

**BRAC University**  
**MAT-215**  
**Practice Sheet # 5**

1. Expand each of the following functions in a Taylor series about the indicated points:

(i)  $e^{-z}$  at  $z = 0$

(ii)  $\cos z$  at  $z = \frac{\pi}{2}$

(iii)  $z^3 - z^2 + 4z - 2$  at  $z = 2$

(iv)  $ze^{2z}$  at  $z = -1$ .

2. Expand  $f(z) = \frac{z}{(z-1)(2-z)}$  in a

Laurent series valid for

(i)  $|z| < 1$

(ii)  $1 < |z| < 2$

(iii)  $|z| > 2$

(iv)  $|z-1| > 1$

(v)  $0 < |z-2| < 1$ .

3. Expand  $f(z) = \frac{1}{z(z-2)}$  in a

Laurent series valid for

(i)  $0 < |z| < 2$

(ii)  $|z| > 2$

4. Evaluate  $\oint_C \frac{z^2}{2z^2 + 5z + 2} dz$  using the residue at the poles, where  $C$  is the unit circle

$|z| = 1$ .

5. Evaluate  $\oint_C \frac{z^2 + 4}{z^3 + 2z^2 + 2z} dz$  using the residue at the poles, around the circle  $|z| = 3$ .

6. Evaluate  $\oint_C \frac{ze^{i\pi z}}{(z^2 + 2z + 5)(z^2 + 1)^2} dz$  using the residue at the poles, where  $C$  is the upper

half circle of the equation  $|z| = 2$ .

7. Evaluate  $\frac{1}{2\pi i} \oint_C \frac{z^2 - z + 2}{z^4 + 10z^2 + 9} dz$  using the residue at the poles, around the circle  $C$  with

the equation  $|z| = 4$ .