BRAC University (Department of Computer Science and Engineering)
CSE 330 (Numerical Methods) for Spring 2024 Semester
Quiz3 [CO1]

Student ID:

Name:

Full Marks: 10

Section: 08 **Duration:15 minutes**

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1) Consider the following table and use if for central difference for f'(1.25):

x	1.10	1.25	1.40
f(x)	-12.52	-14.24	-17.11

a) State the value of h. **[1]**
b) Using the table above, find the f'(1.25) using the central difference method. Round the value to *4 significant figure* **[2]**
c) If the actual value is -16 find the absolute and relative error. Round the value to *3 significant figure* **[1]**

2) Find the forward difference and truncation error for **ln(x) at x=4**, using the corresponding values of **h = 0.2, 0.02**. **[2]**
3) Find the bases of hermite interpolation using the nodes **(0,π/2)** using the function, **f(x) = cos(x). [4]**
4) From question 3, find the hermite polynomial. **[Bonus 1]**

Quiz 3

1(a)

$h = 0.15$

(b)

$$f'(1.25) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$$= \frac{f(1.25+0.15) - f(1.25-0.15)}{2(0.15)}$$

$$= \frac{-17.11 - (-12.52)}{0.30}$$

$$= -15.3$$

(c)

Absolute error value $= |16 - (-15.3)|$

$= 0.700$

Relative error $= \dfrac{0.7}{\cancel{|-15.3|}\ |-16|}$

$= 0.04375$

$\simeq 0.0438$

(2)

$y = \ln(x)$

$\frac{dy}{dx} = \frac{1}{x}$

at $x = 4$,

$\frac{1}{x} = \frac{1}{4} = 0.25$

forward difference $= \frac{f(x+h) - f(x)}{h}$

at $x = 4$, $h = 0.2$ $\frac{\ln(x+h) - \ln(x)}{h}$

	forward diff.	Truncation error.
0.2	0.243951	0.006049
0.02	0.249377	0.000623

(3)

$f(x) = \cos(x)$. $\qquad f'(x) = -\sin(x)$

	x	$f(x)$	$f'(x)$
$x_0 \rightarrow$	0	1	0
$x_1 \rightarrow$	$\frac{\pi}{2}$	0	-1

$$P_3 = h_0(x) f(x_0) + \cancel{\hat{h}_0(x) f'(x_0)} + \cancel{h_1(x) f(x_1)} + \hat{h}_1(x) f'(x_1)$$

$$P_3 = h_0(x) f(x_0) + \hat{h}_1(x) f'(x_1)$$

<u>h_0</u>

$$l_0(x) = \frac{x - x_1}{x_0 - x_1} = \frac{x - \frac{\pi}{2}}{0 - \frac{\pi}{2}}$$

$$= -\frac{2}{\pi}\left(x - \frac{\pi}{2}\right)$$

$$= \left(-\frac{2x}{\pi} + 1\right)$$

$$l_0'(x) = -\frac{2}{\pi}$$

$\cancel{h_0(x) = \{1 - 2(x - x_0) l}$

$$h_0(x) = \left\{1 - 2(x - x_0)\,(l'(x_0))\right\}(l_0(x))^2$$

$$= \left\{1 - 2(x - 0)\left(-\frac{2}{\pi}\right)\right\}\left(-\frac{2x}{\pi} + 1\right)^2$$

$\underline{\hat{h}_1(x)}$

$$l_1(x) = \frac{x - x_0}{x_1 - x_0} = \frac{x - 0}{\frac{\pi}{2} - 0} = \frac{2x}{\pi}$$

$$\hat{h}_1(x) = (x - x_1)\,(l_1(x))^2$$

$$= \left(x - \frac{\pi}{2}\right)\left(\frac{2}{\pi}x\right)^2$$

(4) . $\underline{\text{Bonus}}$

$$P(x) = 1\left[\left\{1 - 2(x)\left(-\frac{2}{\pi}\right)\right\}\left(-\frac{2x}{\pi} + 1\right)^2\right] + 1\left[\left(x - \frac{\pi}{2}\right)\left(\frac{2x}{\pi}\right)^2\right]$$