Vandermonde Matrix

$$5a + 6b = 15$$
$$3a + 2b = 12$$

We need to two equations to solve, to get value of two unknown variables.

$$\begin{pmatrix} 5 & 6 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 15 \\ 12 \end{pmatrix}$$

$$P_n(x) = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 \cdots\cdots a_n x^n$$

unknown $= a_0, a_1, a_2 \cdots a_n$

$(x_0, x_1, x_2 \cdots\cdots x_n) \Rightarrow n+1$ nodes

$$P_n(x_0) = a_0 + a_1 x_0 + a_2 x_0^2 + a_3 x_0^3 \cdots\cdots a_n x_0^n$$
$$P_n(x_1) = a_0 + a_1 x_1 + a_2 x_1^2 + a_3 x_1^3 \cdots\cdots a_n x_1^n$$
$$\vdots$$
$$P_n(x_n) = a_0 + a_1 x_n + a_2 x_n^2 + a_3 x_n^3 \cdots\cdots a_n x_n^n$$

$$\underbrace{\begin{pmatrix} 1 & x_0 & x_0^2 & \cdots\cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ \vdots & & & & \\ 1 & x_n & x_n^2 & \cdots\cdots & x_n^n \end{pmatrix}}_{\text{Vandermondes Matrix, } V} \underbrace{\begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{pmatrix}}_{\text{coeffecient, } a} = \underbrace{\begin{pmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_n) \end{pmatrix}}_{f}$$

$$V \cdot a = f$$
$$a = V^{-1} \cdot f$$

Ex 1.

	x	$f(x)$
$x_0 \rightarrow$	2	5
$x_1 \rightarrow$	3	6

$$a_0 + a_1 x_0 = f(x_0) \qquad a_0 + a_1 x_1 = f(x_1)$$
$$a_0 + a_1 \cdot 2 = 5 \qquad a_0 + a_1 \cdot 3 = 6$$

$$\begin{bmatrix} 1 & x_0 \\ 1 & x_1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & x_0 \\ 1 & x_1 \end{bmatrix}^{-1} \begin{bmatrix} f(x_0) \\ f(x_1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$= \frac{1}{(1\times3) - (2\times1)} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$= 1 \cdot \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} (3\times5) + (-2\times6) \\ (-1\times5) + (1\times6) \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Ex 2.

x	$f(x)$
25	25
30	35
45	55

$$\begin{pmatrix} 1 & 25 & 25^2 \\ 1 & 30 & 30^2 \\ 1 & 45 & 45^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 25 \\ 35 \\ 55 \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 1 & 25 & 25^2 \\ 1 & 30 & 30^2 \\ 1 & 45 & 45^2 \end{pmatrix}^{-1} \begin{pmatrix} 25 \\ 35 \\ 55 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{27}{2} & -15 & \frac{5}{2} \\ -\frac{3}{4} & \frac{14}{15} & -\frac{11}{60} \\ 0.01 & -\frac{1}{75} & \frac{1}{300} \end{pmatrix} \begin{pmatrix} 25 \\ 35 \\ 55 \end{pmatrix}$$

$$=$$

Ex 1.

	x	$f(x)$
$x_0 \rightarrow$	2	15
$x_1 \rightarrow$	4	20
$x_2 \rightarrow$	6	25

nodes = 3

degree = 2

$$P_2(x) = l_0(x)f(x_0) + l_1(x)f(x_1) + l_2(x)f(x_2)$$

$$l_0 = \left(\frac{x - x_1}{x_0 - x_1}\right) \times \left(\frac{x - x_2}{x_0 - x_2}\right)$$

$$l_1 = \left(\frac{x - x_0}{x_1 - x_0}\right) \times \left(\frac{x - x_2}{x_1 - x_2}\right)$$

$$l_2 = \left(\frac{x - x_0}{x_2 - x_0}\right) \times \left(\frac{x - x_1}{x_2 - x_1}\right)$$

$$l_0 = \left(\frac{x-4}{2-4}\right) \times \left(\frac{x-6}{2-6}\right) = \frac{(x-4)(x-6)}{8}$$

$$l_1 = \left(\frac{x-2}{4-2}\right) \times \left(\frac{x-6}{4-6}\right) = \frac{(x-2)(x-6)}{-4}$$

$$l_2 = \left(\frac{x-2}{6-2}\right) \times \left(\frac{x-4}{6-4}\right) = \frac{(x-2)(x-4)}{8}$$

$$P_2(x) = \left(l_0(x)f(x_0)\right) + \left(l_1(x)f(x_1)\right) + \left(l_2(x)f(x_2)\right)$$

$$P_2(x=5) = \left(\frac{(x-4)(x-6)}{8} \times 15\right) + \left(\frac{(x-2)(x-6)}{-4} \times 20\right) + \left(\frac{(x-2)(x-4)}{8} \times 25\right)$$

$$= \left(\frac{-1}{8} \times 15\right) + \left(\frac{-3}{-4} \times 20\right) + \left(\frac{3}{8} \times 25\right)$$

$$= 22.5$$

Ex 2.

$$f(x) = x^2 \cdot \cos(x)$$

	x	$f(x)$
$x_0 \rightarrow$	0	0
$x_1 \rightarrow$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{32}\pi^2$

nodes = 2

degree(n) = 1

At $\underline{x=4}$,

$$x^2 \cdot \cos(x)$$

$$= \left(\frac{\pi}{4}\right)^2 \cdot \cos\left(\frac{\pi}{4}\right)$$

$$= \frac{\pi^2}{16} \times \frac{\sqrt{2}}{2}$$

$$= \frac{\pi^2 \times \sqrt{2}}{32}$$

$$P_1 = l_0(x)f(x_0) + l_1(x)f(x_1)$$

$$l_0(x) = \left(\frac{x - x_1}{x_0 - x_1}\right) = \left(\frac{x - \frac{\pi}{4}}{0 - \frac{\pi}{4}}\right)$$

$$= -\frac{4x}{\pi} + 1$$

$$l_1(x) = \left(\frac{x - x_0}{x_1 - x_0}\right) = \left(\frac{x - 0}{\frac{\pi}{4} - 0}\right)$$

$$= \frac{4x}{\pi}$$

$$P_1 = \left(l_0(x)f(x_0)\right) + \left(l_1(x)f(x_1)\right)$$

$$= \left(\left(-\frac{4x}{\pi} + 1\right) \times 0\right) + \left(\frac{\cancel{4}x}{\cancel{\pi}} \times \frac{\pi^{\cancel{2}\,1} \times \sqrt{2}}{\cancel{32}\,8}\right)$$

$$= 0 + \frac{\sqrt{2}}{8}x$$

$$= \frac{\sqrt{2}}{8}x$$