Network Theorems

- ILOs – Day6
  - State and explain DC network theorems
    - Superposition theorem
    - Thevenin’s theorem

Superposition Theorem

- Used for solution of complicated circuits
- Solution of circuits involving more than one voltage and/or current source

Superposition Theorem

- *The Theorem states that:*

- In a linear bilateral circuit containing several sources, the current and voltage for any element in the circuit is the algebraic summation of current and voltage in that element produced by each source acting independently.

Superposition Theorem

In a linear bilateral circuit containing several sources, the current and voltage for any element in the circuit is the algebraic summation of current and voltage in that element produced by each source acting independently.

Steps:

- Linear & bilateral circuit
- Circuit having more than one energy source
- Want to find current or voltage in one element
- Find current or voltage in that element due to one source at a time
- Then algebraically add the individual results

Steps:

- Linear & bilateral circuit
- Circuit having more than one energy source
- Want to find current or voltage in one element
- Find current or voltage in that element due to one source at a time
- Then algebraically add the individual results

Find current or voltage in that element due to one source at a time

- To calculate the contribution of each source independently, all other sources must be **made inactive and removed**

- When removing a voltage source, its voltage must be set to zero
- This is equivalent to **replacing the voltage source with a short circuit**

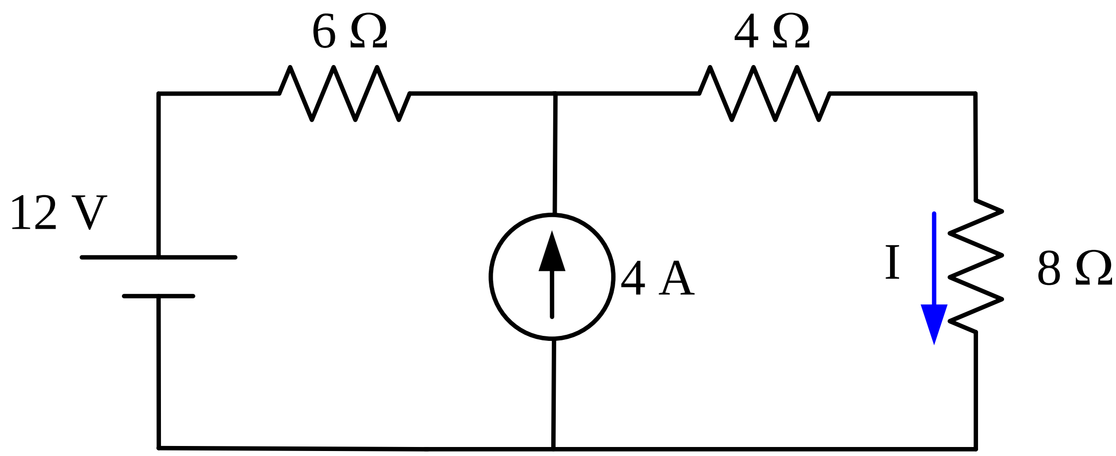
- When removing a current source, its current must be set to zero
- This is equivalent to **replacing the current source with an open circuit**

Find current or voltage in that element due to one source at a time

- If the voltage and current sources are not ideal, i.e. if they have internal resistances, those internal resistances must remain in the circuit and should be considered during calculation.

