Integrály

př: $\int (x^2-3x+6)\,dx = \int x^2\,dx - \int 3x\,dx + \int 6\,dx = \int x^2\,dx - 3\int x\,dx + 6\cdot\int 1\,dx =$

$= \frac{x^3}{3} - \frac{3x^2}{2} + 6x + C$

př: $\int (4x-3)^3\,dx = \int (4x-3)^2(4x-3)\,dx = \int (16x^2 - 2\cdot 4\cdot 3\cdot x + 9)(4x-3)\,dx =$

$= \int 64x^3 - 48x^2 - 96x^2 + 36x + 72x - 27\,dx =$

$= \int 64x^3 - 144x^2 + 108x - 27\,dx = \frac{64x^4}{4} - \frac{144x^3}{3} + \frac{108x^2}{2} - 27x =$

$= 16x^4 - 48x^3 + 54x^2 - 27x + C$

př: $\int \frac{2x^5 - 3x^3 + 1}{4x^3}\,dx = \int \frac{2x^5}{4x^3} - \frac{3x^3}{4x^3} + \frac{1}{4x^3}\,dx = \int \frac{x^2}{2} - \frac{3}{4} + \frac{1}{4x^3}\,dx =$

$= \frac{x^3}{6} - \frac{3x}{4} + \frac{1}{8x^2} + C$

př: $\int \frac{x^2\cdot\sqrt[3]{x^2} - 5\cdot\sqrt[4]{x}}{x\cdot\sqrt{x}}\,dx = \int \frac{x^2\cdot\sqrt[3]{x}}{x\cdot\sqrt{x}} - \frac{5\sqrt[4]{x}}{x\sqrt{x}}\,dx = \int \frac{x\cdot x^{\frac{1}{3}}}{x^{\frac{1}{2}}} - \frac{5\cdot x^{\frac{1}{4}}}{x\cdot x^{\frac{1}{2}}}\,dx =$

$= \int x\cdot x^{\frac{1}{6}} - \frac{5}{x}\cdot x^{-\frac{1}{4}}\,dx = \int x^{\frac{7}{6}} - \frac{5}{x\cdot x^{\frac{1}{4}}}\,dx = \int x^{\frac{7}{6}} - \frac{5}{x^{\frac{5}{4}}}\,dx = \frac{x^{\frac{13}{6}}}{\frac{13}{6}} - \frac{5}{\frac{1}{4}\cdot x^{\frac{1}{4}}} =$

$= \frac{6x^{\frac{13}{6}}}{13} - \frac{20}{x^{\frac{1}{4}}} + C$

př: $\int 2^x + e^{x+3}\,dx = \frac{2^x}{\ln 2} + e^{x+3} + C$

př: $\int \frac{7x^2 - 4x + 3}{x}\,dx = \int \frac{7x^2}{x} - \frac{4x}{x} + \frac{3}{x}\,dx = \int 7x - 4 + \frac{3}{x}\,dx = \frac{7x^2}{2} - 4x + 3\ln|x| + C$

př: $\int 2\sin x + 3\cos x \, dx = 2\cdot(-\cos x) + 3\cdot \sin x + C$

př: $\int tg^2 x \, dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1-\cos^2 x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} - 1 \, dx = tg\, x - x + C$

př: $\int 2x - 3 \, dx = \frac{2x^2}{2} - 3x = x^2 - 3x + C$ .... najít takovou prim. fci, aby procházela bodem $A[1;4]$

$$y = x^2 - 3x + C$$
$$4 = 1^2 - 3.1 + C$$
$$4 = -2 + C$$
$$C = 6 \quad \Rightarrow x^2 - 3x + 6$$

př: $\int \frac{x^3 - 4x^2 + 6x}{x^2} dx = \int \frac{x^3}{x^2} - \frac{4x^2}{x^2} + \frac{6x}{x^2} dx = \int x - 4 + \frac{6}{x} dx = \frac{x^2}{2} - 4x + 6\ln|x| + C$

substituce:

př: $\int (3x-4)^7 dx = \int t^7 \cdot \frac{1}{3} dt = \frac{1}{3}\int t^7 dt = \frac{1}{3}\cdot\frac{t^8}{8} = \frac{t^8}{24} = \frac{(3x-4)^8}{24} + C$

$t = 3x - 4$

$dt = 3dx$

$dx = \frac{1}{3} dt$

př: $\int \frac{1}{(x+1)^3} dx = \int \frac{1}{t^3} dt = \int t^{-3} dt = \frac{t^{-2}}{-2} = \frac{(x+1)^{-2}}{-2} = \frac{-1}{2(x+1)^2} + C$

$t = x + 1$

$dt = 1dx$

př: $\int x \cdot \sqrt{1-x^2} dx = \int x \cdot t^{\frac{1}{2}} \cdot (-2x) dx = \int x \sqrt{1-x^2} \cdot \frac{1}{-2x} dt = -\frac{1}{2} \int \sqrt{1-x^2} dt =$

$t = 1 - x^2$

$dt = -2x dx$

$dx = -\frac{dt}{2x}$

$= -\frac{1}{2}\int \sqrt{t} dt = -\frac{1}{2}\int t^{\frac{1}{2}} dt = -\frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{1}{2} \cdot \frac{2t^{\frac{3}{2}}}{3} =$

$= -\frac{2t^{\frac{3}{2}}}{6} = -\frac{t^{\frac{3}{2}}}{3} = -\frac{\sqrt{1-x^2}^3}{3}$

pr: $\int \sin 5x \, dx = \frac{1}{5} \cos x + C$

pr: $\int 3e^{-x} \, dx = 3e^{-x} + C$

pr: $\int tg\, x \, dx = \int \frac{\sin x}{\cos x} dx = \int \frac{\sin x}{t} dx = \int \frac{\sin x}{t} \cdot \frac{1}{-\sin x} dt =$

