$\boxed{\text{JP-5}}$ Exponenciální a logaritmické rce a nrce

(1)

a) $2^{3x-1} \cdot 4 = 8^{x+1} \cdot \left(\frac{1}{2}\right)^x$

$2^{3x-1} \cdot 4 = 2^{3x+3} \cdot \frac{1^x}{2^x}$

$\frac{2^{3x-1}}{2^{3x+3}} \cdot 4 = \frac{1}{2^x}$

$2^{-4+x} = \frac{1}{4}$

$-4+x = \log_2 \frac{1}{4}$

$-4+x = -2$

$x = 2$

b) $\sqrt[4]{4^x} \cdot \sqrt[3]{2^{x-3}} = \sqrt[6]{16}$

$4^{\frac{x}{4}} \cdot 2^{\frac{x-3}{3}} = 16^{\frac{1}{6}}$

$2^{\frac{2x}{4}} \cdot 2^{\frac{x-3}{3}} = 2^{\frac{4}{6}}$

$2^{\frac{x}{2} + \frac{x-3}{3}} = 2^{\frac{4}{6}}$

$\frac{3x+2x-6}{6} = \frac{4}{6}$

$5x = 4+6$

$x = 2$

c) $\frac{1}{3^x} = \frac{1}{\sqrt{3}} \cdot \sqrt[6]{27^{3-3x}} \cdot \left(\frac{1}{9}\right)^{x+3}$

$\frac{1}{3^x} = \frac{1}{3^{\frac{1}{2}}} \cdot 3^{\frac{9-9x}{6}} \cdot \frac{1}{9^{x+3}}$

$3^{-x} = \frac{3^{\frac{3-3x}{2}}}{3^{\frac{1}{2}}} \cdot 3^{-2x-6}$

$3^{-x} = 3^{\frac{2-3x}{2} + (-2x-6)}$

$-x = \frac{2-3x}{2} - 2x - 6$

$-2x = 2-3x-4x-12$

$-2x+7x = 2-12$

$5x = -10$

$x = -2$

d) $0{,}25 \cdot \left(\frac{1}{4}\right)^{2x} = 1$

$\frac{1}{4^{2x}} = \frac{1}{0{,}25}$

$4^{-2x} = 4^1$

$-2x = 1$

$x = -0{,}5$

$\log_2 8 = 3 \quad 2^3 = 8$

e) $2 \cdot 0{,}5^{x^2+\frac{8}{3}x} = \frac{8}{\sqrt[3]{4}}$

$0{,}5^{x^2+\frac{8}{3}x} = \frac{4}{4^{\frac{1}{3}}} = 4^{\frac{2}{3}}$

$x^2+\frac{8}{3}x = \log_{0{,}5} 4^{\frac{2}{3}}$

$x^2+\frac{8}{3}x = -\frac{4}{3}$

$x_1 = -\frac{2}{3};\ x_2 = -2$

f) $5^x \cdot 2^x = 100^{x-1}$

$5^x \cdot 2^x = 100^x \cdot 100^{-1}$

$\frac{5^x \cdot 2^x}{100^x} = \frac{1}{100}$

$\frac{10^x}{10^{2x}} = \frac{1}{100}$

$10^{-x} = \frac{1}{100}$

$\frac{1}{10^x} = \frac{1}{10^2}$

$x = 2$

g) $\frac{81}{16} = \left(\frac{2}{3}\right)^x \cdot \left(\frac{9}{4}\right)^{x+1}$

$\frac{9^2}{4^2} = \frac{2^x}{3^x} \cdot \frac{9^{x+1}}{4^{x+1}}$

$\left(\frac{9}{4}\right)^2 = \left(\frac{3}{2}\right)^{-x} \cdot \left(\frac{9}{4}\right)^{x+1}$

$\left(\frac{3}{2}\right)^4 = \left(\frac{3}{2}\right)^{-x} \cdot \left(\frac{3}{2}\right)^{2x+2}$

