A

BRAC UNIVERSITY
Department of Computer Science and Engineering

Examination: Mid Semester Exam | Semester: Summer 2023
Duration: 1 Hour 30 Minutes | Full Marks: 40

CSE 221: Algorithms

Answer the following questions.
Figures in the right margin indicate marks.

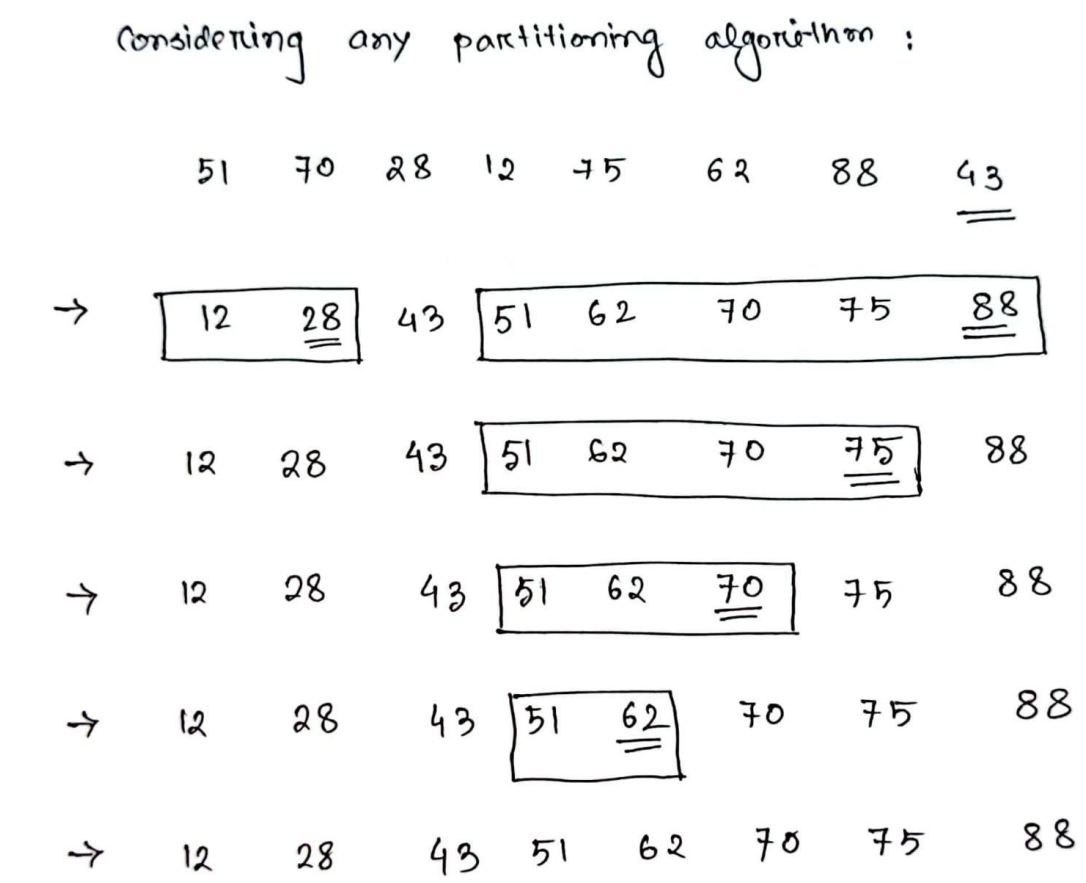
Name:	ID:	Section:

1	**a. CO2**	**Find** the time-complexity of the following task in terms of number of students. You are given a student attendance sheet. Each student has a unique integer ID. You have to count the number of students having an even number as ID. The list is sorted but the IDs are not necessarily consecutive. So you check each ID one by one. **Solution:** If number of students is N, then **O(N)**	**03**

