Chapter - 2

1. Consider the following nodes:

Sl.	x	Y
1	-1	2.2
2	0	10.6
3	1	17.0
4	2	22.4

a) What will be the degree of interpolating polynomial for the above nodes?
b) How many lagrange bases will be there for the interpolating polynomial of the above nodes? Find them.
c) Find the interpolating polynomial using lagrange method.
d) Find Pn (3.2).

2. Consider the data points (-1, 8), (0, 4) and (1, 16). Using Newton's divided difference method,
a) Find the coefficients a_k for k = 0, 1, 2.
b) Write the interpolating polynomial using the above method.
c) If we add another node (2, 24), what will be the interpolating polynomial then?
d) Find Pn (-0.9).

3. Compute the upper bound of interpolation error for $f(x) = 2\sin(x) - 3\cos(x)$ where $x \in \{-\pi/4, 0, \pi/4\}$ within [-4, 5]. Consider up to 4 significant figures.

4. Consider the following dataset:

x	f(x)	f'(x)
0.1	-0.62	3.46
0.2	-0.30	3.15

a) If we want to construct a 3 degree polynomial using the above information, which method should be used?
b) Compute the bases of the method you chose in (a).

c) Write the polynomial and find the value at x = 2.

5. Consider the function f(x) = e^{-3x^2} which shows Runge Phenomenon when it is interpolated by equally spaced nodes.

(a) What is the name of the way to overcome this problem?

(b) Calculate the five nodes within [-2, 2] using the way you mentioned in (a).

(c) Calculate the five nodes within [-2, 4] using the way you mentioned in (a).

6.

Consider the Runge function $f(x) = e^{-\frac{1}{x^2}}$ which shows Runge phenomenon/divergence behavior when $f(x)$ is interpolated by the equally-spaced nodes over some interval. To overcome this divergence, we may interpolate $f(x)$ with Chebyshev nodes.

(a) (2 marks) Exactly what property of Chebyshev nodes allows us to overcome the Runge phenomenon?

(b) (3 marks) Find six Chebyshev nodes over the interval $[-2.5, 2.5]$.

(c) (5 marks) Find the interpolating polynomial using the Lagrange Form with the first 3 nodes found in part-(b), and simplify the expression of the interpolation polynomial to the standard form.

(d) Now find six Chebyshev's nodes over the interval of [-1.5. 3.5]

7. Consider the data points (-1, 9), (0, 3) and (1, 18). Using Vandermonde matrix method,

a) What will be the degree of the polynomial constructed using above information?

b) Find the interpolating polynomial using the above information.

c) Find Pn(3).

Chapter - 3

1. Consider the function $f(x) = e^{x} + x\ln(x)$. Now answer the following:

- **a)** Find numerical derivative f'(3) using Forward, Backward and Central Difference methods with step size, h = 0.1. Keep up to 5 significant figures.
- **b)** Find the truncation error for the three methods in the above case. Keep up to 4 significant figures.
- **c)** Compute the upper bound of truncation error for Forward, Backward and Central Difference methods when x = 2 and h = 0.2. Keep up to 3 decimal places.
- **d)** Compute the upper bound of truncation error Forward Difference when $x = 2.3 \; and \; h = 0.1$ within [1.3, 1.7]. Keep up to 4 significant figures

2. Deduce an expression for D^1_h from D_h by replacing h by 2h/3 using the Richardson Extrapolation method.

3. Consider the function $f(x) = x^3 cos(x) - e^x + sin(x) - ln(x)$. Now answer the following:

- **a)** Compute $D^1_{0.2}$ at x = 3.4 using Richardson Extrapolation method up to 4 significant figures.
- **b)** Compute $D^2_{0.2}$ at x = 3.1 using Richardson Extrapolation method up to 4 significant figures.

Chapter - 1

1. In the classes, we discussed three forms of floating number representations as shown below,

(1) Standard/General/Lecture Note Form, (2) Normalized Form, (3) Denormalized Form

Now, let's consider **$\beta = 2$, $m = 4$ and $-3 \leq e \leq 4$.** Based on these, answer the following:

- **a)** What are the maximum and minimum numbers that can be stored in the system by these three forms defined (without negative support)?
- **b)** How many numbers can be represented in the system by these three forms?
- **c)** If the above system has negative support, find the maximum and minimum numbers that can be stored by the three forms defined.
- **d)** If the above system has negative support, find how many numbers can be represented by the three forms.
- **e)** Show that, for e = -2, all the numbers have equal spacing.

2. Consider the real number x = 1.42 and y = 1.34 where **$\beta = 2$, $m = 4$ and $-2 \leq e \leq 3$** for Standard form.

a) Find fl(x) and fl(y).

b) Find the rounding and absolute errors for both cases.

c) Find fl(x*y).

d) Find the round and absolute error for (c)

e) What will be machine epsilon (ϵ_m) value for the above system?

3. Consider the quadratic equation $x^2 - 16x + 5 = 0$. Explain how the loss of significance occurs in finding the roots of the quadratic equation if we restrict to 5 significant figures. Discuss how to avoid this and find the roots.