

CSE460: VLSI Design

Lecture 8: Logic Function Synthesis using k-map

Review: Logic Function Synthesis using k-map

- The Karnaugh map (or k-map) is an **alternative to the truth-table** form for representing a function
- The map consists of cells that correspond to the rows of the truth table

x_1	x_2	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

x_1	x_2	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

x_1	x_2	x_3	x_4	
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
0	1	0	0	m_4
0	1	0	1	m_5
0	1	1	0	m_6
0	1	1	1	m_7
1	0	0	0	m_8
1	0	0	1	m_9
1	0	1	0	m_{10}
1	0	1	1	m_{11}
1	1	0	0	m_{12}
1	1	0	1	m_{13}
1	1	1	0	m_{14}
1	1	1	1	m_{15}

Logic Function Synthesis (SOP) using k-map

Consider a logic function, $Y = f(A,B)$

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

- **Minterm (SOP)** - Group 2^n no. of 1 and skip 0. $n = 0,1,2,3,\dots$ (Make groups larger as possible)
- Grouping can be square/rectangle shaped or along row/column (Not along diagonal). Groups can be overlapped.
- The inputs which are varying in a group, can be omitted. The others (**fixed**) can be written as **product** form. **Invert** the variables which are fixed with the value **0**.
- Don't cares (d) can be used 0/1 as per convenience.

K-map:

		B	
		0	1
A	0		
	1		

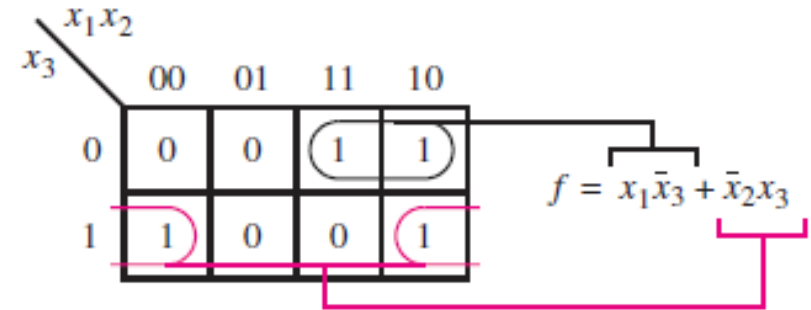
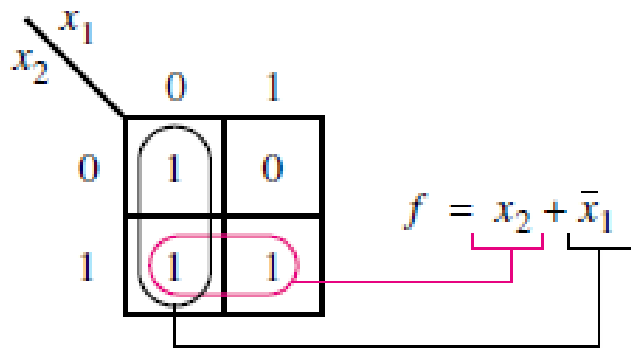
A omitted.
B fixed with 1.
 $Y = B$

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

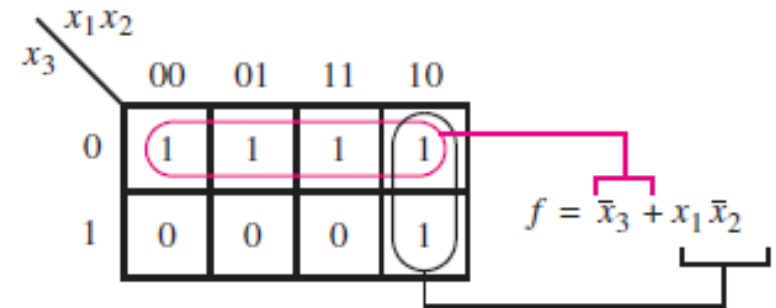
$$Y = B'$$

Logic Function Synthesis (SOP) using k-map

- **Minterm (SOP)** - Group 2^n no. of 1 and no 0. $n = 0,1,2,3,\dots$ (Make groups larger as possible)
- Grouping can be square/rectangle shaped or along row/column (Not along diagonal). Groups can be overlapped.
- The inputs which are varying in a group, can be omitted.
- Don't cares (d) can be used 0/1 as per convenience.
- **Edges are connected.**



