

Practice Problem Set

1. You can attempt to solve these problems with the help of the materials given in your google classroom.
2. You don't need to submit your solutions. However you can share with your classmates if you want.

1. Take $f(x)$ and an interval I as input. Plot both $f(x)$ and $f'(x)$ in the same graph.
2. Plot the following functions and find the maxima,minima in the given interval
 - (a) $f(x) = 2x^3 - 6x^2 + 2x + 1, x \in [-5, 5]$
 - (b) $f(x) = x^4, x \in [-1, 1]$
 - (c) $f(x) = \sin(2x) + \cos(3x), x \in [0, \frac{3\pi}{4}]$

3. A projectile's equation of motion is given by

$$x' = 3, y' = 4 - 10t, x(0) = y(0) = 0$$

Integrate using sympy and find x, y . Plot x vs y for $t \in [0, 0.8]$

4. Find the following indefinite integrals using sympy:

- (a) $\int e^{-x} x^2 \cos(x) dx$

- (b) $\int \frac{\sqrt{x^2-16}}{x} dx$

- (c) $\int \frac{x^2+x-2}{3x^3-x^2+3x-1} dx$

5. Differential equation of damped oscillation is given by:

$$\left\{ \frac{d^2}{dt^2} + 2\gamma \frac{d}{dt} + 1 \right\} x(t) = 0$$

Plot x vs t for $\gamma = 0, 0.5, 1, 2$ (use Matplotlib subplot).

6. Find the following definite integrals using sympy:

- (a) $\int_{-\infty}^{\infty} \sin(x)/x$

- (b) $\int_0^1 \frac{x^3}{\sqrt{1-x^3}} dx$

- (c) $\int_0^{\infty} \frac{\ln(x)}{x^2+a^2} dx$

7. Numerically evaluate the following definite integrals upto three decimal points (You cannot use sympy or scipy):

(a) $\int_0^1 x^x dx$

(b) $\int_0^\pi x \sin(x) dx$

(c) $\int_0^\infty \sin(x^2) dx$

8. Consider the equation of a freely falling object:

$$\frac{dv}{dt} = 10 - v^2, v(0) = 5$$

- (a) Solve the differential equation using sympy.

- (b) Plot v vs t .

9. Suppose the populations of rabbits and wolves are described by Lotka-Volterra equations given below:

$$dR/dt = R(0.08 - 0.001W)$$

$$dW/dt = W(-0.02 + 0.00002R)$$

Start with 1000 rabbits and 40 wolves, i.e., $R(0) = 1000, W(0) = 40$ and solve these equations using Euler's method.

- (a) Plot W vs t .

- (b) Plot R vs t .

- (c) Plot W vs R .

10. Solve the following ordinary differential equations using Euler's method. Plot y vs x for $x \in [0, \pi]$

(a) $y' + xy = \sin(x), y(\frac{\pi}{2}) = 0$

(b) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = 0, y''(0) = -1, y'(0) = -3, y(0) = 7$