①

ⓐ $A = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$; $b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$

ⓑ $A^{(1)} = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$

$m_{21} = \frac{a_{21}}{a_{11}} = \frac{3}{1} = 3$

$m_{31} = \frac{a_{31}}{a_{11}} = \frac{4}{1} = 4$

$F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

$A^{(2)} = F^{(1)} \times A^{(1)} = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{bmatrix}$

$m_{32} = \frac{a_{32}}{a_{22}} = \ldots$

$F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -19/16 & 1 \end{bmatrix}$

$A^{(3)} = F^{(2)} \times A^{(2)} = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -0.0625 \end{bmatrix} = U$

$L = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 19/16 & 1 \end{bmatrix}$

Now, $Ly = b \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 19/16 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$

we get, $y_1 = 10$

$y_2 = 6 - 3y_1 = -24$

$y_3 = 9 - 4y_1 - \frac{19}{16}y_2 = -5/2$

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Now, $Ux = y$

$$\Rightarrow \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -0.0625 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -24 \\ -5/2 \end{bmatrix}$$

we get,

$$\therefore \frac{1}{-16} x_3 = -5/2$$

$$\Rightarrow x_3 = 40$$

$$\therefore -16x_2 - 5x_3 = -24$$

$$\Rightarrow x_2 = \frac{-24 + 5x_3}{-16} = -11$$

$$\therefore x_1 + 6x_2 + 2x_3 = 10$$

$$\Rightarrow x_1 = 10 - 6x_2 - 2x_3$$

$$= \cancel{24} - 4$$

$$\therefore (x_1, x_2, x_3) = -4, -11, 40$$

(3)

(a) Given,

$$6x_2 + 2x_3 = 10$$
$$3x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 5x_2 + 2x_3 = 9$$

$$A = \begin{bmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

(b) Yes, matrix A has pivoting problem. Along the diagonal, element A_{11} is 0. We know, if any diagonal element is 0, multiplier for subsequent row operation will be undefined. As such, GE method will fail.

(c) Swapping row 1 and 2, we get, augmented matrix,

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 4 & 5 & 2 & 9 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & 7/3 & 2/3 & 1 \end{array}\right] \quad \left[R_3 = R_3 - \left(\tfrac{4}{3}\right) R_1\right]$$

$$= \left[\begin{array}{ccc|c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & 0 & -1/9 & -26/9 \end{array}\right] \quad \left[R_3 = R_3 - \left(\frac{7/3}{6}\right) R_2\right]$$

(d)

$\therefore -1/9\, x_3 = -26/9 \implies x_3 = 26$

$\therefore 6x_2 + 2x_3 = 10 \implies x_2 = \dfrac{10 - (2 \times 26)}{6} = -7$

$\therefore 3x_1 + 2x_2 + x_3 = 6 \implies x_1 = \dfrac{6 - 2x_2 - x_3}{3} = \dfrac{6 + 14 - 26}{3}$

$= \dfrac{-6}{3} = -2$

$(x_1, x_2, x_3) = (-2, -7, 26)$