$4 = \cancel{2x+2x}$
$2x+2-x$

$4-2 = x$

$2 = x$

h) $\frac{3^x}{2 \cdot 3^{\sqrt{3}}} = 4,5$

$3^x \cdot 3^{-\sqrt{3}} = 9$

$3^{x-\sqrt{3}} = 3^2$

$x - \sqrt{3} = 2$

$x = 2+\sqrt{3}$

② a) $3^x + 3^{x+1} = 108$

$3^x + 3^x \cdot 3^1 = 108$

$3^x \cdot (1+3) = 108$

$3^x = 27$

$x = 3$

b) $2^{x+1} + 2^{x-1} + 2^{x+3} = \frac{21}{8}$

$2^x \cdot 2 + 2^x \cdot \frac{1}{2} + 2^x \cdot 8 = \frac{21}{8}$

$2^x \left(2 + \frac{1}{2} + 8\right) = \frac{21}{8}$

$2^x = \frac{21}{8} : 10,5$

$2^x = \frac{1}{4}$

$x = -2$

c) ~~$7 \cdot 4^{-x+2} = 3 \cdot 4^{-x+3} - 5$~~

~~$7 \cdot 4^{-x} \cdot 16 = 3 \cdot 4^{-x} \cdot 64 - 5$~~

~~$4^{-x} \cdot 112 = 192 \cdot 4^{-x} - 5$~~

~~$112 \cdot 4^{-x}$~~
~~$192 \cdot 4^{-x} = -5$~~

~~$4^{-x}(112 - 192) = -5$~~

~~$4^{-x} = -5 \cdot (-80)$~~

~~$4^{-x} = 400$~~

~~$-x = \log_4 400$~~

$\log_2 8 = 3$

⑥

c) $7\cdot 4^{-x+2}=3\cdot 4^{-x+3}-5$

$7\cdot 4^{-x}\cdot 4^{2}-3\cdot 4^{-x}\cdot 4^{3}=-5$

$7\cdot 4^{-x+2}-3\cdot 4^{-x+2}\cdot 4=-5$

$4^{-x+2}(7-3\cdot 4)=-5$

$4^{-x+2}(-5)=-5$

$4^{-x+2}=1$

$4^{-x+2}=4^{0}$

$x=2$

d) $\boxed{\frac{4}{5}\cdot 5^{0}+5^{-1}}-25^{x}+20\cdot 25^{x-1}=0$

$1-25^{x}+20\cdot 25^{x-1}$

$25^{x-1}(-25+20)=-1$

$5^{2(x-1)}\cdot(-5)=-1$

$5^{2x-2}\cdot(-5)^{1}=-1$

$-5^{2x-1}=-1$

$-5^{2x-1}=-5^{0}$

$2x-1=0$

$x=0{,}5$

e) $3^{x}\cdot\left(\frac{1}{2}\right)^{x}+3^{x+1}\cdot\left(\frac{1}{2}\right)^{x+1}=\frac{5}{3}$

$\left(\frac{3}{2}\right)^{x}+\left(\frac{3}{2}\right)^{x+1}=\frac{5}{3}$

$\left(\frac{3}{2}\right)^{x}\cdot\left(1+\frac{3}{2}\right)=\frac{5}{3}$

$\left(\frac{3}{2}\right)^{x}=\frac{2}{3}$

$x=-1$

d) $\frac{4}{5}\cdot 5^{0}+5^{-1}-25^{x}+20\cdot 25^{x-1}=0$

$\boxed{\frac{4}{5}\cdot 5^{0}+5^{-1}}-5^{2x}+20\cdot 5^{2x-2}=0$

$\cancel{1-5^{2x}\left(1+\frac{20}{25}\right)=0}$

$\cancel{+5^{2x}=\frac{-1}{1+\frac{20}{25}}=+\frac{5}{9}}$

$\cancel{2x=\log_{5}\left(\frac{5}{9}\right)}$

$-5^{2x}+20\cdot 5^{2x-2}=1$

$\cancel{5^{2x}(1+20)}\quad 5^{2(x-1)}$

$-25^{x}+20\cdot\cancel{\ldots}=1$

$-25^{x}+20\cdot 25^{x}\cdot 25^{-1}=1$

$25^{x}(-1+20\cdot 25^{-1})=1$

$25^{x}\cdot\frac{4}{5}=1$

$25^{x}=\frac{5}{4}$

$\cancel{x=\log_{25}\frac{5}{4}}$

$\cancel{5^{2x}=\frac{5}{4}}$

$\cancel{x^{2}=\log_{5}\frac{5}{4}}$

To je taková kokotina.