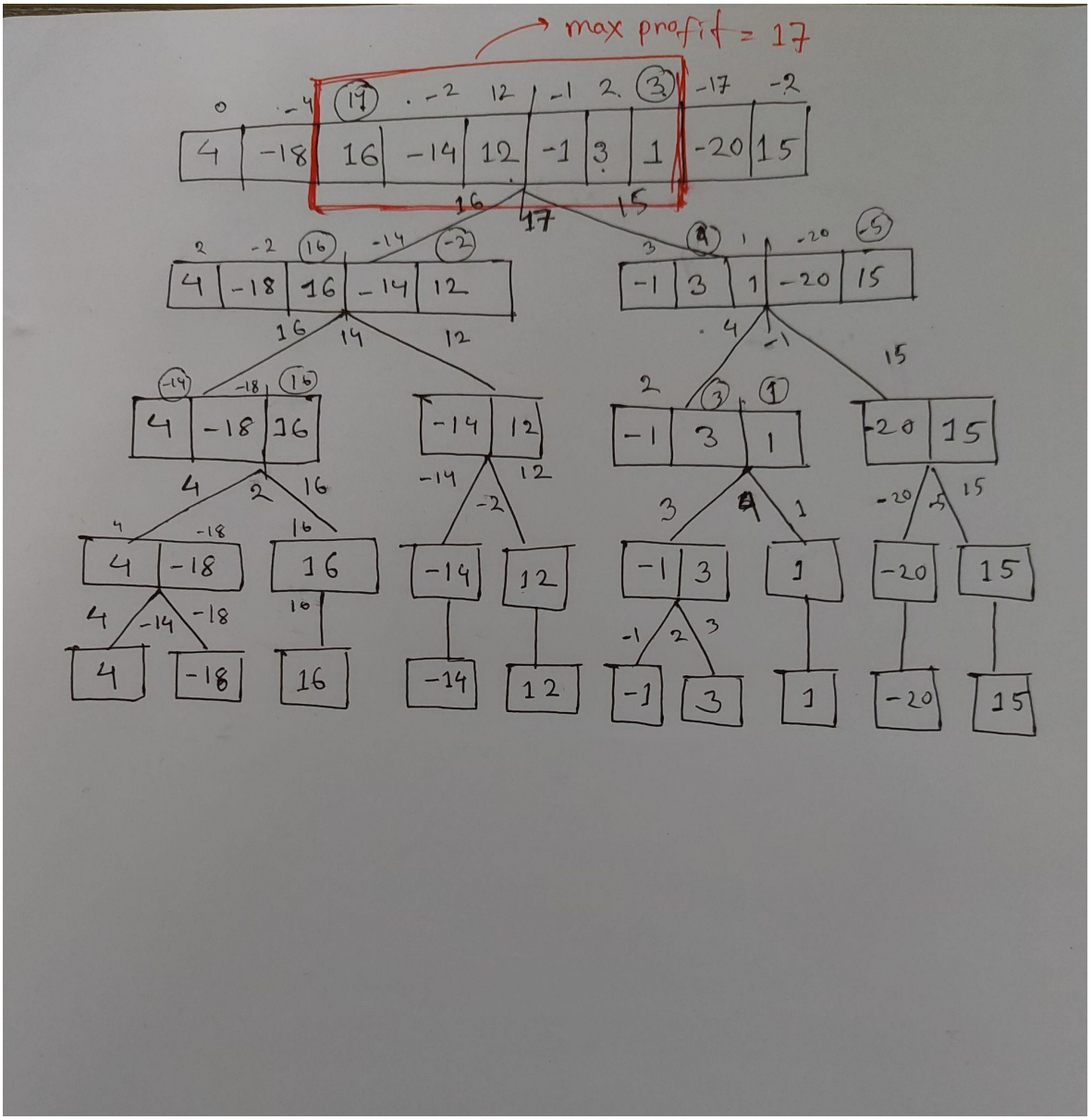
	b. CO2	**Write** the asymptotic time complexity of the following code snippet. Show your works/reasoning.
		
		`1. for i in range (1,n)`
		`2.  j= 1`
		`3.  while j < i*i`
		`4.  j= j+1`
		
		Solution:
		T(N) = 1^2 + 2^2 + 3^2 + … + N^2 = N(N+1)(2N+1)/6
		= **O(N^3)**
		The explanation that the second (nested) loop will run for n*n times in the last iteration, can also be accepted.
			02

	c. CO2	**Express** the following running time $T(n)$ with an asymptotic bound. $T(n) = 8T(\frac{n}{4}) + n\sqrt{n}$ Any method is acceptable as long as you show calculations. **Solution:** a= 8, b= 4, c_{crit} ($\log_b a$) = 3/2, c (degree of n) = 1.5; so it falls into the case #2: f(n) = Θ ($n^{c\_crit} * \log^k n$) where k = 0 So, T(n) = Θ ($n^{c\_crit} * \log^{k+1} n$) = **Θ($n^{3/2} * \log n$)**	**05**

2	**a.** **CO3**	You are given an array containing N distinct integers in ascending order. You have to find out the first missing number from the increasing sequence. For example: [3, 4, 5, 6, 9, 10]. The first missing number in this sequence is 7. [Explanation : If A_i and A_{i+1} are two consecutive integers from the list and their difference $(A_{i+1} - A_i)$ is more than 1, then we say A_i+1 is a missing number.] Now that you have understood the problem completely, you are trying to solve it efficiently. Your friend, Jack, said that by iterating through all the numbers from the beginning, you can find the first missing number in linear time $[O(N)]$. But, you need a more efficient one. Now, **answer** the questions below. a) If N = 9 and the maximum number (obviously the last-indexed one) in the array is 15, which one among 6, 8, 10 might be the first number (obviously the minimum one) in the array? b) Explain why you discarded the other two numbers in question no. (a). c) Assume N = 7 and the minimum and maximum numbers are 2 and 12 respectively. You found the middle-index value to be 5. On which side (left/right) of 5, do you expect to find the first missing number? Only mention 'left' or 'right'. d) Explain why you discarded the other half of the array in question no. (c). e) Assume N = 7 and the minimum and maximum numbers are 8 and 19 respectively. The missing elements are 11, 13, 14, 17, 18. What will be the value of the middle-index element? Determine the absolute difference between the middle-index value and the maximum number. **Solutions:** a) 6 b) If we had chosen 8 or 10, then there should have been at most N = (15-8+1) = 8 or 6 elements. But, here N = 9. c) right. d) Because, the left side of 5 should have at least 3 distinct integers. The first number is 2, the other two candidates are 3,4. So, there is no chance of a missing number existing on the left of 5. e) From the given information, the array is supposed to be : [8 9 10 12 15 16 19]. So, the answers are : 12 and 7	**01** **01** **01** **01** **02**
	b. **CO1**	**Simulate** the Quick Sort algorithm by showing the steps on the following array, always choosing the last element as the pivot element. You can partition the array at each step choosing any suitable algorithm, *but you do not need to show the partitioning steps*. 51 | 70 | 28 | 12 | 75 | 62 | 88 | 43 **Solution:**	**04**

