Kirchhoff's Laws

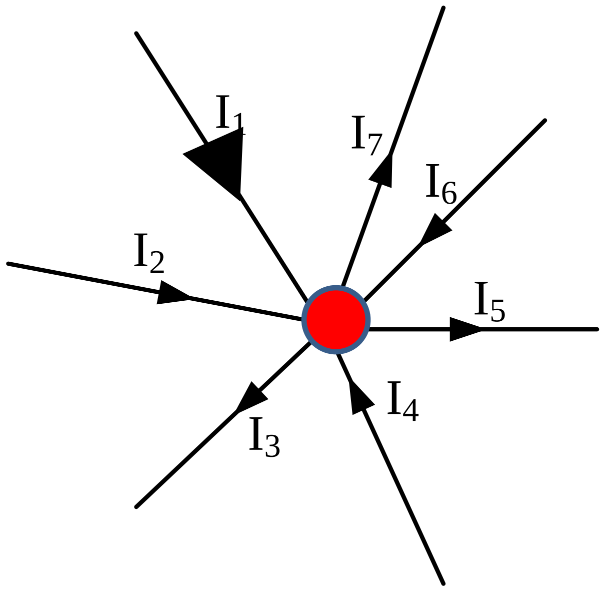
- ILO – Day5
  - State, explain and apply
    - Kirchhoff’s current law (KCL)
    - Kirchhoff’s voltage law (KVL)

Kirchhoff’s Current Law (KCL)

- *According to KCL:*
- *“the **algebraic summation** of all currents at a node in a circuit is **zero**”*
- Node means junction point where more than one elements are connected
- *If the incoming currents towards the node are shown with +ve sign, the outgoing currents flowing away from the node should be –ve*
- *(This only is a convention, and the opposite sense can also be used)*

Kirchhoff's Current Law (KCL)

- *If the incoming currents towards the node are shown with +ve sign, the outgoing currents flowing away from the node should be –ve.*



Currents in the node:

(*maintain polarity of the currents)*

$+I_1$ (incoming)

$+I_2$ (incoming)

$-I_3$ (outgoing)

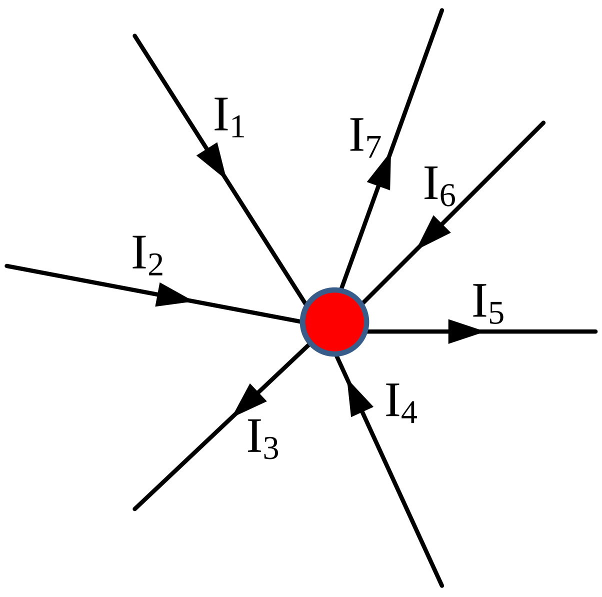
$+I_4$ (incoming)

$-I_5$ (outgoing)

$+I_6$ (incoming)

$-I_7$ (outgoing)

Kirchhoff's Current Law (KCL)



$+I_1$ (incoming)

$+I_2$ (incoming)

$-I_3$ (outgoing)

$+I_4$ (incoming)

$-I_5$ (outgoing)

$+I_6$ (incoming)