$t = \cos x$

$dt = -\sin x \, dx$ $\quad = \int -\frac{1}{t} dt = -\ln|t| = -\ln|\cos x| + C$

$dx = \frac{dt}{-\sin x}$

pr: $\int \frac{5x}{3x^2+1} dx = \int \cancel{} \frac{5x}{t} \cdot \frac{1}{6} dt = \int \frac{5}{6} \frac{x}{t} dt = \frac{5}{6} \int \frac{x}{t} dt$

$t = 3x^2+1$

$dt = 6x \, dx$

$dx = \frac{1}{6} dt$

pr: $\int \sin^3 x \, dx = \int (1-\cos^2 x) \cdot \sin x \, dx = \int (1-t^2) \cdot \sin x \frac{dt}{-\sin x} = \int (1-t^2) \cdot -1 \, dt =$

$t = \cos x$

$dt = -\sin x \, dx$ $\quad = \cancel{} \int t^2 - 1 \, dt = \frac{t^3}{3} - t = \cancel{} = \frac{\cos^3 x}{3} - \cos x + C$

$dx = \frac{dt}{-\sin x}$

pr: $\int \cos^5 x \, dx = \int \cos^4 x \cdot \cos x \, dx = \int (1-\sin^2 x)^2 \cdot \cos x \, dx = \int (1-t^2)^2 \cdot \cos x \frac{dt}{\cos x} =$

$t = \sin x$

$= \int (1-t^2)^2 (+1) \, dt = \int 1 - 2t^2 + t^4 \, dt = t - \frac{2t^3}{3} + \frac{t^5}{5} =$

$dt = \cos x \, dx$

$dx = \frac{dt}{\cos x}$ $\quad = \sin x - \frac{2\sin^3 x}{3} + \frac{\sin^5 x}{5} + C$

pr: $\int \sin^3 x \cos^2 x \, dx = \int (1-\cos^2 x) \sin x \cdot \cos^2 x \, dx = \int (1-t^2) \cdot t^2 \sin x \frac{dt}{-\sin x} =$

$t = \cos x$

$dt = -\sin x \, dx$ $\quad = \int (t^2 - t^4) \cdot (-1) \, dt = \int t^4 - t^2 \, dt = \frac{t^5}{5} - \frac{t^3}{3} = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$

$dx = \frac{dt}{-\sin x}$

př: $\int \sin^3 x \cdot \cos^3 x \, dx = \int (1-\cos^2 x) \cdot \sin x \cdot \cos^3 x = \int (1-t^2) \cdot t^3 \cdot \sin x \frac{dt}{-\sin x} =$

$t = \cos x$

$dt = -\sin x \, dx$

$dx = \frac{dt}{-\sin x}$

$= \int (t^3 - t^5)(-1)\, dt = \int t^5 - t^3 \, dt = \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$

per partes:

př: $\int x \cdot \sin x \, dx = x \cdot (-\cos x) - \int 1 (-\cos x) dx = -x \cdot \cos x - (-\sin x) =$

$u = x \quad v' = \sin x$

$u' = 1 \quad v = -\cos x$

$= -x \cos x + \sin x + C$

př: $\int x \cdot e^x dx = x \cdot e^x - \int e^x dx = x \cdot e^x - e^x + C$

$u = x \quad v' = e^x$

$u' = 1 \quad v = e^x$

parciální zlomky:

$$\frac{10x-1}{(2x+1)(2x-1)} = \frac{A}{2x+1} + \frac{B}{2x-1}$$

$x = -\frac{1}{2}$:

$-5-1 = A(-1-1)$

$A = 3$

$x = \frac{1}{2}$:

$5-1 = B(1+1)$

$B = 2$

$$\int \frac{10x-1}{(2x+1)(2x-1)} dx = \int \frac{3}{2x+1} + \frac{2}{2x-1} dx = \int \frac{3}{t} + \frac{2}{s} dx = \int \frac{3}{t} \cdot \frac{1}{2} dt + \int \frac{2}{s} \frac{1}{2} ds =$$

$t = 2x+1 \quad dt = 2dx \quad dx = \frac{dt}{2}$

$s = 2x-1 \quad ds = 2dx \quad dx = \frac{ds}{2}$

$= \frac{3}{2} \int \frac{1}{t} dt + \int \frac{1}{s} ds = \frac{3}{2} \ln|t| + \ln|s| =$

$= \frac{3}{2} \ln|2x+1| + \ln|2x-1| + C$

pr: $\int \frac{4x+16}{x^2+4x-12}dx = \int \frac{4x+16}{(x-2)(x+6)}dx = \int \frac{A}{x-2} + \frac{B}{x+6}dx = \int \frac{3}{(x-2)} + \frac{1}{x+6}dx$

$x=2$:

$4x+16 = A(x+6)+B(x-2)$

$24 = 8A$

$A = 3$

$x=-6$

$-24+16 = 0A + (-8)B$

$B = 1$

$= \int \frac{3}{s}ds + \int \frac{1}{t}dt = 3\ln|s| + \ln|t| = 3\ln|x-2| + \ln|x+6| + C$

$t = x+6$ $\quad s = x-2$

$dt = dx$ $\quad ds = dx$

pr: $\int \frac{4}{x^2-4x}dx = \int \frac{4}{x(x-4)}dx = \int \frac{A}{x} + \frac{B}{x-4}dx = \int -\frac{1}{x} + \frac{1}{x-4}dx = -\ln|x| + \ln|x-4| + C$

$x_1 = 0$:

$4 = A(x-4)$

$A = -1$

$x_2 = 4$:

$4 = 4B$

$B = 1$

pr: $\int \frac{7}{(x-2)(x+5)}dx = \int \frac{A}{x-2} + \frac{B}{x+5}dx = \int \frac{1}{x-2} - \frac{1}{x+5}dx = \ln|x-2| - \ln|x+5| =$

$= \ln\left|\frac{x-2}{x+5}\right| + C$

$x_1 = 2$:

$7 = A(2+5)$

$A = 1$

$x_2 = -5$:

$7 = (-5-2)B$

$B = -1$

pr: $\frac{7x}{6x^2-x-2} = \frac{7x}{(3x-2)(2x+1)} = \frac{A}{3x-2} + \frac{B}{2x+1} = \frac{2}{3x-2} + \frac{1}{2x+1}$

$x_1 = \frac{2}{3}$:

$\frac{14}{3} = A(\frac{4}{3}+1)$

$\frac{14}{3} = A\frac{7}{3}$

$A = 2$

$x_2 = -\frac{1}{2}$

$-\frac{7}{2} = B(-\frac{3}{2}-2)$

$B = 1$

pr: $\frac{-9x-3}{x^3+2x^2-3x} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{x} = \frac{-3}{x-1} + \frac{2}{x+3} + \frac{1}{x}$

$x = 1:$
$-9-3 = A \cdot 4 \cdot 1$
$-12 = 4A$
$A = -3$

$x = -3:$
$27-3 = B(-3-1)(-3)$
$24 = 12B$
$B = 2$

$x = 0:$
$-3 = C(-1)\cdot 3$
$C = 1$

pr: $\int \frac{6x^2+x-3}{3x^2-2x-1}\,dx = \int \frac{A}{x-1} + \frac{B}{3x+1}\,dx = \int \frac{1}{x-1} + \frac{2}{3x+1}\,dx = \ln|x-1| + 2\ln|3x+1|$

$x = 1:$
$6+1-3 = A \cdot 4$
$A = 1$

$x = -\frac{1}{3}:$
$\frac{2}{3} - \frac{1}{3} - 3 = -\frac{4}{3}B$
$-\frac{8}{3} = -\frac{4}{3}B$
$B = 2$

určitý integrál:

pr: $\int_2^3 5x^2\,dx = \left[5\frac{x^3}{3}\right]_2^3 = 5\frac{3^3}{3} - 5\cdot\frac{2^3}{3} = 45 - \frac{40}{3} = \frac{95}{3}$

pr: $\int_0^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 = 1$

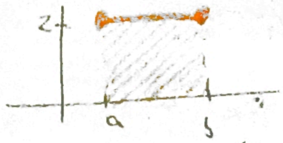
pr: $\int_{-5}^{2} 2x-3\,dx = [x^2-3x]_{-5}^{2} = (4-6) - (25+15) = -42$

pr: $\int_a^b 2\,dx = [2x]_a^b = 2a - 2b = 2(a-b)$

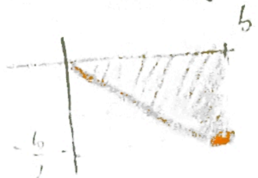
pr: $\int_0^b -\frac{x}{2}\,dx = \left[-\frac{x^2}{4}\right]_0^b = -\frac{b^2}{4}$

pr: $\int_1^3 (x-2)\,dx = \left[\frac{x^2}{2} - 2x\right]_1^3 = \left(\frac{9}{2}-6\right) - \left(\frac{1}{2}-2\right) = 0$

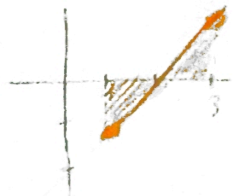
per partes²



př: $\int_a^b 2\,dx = [2x]_a^b = 2(b-a)$



př: $\int_0^b -\frac{x}{2}\,dx = \left[-\frac{x^2}{4}\right]_0^b = -\frac{b^2}{4}$



př: $\int_1^3 (x-2) = \left[\frac{x^2}{2}-2x\right]_1^3 = \left(\frac{9}{2}-6\right)-\left(\frac{1}{2}-2\right) = 0$

př: $\int_0^2 3x^3-2x+5\,dx = \left[\frac{3x^4}{4}-x^2+5x\right]_0^2 = 18-0 = 18$

př: $\int_1^4 -x+\frac{4}{x}\,dx = \left[-\frac{x^2}{2}+4\ln|x|\right]_1^4 = \left(-8+4\ln|4|\right)-\left(-\frac{1}{2}+0\right) = -7{,}5+4\ln|4|$

př: $\int_1^8 \frac{dx}{\sqrt[3]{x^4}} = \int_1^8 \frac{1}{\sqrt[3]{x^4}}\,dx = \int_1^8 x^{-\frac{4}{3}}\,dx = \left[\frac{x^{-\frac{1}{3}}}{-\frac{1}{3}}\right]_1^8 = \left[-3x^{\frac{1}{3}}\right]_1^8 = -6-(-3) = -3$

př: $\int_{-3}^{-1} \frac{dx}{x^2} = \int_{-3}^{-1} \frac{1}{x^2}\,dx = \int_{-3}^{-1} x^{-2}\,dx = \left[\frac{x^{-1}}{-1}\right]_{-3}^{-1} = \left[-\frac{1}{x}\right]_{-3}^{-1} = 1-\left(+\frac{1}{3}\right) = \frac{2}{3}$

př: $\int_1^4 \sqrt{x}\,(1+2\sqrt{x})\,dx = \int_1^4 \sqrt{x}+2x\,dx = \int_1^4 x^{\frac{1}{2}}+2x\,dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+x^2\right]_1^4 =$