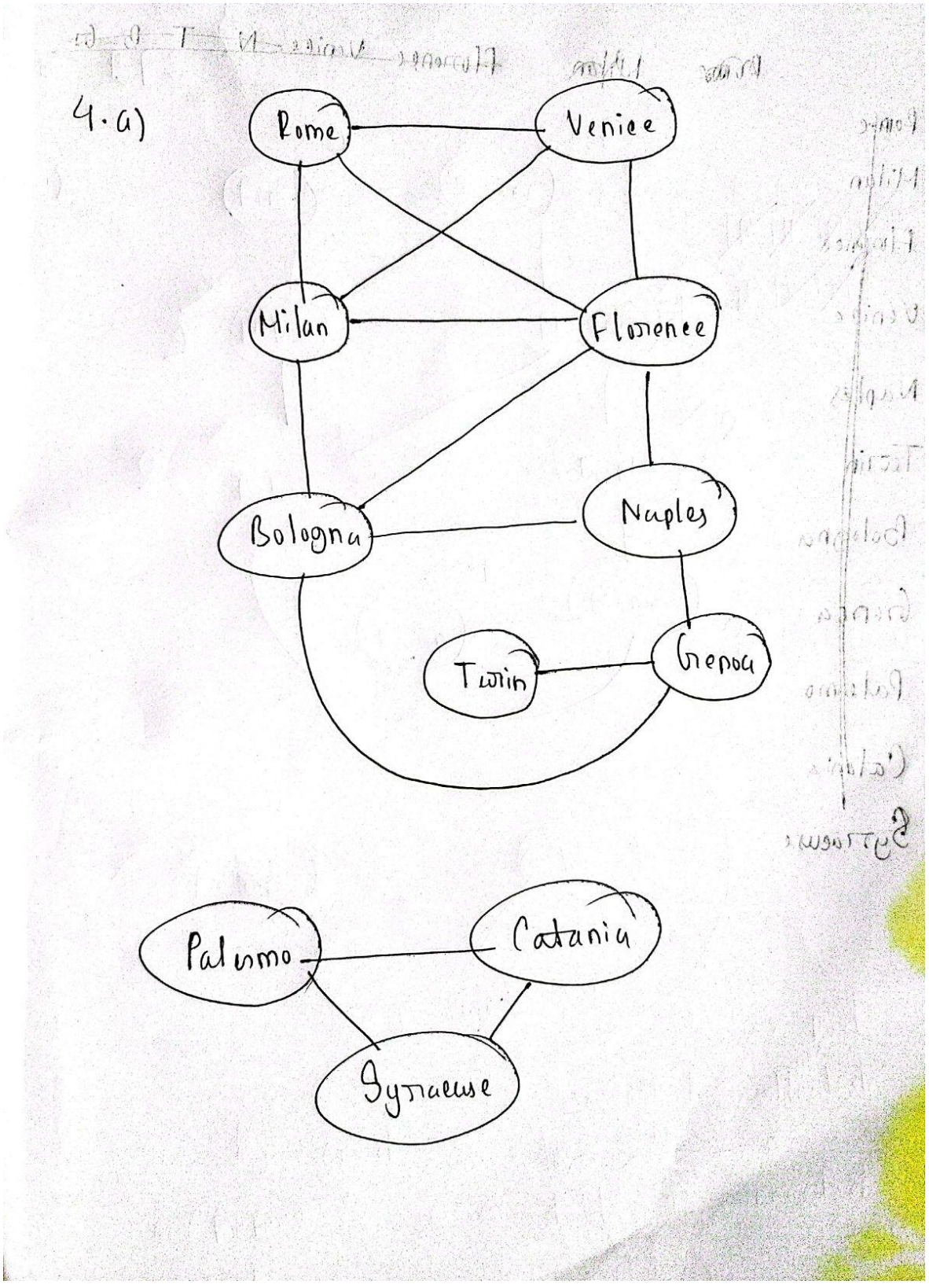


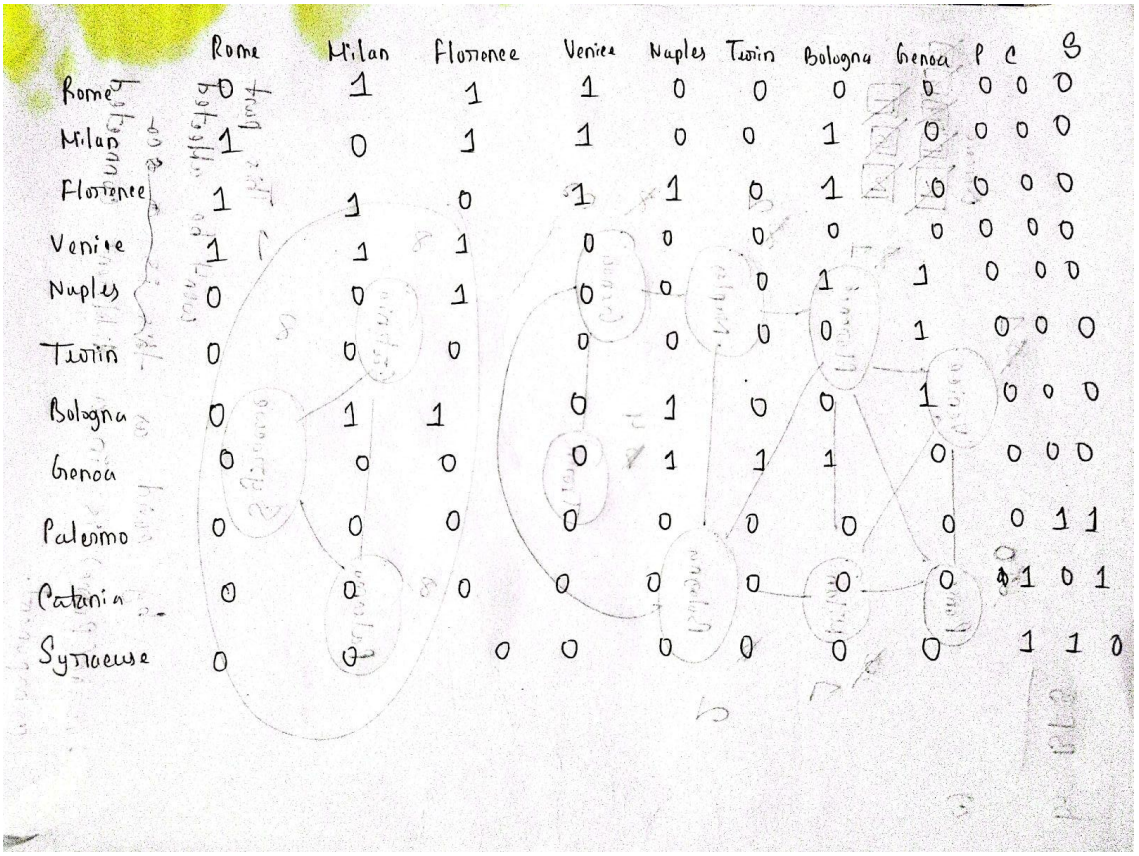
3	**CO1**	Imagine you're a farmer named Sam who grows different types of vegetables in several continuous fields. Each field can grow a specific type of vegetable, and based on the market value of that vegetable, each field will bring a certain profit (positive integer) or loss (negative integer). The profit or loss is estimated and noted for each field. Sam can start farming from any field, but once he starts, he must continue farming the next fields in sequence without skipping any, until he decides to stop, because his tractor can only move to the next adjacent field and cannot skip fields. Sam needs to choose which sequence of fields to farm to maximize his profits. The following array represents the estimated profits or losses for ten consecutive fields on Sam's farm, based on the types of vegetables that can be grown in each field and their respective market values. [4, -18, 16, -14, 12, -1, 3, 1, -20, 15] a) Can you help Sam decide which sequence of fields to farm for maximum profit using an efficient algorithm? What would the profit amount be? **Show** a simulation of your proposed algorithm. You must show which sequence of fields he needs to select in order to achieve this maximum profit. b) **Calculate** the time complexity of your algorithm using proper mathematical logic. An efficient algorithm should have time complexity less than or equal to O($N * log(N)$) where N is the number of fields.	**06** **04**

		Solution:  **b) If Divide and Conquer** $T(n) = 2T(n/2) + \Theta(n)$ Time Complexity : O(nlogn) **If Kadane's Algo:** Time Complexity: O(n)	
4	**CO1**	The cities in Italy are connected in such a way that their adjacency list representation looks as follows- Rome: Milan, Venice, Florence Milan: Rome, Florence, Bologna, Venice Florence: Milan, Naples, Rome, Bologna, Venice Venice: Rome, Milan, Florence Naples: Florence, Bologna, Genoa Turin: Genoa Bologna: Naples, Genoa, Milan, Florence Genoa: Turin, Bologna, Naples	

