$	$-$	$+$	$-$	$+$

$K = (-2;0\rangle \cup \langle 7;\infty)$

c) $\dfrac{3}{x-5} \leq 2$

$\boxed{x > 5}$	$\boxed{x < 5}$
$3 \leq 2x - 10$	$3 \geq 2x - 10$
$\frac{13}{2} \leq x$	$\frac{13}{2} \geq x$
$\boxed{6{,}5 \leq x}$	$\boxed{6{,}5 \geq x}$



$K = (-\infty;5) \cup \langle 6{,}5;\infty)$

1. a) $\lim_{x\to3} 3x-4 = 9-4 = 5$

b) $\lim_{x\to-1}\frac{x^2-1}{x+1} = \lim_{x\to-1}\frac{\cancel{(x+1)}\cdot(x-1)}{\cancel{(x+1)}} = -2$

. a) $\lim_{x\to-3} 7x+10 = -21+10 = -11$

b) $\lim_{x\to0}\frac{x+\sqrt{x+4}}{1-\sqrt{9-x}} = \frac{0+2}{1-3} = \frac{2}{(-2)} = -1$

c) $\lim_{x\to0}\frac{x+3\sin x}{1+\cos x} = \frac{0+0}{1+1} = 0$

d) $\lim_{x\to1}\frac{\log(x+9)}{\log(x+99)} = \frac{\log_{10}10}{\log_{10}100} = \frac{1}{2}$

. a) $\lim_{x\to2}\frac{x^2-1}{x^2-3x+2} = \lim_{x\to2}\frac{(x+1).\cancel{(x-1)}}{\cancel{(x-1)}.(x-2)} = \lim_{x\to2}\frac{x+1}{x-1} \Rightarrow$ limita neexistuje

b) $\lim_{x\to0}\frac{1-(x-1)^2}{x^2+4x} = \lim_{x\to0}\frac{\cancel{1}-x^2+2x\cancel{-1}}{x.(x+4)} = \lim_{x\to0}\frac{x(x+2)}{x.(x+4)} = \frac{2}{4} = \frac{1}{2}$

c) $\lim_{x\to0}\frac{4-x}{2-\sqrt{x}} = \frac{4}{2} = 2$

d) $\lim_{x\to7}\frac{\sqrt{x-3}-2}{x-7} = \lim_{x\to7}\frac{x-3-4}{(x-7).(\sqrt{x-3}+2)} = \lim_{x\to7}\frac{\cancel{(x-7)}}{\cancel{(x-7)}.(\sqrt{x-3}+2)} = \frac{1}{2+2} = \frac{1}{5}$

. a) $\lim_{x\to0}\frac{\sin x}{2x} = \lim_{x\to0}\frac{\sin x}{x}\cdot\lim_{x\to0}\frac{1}{2} = \frac{1}{2}$

b) $\lim_{x\to0}\frac{3x+\sin x}{x} = \lim_{x\to0}\frac{3\cancel{x}}{\cancel{x}} + \lim_{x\to0}\frac{\sin x}{x} = 3+1 = 4$

c) $\lim_{x\to0}\frac{1-\cos x}{x^2} = \lim_{x\to0}\frac{1-\cos^2 x}{x^2.(1+\cos x)} = \lim_{x\to0}\frac{\sin^2 x}{x^2}\cdot\frac{1}{1+\cos x} = 1.1.\frac{1}{2} = \frac{1}{2}$

d) $\lim_{x\to0}\frac{\sin 2x}{3x^2+x} = \lim_{x\to0}\frac{2\sin x\cdot\cos x}{x(3x+1)} = 1.\frac{2.1}{1} = 2$

. a) $\lim_{x\to\infty}\frac{x+3}{x-2} = \lim_{x\to\infty}\frac{x+3}{x} : \frac{x+2}{x} = \lim_{x\to\infty}\left(\frac{x}{x}+\frac{3}{x}\right):\left(\frac{x}{x}+\frac{2}{x}\right) = (1+0):(1+0) = 1$

b) $\lim_{x\to\infty}\frac{3x^2-4x+1}{2-x^2} = \lim_{x\to\infty}\left(\frac{3x^2-4x+1}{x^2} : \frac{2-x^2}{x^2}\right) =$

$= \lim_{x\to\infty}\left[\left(3-\frac{4}{x}+\frac{1}{x^2}\right):\left(\frac{2}{x^2}-1\right)\right] = (3-0+0):(0-1) = -3$

c) $\lim_{x\to\infty}\frac{(1+5x^4)x^2}{(3x^2+1)^3} = \lim_{x\to\infty}\left[\frac{(1+5x^4)\cdot\cancel{x^2}}{x^{\cancel{6}4}} : \frac{(3x^2+1)^3}{(x^2)^3}\right] =$

$= \lim_{x\to\infty}\left[\left(\frac{1}{x^4}+5\right):\left(\frac{3x^2+1}{x^2}\right)^3\right] = \lim_{x\to\infty}\left[\left(\frac{1}{x^4}+5\right):\left(3+\frac{1}{x^2}\right)^3\right] =$

$= (0+5):(3+0)^3 = \frac{5}{27}$

. a) $\lim_{x\to1+}\frac{x+3}{x-1} = \frac{4^+}{0^+} = \infty$

b) $\lim_{x\to1-}\frac{x+3}{x-1} = \frac{4^-}{0^-} = -\infty$

c) $\lim_{x\to3+}\frac{1}{(x-3)^2} = \frac{1}{0^+}\cdot\frac{1}{0^+} = \infty.\infty = \infty$

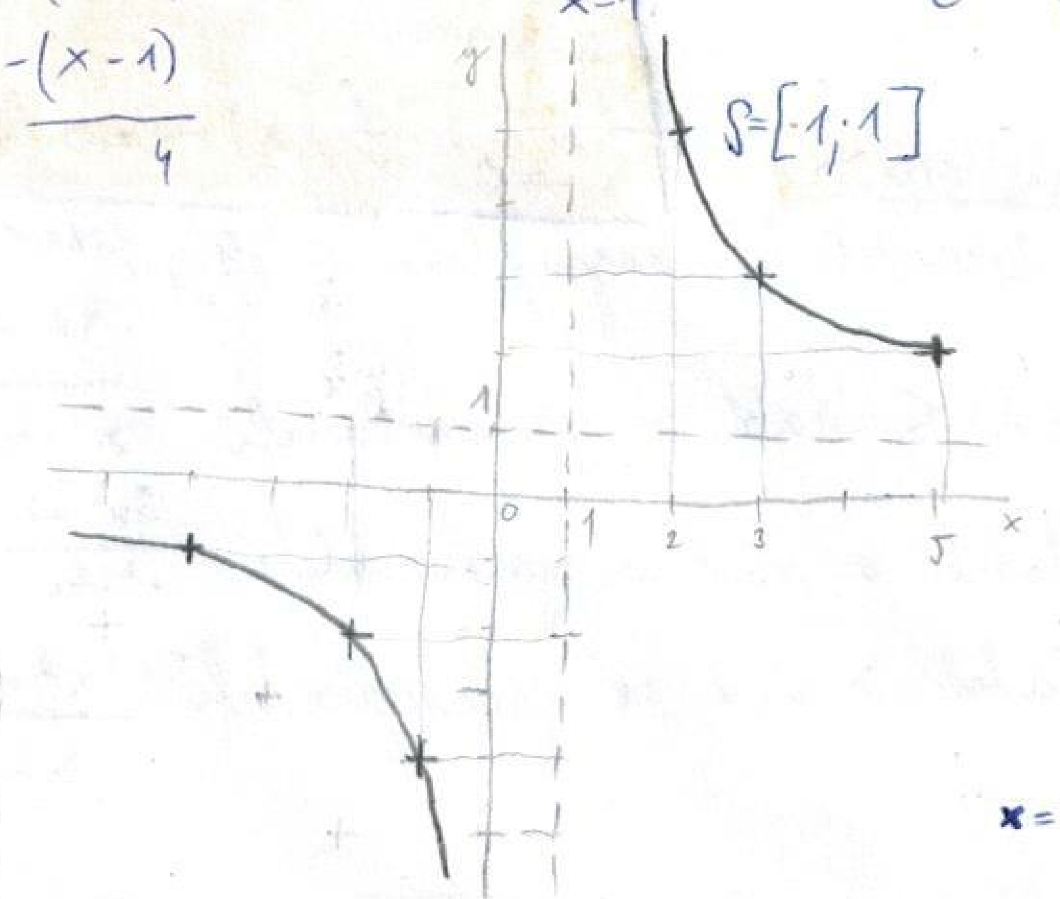
d) $\lim_{x\to3^-}\frac{1}{(x-3)^2} = \frac{1}{0^-}\cdot\frac{1}{0^-} = (-\infty).(-\infty) = \infty$

e) $\lim_{x\to0+}\ln x - \cotg x = -\infty-\infty = -\infty$

7. a) $y = \frac{x+3}{x-1} = 1+\frac{4}{x-1}$ $a_1 = -\frac{d}{c} = \frac{1}{1} = 1$ $a_2 = \frac{a}{c} = \frac{1}{1} = 1$

$(x+3):(x-1) = 1+\frac{4}{x-1}$
$-(x-1)$
4

S=[1;1]