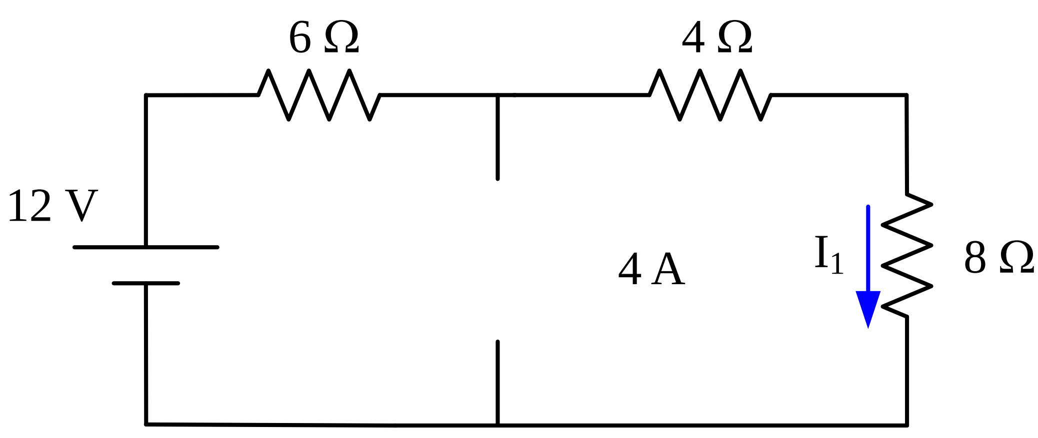
- In such cases, the voltage and current sources to be removed must be **replaced by their individual internal resistances** rather than simply short circuiting or open circuiting respectively.

Calculate the current *I* in the following circuit using superposition theorem.



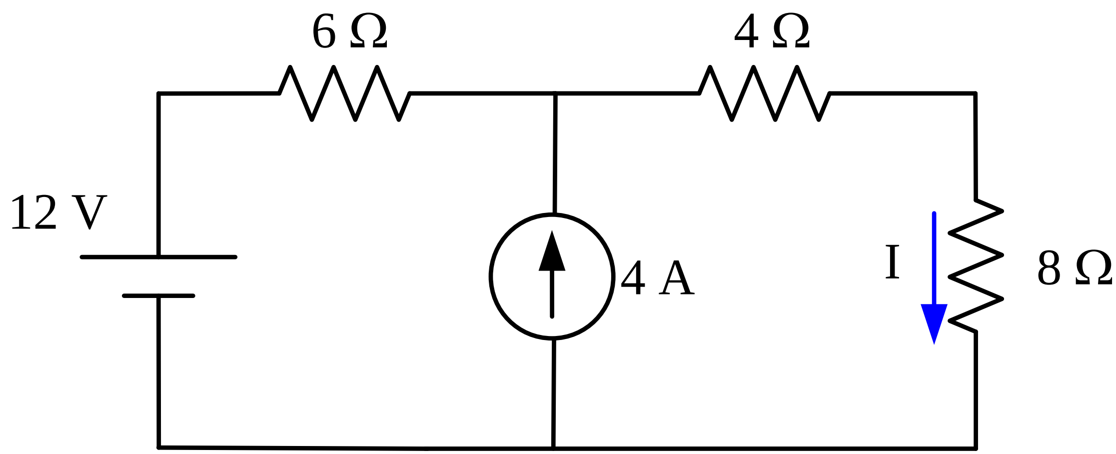
Step 1:

Consider the 12 V voltage source first and remove the 4 A current source (replace it by an open circuit):



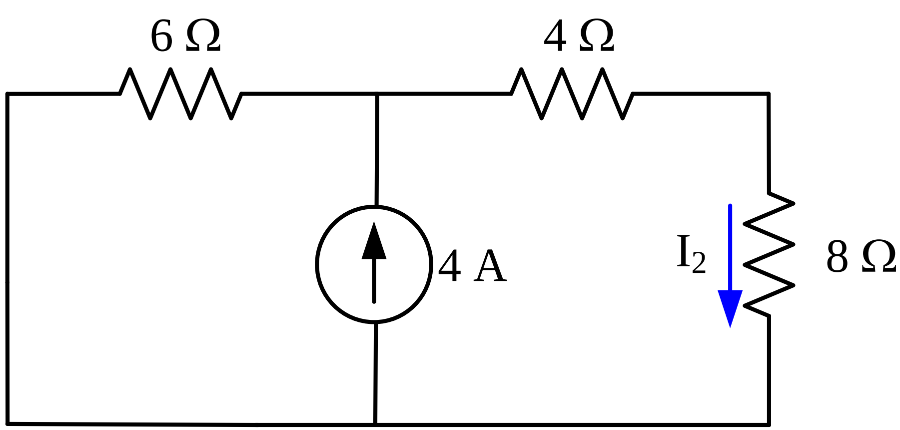
$$I_1 = \frac{12}{6+4+8} = \frac{12}{18} = \frac{2}{3}\,A$$

Calculate the current *I* in the following circuit using superposition theorem.