(a) The function of Figure 2.23



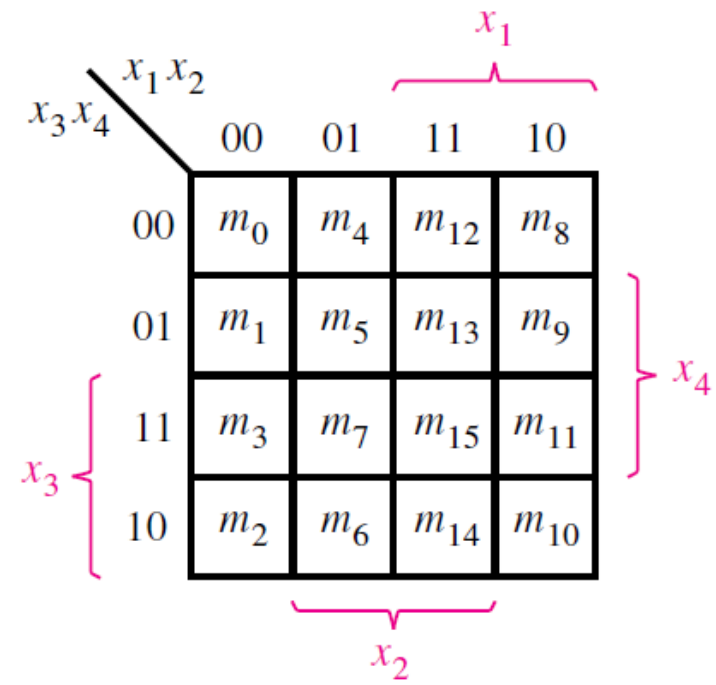
(b) The function of Figure 2.48

Logic Function Synthesis (SOP) using k-map

Consider the following function (where m= minterms, D = don't cares):

$$f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x1	x2	x3	x4	Minterms
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Logic Function Synthesis (SOP) using k-map

Consider the following function (where m= minterms, D = don't cares):

$$f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

$x_1 x_2$ $x_3 x_4$					
		00	01	11	10
00	00				
	01				
	11				
	10				

$x_1 x_2$ $x_3 x_4$					
		00	01	11	10
00	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

Diagram illustrating the K-map for the function $f(x_1, \dots, x_4)$. The K-map is a 4x4 grid with rows labeled $x_3 x_4$ (00, 01, 11, 10) and columns labeled $x_1 x_2$ (00, 01, 11, 10). The minterms are labeled m_0 through m_{15} in the cells. The function is defined as $f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$. The minterms 2, 4, 5, 6, and 10 are marked with 1s in the K-map. The don't care terms 12, 13, 14, and 15 are marked with Xs in the K-map. The K-map is grouped into four groups of four cells each, labeled x_1 , x_2 , x_3 , and x_4 .

Logic Function Synthesis (SOP) using k-map

Consider the following function (where m= minterms, D = don't cares):

$$f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

$x_1 x_2$		$x_3 x_4$			
		00	01	11	10
00	0	1	d	0	
01	0	1	d	0	
11	0	0	d	0	
10	1	1	d	1	

Groupings and labels:

- Group 1 (vertical, $x_2 \bar{x}_3$): Cells (01, 00), (01, 01), (01, 11), (01, 10).
- Group 2 (horizontal, $x_3 \bar{x}_4$): Cells (10, 00), (10, 01), (10, 11), (10, 10).

$x_3 x_4$		$x_1 x_2$			
		00	01	11	10
x_3	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

Diagram illustrating a 4x4 Karnaugh map for variables x_1, x_2, x_3, x_4 . The map is organized into groups of four cells, each group representing a prime implicant. The groups are labeled with their corresponding minterm indices (m_0 through m_{15}) and the variables they cover:

- Group 1 (Top-left): m_0, m_4, m_{12}, m_8 (Covers $x_1 x_2$)
- Group 2 (Top-right): m_4, m_5, m_{13}, m_9 (Covers $x_1 x_3$)
- Group 3 (Bottom-left): m_1, m_5, m_{13}, m_9 (Covers $x_1 x_3$)
- Group 4 (Bottom-right): m_3, m_7, m_{15}, m_{11} (Covers $x_1 x_2$)
- Group 5 (Left): m_0, m_1, m_3, m_2 (Covers $x_3 x_4$)
- Group 6 (Right): m_8, m_9, m_{11}, m_{10} (Covers $x_3 x_4$)
- Group 7 (Top): m_0, m_1, m_2, m_3 (Covers $x_3 x_4$)
- Group 8 (Bottom): m_4, m_5, m_6, m_7 (Covers $x_3 x_4$)

Logic Function Synthesis (SOP) using k-map

Consider the following function (where m= minterms, D = don't cares):

$$f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

$x_1 x_2$ $x_3 x_4$		00	01	11	10
00	0	1	d	0	
01	0	1	d	0	
11	0	0	d	0	
10	1	1	d	1	

$$f = x_2 \bar{x}_3 + x_3 \bar{x}_4$$