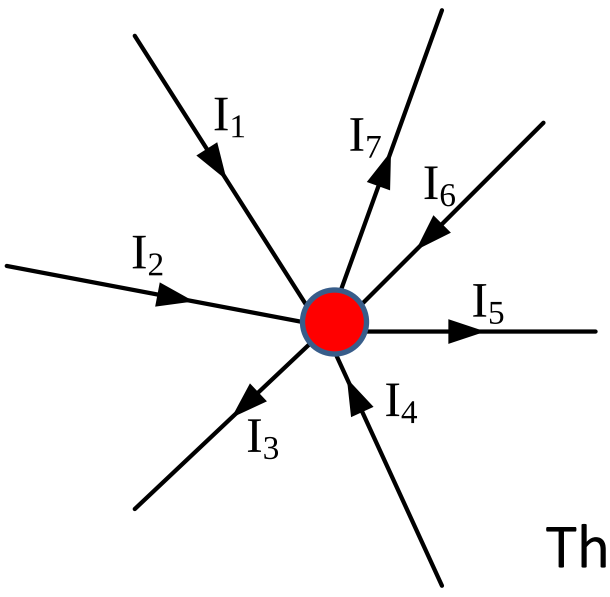
$-I_7$ (outgoing)

Summation of all currents in the node

$$I_1 + I_2 - I_3 + I_4 - I_5 + I_6 - I_7$$

According to KCL: $\boxed{I_1 + I_2 - I_3 + I_4 - I_5 + I_6 - I_7 = 0}$

Kirchhoff's Current Law (KCL)



$$\boxed{I_1 + I_2 - I_3 + I_4 - I_5 + I_6 - I_7 = 0}$$

Rearrange the equation

$$\boxed{I_1 + I_2 + I_4 + I_6 = I_3 + I_5 + I_7}$$

The above equation gives an alternate statement of the KCL as:

"Algebraic summation of all ***incoming currents*** *to a node must equal the algebraic summation of all* ***outgoing currents*** *from that node."*

Kirchhoff's Voltage Law (KVL)

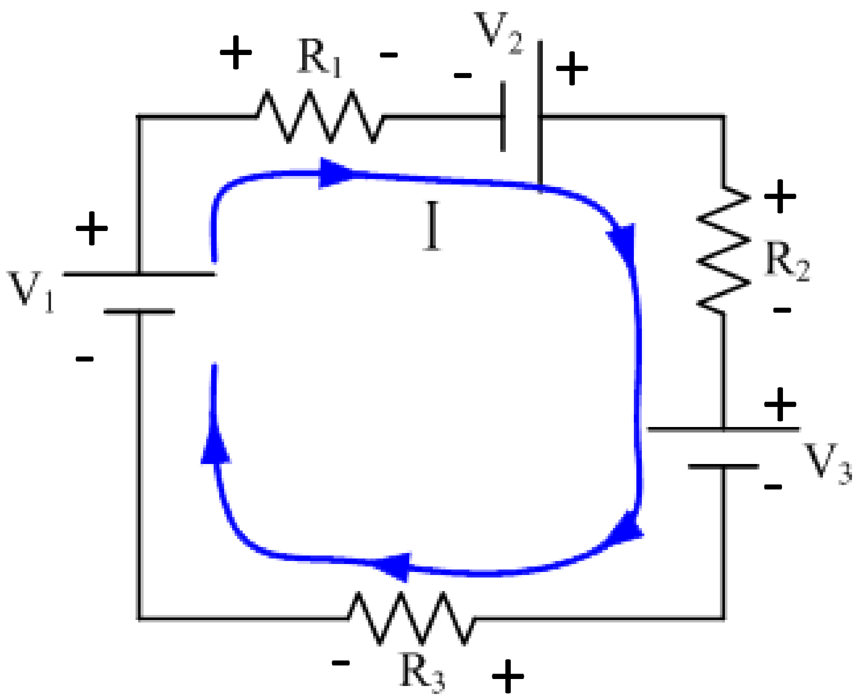
- *According to KVL:*

- *"the **algebraic summation** of all voltages in a closed circuit (loop/mesh) traversed in a single direction is **zero**"*

- *When current flows from +ve potential to –ve potential, we call that voltage drop and represent by –ve sign, while when the current flows from –ve potential to +ve potential, we call that voltage rise and represent it by +ve sign.*
- *This only is a convention, and the opposite sense can also be used.*

Kirchhoff's Voltage Law (KVL)

- *When current flows from +ve potential to –ve potential, we represent it by –ve sign (voltage drop), while when the current flows from –ve potential to +ve potential, we represent it by +ve sign (voltage rise).*