		Palermo:Catania, Syracuse Catania: Palermo, Syracuse Syracuse: Catania, Palermo Suppose Rome suddenly gets affected with covid-19. The rest of the cities are non-affected. You are asked to find out the time steps the covid virus will take to affect all the non-affected cities if it takes one time step to travel from one city to another. So, one affected city will affect all of its neighbors in one time step. a) **Draw** the graph and create the adjacency matrix for the graph. b) **Mention** the name of the graph traversal algorithm you think is suitable for solving the problem. c) **Simulate** the suitable algorithm to find the time steps needed for each non affected city to be affected. Choose the smallest (in alphabetical order) vertex when there is a choice.	**04** **01** **05**

Solutions:

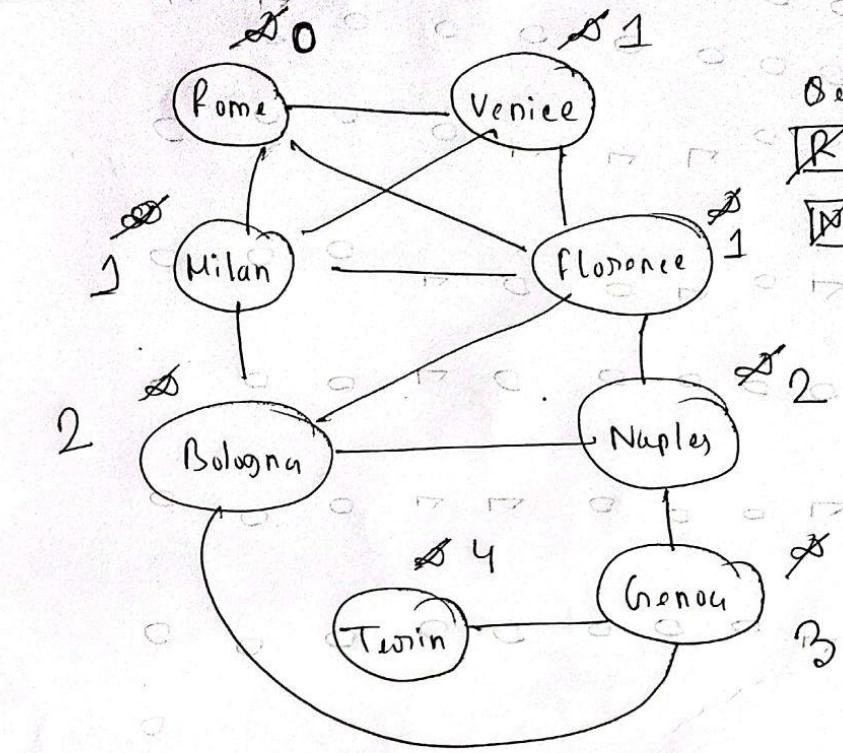

4.a)
Rome
Venice
Milan
Florence
Bologna
Naples
Turin
Genoa
Palermo
Catania
Syracuse



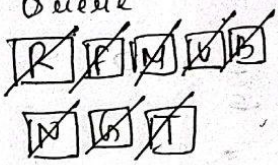
	Rome	Milan	Florrence	Venice	Naples	Turin	Bologna	Genoa	P	C	B
Rome	0	1	1	1	0	0	0	0	0	0	0
Milan	1	0	1	1	0	0	1	0	0	0	0
Florrence	1	1	0	1	1	0	1	0	0	0	0
Venice	1	1	1	0	0	0	0	0	0	0	0
Naples	0	0	1	0	0	0	1	1	0	0	0
Turin	0	0	0	0	0	0	0	1	0	0	0
Bologna	0	1	1	0	1	0	0	1	0	0	0
Genoa	0	0	0	0	1	1	1	0	0	0	0
Palermo	0	0	0	0	0	0	0	0	0	1	1
Catania	0	0	0	0	0	0	0	0	1	0	1
Syracuse	0	0	0	0	0	0	0	0	1	1	0

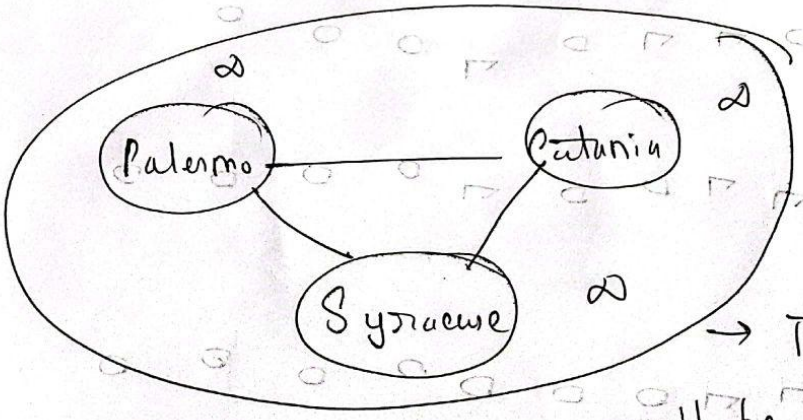
b) BFS

c)



Queue





→ This part won't be affected by covid as ~~this is~~ co-source (Rome) is in a different connected component.

