Electromagnetism

Day 23

ILOs – Day 23 (Tutorial 2)

- Solve numerical problems related to:
  - Electromagnetic induction
  - Inductance
  - Energy stored in magnetic field
  - Lifting force of a magnet

#1) A coil of resistance 100 Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 Ω resistance is connected in series with it. Find the average e.m.f. and the current if the coil is moved in 1/10th second from the given field to a field of 0.2 mWb.

Induced e.m.f. $= N\dfrac{d\phi}{dt}$

$$d\phi = (1 - 0.2) = 0.8\ \mathrm{mWb}$$

$$dt = \frac{1}{10} = 0.1\ s$$

Number of turns, N = 100

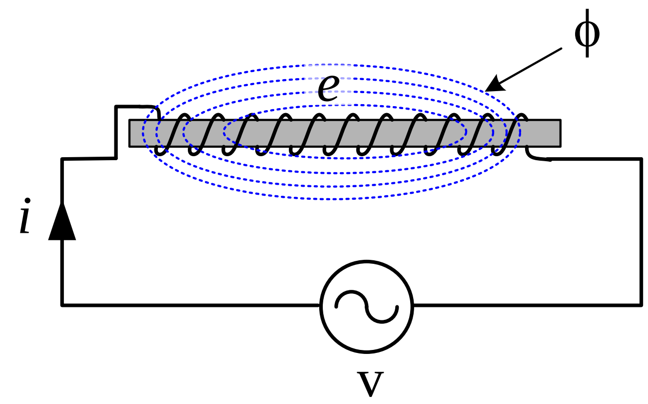
$\therefore$ Induced e.m.f. $e = 100 \times \dfrac{0.8 \times 10^{-3}}{0.1} = \boxed{0.8\ \mathrm{V}}$

Total resistance = 100 + 400 = 500 Ω

$\therefore$ Circuit current = 0.8/500 = $\boxed{1.6\ \mathrm{mA}}$

#2) An iron core solenoid of mean diameter 2 cm and length 2 m is uniformly wound with 1500 turns. What potential difference exists at the terminals of the solenoid when the coil current is increasing at a rate of 20 A/sec? Relative permeability of the iron core is 400.

$$L = \frac{N^2 \mu_0 \mu_r a}{l}$$



Given:

N = 1500, l = 2 m, μ_r = 400

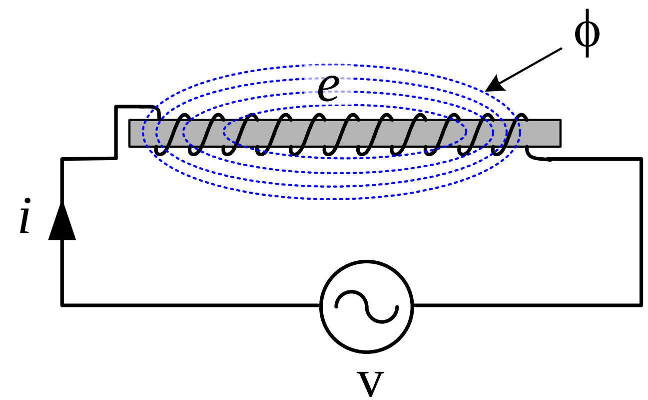
a = $\pi \frac{d^2}{4} = \pi \frac{(0.02)^2}{4} = 0.000314\ m^2$

$$\therefore L = \frac{1500^2 \times 4\pi \times 10^{-7} \times 400 \times 0.000314}{2} = 0.178\ H$$

#2) An iron core solenoid of mean diameter 2 cm and length 2 m is uniformly wound with 1500 turns. What potential difference exists at the terminals of the solenoid when the coil current is increasing at a rate of 20 A/sec? Relative permeability of the iron core is 400.

$\therefore$ The potential difference across the solenoid will be:

$$V = L\frac{di}{dt} = 0.178 \times 20 = \boxed{3.56\,V}$$



#3) The field winding of a d.c. electromagnet is wound with 960 turns and has resistance of 50 Ω when the exciting voltages is 230 V, the magnetic flux linking the coil is 0.005 Wb. Calculate the self-inductance of the coil and the energy stored in the magnetic field.

Self inductance $\boxed{L = N\dfrac{\phi}{i}}$

Current through coil $i = \dfrac{V}{R} = \dfrac{230}{50} = 4.6 \text{ A}$

Given, flux ϕ = 0.005 WB

Number of turns *N* = 960

$$\therefore L = N\frac{\phi}{i} = 960 \times \frac{0.005}{4.6} = \boxed{1.043 \text{ H}}$$

Energy stored $= \dfrac{1}{2}Li^2 = \dfrac{1}{2} \times 1.043 \times 4.6^2 = \boxed{11.04 \text{ J}}$

#4) A flux of 0.5 mWb is produced by a coil of 900 turns wound on a ring with a current of 3 A in it. Calculate (i) the inductance of the coil (ii) the e.m.f. induced in the coil when the current is switched off, assuming the current to fall to zero in 1 ms and (iii) the mutual inductance between the coils, if a second coil of 600 turns is uniformly wound over the first coil.

