Aitken's Acceleration

Aitken's acceleration works on top of other method (bisection, fixed point or newton raphson). Aitken's acceleration can not work on its own

Aitken's acceleration speeds up the process so the convergence happens even quicker ie we find the roots more quickly / early.

$$\chi_{K+2} = \chi_{K} - \frac{\left(\chi_{K+1} - \chi_{K}\right)^{2}}{\chi_{K+2} - 2\chi_{K+1} + \chi_{K}}$$

Every 3rd iteration is Aitken's Acceleration

Aitken's Acceleration

Usually Aitken's Acceleration runs on Newton Raphson Method (NR). The following is the format used by

No NR NR NR

Aitken's Acceleration.

$$\frac{1}{2} \frac{NR}{NR} \times_{3} \frac{NR}{NR} \times_{4}$$
Aitken's Accelerate
$$\frac{1}{2} \frac{1}{NR} \times_{4}$$

$$\frac{1}{2} \frac{1}{NR} \times_{4}$$
Aitken's Accelerate
$$\frac{1}{2} \frac{1}{NR} \times_{6}$$

$$\frac{1}{2} \frac{1}{NR} \times_{6}$$

$$\frac{1}{2} \frac{1}{NR} \times_{6}$$

$$\frac{1}{2} \frac{1}{NR} \times_{6}$$

$$\frac{1}{2} \frac{1}{NR} \times_{6}$$
Accelerate
$$\frac{1}{2} \frac{1}{NR} \times_{6}$$

$$\frac{1}$$

$$\chi_{(+)} = \chi_{(+)} + \frac{f(\chi_{(+)})}{f'(\chi_{(+)})}$$

$$\chi_{(+)} = 0.7689 - \frac{(\chi_{(+)})^2 - \chi_{(+)}}{2(\chi_{(+)})^2 - 2e^{-(\chi_{(+)})}} + \frac{e^{-2\chi_{(+)}}}{2(\chi_{(+)})^2 - 2e^{-(\chi_{(+)})}}$$

$$= 0.7689 - \frac{0.038}{2(\chi_{(+)})^2 - 2e^{-(\chi_{(+)})}} = 0.6648$$

$$|\chi_{(+)}| = \chi_{(+)} + \frac{f(\chi_{(+)})^2}{2(\chi_{(+)})^2 - 2e^{-(\chi_{(+)})}}$$

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$$= 0.7689 - \frac{0.038}{2(\chi_{(+)})^2 - 2e^{-(\chi_{(+)})}}$$

$$= 0.6648$$

$$|\chi_{(+)}| = \chi_{(+)} + \frac{f(\chi_{(+)})^2}{2(\chi_{(+)})^2 - 2e^{-(\chi_{(+)})}}$$

$$= \chi_{(+)} + \chi_{(+)} + \frac{f(\chi_{(+)})^2}{2(\chi_{(+)})^2 - 2e^{-(\chi_{(+)})}}$$

$$= \chi_{(+)} + \chi_{(+)} + \chi_{(+)} + \frac{f(\chi_{(+)})^2}{2(\chi_{(+)})^2 - 2e^{-(\chi_{(+)})}}$$

$$= \chi_{(+)} + \chi_$$

0.7689

0.6646

0.3995

0.0933

0.02253

 $N_{\mathfrak{a}}$

 N_0

 $\partial \mathcal{N}$

 $1 - \frac{0.3995}{1.7293} = 0.7689$

Aitken's Acceleration (first time)

Here for the value of xwe are using
$$\hat{\chi}_2$$
 not $\hat{\chi}_2$ $\hat{\chi}_3$ $\hat{\chi}_4$ $\hat{\chi}_5$ $\hat{\chi}_6$ $\hat{\chi}_6$

Value of k is marked in blue

Aitken's Acceleration (First Time) - Iteration 3

ව

2

 \mathcal{X}_{O}

N,

N2

$$= 1 - \frac{\left(0.7689 - 1\right)^{2}}{\left[8.6646 - 2(0.7689) + 1\right]}$$

$$= 0.5788$$

We are using $\hat{\chi}_{2}$ instead of χ_{2} instead of χ_{2} to accelerate the process.

$$\hat{\chi}_{2+1} = \chi_{1} + \frac{1}{2} + \frac{1$$

= 0.5736

Iteration 5

Same 35 iteration 1 and 2.

$\chi_{K+2} = \chi_4$

Aitken's Acceleration (Second Time)

Value of k is marked in blue
$$\frac{1}{2} + 2 = \frac{1}{2} - \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{2}$$
Value of k is marked in blue

 $(0.5730 - 0.5788)^{2}$

0.5701 -2(0.5730)+0.5788

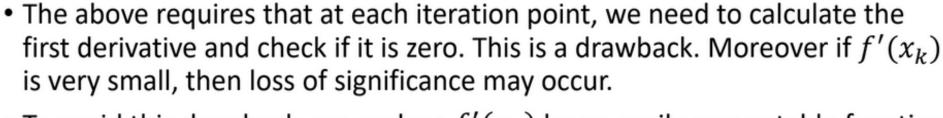
0.5672

= 8.5788 -

Secant

• Recall the Newton's method of finding the root. The iteration is: $f(x_k)$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}.$$



- To avoid this drawback, we replace f'(x_k) by an easily computable function g_k which is approximately equal to f'(x_k). This technique is known as the Quasi-Newton Method.
 In particular if we choose g_k to be the backward difference between the
- nearest iteration points, then it is called the Secant Method.

