Newton's Method

$$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) \dots a_n(x-x_0)(x-x_1)\dots(x-x_{n-1})$$

$a_0 = f[x_0]$ $\quad a_3 = f[x_0, x_1, x_2, x_3]$

$a_1 = f[x_0, x_1]$ $\quad a_n = f[x_0, x_1, \dots x_n]$

$a_2 = f[x_0, x_1, x_2]$

x	$f(x)$
-1	5
0	1
1	3
2	11

nodes = 4

n degree = 3

$$P_3(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1) + f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2)$$

$x_0 = -1 \quad f[x_0] = 5$

$x_1 = 0 \quad f[x_1] = 1$

$x_2 = 1 \quad f[x_2] = 3$

$x_3 = 2 \quad f[x_3] = 11$

$f[x_0, x_1] = \frac{1-5}{0-(-1)} = -4$

$f[x_1, x_2] = \frac{3-1}{1-0} = 3$

$f[x_2, x_3] = \frac{11-3}{2-1} = 8$

$f[x_0, x_1, x_2] = \frac{3-(-4)}{1-(-1)} = 3$

$f[x_1, x_2, x_3] = \frac{8-3}{2-0} = 3$

$f[x_0, x_1, x_2, x_3] = \frac{3-3}{2-(-1)} = 0$

Plugging into $P_3(x)$ formulae

$= 5 + (-4)(x+1) + 3(x+1)(x) + 0$

Now introduce a new node $(4, 20)$

Time	Velocity
10	235
15	362
20	520

$$P_2(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

$x_0 = 10 \quad f[x_0] = 235$

$x_1 = 15 \quad f[x_1] = 362$

$x_2 = 20 \quad f[x_2] = 520$

$\frac{362-235}{15-10} = 25.4$

$\frac{520-362}{20-15} = 31.6$

$\frac{31.6-25.4}{20-10} = 0.62$

Plugging in $P_2(x)$ formulae,

$= 235 + 25.4(x-10) + 0.383(x-15)$

Cauchy's Theorem

$$\left| f(x) - P_n(x) \right| = \frac{f^{n+1}(\xi)}{(n+1)!}(x-x_0)(x-x_1)\dots(x-x_n)$$

$f(x) = \cos(x) \quad \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$ $\quad x/\xi \in [1,1]$

$$\left| f(x) - P_2(x) \right| = \frac{f^3(\xi)}{3!}(x-x_0)(x-x_1)(x-x_2)$$

$$= \left|\frac{\sin(\xi)}{3!}\right| \underbrace{\left|\left(x - -\frac{\pi}{2}\right)(x-0)\left(x-\frac{\pi}{2}\right)\right|}_{\omega(x)}$$

$$= \frac{\sin(1)}{3!} \times 0.383 =$$

$\omega(x) = \left(x + \frac{\pi}{4}\right)(x)\left(x - \frac{\pi}{4}\right)$

$= x^3 - \frac{\pi^2}{16}x$

$\omega'(x) = 3x^2 - \frac{\pi^2}{16}$

$\omega'(x) = 0$

$x = \pm \frac{\pi}{4\sqrt{3}}$

X	$\lvert\omega(x)\rvert$
$-\frac{\pi}{4\sqrt{3}}$	0.186
$\frac{\pi}{4\sqrt{3}}$	0.186
-1	0.383 ✓
1	0.383 ✓