p: f: $y=(12-x)\cdot\sqrt{24x-144}=(12-x)\cdot(24x-144)^{\frac{1}{2}}$

$$f'(x)=-1\cdot\left(\frac{1}{2}(24x-144)^{-\frac{1}{2}}\right)\cdot 24\ \cancel{\quad}\cdot(12-x)\cdot(24x-144)^{\frac{1}{2}}=$$

$$=-\frac{1}{2}\frac{1}{\sqrt{24x-144}}\cdot 24\cdot(12-x)\cancel{\quad}\sqrt{24x-144}=$$

$$=-12\cdot\frac{\sqrt{24x-144}}{\sqrt{24x-144}}\cdot(12-x)=-12\cdot(12-x)=144-12x=12(12-x)$$

$$f'(x) = \frac{2x(x-1) - x^2(1)}{(x-1)^2} = \frac{2x^2-2x-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$$

$$x^2 - 2x = 0$$
$$x(x-2) = 0$$
$$x_1 = 0 \; ; \; x_2 = 2$$

		0	1	2
$f'(x)$	+	−	−	+
f	↗	↘	↘	↗

$[0;0]$
$(2;4)$

$$f''(x) = 2x - 2$$
$$2x - 2 = 0$$
$$2x = 2$$
$$x = 1$$

		1
$f''(x)$	−	+
f	∩	∪

$$\lim_{x\to\infty} \frac{x^2}{x-1} = \infty$$

$$\lim_{x\to-\infty} \frac{x^2}{x-1} = -\infty$$

asymptoty:

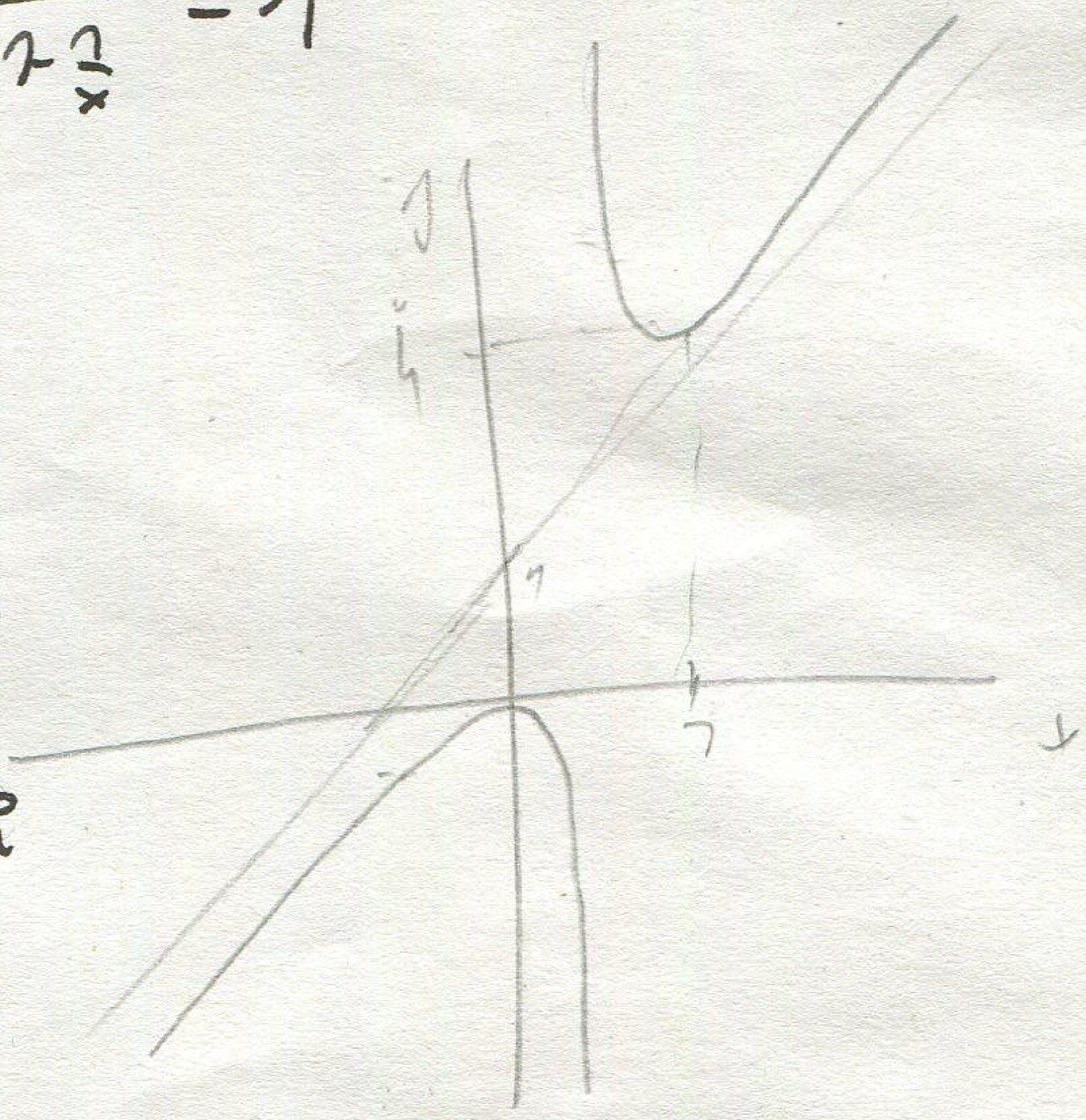
$$\frac{x^2}{x-1} : \frac{x-1}{1}$$

$$a = \lim_{x\to\infty} \frac{f(x)}{x} = \cancel{\ldots}$$

$$= \lim_{x\to\infty} \frac{x}{x-1} = \lim_{x\to\infty} \frac{1}{1-\frac{1}{x}} = 1$$

$$b = \lim_{x\to\infty} [f(x) - ax] = 1$$

$$y = 1x + 1$$



neomezená, neextrémní

př: $f: y = \frac{x}{1+x^2}$ $D_f = \mathbb{R}$

$$f'(x) = \cancel{\frac{x\cdot 2x - 1\cdot(1+x^2)}{(1+x^2)^2} = \frac{2x^2-1-x^2}{} = \frac{x^2-1}{(1+x^2)^2}}$$

$$= \frac{1\cdot(1+x^2) - x\,(2x)}{(1+x^2)^2} = \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$f''(x) = \frac{(-2x)\cdot(1+x^2)^2 - (1-x^2)\cdot 2\cdot(1+x^2)\cdot 2x}{(1+x^2)^4} =$$

$$= \frac{\cancel{(1+x^2)}\cdot 2x\cdot[-(1+x^2)-(1-x^2)\cdot 2]}{\cancel{(1+x^2)}\,(1+x^2)^3} = \frac{2x\,(-1-x^2-2+2x^2)}{(1+x^2)^3} =$$

$$= \frac{(x^2-3)\cdot 2x}{(1+x^2)^3}$$

nulové body: $2x\,(x^2-3) = 0$

$x_1 = 0$

$x_2 = \pm\sqrt{3}$

		$-\sqrt{3}$		0		$\sqrt{3}$	
$f''(x)$	$-$		$+$		$-$		$+$
f	$\cap$		$\cup$		$\cap$		$\cup$

př: $f: y = \frac{1}{x^2+1}$ $D_f = \mathbb{R}$

$$f'(x) = \frac{0\cdot(x^2+1) - 1\cdot(2x+0)}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$$

$$f''(x) = \frac{-2\cdot(x^2+1)^2 - (-2x)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4} = \frac{2\cdot\cancel{(x^2+1)}\cdot[-(x^2+1)+4x^2]}{\cancel{(x^2+1)}(x^2+1)^3} =$$

$$= \frac{2\cdot(-x^2-1+4x^2)}{(x^2+1)^3} = \frac{6x^2-2}{(x^2+1)^3}$$

$6x^2 - 2 = 0$

$6x^2 = 2$

$x^2 = \frac{1}{3}$

$x = \pm\frac{\sqrt{3}}{3}$

		$-\frac{\sqrt{3}}{3}$		$\frac{\sqrt{3}}{3}$	
$f''(x)$	$+$		$-$		$+$
f	$\cup$		$\cap$		$\cup$

př: $f: y = x^3 - 6x^2 + 9x$

$f'(x) = 3x^2 - 12x + 9$

$f''(x) = 6x - 12$

sudá / lichá?

$f(x) = f(-x)$ ×

$-f(x) = f(-x)$ ×

ASAL

$P_x; P_y$:

$y = 0$:

$x_1 = 3; x_2 = 0$

$[3;0]$

$[0;0]$

$x = 0$:

$[0;0]$

$3x^2 - 12x + 9 = 0$

$x_1 = 3; x_2 = 1$

		1	3
$f'(x)$	+	−	+
f	↗	↘	↗

$6x - 12 = 0$

$6x = 12$

$x = 2$

		2
$f''(x)$	−	+
f	∩	∪

$\lim_{x \to +\infty} (x^3 - 6x^2 + 9x) = \infty$

$\lim_{x \to -\infty} (x^3 6x^2 + 9x) = -\infty$

$\Rightarrow$ nemá globální extrémy

asymptoty: $y = ax + b$

$a = \lim_{x \to \infty} \frac{f(x)}{x} = \infty$

$b = \lim_{x \to \infty} (f(x) - ax) = \infty - \infty$

nemá asymptoty

př: $f: y = \frac{x^2}{x-1}$ $\quad D_f = \mathbb{R} - \{1\}$

$f(x) \neq f(-x)$

$-f(x) \neq f(-x)$

ani sudá ani lichá

P_x:

$0 = \frac{x^2}{x-1}$

$x = 0$

$[0;0]$

P_y:

$y = \frac{0^2}{0-1}$

$[0;0]$