

**BRAC University**  
**MAT-215**  
**Practice Sheet # 4    PART A**

1. Evaluate  $\int_{(0,1)}^{(2,5)} (3x + y)dx + (2y - x)dy$  along
  - (a) the curve  $y = x^2 + 1$
  - (b) the straight line joining  $(0,1)$  and  $(2,5)$
  - (c) the straight lines from  $(0,1)$  to  $(0,5)$  and then from  $(0,5)$  to  $(2,5)$
  - (d) the straight lines from  $(0,1)$  to  $(2,1)$  and then from  $(2,1)$  to  $(2,5)$ .
2. Evaluate  $\oint_C (x + 2y)dx + (y - 2x)dy$  around the ellipse  $C$  defined by  $x = 4\cos\theta$ ,  $y = 3\sin\theta$ ,  $0 \leq \theta \leq 2\pi$  if  $C$  is described in a counterclockwise direction.
3. Evaluate  $\int_C (x^2 - iy^2)dz$  along
  - (a) the parabola  $y = 2x^2$  from  $(1,1)$  to  $(2,8)$
  - (b) the straight lines from  $(1,1)$  to  $(1,8)$  and then from  $(1,8)$  to  $(2,8)$
  - (c) the straight line from  $(1,1)$  to  $(2,8)$ .
4. Evaluate  $\oint_C |z|^2 dz$  around the square with vertices at  $(0,0), (1,0), (1,1), (0,1)$ .
5. Evaluate  $\int_C (z^2 + 3z)dz$ 
  - (a) along the circle  $|z| = 2$  from  $(2,0)$  to  $(0,2)$  in a counter clockwise direction.
  - (b) the straight line from  $(2,0)$  to  $(2,2)$  and then from  $(2,2)$  to  $(0,2)$ .
6. Evaluate  $\int_i^{2-i} (3xy + iy^2)dz$ 
  - (a) along the straight line joining  $z = i$  and  $z = 2 - i$
  - (b) along the parabola  $x = 2t - 2$ ,  $y = 1 + t - t^2$ .
7. Evaluate  $\oint_C (\bar{z})^2 dz$  around the circles (a)  $|z| = 1$  and (b)  $|z - 1| = 1$ .
8. Evaluate  $\oint_C \frac{dz}{z - 2}$  around (a) the circle  $|z - 2| = 4$  (b) the circle  $|z - 1| = 9$ .

9. Evaluate  $\oint_C (5z^4 - z^3 + 2) dz$  around the circle  $|z| = 1$ .

**(Cauchy's Integral Formula) PART B**

1. Evaluate (a)  $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$  (b)  $\oint_C \frac{e^{2z}}{(z+1)^4} dz$

where  $C$  is the circle  $|z| = 3$ .

2. Evaluate  $\oint_C \frac{e^z}{(z^2 + \pi^2)^2} dz$

where  $C$  is the circle  $|z| = 4$ .

3. Evaluate  $\oint_C \frac{e^{3z}}{z - \pi i} dz$

where  $C$  is the circle  $|z - 1| = 4$ .

4. Evaluate  $\frac{1}{2\pi i} \oint_C \frac{e^{zt}}{(z^2 + 1)^2} dz$  if  $t > 0$  and  $C$  is the circle  $|z| = 3$ .