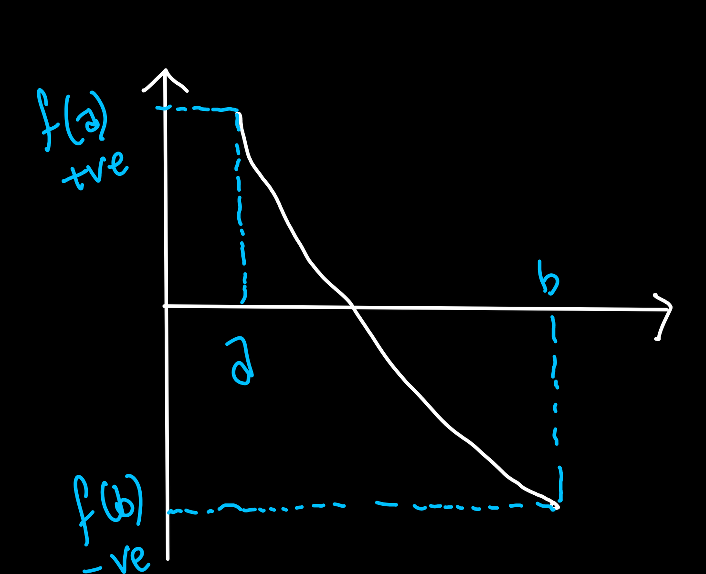
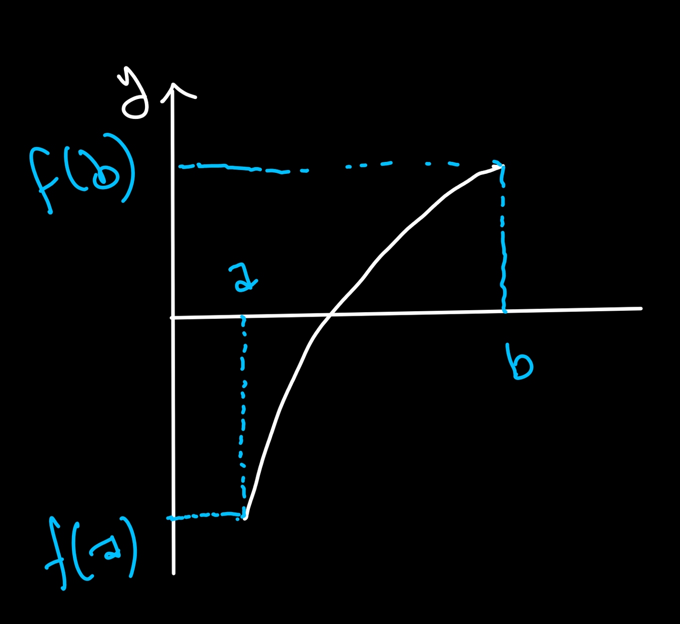
Bisection Method

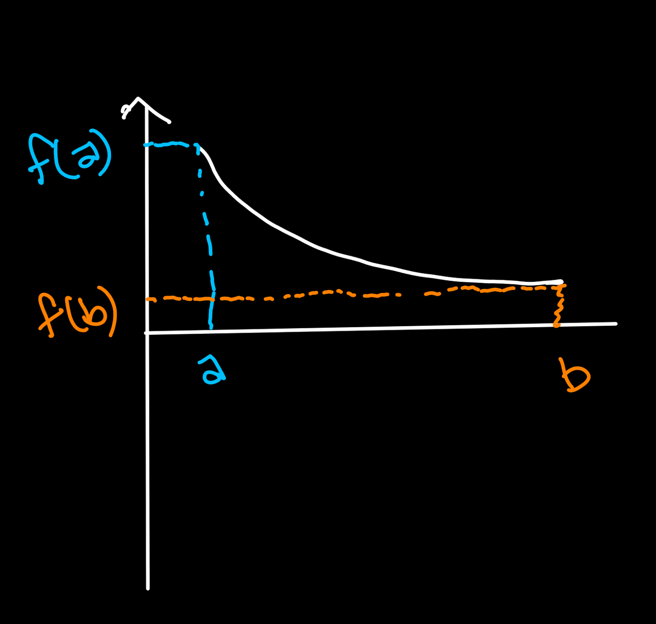
This method only works when there is ONLY one root inside the interval



