Day 34
DC Machine Operation

DC Machine – Day 3

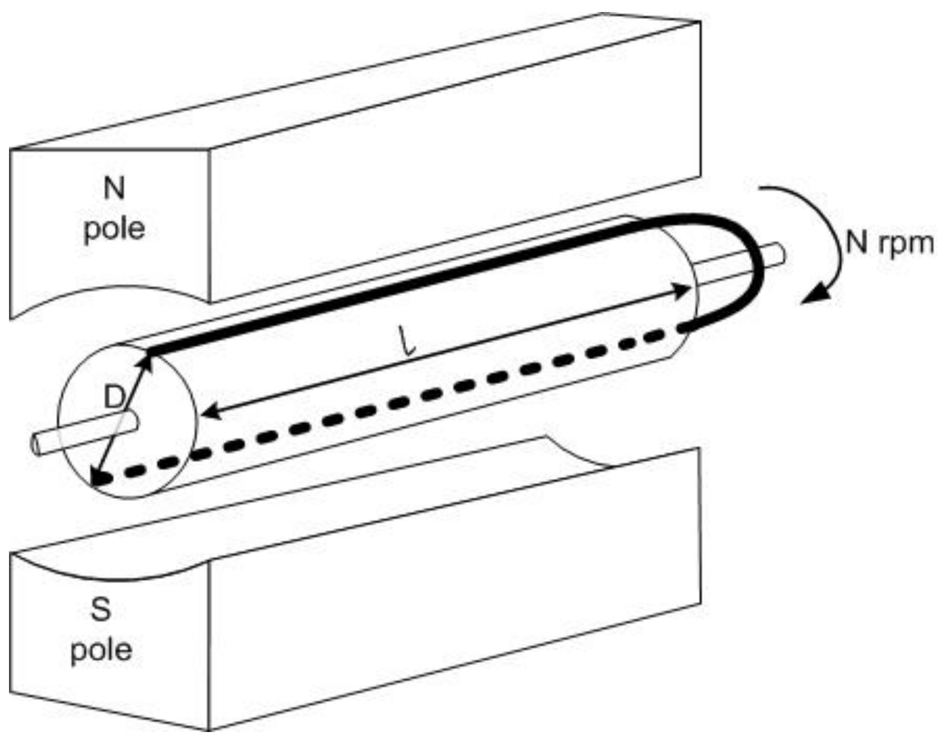
Day 3 Topics

Operation of DC machine

- Generation of EMF (Generator)
- Generation of torque (Motor)
- Circuit Symbol of DC machine
- Types of excitation

EMF induced in DC generator

- Consider the single coil with two coil sides (two conductors) in a 2-pole DC generator
- Flux density is not uniform throughout the surface of the armature
- The flux density is maximum under the poles, but reduces to zero in the interpolar region



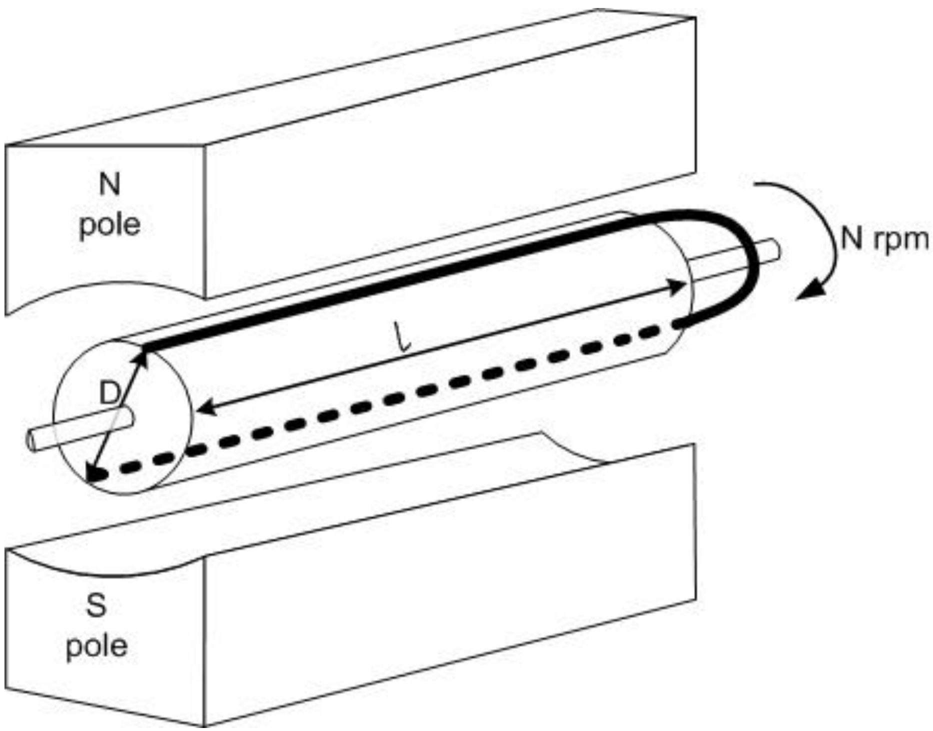
- As the armature rotates, flux linked by the conductors will also vary accordingly
- Thus, EMF induced in a conductor varies according to its relative position with respect to the poles

EMF induced in DC generator

- Thus, average value of the EMF induced in a single conductor is:

$$e_{av} = B_{av} l v$$

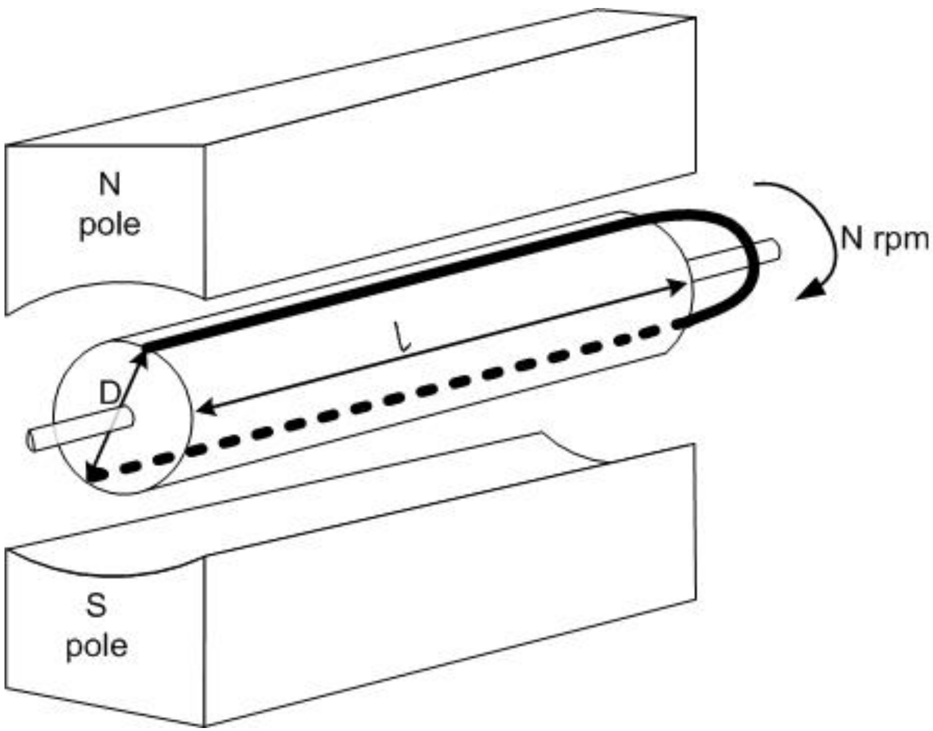
- Where, B_{av} is the average value of flux density around the armature surface, l is the conductor length, and v is the linear velocity (tangential velocity) of the conductor