f) $2^{2x}\cdot 5^{x}-2^{2x-1}\cdot 5^{x+1}=-600$

$2^{2x-1}(2\cdot 5^{x}-5^{x+1})=-600$

$2^{2x-1}(5^{x}(2-5))=-600$

$2^{2x-1}\cdot 5^{x}=-\frac{600}{-3}=200$

$2^{2x-1}\cdot 5^{x}=200$

$2^{2x}\cdot\frac{1}{2}\cdot 5^{x}=200$

$4^{x}\cdot 5^{x}=400$

$20^{x}=400$

$x=2$

3

a) $4^{2x}-6\cdot 4^{x}+8=0$

$4^{x^2}-6\cdot 4^{x}+8=0$

$4^{x}=t$

$t^2-6t+8=0$

$t_1=4;\ t_2=2$

$4^{x_1}=4 \Rightarrow x_2=1$

$4^{x_2}=2 \Rightarrow x_2=\frac{1}{2}$

b) $\frac{2}{4}\cdot 2^{x}+\frac{1}{2}\cdot 4^{x}=9$

$\frac{1}{2}2^{x^2}+\frac{1}{4}2^{x}-9=0$

$2^{x}=t$

$\frac{1}{2}t^2+\frac{1}{4}t-9=0$

$t_1=4;\ t_2=-\frac{9}{2}$

$2^{x_1}=4 \Rightarrow x_1=2$

$2^{x_2}=-\frac{9}{2} \Rightarrow x_2=$ nič

c) $9\cdot 3^{x}+3^{-x}=10$

$9\cdot 3^{x}+\frac{1}{3^{x}}=10$

$t=3^{x}$

$9t+\frac{1}{t}=10$

$\frac{9t^2+1-10t}{t}=0$

$9t^2-10t+1=0$

$t_1=1;\ t_2=\frac{1}{9}$

$3^{x_1}=1 \Rightarrow x_1=0$

$3^{x_2}=\frac{1}{9} \Rightarrow x_2=-2$

d) $9^{x-0,5}+9^{0,5-x}=\frac{10}{3}$

$9^{x-0,5}+\frac{1}{9^{x-0,5}}-\frac{10}{3}=0$

$9^{x-0,5}=t$

$t+\frac{1}{t}-\frac{10}{3}=0$

$t^2+1-\frac{10t}{3}=0$

$t_1=3;\ t_2=\frac{1}{3}$

$9^{x_1-0,5}=3^{1}$ $\quad$ $9^{x_2-0,5}=\frac{1}{3}$

$3^{2(x_1-0,5)}=3^{1}$ $\quad$ $x_2=0$

$2x_1-1=1$

$2x_1=2$

$x_1=1$

e) $9\cdot 3^{x}+3^{-x}=10$

$9\cdot 3^{x}+\frac{1}{3^{x}}-10=0$

$3^{x}=t$

$9t+\frac{1}{t}-10=0$

$9t^2-10t+1=0$

$t_1=1;\ t_2=\frac{1}{9}$

$3^{x_1}=1$ $\quad$ $3^{x_2}=\frac{1}{9}$

$x_1=0$ $\quad$ $x_2=-2$

f)

$5^{2x}\cdot(5^{2x}-5)=3(5^{2x+1}+5^{2x})+50$

$5^{2x}(5^{2x}-5)=3(5^{2x}(5+1))+50$

$5^{2x}(5^{2x}-5)=5^{2x}\cdot 18+50$

$5^{2x}=t$

$t(t-5)=18t+50$

$t^2-5t-18t-50=0$

$t_1=25;\ t_2=-2$

$5^{2x_1}=25$

$x_1=1$

$5^{2x}=-2$

$25^{x}=-2$

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8

8 a)

$2^x + 5\cdot 3^y = 53$

$7\cdot 2^x - 3^y = 47$

$2^x = s;\ 3^y = t$

$s + 5t = 53$

$7\cdot s - t = 47$

$s = 53 - 5t$

$7(53-5t) - t = 47$

$371 - 35t - t = 47$

$-36t = -324$

$t = 9$

$s = 53 - 45 = 8$

$2^x = 8 \Rightarrow \underline{\underline{x = 3}}$

$3^y = 9 \Rightarrow \underline{\underline{y = 2}}$

d) $16^{x+y} = 8$

$16^x + 16^y = 6$

$16^{xy} = 6$

$x + y = \frac{3}{4}$

$x = \frac{3}{4} - y$

$16^{\left(\frac{3}{4}-y\right)y} = 6$

$16^{\frac{3y}{4} - y^2} = 6$

$\frac{3y}{4} - y^2 = \log_{16} 6$

$-y^2 + \frac{3y}{4} - \log_{16} 6 = 0$

$y_1 = 1{,}26\ldots \quad \Leftarrow x; y$

$y_2 = -0{,}512\ldots$

b)

