BRAC UNIVERSITY
Inspiring Excellence

STA201 Assignment 4

Random Variables

1. A discrete random variable X has the following probability mass function

$$P(X=x)=\begin{cases}2kx & x=2,4,6\\ k(x+2) & x=8\\ 0 & otherwise\end{cases}$$

where k is a constant

- **a.** Show that $k=\frac{1}{34}$
- **b.** Find the exact value of $P(4<x\leq 8)$
- **c.** Find the exact value of $P(2<x<4)$
- **d.** What is the expected value of the random variable X?
- **e.** What is the variance of the random variable X?
- **f.** Determine $Var(5-3X)$

2. In commuting to work, a professor must first get on a bus near her house and then transfer to a second bus. The total waiting time Y in minutes can be shown to have the following PDF

$$f(Y=y)=\begin{cases}\frac{1}{25}y & 0\leq y<5\\ \frac{2}{5}-\frac{1}{25}y & 5\leq y\leq 10\\ 0 & otherwise\end{cases}$$

- **a.** What is the probability that total waiting time is at most 8 min?
- **b.** What is the probability that total waiting time is either less than 2 min or more than 6 min?
- **c.** What is the expected total waiting time for the professor?
- **d.** Determine the standard deviation in the total waiting time.

3. A certain market has both an express checkout line and a super-express checkout line. Let A denote the number of customers in line at the express checkout at a particular time of day, and let B denote the number of customers in line at the super-express checkout at the same time. Suppose the joint PMF of A and B is as given in the following table.

A \ B	0	1	2	3
0	0.08	0.07	0.04	0
1	0.06	0.15	0.05	0.04
2	0.05	0.04	0.1	0.06
3	0	0.03	0.04	0.07
4	0	0.01	0.05	0.06

- **a.** What is $P(A=B)$, that is, the probability that the numbers of customers in the two lines are identical?
- **b.** What is the probability that the total number of customers in the two lines is exactly four? At least four?
- **c.** Determine the marginal PMF of A and B and then calculate the expected number of customers in line at the express checkout.
- **d.** If at a given time there are 3 customers in line at the express checkout, what is the probability of 2 customers being in line at the super-express checkout?
- **e.** By inspection of the probabilities $P(A=4)$, $P(B=0)$, and $P(A=4,B=0)$, are A and B independent random variables? Explain.

Discrete Probability Distributions

4. There are 4 red balls, 6 blue balls and 2 white balls in a bag. Suppose on every single turn, you randomly select a ball, see the color of it and put it back in the bag. You keep doing this repeatedly.
   a. What is the probability that you get the first red ball on the 5th turn?
   b. How many turns are expected to get one non-white ball?
   c. What is the variance of the number of turns required to get one blue ball?

5. There are 4 red balls, 6 blue balls and 2 white balls in a bag. Suppose on every single turn, you randomly select a ball, see the color of it and put it back in the bag. Let's say, you do this 6 times.
   a. What is the probability that you get exactly 3 blue balls after 6 turns?
   b. What is the probability that you pick more than 4 blue balls after 6 turns?
   c. What is the mean number of red balls picked after 48 turns?
   d. What is the standard deviation of the number of white balls picked after 36 turns?

6. Suppose on average, Nepal experiences 6 earthquakes per year.
   a. What is the mean number of earthquakes in Nepal in the first four month of a year?
   b. What is the probability that there'll be 7 earthquakes in Nepal in the next two years?
   c. What is the probability that there'll be at least 9 earthquakes in Nepal in 2021?

Continuous Probability Distributions

7. The home loan department of BRAC Bank Limited sanction a significant number of loans per month. In this month the amount of money requested on home loan applications at a Bank follow a normal distribution with a mean of Tk. 73 lacs and a standard deviation of Tk. 22 lacs. A loan application is received this morning. Find the probability that:
   a. The amount requested is Tk. 75 lacs or more?
   b. The amount requested is Tk. 45 lacs or less?
   c. The amount requested is between Tk. 55 lacs and Tk. 90 lacs?
   d. The amount requested is exactly Tk. 65 lacs?
   e. The amount requested that would be in the lowest 8%?
   f. Also, what is the the median and mode amount?

8. A TV manufacturing company is planning to launch a new type of product recently. To check the lifespan status of their previous products they conduct a study. And they found that the lifetime of their previous products (plasma TV sets) follows an exponential distribution with a mean of 100,000 hours. Compute the probability a television set:
   a. Fails in less than 10,000 hours.
   b. Lasts more than 120,000 hours.
   c. Fails between 60,000 and 100,000 hours of use.
   d. Find the 90th percentile. So, 10 percent of the TV sets last more than what length of time?