EMF induced in DC generator

- If ϕ is the flux per pole and P is the total number of poles in the machine, then total flux around the armature surface is = $\phi \times P$
- Thus, average value of flux density on armature surface is given by:

$$B_{av} = \frac{Total\ \ flux}{Total\ surface\ area\ of\ \ armature} = \frac{\phi P}{\pi D l}$$



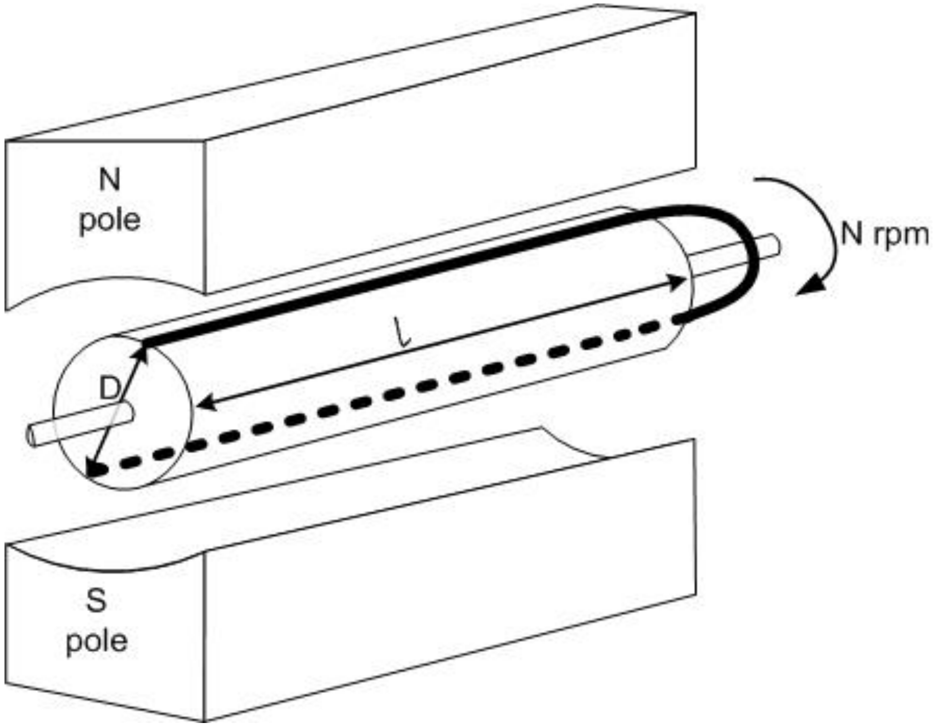
EMF induced in DC generator

$$B_{av} = \frac{\phi P}{\pi D l}$$

Thus, induced EMF per conductor is: $e_{av} = B_{av} l v = \frac{\phi P}{\pi D l} l v = \frac{\phi P v}{\pi D}$

When N = Rotational speed of armature in rpm, then angular speed (rad/s)is:

$$\omega = \frac{2\pi N}{60} \text{ rad/s}$$



Thus, linear speed (m/s)

$$v = \omega \times \frac{D}{2} = \frac{2\pi N}{60} \times \frac{D}{2} = \frac{\pi D N}{60} \; m/s$$

$\therefore$ Induced EMF per conductor:

$$e_{av} = \frac{\phi P v}{\pi D}$$

$$e_{av} = \frac{\phi P}{\pi D} \times \frac{\pi D N}{60} = \frac{\phi P N}{60}$$

EMF induced in DC generator

Induced EMF per conductor: $e_{av} = \dfrac{\phi PN}{60}$

If there are *Z* number of total conductors on the armature surface and *a* is the number of parallel paths, then the effective number of conductors per parallel path is = *Z/a*

a = P for Lap winding
a = 2 for wave winding

Thus, total EMF induced in a DC generator is:

$$E = e_{av} \times \frac{Z}{a} = \frac{\phi PN}{60} \times \frac{Z}{a} = \frac{\phi PNZ}{60a}$$

This induced EMF equation in a DC generator is popularly represented by the expression:

$$E = K\phi N$$

$K = \dfrac{PZ}{60a}$ is constant for a given machine and is called the generator constant or speed constant

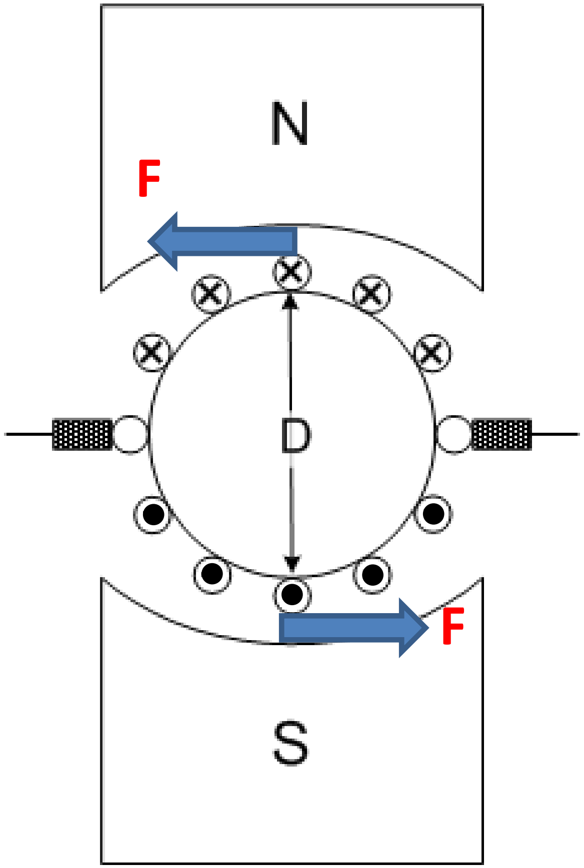
Torque developed in DC motor

- In DC motors, electrical energy is converted to mechanical energy which is available in the form of a torque at the output shaft fitted to the armature

- As soon as electric supply is given and current flows through the armature, a force is developed on the armature conductors due to interaction of the main flux and the armature current

