Integration

1. Find the exact integration for the following expression for the interval [0,2]

$$f(x) = e^{2x}$$

$$I(f) = \int_0^2 e^{2x}$$

$$= \left[\frac{e^{2x}}{2}\right]_0^2$$

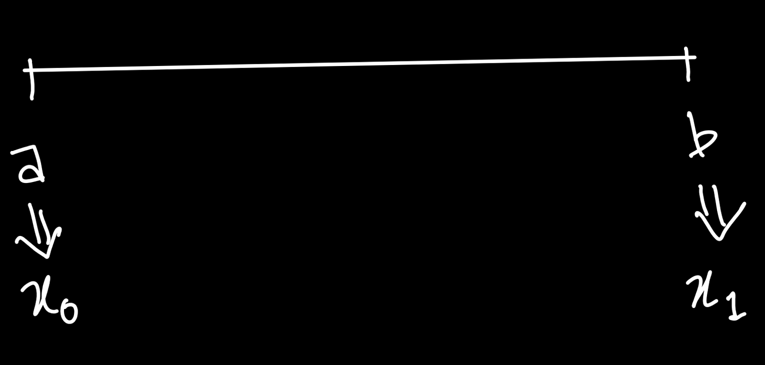
$$= \left[\frac{e^{2(2)}}{2} - \frac{e^{2(0)}}{2}\right]$$

$$= 26.80 \quad \leftarrow \text{Actual}$$

2. Closed Newton Cotes formulae with degree (n) = 1 (Trapezium Rule)

degree $(n) = 1$

nodes $= 2 \rightarrow \{x_0, x_1\}$



Trapezium Rule Formulae:

$$\boxed{I_1(f) = \frac{b-a}{2}\left[f(a) + f(b)\right]}$$

$f(x) = e^{2x}$. Interval : [0,2]

$$I_1(f) = \frac{b-a}{2}\left[f(a) + f(b)\right]$$

$$= \frac{2-0}{2}\left[e^{2(0)} + e^{2(2)}\right]$$

$$= 1\left[55.598\right]$$

$$= 55.598$$

Absolute Error = $|\text{Actual Value - Approximate Value}|$

$$= |26.80 - 55.598|$$

$$= 28.798$$

Relative Error = $\dfrac{|\text{Actual Value - Approximate Value}|}{|\text{Actual Value}|}$

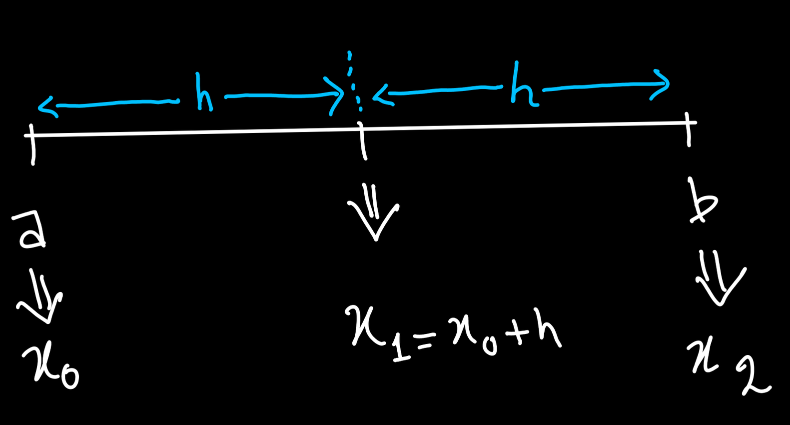
$$= \frac{|26.80 - 55.598|}{|26.80|}$$

$$= 1.074$$

3. Closed Newton Cotes formulae with degree (n) = 2 (Simpson Rule)

degree $(n) = 2$

nodes $= 3 \rightarrow \{x_0, x_1, x_2\}$



$$m = x_1 = \frac{a+b}{2}$$

Simpson Rule Formulae:

$$\boxed{I_2(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]}$$

$f(x) = e^{2x}$. Interval : [0,2]

$$f\left(\frac{a+b}{2}\right) = f\left(\frac{0+2}{2}\right) = f(1)$$

$$I_2(f) = \frac{2-0}{6}\left[e^{2(0)} + 4e^{2(1)} + e^{2(2)}\right]$$

$$= \frac{2}{6}\left[85.154\right]$$

$$= 28.384$$

Absolute Error = $|\text{Actual Value - Approximate Value}|$

$$= |26.80 - 28.384|$$

$$= 1.584$$

Relative Error = $\dfrac{|\text{Actual Value - Approximate Value}|}{|\text{Actual Value}|}$

