$$h_k = \left\{ 1 - 2(x - x_k)\, l'_k(x_k) \right\} \left(l_k(x) \right)^2$$

$$\hat{h}_k = (x - x_k) \left(l_k(x) \right)^2$$

Ex - 1

f(x) = sin(x)

$$f'(x) = \cos(x)$$

	x	$f(x)$	$f'(x)$
$x_0 \rightarrow$	0	0	1
$x_1 \rightarrow$	$\frac{\pi}{2}$	1	0

nodes = 2

n = 1

degree = 2n+1 = 2(1) +1 = 3

$$P_{2n+1} = P_{2(1)+1} = P_3$$

$$P_3 = h_0(x)\cancel{f(x_0)} + h_1(x) f(x_1) + \hat{h}_0(x) f'(x_0) + \cancel{\hat{h}_1(x) f'(x_1)}$$

$$f(x_0) = 0$$

$$f'(x_1) = 0$$

Putting the 0 values in the above formulae to simplify it

$$= h_1(x) f_1(x) + \hat{h}_0(x) f(x_0)$$

$h_1(x)$

$$l_1(x) = \left(\frac{x - x_0}{x_1 - x_0}\right) = \frac{x - 0}{\frac{\pi}{2} - 0} = \frac{2}{\pi}x$$

$$l_1'(x) = \frac{2}{\pi}$$

$$h_1(x) = \left[1 - 2\left(x - \frac{\pi}{2}\right)\left(\frac{2}{\pi}\right)\right] \left(\frac{2}{\pi}x\right)^2$$

$\hat{h}_0(x)$

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} = \frac{x - \frac{\pi}{2}}{0 - \frac{\pi}{2}} = \frac{1 - 2x}{\pi}$$

$$\hat{h}_0(x) = (x - 0)\left(\frac{1 - 2x}{\pi}\right)^2$$

$$P_3 = h_1(x) f_1(x) + h_0(x) f(x_0)$$

$$= \underbrace{\left[1 - 2\left(x - \tfrac{\pi}{2}\right)\left(\tfrac{2}{\pi}\right)\right]\left(\tfrac{2}{\pi}x\right)^2}_{h_0(x)} \times \underbrace{1}_{f(x_0)} + \underbrace{(x - 0)\left(\tfrac{1 - 2x}{\pi}\right)^2}_{\hat{h}_0(x)} \times \underbrace{1}_{f'(x_0)}$$

Ex - 2

	x	$f(x)$	$f'(x)$
$x_0 \rightarrow$	-1	1	2
$x_1 \rightarrow$	0	0	2
$x_2 \rightarrow$	1	1	0

nodes =3

n = 2

degree = 2n+1 = 2(2) +1 = 5

$$P_{2n+1} = P_{2(1)+1} = P_5$$

$$P_5 = h_0(x) f(x_0) + h_1(x)\cancel{f(x_1)} + h_2(x) f(x_2) + \hat{h}_0(x) f'(x_0) + \hat{h}_1(x) f'(x_1) + \cancel{\hat{h}_2(x) f'(x_2)}$$

$$f(x_1) = 0$$

$$f'(x_2) = 0$$

Putting the 0 values in the above formulae to simplify it

$$= h_0(x) f(x_0) + h_2(x) f(x_2) + \hat{h}_0(x) f'(x_0) + \hat{h}_1(x) f'(x_1)$$

$h_0 x$:

$$l_0(x) = \left(\frac{x - x_1}{x_0 - x_1}\right) \times \left(\frac{x - x_2}{x_0 - x_2}\right) = \left(\frac{x - 0}{-1 - 0}\right) \times \left(\frac{x - 1}{-1 - 1}\right)$$

$$= \frac{1}{2}x(x - 1)$$

$$l_0(x) = \frac{x^2}{2} - \frac{1}{2}x$$

$$l_0'(x) = x - \frac{1}{2}$$

putting x =-1 as the value of x_1 = -1

$$l_0(x_1) = -1 - \frac{1}{2} = -\frac{3}{2}$$

$$h_0(x) = \left[1 - 2(x - -1)\left(-\frac{3}{2}\right)\right]\left(\frac{1}{2}x^2 - \frac{1}{2}x\right)$$

$$= \left[1 + 3(x + 1)\right]\left(\frac{1}{2}x^2 - \frac{1}{2}x\right)$$

$h_2(x)$:

$$l_2(x) = \left(\frac{x - x_0}{x_2 - x_0}\right) \times \left(\frac{x - x_1}{x_2 - x_1}\right) = \left(\frac{x - -1}{1 - -1}\right)\left(\frac{x - 0}{1 - 0}\right)$$

$$= \frac{1}{2}x(x + 1)$$

$$l_2(x) = \frac{1}{2}x^2 + \frac{1}{2}x$$

$$l_2'(x) = x + \frac{1}{2}$$

putting x =1 as the value of x_2 = 1

$$l_2(x_2) = 1 + \frac{1}{2} = \frac{3}{2}$$

$$h_2(x) = \left[1 - 2(x - 1)\left(\frac{3}{2}\right)\right]\left(\frac{1}{2}x^2 + \frac{1}{2}x\right)^2$$

$$= \left[1 - 3(x - 1)\right]\left(\frac{1}{2}x^2 + \frac{1}{2}x\right)^2$$

$\hat{h}_0(x)$:

$$l_0(x) = \frac{1}{2}x^2 - \frac{1}{2}x$$

$$\hat{h}_0(x) = (x - x_0)\left(l_0(x)\right)^2$$

$$= (x + 1)\left(\frac{1}{2}x^2 - \frac{1}{2}x\right)^2$$

$\hat{h}_1(x)$:

$$l_1(x) = \left(\frac{x - x_0}{x_1 - x_0}\right) \times \left(\frac{x - x_2}{x_1 - x_2}\right)$$

$$= \left(\frac{x - -1}{0 - -1}\right) \times \left(\frac{x - 1}{0 - 1}\right)$$

$$= (x + 1)(-x + 1) = (1 + x)(1 - x)$$

$$= 1 - x^2$$

$$\hat{h}_1(x) = (x - x_1)\left(l_1(x)\right)^2 = (x - 0)(1 - x)^2$$

$$= x(1 - x)^2$$

$$P_5(x) = h_0(x) f(x_0) + h_2(x) f(x_2) + \hat{h}_0(x) f'(x_0) + \hat{h}_1(x) f'(x_1)$$

$$= \left[1 + 3(x + 1)\right]\left(\frac{1}{2}x^2 - \frac{1}{2}x\right) \times 1 + \left[1 - 3(x - 1)\right]\left(\frac{1}{2}x^2 + \frac{1}{2}x\right)^2 \times 1$$

$$(x + 1)\left(\frac{1}{2}x^2 - \frac{1}{2}x\right)^2 \times 2 + x(1 - x)^2 \times 2$$