STA201: Elements of Statistics and Probability

Quiz 02

Section: 20

Date: 29/11/2022

1. Every year, many people hiking the Australian Outbacks are reported to be lost, and around 80% of them are eventually found by rescue missions. 60% of the people who were rescued were found to carry an emergency GPS locator, whereas 35% of the people who were not rescued were known to carry a similar locator.

0.8 found, 0.2 Not-
0.6 carry, 0.4 not; 0.35 carry, 0.65

a. If a lost hiker is carrying an emergency GPS locator, what is the probability of them being rescued?[4 marks]

$$P(R|G) = \frac{0.\cancel{4}6 \times 0.8}{(0.\cancel{4}6 \times 0.8) + (0.2 \times .35)} = 0.82$$

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b. If a hiker gets lost without a GPS locator, what is the probability that they will not get rescued?[4 marks]

$$P(R|NG) = \frac{0.2 \times .65}{(0.2 \times .65) + (.8 \times 0.4)} = 0.289$$

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c. If the probability of carrying a GPS locator is 65%, what is the probability that a hiker was carrying a GPS locator and was found by rescued missions?[4 marks]

2. The discrete random variable X has the probability distribution given by:

x	7	9	12	13	15
P(X=x)	u/2 $0.3/2$	0.05	0.35	u 0.3	0.15

a. Determine the following probability: P(X ≤ 13). [3 marks]

$\frac{u}{2} + 0.05 + 0.35 + u + 0.15 = 1$

$\Rightarrow \frac{u}{2} + u = 0.45$

$\Rightarrow 3u = 0.9$

$\therefore u = 0.3$

$P(X \leq 13) = \frac{0.3}{2} + 0.05 + 0.35 + 0.3$

$= 0.85$

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b. What is the expected value of the random variable X? [3 marks] Check

$E(X) = (7 \times \frac{0.3}{2}) + (9 \times 0.05) + (12 \times 0.35) + (13 \times 0.3) + (15 \times 0.15)$

$= 1.05 + 0.45 + 4.2 + 3.9 + 2.25$

$= 11.85$

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c. What is the variance of the random variable X? [3 marks]

$Var(X) = [E(x^2)] - [E(X)]^2$

$= \cancel{3.1575 - (11.85)^2}$

$= 146.25 - (11.85)^2 = 5.83$

03

$E(X^2) = (7^2 \times (\frac{0.3}{2})) + (9^2 \times (0.05)) + (12^2 \times (0.35)) + (13^2 \times (0.3)) + (15^2 \times (0.15))$

$= \cancel{.1575 + 0.0225 + 1.47 + 1.17 + 0.3375}$

$= \cancel{3.1575} = 146.25$

d. Find the expectation and standard deviation of the random variable Y, where $Y = \frac{5X}{6} - 4$. [3 marks]

$E(Y) = \frac{5}{6} E(X) - 4$

$= \frac{5}{6} \times 11.85 - 4$

$= 5.875$

$Var(Y) = (\frac{5}{6})^2 \times 11.85$

$= 8.229$

Standard deviation $= \sqrt{Var(Y)}$

$= 2.86$

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3. Letting Y denote the random variable that is defined as the sum of two fair dice, find the probabilities associated with ~~X~~. [5 marks]

Y

Y	2	3	4	5	6
	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$

7	8	9	10	11	12
$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

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(b) ~~1~~,2

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,
2	2,1	2,2	2,3	2,4	2,5	2
3	3,1	3,2	3,3	3,4	3,5	
4	4,1	4,2	4,3	4,4	4,5	
5	5,1	5,2	5,3	5,4	5,5	
6	6,1	6,2	6,3	6,4	6,5	

4. Consider the following probability density function

$f(x) = \{x^2 - 1, \ -2 \le x < 0 \qquad \frac{1}{2}x + 2, \ 0 \le x < 2 \qquad 0, \ otherwise$

a. Determine the value of $P(-1 \le x \le 2)$ [6 marks]

$$P(-1 \le x \le 2) = \int_{-2}^{0} x^2 - 1 \, dx + \int_0^2 \frac{1}{2}x + 2$$

-1

$$= \left[\frac{x^3}{3} - x\right]_{-2}^{0} + \left[\frac{1}{2}\frac{x^2}{2} + 2x\right]_0^2$$

-1

$$= \left(\frac{(-2)^3}{3} - (-2)\right) + \left(\frac{1}{2}\frac{2^2}{2} + 2\times 2\right)$$

$$= \left(\frac{8}{3} + 2\right) + 5$$

$$= \frac{14}{3} + 5$$

$$= \frac{29}{3} = 9.67$$

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b. Determine the expectation of the random variable. [6 marks]

$$E(x) = \int_{-2}^{0} x(x^2-1) + \int_{0}^{2} x\left(\frac{1}{2}x+2\right)$$

$$= \int_{-2}^{0} (x^3-x)\,dx + \int_{0}^{2} \frac{1}{2}x^2+2x\,dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-2}^{0} + \left[\frac{1}{2} \times \frac{x^3}{3} + x^2\right]_{0}^{2}$$

$$= 8.67$$

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