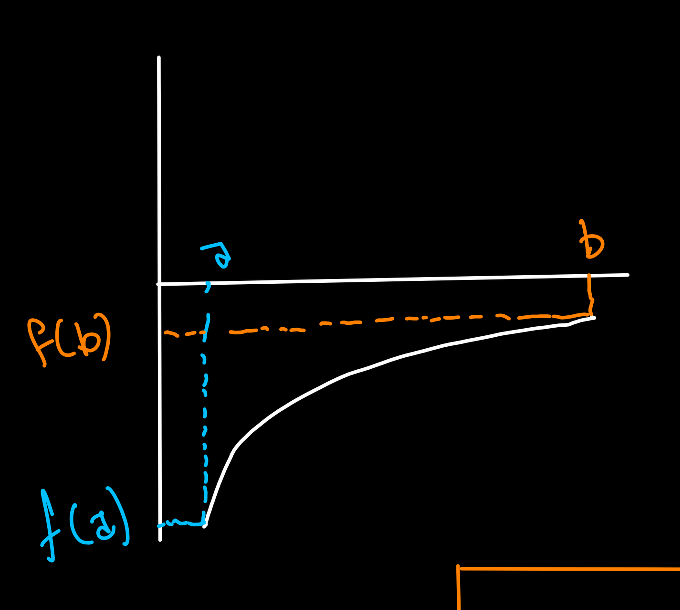
$$f(a)\times f(b) = +ve \times -ve$$
$$= -ve.$$



$$f(a)\times f(b) = -ve \times +ve$$
$$= -ve$$



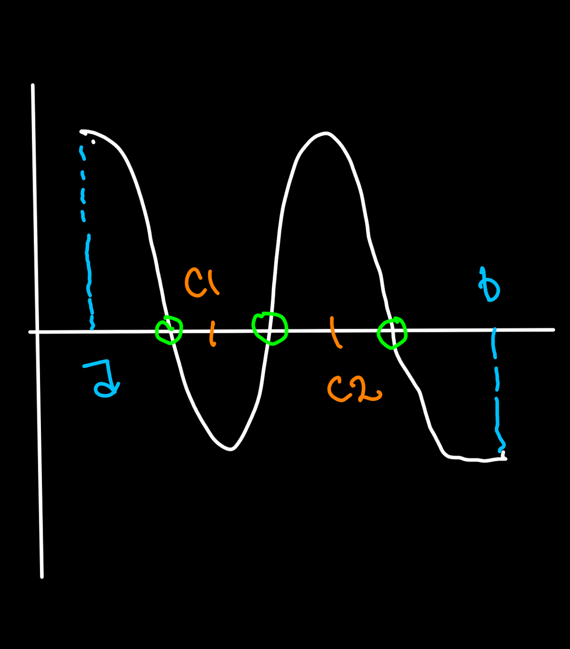
$$f(a)\times f(b) = +ve \times +ve$$
$$= +ve$$



