BRAC UNIVERSITY

Inspiring Excellence

STA201: Elements of Statistics and Probability
Assignment 03

1. A consulting group was hired by the Human Resources Department to survey company employees regarding their degree of satisfaction with their quality of life. A special index, called the index of satisfaction, was used to measure satisfaction. Six factors were studied, namely, age at the time of first marriage (x_1), annual income (x_2), number of children living (x_3), value of all assets (x_4), status of health in the form of an index (x_5), and the average number of social activities per week—such as bowling and dancing (x_6). Suppose the equation is:

$$\hat{y} = 16.24 + 0.017x_1 + 0.0028x_2 + 42x_3 + 0.0012x_4 + 0.19x_5 + 26.8x_6$$

a. What is this equation called?
b. How many dependent and independent variables are there?
c. What are the numbers 16.24 and 0.017 called?
d. Interpret the model.
e. What is the estimated index of satisfaction for a person who first married at 18, has an annual income of $26,500, has three children living, has assets of $156,000, has an index of health status of 141, and has 2.5 social activities a week on the average?
f. Which would add more to satisfaction, an additional income of $10,000 a year or two more social activities a week?

2. Designers of backpacks use exotic material to make packs that fit comfortably and distribute weight to eliminate pressure points. For fitting a regression model of price of backpack on the capacity (cubic inches) and comfort rating of backpacks, data for 10 backpacks are used. Comfort was measured using a rating from 1 to 5, with a rating of 1 denoting average comfort and a rating of 5 denoting excellent comfort. The output of the regression model is as follows:

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Residuals:
   Min     1Q Median     3Q    Max
-84.12 -27.18  10.61  36.90  48.26

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 356.12083  197.17401   1.806 0.113859
x1           -0.09874    0.04588  -2.152 0.068372 .
x2          122.86721   21.79975   5.636 0.000786 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 51.14 on 7 degrees of freedom
Multiple R-squared:  0.8318,    Adjusted R-squared:  0.7838
F-statistic: 17.31 on 2 and 7 DF,  p-value: 0.00195
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a. Determine the estimated regression equation that can be used to predict the price of a backpack given the capacity and the comfort rating.
b. Interpret the model.
c. Predict the price for a backpack with a capacity of 5500 cubic inches and a comfort rating of 4.5.
d. Comment on the goodness of fit of the model.

3. In a simultaneous throw of a pair of fair 6-sided dice, find the probability of getting:
    a. A sum of 8
    b. A doublet (two dice landing on the same value)
    c. A sum greater than 5
    d. A sum less than 4 or greater than 8
    e. An even number on the first die
    f. An odd number on one and an even number on the other
    g. At least one 6
    h. At least one 6, if the two faces are different

4. A bag contains 30 balls numbered 1 through 30. Suppose drawing an even numbered ball is considered a 'Success'.
    a. Two balls are drawn from the bag with replacement. Find the probability of getting:
    i. Two successes
    ii. exactly one success
    iii. at least one success
    iv. no successes

    b. Find the same four probabilities for the experiment where the two balls are drawn without replacement.

5. Suppose, you are rolling two regular six-sided dice and two four-sided dice together. Let's say, the sum of the numbers appearing on the two regular six-sided dice is 'A' and the sum of the numbers appearing on the two four-sided dice is 'B'. What is the probability that the product of A and B is 12?

6. Assume that the chances of a patient suffering from high blood pressure is 60%. It is also assumed that a course of meditation reduces the risk of high blood pressure by 45% and prescription of certain drugs reduces its chances by 55%. At a time, a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random does not suffer from high blood pressure. Find the probability that the patient chose a course of meditation?

7. Bag A contains 6 red and 7 black balls and Bag B contains 9 red and 6 black balls. One ball is transferred from Bag A to Bag B and then a ball is drawn from Bag B. The ball so drawn is found to be black in color. Find the probability that the transferred ball was red.