Put the voltage source polarities

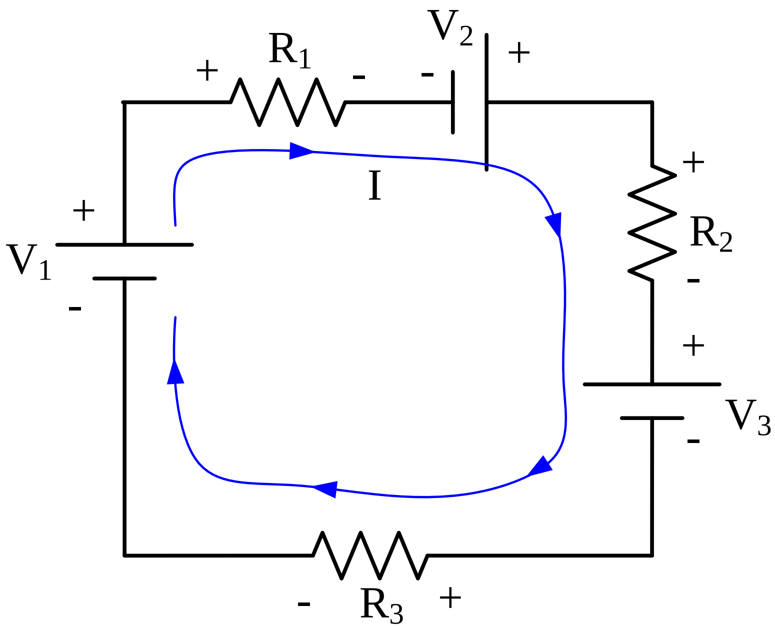
Draw current in the loop. You can choose any direction, clockwise, or anti-clockwise

Through a passive element (like resistance), current always flows from **higher to lower potential**, i.e. there is always a **drop in voltage** as current passes through a resistance

Put the voltage polarities across each resistance

Kirchhoff's Voltage Law (KVL)

Voltage across a resistance is: $V = IR$



Along the direction of current:

$+V_1$ (voltage rise)

$-IR_1$ (voltage drop)

$+V_2$ (voltage rise)

$-IR_2$ (voltage drop)

$-V_3$ (voltage drop)

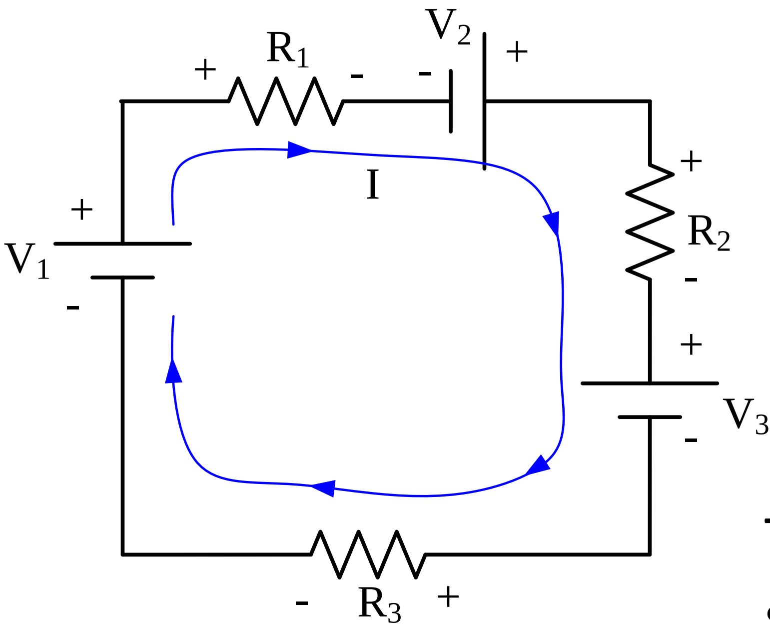
$-IR_3$ (voltage drop)

Algebraic summation of all voltages in the loop:

$$+V_1 - IR_1 + V_2 - IR_2 - V_3 - IR_3$$

According to KVL: $\boxed{+V_1 - IR_1 + V_2 - IR_2 - V_3 - IR_3 = 0}$

Kirchhoff's Voltage Law (KVL)



$$+V_1 - IR_1 + V_2 - IR_2 - V_3 - IR_3 = 0$$

Rearrange the equation

$$V_1 + V_2 = V_3 + IR_1 + IR_2 + IR_3$$

The above equation gives an alternate statement of the KVL as:

"Algebraic summation of all the ***voltage rises*** *in a closed circuit (path) must equal the algebraic summation of all the* ***voltage drops*** *in that path."*

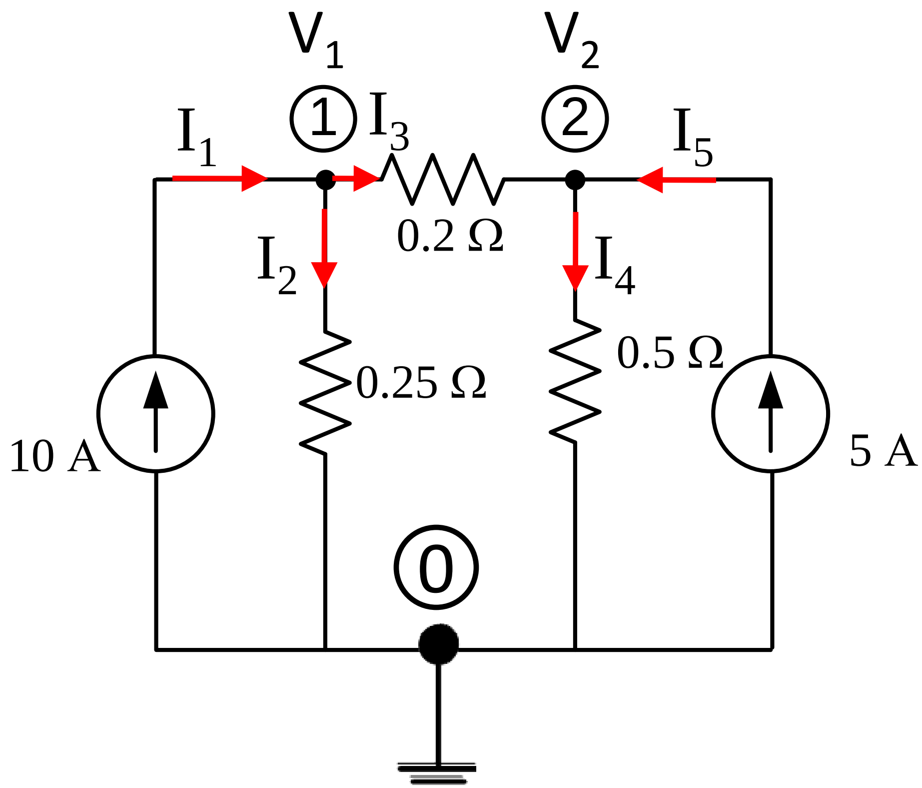
Applications of KCL & KVL

Application of KCL – The Nodal analysis

One of the methods for analyzing electric circuits for finding out unknown voltages and currents is the nodal analysis method.

- Nodal analysis is based on KCL
- This method is commonly used for circuits that have large number of parallel branches
- All the nodes are given names or numbers
- All the nodes are given different voltage symbols
- One of the nodes is called the ***reference node***
  - *Generally the node at zero potential is called the reference node.*
- Voltages at all other nodes are measured with respect to this reference node voltage
- All node voltages are expressed in terms of the branch currents
- The simultaneous equations are solved to find out values of the node voltages and branch currents