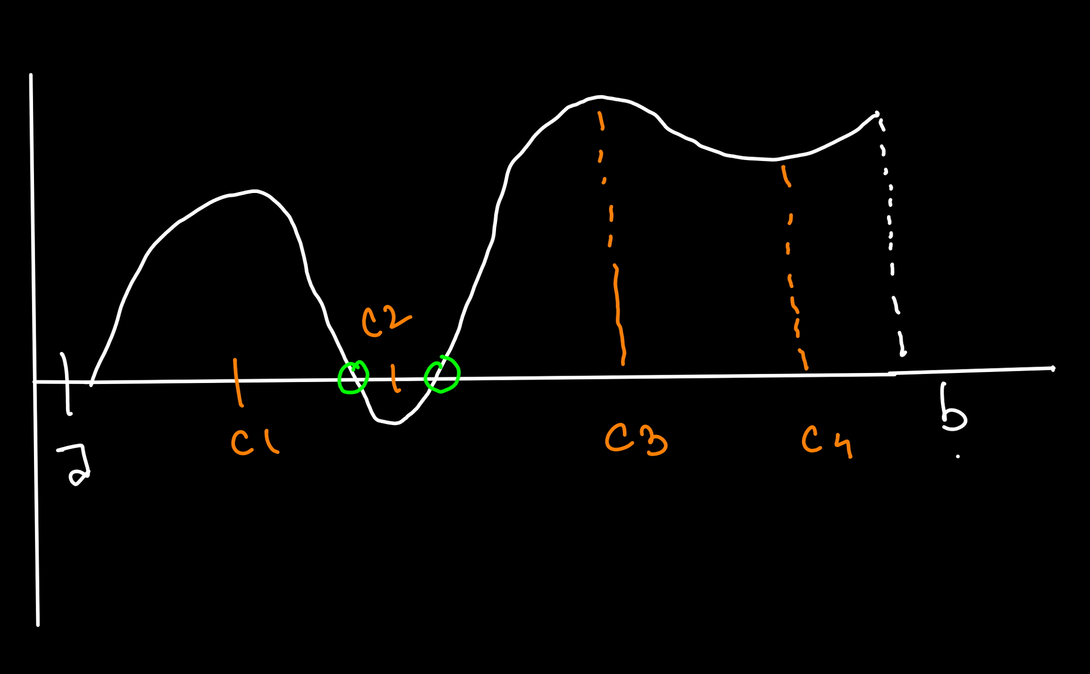
$$f(a)\times f(b) = -ve \times -ve$$
$$= +ve$$

if $\boxed{\begin{array}{ll} f(a)\times f(b) = +ve, & \text{No roots exist} \\ f(a)\times f(b) = -ve, & \text{Roots Exist} \end{array}}$

Given there are multple roots inside the interval, we have to update the interval like the following 2 graphs



$$[a,b] = [a,c_1]\cup[c_1,c_2]\cup[c_2,b]$$



$$[a,b] = [a,c_1]\cup[c_1,c_2]\cup[c_2,c_3]$$

The Algo

$f(x) =$ The function

$[a, b] \rightarrow$ interval

$a \rightarrow x_L$, $b \rightarrow x_U$

Step 1(optional): Checking sign changes in the given interval.