$2^{x+1} + 3^y = 31$

$2^x - 3^{y-2} = -1$

$2^x \cdot 2 + 3^{y-2} \cdot 3^2 = 31$

$2^x - 3^{y-2} = -1$

$2^x = s;\ 3^{y-2} = t$

$2s + 9t = 31$

$s - t = -1$

$s = t - 1$

$2t - 2 + 9t = 31$

$11t = 33$

$t = 3$

$s = 2$

$2^x = 2 \Rightarrow \underline{\underline{x = 1}}$

$3^{y-2} = 3 \Rightarrow \underline{\underline{y = 3}}$

$\log_2 s = 3 \quad 2^3 = 8$

$2^{4x+4y} = 2^3$

$4x + 4y = 3$

$2^{4xy} = 2\cdot 3$

$4x = 3 - 4y$

$2^{(3-4y)y} = 6$

$2^{3y - 4y^2} = 6$

$3y - 4y^2 = \log_2 6$

$16^{x+y} = 8$

$16^x \cdot 16^y = 8$

$16^{xy} = 6$

$16^x = t \quad t \cdot 16^y = 8$

$t^y = 6$

g) $2\cdot 2^{x-y} + 2^{x+y-1} = 20$

$10\cdot 2^{x-y-1} - 2^{x+y} = -22$

$2\cdot 2\cdot 2^{x-y-1} + 2^{x+y-1} = 20$

$10\cdot 2^{x-y-1} + 2^{x+y-1}\cdot 2 = -22$

$2^{x-y-1} = s;\ 2^{x+y-1} = l$

$2\cdot 2\cdot s + l = 20$

$10\cdot s + 2\cdot l = -22$

$l = 20 - 4s$

$10s + 2(20-4s) = -22$

$10s + 40 - 8s = -22$

$2s = -22 + 40$

$s = 9$

$l = 20 - 36 = -16$

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$2\cdot 2^{x-y} + 2^{x+y-1} = 20$

$10\cdot 2^{x-y-1} + 2^{x+y} = -22$

$4\cdot 2^{x-y-1} + 2^{x+y-1} = 20$

$10\cdot 2^{x-y-1} + 2\cdot 2^{x+y-1} = -22$

$2^{x-y-1} = s;\ 2^{x+y-1} = t$

$4s + t = 20$

$10s + 2t = -22$

$t = 20 - 4s$

$10s + 2(20-4s) = -22$

$10s + 40 - 8s = -22$

$2s = -62$

$s = -31$ nope

$4(-31) + t = 20$

$t = 144$

d) ~~$16^{x+y}=8$~~

~~$16^{x+y}=6$~~

~~$16^{x} \cdot 16^{y}=8$~~

~~$16^{x}=\frac{8}{16^{y}}$~~

~~$16^{\frac{8}{16^x}+y}=8$~~

~~$16^{x+\frac{8}{16^y}}=6$~~

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9 a) $2^3=8 \quad \log_2 8=3$