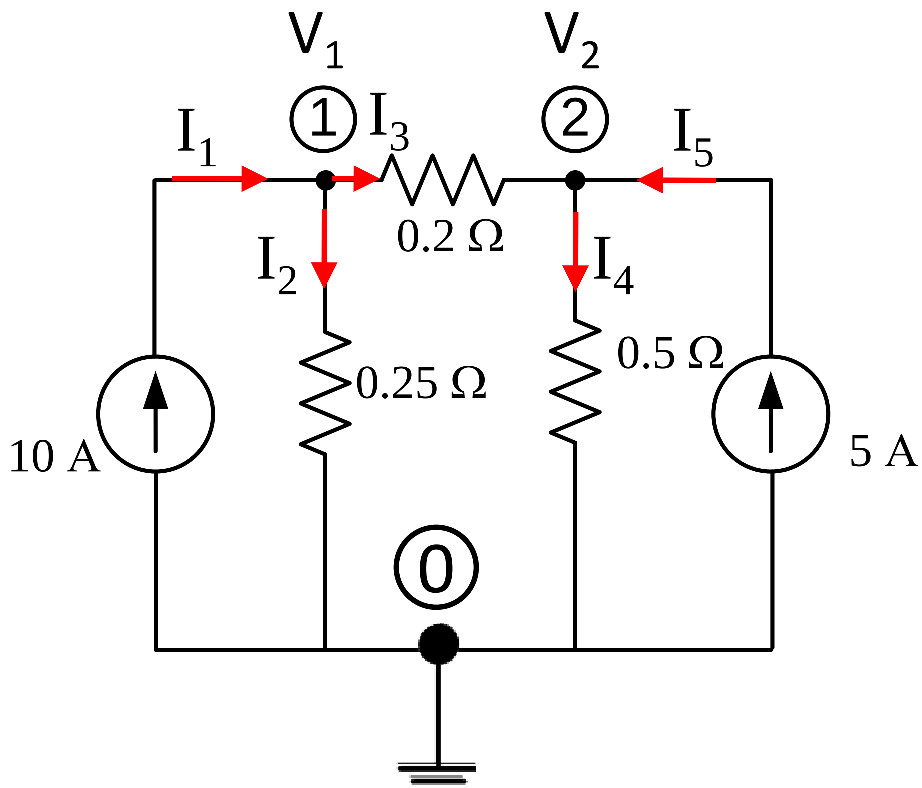
Find the voltages at node 1 and 2



- Put voltage symbols V_1 and V_2 at nodes 1 and 2
- Mark the reference node (common node) with zero potential (Ground symbol)
- Mark all the branch currents

- Though current directions can be drawn arbitrarily, it is recommended to follow the direction of current as per polarity of the source present in that branch
- *Current flows out of the arrowhead in a current source and + terminal of a voltage source*

Find the voltages at node 1 and 2



KCL at Node 1

$$I_1 = I_2 + I_3$$

$$10 = \frac{V_1 - 0}{0.25} + \frac{V_1 - V_2}{0.2}$$

$$\boxed{9V_1 - 5V_2 = 10}$$

KCL at Node 2

$$I_3 + I_5 = I_4$$

$$\frac{V_1 - V_2}{0.2} + 5 = \frac{V_2 - 0}{0.5}$$

$$\boxed{-5V_1 + 7V_2 = 5}$$

Solving the two equations:

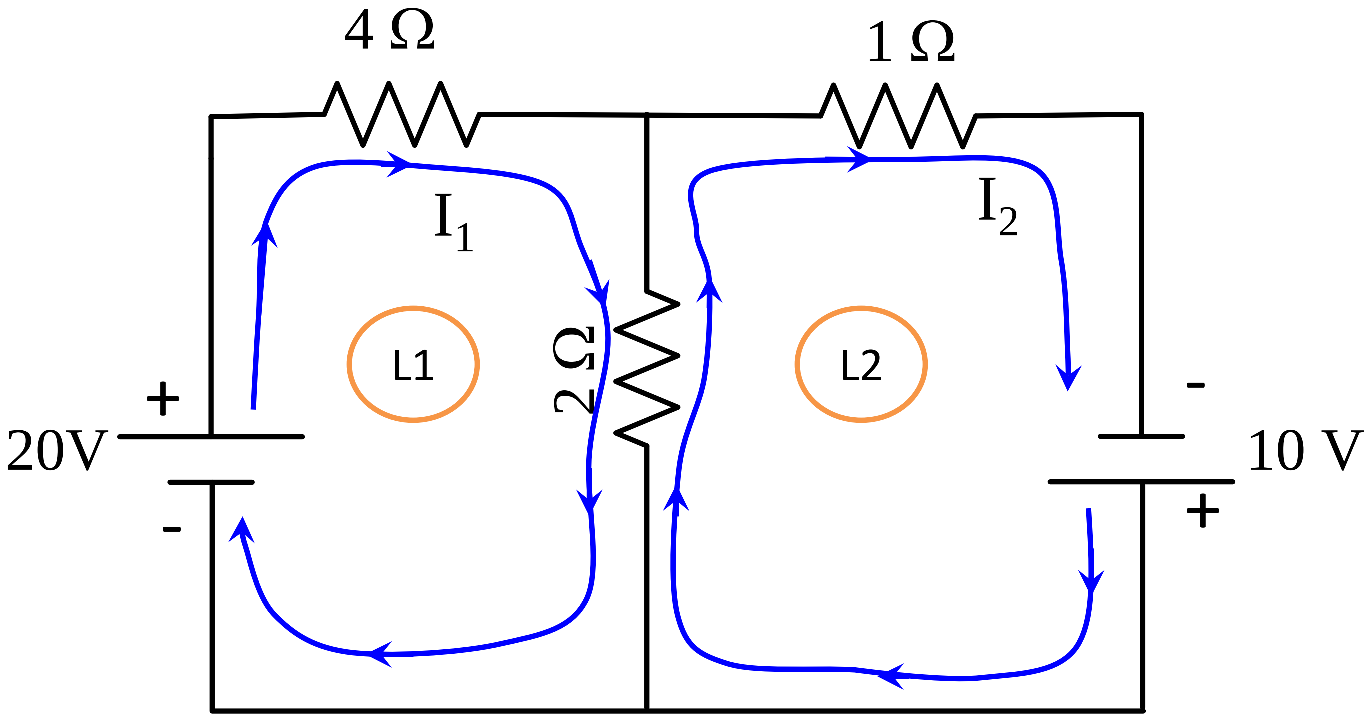
$V_1 = 2.5\,V$ $\quad$ $V_2 = 2.5\,V$

Application of KVL – The Mesh (or Loop) analysis

Another method for analyzing electric circuits is the mesh (or loop) analysis method.

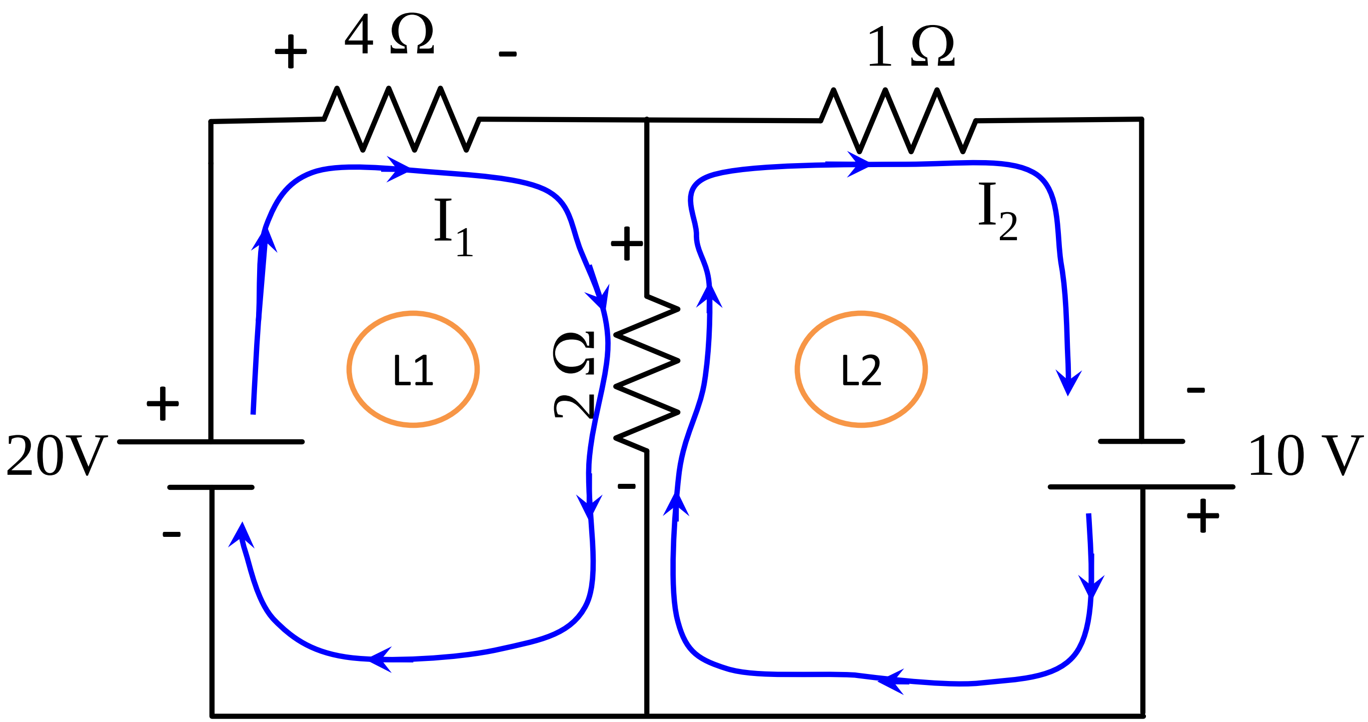
- Mesh analysis is based on KVL
- This method is commonly used for circuits that have large number of branches connected in series
- All currents in different loops around the circuit are given different names or symbols
- Voltage across all branches in a loop based on the current in that loop are calculated; due regard is given to the direction of current in that loop
- Equate the algebraic summation of all voltage drops in the mesh to the summation of all voltage sources present in that mesh
- If no voltage sources are present in that mesh, then equate the summation of total voltage drop to zero
- Thus, a number of simultaneous equations are obtained in terms of the mesh currents
- Solve these equations to get the unknown mesh currents

