1. Use Hermite polynomial that agrees with the data listed below to find the approximation of f(1.5)

k	x_k	$f(x_k)$	$f'(x_k)$
0	1.3	0.6200860	-0.5220232
1	1.6	0.4554022	-0.5698959
2	1.9	0.2818186	-0.5811571

2. Consider the data points $(-1, 8)$, $(0, 4)$ and $(1, 16)$ and . Using these data and Newton's divided/difference method, answer the following:
   (a) (6 marks) Compute the coefficients a_k for $k = 0, 1, 2$. Also write the interpolating polynomial.
   (b) (2 marks) Using the polynomial you found in the previous part, find $p_n(0.5)$ and $p_n(-0.9)$.

3. (a) (6 marks) Compute the upper bound of the error using Cauchy's Theorem for $f(x) = \sin^2(x/2)$ with nodes $\{-\pi/3, 0, \pi/3\}$ within the interval $[-1.2, 1.2]$.
   (b) (2 marks) Why are Chebyshev nodes an optimal choice in Interpolation?

4.

Sl.	x	Y
1	-1	2.2
2	0	10.6
3	1	17.0
4	2	22.4

a. Use the nodes given above to set up a lagrange polynomial of appropriate degree. Find an approximate value of Pn(2.5)

b. Use the nodes given above to set up a lagrange polynomial of appropriate degree. Find an approximate value of Pn(2.5)

5.

. Consider the following data set:

x	$f(x)$	$f'(x)$
0.1	-0.62050	3.58502
0.2	-0.28340	3.14033

Answer the following based on the above data:

(a) (8 marks) Compute the Hermite bases: $h_0(x)$, $h_1(x)$, $\hat{h}_0(x)$ and $\hat{h}_1(x)$.

(b) (2 marks) Write the Hermite polynomial and find the value at $x = 0.15$.

6.

Let f(x) = Cos(x) and the nodes are {-pi/4,0,pi/4). Use Cauchy's theorem to find the upper bound of the error

7. Practice Question From AMK Sir [Section 1 and 2]

. (4 marks) Consider the function $f(x) = e^{-x}$ with nodes at 0, 1 and 2 in the interval $[-0.25, 2.25]$. Working to 3 significant figures, compute the upper bound of the estimated error if $f(x)$ is interpolated by a degree two polynomial.

Solution: The formula for upper bound of error is

$$\begin{aligned} \text{Upper Bound} &= \left| \frac{1}{(n+1)!} f^{(n+1)}(\xi)\,(x-x_0)(x-x_1)\cdots(x-x_n) \right|_{\max}, \\ &\leq \frac{1}{6} \left| f^{(n+1)}(\xi) \right|_{\max} \cdot \left| (x-x_0)(x-x_1)\cdots(x-x_n) \right|_{\max}, \qquad \text{where } \xi \in I. \end{aligned}$$

Here, we have, $n = 2$ (because of three nodes), $x_0 = 0$, $x_2 = 2$, and the interval $I = [-0.25, 2.25]$. Now the maximum values are,

$$\left| f^{(3)}(\xi) \right|_{\max} = \left| \frac{d^3}{d\xi^3}(e^-\xi) \right|_{\max} = e^{-(-0.25)} = 1.28\ .$$

Also, we have, $w_3(x) \equiv (x-x_0)(x-x_1)(x-x_2) = x(x-1)(x-2) = x^3 - 3x^2 + 2x$. Hence the critical points for $w_3(x)$ are:

$$w_3'(x) = 0 \Longrightarrow 3x^2 - 6x + 2 = 0 \quad \Longrightarrow \quad x = 1 \pm \frac{1}{\sqrt{3}}\ .$$

Now, the values of $w_3(x)$ at the critical ponts and at the end points are:

$$w_3\left(1 \pm 1/\sqrt{3}\right) = \mp 0.385, \quad w_3(-0.25) = -0.703 \quad \text{and} \quad w_3(2.25) = 0.703 \quad \Longrightarrow \quad \left| w_3(x) \right|_{\max} = 0.703.$$

Therefore, the upper bound is

$$\text{Upper Bpound of Error} \leq \frac{1}{6} \times 1 \times 0.703 = 0.117\ .$$