B

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CSE 221: Algorithms

Answer the following questions.
Figures in the right margin indicate marks.

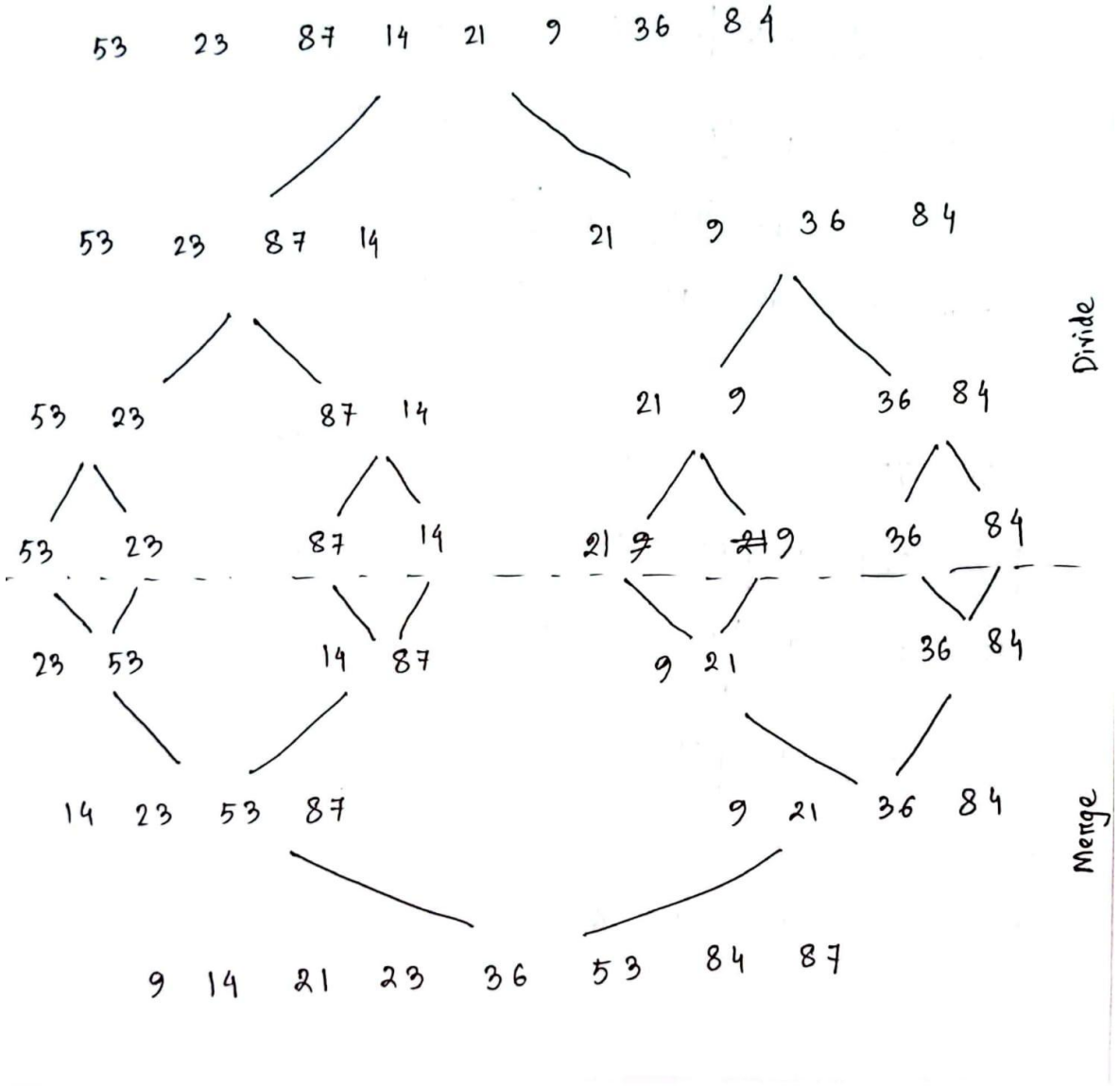
Name:	ID:	Section:

1	**a. CO2**	**Find** the time-complexity of the following task in terms of number of students. You are given a student attendance sheet. Each student has a unique integer ID. You have to count the number of students having an ID which is divisible by 3. The list is sorted but the IDs are not necessarily consecutive. So you check each ID one by one. **Solution:** If number of students is N, then **O(N)**	**03**