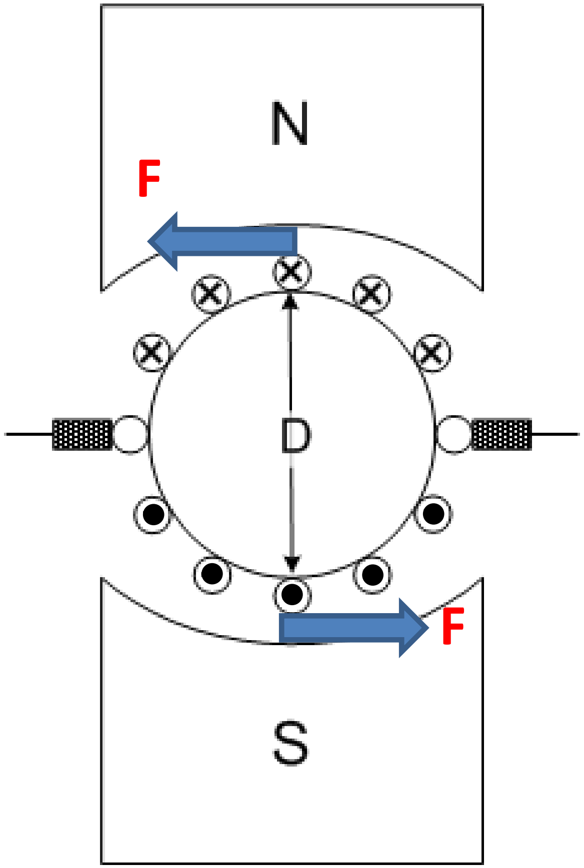
- The direction of this force acting on the armature conductors can be determined using Fleming's left hand rule

Torque developed in DC motor



- Say, conductors under N pole carry ‘cross’ current
- Those under the S pole carry ‘dot’ current
- Fleming’s left hand rule
- The two diametrically opposite conductors experience force in opposite directions
- These two forces are separated by the distance D
- Thus, a torque is developed and the armature rotates

Torque developed in DC motor



- Average force acting on a single conductor:

$$F_{av} = B_{av} I_C l$$

B_{av} = Average flux density on armature surface

I_C = Current per conductor

l = Active length of conductor

$$B_{av} = \frac{Total\ \ flux}{Total\ surface\ area\ of\ \ armature} = \frac{\phi P}{\pi D l}$$

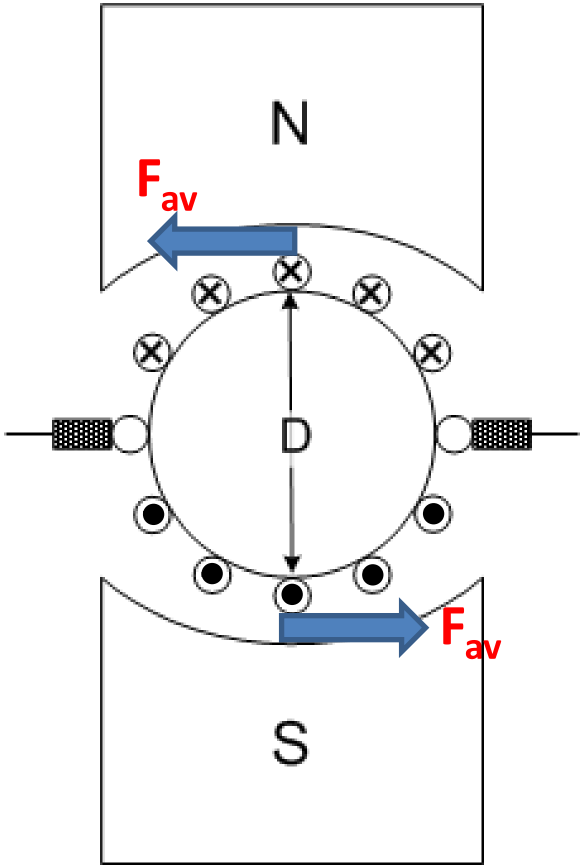
ϕ = Flux per pole

P = Number of poles

D = Armature diameter

$$\therefore \quad F_{av} = \frac{\phi P}{\pi D l} \times I_C \times l = \frac{\phi P I_C}{\pi D}$$

Torque developed in DC motor



$$F_{av} = \frac{\phi P I_C}{\pi D}$$

Two such conductors placed in diametrically opposite location on armature surface will experience the same force F_{av} and thus produce a torque:

$$T_{av} = F_{av} \times D = \frac{\phi P I_C}{\pi D} \times D = \frac{\phi P I_C}{\pi}$$

If there is Z number of total conductors on the armature surface, then the total torque produced by them is (two opposite conductors taken at a time):

$$T = \frac{\phi P I_C}{\pi} \times \frac{Z}{2}$$

Torque developed in DC motor

$$T = \frac{\phi P I_C}{\pi} \times \frac{Z}{2}$$

If a is the number of parallel paths, then the total armature current I_a gets divided into those a number of parallel paths and current through each conductor $I_C = I_a/a$

a = P for Lap winding, a = 2 for wave winding

Thus, total torque developed: $T = \frac{\phi P}{\pi} \times \frac{I_a}{a} \times \frac{Z}{2}$ $= \frac{\phi P I_a Z}{2\pi a}$

This developed torque equation in a DC motor is popularly represented by the expression:

$$T = K_t \phi I_a$$

$K_t = \frac{PZ}{2\pi a}$ is constant for a given machine and is called the motor constant or the torque constant

Counter Torque and Back EMF

- In a ***DC generator*** the armature is rotated by an external mechanical prime mover in the magnetic field developed by the main poles
- Thus an **EMF is induced** in the armature coils
- In addition to the EMF, a generator develops a torque also

$$T = K_t \phi I_a$$

- According to Lenz's law, direction of this developed torque is opposite to the direction of rotation
  - i.e. the developed torque opposes the input torque provided by the prime mover
- This opposing torque developed in a DC generator is called the ***counter-torque*** or ***back-torque***

A DC Generator thus produces both EMF and torque
- EMF in forward direction
- Torque in reverse direction (back torque)

Counter Torque and Back EMF

- In ***DC motor***, the armature is fed by an external DC voltage source
- As current passes through the armature coils, due to interaction of this current and the main pole flux, a **torque is developed** on the armature
- This torque makes the armature to rotate
- In addition to the torque, a motor generates an EMF also

$$E_b = K\phi N$$

- According to Lenz's law, direction of this EMF in armature is such that it opposes the applied voltage
- That is why this induced EMF is called ***counter EMF*** or ***back EMF***

A DC Motor thus produces both torque and EMF

- Torque in forward direction
- EMF in reverse direction (back EMF)

Relationship between Induced EMF and Developed Torque

$$T = \frac{\phi P Z I_a}{2\pi a} = \frac{PZ}{2\pi a} \times \phi \times I_a = K_t \phi I_a \qquad K_t = \frac{PZ}{2\pi a}$$

$$E = \frac{\phi P Z N}{60a} = \frac{PZ}{2\pi a} \times \phi \times \left(\frac{2\pi N}{60}\right) = K_t \phi \omega$$

Mechanical power

$$\frac{T}{E} = \frac{K_t \phi I_a}{K_t \phi \omega} = \frac{I_a}{\omega} \Rightarrow T \times \omega = E \times I_a$$

Electrical power

- This is the power balance equation of a DC machine
- It shows that in case of a generator, the mechanical power input is equal to the electrical power developed in the armature
- In case of a motor, the electrical power input is equal to the mechanical power developed at the armature

#1) The armature of a 4-pole DC machine has 150 turns and rotates at 1200 rpm. The EMF generated on open circuit is 400 V. Find the useful flux per pole when the armature is (i) lap connected (ii) wave connected

