# LU Decomposition

$$x_1 + 2x_2 + x_3 = 0$$
$$x_1 - 2x_2 + 2x_3 = 4$$
$$2x_1 + 12x_2 - 2x_3 = 4$$

$$\rightarrow A^{(1)} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{bmatrix}$$

$$R_2 = R_2 - \underbrace{\left(\tfrac{1}{1}\right)}_{m_{21}=1} R_1$$

$$R_3 = R_3 - \underbrace{\left(\tfrac{2}{1}\right)}_{m_{31}=2} R_1$$

For $F^{(1)}$ find:

1) Find the multiplier for every element under Row 1, Column 1 element
2) Then change the sign and add it to F(1) according to the element position in A(1)
3) The diagonal of F(1) will be 1. And all other elements will be zero.

$$F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Notice how the sign has changed in the multiplier

$$A^{(2)} = F^{(1)} \times A^{(1)}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & 12 & -2 \end{bmatrix}$$

$$A^{(2)} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 8 & -4 \end{bmatrix}$$

This must be 0

For F(2) find:

1) Find the multiplier of the elements under row 2, column 2.
2) Then change the sign and add it to F(2) according to the element position in A(2)
3) The diagonal of F(2) will be 1. And all other elements will be zero.

$$R_3 = R_3 - \underbrace{\left(\tfrac{8}{-4}\right)}_{m_{23}=-2} R_2$$

$$F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A^{(3)} = F^{(2)} \times A^{(2)}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 8 & -4 \end{bmatrix}$$

$$U = A^{(3)} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

At this point, you MUST get a U matrix.

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$$Ax = b$$
$$\underbrace{LUx}_{y} = b$$
$$Ly = b$$
$$y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$Ux = y$$
$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

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To make L:

1) Directly place the multiplier in their respective position
2) Diagonal elements are 1. All other are zero.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix}$$

$$Ly = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix}$$

① $1y_1 = 0$

$y_1 = 0$

② $1y_1 + 1y_2 = 4$

$1(0) + 1y_2 = 4$

$y_2 = 4$

③ $2y_1 - 2y_2 + 1y_3 = 4$

$2(0) - 2(4) + 1y_3 = 4$

$y_3 = 12$

$$y = \begin{pmatrix} 0 \\ 4 \\ 12 \end{pmatrix}$$

$$Ux = y$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 12 \end{pmatrix}$$

① $-2x_3 = 12$

$x_3 = -6$

② $-4x_2 + x_3 = 4$

$-4x_2 + (-6) = 4$

$x_2 = \frac{4+6}{-4}$

$x_2 = -2.5$

③ $x_1 = 11$

$$x = \begin{pmatrix} 11 \\ -2.5 \\ -6 \end{pmatrix}$$

# Summary

$A^{(1)}$

$\downarrow$

$F^{(1)}$

$\downarrow$

$A^{(2)} = F^{(1)} \times A^{(1)}$

$\downarrow$

$F^{(3)}$

$\downarrow$

$U \rightarrow A^{(3)} = F^{(2)} \times A^{(2)}$

$Ly = b$ → Find

$Ux = y$ → Find

## Show Det(L) always is 1

$$\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$

Determinant

$$1\Big[(1\times1) - \underbrace{(c\times0)}_{\text{Always } 0}\Big] - \underbrace{0\Big[(a\times1) - (b\times0)\Big]}_{\text{Always } 0} + \underbrace{0\Big[(a\times c) - (b\times1)\Big]}_{\text{Always } 0}$$

$$= 1[1]$$

$$= 1. \quad (\text{shown}).$$