MAT120

Integral Calculus and Differential Equations

Assignment 01

Summer 23

Instructions:
Submit your assignment in A4 paper.Assignment should be handwritten.Please write your Name, ID and Section on the cover page of your assignment answer script. No late submission will be allowed after the class time.

Solve all problems.Answer the questions by yourself. Plagiarism will lead to an F grade in the course. Each question carries 10 mark. Total marks is 80 and it will be converted to 10.

1. Evaluate
$$\int_{\sqrt{e}}^{e} \frac{\ln x}{x^2} \, dx$$

2. Find a reduction formula for $\int \sec^n x \, dx$. By using this formula evaluate $\int \sec^5 x \, dx$.

3. Sketch the region whose signed area is represented by the definite integral, and evaluate the integral using an appropriate formula from geometry.

(a) $\int_{-1}^{2} |2x - 3| \, dx$

(b) $\int_{0}^{10} \sqrt{10x - x^2} \, dx$

4. Evaluate by using partial fraction decomposition

$$\int \frac{dx}{x^3 + 2x}$$

5. Evaluate the following improper integral. Also state that whether the integral is convergent or divergent.

$$\int_{-\infty}^{+\infty} \frac{e^{-t}}{1 + e^{-2t}} \, dt$$

6. Show that,

$$\Gamma(n) = 2 \int_{0}^{\infty} e^{-x^2} x^{2n-1} \, dx$$

7. Show that,

$$\int_{0}^{1} x^6 (1 - x^2)^{\frac{1}{2}} \, dx = \frac{5\pi}{256}$$

8. Find the length of the curve $x = e^t(\sin t + \cos t)$, $y = e^t(\cos t - \sin t)$, $(0 \leq t \leq 4)$.