Working formula?? $E = \frac{\phi P N Z}{60a}$

Unknown quantity?? ϕ

Given,

Number of poles P = 4

Number of turns = 150, $\therefore$ number of conductors Z = 2×150 = 300

Speed N = 1200 rpm

Open circuit voltage = Induced EMF = 400 V

(i) For lap connection, number of parallel paths: a = P = 4

$$E = \frac{\phi P N Z}{60a} \Rightarrow 400 = \frac{\phi \times 4 \times 1200 \times 300}{60 \times 4} \Rightarrow \phi = 0.067 \text{ Wb (per pole)}$$

(ii) For wave connection, number of parallel paths: a = 2

$$E = \frac{\phi P N Z}{60a} \Rightarrow 400 = \frac{\phi \times 4 \times 1200 \times 300}{60 \times 2} \Rightarrow \phi = 0.033 \text{ Wb (per pole)}$$

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48. A 8 pole, DC generator has a simplex wave-wound armature containing 32 coils of 6 turns each. Its flux per pole is 0.06 Wb. The machine is running at 250 rpm. The induced armature voltage is

(a) 96V (b) 192V (c) 384V (d) 768V

Given, P = 8

Z = 32x2x6 = 384

ϕ = 0.06 Wb

N = 250 rpm

a = 2 (wave connected)

ANS: (c)

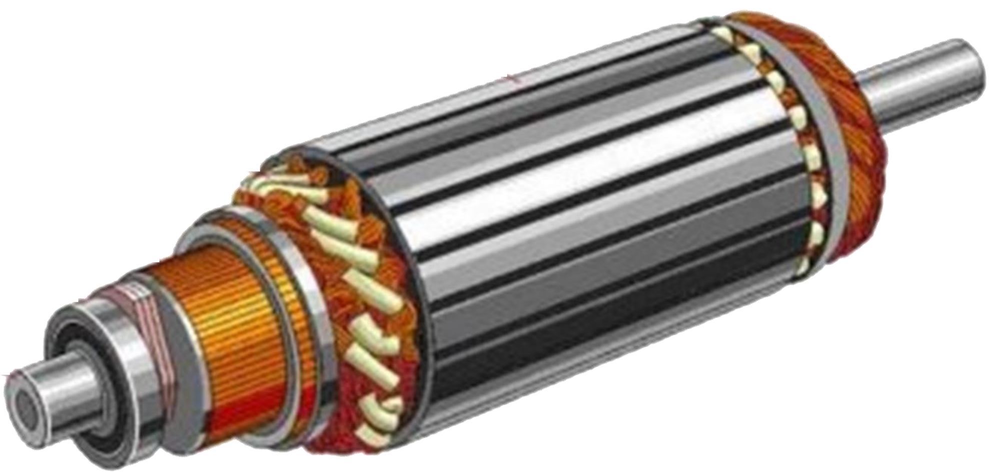
$$E = \frac{\phi PNZ}{60a} = \frac{0.06 \times 8 \times 250 \times 384}{60 \times 2} = 384\,V$$

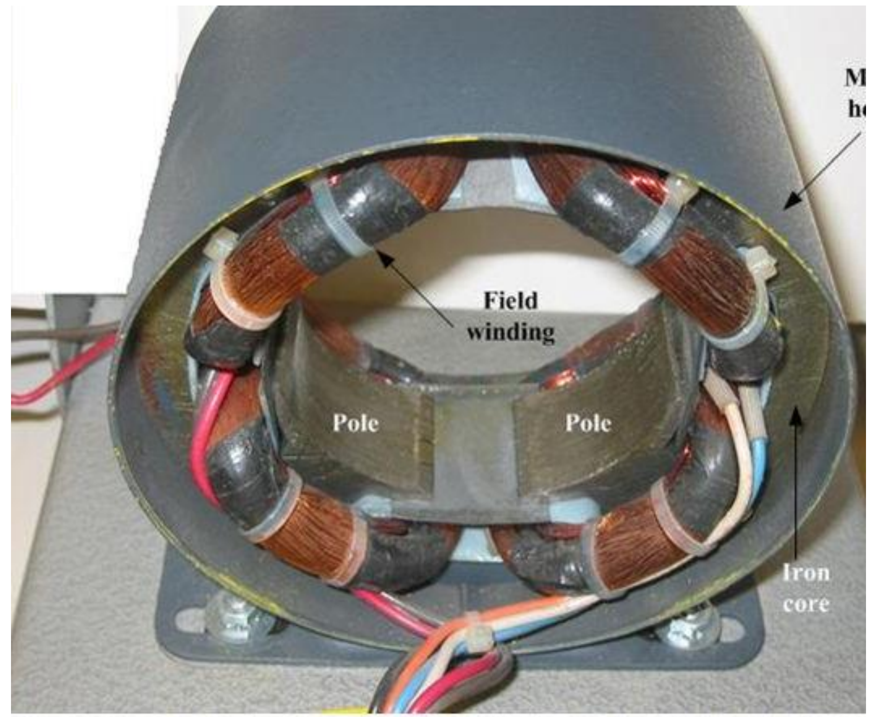
DC Machine – Day 3

Day 1 Recapitulation

Construction of DC machine

- Stator (poles) + yoke +…..
- Rotor (armature) + ….