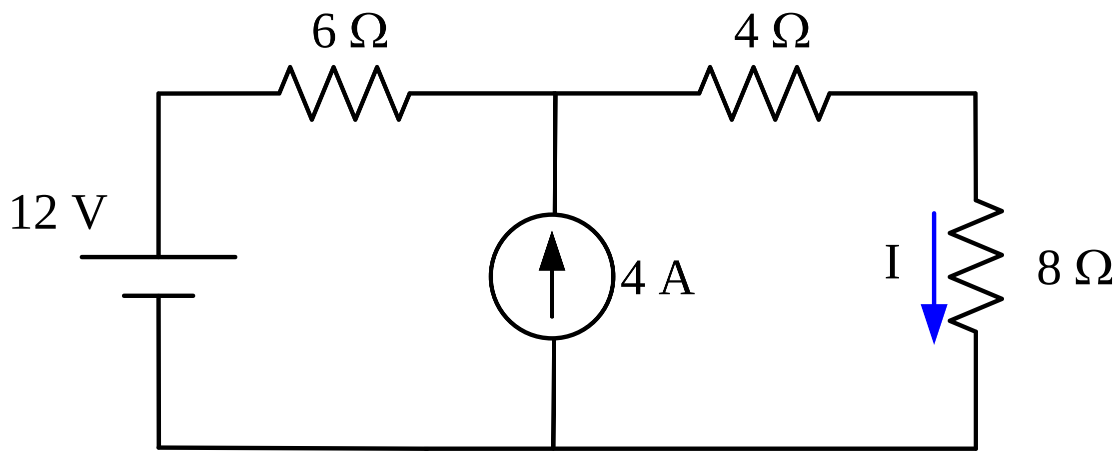
Step 2:

Now consider the 4 A current source and remove the 12 V voltage source (replace by a short circuit):



$$I_2 = 4 \times \frac{6}{6 + (4 + 8)} = 4 \times \frac{6}{18} = \frac{4}{3}\,A$$

Calculate the current *I* in the following circuit using superposition theorem.



Step 3:

Use superposition theorem to calculate total current:

With only 12V acting alone: $I_1 = \frac{2}{3}\,A$ ↓

With only 4 A acting alone: $I_2 = \frac{4}{3}\,A$ ↓

$\therefore$ Total current through 8 ohm is: $I = I_1 + I_2 = \frac{2}{3} + \frac{4}{3} = 2\ A$

Thevenin’s Theorem

- Used for solution of complicated circuits
- It allows to replace a complicated circuit with a simple equivalent circuit containing only a **single voltage source in series with a single resistor**