$\log_2(x+1)=3$

$\cancel{\log_2 2x}\quad x=7$

b) $4\cdot\log_3(2x-1)=12$

$\log_3(2x-1)=3$

$3^3=2x-1$

$28=2x$

$x=14$

c) $\log_{\frac{1}{2}}(2-x)=-2$

$\frac{1}{2}^{-2}=2-x$

$4=2-x$

$x=-2$

d) $\log_4(5x-4)=2$

$4^2=5x-4$

$20=5x$

$x=4$

e) $\log_2\left(\log_3 \log_{\frac{1}{2}} x\right)=0$

$2^0=\log_3\log_{\frac{1}{2}}x=1$

$3^1=\log_{\frac{1}{2}}x$

$x=\left(\frac{1}{2}\right)^3=\frac{1}{8}$

f) $\log_{\frac{1}{2}}\left(\log_3(1+20\log_2 x)\right)=-2$

$\log_3(1+20\log_2 x)=4$

$20\log_2 x=80$

$\log_2 x=4$

$x=16$

g) $\log_2[14+2\log_7(1+2\log_{\frac{1}{2}}x)]=4$

$14+2\log_7(1+2\log_{\frac{1}{2}}x)=16$

$\log_7(1+2\log_{\frac{1}{2}}x)=1$

$1+2\log_{\frac{1}{2}}x=7$

$\log_{\frac{1}{2}}x=3$

$x=\frac{1}{8}$

h) $\log_9(3\log_2(1+\log_3(1-2\log_3 x)))=\frac{1}{2}$

$\log_2(1+\log_3(1-2\log_3 x))=1$

$\log_3(1-2\log_3 x)=1$

$1-2\log_3 x=3$

$x=\frac{1}{3}$

$\log_a(xy)=\log_a x+\log_a y$

$\log_a(x^r)=r\cdot\log_a x$

$\log_y x=\frac{\log_a x}{\log_a y}$

10

(10) a) $\log_5(x^2+2x) = \log_5(-3x)$

$\log_5(x\cdot(x+2)) \cancel{= \log_5(-3) +} \log_5(-x)$

$\cancel{\log_5 -x + \log_5(x+2) =}$

$x^2+2x = -3x$

$x^2+5x=0$

$x_1=-5$

$\cancel{x_2=0}$

b) $\log x^2 = \log(4-x^2)$

$\cancel{x^2=4-2x}$

$\cancel{x^2+2x-4=0}$

$x^2=4-x^2$

$2x^2=4$

$x^2=2$

$x=\pm\sqrt{2}$

c) $x^2-5x = 5x+11$

$x^2-10x-11=0$

$x_1=11;\ x_2=-1$

d) $x^2-x=x$

$x^2-2x=0$

$x_1=2$

(11) a) $\log x = 2\log 5+\log 4$

$\log x=2$

$x=100$

b) $\dfrac{\log_3 x}{1+\log_3 2}=2$

$\dfrac{\log_3 x}{\log_3 3+\log_3 2}=2$

$\dfrac{\log_3 x}{\log_3 6}=2$

$\log_3 x = \log_3 9\cdot\log_3 6$

$\cancel{\log_3 x = \log_3 54}$

$\cancel{x=54}$

$x=3^{\log_3 9\cdot\log_3 6}$

$x=36$

c) $\log_6(x+1)+\log_6 x=1$

$\log_6(x+1)+\log_6 x=\log_6 6$

$\cancel{x+1+x=6}$

$\cancel{x=2{,}5}$

$\log_6(x^2+x)=\log_6 6$

$x^2+x-6=0$

$x_1=2$

$\cancel{x_2=-3}$

e) $\log(x+3)=\log x+\log 3$

$\log(x+3)=\log(3x)$

$x+3=3x$

$-2x=-3$

$x=\frac{3}{2}$

d) $\log_2(x+7) - \log_2 x = 3$

$\log_2(x+7) = \log_2 8 + \log_2 x$

$x + 7 = 8x$

$7 = 7$

$\underline{\underline{x = 1}}$

f) $\log_8\sqrt{x+30} + \log_8\sqrt{x} = \cancel{1} = 1$

$\log_8\left(\sqrt{x+30}\cdot\sqrt{x}\right) = \log_8 8$

$\log_8\sqrt{x^2+30x} = \log_8 8$

$\sqrt{x^2+30x} - 8 = 0$

$x^2 + 30x = 8^2$

$x^2 + 30x - 64 = 0$

$\underline{\underline{x_1 = 2}};\ \cancel{x_2 = -32}$

g) $\log x^5 - \log x^4 + \log x^3 = 12$

$5\log x - 4\log x + 3\log x = 12$

$4\log x = 12$

$\log x = 3$

$\underline{\underline{x = 1000}}$

i) $3\log(2x^2) + 2\log(3x^3) = 5\log x + 2\log 6x^3$

$6\log 2 + 6\log x + 6\log 3 + 6\log x = 5\log x + 6\log 6x$

$\cancel{6\log 6 + 12\log(x)} = 11\log x + 6\log 6 + 6\log x$

$12\log x = 11\log x$

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$3\log 2 + 6\log x + 2\log 3 + 6\log x = 5\log x + 2\log 6 + 6\log x$