$$f(a)\times f(b) = -ve\ ?$$

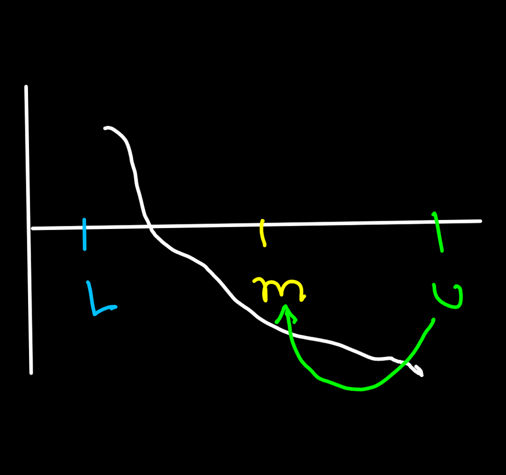
if Yes, then root exists

Step 2: Find Point the midpoint

$$x_m = \frac{x_L + x_U}{2}$$

Step 3: Updating the points

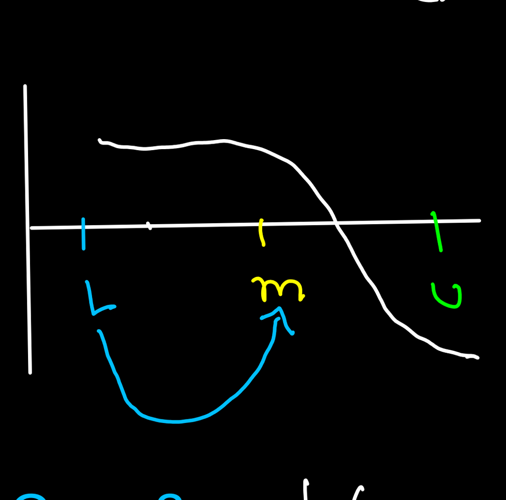
Case 1: $f(x_l)\times f(x_m) < 0$



$x_l = x_l$

updated $\rightarrow x_U = x_m$

Case 2: $f(x_l)\times f(x_m) > 0$

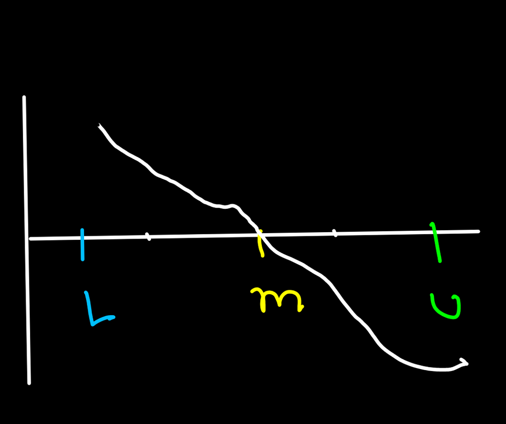


updated $\rightarrow x_L = x_m$

$x_U = x_U$

Case 3: $f(x_l)\times f(x_m) = 0$

root



We got the root and we stop the iteration

Step 4: For next iteration go to step 2

Q1) $f(x) = x^3 - 0.165x^2 + 3.993\times 10^{-4}$

$[0, 0.11]$, $0 \rightarrow x_L$, $0.11 \rightarrow x_U$

Iteration 1

$$x_m = \frac{0+0.11}{2} = 0.055$$

$$f(x_l)\times f(x_m) = f(0)\,f(0.055)$$
$$= 3.993\times 10^{-3} \times 6.655\times 10^{-4}$$
$$= +ve$$

$x_l = x_m$ ✓ updated

$x_U = x_U$

The root exists inside lower and mid interval

$[0.055, 0.11]$, $0.055 \rightarrow x_L$, $0.11 \rightarrow x_U$

Iteration 2

$$x_m = \frac{0.055+0.11}{2} = 0.0825$$

$$f(x_l)\times f(x_m) = f(0.055)\times f(0.0825)$$
$$= 6.665\times 10^{-4} \times -1.622\times 10^{-4}$$
$$= -ve.$$

The root exists inside mid and upper interval

$x_L = 0.055$

$x_U = 0.0825$ ✓ Updated

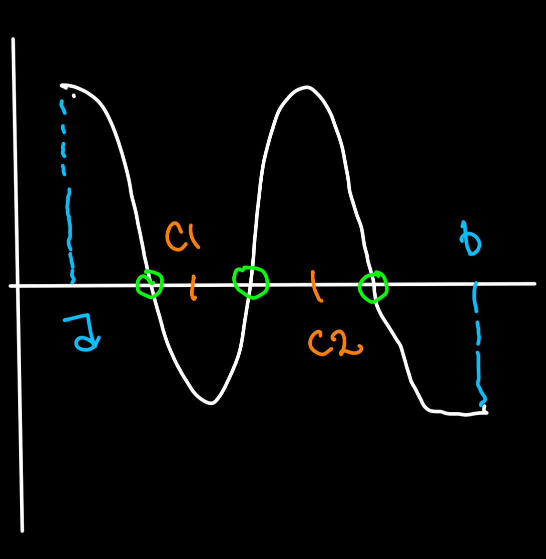
$$[0.055,\ 0.0825]$$

Q2) Find the interval from the equation/ expression. Find the sub-intervals so that there is only one root within each sub-interval.

$$x^3 + 5x^2 + 5x = 0$$

Interval $= [-\alpha, \alpha]$

We can not have multiple roots within the interval for Interval Bisection Method.