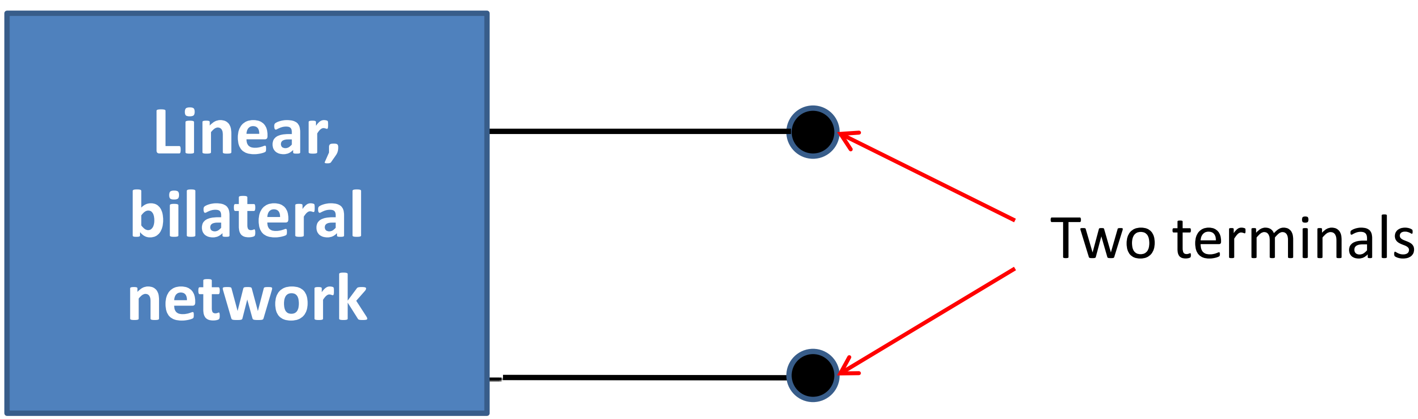
Thevenin's Theorem

- *The Theorem states that:*

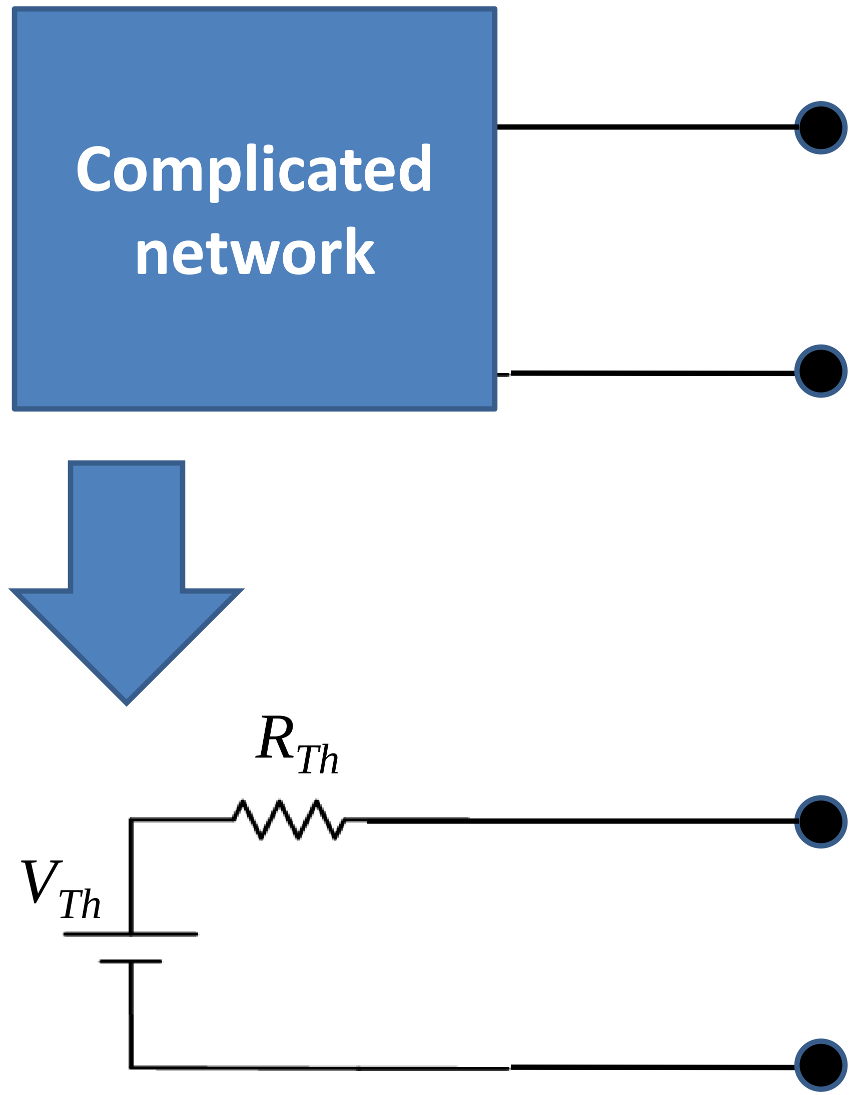
- Any linear, bilateral, active network **between two terminals** can be replaced by an equivalent circuit consisting of a single voltage source (**called the Thevenin's voltage source, V_{Th}**) in series with a resistance (**called the Thevenin's resistance, R_{Th}**).