$= \left[\frac{2x^{\frac{3}{2}}}{3}+x^2\right]_1^4 = \frac{59}{3}$

př: $\int_a^{2a} \frac{dx}{2\sqrt{ax}} = \int_a^{2a} \frac{1}{2\sqrt{ax}}\,dx = \cancel{\left[\ln|2\sqrt{ax}|\right]_a^{2a} = \ln|2\sqrt{2a^2}| - \ln|2\sqrt{a^2}| = \ln\left|\frac{2a\sqrt{2}}{2a}\right| = \ln|\sqrt{2}|}$

$= \frac{1}{2}\int_a^{2a} \frac{1}{\sqrt{ax}}\,dx = \frac{1}{2}\int_a^{2a} \frac{1}{\sqrt{a}}\cdot\frac{1}{\sqrt{x}}\,dx = \frac{1}{2\sqrt{a}}\int_a^{2a} \frac{1}{\sqrt{x}}\,dx = \frac{1}{2\sqrt{a}}\left[2\sqrt{x}\right]_a^{2a} = \frac{1}{2\sqrt{a}}\cdot 2\sqrt{2a} - \frac{1}{2\sqrt{a}}\,2\sqrt{a} =$

$= \sqrt{2}-1$

substituce:

př: $\int_{-1}^{1}(x+1)^3 dx = \int_{0}^{2} t^3 dt = \left[\frac{t^4}{4}\right]_0^2 = 4 - 0 = 4$

$t = x+1$

$dt = dx$

$x = 1 \Rightarrow t = x+1 = 2$

$x = -1 \Rightarrow t = x+1 = 0$

př: $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2 x \cdot \cos x \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} t^2 \cdot \cos x \cdot \frac{1}{\cos x} dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} t^2 dt = \left[\frac{t^3}{3}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{7}{24}$

$t = \sin x$

$dt = \cos x \, dx$ $\quad t = \sin\frac{\pi}{2} = 1$

$dx = \frac{dt}{\cos x}$ $\quad t = \sin\frac{\pi}{6} = \frac{1}{2}$

př: $\int_{-4}^{2} \sqrt{17+4x} \, dx = \int_{1}^{25} \sqrt{t} \cdot \frac{1}{4} dt = \frac{1}{4}\int_{1}^{25} t^{\frac{1}{2}} dt = \frac{1}{4}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^{25} = \frac{1}{4}\left[\frac{2t^{\frac{3}{2}}}{3}\right]_1^{25} = \frac{62}{3}$

$t = 17+4x$ $\quad 17+4\cdot 2 = 25$

$dt = 4\,dx$ $\quad 17 - 16 = 1$

$dx = \frac{dt}{4}$

př: $\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{1+\sin x} dx = \int_{1}^{2} \frac{\cos x}{t} \cdot \frac{1}{\cos x} dt = \int_{1}^{2} \frac{1}{t} dt = \left[\ln|t|\right]_1^2 = \ln|2|$

$t = 1+\sin x$ $\quad 1+\sin\frac{\pi}{2} = 2$

$dt = \cos x \, dx$ $\quad 1+\sin 0 = 1$

$dx = \frac{dt}{\cos x}$

př: $\int_{0}^{1} \frac{e^x}{e^x+1} dx = \int_{2}^{e+1} \frac{e^x}{t} \cdot \frac{1}{e^x} dt = \int_{2}^{e+1} \frac{1}{t} dt = 0{,}62\ldots$

$t = e^x+1$ $\quad e^1+1 = e+1$

$dt = e^x dx$ $\quad e^0+1 = 2$

$dx = \frac{dt}{e^x}$

per partes²

$\text{pp:} \int \ln x \, dx = \ln x \cdot x - \int \frac{1}{x} \cdot x \, dx = \ln x \cdot x - x = x(\ln x - 1) + C$

$u = \ln x \quad v' = 1$
$u' = \frac{1}{x} \quad v = x$

$\text{pp:} \int x^3 \ln x \, dx = \ln x \cdot 3x^2 - \int \frac{1}{x} \cdot 3x^2 \, dx = 3 \ln x \cdot x^2 - 3 \int x \, dx =$
$= 3 \ln x \cdot x^2 - \frac{3x^2}{2}$

$u = \ln x \quad v' = x^3$
$u' = \frac{1}{x} \quad v = 3x^2$

$\text{pp:} \int x \cdot \ln^2 x \, dx = \ln^2 x \cdot \frac{x^2}{2} - \int 2 \ln x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx = \ln^2 x \cdot \frac{x^2}{2} - 2 \int \ln x \cdot \frac{1}{2} x \, dx =$
$= \ln^2 x \cdot \frac{x^2}{2} - \int \ln x \cdot x \, dx = \ln^2 x \cdot \frac{x^2}{2} - ($
$(\ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx =$
$= \ln^2 x \cdot \frac{x^2}{2} \ln x \cdot \frac{x^2}{2} + \frac{1}{2} \cdot \frac{x^2}{2} =$
$= \frac{x^2}{2} (\ln^2 x - \ln x + \frac{1}{2})$

$u = \ln^2 x \quad v' = x$
$u' = 2 \cdot \ln x \cdot \frac{1}{x} \quad v = \frac{x^2}{2}$
$u = \ln x \quad v' = x$
$u' = \frac{1}{x} \quad v = \frac{x^2}{2}$

... something went wrong... takže, bylo to dobře, [illegible]

$\int x \cdot \ln^2 x \, dx = \ln^2 x \cdot \frac{x^2}{2} - \int 2 \cdot \ln x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx = \ln^2 x \cdot \frac{x^2}{2} - 2 \cdot \frac{1}{2} \int \ln x \cdot x \, dx =$
$= \ln^2 x \cdot \frac{x^2}{2} - \int \ln x \cdot x \, dx =$
$= \ln^2 x \cdot \frac{x^2}{2} - (\ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx) =$
$= \ln^2 x \cdot \frac{x^2}{2} - \ln x \cdot \frac{x^2}{2} + \frac{1}{2} \int x \, dx =$
$= \frac{\ln^2 x \cdot x^2}{2} - \frac{\ln x \cdot x^2}{2} + \frac{1}{2} \cdot \frac{x^2}{2} =$
$= \frac{x^2}{2} (\ln^2 x - \ln x + \frac{1}{2})$