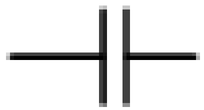
Circuit Symbol
of
DC machine

Circuit model of DC machine

- Circuit model useful for analyzing operation and performance

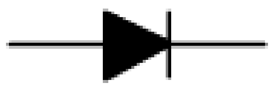
Resistance

Capacitance



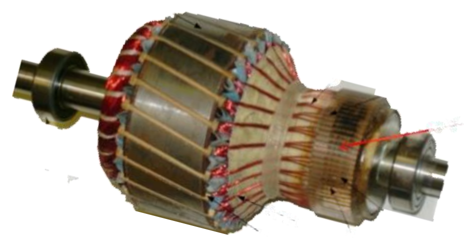
Inductance

Diode

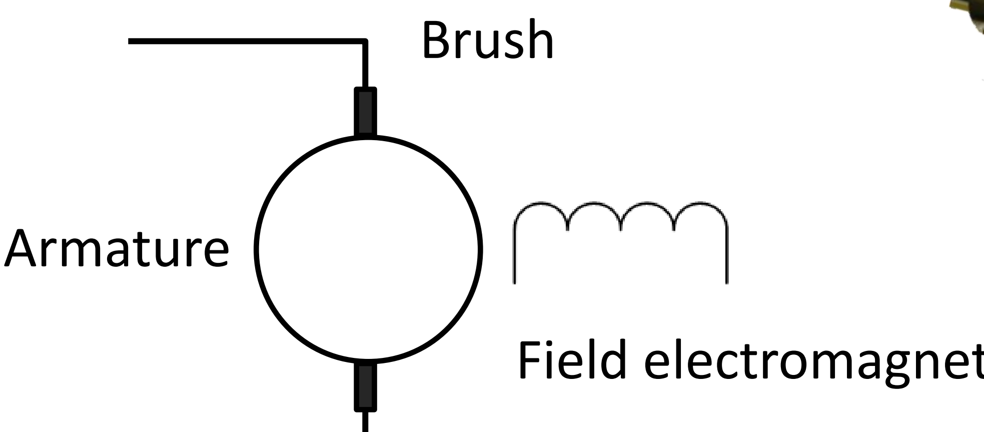


DC Machine

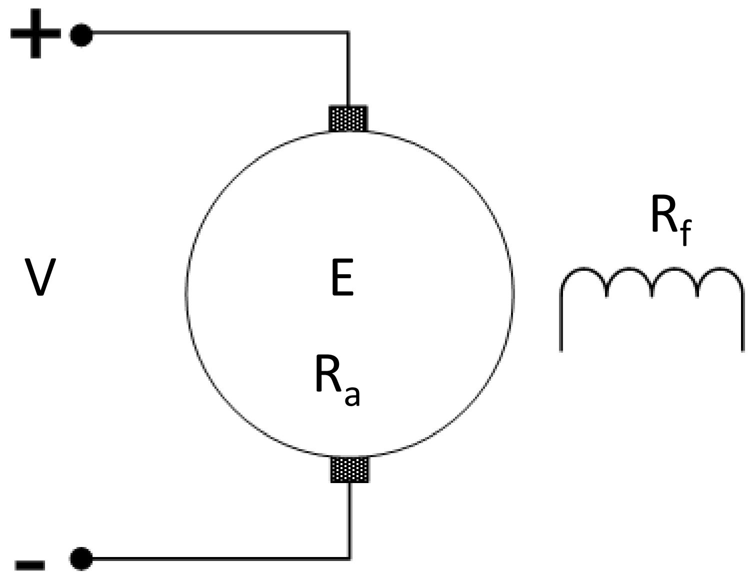
Poles (electromagnet)



Armature



Circuit model of DC machine



Induced EMF for generator = E

(or Back EMF for a motor)

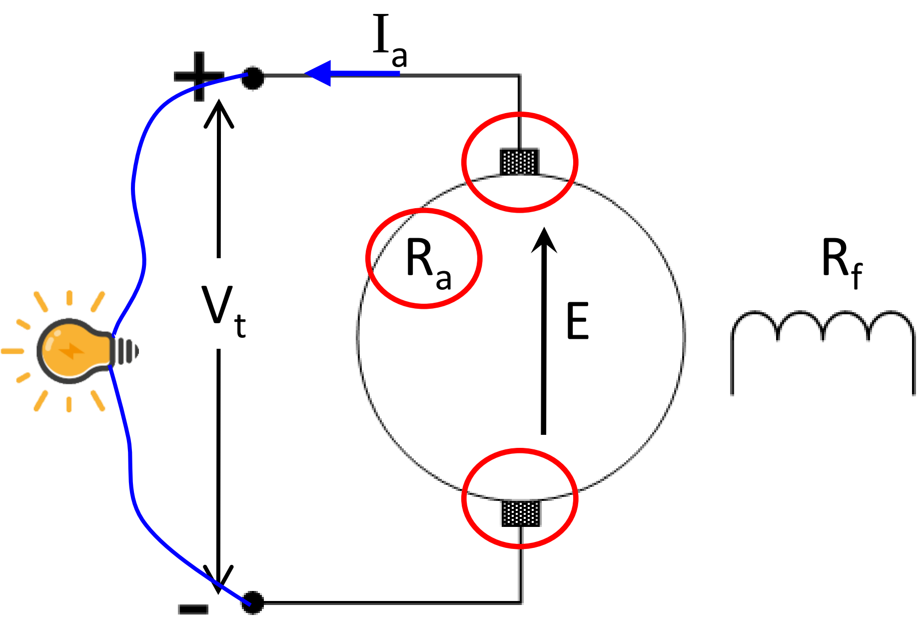
Armature winding resistance = R_a

Field winding resistance = R_f

Terminal voltage = V

(Output voltage for generator and input voltage for motor)

Circuit model of DC machine



Generator Mode

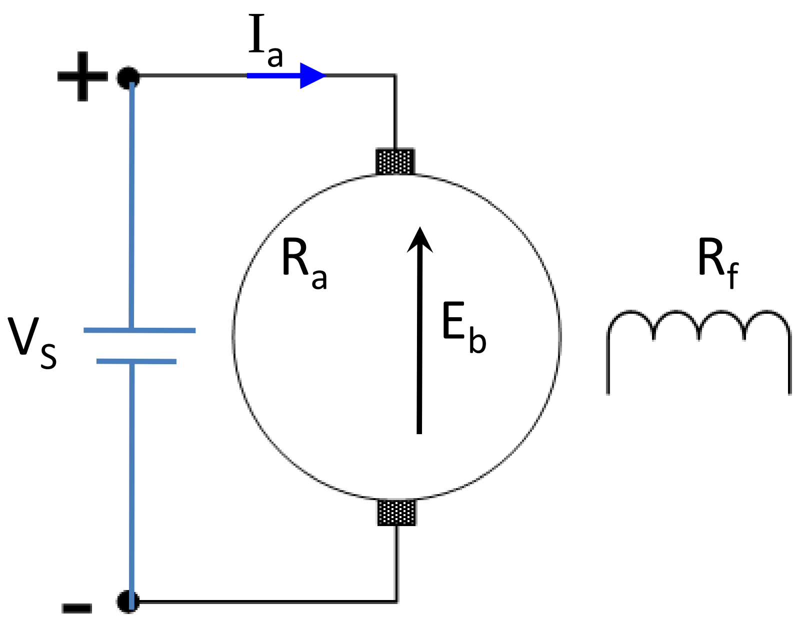
- The induced EMF E is available at the generator output terminals when it is under no-load, that is, its terminals are open circuited
- When some load is connected to the generator output terminals, an armature current I_a starts flowing out

- The output terminal voltage V_t now becomes lower than the induced EMF E due to voltage drop in the armature resistance R_a
- In addition, there is a voltage drop of $V_b \approx 1$ V per brush (*which sometimes is neglected*)

$$V_t = E - I_a R_a - V_b \approx E - I_a R_a \quad (V_b \text{ neglected})$$

- Thus, in a DC generator, Induced *EMF* $\geq$ *Terminal voltage*

Circuit model of DC machine



Motor Mode

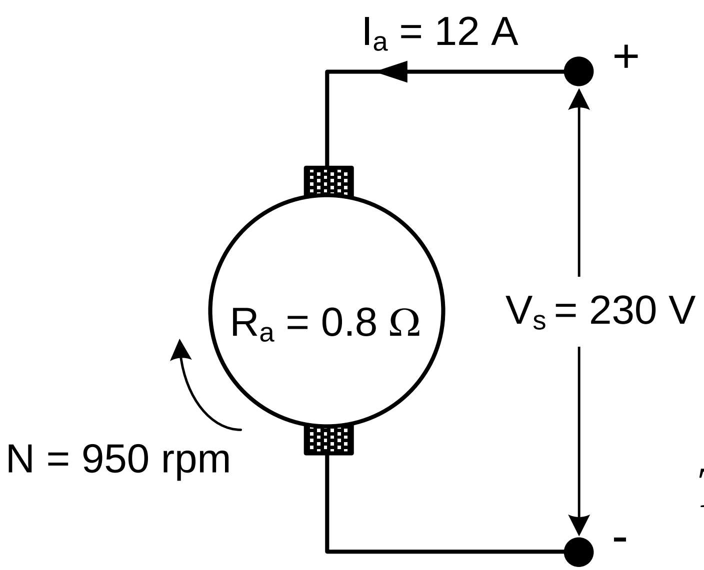
- DC supply V_s is given to the terminals of the machine
- This drives the armature current I_a inwards from the terminals towards the machine armature
- Electrical energy thus flows from external supply towards the armature of the machine