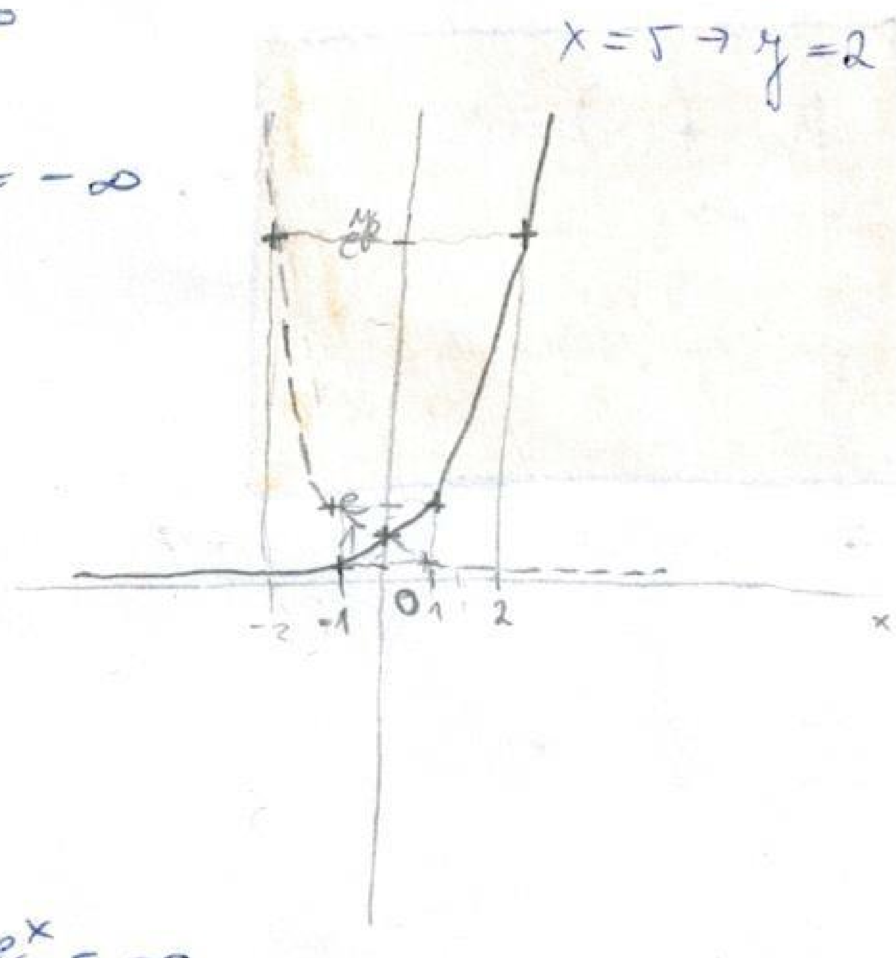


$x=2 \to y=5$
$x=3 \to y=3$
$x=5 \to y=2$

$\lim_{x\to1+}\frac{x+3}{x-1} = \infty$

b) $\lim_{x\to1^-}\frac{x+3}{x-1} = -\infty$

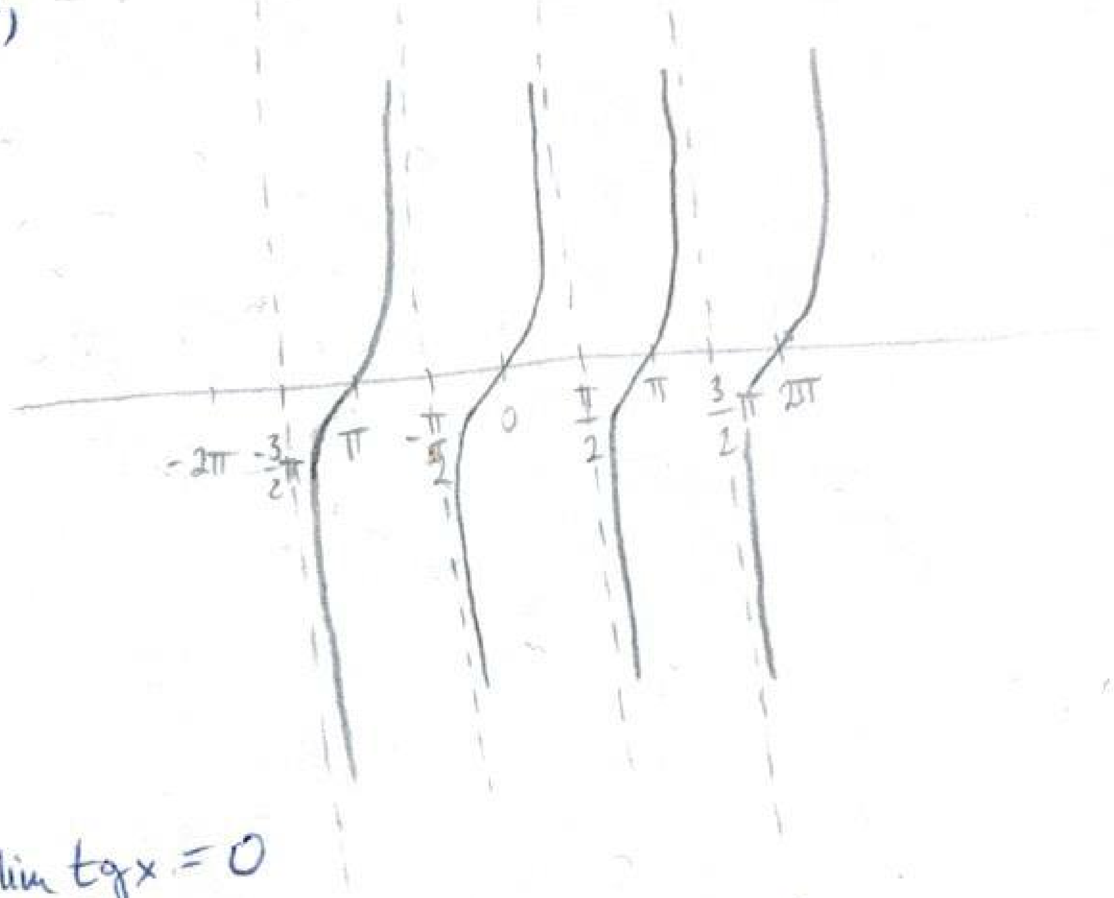
c)



$\lim_{x\to\infty} e^x = \infty$

d) $\lim_{x\to\infty} e^{-x} = 0$

e)



$\lim_{x\to\pi+} tg\, x = 0$

$f_1: y = \frac{1}{x-3}$ $\lim_{x\to3+}\frac{1}{x-3} = \infty$

V: $a_1 = 3(x)$ $\lim_{x\to\infty}\frac{1}{x-3} = 0$
H: $a_2 = 0(y)$ $\lim_{x\to-\infty}\frac{1}{x-3} = 0$

$f_2: y = \frac{x^2-2x}{x} = x-2$

→ nemá asymptoty

$f_3: y = \frac{1}{x^2-5}$

V: $a_1 = \sqrt{5}(x)$ $\lim_{x\to\sqrt{5}+}\frac{1}{x^2-5} = \infty$
$a_2 = -\sqrt{5}(x)$ $\lim_{x\to(-\sqrt{5})+}\frac{1}{x^2-5} = \infty$

H: $a_3 = 0(y)$ $\lim_{x\to\infty}\frac{1}{x^2-5} = 0$

$\lim \frac{1}{\dots}$

$f_4: y = \frac{x^2}{x+10}$

V: $a_1 = -10$ $\lim_{x\to-10^+}\frac{x^2}{x+10} = \infty$

H: $\lim_{x\to\infty}\frac{x^2}{x+10} = \infty$

$\lim_{x\to-\infty}\frac{x^2}{x+10} = -\infty$

→ nemá horizontální

VERTIKÁLNÍ ASYMPTOTA

Funkce f má vertikální asymptotu v „a", jestliže alespoň jedna jednostranná limita f v bodě „a" je nevlastní ($-\infty/\infty$). $\boxed{x=a}$

V praxi jsou to nulové body jmenovatele.

$$\lim_{x \to a^+} f(x) = \infty / -\infty$$
$$\lim_{x \to a^-} f(x) = \infty / -\infty$$

HORIZONTÁLNÍ ASYMPTOTA

Funkce f má horizontální asymptotu v „a", pokud má v nekonečnu/mínus nekonečnu vlastní limitu „a". $\boxed{y=a}$

řád v čitateli < řád ve jmenovateli $\frac{4x+2}{x^2+4x-5}$ → horizontální asymptota existuje, $y=0$

řád v čitateli = řád ve jmenovateli $\frac{3x^2+2}{x^2+4x-5}$ → horizontální asymptota existuje, $y =$ podíl koeficientů vedoucích členů

řád v čitateli > řád ve jmenovateli $\frac{3x^2-2x+1}{x-1}$ → horizontální asymptota neexistuje, existuje pouze šikmá asymptota

$$\lim_{x \to \infty} f(x) = a$$
$$\lim_{x \to -\infty} f(x) = a$$

LIMITA FUNKCE VE VLASTNÍM BODĚ

19.1.1

a) $\lim\limits_{x\to5}\frac{x+3}{7}=\frac{8}{7}$

$\lim\limits_{x\to\frac34\pi}(\sin x+\cos x)=\lim\limits_{x\to\frac34\pi}\sin x+\lim\limits_{x\to\frac34\pi}\cos x=\sin\frac34\pi+\cos\frac34\pi=\sin135^\circ+\cos135^\circ=\sin45^\circ-\cos45^\circ=\frac{\sqrt2}{2}-\frac{\sqrt2}{2}=0$

$\lim\limits_{x\to0}\frac{x^2+2x-1}{x+1}=-1$

$\lim\limits_{x\to1}(2^x-3^x)=\lim\limits_{x\to1}2^x-\lim\limits_{x\to1}3^x=2-3=-1$

$\lim\limits_{x\to0}\frac{\cos2x+\sin2x}{x+1}=\cos0+\sin0=1+0=1$

$\lim\limits_{x\to1}(\log10x-\ln x)=\log10-\ln1=1-0=1$

19.1.2

a) $\lim\limits_{x\to2}\frac{x^2-4}{x-2}=\lim\limits_{x\to2}\frac{(x+2)\cdot\cancel{(x-2)}}{\cancel{(x-2)}}=4$

$\lim\limits_{x\to1}\frac{x^2-1}{x^3-1}=\lim\limits_{x\to1}\frac{(x+1)\cdot\cancel{(x-1)}}{\cancel{(x-1)}\cdot(x^2+x\cdot1+1^2)}=\frac{2}{1+1+1}=\frac{2}{3}$

$\lim\limits_{x\to-3}\frac{4x^2-36}{x+3}=\lim\limits_{x\to-3}\frac{4(x^2-9)}{x+3}=\lim\limits_{x\to-3}\frac{4\cdot\cancel{(x+3)}\cdot(x-3)}{\cancel{(x+3)}}=4\cdot(-6)=-24$

$\lim\limits_{x\to-1}\frac{x^2+2x+1}{x^3+1}=\lim\limits_{x\to-1}\frac{(x+1)^{\cancel{2}}}{\cancel{(x+1)}\cdot(x^2-x\cdot1+1^2)}=\lim\limits_{x\to-1}\frac{x+1}{x^2-x+1}=\frac{0}{1+1+1}=0$

$\lim\limits_{x\to-2}\frac{x^4-16}{x^3+8}=\lim\limits_{x\to-2}\frac{(x^2+2^2)\cdot(x^2-2^2)}{(x+2)\cdot(x^2-2x+2^2)}=\lim\limits_{x\to-2}\frac{(x^2+2^2)\cdot\cancel{(x+2)}\cdot(x-2)}{\cancel{(x+2)}\cdot(x^2-2x+2^2)}=\frac{(4+4)\cdot(-4)}{4+4+4}=\frac{-32}{12}=-\frac{8}{3}$

$\lim\limits_{x\to3}\frac{x^2-6x+9}{81-x^4}=\lim\limits_{x\to3}\frac{(x-3)^2}{3^4-x^4}=\lim\limits_{x\to3}\frac{(x-3)^2}{(3^2+x^2)\cdot(3^2-x^2)}=\lim\limits_{x\to3}\frac{(x-3)^2}{(3^2+x^2)\cdot(3+x)\cdot(3-x)}=\lim\limits_{x\to3}\frac{(x-3)^{\cancel{2}}}{(-1)\cdot\cancel{(x-3)}\cdot(x+3)\cdot(x^2+3^2)}=$

