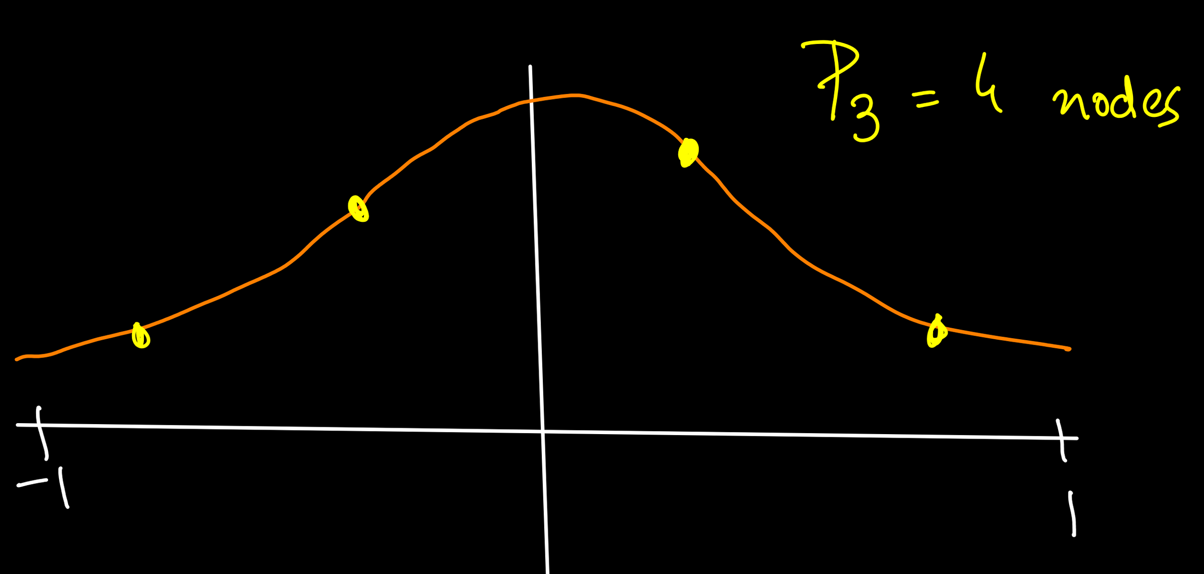
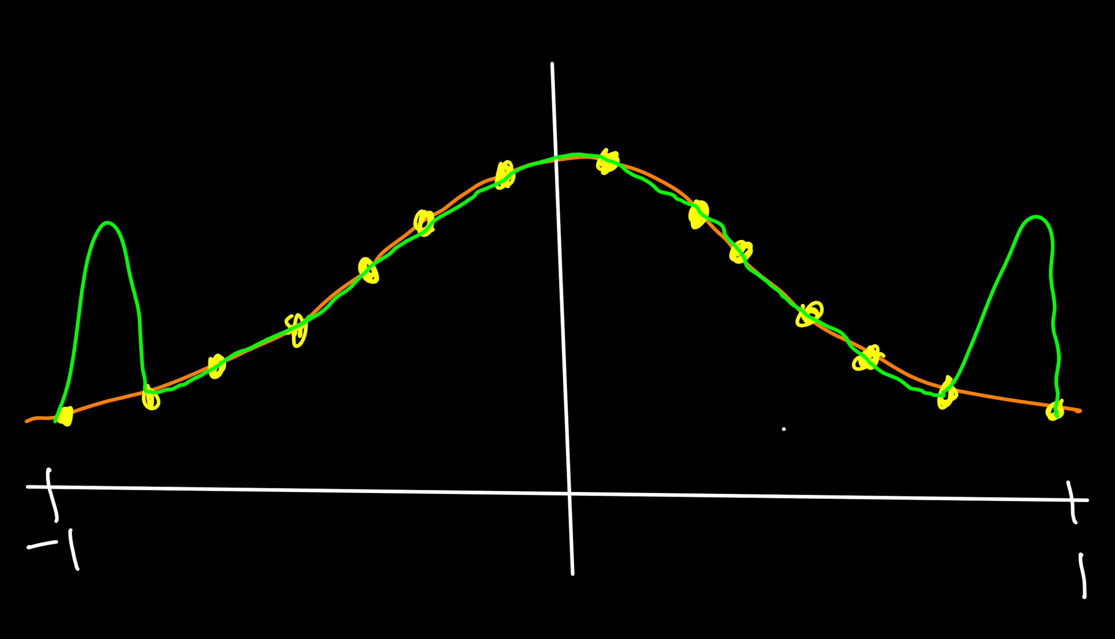
Chevyshev's Nodes

$$f(x) = \frac{1}{1+25x^2}$$





Runge Phenomena

Depends on the function (symmetric function)