- Due to rotation of the armature in the magnetic field set up by the poles, **back EMF** (E_b)is induced in the armature
- This back EMF is equal and opposite to the remaining amount of supply voltage available at armature after drop in the armature circuit resistance

$$E_b = V_S - I_a R_a - V_b \approx V_S - I_a R_a \quad (V_b \text{ neglected})$$

- Thus, in a DC motor, *Terminal voltage* $\geq$ *Back EMF*

#1) The armature supply voltage of a DC motor is 230 V. The armature current is 12A, the armature resistance is 0.8 Ω, and the speed is 950 rpm. Calculate (i) the induced EMF (ii) the electromagnetic torque



(i) Induced EMF (back EMF)

$$E_b = V_S - I_a R_a = 230 - 12 \times 0.8 = 220.4\,V$$

(ii) Electromagnetic torque

$$T \times \omega = E_b \times I_a$$

$$T = \frac{E_b \times I_a}{\omega} = \frac{E_b \times I_a}{\frac{2\pi N}{60}} = \frac{220.4 \times 12}{\frac{2\pi \times 950}{60}} = 26.59\ \text{N-m}$$

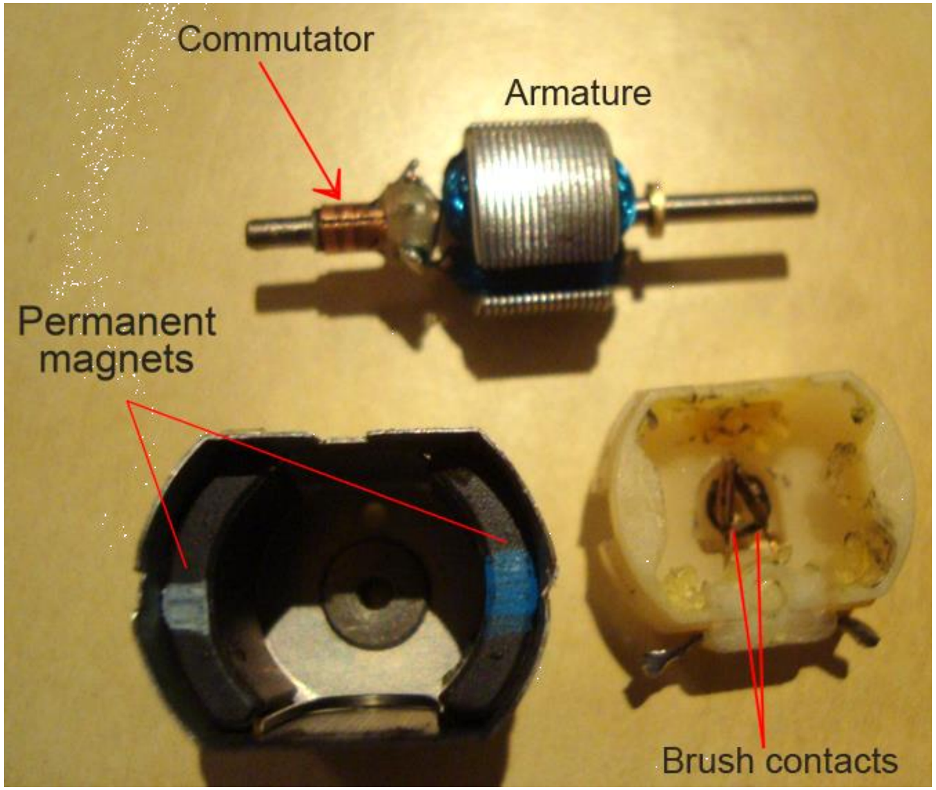
Step1: Draw the circuit
Step 2: Identify the working formulae
Step 3: Identify the known quantities
Step 4: Calculate the unknown

Types of Excitation
in
DC machine

Excitation in DC machines

Excitation in a DC machine means the way in which the main poles are excited to produce magnetic flux

- The easiest way is to use permanent magnets to work as the North and South poles
- Such machines are called *permanent magnet DC machine*



- Strength of the magnetic poles in such permanent magnet machines remain constant
- However, permanent magnets are very costly
- So, *permanent magnets* are used only for small DC machines

Excitation in DC machines

- All bigger size and industrial DC machines use electromagnets as their main poles
- Coils made of copper conductors are wound over the pole body
- The field coils are energized from DC voltage source
- Current passing through these field windings provides the required MMF that creates the magnetic flux