$=\frac{0}{(-1)\cdot6\cdot18}=0$

19.1.3

a) $\lim\limits_{x\to2}\frac{x^2-5x+6}{x^2-3x+2}=\lim\limits_{x\to2}\frac{\cancel{(x-2)}\cdot(x-3)}{(x-1)\cdot\cancel{(x-2)}}=\lim\limits_{x\to2}\frac{x-3}{x-1}=\frac{-1}{1}=-1$

$\lim\limits_{x\to3}\frac{x^2+2x-15}{x^2-8x+15}=\lim\limits_{x\to3}\frac{\cancel{(x-3)}\cdot(x+5)}{\cancel{(x-3)}\cdot(x-5)}=\frac{8}{-2}=-4$

$\lim\limits_{x\to-\frac12}\frac{2x^2+7x+3}{2x^2+9x+4}=\lim\limits_{x\to-\frac12}\frac{2x^2+6x+x+3}{2x^2+8x+x+4}=\lim\limits_{x\to-\frac12}\frac{2x(x+3)+(x+3)}{2x(x+4)+(x+4)}=\lim\limits_{x\to-\frac12}\frac{(x+3)\cdot\cancel{(2x+1)}}{(x+4)\cdot\cancel{(2x+1)}}=\frac{-\frac12+\frac62}{-\frac12+\frac82}=\frac{\frac52}{\frac72}=\frac57$

$\lim\limits_{x\to6}\frac{5x+6-x^2}{7x-6-x^2}=\lim\limits_{x\to6}\frac{-x^2+5x+6}{-x^2+7x-6}=\lim\limits_{x\to6}\frac{-x^2+6x-x+6}{-x^2+6x+x-6}=\lim\limits_{x\to6}\frac{-x(x-6)+(-1)\cdot(x-6)}{-x(x-6)+(x-6)}=\lim\limits_{x\to6}\frac{\cancel{(x-6)}\cdot(-x-1)}{\cancel{(x-6)}\cdot(-x+1)}=\frac{-7}{-5}=\frac75$

$\lim\limits_{x\to-2}\frac{x^2-x-6}{x^3+3x^2+2x}=\lim\limits_{x\to-2}\frac{(x-3)\cdot(x+2)}{x\cdot(x^2+3x+2)}=\lim\limits_{x\to-2}\frac{(x-3)\cdot\cancel{(x+2)}}{x\cdot(x+1)\cdot\cancel{(x+2)}}=\frac{-5}{(-2)\cdot(-1)}=-\frac52$

$\lim\limits_{x\to1}\frac{x^4-1}{2x^2-x-1}=\lim\limits_{x\to1}\frac{(x^2+1^2)\cdot(x^2-1^2)}{2x^2-2x+x-1}=\lim\limits_{x\to1}\frac{(x^2+1^2)\cdot(x+1)\cdot(x-1)}{2x(x-1)+(x-1)}=\lim\limits_{x\to1}\frac{(x^2+1^2)\cdot(x+1)\cdot\cancel{(x-1)}}{\cancel{(x-1)}\cdot(2x+1)}=\frac{2\cdot2}{2+1}=\frac43$

$$\lim_{x\to\frac{1}{3}} \frac{3x^2+5x-2}{27x^3-1} = \lim_{x\to\frac{1}{3}} \frac{3x(x+2)-x-2}{(3x)^3-1^3} = \lim_{x\to\frac{1}{3}} \frac{\cancel{3x}(x+2)+\cancel{(-1)}.(x+2)}{\cancel{(3x-1)}.(27x^3+3x\cdot1+1^2)} = \frac{\frac{1}{3}+\frac{6}{3}}{27.\frac{1}{27}+\frac{3}{3}+\frac{3}{3}} = \frac{\frac{7}{3}}{\frac{3+3+3}{3}} =$$

$$= \underline{\underline{\frac{7}{9}}}$$

$$\lim_{x\to\sqrt{3}} \frac{x^4+x^2-12}{x^4-2x^2-3} = \lim_{x\to\sqrt{3}} \frac{a^2+a-12}{x^4-3x^2+x^2-3} = \lim_{x\to\sqrt{3}} \frac{(a+4).(a-3)}{x^2(x^2-3)+(x^2-3)} = \lim_{x\to\sqrt{3}} \frac{(x^2+4).\cancel{(x^2-3)}}{\cancel{(x^2-3)}.(x^2+1)} = \frac{3+4}{3+1} = \underline{\underline{\frac{7}{4}}}$$

$$\lim_{x\to 2} \frac{x^3-2x^2-4x+8}{x^4-8x^2+16} = \lim_{x\to 2} \frac{x^2(x-2)-4.(x-2)}{a^2-8a+16} = \lim_{x\to 2} \frac{(x-2).(x^2-2^2)}{(a-4)^2} = \lim_{x\to 2} \frac{(x-2).\cancel{(x^2-2^2)}}{(x^2-2^2)\cancel{2}} = \lim_{x\to 2} \frac{\cancel{(x-2)}}{(x+2).\cancel{(x-2)}} = \underline{\underline{\frac{1}{4}}}$$

$$\lim_{x\to 3} \frac{(x-1)^3-8}{3x^2-10x+3} = \lim_{x\to 3} \frac{(x-1-2).[(x-1)^2+2.(x-1)+2^2]}{3x^2-9x-x+3} = \lim_{x\to 3} \frac{(x-3).[x^2-2x+1+2x-2+4]}{3x(x-3)+(-1).(x-3)} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x^2+3)}{\cancel{(x-3)}.(3x-1)} =$$

$$= \lim_{x\to 3} \frac{(x^2+3)}{(3x-1)} = \frac{9+3}{9-1} = \frac{12}{8} = \underline{\underline{\frac{3}{2}}}$$

9.1.4

a) $$\lim_{x\to\frac{\pi}{6}} \frac{2\sin^2x+\sin x-1}{2\sin^2x-5\sin x+2} = \lim_{x\to\frac{\pi}{4}} \frac{2a^2+a-1}{2a^2-5a+2} = \lim_{x\to\frac{\pi}{4}} \frac{2a^2-a+2a-1}{2a^2-a-4a+2} = \lim_{x\to\frac{\pi}{4}} \frac{a(2a-1)+(2a-1)}{a(2a-1)+(-2).(2a-1)} = \lim_{x\to\frac{\pi}{4}} \frac{\cancel{(2a-1)}.(a+1)}{\cancel{(2a-1)}.(a-2)} =$$

$$= \frac{\frac{1}{2}+1}{\frac{1}{2}-2} = \frac{\frac{3}{2}}{-\frac{3}{2}} = \underline{\underline{-1}}$$

b) $$\lim_{x\to\frac{\pi}{4}} \frac{tg^2x+3tgx-4}{tg^2x+4tgx-5} = \lim_{x\to\frac{\pi}{4}} \frac{a^2+3a-4}{a^2+4a-5} = \lim_{x\to\frac{\pi}{4}} \frac{a^2+4a-a-4}{a^2+5a-a-5} = \lim_{x\to\frac{\pi}{4}} \frac{a(a+4)-(a+4)}{a(a+5)-(a+5)} = \lim_{x\to\frac{\pi}{4}} \frac{(a+4).\cancel{(a-1)}}{(a+5).\cancel{(a-1)}} =$$

$$= \frac{1+4}{1+5} = \underline{\underline{\frac{5}{6}}}$$

c) $$\lim_{x\to\pi} \frac{\cos^2x-3\cos x-4}{\cos^2x-4\cos x-5} = \lim_{x\to\pi} \frac{a^2-3a-4}{a^2-4a-5} = \lim_{x\to\pi} \frac{a^2-4a+a-4}{a^2-5a+a-5} = \lim_{x\to\pi} \frac{a(a-4)+(a-4)}{a(a-5)+(a-5)} = \lim_{x\to\pi} \frac{(a-4).\cancel{(a+1)}}{(a-5).\cancel{(a+1)}} = \frac{-1-4}{-1-5} = \underline{\underline{\frac{5}{6}}}$$

d) $$\lim_{x\to-\frac{\pi}{4}} \frac{4+2\cot g x-2\cot g^2x}{\cot g^2x-1} = \lim_{x\to-\frac{\pi}{4}} \frac{-2a^2+2a+4}{a^2-12} = \lim_{x\to-\frac{\pi}{4}} \frac{-2a+4a-2a+4}{(a+1).(a-1)} = \lim_{x\to-\frac{\pi}{4}} \frac{a(-2a+4)+(-2a+4)}{(a+1).(a-1)} =$$

$$= \lim_{x\to-\frac{\pi}{4}} \frac{\cancel{(a+1)}.(-2a+4)}{\cancel{(a+1)}.(a-1)} = \frac{-2.(-1)+4}{-1-1} = \frac{2+4}{-2} = \frac{6}{-2} = \underline{\underline{-3}}$$

9.1.5

a) $\lim\limits_{x\to 3} \dfrac{x^4-x^3-5x^2-2x-3}{x-3} = \lim\limits_{x\to 3} x^3+2x^2+x+1 = 27+18+3+1 = \underline{\underline{49}}$

$(x^4-x^3-5x^2-2x-3):(x-3) = x^3+2x^2+x+1$

$-(x^4-3x^3)$

$2x^3-5x^2-2x-3$

$-(2x^3-6x^2)$

x^2-2x-3

$-(x^2-3x)$

$x-3$

$-(x-3)$

0

b) $\lim\limits_{x\to 2} \dfrac{x^4-2x^3+2x^2-5x+2}{x-2} = \lim\limits_{x\to 2} x^3+2x-1 = 8+4-1 = \underline{\underline{11}}$

$(x^4-2x^3+2x^2-5x+2):(x-2) = x^3+0+2x-1$

$-(x^4-2x^3)$

$0x^3+2x^2-5x+2$

$-(0x^3+0x^2)$

$2x^2-5x+2$

$-(-2x^2-4x)$

$-x+2$

$-(-x+2)$

0

c) $\lim\limits_{x\to 1} \dfrac{x^3-3x+2}{x^4-4x+3} = \lim\limits_{x\to 1} \dfrac{x\cdot(x^2-3)+2}{x(x^3-4)+3} =$

$= \lim\limits_{x\to 1} \dfrac{x(x^2-4)+x+2}{x\cdot(x^3-1)-3x+3} = \lim\limits_{x\to 1} \dfrac{x\cdot(x+2)\cdot(x-2)+(x+2)}{x\cdot(x^3-1)-3\cdot(x-1)} = \lim\limits_{x\to 1} \dfrac{(x+2)\cdot(x\cdot(x-2)+1)}{x\cdot(x-1)\cdot(x^2+x+1)-3\cdot(x-1)} = \lim\limits_{x\to 1} \dfrac{(x+2)\cdot(x^2-2x+1^2)}{(x-1)\cdot[(x^2+x+1)\cdot x-3]} =$

