**BRAC University (Department of Computer Science and Engineering)**
**CSE 330 (Numerical Methods) for Spring 2024 Semester**
**Quiz3 [CO1]**

**Student ID:**

**Name:**

**Full Marks: 10**
**Section: 08** **Duration:15 minutes**

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**1)** Consider the following table and use if for central difference for f'(1.25):

| x | 1.10 | 1.25 | 1.40 |
|---|---|---|---|
| f(x) | 12.52 | 14.24 | 17.11 |

**a)** State the value of h. **[1]**
**b)** Using the table above, find the f'(1.25) using the central difference method. Round the value to *3 significant figure* **[2]**
**c)** If the actual value is 16 find the absolute and relative error. Round the value to *3 significant figure* **[1]**

**2)** Find the forward difference and truncation error for **ln(x) at x=5**, using the corresponding values of **h = 0.2, 0.02**. **[2]**
**3)** Find the bases of hermite interpolation using the nodes **(π,1.5π)** using the function, **f(x) = cos(x). [4]**
**4)** From question 3, find the hermite polynomial. **[Bonus 1 ]**

1.a) $h = 0.15$

(b) $f'(1.25) = \dfrac{f(x_0+h) - f(x_0-h)}{2h}$

$= \dfrac{f(1.25+0.15) - f(1.25-0.15)}{2(0.15)}$

$= \dfrac{17.11 - (12.52)}{0.30}$

$= 15.3.$

(c) Absolute error $= |16 - 15.3|$

$= 0.7$

Relative error $= \dfrac{0.7}{16} = 0.04375$

$\approx 0.0438.$

(2) $y = \ln(x)$

$\frac{dy}{dx} = \frac{1}{x}$

forward difference $= \frac{f(x+h) - f(x)}{h}$

at $x = 5$,

$\frac{1}{x} = \frac{1}{5} = 0.2$

at $x = 5$, $h = 0.2$, $h = 0.02$

$\frac{\ln(x+h) - \ln(x)}{h}$

| | Forward diff | Truncation error |
|---|---|---|
| 0.2 | 0.19610 | 0.0039 |
| 0.02 | 0.19960 | 0.0004 |

(3) $f(x) = \cos x$

$f'(x) = -\sin x$

| | | $f(x)$ | $f'(x)$ |
|---|---|---|---|
| $x_0$ | $\frac{\pi}{2}$ | -1 | 0 |
| $x_1$ | $\frac{3}{2}\pi$ | 0 | 1 |

$$P_3 = h_0(x)\cdot f(x_0) + h_1(x) f(x_1) + \hat{h}_0 f'(x_0) + \hat{h}_1(x_1) f'(x_1)$$

$$= h_0(x) f(x_0) + \hat{h}_1(x_0)\cdot f'(x_1)$$

## $h_0$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} = \frac{x - \frac{3}{2}\pi}{\pi - \frac{3}{2}\pi} = \frac{x - 1.5\pi}{-0.5\pi}$$

$$l_0'(x) = -\frac{2}{\pi} \qquad = \frac{-2x}{\pi} + 3$$

$$h_0(x) = \left[1 - 2(x - x_0)\left(l_0'(x_0)\right)\right]\left(l_0(x)\right)^2$$

$$= \left[1 - 2(x-\pi)\left(-\frac{2}{\pi}\right)\right]\left[-\frac{2x}{\pi} + 3\right]^2$$

$\hat{h}_1(x)$:

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} = \frac{x - \pi}{1.5\pi - \pi} = \frac{x - \pi}{0.5\pi}$$

$$= \frac{2x}{\pi} - 2$$

$$\hat{h}_1(x_1) = \cancel{[x - x_0]}\,[x - x_1]\,(l_1(x))^2$$

$$= (x - 1.5\pi)\left(\frac{2x}{\pi} - 2\right)^2$$

(4) Bonus

$$P_3 = -1\left[\left\{1 - 2(x - 1.5\pi)\left(-\frac{2}{\pi}\right)\right\}\left(-\frac{2x}{\pi} + 3\right)^2\right]$$

$$+ 1\left[(x - 1.5\pi)\left(\frac{2x}{\pi} - 2\right)^2\right]$$