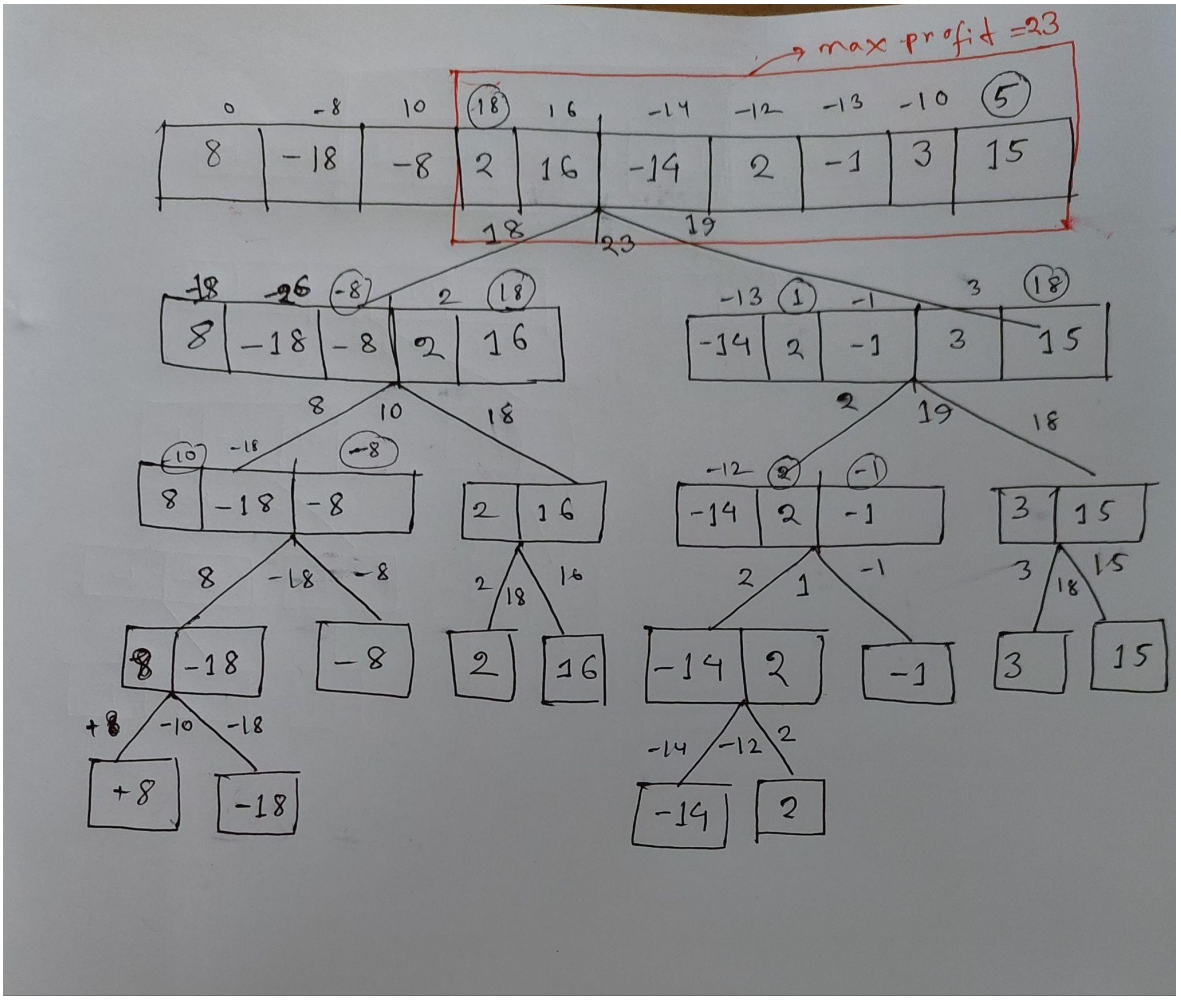
	b. CO2	**Write** the asymptotic time complexity of the following code snippet. Show your works/reasoning.
		
		`1. for i in range (1,n)`
		`2.  j= 1`
		`3.  while j*j < i`
		`4.  j= j+1`
		
		Solution:
		T(N) = 1^0.5 + 2^0.5 + 3^0.5 + … + N^0.5 = $\frac{2}{3}N^{1.5}$ + C [by integrating $\sqrt{x}dx$]
		= **O(N*sqrt(N))**
		The explanation that the second (nested) loop will run for sqrt(n) times in the last iteration, can also be accepted.
			02

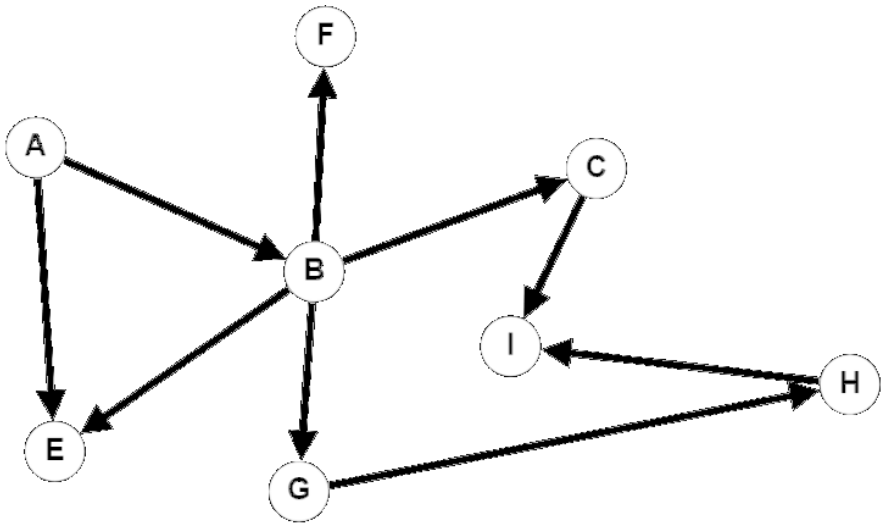
	c. CO2	**Express** the following running time $T(n)$ with an asymptotic bound. $$T(n) = 4T(\frac{n}{4}) + \sqrt{n}$$ Any method is acceptable as long as you show calculations. **Solution:** a= 4, b= 4, c_{crit} ($\log_b a$) = 1, c (degree of n) = 0.5; so it falls into the case **$c < c_{crit}$** So, T(n) = Θ (n^{c_crit}) = **Θ(n)**	**05**