$= \lim\limits_{x\to 1} \dfrac{(x+2)\cdot\cancel{(x-1)}\cdot(x-1)}{\cancel{(x-1)}\cdot[x\cdot(x^2+x+1)-3]} = \lim\limits_{x\to 1} \dfrac{x^2+x-2}{x\cdot(x^2+x-2)+3x-3} = \lim\limits_{x\to 1} \dfrac{(x-1)\cdot(x+2)}{x\cdot(x-1)\cdot(x+2)+3\cdot(x-1)} = \lim\limits_{x\to 1} \dfrac{\cancel{(x-1)}\cdot(x+2)}{\cancel{(x-1)}\cdot[x\cdot(x+2)+3]} =$

$= \dfrac{3}{3+3} = \underline{\underline{\dfrac{1}{2}}}$

d) $\lim\limits_{x\to -1} \dfrac{x^3+3x^2-2}{x^3+2x+3} = \lim\limits_{x\to -1} \dfrac{x(x^2+3x)-2}{x(x^2+2)+3} = \lim\limits_{x\to -1} \dfrac{x\cdot(x^2+3x+2)-2x-2}{x(x^2-1)+3x+3} = \lim\limits_{x\to -1} \dfrac{x\cdot(x+2)\cdot(x+1)-2\cdot(x+1)}{x\cdot(x^2-1)+3(x+1)} =$

$= \lim\limits_{x\to -1} \dfrac{(x+1)\cdot[x\cdot(x+2)-2]}{x\cdot(x+1)\cdot(x-1)+3\cdot(x+1)} = \lim\limits_{x\to 1} \dfrac{\cancel{(x+1)}\cdot[x\cdot(x+2)-2]}{\cancel{(x+1)}\cdot[x\cdot(x-1)+3]} = \dfrac{-1\cdot(-1+2)-2}{-1\cdot(-1-1)+3} = \underline{\dfrac{-3}{5}}$

19.1.6

a) $\lim\limits_{x\to 3} \dfrac{x-3}{\sqrt{x+1}-2} = \lim\limits_{x\to 3} \dfrac{(x-3)\cdot(\sqrt{x+1}+2)}{(\sqrt{x+1}-2)\cdot(\sqrt{x+1}+2)} = \lim\limits_{x\to 3} \dfrac{(x-3)\cdot(\sqrt{x+1}+2)}{x+1-2^2} = \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}\cdot(\sqrt{x+1}+2)}{\cancel{(x-3)}} = \underline{4}$

b) $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{x+9}-3} = \lim\limits_{x\to 0} \dfrac{x\cdot(\sqrt{x+9}+3)}{(\sqrt{x+9}-3)\cdot(\sqrt{x+9}+3)} = \lim\limits_{x\to 0} \dfrac{x(\sqrt{x+9}+3)}{x+9-3^2} = \lim\limits_{x\to 0} \dfrac{\cancel{x}(\sqrt{x+9}+3)}{\cancel{x}} = \underline{6}$

c) $\lim\limits_{x\to 2} \dfrac{4-x^2}{\sqrt{2x}-2} = \lim\limits_{x\to 2} \dfrac{(2+x)\cdot(2-x)\cdot(\sqrt{2x}+2)}{(\sqrt{2x}-2)\cdot(\sqrt{2x}+2)} = \lim\limits_{x\to 2} \dfrac{(2+x)\cdot(2-x)\cdot(\sqrt{2x}+2)}{2x-2^2} = \lim\limits_{x\to 2} \dfrac{(x+2)\cdot\cancel{(2-x)}\cdot 4}{-2\cancel{(x+2)}} = \dfrac{16}{-2} = \underline{\underline{-8}}$

d) $\lim\limits_{x\to -2} \dfrac{2-\sqrt{6+x}}{x+2} = \lim\limits_{x\to -2} \dfrac{(2-\sqrt{6+x})\cdot(2+\sqrt{6+x})}{(x+2)\cdot(2+\sqrt{6+x})} = \lim\limits_{x\to -2} \dfrac{4-6-x}{(x+2)\cdot(2+\sqrt{6+x})} = \lim\limits_{x\to -2} \dfrac{\cancel{-x-2}}{(-1)\cdot\cancel{(-x-2)}\,(2+\sqrt{6+x})} = \dfrac{-1}{2+2} = \underline{\underline{-\dfrac{1}{4}}}$

e) $\lim\limits_{x\to 10} \frac{\sqrt{x-1}-3}{x-10} = \frac{(\sqrt{x-1}-3)\cdot(\sqrt{x-1}+3)}{\lim\limits_{x\to 10}(x-10)\cdot(\sqrt{x-1}+3)} = \lim\limits_{x\to 10} \frac{\cancel{(x-1-9)}}{\cancel{(x-10)}(\sqrt{x-1}+3)} = \frac{1}{6}$

f) $\lim\limits_{x\to 3} \frac{\sqrt{x+6}-3\cdot\sqrt{x-2}}{x^2-9} = \lim\limits_{x\to 3} \frac{x+6-9\cdot(x-2)}{(x+3)\cdot(x-3)\cdot(\sqrt{x+6}+3\cdot\sqrt{x-2})} = \lim\limits_{x\to 3} \frac{x-9x+6+18}{-\|-} = \lim\limits_{x\to 3}\frac{-8x+24}{-\|-} = \lim\limits_{x\to 3}\frac{-8\cancel{(x-3)}}{-\|-\cancel{(x-3)}} =$

$= \lim\limits_{x\to 3} \frac{-8}{(x+3)\cdot(\sqrt{x+6}+3\cdot\sqrt{x-2})} = \frac{-8}{6\cdot(3+3\cdot 1)} = \frac{-8}{36} = \frac{-2}{9}$

$\lim\limits_{x\to 0} \frac{\sqrt{(1+x)^3}-1}{x} = \lim\limits_{x\to 0} \frac{(1+x)^3-1}{x\cdot[\sqrt{(1+x)^3}+1]} = \lim\limits_{x\to 0} \frac{(\cancel{x+1-1})\cdot[(x+1)^2+(x+1)\cdot 1+1^2]}{\cancel{x}\cdot[\sqrt{(1+x)^3}+1]} = \frac{1+1+1}{2} = \frac{3}{2}$

$\lim\limits_{x\to 4} \frac{\sqrt{1+2x}-3}{\sqrt{x}-2} = \lim\limits_{x\to 4} \frac{1+2x-9}{(\sqrt{x}-2)\cdot[\sqrt{1+2x}+3]} = \lim\limits_{x\to 4} \frac{2\cdot\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(\sqrt{x}-2)\cdot(\sqrt{x}+2)}[\sqrt{1+2x}+3]} = \frac{2\cdot(2+2)}{3+3} = \frac{8}{6} = \frac{4}{3}$

$\lim\limits_{x\to 9} \frac{\sqrt{x}-3}{\sqrt{2x}-3\sqrt{2}} = \lim\limits_{x\to 9} \frac{(\sqrt{x}-3)\cdot(\sqrt{2x}+3\sqrt{2})}{2x-9\cdot 2} = \lim\limits_{x\to 9} \frac{(\sqrt{2x}+3\sqrt{2})\cdot\cancel{(\sqrt{x}+3)}\cdot\cancel{(\sqrt{x}-3)}}{2\cancel{(x-9)}\cancel{(\sqrt{x}+3)}} = \frac{\sqrt{18}+3\sqrt{2}}{2\cdot 6} = \frac{3\sqrt{2}+3\sqrt{2}}{2\cdot 6} = \frac{6\sqrt{2}}{6\cdot 2} = \frac{\sqrt{2}}{2}$

$\lim\limits_{x\to 1} \frac{2-\sqrt{x+3}}{\sqrt{x}-1} = \lim\limits_{x\to 1} \frac{4-x-3}{(\sqrt{x}-1)\cdot(2+\sqrt{x+3})} = \lim\limits_{x\to 1} \frac{\cancel{(-x+1)}\cdot(\sqrt{x}+1)}{\cancel{(x-1)}\cdot(2+\sqrt{x+3})} = \lim\limits_{x\to 1} \frac{(-1)\cdot(\sqrt{x}+1)}{2+\sqrt{x+3}} = \frac{(-1)\,2}{2+2} = -\frac{1}{2}$

9.1.7

$\lim\limits_{x\to \frac{\pi}{4}} \frac{\sin x-\cos x}{1-\mathrm{tg}\,x} = \lim\limits_{x\to\frac{\pi}{4}} \frac{\sin x-\cos x}{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}} = \lim\limits_{x\to\frac{\pi}{4}} \frac{\cancel{(\sin x-\cos x)}}{1} : \frac{(-1)\cdot\cancel{(\sin x-\cos x)}}{\cos x} = \lim\limits_{x\to\frac{\pi}{4}} -\cos x = -\frac{\sqrt{2}}{2}$

$\lim\limits_{x\to\frac{\pi}{4}} \frac{\sin x-\cos x}{\cos 2x} = \lim\limits_{x\to\frac{\pi}{4}} \frac{\sin x-\cos x}{\cos^2 x-\sin^2 x} = \lim\limits_{x\to\frac{\pi}{4}} \frac{\sin x-\cos x}{(\cos x+\sin x)\cdot(\cos x-\sin x)} = \lim\limits_{x\to\frac{\pi}{4}} \frac{-1}{\sin x+\cos x} = \frac{-1}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}} = \frac{-1}{\frac{2\sqrt{2}}{2}} =$

$= \frac{-1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

c) $\lim\limits_{x\to\frac{\pi}{2}} \left(\frac{\sin x}{\cos^2 x}-\mathrm{tg}^2 x\right) = \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x} = \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x(1-\sin x)}{\cos^2 x} = \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x\cdot(1-\sin x)\cdot(1+\sin x)}{\cos^2 x\cdot(1+\sin x)} =$

$= \lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x\cdot\cancel{\cos^2 x}}{\cancel{\cos^2 x}\cdot(1+\sin x)} = \frac{1}{1+1} = \frac{1}{2}$

$\lim\limits_{x\to 0} \frac{1-\cos 2x+\mathrm{tg}^2 x}{\sin^2 x} = \lim\limits_{x\to 0} \frac{1-(\cos^2 x-\sin^2 x)+\mathrm{tg}^2 x}{\sin^2 x} = \lim\limits_{x\to 0} \frac{1-\cos^2 x+\sin^2 x+\frac{\sin^2 x}{\cos^2 x}}{\sin^2 x} = \lim\limits_{x\to 0} \frac{\sin^2 x+\sin^2 x+\frac{\sin^2 x}{\cos^2 x}}{\sin^2 x} =$

$= \lim\limits_{x\to 0} \frac{\cancel{\sin^2 x}\left(1+1+\frac{1}{\cos^2 x}\right)}{\cancel{\sin^2 x}} = 1+1+\frac{1}{1} = 3$

