26.11.23

CSE 461

"Lecture-11"

"Introduction to Control Theory Part-2".

→ Time Domain to Frequency Domain:

~~signals are~~

we use Laplace Transformation to convert time domain to frequency domain.

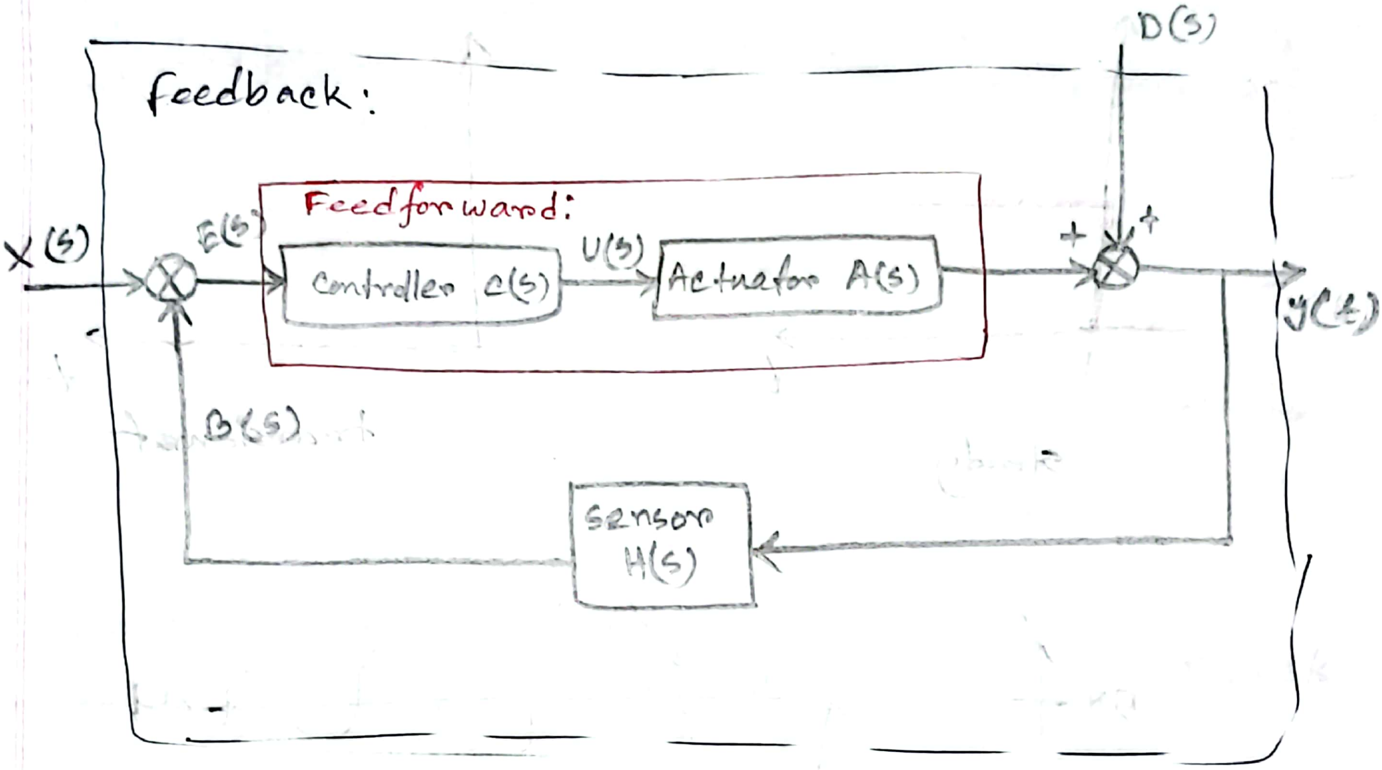
$\cancel{X(s) \xrightarrow{L.T} X}$

$$X(t) \xrightarrow{L.T.} X(s)$$

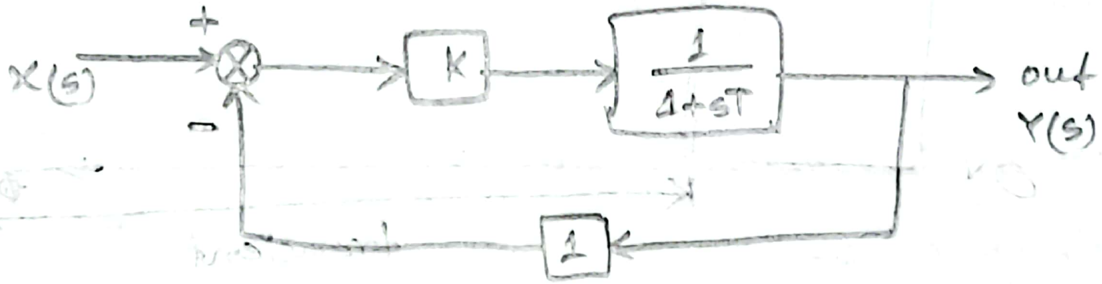
→ Key transfer function:

Feedback: $$\frac{Y(s)}{X(s)} = \frac{C(s) * A(s)}{1 + C(s) * A(s) * H(s)}$$

Feedforward: $\dfrac{Y(s)}{E(s)} = C(s)\,A(s)$



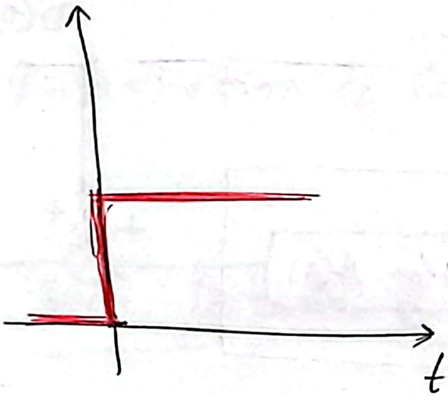
Ex:



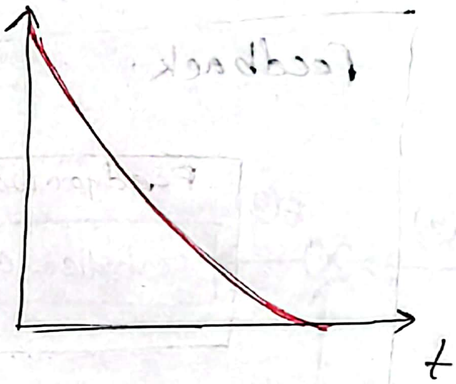
here, $Y(s) = \dfrac{K}{1+sT} * \left[X(s) - Y(s)\right]$

$\Rightarrow \dfrac{Y(s)}{X(s)} = \dfrac{K}{1+K+sT}$

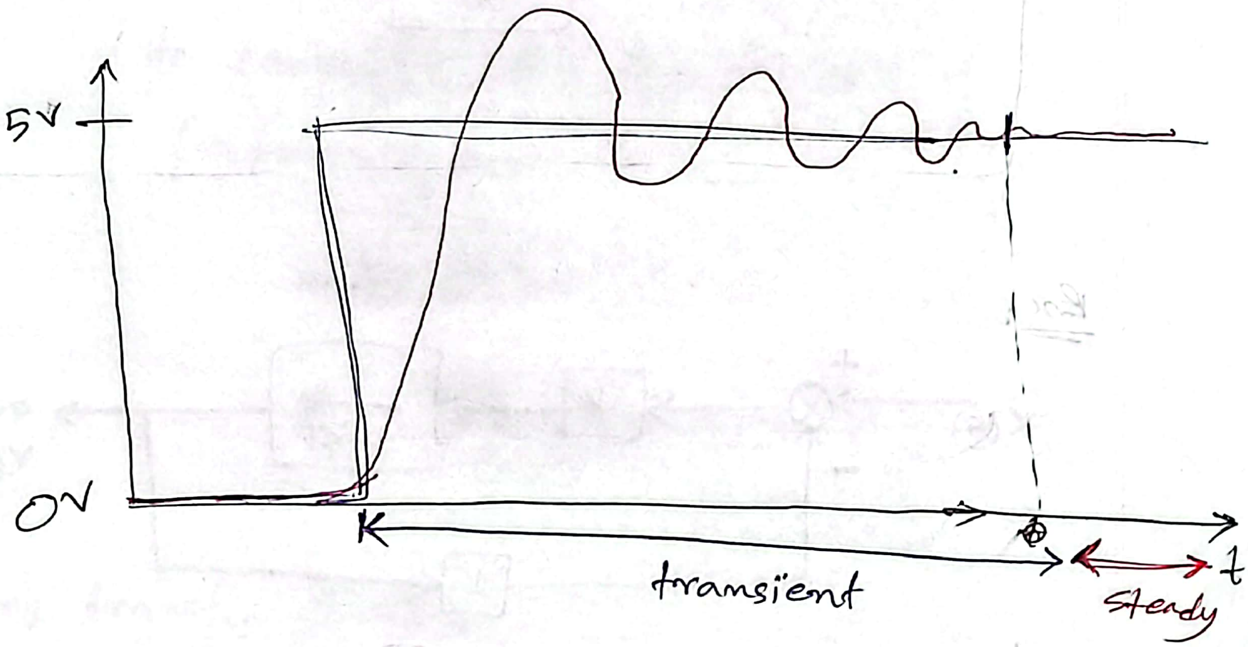
→ Steady State VS Transient :



