CSE 330: Spring 2024
Assignment-6 [CO4]
Total Marks: 20

1. (5 marks) Verify that the matrix below for a linear system consists of **orthonormal vectors**.

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{\sqrt{2}} \end{bmatrix}$$

2. Consider a set of four data points: **f(0) = 3, f(4) = −2, f(−1) = 2, f(1) = 1**. In the following, you are asked to find the **best fit polynomial of degree 2** by using the **Discrete square/Least square approximation** method as follows:

a. (2 marks) From the given data, write down the matrices A, b and x.
b. (3 marks) Evaluate A^TA and det(A^TA).
c. (2 marks) Compute the best-fit polynomial of degree 2.

3. Consider the coordinates: **(x, f(x)) = (0, 1),(0.5, 1.4),(1, 1.7),(1.5, 2)**. In the following, you are asked to construct the **best-fit linear polynomial** by using the **QR-decomposition** method as follows:

a. (2 marks) Construct the matrices A, b and x.
b. (3 marks) Evaluate the orthonormal vectors q1 and q2, and construct the matrix Q.
c. (2 marks) Compute the matrix R.
d. (3 marks) Using Q and R, evaluate the matrix x, and hence compute the best-fit linear polynomial.

4. A function is given by **$f(x) = e^{0.5x} + \sin x$** which is to be integrated on the interval [0, 2].

a. (2 marks) Evaluate the **exact integral I(f)**.
b. (2 marks) Compute the numerical integral by using the **Newton-Cotes formula with n = 2**.
c. (4 marks) Evaluate the numerical integral $\mathbf{C_{1,4}}$ by using the **Composite Newton-Cotes** formula and also find the percentage relative error.