$\lim\limits_{x\to\frac{\pi}{2}} \frac{\sin 2x\cdot\cos x}{1+\cos 2x} = \lim\limits_{x\to\frac{\pi}{2}} \frac{2\sin x\cdot\cos^2 x}{1+\cos^2 x-\sin^2 x} = \lim\limits_{x\to\frac{\pi}{2}} \frac{\cancel{2}\sin x\cdot\cancel{\cos^2 x}}{\cancel{\cos^2 x+\cos^2 x}} = \lim\limits_{x\to\frac{\pi}{2}} \sin x = 1$

$\lim\limits_{x\to 0} \frac{\sin^2 2x}{1-\cos 2x} = \lim\limits_{x\to 0} \frac{(2\sin x\cdot\cos x)^2}{1-\cos^2 x+\sin^2 x} = \lim\limits_{x\to 0} \frac{2\cdot\cancel{2}\cancel{\sin^2 x}\cdot\cos^2 x}{\cancel{2\sin^2 x}} = 2\cdot\cos 0\cdot\cos 0 = 2\cdot 1\cdot 1 = 2$

$\lim\limits_{x\to\pi} \frac{\sin^2 x}{1+\cos x} = \lim\limits_{x\to\pi} \frac{(\sin^2 x)\cdot(1-\cos x)}{1-\cos^2 x} = \lim\limits_{x\to\pi} \frac{\cancel{\sin^2 x}\cdot(1-\cos x)}{\cancel{\sin^2 x}} = 1-\cos\pi = 1-(-1) = 2$

$\lim\limits_{x\to 0} \frac{1-\cos x}{\sin x} = \lim\limits_{x\to 0} \frac{1-\cos^2 x}{\sin x\cdot(1+\cos x)} = \lim\limits_{x\to 0} \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}\cdot(1+\cos x)} = \frac{0}{2} = 0$

$\lim\limits_{x\to-\frac{\pi}{4}} \frac{1+\mathrm{cotg}\,x}{\sin x+\cos x} = \lim\limits_{x\to-\frac{\pi}{4}} \frac{\frac{\cancel{\sin x+\cos x}}{\sin x}}{\cancel{\sin x+\cos x}} = \frac{1}{\sin(-\frac{\pi}{4})} = -\frac{2}{\sqrt{2}} = -\frac{2\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$

$\mathrm{tg}\,x-1 \quad \frac{\frac{\sin x-\cos x}{\cos x}}{\cos x-\sin x} = \lim \quad \cancel{\sin x-\cos x} \quad \sin x \quad \sin(\frac{\pi}{4}) \quad 1$

a) $\lim\limits_{x\to0}\frac{tg^2x}{1-\sqrt{\cos2x}}=\lim\limits_{x\to0}\frac{tg^2x\cdot(1+\sqrt{\cos2x})}{1-\cos2x}=\lim\limits_{x\to0}\frac{\frac{\sin^2x\cdot(1+\sqrt{\cos2x})}{\cos^2x}}{1-\cos^2x+\sin^2x}=\lim\limits_{x\to0}\frac{\cancel{\sin^2x}\cdot(1+\sqrt{\cos2x})}{\cos^2x}\cdot\frac{1}{2\cancel{\sin^2x}}=$

$=\frac{1+1}{2.1.1}=\frac{2}{2}=\underline{\underline{1}}$

) $\lim\limits_{x\to0}\frac{\sin^2x}{\sqrt3-\sqrt{2+\cos x}}=\lim\limits_{x\to0}\frac{\sin^2x\cdot(\sqrt3+\sqrt{2+\cos x})}{3-2-\cos x}=\lim\limits_{x\to0}\frac{\cancel{\sin^2x}\,(\sqrt3+\sqrt{2+\cos x})\cdot(1+\cos x)}{\cancel{(1-\cos x)\cdot(1+\cos x)}}=$

$=(\sqrt3+\sqrt3)\cdot2=\underline{\underline{4\sqrt3}}$

) $\lim\limits_{x\to0}\frac{tgx}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}=\lim\limits_{x\to0}\frac{tgx\cdot(\sqrt{1-\sin x}+\sqrt{1+\sin x})}{1-\sin x-1-\sin x}=\lim\limits_{x\to0}\frac{\frac{\cancel{\sin x}}{\cos x}\cdot(\sqrt{1-\sin x}+\sqrt{1+\sin x})}{-2\cdot\cancel{\sin x}}=$

$=\frac{\sqrt{1-0}+\sqrt{1+0}}{1}\cdot\frac{1}{-2}=\frac{2}{1}\cdot\frac{1}{-2}=\underline{-1}$

) $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos2x}{\sqrt{\sin x}-\sqrt{\cos x}}=\lim\limits_{x\to\frac{\pi}{4}}\frac{(\cos^2x-\sin^2x)\cdot(\sqrt{\sin x}+\sqrt{\cos x})}{(\sin x-\cos x)}=\lim\limits_{x\to\frac{\pi}{4}}\frac{(\cos x+\sin x)\cdot\cancel{(\cos x-\sin x)}\cdot(\sqrt{\sin x}+\sqrt{\cos x})}{(-1)\cdot\cancel{(-\sin x+\cos x)}}=$

$=(-1)\cdot\left(2\cdot\frac{\sqrt2}{2}\right)\cdot\left(\sqrt{\frac{\sqrt2}{2}}+\sqrt{\frac{\sqrt2}{2}}\right)=-\sqrt2\cdot\left(2\cdot\frac{\sqrt[4]{2}}{\sqrt2}\right)=\underline{\underline{-2\sqrt[4]{2}}}$

)

$\lim\limits_{x\to\pi}\frac{1-\sqrt{\cos x+2}}{\sin^22x}=\lim\limits_{x\to\pi}\frac{1-\cos x-2}{(2\sin x\cos x)^2\cdot(1+\sqrt{\cos x+2})}=\lim\limits_{x\to\pi}\frac{-1\cdot\cancel{(1+\cos x)}\cdot\cancel{(1-\cos x)}}{4\cdot\sin^2x\cdot\cos^2x\cdot(1+\sqrt{\cos x+2})\cdot(1-\cos x)}=$

$=\frac{-1}{4.1.(1+1).(1+1)}=\underline{\underline{-\frac{1}{16}}}$

)

$\lim\limits_{x\to\frac{\pi}{6}}\frac{2\sin^2x-\cos2x}{\sqrt{2\sin x}-1}=\lim\limits_{x\to\frac{\pi}{6}}\frac{(2\sin^2x-\cos^2x+\sin^2x)\cdot(\sqrt{2\sin x}+1)}{2\sin x-1}=\lim\limits_{x\to\frac{\pi}{6}}\frac{(2\sin^2x-(1-\sin^2x)+\sin^2x)\cdot(\sqrt{2\sin x}+1)}{2\sin x-1}=$

$=\lim\limits_{x\to\frac{\pi}{6}}\frac{(2\sin^2x-1+\sin^2x+\sin^2x)\cdot(\sqrt{2\sin x}+1)}{2\sin x-1}=\lim\limits_{x\to\frac{\pi}{6}}\frac{\left[(2\sin x)^2-1^2\right]\cdot(\sqrt{2\sin x}+1)}{2\sin x-1}=\lim\limits_{x\to\frac{\pi}{6}}(2\sin x+1)(\sqrt{2\sin x}+1)=$

$=\left(2\cdot\frac12+1\right)\left(\sqrt{2\cdot\frac12}+1\right)=2\cdot(1+1)=\underline{\underline{4}}$

1.9

a) $\lim\limits_{x\to0}\frac{\sin2x}{x}=\lim\limits_{x\to0}\frac{2\sin x\cos x}{x}=\lim\limits_{x\to0}\frac{\sin x}{x}\cdot2\cdot\cos=1.2.1=\underline{\underline{2}}$

b) $\lim\limits_{x\to0}\frac{\sin8x}{x}=\lim\limits_{x\to0}\frac{8}{8}\cdot\frac{\sin8x}{x}=\lim\limits_{x\to0}8\cdot\frac{\sin8x}{8x}=\lim\limits_{x\to0}8\cdot\frac{\sin a}{a}=8.1=\underline{\underline{8}}$

c) $\lim\limits_{x\to0}\frac{2x}{\sin3x}=\lim\limits_{x\to0}\frac33\cdot2\cdot\frac{x}{\sin3x}=\lim\limits_{x\to0}\frac23\cdot\frac{a}{\sin a}=\frac23\cdot1=\underline{\underline{\frac23}}$

d) $\lim\limits_{x\to0}\frac{\sin x+\sin3x}{x}=\lim\limits_{x\to0}\frac{\sin x}{x}+\frac{\sin3x}{3x}\cdot3=1+1.3=\underline{\underline{4}}$

d) $\lim\limits_{x\to0}\frac{\sin^2x}{4x^2}=\lim\limits_{x\to0}\frac{\sin x\cdot\sin x}{2x\cdot2x}=\lim\limits_{x\to0}\frac14\cdot\frac{\sin x}{x}\cdot\frac{\sin x}{x}=\underline{\underline{\frac14}}$

i) $\lim\limits_{x\to 0} \frac{5x+\sin 7x}{2x} = \lim\limits_{x\to 0} \frac{5x}{2x} + \frac{\sin 7x}{2x} = \lim\limits_{x\to 0} \frac{5}{2} + \frac{1}{2}\cdot\frac{7}{1}\cdot\frac{\sin 7x}{7x} = \frac{5}{2} + \frac{7}{2} = \frac{12}{2} = 6$

j) $\lim\limits_{x\to 0} \frac{\sin^2 x + x}{2x} = \lim\limits_{x\to 0} \frac{\sin^2 x}{2x} + \frac{\cancel{x}}{2\cancel{x}} = \lim\limits_{x\to 0} \frac{\sin x\cdot\sin x}{x}\cdot\frac{x}{x}\cdot\frac{1}{2} + \frac{1}{2} = \lim\limits_{x\to 0} \frac{x}{2} + \frac{1}{2} = 0 + \frac{1}{2} = \frac{1}{2}$

k) $\lim\limits_{x\to 0} \frac{\cos^2 x - 1 + \sin 2x}{x} = \lim\limits_{x\to 0} \frac{(-1)\cdot\sin^2 x + \sin 2x}{x} = \lim\limits_{x\to 0} \frac{\sin x\cdot\sin x}{x} + \frac{\sin 2x}{x} = \lim\limits_{x\to 0} x\cdot\frac{\sin x\cdot\sin x}{x\cdot x} + 2\cdot\frac{\sin 2x}{2x} =$

$= 0 + 2\cdot 1 = 2$

l) $\lim\limits_{x\to 0} \frac{x^3 - x\cdot\sin x}{x^2} = \lim\limits_{x\to 0} \frac{x^2 - \sin x}{x} = \lim\limits_{x\to 0} \frac{x^{\cancel{2}}}{\cancel{x}} - \frac{\sin x}{x} = 0 - 1 = -1$

