

BRAC University
MAT-215
Practice Sheet # 3

1. Show that:

$$(i) \operatorname{Exp}(2 \pm 3\pi i) = -e^2 \quad (ii) \operatorname{Exp}\left(\frac{2 + \pi i}{4}\right) = \sqrt{\frac{e}{2}}(1 + i) \quad (iii) \operatorname{Exp}(z + \pi i) = -e^z.$$

2. Find all values of z such that:

$$(i) e^z = -2 \quad (ii) e^z = 1 + \sqrt{3}i \quad (iii) e^{2z-1} = 1 \quad (iv) e^z = -1.$$

3. Prove that:

$$(i) \sin z = \sin x \cosh y + i \cos x \sinh y$$

$$(ii) \cos z = \cos x \cosh y - i \sin x \sinh y$$

$$(iii) \sin(z + 2\pi) = \sin z$$

$$(iv) \cos(z + 2\pi) = \cos z.$$

4. Prove that: (i) $\sinh z = \sinh x \cos y + i \cosh x \sin y$

$$(ii) \cosh z = \cosh x \cos y + i \sinh x \sin y.$$

5. Show that: (i) $\sin^{-1} z = -i \ln[i z \pm (1 - z^2)^{1/2}]$

$$(ii) \cos^{-1} z = -i \ln[z \pm i(1 - z^2)^{1/2}].$$

6. Solve the following equations: (i) $\cosh z = \frac{1}{2}$ (ii) $\sinh z = i$.

7. Show that: (i) $\ln(-1 + \sqrt{3}i) = \ln 2 + 2\left(n + \frac{1}{3}\right)\pi i$

$$(ii) \ln(1 - i) = \frac{1}{2} \ln 2 + \left(2n + \frac{7}{4}\right)\pi i$$

$$(iii) \ln(i^{1/2}) = \left(n + \frac{1}{4}\right)\pi i.$$