steady

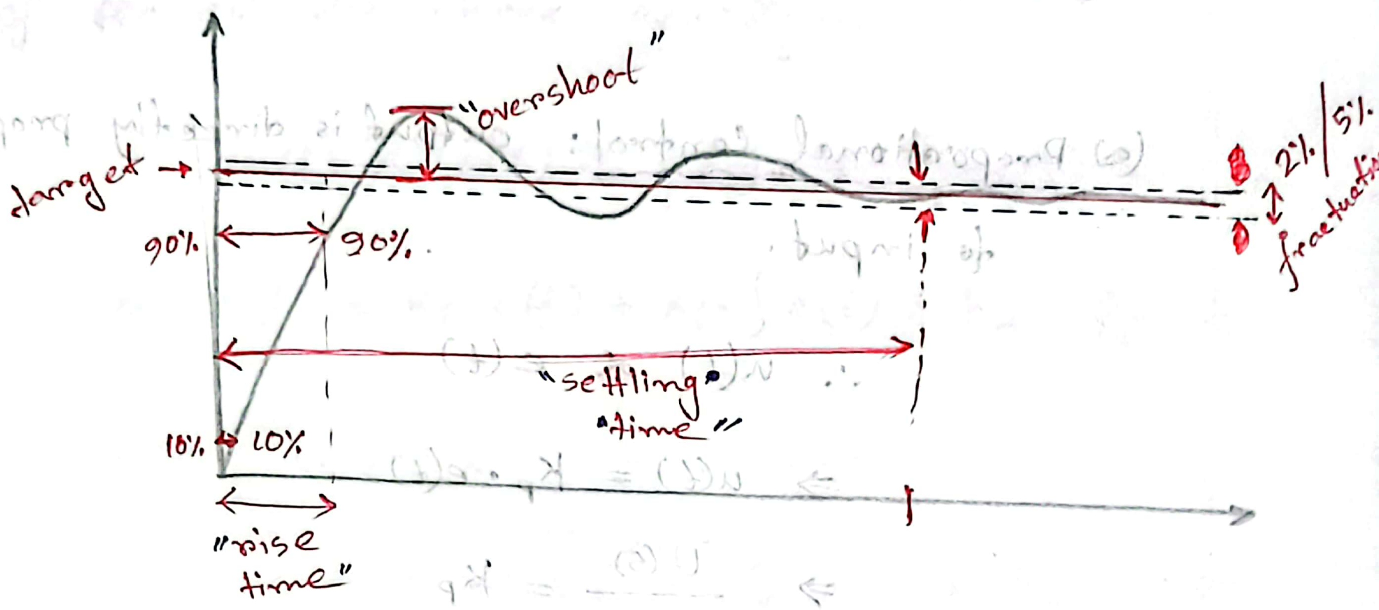


transient



Our goal is to reduce the time for the transient state.

→ System Response Graph:



(a) Overshoot: percentage of the highest peek value and target value.

$$\therefore \text{overshoot} = \frac{\text{highest peek} - \text{target}}{\text{target}} \times 100\%$$

(b) Rise time: time to reach from 10% to 90% of the target value. Difference btween them,

∴ rise time = (90% time − 10% time) of target

(c) Settling time: time to reach within 2% or 5% fractuation where the graph gets steady.

∴ settling time = (2%/5%/given value time)

→ PID Controller:

(a) **Proportional Control:** output is directly proportional to input.

$$\therefore \; u(t) \propto e(t)$$

$$\Rightarrow u(t) = K_p \cdot e(t)$$

$$\Rightarrow \boxed{\frac{U(s)}{E(s)} = K_p}$$

(b) **Integral Control:** output is the integral of input.

$$\therefore \; u(t) = K_i \cdot \int e(t)\,dt$$

$$\Rightarrow \boxed{\frac{U(s)}{E(s)} = \frac{K_i}{s}}$$

(c) **Differential Control:** output is the differential of input.

$$\therefore \; u(t) = K_d \cdot \frac{d}{dt} e(t)$$

$$\Rightarrow \boxed{\frac{U(s)}{E(s)} = K_d \cdot s}$$

So, it produces an output which is the combination of ~~the~~ all the three equations,

$$u(t) \propto e(t) + \int e(t) + \frac{d}{dt} e(t)$$

$$\Rightarrow u(t) = K_p \cdot e(t) + K_i \cdot \int e(t) + K_d \cdot \frac{d}{dt} e(t)$$

Using Laplace Transformation,

$$U(S) = K_p \cdot E(S) + \frac{K_i}{S} \cdot E(S) + K_d S \cdot E(S)$$

$$\Rightarrow U(S) = E(S) \left(K_p + \frac{K_i}{S} + K_d S \right)$$

$$\Rightarrow \frac{U(S)}{E(S)} = K_p + \frac{K_i}{S} + K_d S$$

$$\Rightarrow \boxed{\frac{U(S)}{E(S)} = \frac{K_p S + K_i + K_d S^2}{S}}$$

So, the K_p, K_i, K_d is the tuner. We have to tune the values of these three parameters to tune the PID.

How to get the values?

⇒ Assume that "K" is the Gain and the ~~oscilli~~ oscillation period is "p". Then we will use,

→ for $\boxed{P}$ controller,

$$\boxed{K_p = 0.5K}$$

→ for $\boxed{PI}$ controller,

$$\boxed{\begin{aligned} &K_p = 0.45K \\ &K_i = 1.2/p \end{aligned}}$$

→ for $\boxed{PID}$ controller,

$$\boxed{\begin{aligned} &K_p = 0.6K \\ &K_i = 2/p \\ &K_d = P/8 \end{aligned}}$$

have to memorize these equation