9.1.10

a) $\lim\limits_{x\to 0} \frac{1-\cos 2x}{x\cdot\sin x} = \lim\limits_{x\to 0} \frac{1-\cos^2 x + \sin^2 x}{x\cdot\sin x} = \lim\limits_{x\to 0} \frac{2\sin^{\cancel{2}} x}{x\cdot\cancel{\sin x}} = 2\cdot 1 = 2$

b) $\lim\limits_{x\to 0} \frac{\operatorname{tg} x - \sin 2x}{x} = \lim\limits_{x\to 0} \frac{\frac{\sin x}{\cos x} - 2\sin x\cos x}{x} = \lim\limits_{x\to 0} \frac{\sin x\left(\frac{1}{\cos x} - 2\cos x\right)}{x} = 1\cdot(1 - 2\cdot 1) = -1$

c) $\lim\limits_{x\to 0} \frac{1-\cos x}{x^2} = \lim\limits_{x\to 0} \frac{1-\cos^2 x}{x^2\cdot(1+\cos x)} = \lim\limits_{x\to 0} \frac{\sin^2 x}{x^2}\cdot\frac{1}{1+\cos x} = 1\cdot\frac{1}{1+1} = \frac{1}{2}$

d) $\lim\limits_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1} = \lim\limits_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{x+1-1} = \lim\limits_{x\to 0} \frac{4}{1}\cdot\frac{\sin 4x}{4x}\cdot(\sqrt{x+1}+1) = 4\cdot 1\cdot(1+1) = 8$

e) $\lim\limits_{x\to 0} \frac{\sqrt{x+5}-\sqrt{5}}{\sin x} = \lim\limits_{x\to 0} \frac{x+5-5}{\sin x}\cdot\frac{1}{\sqrt{x+5}+\sqrt{5}} = 1\cdot\frac{1}{\sqrt{5}+\sqrt{5}} = \frac{1}{2\sqrt{5}} = \frac{\sqrt{5}}{2\cdot 5} = \frac{\sqrt{5}}{10}$

f) $\lim\limits_{x\to 0} \frac{1-\sqrt{\cos 2x}}{x^2} = \lim\limits_{x\to 0} \frac{1-\cos 2x}{x^2}\cdot\frac{1}{(1+\sqrt{\cos 2x})} = \lim\limits_{x\to 0} \frac{1-\cos^2 x + \sin^2 x}{x^2}\cdot\frac{1}{(1+\sqrt{\cos 2x})} = \lim\limits_{x\to 0} \frac{2\sin^2 x}{x^2}\cdot\frac{1}{1+\sqrt{\cos 2x}} =$

$= 2\cdot 1\cdot\frac{1}{1+1} = 1$

g) $\lim\limits_{x\to 0} \frac{\sqrt{1-\cos^2 x}}{|x|} = \lim\limits_{x\to 0} \frac{1-\cos^2 x}{|x|}\cdot\frac{1}{\sqrt{1-\cos^2 x}} = \lim\limits_{x\to 0} \frac{\sin^2 x}{|x|\cdot|x|}\cdot\frac{|x|}{\sqrt{\sin^2 x}} = 1$

h) $\lim\limits_{x\to 0} \frac{\sqrt{1-\cos 2x}}{|x|} = \lim\limits_{x\to 0} \frac{\sqrt{1-\cos^2 x + \sin^2 x}}{|x|} = \lim\limits_{x\to 0} \frac{\sqrt{\sin^2 x + \sin^2 x}}{|x|} = \lim\limits_{x\to 0} \frac{\sqrt{2}\cdot|\sin x|}{|x|} = \sqrt{2}$

i) $\lim\limits_{x\to 0} \frac{\sqrt{x^2 + 3\sin^2 x}}{\sqrt{\sin^2 x}} = \lim\limits_{x\to 0} \sqrt{\frac{x^2 + 3\sin^2 x}{\sin^2 x}} = \lim\limits_{x\to 0} \sqrt{\frac{x\cdot x}{\sin x\cdot\sin x} + \frac{3\cdot\cancel{\sin^2 x}}{\cancel{\sin^2 x}}} = \lim\limits_{x\to 0} \sqrt{1\cdot 1 + 3} = 2$

LIMITA FUNKCE V NEVLASTNÍM BODĚ

a) $\lim\limits_{x\to\infty} \frac{x+5}{3x-6} = \underline{\underline{\frac{1}{3}}}$

b) $\lim\limits_{x\to\infty} \frac{3x^2+1}{x^2+x-2} = \underline{\underline{3}}$

c) $\lim\limits_{x\to\infty} \frac{2+4x}{3-7x} = \underline{\underline{-\frac{4}{7}}}$

d) $\lim\limits_{x\to-\infty} \frac{2x^4-x^3+4}{5x^4+x^3+2} = \underline{\underline{\frac{2}{5}}}$

e) $\lim\limits_{x\to-\infty} \frac{2^{x+3}+4}{2^{x-1}+1} = \lim\limits_{x\to-\infty} \frac{2^x\cdot 2^3+4}{2^x\cdot 2^{-1}+1} = \lim\limits_{x\to-\infty} \frac{\frac{2^x\cdot 2^3+4}{2^x}}{\frac{2^x\cdot 2^{-1}+1}{2^x}} = \lim\limits_{x\to-\infty} \frac{2^3+\frac{4}{2^x}}{2^{-1}+\frac{1}{2^x}} = \frac{2^3}{2^{-1}} = \underline{\underline{16}}$

f) $\lim\limits_{x\to\infty} \frac{\log x+5}{3\log x-1} = \lim\limits_{x\to\infty} \frac{\frac{\log x+5}{\log x}}{\frac{3\log x-1}{\log x}} = \frac{1+\frac{5}{\log x}}{3-\frac{1}{\log x}} = \underline{\underline{\frac{1}{3}}}$

8.2.12

a) $\lim\limits_{x\to\infty} \frac{x^3+x^2}{x^2-1} = \underline{\underline{\infty}}$

b) $\lim\limits_{x\to\infty} \frac{x+2}{x^2+3} = \underline{\underline{0}}$

c) $\lim\limits_{x\to-\infty} \frac{x^2-3}{x^3-3} = \underline{\underline{0}}$

d) $\lim\limits_{x\to-\infty} \frac{x^4+3x^2+5}{3-x} = \underline{\underline{\infty}}$

f) $\lim\limits_{x\to-\infty} \frac{0{,}1^x+3}{0{,}1^{3x}+3} = \lim\limits_{x\to-\infty} \frac{0{,}1^x+3}{0{,}1^x\cdot 0{,}1^x\cdot 0{,}1^x+3} = \lim\limits_{x\to-\infty} \frac{1+\frac{3}{0{,}1^x}}{0{,}1^x\cdot 0{,}1^x+\frac{3}{0{,}1^x}} = \frac{1+0}{\infty+0} = \underline{\underline{0}}$

e) $\lim\limits_{x\to\infty} \frac{2^{2x}}{2^x-1} = \lim\limits_{x\to\infty} \frac{2^x\cdot 2^x}{2^x-1} = \lim\limits_{x\to\infty} \frac{2^x}{1-\frac{1}{2^x}} = \underline{\underline{\infty}}$

8.2.13

a) $\lim\limits_{x\to\infty} \sqrt{\frac{2x+3}{x-1}} = \underline{\underline{\sqrt{2}}}$

b) $\lim\limits_{x\to\infty} \sqrt{\frac{4x^2-x}{3x^2}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = \underline{\underline{\frac{2\sqrt{3}}{3}}}$

c) $\lim\limits_{x\to\infty} \sqrt{\frac{x+1}{2x-1}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \underline{\underline{\frac{\sqrt{2}}{2}}}$

d) $\lim\limits_{x\to\infty} \frac{\sqrt{2x^2+x}}{x} = \underline{\underline{\sqrt{2}}}$

e) $\lim\limits_{x\to\infty} \frac{\sqrt{x^2+2}}{x+1} = \underline{\underline{1}}$

f) $\lim\limits_{x\to\infty} \frac{\sqrt{x}+3}{\sqrt{x}} = \underline{\underline{1}}$

g) $\lim\limits_{x\to\infty} \frac{\sqrt{x+2}+3\sqrt{x^2-6}}{2x+1} = \underline{\underline{\frac{3}{2}}}$

h) $\lim\limits_{x\to\infty} \sqrt{x}\left(\sqrt{x+2}-\sqrt{x}\right) = \lim\limits_{x\to\infty} \sqrt{x\cdot(x+2)}-x = \lim\limits_{x\to\infty} \sqrt{x^2+2x}-x = \lim\limits_{x\to\infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x} = \lim\limits_{x\to\infty} \frac{2\cancel{x}}{x\left(1+\frac{\sqrt{x^2+2x}}{x}\right)} = \lim\limits_{x\to\infty} \frac{2}{1+\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}}} =$

$= \lim\limits_{x\to\infty} \frac{2}{1+\sqrt{1+\frac{2}{x}}} = \frac{2}{1+\sqrt{1+0}} = \frac{2}{2} = \underline{\underline{1}}$

i) $\lim\limits_{x\to\infty}\left(2x-\sqrt{4x^2+3x}\right) = \lim\limits_{x\to\infty}\dfrac{(2x)^2-\cancel{4x^2}-3x}{2x+\sqrt{4x^2+3}} = \dfrac{-3x}{\lim\limits_{x\to\infty} 2x+\sqrt{4x^2+3}} = \lim\limits_{x\to\infty}\dfrac{-3x}{x\cdot\left(2+\sqrt{\frac{4x^2+3}{x}}\right)} = \lim\limits_{x\to\infty}\dfrac{-3}{2+\sqrt{\frac{4x^2+3}{x^2}}} = \lim\limits_{x\to\infty}\dfrac{-3}{2+\sqrt{4+\frac{3}{x^2}}} =$

$= -\dfrac{3}{2+\sqrt{4+0}} = -\dfrac{3}{4}$

j) $\lim\limits_{x\to\infty} x\cdot\left(\sqrt{x^2+1}-x\right) = \dfrac{x\cdot(x^2+1-x^2)}{\lim\limits_{x\to\infty}\sqrt{x^2+1}+x} = \lim\limits_{x\to\infty}\dfrac{x}{\sqrt{x^2+1}+x} = \lim\limits_{x\to\infty}\dfrac{\frac{x}{x}}{\frac{\sqrt{x^2+1}+x}{x}} = \lim\limits_{x\to\infty}\dfrac{1}{\sqrt{\frac{x^2+1}{x^2}}+\frac{x}{x}} = \lim\limits_{x\to\infty}\dfrac{1}{\sqrt{1+\frac{1}{x^2}}+1} =$