Thevenin's Theorem

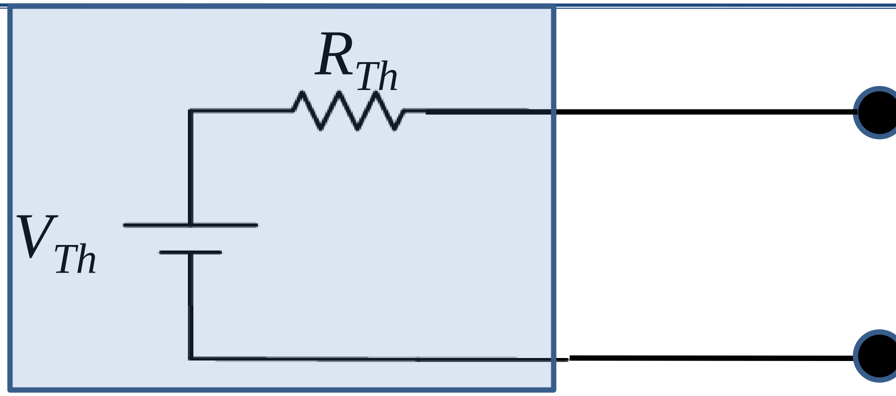
- *The Theorem states that:*
- Any linear, bilateral, active network
- **between two terminals**
- can be replaced by an equivalent circuit
- consisting of a single voltage source (**called the Thevenin's voltage source, V_{Th}**)
- in series with a resistance (**called the Thevenin's resistance, R_{Th}**).