$u = \ln^2 x \quad v' = x$
$u' = 2 \ln x \cdot \frac{1}{x} \quad v = \frac{x^2}{2}$
$u = \ln x \quad v' = x$
$u' = \frac{1}{x} \quad v = \frac{x^2}{2}$

$\text{pp:} \int x^2 \cdot e^x \, dx = x^2 \cdot e^x - \int 2x \cdot e^x \, dx = x^2 e^x - 2 \int x \cdot e^x \, dx =$
$= x^2 \cdot e^x - 2 \cdot (x \cdot e^x - \int e^x dx) =$
$= x^2 \cdot e^x - 2x \cdot e^x + 2e^x =$
$= e^x (x^2 - 2x + 2)$

$u = x^2 \quad v' = e^x \qquad u = x \quad v' = e^x$
$u' = 2x \quad v = e^x \qquad u' = 1 \quad v = e^x$

$\text{pp:} \int e^x \sin x \, dx = \sin x \cdot e^x - \int \cos x \, e^x \, dx =$
$= \sin x \cdot e^x - (\cos x \cdot e^x - \int -\sin x \cdot e^x \, dx =$
$= \sin x \cdot e^x (\cos x \cdot e^x - \cos x \cdot e^x) = \sin x \cdot e^x$

$u = \sin x \quad v' = e^x$
$u' = \cos x \quad v = e^x$
$u = \cos x \quad v' = e^x$
$u' = -\sin x \quad v = e^x$

..... nope.

pr: $\int e^x \cdot \sin x \, dx = -e^x \cdot \cos x - \int e^x(-\cos x)\,dx = -e^x \cdot \cos x - (-1)\int e^x \cos x \, dx =$

$u = e^x \quad v' = \sin x$

$u' = e^x \quad v = -\cos x$

$= -e^x \cos x + \int e^x \cos x \, dx = -e^x \cdot \cos x + e^x \cdot \sin x - \int e^x \cdot \sin x \, dx =$

$= -e^x \cos x + e^x \cdot \sin x - e^x \cdot \cos x = -2e^x \cdot \cos x + e^x \sin x$

$u = e^x \quad v' = \cos x$

$u' = e^x \quad v = \sin x$

per partes:

$$\int a \cdot b \, dx = a \cdot \int b \, dx - \int (a)' \cdot \int b \, dx$$

$u = a \quad v' = b$

$u' = (a)' \quad v' = \int b \, dx$

[illegible]

[illegible]

[illegible]

Integrály, per partes

př: $\int \frac{10x-1}{(2x+1)(2x-1)}\,dx = \int \frac{A}{2x+1} + \frac{B}{2x-1}\,dx = \int \frac{2}{2x+1} + \frac{3}{2x-1}\,dx =$

$A(2x-1) + B(2x+1) = 10x-1 \qquad x_1 = \frac{1}{2};\ x_2 = -\frac{1}{2}$

x_1: $B(2x+1) = 10x-1$
$B(1+1) = 10x-1$
$\cancel{2B}$
$2B = 4$
$B = 2$

x_2: $A(2x-1) = 10x-1$
$A\cdot(-2) = \cancel{10} - 5 - 1$
$-2A = -6$
$A = 3$

subst:
$t = 2x+1$
$dt = 2dx$
$dx = \frac{1}{2}dt$

$f = 2x-1$
$df = 2dx$

$= \int \frac{2}{t} + \cancel{\frac{3}{f}}$

$\cancel{D = \frac{-4^2 \pm \sqrt{4\cdot(-6)}}{2} = \frac{-16 \pm \sqrt{-24}}{2}}$

př: $\int \frac{4x+16}{x^2-4x-12}\,dx = \int \frac{A}{x-6} + \frac{B}{x+2}\,dx =$

$x_1 = 6$
$x_2 = -2$
$A(x+6) + B(x+2) = 4x+16$

x_1: $8B = 24+16$
$B = \cancel{10}\,5$

x_2: $-8A = 8$
$A = -1$

$= \int \frac{-1}{x-6} + \frac{5}{x+2}\,dx =$

$= -1\cdot \ln|x-6| + 5\cdot \ln|x+2| + C$

$x_1 = -6$
$x_2 = 2$

x_1: $-8B = -8$
$B = 1$

x_2: $8A = 24$
$A = 3$

$= \int \frac{3}{x-2} + \frac{1}{x+6}\,dx =$

$= 3\cdot \ln|x-2| + \ln|x+6| + C$

pr:

$$\int \frac{4}{x^2-4}\,dx = \int \frac{A}{x+2} + \frac{B}{x-2}\,dx = \int \frac{-1}{x+2} + \frac{1}{x-2}\,dx = -\ln|x+2| + \ln|x-2| + C$$

$(x+2)(x-2)$

$A\cdot(x-2) + B\cdot(x+2) = 4$

$x_1 = 2:$ $\quad$ $x_2 = -2:$

$4B = 4$ $\quad$ $-4A = 4$

$B = 1$ $\quad$ $A = -1$

pr:

$$\int \frac{x}{x^2-4x}\,dx = \int \frac{A}{x} + \frac{B}{x-4}\,dx = \int \frac{-1}{x} + \frac{1}{x-4}\,dx = -\ln|x| + \ln|x-4| + C$$