$= \dfrac{1}{\sqrt{1+0}+1} = \dfrac{1}{2}$

JEDNOSTRANNÉ LIMITY

9.3.14

a) $\lim\limits_{x\to 5^+}\dfrac{2x+1}{x-5} = \dfrac{11^+}{0^+} = \infty$

) $\lim\limits_{x\to 5^-}\dfrac{2x+1}{x-5} = \dfrac{11^-}{0^-} = -\infty$

) $\lim\limits_{x\to 5}\dfrac{2x+1}{x-5}$ = neexistuje

) $\lim\limits_{x\to 0^+}\dfrac{x+3}{x} = \dfrac{3^+}{0^+} = \infty$

) $\lim\limits_{x\to 0^-}\dfrac{x^2-5}{x^2} = \dfrac{(-5)^-}{0^-} = -\infty$

) $\lim\limits_{x\to 2^+}\dfrac{7}{(2-x)^3} = \dfrac{7}{0^-} = -\infty$

) $\lim\limits_{x\to 1^-}\dfrac{x^2+x+1}{x^2-2x+1} = \dfrac{1^-+1^-+1}{1^--2\cdot(1^-)+1} = \dfrac{3^-}{1-1^-} = \dfrac{3^-}{0^+} = \infty$

) $\lim\limits_{x\to 1^+}\dfrac{5x}{1-x^2} = \dfrac{5^+}{1-(1^+)} = \dfrac{5^+}{0^-} = -\infty$

) $\lim\limits_{x\to 3^-}\dfrac{2x^2+6}{x^2-9} = \dfrac{2\cdot(3^-)+6}{(3^-)-9} = \dfrac{6^-+6}{3^--9} = \dfrac{12^-}{(-6)^-} = -\infty$

9.3.15

) $\boxed{f_1: y = \dfrac{x+3}{x-2} = 1+\dfrac{5}{x-2}}$

$D_{f_1} = R-2$

bod nespojitosti: 2

$\lim\limits_{x\to 2^-}\dfrac{x+3}{x-2} = \dfrac{5^-}{0^-} = -\infty$

$\lim\limits_{x\to 2^+}\dfrac{x+3}{x-2} = \dfrac{5^+}{0^+} = \infty$

$\lim\limits_{x\to -\infty}\dfrac{x+3}{x-2} = \dfrac{1}{1} = 1$

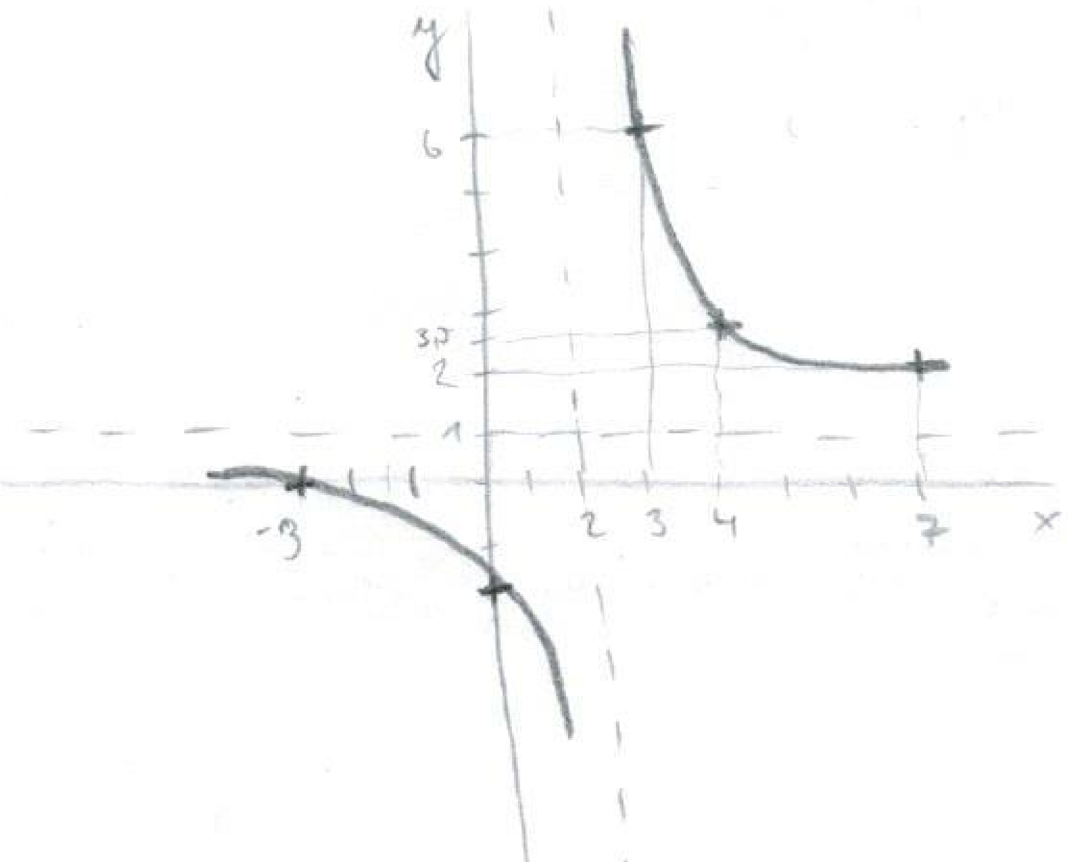
$\lim\limits_{x\to \infty}\dfrac{x+3}{x-2} = \dfrac{1}{1} = 1$

$S = [2;1]$ $a_1: x = -1$

$(x+3):(x-2) = 1+\dfrac{5}{x-2}$

$\underline{-(x-2)}$

5



$P_x = [0;?] = [0;-3]$

$0 = \dfrac{x+3}{x-2}$

$x = -3$

$P_y = [?;0] = [-\frac{3}{2};0]$

$y = \dfrac{0+3}{0-2}$

$y = -\dfrac{3}{2}$

b) $f_2: y = \dfrac{2x+3}{x-2} =$

$D_{f_2} = R-2$

bod nespojitosti: 2

$\lim\limits_{x\to 2^-}\dots = $ -||-

$\lim\limits_{x\to 2^+}\dots = $ -||-

$\lim\limits_{x\to -\infty}\dfrac{2x+3}{x-2} = 2$

$\lim\limits_{x\to \infty}\dfrac{2x+3}{x-2} = 2$

$S =$

$(2x+3):(x-2) =$

1. Vypočítejte:

$$(a)\quad \lim_{x\to\infty} \frac{x-3}{x} =$$
$$(b)\quad \lim_{x\to\infty} \frac{x-5x^2+3}{x^2} =$$
$$(c)\quad \lim_{x\to\infty} \frac{x-5x^2+3}{x} =$$
$$(d)\quad \lim_{x\to\infty} \frac{(x-3)(x+2)}{x^2} =$$
$$(e)\quad \lim_{x\to\infty} \frac{2x^3-4x-3}{1-2x^2+5x^3} =$$

2. Vypočítejte:

$$(a)\quad \lim_{x\to\infty} \frac{(2x-4)(3x+7)(x-1)}{3x(x+2)(4x+5)} =$$
$$(b)\quad \lim_{x\to\infty} \frac{(2-x)(1-3x)(4+x)}{(15x-2)(3+2x)(x-6)} =$$
$$(c)\quad \lim_{x\to\infty} \frac{(3-x^2)(4+x^3)(3-7x)}{(3+x-x^2)(2+5x)(3+x^3)} =$$
$$(d)\quad \lim_{x\to\infty} \frac{(2-5x^2)(3+x^5)(x^3+5x-x^4)}{(3-x+x^6)(2-x^3)(4-9x^2)} =$$
$$(e)\quad \lim_{x\to\infty} \frac{(4+3x^2)(5+4x^3)}{(6+5x^4)(7-x)} =$$

3. Vypočítejte:

$$(a)\quad \lim_{x\to\infty} \frac{(4x-3)^2(3x+1)^5}{(7-6x^2)^3} =$$
$$(b)\quad \lim_{x\to\infty} \frac{(x^2-4)^6(3x+3x^2-1)^7}{(x^4-2x+3)^3(x-4)^2(9x^3-2x)^4} =$$
$$(c)\quad \lim_{x\to\infty} \frac{(4-3x^2)^3(1-2x)^4(x^2-7)^6}{(2x^2-5x+7)^5(9x^4-2)^2\cdot x^4} =$$
$$(d)\quad \lim_{x\to\infty} \frac{(2x^2+3)^3(3x^3-4)^4}{(5x^5-6)^6(6x^6-7)^7} =$$
$$(e)\quad \lim_{x\to\infty} \frac{(3x-2)^7(4-x^3)}{(2-5x^2)^3(4x-3)} =$$

4. Vypočítejte:

$$(a)\quad \lim_{x\to\infty} \frac{\sqrt{2x^3-1}}{x^{\frac{3}{2}}} =$$
$$(b)\quad \lim_{x\to\infty} \left(3x-\sqrt{9x^2-10x+1}\right) =$$
$$(c)\quad \lim_{x\to\infty} \frac{\sqrt{x^2-7}+\sqrt{x^2+7}}{x} =$$
$$(d)\quad \lim_{x\to\infty} \frac{\sqrt[3]{x}-2\sqrt{x}+1}{\sqrt[3]{x}+\sqrt{x}-2} =$$

.a) $\lim\limits_{x\to\infty} \frac{\ldots}{x} = \lim\limits_{x\to\infty} \left(\frac{x}{x} - \frac{3}{x}\right) = 1 - 0 = \underline{\underline{1}}$

b) $\lim\limits_{x\to\infty} \frac{x - 5x^2 + 3}{x^2} = \lim\limits_{x\to\infty} \left(-5 \cdot \frac{x^2}{x^2} + \frac{x}{x^2} + \frac{3}{x^2}\right) = \lim\limits_{x\to\infty} \left(-5 + \frac{1}{x} + \frac{3}{x^2}\right) = -5 + 0 + 0 = \underline{\underline{-5}}$

c) $\lim\limits_{x\to\infty} \frac{x - 5x^2 + 3}{x} = \lim\limits_{x\to\infty} \left(-5 \cdot \frac{x^2}{x} + \frac{x}{x} + 3\right) = \lim\limits_{x\to\infty} (-5 \cdot x + 1 + 3) = \underline{\underline{-\infty}}$

d) $\lim\limits_{x\to\infty} \frac{(x-3)\cdot(x+2)}{x^2} = \lim\limits_{x\to\infty} \frac{x^2 - 3x + 2x - 6}{x^2} = \lim\limits_{x\to\infty} \left(\frac{x^2}{x^2} - \frac{x}{x^2} - \frac{6}{x^2}\right) = 1 - 0 - 0 = \underline{\underline{1}}$

e) $\lim\limits_{x\to\infty} \frac{2x^3 - 4x - 3}{1 - 2x^2 + 5x^3} = \underline{\underline{\frac{2}{5}}}$