$3\log 2 + 2\log 3 + 6\log(x^2)$

$3\log 2 + 2\log 3 + 12\log x = 5\log(x^2) + \log x + 2\log 6$

$3\log 2 + 2\log 3 + 12\log x = 11\log x + 2\log 6$

$\log_{10} x = (2\log 6 - 3\log 2 - 2\log 3)$

$\cdot 11$

$-0{,}3\ldots$

$10^{-0{,}3\ldots} = x$

$\underline{\underline{x = \frac{1}{2}}}$

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h) $\log\sqrt{x} + \log\frac{1}{x^2} - \log x^3 + \frac{11}{2} = \frac{\log x^2}{1+\log 10}$

$\log\left(\frac{\sqrt{x}}{x^2}\right) - \log x^3 + \frac{11}{2} = \frac{\log x^2}{\log 100}$

$\log\frac{\sqrt{x}}{x^2} - \log x^3 + \frac{11}{2} = \frac{2\cdot\log x}{2}$

$\log\frac{\sqrt{x}}{x^2} + \frac{11}{2} = \log(x\cdot x^3)$

$\log\frac{\sqrt{x}}{x^2} + \frac{11}{2} = 4\log x$

$\log\frac{x^{\frac{1}{2}}}{x^2} + \frac{11}{2} = 4\log x$

$\log\left(x^{-\frac{3}{2}}\right) + \frac{11}{2} = 4\log x$

$-\frac{3}{2}\cdot\log x + \frac{11}{2} = 4\log x$

$-3\log x + 11 = 8\log x$

$11 = 11\log x$

$\log x = 1$

$\underline{\underline{x = 10}}$

12

(10) a) $\log_5(x^2+2x) = \log_5(-3x)$

$\cancel{\log_5(x\cdot(x+2)) = \log_5(-3) + \log_5(-x)}$

$\cancel{\log_5 -x + \log_5(x+2) =}$

$x^2+2x = -3x$

$x^2+5x=0$

$x_1 = -5$

$\cancel{x_2 = 0}$

b) $\log x^2 = \log(4-x^2)$

$\cancel{x^2 = 4-2x}$

$\cancel{x^2+2x-4=0}$

$x^2 = 4-x^2$

$2x^2=4$

$x^2=2$

$x = \pm\sqrt{2}$

c) $x^2-5x = 5x+11$

$x^2-10x-11=0$

$x_1 = 11;\ x_2 = -1$

d) $x^2-x=x$

$x^2-2x=0$

$x_1 = 2$

(11) a) $\log x = 2\log 5 + \log 4$

$\log x = 2$

$x=100$

b) $\dfrac{\log_3 x}{1+\log_3 2} = 2$

$\dfrac{\log_3 x}{\log_3 3+\log_3 2} = 2$

$\dfrac{\log_3 x}{\log_3 6} = 2$

$\log_3 x = \log_3 9 \cdot \log_3 6$

$\cancel{\log_3 x = \log_3 54}$

$\cancel{x = 54}$

$x = 3^{\log_3 9\cdot\log_3 6}$

$x = 36$

c) $\log_6(x+1) + \log_6 x = 1$

$\log_6(x+1)+\log_6 x = \log_6 6$

$\log_6(x^2+x) = \log_6 6$

$x^2+x-6=0$

$x_1 = 2$

$\cancel{x_2 = -3}$

d) $\log(x+3) = \log x + \log 3$

$\log(x+3) = \log(3x)$

$x+3 = 3x$

$-2x = -3$

$x = \frac{3}{2}$

j) $\frac{1}{2}\cdot(3\log 5 - 1 - \log x) = 2 - \log 5$

$\frac{3}{2}\log 5 - \frac{1}{2} - \frac{1}{2}\log x = 2 - \log 5$

$\frac{5}{2}\log 5 - \frac{1}{2} - 2 = \frac{1}{2}\log x$

$5\log 5 - 1 - 4 = \log x$

$5\log 5 - 5 = \log x$

$x = 10^{5\log 5 - 5}$

$x = \frac{1}{32}$

k) $\log_4(3x+2) - 2\log_4 x = \underbrace{2 - \log_4 8}_{=\frac{1}{2}}$

$\log_4(3x+2) = 2\log_4 x + \log_4 2$

$\log_4((3x+2)) = \log_4(x^2\cdot 2)$