(i) Self inductance of the first coil $= L = N\frac{\phi}{i} = 900\times\frac{0.5\times10^{-3}}{3} = \boxed{0.15\ \mathrm{H}}$

(ii) EMF induced in the coil $= L\frac{di}{dt} = 0.15\times\frac{(3-0)}{1\times10^{-3}} = \boxed{450\ \mathrm{V}}$

(iii) Mutual inductance

$$M = N_2\frac{\phi_1}{i_1} = 600\times\frac{0.5\times10^{-3}}{3} = \boxed{0.1\ \mathrm{H}}$$

#5) Two coils having 30 and 600 turns respectively are wound side-by-side on a closed iron circuit of area of cross-section 100 sq.cm. and mean length 200 cm. Estimate the mutual inductance between the coils if the relative permeability of the iron is 2000. If a current of zero ampere grows to 20 A in a time of 0.02 second in the first coil, find the e.m.f. induced in the second coil.

(i) Mutual inductance between the coils

$$M = \frac{N_1 N_2}{S} = \frac{N_1 N_2}{\frac{l}{\mu_0 \mu_r a}} = \frac{30 \times 600}{\frac{200 \times 10^{-2}}{4\pi \times 10^{-7} \times 2000 \times 100 \times 10^{-4}}} = \boxed{0.226 \text{ H}}$$

(ii) EMF induced in the 2nd coil

$$e_2 = M \frac{di_1}{dt} = 0.226 \times \frac{20}{0.02} = \boxed{226 \text{ V}}$$

#6) Two coils, A of 12,500 turns and B of 16,000 turns, lie in parallel planes so that 60 % of flux produced in A links coil B. It is found that a current of 5A in A produces a flux of 0.6 mWb while the same current in B produces 0.8 mWb. Determine (i) mutual inductance and (ii) coupling coefficient.

(i) Flux produced by coil A, $\phi_1 = 0.6\times10^{-3}$ Wb

Since only 60% of this flux links with coil B, mutual inductance between the two coils is:

$$\boxed{M = N_2 \frac{0.6\phi_1}{i_1} = 16000\times\frac{0.6\times0.6\times10^{-3}}{5} = 1.152 \text{ H}}$$

(ii) Self-inductance of the two coils:

$$L_A = N_A \frac{\phi_A}{i_A} = 12500\times\frac{0.6\times10^{-3}}{5} = 1.5 \text{ H}$$

$$L_B = N_B \frac{\phi_B}{i_B} = 16000\times\frac{0.8\times10^{-3}}{5} = 2.56 \text{ H}$$

Coefficient of coupling:

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

$$\boxed{= \frac{1.152}{\sqrt{1.5\times2.56}} = 0.587}$$

#7) Two identical 750 turn coils A and B lie in parallel planes. A current changing at the rate of 1500 A/s in A induces an e.m.f. of 11.25 V in B. Calculate the mutual inductance of the arrangement. If the self-inductance of each coil is 15 mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.

$$e_M = M\frac{di_1}{dt} \Rightarrow M = \frac{e_M}{\frac{di_1}{dt}} = \frac{11.25}{1500} = 7.5\times10^{-3} = 7.5\,\text{mH}$$

It is given that self-inductance of coil A is 15 mH, and it has 750 turns.

Since we have the relation: $L_A = N_A\dfrac{\phi_A}{i_A} \Rightarrow \dfrac{\phi_A}{i_A} = \dfrac{L_A}{N_A} = \dfrac{15\times10^{-3}}{750} = 2\times10^{-5}\ \text{Wb/A}$

Coefficient of coupling: $K = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{7.5}{\sqrt{15\times15}} = 0.5$

$\therefore$ The percentage of the flux which links the turns of B = 50%

#8) Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9 H and for (b) 0.7 H. Find the self-inductances of the two coils and the mutual inductance between them.

(a) Additive: $L = L_1 + L_2 + 2M \Rightarrow 1.9 = L_1 + L_2 + 2M$

(b) Subtractive: $L = L_1 + L_2 - 2M \Rightarrow 0.7 = L_1 + L_2 - 2M$

Subtracting (a) from (b), we get: $\boxed{4M = 1.2 \Rightarrow M = 0.3\,\text{mH}}$

