Chapter-2

4) a) If we want to construct a 3 degree polynomial, we would need to do Hermite Interpolation.

x	$f(x)$	$f'(x)$
0.1	-0.62	3.46
0.2	-0.30	3.15

$n = 1$

$P_{2n+1} = P_{2(1)+1} = P_3$

b) $P_3(x) = h_0(x)\,f(x_0) + \hat{h}_0(x)\,f'(x_0) + h_1(x)\,f(x_1) + \hat{h}_1(x)\,f'(x_1)$

$h_k(x) = \{1 - 2(x - x_k)\,l'_k(x_k)\}\,(l_k(x))^2$

$\hat{h}_k(x) = (x - x_k)\,(l_k(x))^2$

$l_1(x) = \dfrac{x - x_0}{x_1 - x_0} = \dfrac{x - 0.1}{0.2 - 0.1} = \dfrac{1}{10}(x - 0.1)$

$l'_1(x) = \dfrac{1}{10}(1) = \dfrac{1}{10}$

$l_0(x) = \dfrac{x - x_1}{x_0 - x_1} = \dfrac{x - 0.2}{0.1 - 0.2} = -\dfrac{1}{10}(x - 0.2)$

$l'_0(x) = -\dfrac{1}{10}(1) = -\dfrac{1}{10}$

$\hat{h}_0(x) = (x - x_0)(l_0(x))^2$

$\hat{h}_0(x) = (x - 0.1)\left(-\dfrac{1}{10}(x - 0.2)\right)^2$

$\hat{h}_1(x) = (x - x_1)(l_1(x))^2$

$= (x - 0.2)\left(\dfrac{1}{10}(x - 0.1)\right)^2$

$$h_0(x) = \{1 - 2(x - x_0)\, l_0'(x_0)\}\cdot(l_0(x))^2$$

$$h_0(x) = \{1 - 2(x - 0.1)\left(\frac{-1}{10}\right)\}\left(\frac{-1}{10}(x - 0.2)\right)^2$$

$$h_1(x) = \{1 - 2(x - x_1)\, l_1'(x_1)\}(l_1(x))^2$$

$$h_1(x) = \{1 - 2(x - 0.2)\left(\frac{1}{10}\right)\}\{\frac{1}{10}(x - 0.1)\}^2$$

$$P_3(x) = \{1 - 2(x - 0.1)\left(\frac{-1}{10}\right)\}\left(\frac{-1}{10}(x - 0.2)\right)^2$$

$$(-0.62) + \{1 - 2(x - 0.2)\left(\frac{1}{10}\right)\}\{\frac{1}{10}(x - 0.1)\}^2$$

$$(-0.30) + (x - 0.1)\left(\frac{-1}{10}(x - 0.2)\right)^2$$

$$(3.46) + (x - 0.2)\left(\frac{1}{10}(x - 0.1)\right)^2 (3.15)$$

(c) At $x = 2$,

$$P_3(2) = \{1 - 2(2 - 0.1)\left(\frac{-1}{10}\right)\}\left(\frac{-1}{10}(2 - 0.2)\right)^2(-0.62)$$

$$+ \{1 - 2(2 - 0.2)\left(\frac{1}{10}\right)\}\{\frac{1}{10}(2 - 0.1)\}^2(-0.30)$$

$$+ (2 - 0.1)\left(\frac{-1}{10}(2 - 0.2)\right)^2(3.46) +$$

$$(2 - 0.2)\left(\frac{1}{10}(2 - 0.1)\right)^2(3.15)$$

$$= -0.02772 + (-6.9312 \times 10^{-3}) + 0.2129976$$

$$+ 0.204687$$

$$= 0.3830334$$

(5) $f(x) = e^{-3x^2}$

(a) The way to overcome this problem is by choosing Chebyshev nodes. We choose more nodes on the edges ~~for Runge~~ to reduce the error caused by spikes on the edges for Runge functions.

(b) $I = [-2, 2]$ $n = 5$.

$$x_j = r\cos\left[\frac{(2j+1)\pi}{2(5+1)}\right]$$

$j = 0, 1, 2, 3, 4, 5, 6$

$$x_0 = 2\cos\left[\frac{(2\times 0+1)\pi}{12}\right] = 2\cos\left(\frac{\pi}{12}\right)$$

$$x_1 = 2\cos\left[\frac{(2\times 1+1)\pi}{12}\right] = 2\cos\left(\frac{3\pi}{12}\right)$$

$$x_2 = 2\cos\left[\frac{(2\times 2+1)\pi}{12}\right] = 2\cos\left(\frac{5\pi}{12}\right)$$

$$x_3 = 2\cos\left[\frac{(2\times 3+1)\pi}{12}\right] = 2\cos\left(\frac{7\pi}{12}\right)$$

$$x_4 = 2\cos\left[\frac{(2\times 4+1)\pi}{12}\right] = 2\cos\left(\frac{9\pi}{12}\right)$$

$$x_5 = 2\cos\left[\frac{(2\times 5+1)\pi}{12}\right] = 2\cos\left(\frac{11\pi}{12}\right)$$

$$x_6 = 2\cos\left[\frac{(2\times 6+1)\pi}{12}\right] = 2\cos\left(\frac{13\pi}{12}\right)$$

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(c) $I = [-2, 4]$ The range is assymetric so we need to shift the interval.

$$x_j = \frac{b-a}{2} \cos\left[\frac{(2j+1)\pi}{2(n+1)}\right] + \left(\frac{a+b}{2}\right)$$

$$= \frac{4-(-2)}{2} \cos\left[\frac{(2j+1)\pi}{2(5+1)}\right] + \left(\frac{-2+4}{2}\right)$$

$$= 3\cos\left[\frac{(2j+1)\pi}{12}\right] + 1$$

Find $x_0, x_1, x_2, x_3, x_4, x_5$ and x_6 like part b using the above formula.

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