$$[a,b] = [a,c_1] \cup [c_1,c_2] \cup [c_2,b]$$

Turning point is the boundary of the interval

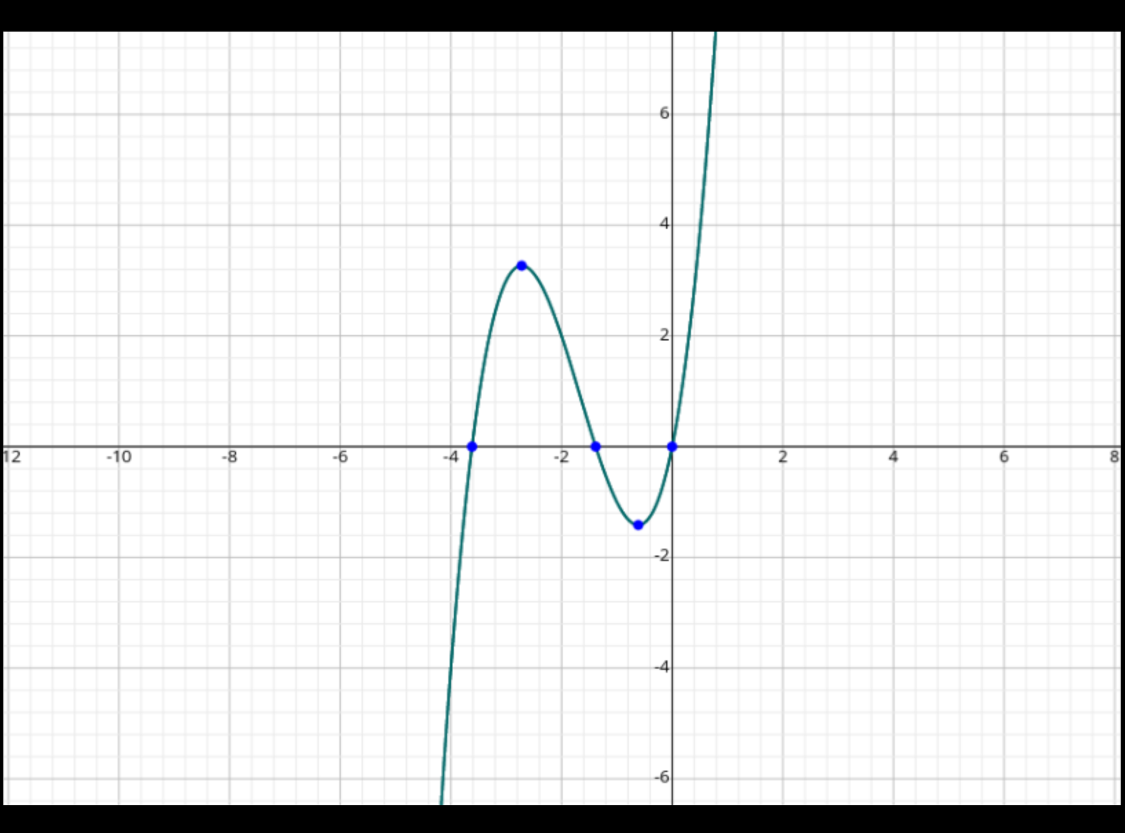
$$f'(x) = 3x^2 + 10x + 5$$

$$f'(x) = 0$$

$$x = -0.6126,\ -2.721$$

Updated Intervals:

$$[-\alpha, -2.721] \cup [-2.721, -0.6126] \cup [-0.6126, \alpha]$$



Q) Compute the minimum number of iteration using the given info

formulae:-

$$n \geq \frac{\log(|b_0 - a_0|) - \log(\delta)}{\log(2)} - 1$$

$$a_0 = 1.5$$

$$b_0 = 3$$

$$\delta = t_m = 1.1 \times 10^{-16}$$

$$n \geq \frac{\log(|3 - 1.5|) - \log(1.1 \times 10^{-16})}{\log(2)} - 1$$

$n \geq 53$ iterations.