Excitation in DC machines

Types of excitation

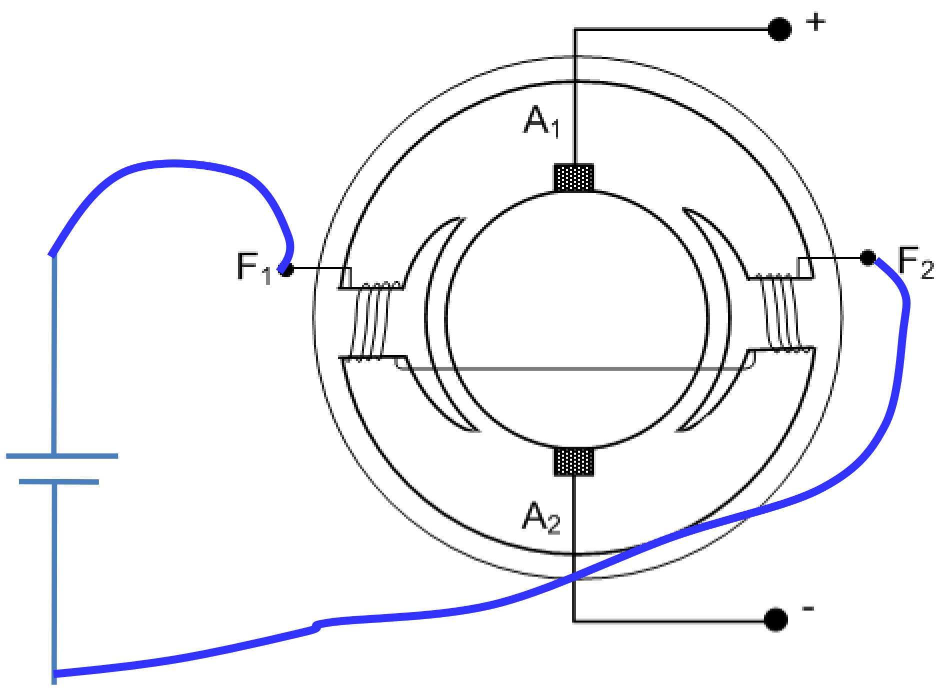
Separate excitation

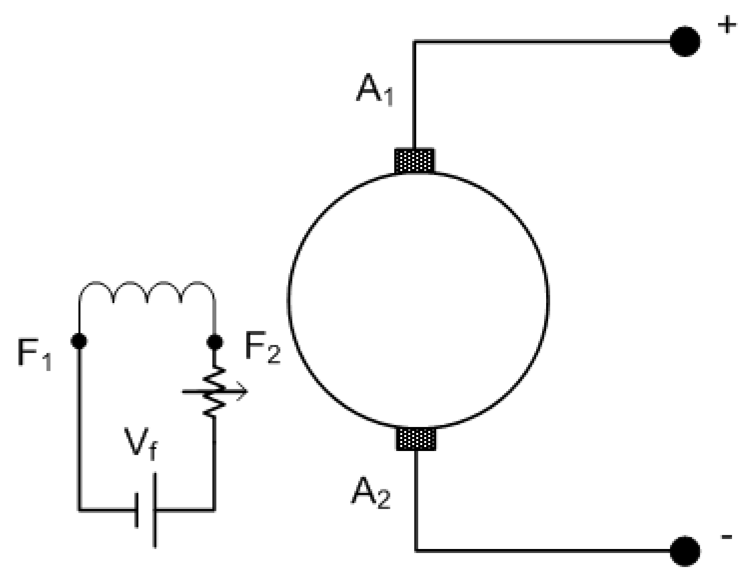
Self excitation

Excitation in DC machines

Separate excitation

- In separately excited DC machines, the field winding is supplied from a separate DC voltage source such as a battery or a rectified AC voltage source
- A rheostat is generally connected in the field circuit for controlling the field current and hence the main pole flux





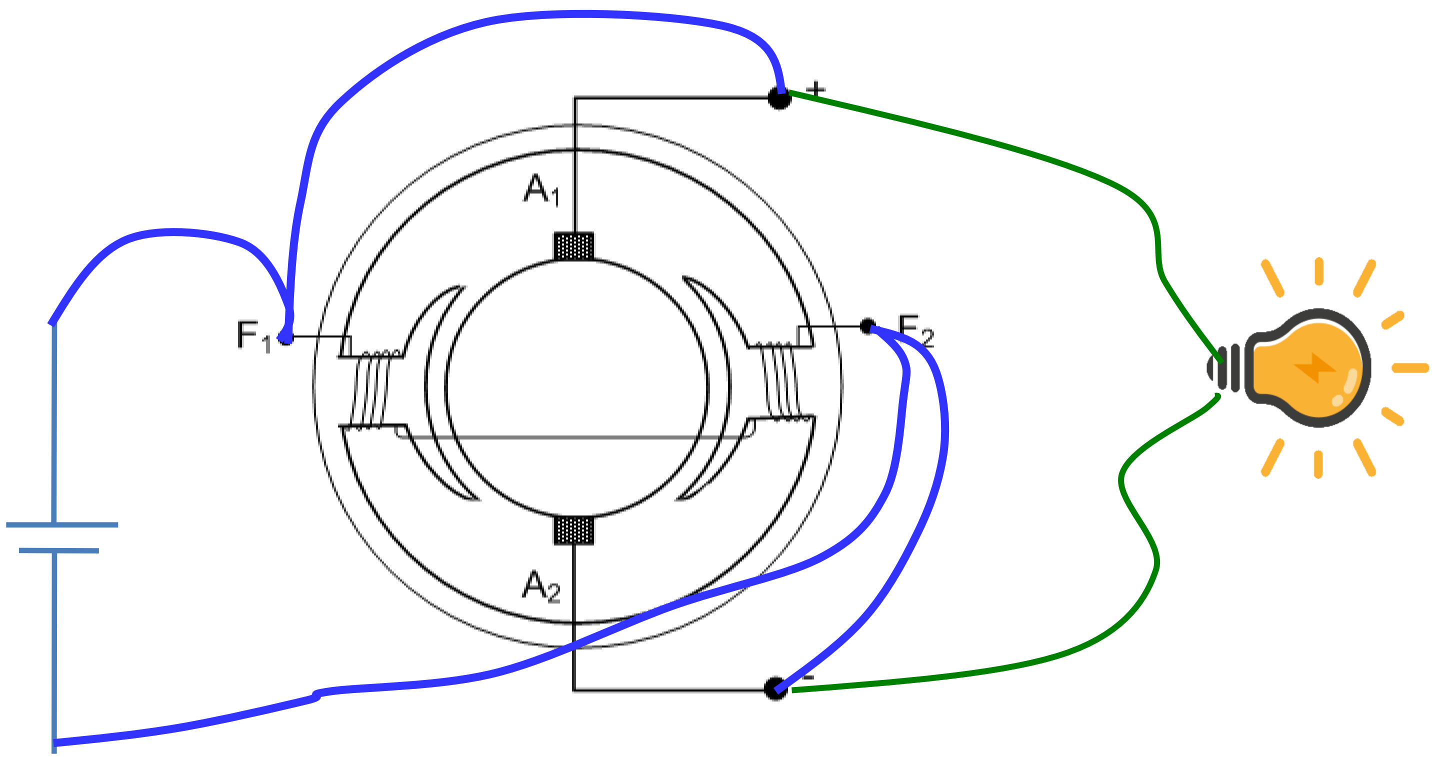
Excitation in DC machines

Self excitation

- In self-excited DC machines, no separate DC voltage source is used for supplying the field coils

Q1) Generator is producing what type of voltage? AC or DC?

Q2) Field winding requires what type of voltage? AC or DC?



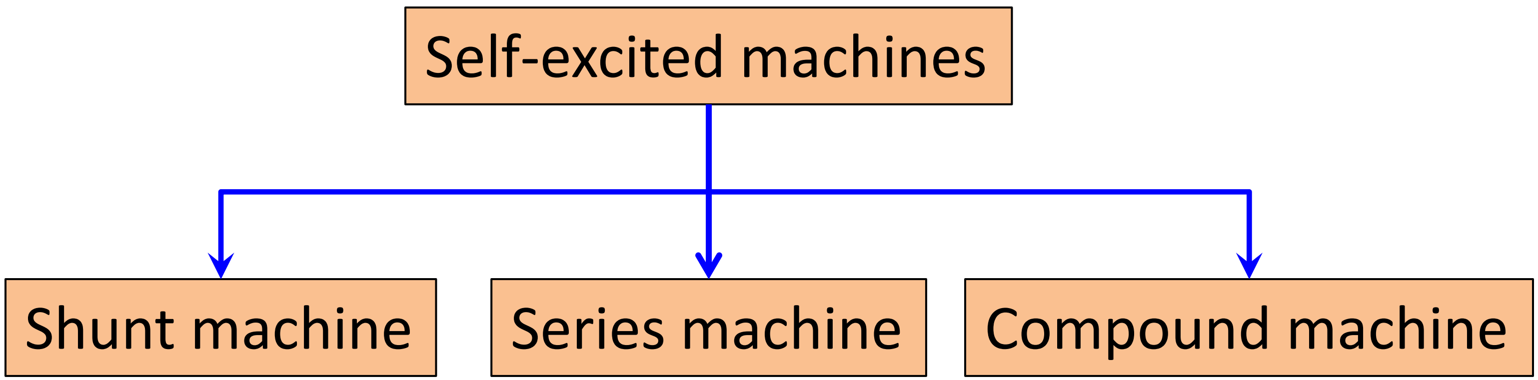
Excitation in DC machines

Self excitation

- In self-excited DC machines, no separate DC voltage source is used for supplying the field coils
- In self-excited generators, a part or whole of the generated voltage or current in the armature is used for exciting the field winding
- Such a connection enables the machine to develop its own field, and hence the name *self-excitation*
- In such machines, the field winding is interconnected with the armature