~~$9x^2+2\cdot3\cdot2\cdot x+4 = 2x^2$~~

~~$7x^2+12x+4=0$~~

~~$x_1 = \frac{-6+2\sqrt{2}}{7}$; $x_2 = \frac{-6-2\sqrt{2}}{7}$~~

~~$3x+2=2x$~~ blbě again

~~$x=-2$~~

$-2x^2+3x+2=0$

$x_1=2$; ~~$x_2=-\frac{1}{2}$~~

l) $1+\log_3(5-x) - \log_3(2x-1) = \log_3(2x-1)$

$\log_3(3\cdot(5-x)) = 2\log_3(2x-1)$

$15-3x = 4x^2 - 2\cdot 2x\cdot 1 + 1$

$-4x^2-3x+14=0$

... nope.

$\Rightarrow 4x^2-4x+1-15+3x=0$

$4x^2-x-14=0$

$x_1=2$; ~~$x_2=-\frac{7}{4}$~~

12 a) $\log_2\left(\frac{3-x}{x+3}\right) = -2$

$\log_2(3-x) - \log_2(x+3) = \log_2\left(\frac{1}{4}\right)$

$\frac{3-x}{x+3} = \frac{1}{4}$

$3-x = \frac{x+3}{4}$

$12-4x = x+3$

$-5x = -9$

$x = \frac{9}{5}$

b) $\log_3\left(\frac{6x-2}{x-3}\right) = 2$

$\log_3\left(\frac{6x-2}{x-3}\right) = \log_3 9$

$6x-2 = 9x-27$

$-3x = -25$

$x = \frac{25}{3}$

c) $\log_{\frac{1}{2}}\frac{x}{x+14} = \frac{\log 125}{\log 5} = 3$

$\frac{x}{x+14} = \frac{1}{8}$

$x = \frac{x+14}{8}$

$8x - x - 14 = 0$

$7x = 14$

$x = 2$

d) $\log_7 \frac{x^2+1}{x^2-1} = 1$

$\frac{x^2+1}{x^2-1} = 7$

$x^2+1 = 7x^2-7$

$-6x^2 = -6$

$x^2 = 1$

$x = \pm 1$

Well.... obviously.... not.

20: $\log_7(x^2+1) - \log_7(x^2-1) = \log_7 7$

$\log_7(x^2+1) = \log_7 7 + \log_7(x^2-1)$

$\log_7(x^2+1) = \log_7(7\cdot(x^2-1))$

$x^2+1 = 7x^2-7$

... here we go again...

$8 = 6x^2$

...aha neumim psat...

$x^2 = \frac{8}{6}$

$x = \pm\frac{2\sqrt{3}}{3}$

(13) Mi má to nebaví....

...[illegible] ..... [illegible]

a) $\frac{\log_3(6x-2)}{\log_3(x-3)} = 2$

$\log_3(6x-2) = 2\cdot\log_3(x-3)$

$6x-2 = (x-3)^2$ $x^2-3\cdot 1x\cdot 2+3^2$ WTF PROČ TO NEFUNGUJE

$6x-2 = x^2-3\cdot 2\cdot x+9$

$-x^2+6x+6x-2-9=0$

aha, jsem debil.

$-x+12x-11=0$

$x_1 = 11$

$\cancel{x_2 = 1}$

b) [illegible]

$\frac{\log_5(x-\frac{1}{4})}{\log_5(x+\frac{7}{2})} = -1$

$\log_5(x-\frac{1}{4}) = -\log_5(x+\frac{7}{2})$

$x-\frac{1}{4} = \frac{1}{x+\frac{7}{2}}$

$(x-\frac{1}{4})(x+\frac{7}{2}) = 1$

$x^2+\frac{7x}{2}-\frac{x}{4}-\frac{7}{8}-1=0$

$x^2+\frac{14x-x}{4}-\frac{7}{8}-\frac{8}{8}=0$

$8x^2+26x+1=0$

$8x^2+26x-15=0$

$x_1 = \frac{1}{2}$; $\cancel{x_2 = -\frac{15}{4}}$

c) $\frac{2\log(3x)}{\log(2-7x)} = 1$

$\log(9x^2) = \log(2-7x)$

$9x^2+7x-2=0$

$x_1 = \frac{2}{9}$; $\cancel{x_2 = -1}$

d) $\frac{\log x}{\log(x-2)} = \frac{\log 9}{\log 3}$

$x = (x-2)^2$

$0 = x^2-4x+4-x$

$0 = x^2-5x+4$

$x_1 = 4$; $\cancel{x_2 = 1}$

[illegible]