Putting this value of *M* in (a) we have: $L_1 + L_2 = 1.3$

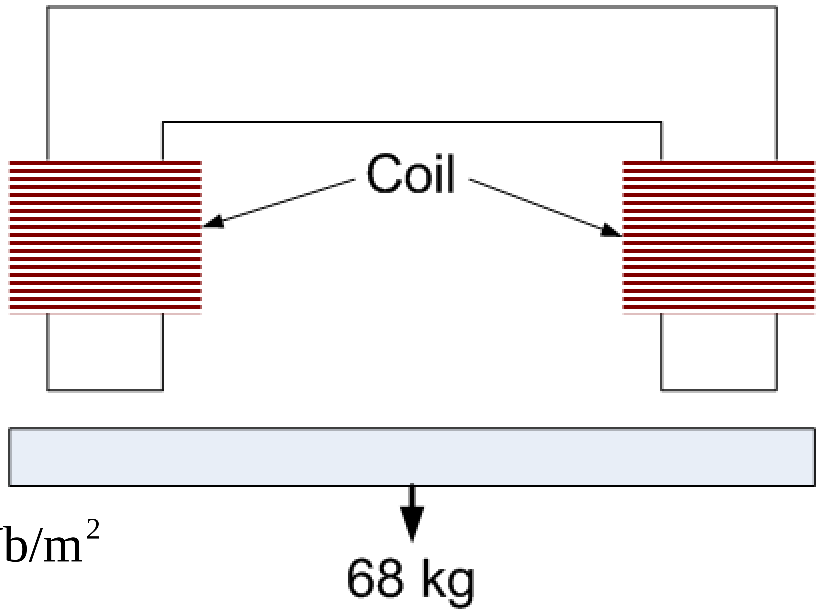
Given, Coefficient of coupling, *K* = 0.5

$$\Rightarrow K = \frac{M}{\sqrt{L_1 L_2}} = 0.5 \Rightarrow \frac{0.3}{\sqrt{L_1 L_2}} = 0.5 \Rightarrow \sqrt{L_1 L_2} = 0.6 \Rightarrow L_1 L_2 = 0.36$$

Now, solving the two simultaneous equations involving L_1 and L_2, we get:

$\boxed{L_1 = 0.9\,\text{H}}$ and $\boxed{L_2 = 0.4\,\text{H}}$

#9) A horse-shoe magnet is formed out of a bar of wrought iron 45.7 cm long, having a cross-section of 6.45 cm^2. Exciting coils of 500 turns are placed on each limb and are connected in series. Find the exciting current necessary for the magnet to lift a load of 68 kg assuming that the load has negligible reluctance and makes close contact with the magnet. Relative permeability of iron = 700.



Force of attraction of each pole:

$$F = \frac{68}{2} = 34 \text{ kg} = 34 \times 9.81 = 333.54 \text{ N}$$

Cross section of each pole:

$$A = 6.45 \text{ cm}^2 = 6.45 \times 10^{-4} \text{ m}^2$$

Using the magnetic pulling force relation:

$$F = \frac{B^2 a}{2\mu_0} \Rightarrow 333.54 = \frac{B^2 \times 6.45 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} \Rightarrow B = 1.14 \text{ Wb/m}^2$$

Thus, magnetizing force of the horse-shoe magnet:

$$H = \frac{B}{\mu_0 \mu_r} = \frac{1.14}{4\pi \times 10^{-7} \times 700} = 1296 \text{ AT/m}$$

Since reluctance of the load is negligible, and is making close contact with the magnet (very small air gap), then total magnetic path length can be taken as to be equal to the length of the horse shoe magnet:

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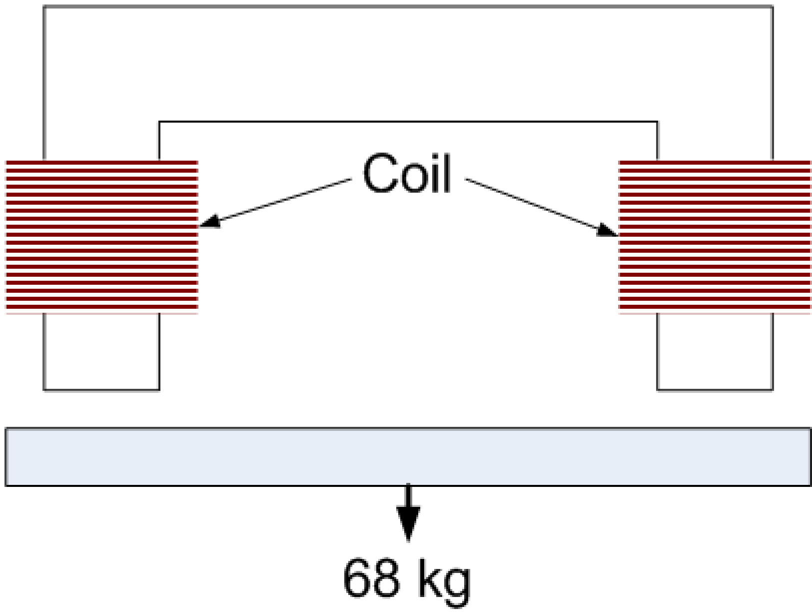

Thus, $l = 45.7 \text{ cm} = 0.457 \text{ m}$

With H = 1296 AT/m, and length l = 0.457 m, total AT required:

$$AT = H \times l = 1296 \times 0.457 = 592.3$$

Total number of turns in the magnet coil:

N = 500×2 = 1000

Thus, Exciting current $= \dfrac{AT}{N} = \dfrac{592.3}{1000} = \boxed{0.5923 \text{ A}}$