Find the currents in the two loops



- Mark the voltage source polarities
- Draw two currents I_1 and I_2 in the two loops.
- It is easier if we take the current in a loop to start from the +ve side of the source present in that loop

Find the currents in the two loops



KVL in Loop 1: (mark voltage polarities across resistances along I_1)

$$20 - 4I_1 - 2(I_1 - I_2) = 0$$

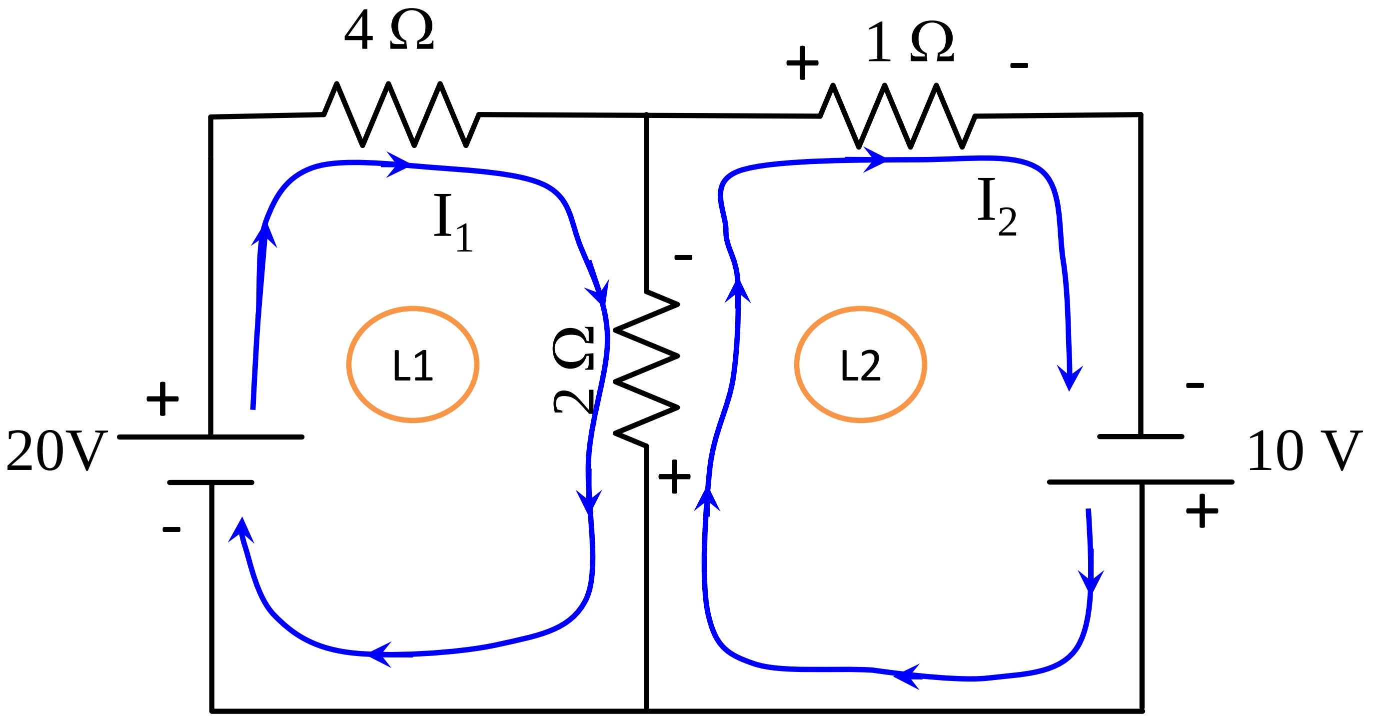
$$20 - 4I_1 - 2I_1 + 2I_2 = 0$$

$$6I_1 - 2I_2 = 20$$

$$\boxed{3I_1 - I_2 = 10}$$

Note that current through the 2 Ω resistance is combination of I_1 in first loop and I_2 in the 2nd loop