(14)

$y = \log_a x \Leftrightarrow a^y = x$

1. $9^x + 2\cdot 3^x - 3 = 0$

$(3^x)^2 + 2\cdot 3^x - 3 = 0$

$3^x = t$

$t^2 + 2t - 3 = 0$

$t_1 = 1;\ t_2 = -3$

$3^{x_1} = 1 \Rightarrow \underline{x_1 = 0}$

$3^{x_2} = -3 \Rightarrow x_2 = \log_3(-3) \Rightarrow$ nelze

2. $\log_2(x-6) = 3 + \log_2 x$

$\log_2(x-6) = \log_2(8) + \log_2(x)$

$\log_2(x-6) = \log_2(8x)$

$x - 6 = 8x$

podm.: $x > 6$, $x > 0$

$-7x = 6$

$\underline{\underline{x = -\frac{6}{7} \Rightarrow \text{nelze}}}$

3. $3^{2x+1} = 5^x$

$2x+1 = \log_3(5^x)$

$2x+1 = x\cdot\log_3 5$

$\frac{2x+1}{x} = \log_3 5$

$2 + \frac{1}{x} = \log_3 5$

$\frac{1}{x} = \log_3 5 - 2$

$1 = (\log_3 5 - 2)\cdot x$

$x = \frac{1}{\log_3 5 - 2} = \underline{\underline{-1{,}869\ldots}}$

4. $\log_{\frac{1}{2}}(x+3) \geq -2$

$x > 3$

$\log_{\frac{1}{2}}(x+3) \geq \log_{\frac{1}{2}} 4$

$x + 3 \geq 4$

$x \geq 1$

$\underline{\underline{x > 3}}$

1. $9^x + 2\cdot 3^x - 3 = 0$

$(3^2)^x + 2\cdot 3^x - 3 = 0$

$3^{x^2} + 2\cdot 3^x - 3 = 0$

$3^x = t$

$t^2 + 2t - 3 = 0$

$t_1 = 1,\ t_2 = -3$

$3^{x_1} = 1$

$3^{x_2} = -3$

$x = \log_3 1 = 0$

$x = \log_3(-3) \Rightarrow$ nelle

$\underline{\underline{x = 0}}$

2. $\log_2(x-6) = 3 + \log_2 x$

$\log_2(x-6) = \log_2 8 + \log_2 x$

$\log_2(x-6) = \log_2(8x)$

$x - 6 = 8x$

$x - 8x = 6$ $\quad \sqrt{}\ x > 0$

$-7x = 6$

$x = -\frac{6}{7}$

$\underline{\underline{NR}}$

$y = \log_a x \Leftrightarrow a^y = x$

$\log_a(xy) = \log_a x + \log_a y$

$\log_a(x^r) = r\cdot\log_a x$

$\log_y x = \frac{\log_a x}{\log_a y}$

3. $3^{2x+1} = 5^x$

$2x+1 = \log_3 5^x$

$2x+1 = x\cdot\log_3 5$

$\frac{2x+1}{x} = \log_3 5$

$2 + \frac{1}{x} = \log_3 5$

$\frac{1}{x} = \log_3 5 - 2 \quad / \cdot x$

$1 = (\log_3 5 - 2)\cdot x$

$x = \frac{1}{\log_3 5 - 2} = \underline{-1{,}869}$

4. $\log_{\frac{1}{2}}(x+3) \geq -2$

$\log_{\frac{1}{2}}(x+3) \geq \log_{\frac{1}{2}} 4$ $\quad \underline{\underline{x > 3}}$

$x + 3 \geq 4$

$\underline{\underline{x \geq 1}}$

5. $4^{\log_{\frac{1}{2}}(x^2-5x+7)} > 1$

$\log_{\frac{1}{2}}(x^2-5x+7) > \log_4 1$

$\log_{\frac{1}{2}}(x^2-5x+7) > 0$

$\frac{1}{2}^0 = x^2 - 5x + 7$

$1 = x^2 - 5x + 7$

$0 = x^2 - 5x + 6$

$\underline{\underline{x_1 = 3;\ x_2 = 2}}$