

JP-2

Rovnice s neznámou ve jmenovateli

$$\textcircled{18} \text{ a) } \frac{3}{x+2} + \frac{5x}{4-x^2} = \frac{3}{x-2} + \frac{x}{x^2-4}$$

$$\frac{3}{x+2} + \frac{5x}{(2-x)(2+x)} = \frac{3}{x-2} + \frac{x}{(x-2)(x+2)} \quad / (2-x)(x-2)(x+2)$$

$$3(2-x)(x-2) + 5x(x-2) = 3(2-x)(x+2) + x(2-x)$$

$$3(2x - 4 - x^2 + 2x) + 5x^2 - 10x = 3(3 + 2x - x^2 - 2x) + 2x - x^2$$

$$6x - 12 - 3x^2 + 6x + 5x^2 - 10x = 12 + 6x - 3x^2 - 6x + 2x - x^2$$

$$2x^2 + 2x - 12 = -5x^2 + 2x + 12$$

$$6x^2 - 8x - 24 = 0$$

$$x_1 = 4; x_2 = -2$$

$$x \neq \pm 2 \Rightarrow \underline{\underline{\quad}}$$

$$\text{b) } 1 + 3 \left(\frac{1}{2} \cdot \frac{x-3}{x-2} - \frac{2}{x-2} \right) = \frac{15}{2x-x^2}$$

$$1 + \left(\frac{3}{2} \cdot \frac{x-3}{x-2} - \frac{6}{x-2} \right) = \frac{15}{x(2-x)}$$

$$1 + \left(\frac{3(x-3)}{2(x-2)} - \frac{12}{2(x-2)} \right) = \frac{-15}{x(x-2)}$$

$$\frac{2x-4 + 3x-9 - 12}{2(x-2)} = \frac{-15}{x(x-2)}$$

$$2x^2 - 5x + 3x^2 - 9x - 11x = -30$$

$$5x^2 - 25x + 30 = 0$$

$$\underline{\underline{x_1 = 3; x_2 = 2}}$$

$$9 \left(\frac{x}{x-1} - \frac{x+1}{x} \right) : \left(\frac{x}{x+1} - \frac{x-1}{x} \right) = \frac{x+1}{x-1}$$

$$\frac{x^2 - x^2 + 1}{x(x-1)} : \frac{x^2 - x^2 + 1}{x(x+1)} = \frac{(x+1)}{x-1}$$

~~$$\cancel{x(x+1)} \\ \cancel{x(x-1)(x+1)}$$~~

$$\frac{1}{x(x-1)} \cdot \frac{x(x+1)}{1} = \frac{x+1}{x-1}$$

$$\frac{x+1}{x-1} = \frac{x+1}{x-1}$$

...alebo

$$x \neq 0; x \neq \pm 1$$

$$\underline{\underline{R = \mathbb{R} - \{-1, 0, 1\}}}$$

+ mehrmals:

$$\textcircled{19} \quad \text{af } \frac{1}{x} \geq 6$$

$$\begin{aligned} x > 0 \\ \frac{1}{x} &\geq 6 \\ 1 &\geq 6x \\ x &\leq \frac{1}{6} \end{aligned}$$