2	**a.** **CO3**	You are given an array containing N distinct integers in descending order. You have to find out the first missing number from the decreasing sequence. For example: [10, 9, 6, 5, 4, 3]. The first missing number in this sequence is 8. [Explanation : If A_i and A_{i+1} are two consecutive integers from the list and their difference $(A_i - A_{i+1})$ is more than 1, then we say A_i-1 is a missing number.] Now that you have understood the problem completely, you are trying to solve it efficiently. Your friend, Jack, said that by iterating through all the numbers from the beginning, you can find the first missing number in linear time $[O(N)]$. But, you need a more efficient one. Now, **answer** the questions below. a) If N = 9 and the maximum number (obviously the first-indexed one) in the array is 17, which one among 8, 10, 12 might be the last number (obviously the minimum one) in the array? b) Explain why you discarded the other two numbers in question no. (a). c) Assume N = 7 and the maximum and minimum numbers are 10 and 3 respectively. You found the middle-index value to be 6. On which side (left/right) of 6, do you expect to find the first missing number? Only mention 'left' or 'right'. d) Explain why you discarded the other half of the array in question no. (c). e) Assume N = 7 and the maximum and minimum numbers are 23 and 12 respectively. The missing elements are 21, 20, 17, 16, 13. What will be the value of the middle-index element? Determine the absolute difference between the middle-index value and the minimum number. **Solution :** a) 8 b) If we had chosen 10 or 12, then there should have been at most N = (17-10+1) = 8 or 6 elements. But, here N = 9. c) left. d) Because, the right side of 6 should have at least 3 distinct integers. The last number is 3, the other two candidates are 5,4. So, there is no chance of a missing number existing on the right of 6. **<u>Update :</u>** As an argument, it is enough to show that, mid = 6, max number = 10, distance > 3, so we should take the left side and discard the right. e) From the given information, the array is supposed to be : [23 22 19 18 15 14 12]. So, the answers are : 18 and 6	**01** **01** **01** **01** **02**
	b. **CO1**	**Draw** the recursion tree of the Merge Sort algorithm while sorting the following array. Show the sorted lists at each step. 53 | 23 | 87 | 14 | 21 | 9 | 36 | 84	**04**



3	**CO1**	John is an amateur poker player who has decided to try his luck at a local casino. The casino allows players to play in rounds, each leading to a win (positive integer) or a loss (negative integer). John keeps track of his net gain or loss after each round. John knows that poker isn't just about luck—it's about making strategic decisions. And one of the critical decisions he has to make is knowing when to stop playing. However, the casino has a rule that once a player starts playing rounds, they must play the subsequent rounds in order without skipping any, until they decide to stop, as skipping rounds might disrupt the flow of the game and disadvantage other players. With this in mind, John wants to find the optimal sequence of rounds where he can maximize his net winnings. The following array represents the net gain or loss John experiences in ten rounds of poker at the casino. [8, -18, -8, 2, 16, -14, 2, -1, 3, 15] a) Find an efficient algorithm that will help John to figure out which sequence of rounds he should play to maximize his winnings. What would be the exact profit amount? **Show** a	**06**

		simulation of your proposed algorithm. You must show which sequence of rounds he needs to select in order to achieve this maximum profit. b) **Calculate** the time complexity of your algorithm using proper mathematical logic. An efficient algorithm should have time complexity less than or equal to O($N * log(N)$) where N is the number of fields.	**04**
		 **b) If Divide and Conquer** T(n) = 2T(n/2) + Θ(n) Time Complexity : O(nlogn**)** **If Kadane's Algo:** Time Complexity: O(n)	

4	**CO1**	 (a) **Apply** the DFS algorithm on the following graph starting from vertex **A**. Determine start time, finish time, and parent for each of the vertices. Choose the smallest (in alphabetical order) vertex when there is a choice. (b) For each edge, **determine** whether it is a tree edge, forward edge, back edge or cross edge. **Solutions:** **(a)** **(b)** **(A,E)** is a forward edge **(H,I)** is a cross edge All the rest are tree edges	**06** **04**

vertex	parent	start	finish
A	–	1	16
B	A	2	15
C	B	3	6
E	B	7	8
F	B	9	10
G	B	11	14
H	G	12	13
I	C	4	5