Excitation in DC machines

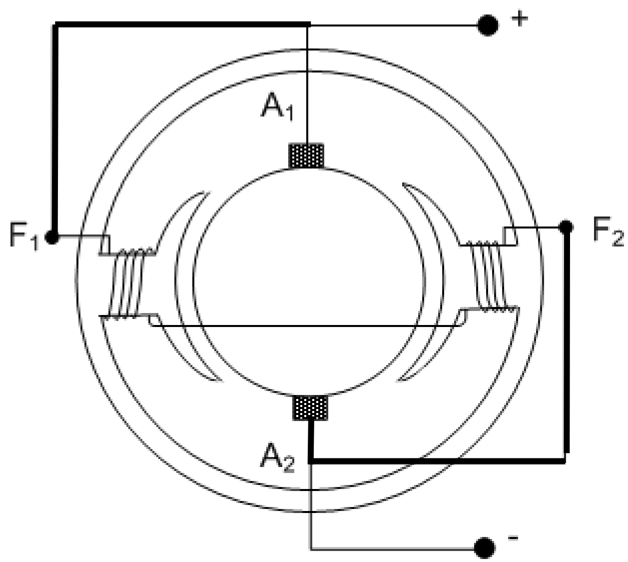
Self excitation



Excitation in DC machines

Shunt machines

- The field coil is connected in parallel (shunt) with the armature so that the **entire armature voltage** is available across the field winding

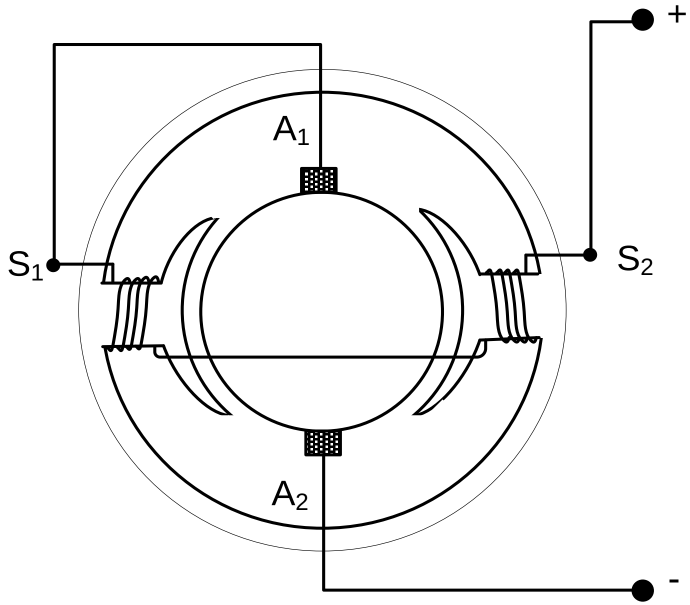


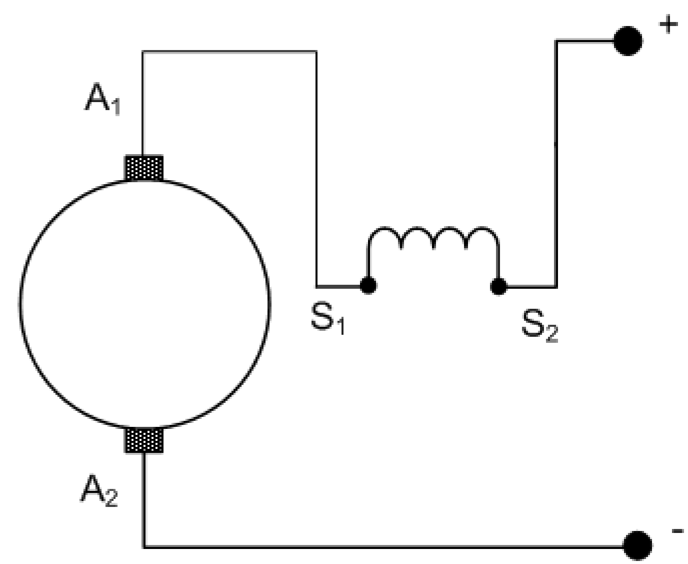
- In shunt generators, a **part of the armature current** branches off to the field winding and the remaining current is fed to the load
- In shunt motors, a **part of the supply current** flows through the field winding and the remaining current goes to the armature

Excitation in DC machines

Series machines

- The field coil is connected in series with the armature so that the **entire armature current** flows through the field winding



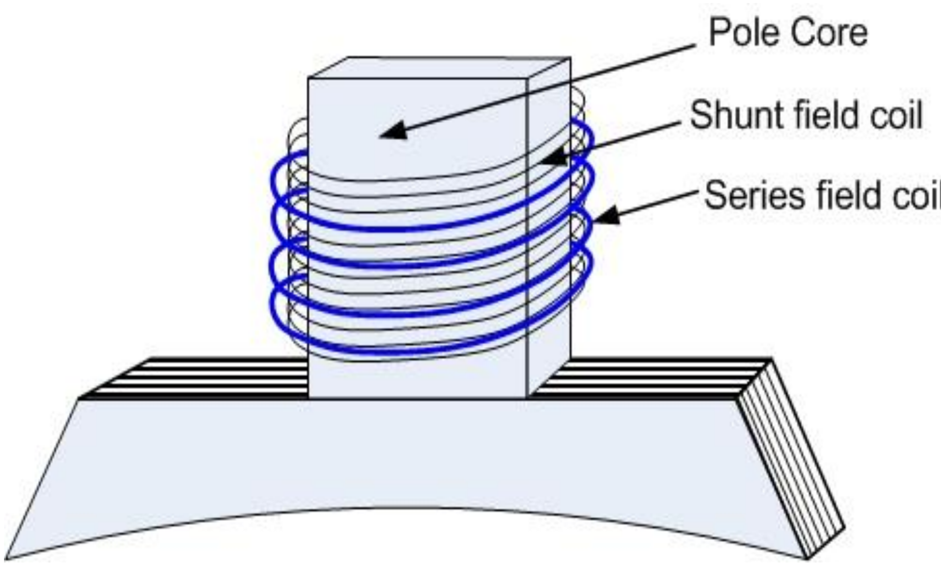


- In series generators, a **part of the EMF induced in armature** is consumed by the field winding (voltage drop) and the remaining voltage is supplied to the load
- In series motors, a **part of the supply voltage** is dropped across the field winding and the remaining is available for the armature

Excitation in DC machines

Compound machines

- A combination of shunt and series excitation can also be used and it is called *compound excitation*
- In compound machines, two field windings (two separate coils on the pole core) are used



- The coil with thinner conductors and large number of turns is used as the shunt field coil
- The coil with thicker conductors and less number of turns is used as the series field coil