$$1) \quad \frac{2x+3}{x-1} < 1$$

$$x+4 < 0 \wedge x-1 > 0$$

$$x < -4 \wedge x > 1$$

$$K = \{-4; 1\}$$

$$x+4 > 0 \wedge x-1 < 0$$

$$x > -4 \wedge x < 1$$

∅

$$\frac{x+4}{x-1} < 0 \quad \Rightarrow K = \{-4; 1\}$$

$$2) \quad \frac{x+3}{x-1} \leq \frac{x+3}{x}$$

$$\begin{cases} x \neq 0 \\ x \neq 1 \end{cases}$$

$$2) \quad \frac{8}{x^2+4x+1} \leq 0$$

$$\frac{8}{(x-(-2+\sqrt{3}))(x-(-2-\sqrt{3}))} \leq 0$$

$$(x+2-\sqrt{3})(x+2+\sqrt{3}) \leq 0$$

$$(x+2-\sqrt{3})(x+2+\sqrt{3}) \leq 0$$

$$\frac{x^3 - x^3 - x^2 + 3x - 3}{x(x-1)} \leq 0$$

$$\frac{x^2 + 3x - (x^2 - x^2 - 3x - 3)}{x(x-1)} \leq 0$$

$$\frac{x^2 + 3x - x^2 + x - 3x - 3}{x(x-1)} \leq 0$$

$$\frac{x+3}{x(x+1)} \leq 0$$

$$x+3 \leq 0 \wedge x(x+1) > 0$$

$$x \leq -3 \wedge x^2 + x > 0$$

$$x+3 \geq 0 \wedge x(x+1) < 0$$

$$x \geq -3 \wedge x(x+1) < 0$$

$$x > 0 \wedge x+1 < 0$$

$$K = (-\infty; -3)$$

$$x > 0 \wedge x > -1$$

$$x < 0 \wedge x+1 < 0$$

$$x < 0 \wedge x < -1$$

$$x > 0 \wedge x < -1$$

$$x < 0 \wedge x+1 > 0$$

$$x < 0 \quad x > 1$$

$$K = (-\infty; -3) \cup (0; 1)$$

g)

$$\frac{\frac{1}{2}x-2}{x+1} + \frac{\frac{1}{2}x+2}{x+1} = 1$$

$$\sqrt{x \neq -1}$$

$$\frac{(\frac{1}{2}x-2)(x+1) + (\frac{1}{2}x+2)(x-1) - (x+1)(x-1)}{(x+1)(x-1)} \geq 0$$

$$\frac{\frac{1}{2}x^2 + \frac{1}{2}x - 2x - 2 + \frac{1}{2}x^2 - \frac{1}{2}x + 2x - 2 - x^2 + x - x + 1}{(x+1)(x-1)} \geq 0$$

$$(x+1)(x-1)$$

$$\frac{-3}{(x+1)(x-1)} \geq 0$$

$$(x+1)(x-1) < 0$$

$$1) 2 \leq \frac{x}{x^2+1}$$

$$0 \leq \frac{x-2(x^2+1)}{x^2+1}$$

$$\frac{x-2x^2-2}{x^2+1} \geq 0$$

$$\frac{2x^2-x+2}{-x^2-1} \geq 0$$

$$2x^2-x+2 \geq 0 \wedge -x^2-1 \geq 0$$

$$-x^2 \geq 1$$

$$2x^2-x+2 \leq 0 \wedge -x^2-1 \leq 0$$

$$\nexists R \quad \downarrow \quad -x^2 \not\leq 1$$

⇒ nebo jinak něco

Rovnice s meznámi a absolutní hodnotou

(22) a) $|x| = 7$

$$\underline{x = \pm 7}$$

b) $|x-1| = 3$

$$\underline{x_1 = 4; x_2 = -2}$$

c) $|x+5| = 1$

$$\underline{x_1 = -4; x_2 = -6}$$

d) $|x+\pi| = 6$

$$\begin{cases} x+\pi = 6 \\ -x-\pi = 6 \end{cases}$$

$$-x = 6-\pi$$

$$\underline{x_1 = 6-\pi}$$

e) $|2x-3| = 6$

$$2x = 9 \rightarrow x_1 = 4.5 \quad -x_2 = 2-2\sqrt{3}+1 \Rightarrow$$

$$-2x = 3 \rightarrow \underline{x_1 = -\frac{3}{2}} \quad x_2 = -3+2\sqrt{3}$$

f) $|6-x| = 2$

$$\underline{x_1 = 4; x_2 = 8}$$

g) $|x-\sqrt{3}+1| = 2-\sqrt{3}$

$$\begin{cases} x-\sqrt{3}+1 = 2-\sqrt{3} \\ x_1 = 1 \end{cases}$$

$$-x+\sqrt{3}-1 = 2-\sqrt{3}$$

$$h) |x - 5\sqrt{3}| = 0$$

$$\underline{x = 5\sqrt{3}}$$

$$i) |4x - 7| = -1$$

$$\underline{\emptyset}$$

$$j) |x - 7| = x - 7$$

$$\cancel{\text{Nicht def.}}$$

$$\underline{x \in \langle 7; \infty \rangle}$$

$$k) |x - 2| = 2 - x$$

$$\begin{cases} x - 2 = 2 - x \\ x + x = 2 + 2 \\ \underline{x_1 = 2} \\ -x + 2 = 2 - x \\ \underline{x \in \mathbb{R} \setminus \{x = 2\}} \end{cases}$$