$x\cdot(x-4)$

$$= \ln\left|\frac{x-4}{x}\right| + C$$

$A(x-4) + Bx = 4$

$x_1 = 0:$ $\quad$ $x_2 = 4:$

$-4A = 4$ $\quad$ $4B = 4$

$A = -1$ $\quad$ $B = 1$

pr:

$$\int \frac{7}{(x-2)(x+5)}\,dx = \int \frac{A}{x-2} + \frac{B}{x+5}\,dx = \int \frac{1}{x-2} + \frac{-1}{x+5}\,dx = \ln|x-2| - \ln|x+5| + C$$

$A(x+5) + B(x-2) = 7$

$$= \ln\left|\frac{x-2}{x+5}\right| + C$$

$x_1 = -5:$ $\quad$ $x_2 = 2:$

$-7B = 7$ $\quad$ $7A = 7$

$B = -1$ $\quad$ $A = 1$

pr:

$$\frac{7x}{6x^2-x-2} = \frac{A}{3x-2} + \frac{B}{2x+1} = \frac{2}{3x-2} + \frac{1}{2x+1}$$

$x_1 = \frac{2}{3}$; $x_2 = -\frac{1}{2}$

$(3x-2)(2x+1)$

$A(2x+1) + B(3x-2) = 7x$

$x_1 = \frac{2}{3}:$ $\quad$ $x_2 = -\frac{1}{2}:$

$\frac{7}{3}A = \frac{14}{3}$ $\quad$ $-\frac{7}{2}B = -\frac{7}{2}$

$A = 2$ $\quad$ $B = 1$

pf: $\dfrac{-9x-3}{x^3+2x^2-3x} = \dfrac{A}{x} + \dfrac{B}{x-1} + \dfrac{C}{x+3} = \dfrac{1}{x} - \dfrac{3}{x-1} + \dfrac{2}{x+3}$

$x \cdot (x^2+2x-3)$

$(x+1)(x-3)$ $\Rightarrow 0;\ +1;\ 3$

$A(x-1)(x+3) + B \cdot x \cdot (x+3) + C \cdot x \cdot (x-1) = -9x - 3$

$x_1 = 0$:

$-3A = -3$

$x_2 = +1$:

$+B \cdot (+4) = -9 \cdot (+1) - 3$

$+4B = -12$

$B = -3$

$x_3 = -3$:

$-3C \cdot (-4) = 27 - 3$

$C = 2$

nope

$(x-1)(x+3)$

$x^2 + 3x - x - 3 = x^2 + 2x - 3$

$A(x-1)(x+3) + B \cdot x \cdot (x+3) + C \cdot x \cdot (x-1) = -9x - 3$

$x = 0;\ 1;\ -3$

$x_1 = 0$:

$A \cdot (-1) \cdot 3 = -3$

$-3A = -3$

$A = 1$

$x_2 = 1$:

$B \cdot 1 \cdot 4 = -9 - 3$

$4B = -12$

$B = -3$

$x_3 = -3$:

$C \cdot (-3) \cdot (-4) = -9 \cdot (-3) - 3$

$12C = 24$

$C = 2$

$$\frac{1}{x} + \frac{-3}{x-1} + \frac{2}{x+3}$$

$$\frac{7}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} =$$

$A(x+5) + B(x-2) = 7$

$x_1 = -5$:

$B(-5-2) = 7$

$B = -1$

$x_2 = 2$:

$A \cdot 7 = 7$

$A = 1$

$$= \frac{1}{x-2} - \frac{1}{x+5}$$

$$\int \frac{x^2+x-3}{3x^2-2x-1}\,dx = \int \frac{A}{x-1} + \frac{B}{3x+1}\,dx = \int \frac{\cancel{4}\,1}{x-1} + \frac{2}{3x+1}\,dx =$$

$-\frac{1}{3}$

$x_1 = 1:$

$A(3\cdot 1+1) = 6+1+3$

$4A = 8\,4$

$A = \cancel{8}\,4$

$x_2 = -\frac{1}{3}:$

$B\cdot\left(-\frac{4}{3}\right) = 6\cdot\frac{1}{9} - \frac{1}{3} + 3$

$\cancel{\frac{4\,12}{3}B = \frac{\cancel{1}\,6 - 3 + 27}{9}}$

$\cancel{-12B =}$

$-\frac{12B}{9} = \frac{6-3+27}{9}$

$-12B = -24$

$B = 2$

$1)\,(3x+1)$

$3x^2 + x - 3x - 1$

$(3x+1) + B\cdot(x-1) = 6x^2 + x + 3$

$3x+1$

$= \cancel{\ln|x-1| + 2\cdot\ln|3x+1|}$ $\frac{2}{3}\cdot \ln|x+\frac{1}{3}| + \ln|x-1| + C$

$3x+1$

$\frac{3}{3}\cdot\left(x+\frac{1}{3}\right)$

$\frac{2}{3}x + \frac{2}{9}$

$\frac{2}{1} = 2\cdot\frac{1}{1}$

$\frac{4x+2}{9}$

$(u\cdot v)' = u'\cdot v + v'\cdot u'$

$u \quad v_1$

$u_1 \quad v$

$$\int u\cdot v'\,dx = u\cdot v - \int u'\cdot v\,dx$$

$u \quad v_1$

$u_1 \quad v$

pr.: $\int \ln x \, dx = x \cdot \ln x - \int \frac{1}{x} \cdot x \, dx = x \ln x - x = \underline{\underline{x \cdot (\ln x - 1) + C}}$

$u = \ln x \quad v' = 1$
$u' = \frac{1}{x} \quad v = x$

pr.: $\int x^3 \ln x \, dx = 3x^2 \ln x - \int \frac{1}{x} \cdot 3x^2 \, dx = 3x^2 \ln x - \int 3x \, dx =$

