Date:......................

Code:

```
sum1 = 0
for (i=1 ; i<=n; i++)
        for (j=1 ; j<=n; j++)
                sum1++
```

Values of i	num of times inner loop runs
1	n
2	n
3	n
⋮	
n	$+\ n$
	$n \times n$
	$= n^2$

Time complexity $O(n^2)$

Date:......................

Code:

```
sum2 = 0
for (i=1; i<=n; i++)
    for (j=1; j<=i; j++)
        sum2++;
```

Values of i	number of times the inner loop will run
1	1
2	2
3	3
⋮	
n	+ n
	$1+2+3+\dots+n$

$$= \frac{n(n+1)}{2}$$

$$= \frac{n^2}{2} + \frac{n}{2}$$

Time complexity: $O(n^2)$

Date:....................

Code:

```
sum1 = 0
for (k=1; k<=n; k*=2)
    for (j=1; j<=n; j++)
        sum1++;
```

Values of k		number of times inner loop runs
1	$= 2^0$	n
2	$= 2^1$	n
4	$= 2^2$	n
8	$= 2^3$	$\vdots$
16	$= 2^4$	$\vdots$
$\vdots$	$\vdots$	$\vdots$
n	$= 2^i$	n

i terms

$+$

$= n\log_2 n$

$n = 2^i$

$\Rightarrow \log_2 n = \log_2 2^i$

$\therefore i = \log_2 n$

$O(n\log_2 n)$

Date:.......................

Code :

```
sum2 = 0
for (k=1; k<=n; k*=2)
    for(j=1; j<=k; j++)
        sum2++;
```

Values of k	Num of times inner loop runs
1	$1 = 2^0$
2	$2 = 2^1$
4	$4 = 2^2$
8	$8 = 2^3$
	$\vdots$
n	$n = 2^{\log_2 n}$

$$2^0 + 2^1 + 2^2 + \dots + 2^{\log_2 n}$$

$$= \frac{a(r^{n+1} - 1)}{r - 1}$$

$$= \frac{2^{\log_2 n + 1} - 1}{2 - 1}$$

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$= 2 \cdot 2^{\log_2 n} - 1$

$= 2n - 1$

Time complexity : $O(n)$

Geometric series sum

☆ $S_n = \dfrac{a(r^{n+1} - 1)}{r - 1}$

a = 1st term

r = common ratio

n = number of terms

☆ $1 + 2 + 3 + \dots + n = \dfrac{n(n+1)}{2}$

☆ $1^2 + 2^2 + 3^2 + \dots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

☆ $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\dfrac{n(n+1)}{2}\right)^2$