$$l) |2x - 5| - |x + 3| = 2 - |x - 5|$$

$$2x - 5 - (x + 3) = 2 - (x - 5)$$

$$2x - 5 - x - 3 = 2 - x + 5$$

$$2x - 2 = 2$$

$$2x = 4$$

$$\underline{x_1 = 2}$$

$$2x - 5 - (x + 3) = 2 - (-x + 5)$$

$$2x - 5 - x - 3 = 2 + x - 5$$

$$x - 7 = -3 + x$$

$$\underline{\emptyset}$$

$$2x - 5 - (-x - 3) = 2 - (x - 5)$$

$$2x - 5 + x + 3 = 2 - x + 5$$

$$3x - 2 = 7 - x$$

$$4x = 8$$

$$\underline{x_2 = 2} \Rightarrow \emptyset$$

$$2x - 5 - (-x - 3) = 2 - (-x + 5)$$

$$2x - 5 + x + 3 = 2 + x - 5$$

$$3x - 2 = 2 + x$$

$$2x = 6$$

$$l) |8 - 5x| = 5x - 8$$

$$\begin{cases} 8 - 5x = 5x - 8 \\ 5x = 8 \\ \underline{x_1 = \frac{8}{5}} \\ -8 + 5x = 5x - 8 \\ \cancel{x_2 = \frac{8}{5}} > \frac{8}{5} \end{cases}$$

$$m) |2x - 5| = 1 - 3x$$

$$-2x - 5 = 1 - 3x$$

$$5x = 6$$

$$\underline{x_1 = \frac{6}{5}}$$

$$1 - 3x \geq 0 \Rightarrow \emptyset$$

$$\underline{x_2 = -5}$$

$$n) |4 - x| - |2x + 3| = ?$$

$$4 - x - (2x + 3) = ?$$

$$4 - x - 2x - 3 = ?$$

$$-3x = 6$$

$$\underline{x_1 = -2}$$

$$4 - x - (-2x - 3) = ?$$

$$4 - x + 2x + 3 = ?$$

$$\underline{x_2 = 0}$$

$$-4 + x - (2x + 3) = ?$$

$$-4 + x - 2x - 3 = ?$$

$$-x = 7$$

$$\underline{x_3 = 14}$$

$$-4 + x - (-2x - 3) = ?$$

$$-4 + x + 2x + 3 = ?$$

$$3x = 8$$

$$\underline{x_4 = \frac{8}{3}}$$

\Downarrow

$$-2x + 4 - (x + 3) = 2 - (x - 5)$$

$$-2x + 4 - x - 3 = 2 - x + 5$$

$$-3x + 1 = 7 - x$$

$$-2x = 6$$

$$\underline{x_5 = -3}$$

$$2x + 4 > 0$$

$$x < 2$$

$$x + 3 > 6$$

$$x > 3$$

$$-2x + 4 > 0$$

$$x > 2$$

$$x > 3$$

$$x > 3$$

$$\emptyset$$

$$\begin{aligned}
 -2x+5 - (x+3) &= 2 - (-x+5) \\
 -2x+5-x-3 &= 2+x-5 \\
 -3x+1 &= -3+x \\
 -2x &= -3 \\
 \underline{x=2} &\Rightarrow \emptyset
 \end{aligned}$$

$$\begin{aligned}
 -2x+5 - (-x-3) &= 2 - (-x+5) \\
 -2x+5+x+3 &= 2+x-5 \\
 -x+8 &= -3+x \\
 -2x &= -3-8 \\
 \underline{x=5} &
 \end{aligned}$$

$$-2x+5 - (-x-3) = 2-(x-5)$$

$$\begin{aligned}
 -2x+5+x+3 &= 2-x+5 \\
 -x+7 &= 7-x
 \end{aligned}$$

also:

$$\underline{\underline{K = \{5, 7\}}}$$

... and now some inequalities:

(23)

a) $|x| \geq 6$

$$\underline{\underline{K = (-\infty; -6) \cup \{6; \infty\}}}$$

b) $|x-3| < 2$



$$\underline{\underline{x \in (1, 5)}}$$

c) $|x+5| \leq 3$

$$\begin{aligned}
 x+5 \leq 3 &\quad |x+5| > 0 \\
 x &> 2
 \end{aligned}$$

$$-x-5 \leq 3$$

$$-x \leq 12$$

$$x \geq -12$$

$$\underline{\underline{K = [-12; 2]}}$$

d) $|x-\sqrt{3}| > 2 + 5\sqrt{3}$

$$\begin{aligned}
 x-\sqrt{3} &> 2 + 5\sqrt{3} \\
 x &> 2 + 6\sqrt{3}
 \end{aligned}$$

$$\underline{\underline{K = (-\infty; 2+6\sqrt{3}) \cup (2+6\sqrt{3}; \infty)}}$$

... d) $3x-1 < x$

e) $3x-1 < x$

$\frac{3x-1 < x}{2x < 1}$

$x < \frac{1}{2}$

$$\underline{\underline{K = \left(-\infty; \frac{1}{2}\right)}}$$

A) ... $|x| > 3(x+3)$

$|x| > 3x+9$

$-4x > 9$

$\underline{\underline{x < -2}}$

$|x-3| < -x-3$

$2x > -3$

$\underline{\underline{x > -1.5}}$

... já si meneši stranou
ale tohle je prosto
nepá strachovat...

