

**CSE 330: Spring 2024**  
**Assignment-4 [CO4]**  
**Total Marks: 30**

1. Consider a function  $f(x) = x^3 + x^2 - 4x - 4$ .

(a) (1 marks) Compute the minimum number of iterations required to find the root within the interval  $[-10, -1.5]$  if the machine epsilon(error bound) is  $1 \times 10^{-2}$ .

(b) (4 marks) Show 5 iterations using the Bisection Method to find the root of the above function within the interval  $[-10, -1.5]$ .

(c) (2 marks) State the exact roots of  $f(x)$  and construct two different fixed point functions  $g(x)$  such that  $f(x) = 0$ .

(d) (3 marks) Compute the convergence rate of each fixed point function  $g(x)$  obtained in the previous part, and state which root it is converging to or diverging.

2. Consider the following function:  $f(x) = xe^x - 1$ .

(a) (4 marks) Find solution of  $f(x) = 0$  up to 5 iterations using Newton's method starting with  $x_0 = 1.5$ . Keep up to four significant figures.

(b) (4 marks) Consider the fixed point function,  $g(x) = \frac{2x+1}{\sqrt{x+1}}$ . Show that to be super-linearly convergent, the root must satisfy  $x^* = \frac{-3}{2}$ .

3. (a) (4 marks) Consider a cubic function,  $f(x) = 2x^3 - 2x - 5$ . Compute a **superlinearly convergent fixed point function  $g(x)$**  for the given function  $f(x)$  using **Newton's method**.

(b) (4 marks) Consider the function  $f(x) = \cos(2x) - \sin(x)$ . Compute the root of the function using Newton's method with Aitken's acceleration and starting point,  $x_0 = 0$ . Consider up to five decimal places. **[Error bound is  $1 \times 10^{-3}$ ]** [You must use **Radian** Mode of calculator]

4. In the interval  $[-4, 4]$ , the function,  $f(x) = x^3 - x^2 - 3x + 2$ , has three roots at 2.000, 0.6180 and -1.618; and **two turning points at  $x = -0.721$  and  $x = 1.387$** .

a) (1 mark) Explain why it might not be possible to find the root of the given function and interval using the interval bisection method.

b) (3 marks) Write down the correct intervals, including the root it contains, such that the problem in the previous part can be avoided.