$$= \frac{|26.80 - 28.384|}{|26.80|}$$

$$= 0.059.$$

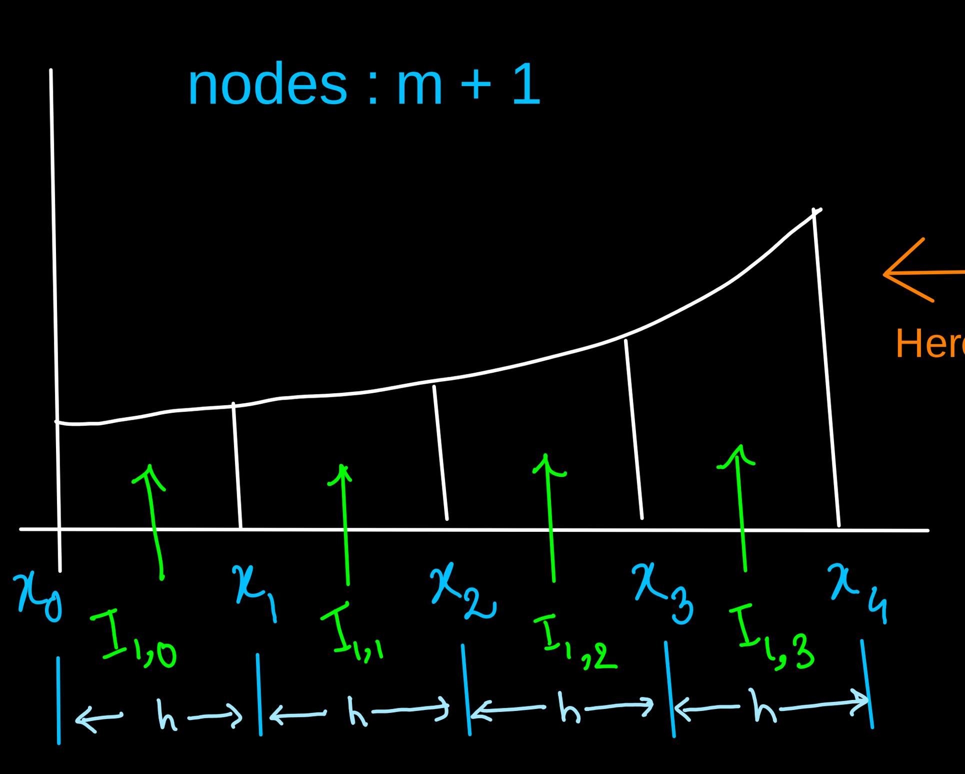
4. Composite Newton Cotes

$[a,b]$ interval split into m sub-intervals

$$h = \frac{b-a}{m}$$

Sub-intervals : m

nodes : m + 1



Here, m = 4, nodes = 5

$$C_{1,m} = \frac{h}{2}\left[\underset{\text{first node}}{1f(x_0)} + \underset{\text{middle nodes}}{2f(x_1) + 2f(x_2)} \cdots\cdots + \underset{\text{Last node}}{1f(x_m)}\right]$$

First and Last Node has coeffecient of 1.

Middle nodes has coeffecient of 2

4.1) Sub-interval = 2 (m = 2)

$$h = \frac{b-a}{m} = \frac{2-0}{2} = 1$$

$$x_0 = a = 0$$
$$x_1 = x_0 + h = 0 + 1 = 1$$
$$x_2 = x_1 + h = 1 + 1 = 2$$

$$C_{1,2} = \frac{h}{2}\left[f(x_0) + 2f(x_1) + f(x_2)\right]$$
$$= \frac{1}{2}\left[e^{2(0)} + 2e^{2(1)} + e^{2(2)}\right]$$
$$= 35.188$$

Absolute Error = $|$Actual Value - Approximate Value$|$

$$= |26.80 - 35.188|$$
$$= 8.388$$

Relative Error = $\dfrac{|\text{Actual Value - Approximate Value}|}{|\text{Actual Value}|}$

$$= \frac{|26.80 - 35.188|}{|26.80|}$$
$$= 0.313$$

4.2) Sub-interval = 3 (m = 3)

$$h = \frac{b-a}{m} = \frac{2-0}{3} = \frac{2}{3}$$

$$x_0 = a = 0$$
$$x_1 = x_1 + h = 0 + \frac{2}{3} = \frac{2}{3}$$
$$x_2 = x_1 + h = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$
$$x_3 = x_2 + h = \frac{4}{3} + \frac{2}{3} = 2$$

$$C_{1,3} = \frac{2/3}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + f(x_3)\right]$$
$$= \frac{2/3}{2}\left[e^{2(0)} + 2e^{2(2/3)} + 2e^{2(4/3)} + e^{2(2)}\right]$$
$$= 30.656$$

Absolute Error = $|$Actual Value - Approximate Value$|$

$$= |26.80 - 30.656|$$
$$= 3.856$$

Relative Error = $\dfrac{|\text{Actual Value - Approximate Value}|}{|\text{Actual Value}|}$

$$= \frac{|26.80 - 30.656|}{|26.80|}$$
$$= 0.143$$

4.3) Sub-interval = 4 (m = 4)

$$h = \frac{b-a}{m} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2}$$

$$x_0 = a = 0$$
$$x_1 = x_1 + h = 0 + \frac{1}{2} = \frac{1}{2}$$
$$x_2 = x_1 + h = \frac{1}{2} + \frac{1}{2} = 1$$
$$x_3 = x_2 + h = 1 + \frac{1}{2} = \frac{3}{2}$$
$$x_4 = x_3 + h = \frac{3}{2} + \frac{1}{2} = 2.$$

$$C_{1,4} = \frac{\frac{1}{2}}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$
$$= \frac{1/2}{2}\left[e^{2(0)} + 2e^{2(\frac{1}{2})} + 2e^{2(1)} + 2e^{2(3/2)} + e^{2(2)}\right]$$
$$= 28.99$$

Absolute Error = $|$Actual Value - Approximate Value$|$

$$= |26.80 - 28.99|$$
$$= 2.19$$

Relative Error = $\dfrac{|\text{Actual Value - Approximate Value}|}{|\text{Actual Value}|}$

$$= \frac{|26.80 - 28.99|}{|26.80|}$$
$$= 0.0817$$