$-1+x > 3(x+3)$

$-1+x > 3x+9$

$-2x > 10$

$\underline{\underline{x < -5}}$

$-1+x > 3(-x-3)$

$-1+x > -3x-9$

$4x > -8$

$\underline{\underline{x > -2}}$

... meneši do intervalu

$\underline{\underline{\emptyset}}$

$$\underline{\underline{K = (-\infty; -2)}}$$

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REJECT MATHS

RETURN TO MONKE

$$g) |2x+1| - 13 \geq x + 2 \quad x \geq -1 \quad \checkmark$$

$$2x+1 - (3-x) \geq x \quad x \leq 2 \quad \checkmark$$

$$\begin{aligned} 2x+1 - 3 + x &\geq x & 2x+1 > 0 \\ 3x - 2 &\geq x & x > \frac{2}{3} \\ 2x &\geq 2 & 3-x > 0 \\ x &\geq 1 & 3 > x \end{aligned}$$

$$\begin{aligned} 2x+1 - (-3+x) &\geq x & 2x+1 > 0 \\ 2x+1 + 3 - x &\geq x & 2x > 1 \\ x + 4 &\geq x & x > \frac{1}{2} \\ \cancel{x+4 \geq x} && \text{not am I doing} \end{aligned}$$

ans:

$$\begin{aligned} -2x-1 - (3-x) &\geq x & 2x+1 < 0 \\ -2x-1 - 3 + x &\geq x & x < \frac{1}{2} \\ -x-5 &\geq x & 3-x > 0 \\ -5 &\geq 2x & 3 > x \\ -2 &\geq x & \end{aligned}$$

$$\begin{aligned} -2x-1 - (-3+x) &\geq x & x \leq \frac{1}{2} \\ -2x-1 + 3 - x &\geq x & 3-x < 0 \\ -3x+2 &\geq x & 3 > x \\ -5x &\geq -2 & \because (-5) \\ x &\leq \frac{1}{2} & \text{mine} \end{aligned}$$

$$\underline{\underline{K = (-\infty; -2) \cup (-1; \infty)}}$$

$$h) x^2 - 3|x+1| - x \leq 0$$

$$x^2 - 3(x+1) - x \leq 0$$

$$x^2 - 3x - 3 - x \leq 0$$

$$x^2 - 4x - 3 \leq 0$$

$$(x - (2+\sqrt{7}))(x - (2-\sqrt{7})) \leq 0$$

$$\bullet x - 2 - \sqrt{7} \leq 0 \wedge x - 2 + \sqrt{7} \geq 0$$

$$x \leq 2 + \sqrt{7} \wedge x \geq 2 - \sqrt{7}$$

$$\bullet x - 2 - \sqrt{7} \geq 0 \wedge x - 2 + \sqrt{7} \leq 0$$

$$x \geq 2 + \sqrt{7} \wedge x \leq 2 - \sqrt{7}$$

$$x^2 - 3(-x-1) - x \leq 0$$

$$x^2 + 3x + 3 - x \leq 0$$

$$x^2 + 2x + 3 \leq 0$$

-\$\mathbb{R}\$

$$\underline{\underline{K = \{2-\sqrt{7}; 2+\sqrt{7}\}}}$$

$$g) i) |x| < x^2 - 6$$

$$-x^2 + |x| + 6 < 0$$

$$-x^2 + x + 6 < 0$$

$$(x-3)(x+2) < 0$$

$$\bullet x-3 < 0 \wedge x+2 > 0$$

$$x < 3 \wedge x > -2$$

$$\bullet x-3 > 0 \wedge x+2 < 0$$

$$x > 3 \wedge x < -2$$

$$-x^2 - x + 6 < 0$$

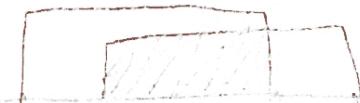
$$(x-2)(x+3) < 0$$

$$\bullet x-2 < 0 \wedge x+3 > 0$$

$$x < 2 \wedge x > -3$$

$$\bullet x-2 > 0 \wedge x+3 < 0$$

$$x > 2 \wedge x < -3$$



$$\underline{\underline{K = (2; 2)}}$$