$u = \ln x \quad v' = x^3$
$u' = \frac{1}{x} \quad v = 3x^2 \qquad = 3x^2 \ln x - \frac{3x^2}{2} =$

pr.: $\int x \ln^2 x \, dx = \ln^2 x \cdot \frac{x^2}{2} - \int \cancel{2} \ln x \cdot \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{\cancel{2}} \, dx = \frac{x^2 \ln^2 x}{2} - \int \ln x \cdot x \, dx =$

$u = \ln^2 x \quad v' = x \qquad u = \ln x \quad v' = x$
$u' = 2 \ln x \cdot \frac{1}{x} \quad v = \frac{x^2}{2} \qquad u' = \frac{1}{x} \quad v = \frac{x^2}{2}$

$= \frac{x^2 \ln^2 x}{2} - \left(\ln x \cdot \frac{x^2}{2} - \cancel{\int \frac{1}{x} \cdot \frac{x^2}{2} \, dx} \right) = \frac{x^2 \ln x}{2} - \left(\frac{x^2 \ln x}{2} - \frac{x^3}{6} \right) =$

$= \cancel{\frac{x^2 \ln x}{2}} \quad \cancel{x^2 \ln^3 x \; x^2 \ln x}$

$= \frac{x^2 \ln^2 x}{2} - \int x \ln x \, dx = \frac{x^2 \ln^2 x}{2} - \left(\ln x \, \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx \right) =$

$u = \ln x \quad v' = x$
$u' = \frac{1}{x} \quad v = \frac{x^2}{2}$

$= \frac{x^2 \ln^2 x}{x} - \left(\ln x \, \frac{x^2}{2} - \int \frac{x}{2} \, dx \right) = \frac{x^2 \ln^2 x}{2} - \left(\frac{x^2 \ln x}{2} - \frac{x^2}{4} \right) =$

$= \frac{x^2 \ln^2 x}{2} - \frac{x^2 \ln x}{2} + \frac{x^2}{4} + C = \underline{\underline{x^2 \cdot \left(\frac{\ln^2 x - \ln x}{2} + \frac{1}{4} \right) + C}}$

pr:

$$\frac{7x+3}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{2}{x+1} + \frac{5}{x-1}$$

$1; -1$ $\quad A(x-1) + B(x+1) = 7x+3$

$x_1 = 1:$ $\quad 0 + 2B = 10$, $\quad B = 5$

$x_2 = -1:$ $\quad -2A = -7+3$, $\quad A = 2$

pr: $\int x^2 \cdot e^x \, dx = x^2 \cdot e^x - \int 2x \cdot e^x \, dx = x^2 e^x - 2\int x \cdot e^x \, dx =$

$u = x^2 \quad v' = e^x$
$u' = 2x \quad v = e^x$

$u = x \quad v' = e^x$
$u' = 1 \quad v = e^x$

$= x^2 e^x - 2 \cdot \left(x \cdot e^x - \int 1 \cdot e^x \, dx\right) = \underline{\underline{x^2 e^x - 2x e^x - 2e^x + C}}$

pr: $\int x^3 \cdot \ln x \, dx = \ln x \cdot 3x^2 - \int \frac{1}{x} \cdot 3x^2 \, dx = 3x^2 \ln x - \int 3x \, dx =$

$u = \ln x \quad v' = x^3$
$u' = \frac{1}{x} \quad v = 3x^2$

$= 3x^2 \ln x - 3\,\frac{x^2}{2} = 3x^2\left(\ln x - \frac{1}{2}\right)$

~~$u = x^3 \quad v'$~~
~~$u' = 3x^2 \quad v =$~~

pr: $\int x \ln x \, dx = \ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx = \frac{x^2 \ln x}{2} - \int \frac{x}{2} dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} =$

$u = \ln x \quad v' = x$
$u' = \frac{1}{x} \quad v = \frac{x^2}{2}$

$= \underline{\underline{\frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) + C}}$

př: $\int x^3 \cdot \ln x \, dx = \ln x \cdot \frac{x^4}{4} - \int \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{4}\,3}}{4} dx = \frac{x^4 \ln x}{4} - \int \frac{x^3}{4} dx =$

$u = \ln x \quad v' = x^3$
$u' = \frac{1}{x} \quad v = \frac{x^4}{4}$

$= \frac{x^4 \ln x}{4} - \frac{1}{4} \int x^3 dx = \frac{x^4 \ln x}{4} - \frac{1}{4} \cdot \frac{x^4}{4} =$

$= \frac{4x^4 \ln x - x^4}{16} = \frac{3x^4 \ln x}{16} + C$

př: $\int x^2 \cdot e^x \, dx = x^2 e^x - \int 2x \cdot e^x \, dx = x^2 e^x - 2 \cdot \int x \cdot e^x \, dx = x^2 \cdot e^x - 2 \cdot (x e^x - \int e^x dx)$

$u = x^2 \quad v' = e^x$
$u' = 2x \quad v = e^x$

$u = x \quad v' = e^x$
$u' = 1 \quad v = e^x$

$= x^2 e^x - 2(x e^x - e^x) = x^2 e^x - 2x e^x - 2e^x = \underline{\underline{e^x (x^2 - 2x - 2) + C}}$

př: $\frac{7x}{6x^2 - x - 2} = \frac{A}{3x-2} + \frac{B}{2x+1} = \frac{2}{3x-2} + \frac{1}{2x+1}$

$x_1 = \frac{2}{3}; x_2 = -\frac{1}{2}$

$(3x - 2)(2x+1)$

$A(2x+1) + B(3x-2) = 7x$

$x_1 = \frac{2}{3}$:

$A\left(\frac{4+3}{3}\right) = \frac{14}{3}$

$A = 2$

$x_2 = -\frac{1}{2}$:

$B\left(-\frac{3-4}{2}\right) = -\frac{7}{2}$

$B = 1$

př: $\int x \cdot \ln^2 x \, dx = \ln^2 x \cdot \frac{x^2}{2} - \int \frac{2 \ln x}{x} \cdot \frac{x^2}{2} dx = \frac{x^2 \ln^2 x}{2} - \int \frac{2 \ln x \cdot x^2}{2x} dx$