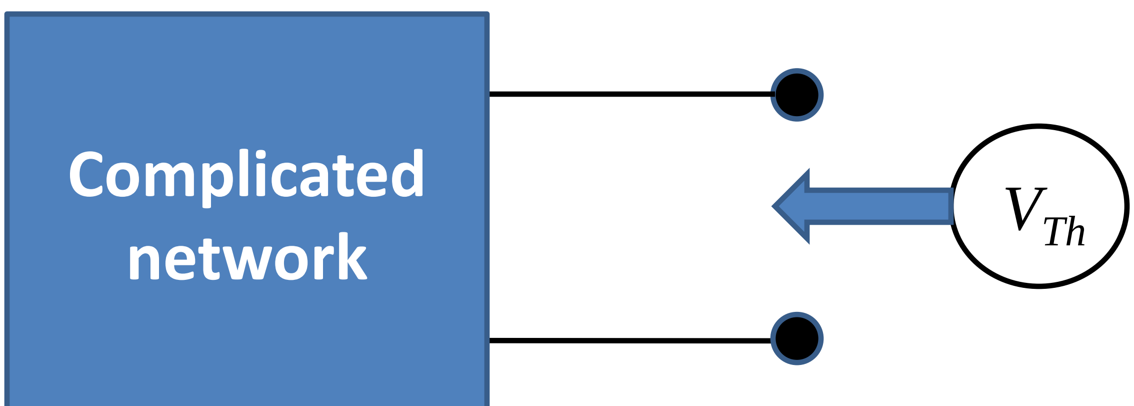
Thevenin's Theorem



Thevenin's Theorem - V_{Th}

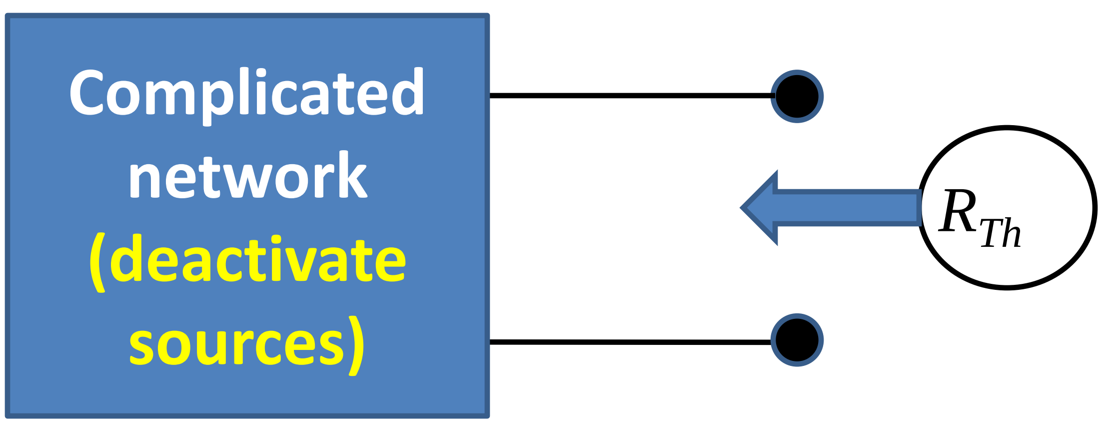


- The Thevenin's voltage V_{Th} is calculated as the **<u>open circuit voltage</u>** between the two terminals
- Make the two terminals **OPEN**
- Measure the voltage across it using a voltmeter



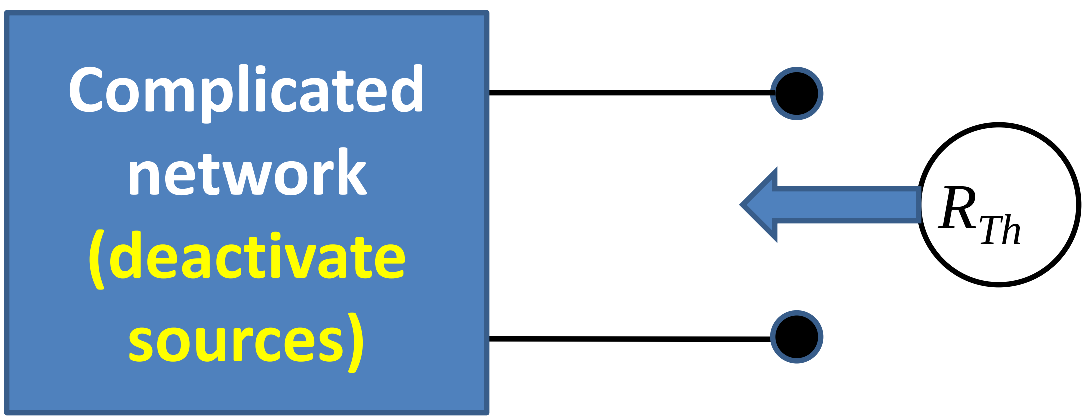
Thevenin's Theorem - R_{Th}

- The Thevenin's resistance R_{Th} is calculated as
- The **<u>equivalent resistance of the network measured between the two open circuited terminals</u>**
- By deactivating all sources in the circuit.