Find the currents in the two loops



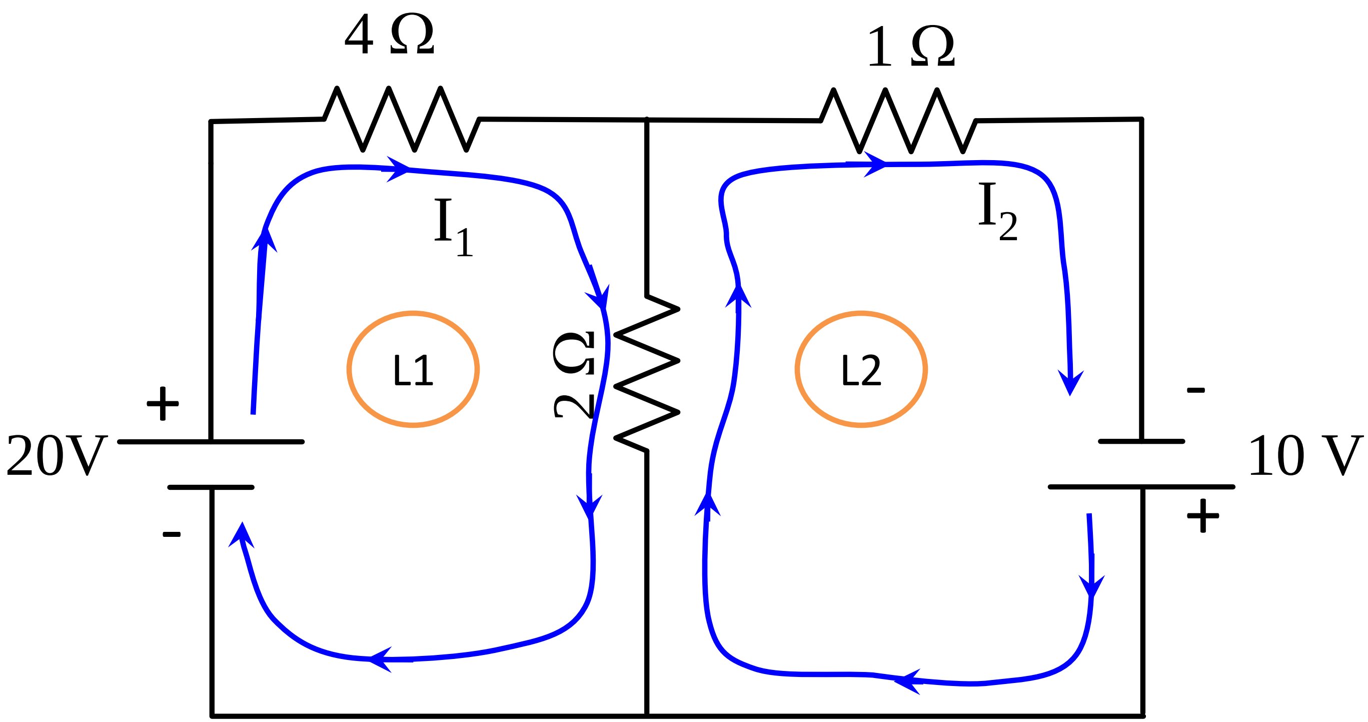
<u>KVL in Loop 2:</u> (mark voltage polarities across resistances along I_2)

$$10 - 2(I_2 - I_1) - 1I_2 = 0$$

$$10 - 2I_2 + 2I_1 - I_2 = 0$$

$$\boxed{2I_1 - 3I_2 = -10}$$

Find the currents in the two loops



Solving the two simultaneous equations:

$$\boxed{3I_1 - I_2 = 10}$$

$$\boxed{2I_1 - 3I_2 = -10}$$

$I_1 = 5.71$ A

$I_2 = 7.14$ A