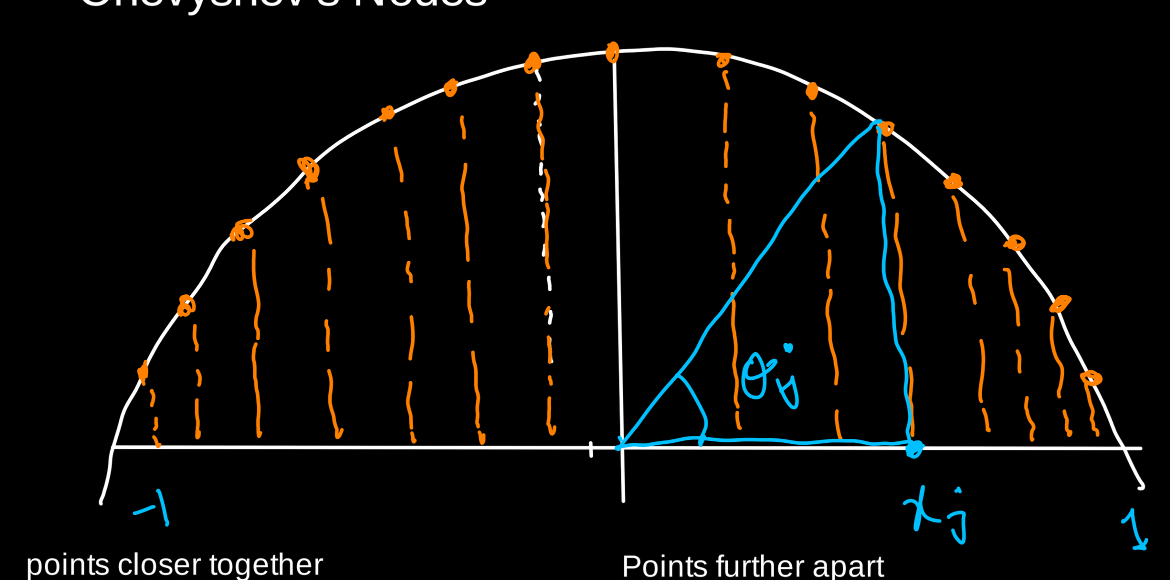
Depends on the nodes (Equally spaced)

Solution

Piece wise Interpolation

Non-Equivalent Nodes (Chevyshev's Nodes)

Chevyshev's Nodes -



points closer together
further away from the centre

Points further apart
near the centre

$$\phi_j = \frac{(2j+1)\pi}{2(n+1)}$$

j = position of node.

n = no. of degree

$$\cos\phi_j = \frac{x_j}{r}$$

here $r = 1$,

$$x_j = \cos\left[\frac{(2j+1)\pi}{2(n+1)}\right] \times r$$

n and r values will change with the given question

Full Formulae to Find Cheyshev's Nodes with/without different nodes.

interval = $[a, b]$

$$\underbrace{\frac{a+b}{2}}_{\text{offset}} + \left[\frac{b-a}{2} \times \cos\left[\frac{(2j+1)\pi}{2(n+1)}\right]\right]$$

Q1) Find the cheyvshev's nodes for the following equation for n=3

$$f(x) = \frac{1}{1+25x^2} \quad [\underset{a}{-1}, \underset{b}{1}]$$

$n=3$, node $=4$

$$x_0 = \underbrace{\frac{-1+1}{2}}_{=0} + \left[\frac{1-(-1)}{2} \times \cos\left[\frac{(2(0)+1)\pi}{2(3+1)}\right]\right] = \cos\frac{\pi}{8}$$

$$x_1 = \underbrace{\frac{-1+1}{2}}_{=0} + \left[\frac{1-(-1)}{2} \times \cos\left[\frac{(2(1)+1)\pi}{2(3+1)}\right]\right] = \frac{\cos 3\pi}{8}$$

$$x_2 = \frac{\cos 5\pi}{8}$$

$$x_3 = \frac{\cos 7\pi}{8}$$

Q2) Use the qs above but use interval [-2,3]

Green value is the only thing that is different in equation

$$x_0 = \underbrace{\frac{-2+3}{2}}_{=\frac{1}{2}} + \left[\frac{3-(-2)}{2} \times \cos\left[\frac{(2(0)+1)\pi}{2(3+1)}\right]\right] =$$

$$= \frac{1}{2} + \left(\frac{5}{2} \times \cos\frac{\pi}{8}\right)$$

$$x_1 = \underbrace{\frac{-2+3}{2}}_{=\frac{1}{2}} + \left[\frac{3-(-2)}{2} \times \cos\left[\frac{(2(1)+1)\pi}{2(3+1)}\right]\right] =$$

$$= \frac{1}{2} + \left(\frac{5}{2} \times \cos\frac{3\pi}{8}\right)$$

$$x_2 = \underbrace{\frac{-2+3}{2}}_{=\frac{1}{2}} + \left[\frac{3-(-2)}{2} \times \cos\left[\frac{(2(2)+1)\pi}{2(3+1)}\right]\right] =$$

$$= \frac{1}{2} + \left(\frac{5}{2} \times \cos\frac{5\pi}{8}\right)$$

$$x_3 = \underbrace{\frac{-2+3}{2}}_{=\frac{1}{2}} + \left[\frac{3-(-2)}{2} \times \cos\left[\frac{(2(3)+1)\pi}{2(3+1)}\right]\right]$$

$$= \frac{1}{2} + \left(\frac{5}{2} \times \cos\frac{7\pi}{8}\right)$$