2. a) $\lim\limits_{x\to\infty} \frac{(2x-4)\cdot(3x+7)\cdot(x-1)}{3x\cdot(x+2)\cdot(4x+5)} = \lim\limits_{x\to\infty} \frac{(6x^2 + 14x - 12x - 28)\cdot(x-1)}{(3x^2 + 6x)\cdot(4x+5)} = \lim\limits_{x\to\infty} \frac{(6x^2 - 2x - 28)\cdot(x-1)}{12x^3 + 15x^2 + 24x^2 + 30x} =$

$= \lim\limits_{x\to\infty} \frac{6x^3 - 2x^2 - 28x - 6x^2 + 2x + 28}{12x^3 + 39x^2 + 30x} = \lim\limits_{x\to\infty} \frac{6x^3 - 8x^2 - 26x + 28}{12x^3 + 39x^2 + 30x} = \frac{6}{12} = \frac{1}{2}$

b) $\lim\limits_{x\to\infty} \frac{(2-x)\cdot(1-3x)\cdot(4+x)}{(15x-2)\cdot(3+2x)\cdot(x-6)} = \lim\limits_{x\to\infty} \frac{3x^3\ldots}{30x^3\ldots} = \underline{\underline{\frac{1}{10}}}$

c) $\lim\limits_{x\to\infty} \frac{(3-x^2)\cdot(4+x^3)\cdot(3-7x)}{(3+x-x^2)\cdot(2+5x)\cdot(3+x^3)} = \lim\limits_{x\to\infty} \frac{7x^6\ldots}{-5x^6\ldots} = \underline{\underline{-\frac{7}{5}}}$

d) $\lim\limits_{x\to\infty} \frac{(2-5x^2)\cdot(3+x^5)\cdot(x^3+5x-x^4)}{(3-x+x^6)\cdot(2-x^3)\cdot(4-9x^2)} = \lim\limits_{x\to\infty} \frac{+5x^{11}\ldots}{+9x^{11}\ldots} = \frac{5}{9}$

e) $\lim\limits_{x\to\infty} \frac{(4+3x^2)\cdot(5+4x^3)}{(6+5x^4)\cdot(7-x)} = \lim\limits_{x\to\infty} \frac{12x^5\ldots}{-5x^5\ldots} = \underline{\underline{-\frac{12}{5}}}$

.a) $\lim\limits_{x\to\infty} \frac{(4x-3)^2\cdot(3x+1)^5}{(7-6x^2)^3} = \lim\limits_{x\to\infty} \frac{a\cdot x^7\ldots}{-b\cdot x^6\ldots} = \underline{\underline{-\infty}}$

b) $\lim\limits_{x\to\infty} \frac{(x^2-4)^6\cdot(3x+3x^2-1)^7}{(x^4-2x+3)^3\cdot(x-4)^2\cdot(9x^3-2x)^4} = \lim\limits_{x\to\infty} \frac{a\cdot x^{26}\ldots}{b\cdot x^{26}\ldots} = \lim\limits_{x\to\infty} \frac{3^7 x^{26}\ldots}{9^4 x^{26}\ldots} = \frac{3^7}{9^4} = \frac{3^7}{3^8} = \underline{\underline{\frac{1}{3}}}$

c) $\lim\limits_{x\to\infty} \frac{(4-3x^2)^3\cdot(1-2x)^4\cdot(x^2-7)^6}{(2x^2-5x+7)^5\cdot(9x^4-2)^2\cdot x^4} = \lim\limits_{x\to\infty} \frac{-a\cdot x^{22}\ldots}{b\cdot x^{22}\ldots} = \lim\limits_{x\to\infty} \frac{(-3)^3\cdot(-2)^4\cdot x^{22}\ldots}{2^5\cdot 9^2\cdot x^{22}\ldots} = \frac{(-1)\cdot 3^3\cdot 2^4\cdot 4}{2^5\cdot 3^4\cdot 4} = \underline{\underline{-\frac{1}{6}}}$

d) $\lim\limits_{x\to\infty} \frac{(2x^2+3)^3\cdot(3x^3-4)^4}{(5x^5-6)^6\cdot(6x^6-7)^7} = \lim\limits_{x\to\infty} \frac{a\cdot x^{18}\ldots}{b\cdot x^{72}\ldots} = \underline{\underline{0}}$

e) $\lim\limits_{x\to\infty} \frac{(3x-2)^7\cdot(4-x^3)}{(2-5x^2)^3\cdot(4x-3)} = \lim\limits_{x\to\infty} \frac{-a\cdot x^{10}}{-b\cdot x^7} = \underline{\underline{\infty}}$

.a) $\lim\limits_{x\to\infty} \frac{\sqrt{2x^3-1}}{x^{\frac{3}{2}}} = \lim\limits_{x\to\infty} \sqrt{\frac{2x^3-1}{x^3}} = \lim\limits_{x\to\infty} \sqrt{\frac{2x^3}{x^3} - \frac{1}{x^3}} = \sqrt{2-0} = \underline{\underline{\sqrt{2}}}$

b) $\lim\limits_{x\to\infty} (3x - \sqrt{9x^2-10x+1}) = \lim\limits_{x\to\infty} \frac{(3x)^2 - 9x^2 + 10x - 1}{3x + \sqrt{9x^2-10x+1}} = \lim\limits_{x\to\infty} \frac{10x\ldots}{6x\ldots} = \frac{10}{6} = \underline{\underline{\frac{5}{3}}}$

c) $\lim\limits_{x\to\infty} \frac{\sqrt{x^2-7} + \sqrt{x^2+7}}{x} = \lim\limits_{x\to\infty} \left(\sqrt{\frac{x^2-7}{x^2}} + \sqrt{\frac{x^2+7}{x^2}}\right) = \lim\limits_{x\to\infty} \left(\sqrt{1 - \frac{7}{x^2}} + \sqrt{1 + \frac{7}{x^2}}\right) = \sqrt{1-0} + \sqrt{1+0} = \underline{2}$

d) $\lim\limits_{x\to\infty} \frac{3\sqrt{x} - 2\sqrt{x} + 1}{\sqrt[3]{x} + \sqrt{x} - 2} = \lim\limits_{x\to\infty} \frac{-2\cdot x^{\frac{1}{2}}\ldots}{1\cdot x^{\frac{1}{2}}\ldots} = \underline{\underline{-2}}$

Vypočítejte:

(a) $\lim\limits_{x\to 3} \dfrac{2x-4}{x+3} =$

(b) $\lim\limits_{x\to 2} \dfrac{x^3-2x^2-4x+8}{3x^3-11x^2+8x+4} =$

(c) $\lim\limits_{x\to 0} \dfrac{(x+1)\sin x}{3x^2+5x} =$

(d) $\lim\limits_{x\to 3+} \dfrac{2}{x-3} - 1 =$

a) $\lim\limits_{x\to 3} \dfrac{2x-4}{x+3} = \dfrac{6-4}{6} = \dfrac{2}{6} = \underline{\underline{\dfrac{1}{3}}}$

b) $\lim\limits_{x\to 2} \dfrac{x^3-2x^2-4x+8}{3x^3-11x^2+8x+4} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}\cdot\cancel{(x-2)}\cdot(x+2)}{(3x+1)\cdot\cancel{(x-2)^2}} = \underline{\underline{\dfrac{4}{7}}}$

c) $\lim\limits_{x\to 0} \dfrac{(x+1)\cdot\ldots}{3x^2+5x} = \lim\limits_{x\to 0} \dfrac{\ldots}{x\cdot(3x+5)} = \underline{\underline{\dfrac{1}{5}}}$

d) $\lim\limits_{x\to 3^+} \dfrac{2}{x-3} - 1 = \dfrac{2}{0^+} - 1 = \underline{\underline{\infty}}$

Rational root theorem:

factor of leading coefficient $= 3 = 3.1 \to 3, 1 = Q$

factor of trailing constant $= 4 = 2.2.1 \to 4, 2, 1 = P$

P	Q	$\frac{P}{Q}$	hodnota výrazu po dosazení	Q.x − P
−1	1	−1	≠0	
−1	3	$-\frac{1}{3}$	0	3x+1
−2	1	−2	≠0	
−2	3	$-\frac{2}{3}$	≠0	
−4	1	−4	≠0	
−4	3	$-\frac{4}{3}$	≠0	
1	1	1	≠0	
1	3	$\frac{1}{3}$	≠0	
2	1	2	0	x−2
2	3	$\frac{2}{3}$	≠0	
4	1	4	≠0	
4	3	$\frac{4}{3}$	≠0	

$(3x^3 - 11x^2 + 8x + 4) : (3x+1) = x^2 - 4x + 4 = (x-2)^2$

$-(3x^3 + x^2)$

$\quad -12x^2 + 8x + 4$

$\quad -(-12x^2 - 4x)$

$\qquad 12x + 4$

$\qquad -(12x + 4)$

$\qquad\quad 0$

[redacted]

Skupina 7

NE

~~ANO~~

ANO ~~NE~~

1. Je pravda, že pokud má funkce v bodě a vlastní limitu, pak je v bodě a spojitá? Je pravda, že pokud je funkce v bodě a spojitá, pak má v bodě a vlastní limitu?

2. Vypočítejte limity:

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{(x-4)\cdot(\sqrt{x}+2)}{(x-4)} = \sqrt{4}+2 = \underline{4}$$

$$\lim_{x\to 0} \frac{2\sin x - 3x}{x-3} = \frac{2\cdot 0 - 0}{-3} = \underline{0}$$

$$\lim_{x\to 0} \frac{e^{2x}-e^{x}}{1-e^{x}} = \lim_{x\to 0} \frac{e^x\cdot e^x - e^x}{1-e^x} = \lim_{x\to 0} \frac{-e^x\cancel{(-e^x+1)}}{\cancel{1-e^x}} = -e^0 = \underline{-1}$$

Skupina 4

1. Definujte spojitost funkce v otevřeném a uzavřeném intervalu.

2. Určete intervaly spojitosti funkcí:

$$f:\ y = \frac{x-3}{x^2-12x+20} = \frac{x-3}{(x-2)\cdot(x-10)} \qquad (-\infty; 2) \cup (2; 10) \cup (10; \infty)$$

$$g:\ y = \frac{1}{|x|-1} \qquad x_1 = 1 \quad x_2 = -1 \qquad (-\infty; -1) \cup (-1; 1) \cup (1; \infty)$$

$$h:\ y = \frac{\operatorname{tg} x}{x} \qquad \left(\left(k\cdot 1\right)\frac{\pi}{2}; (k+1)\cdot\frac{\pi}{2}\right) - \{0\}$$

$$k:\ y = \operatorname{sgn} x \qquad (-\infty; 0) \cup (0; \infty)$$

3. Určete hodnoty reálných parametrů a, b tak, aby funkce $g:\ y = \sqrt{x-a} + \sqrt{b-x}$ byla spojitá právě na intervalu $\langle 5; 10\rangle$.

$D_g = \langle 5; 10\rangle$

~~větší než 5~~ ~~menší než 10~~ $a = 5$ $b = 10$