Least Square Method

$$\underbrace{\begin{bmatrix} 1 & -3 & 9 \\ 1 & 0 & 0 \\ 1 & 6 & 36 \\ 1 & 5 & 25 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}}_{b}$$

$$Ax = b$$

$$x = A^{-1} b$$

$$= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 0 & 0 \\ 1 & 6 & 36 \\ 1 & 5 & 25 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

We can not inverse a NON-square matrix

Fit a degree 2 polynomial through the following nodes.

$(x, P(x)) = (-3, 0), (0, 0), (1, 2), (5, 3)$

Two degree polynomial =

$$a_0 + a_1 x + a_2 x^2$$

$$P_2(x_0) = a_0 + a_1(-3) + a_2(-3)^2 = 0$$

$$P_2(x_1) = a_0 + a_1(0) + a_2(0)^2 = 0$$

$$P_2(x_2) = a_0 + a_1(6) + a_2(6)^2 = 2$$

$$P_2(x_3) = a_0 + a_1(5) + a_2(3)^2 = 3$$

No of unknowns < No of equations

$3(a_0, a_1, a_2) < 4$

$$Ax = b$$

$$\underbrace{\begin{bmatrix} 1 & -3 & (-3)^2 \\ 1 & 0 & (0)^2 \\ 1 & 6 & (6)^2 \\ 1 & 5 & (5)^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \\ 3 \end{bmatrix}}_{b}$$

$$\underbrace{\begin{bmatrix} 1 & -3 & 9 \\ 1 & 0 & 0 \\ 1 & 6 & 36 \\ 1 & 5 & 25 \end{bmatrix}}_{A}$$

$$A^T A x = A^T b$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & 0 & 6 & 5 \\ 9 & 0 & 36 & 25 \end{bmatrix} \begin{bmatrix} 1 & -3 & 9 \\ 1 & 0 & 0 \\ 1 & 6 & 36 \\ 1 & 5 & 25 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & 0 & 6 & 5 \\ 9 & 0 & 36 & 25 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 8 & 70 \\ 8 & 70 & 314 \\ 70 & 314 & 2002 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 27 \\ 147 \end{bmatrix}$$

2. Fit a degree 1 polynomial (straight line/linear polynomial) through the following nodes.

$(x, P(x)) = (-3, 0), (0, 0), (1, 2), (5, 3)$

$$P_1(x) = a_0 + a_1 x$$

$$P_2(x_0) = a_0 + a_1(-3) = 0$$

$$P_2(x_1) = a_0 + a_1(0) = 0$$

$$P_2(x_2) = a_0 + a_1(6) = 2$$

$$P_2(x_3) = a_0 + a_1(5) = 3$$

No of unknowns < No of equations

$2(a_0, a_1) < 4$

$$Ax = b$$

$$\underbrace{\begin{bmatrix} 1 & -3 \\ 1 & 0 \\ 1 & 6 \\ 1 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \\ 3 \end{bmatrix}}_{b}$$

$$A^T A x = A^T b$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & 0 & 6 & 5 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 1 & 0 \\ 1 & 6 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & 0 & 6 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 3 \end{bmatrix}$$