$u = \ln^2 x \quad \ln^2 x$
$u' = (g^2)' \cdot f'$
$2g \cdot \frac{1}{x}$
$= 2 \ln x \cdot \frac{1}{x}$

$v' = x$
$v = \frac{x^2}{2}$

$= \frac{x^2 \ln^2 x}{2} - \int x \cdot \ln x \, dx = \frac{x^2}{2} \ln^2 x - \frac{x^2}{2}$

blbě

pr: $\int \frac{4x+16}{x^2+4x-12}\,dx = \int \frac{A}{x-2} + \frac{B}{x+6}\,dx = \int \frac{3}{x-2} + \frac{1}{x+6}\,dx$ ~~$= \ln|x-2| + \ln|x+6| + C$~~

$x_1 = 2;\ x_2 = -6$

$(x-2)(x+6)$

$A(x+6) + B(x-2) = 4x+16$

$x_1 = 2:$ $8A = 24$ $A = 3$

$x_2 = -6:$ $-8B = -8$ $B = 1$

$= \int \frac{3}{x-2}\,dx + \ln|x+6| = 3\cdot\int \frac{1}{x-2}\,dx + \ln|x+6| = 3\ln|x-2| + \ln|x+6| + C$

pr: $\int x \cdot \sin x\,dx = -x\cos x - \int -1\cos x\,dx = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C$

$u = x$ $\quad v' = \sin x$

$u' = 1$ $\quad v = -\cos x$

pr: $\frac{-9x-3}{x^3+2x^2-3x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+3} = \frac{1}{x} + \frac{-3}{x-1} + \frac{2}{x+3}$

$x(x^2+2x-3)$

$(x-1)(x+3)$

$A(x-1)(x+3) + B\cdot x\cdot(x+3) + C\cdot x\cdot(x-1) = -9x-3$

$x_1 = 0:$ $-1\cdot 3A + 0 + 0 = -3$ $A = 1$

$x_2 = 1:$ $0 + 4B + 0 = -12$ $B = -3$

$x_3 = -3:$ $0 + 0 + -3C\cdot(-4) = 27-3$ $12C = 24$ $C = 2$

$\int \frac{1}{x} + \frac{-3}{x-1} + \frac{2}{x+3}\,dx = \ln|x| - 3\ln|x-1| + 2\ln|x+3|$

$pr: \int x \cdot \ln^2 x \, dx =$

$\cancel{u = \ln^2 x \quad v'}$
$\cancel{u' = 2 \quad v}$ to je sračka.

$\cancel{\int e^x \cdot \sin x \, dx = \sin x \, e^x - \int \cos x \, e^x \, dx = \sin x \, e^x - \sin x \, e^x}$

$\cancel{u = \sin x \quad v' = e^x}$
$\cancel{u' = \cos x \quad v = e^x}$

$\cancel{u = e^x \quad v' = \sin x}$
$\cancel{u' = e^x \quad v = -\cos x}$
$\cancel{= -e^x \cos x - \int e^x (-\cos x) dx =}$

$\cancel{= -e^x \cos x + \cos x \, e^x - \int \sin x \, e^x \, dx =}$

$u = -\cos x \quad e^x$
$\cancel{u' = \sin x} \quad e^x$

$pr:$

$\int e^x \cdot \sin x \, dx = -e^x \cdot \cos x - \int e^x \cdot (-\cos x) \, dx = -e^x \cdot \cos x + \int e^x \cos x \, dx =$

$u = e^x \quad v' = \sin x \qquad u = e^x \quad v' = \cos x$
$u' = e^x \quad v = -\cos x \qquad u' = e^x \quad v = \sin x$

$= -e^x \cdot \cos x + e^x \cdot \sin x - \int e^x \cdot \sin x \, dx = e^x \cdot \cos x + e^x \cdot \sin x + e^x \cdot (+\cos x) =$

$= e^x (-\cos x + \cos x + \sin x$

pr:

$\int x \sin x \, dx = x(-\cos x) - \int 1 \cdot (-\cos x) \, dx = -$

$u = x \quad v' = \sin x$

$u' = 1 \quad v = -\cos x$

$= -x \cos x + \sin x + C$

$\sin \to -\cos \qquad \cos \to \sin$

$-\sin \to \cos \qquad -\cos \to -\sin$

pr:

$\int x^3 \cdot \ln x \, dx = \ln x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx = \frac{(\ln x) \, x^4}{4} - \int \frac{x^{\cancel{4}3}}{4\cancel{x}} dx$

$u = \ln x \quad v' = x^3$

$u' = \frac{1}{x} \quad v = \frac{x^4}{4}$

$= \frac{\ln(x) \cdot x^4}{4} - \int \frac{x^3}{4} dx = \frac{\ln(x) \cdot x^4}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$

$= \frac{x^4 \cdot \ln x}{4} - \frac{x^4}{16} = \frac{4(x^4 \cdot \ln x) - x^4}{16} =$

$= 4x^4 \ln x - x^4 = \frac{x^4 (4 \ln x - 1)}{16} + C$

pr:

$\int \ln x \, dx = x \ln x - \int \frac{1}{x} \cdot x \, dx = x \ln x - x + C = x \cdot (\ln x - 1)$

$u = \ln x \quad v' = 1$

$u' = \frac{1}{x} \quad v = x$

pr:

$\int x^2 \cdot e^x dx = x^2 \cdot e^x - \int 2x \, e^x dx = x^2 e^x - 2 \frac{x^2}{2} \cdot e^x$

$u = x^2 \quad v' = e^x$

$u' = 2x \quad v = e^x$