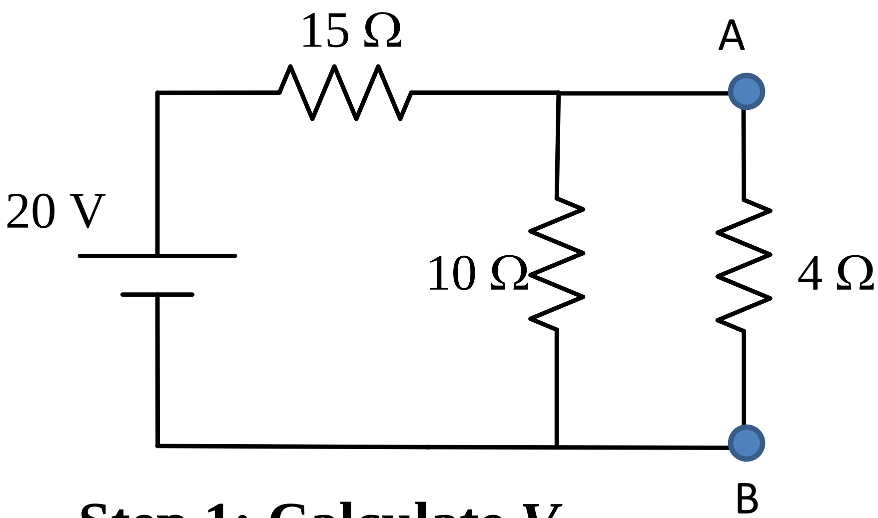


Thevenin's Theorem - R_{Th}

- To make a voltage source inactive, it must be replaced by its internal resistance (**short circuit for an ideal voltage source**)
- To make a current source inactive, it must be replaced by its internal resistance (**open-circuited for an ideal current source**)



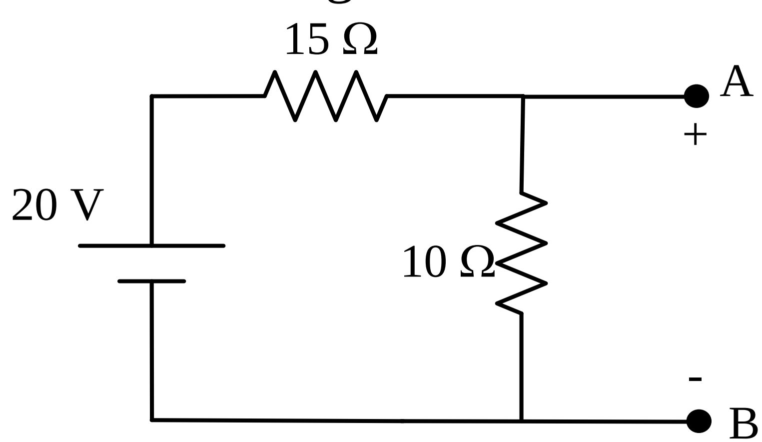
Find current through the 4Ω resistance using Thevenin's theorem



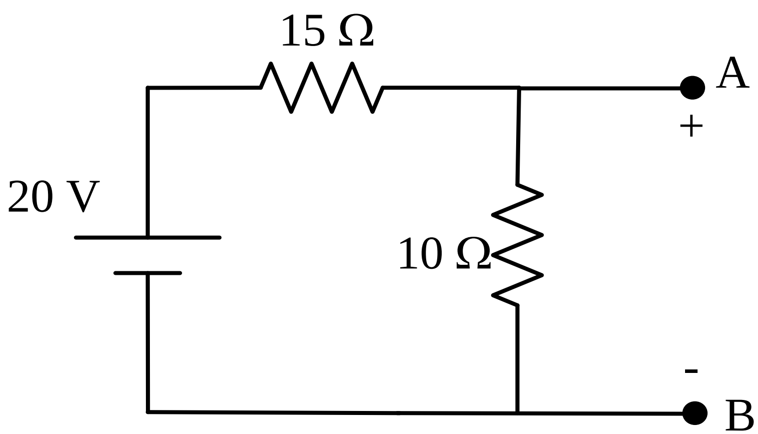
- We are to find out the Thevenin's equivalent circuit of the network across the 4 Ω resistance.

Step 1: Calculate V_{Th}

To calculate V_{Th} we make the terminals A and B open circuited and calculate the open circuit voltage between the two points A and B:



Find current through the 4Ω resistance using Thevenin's theorem



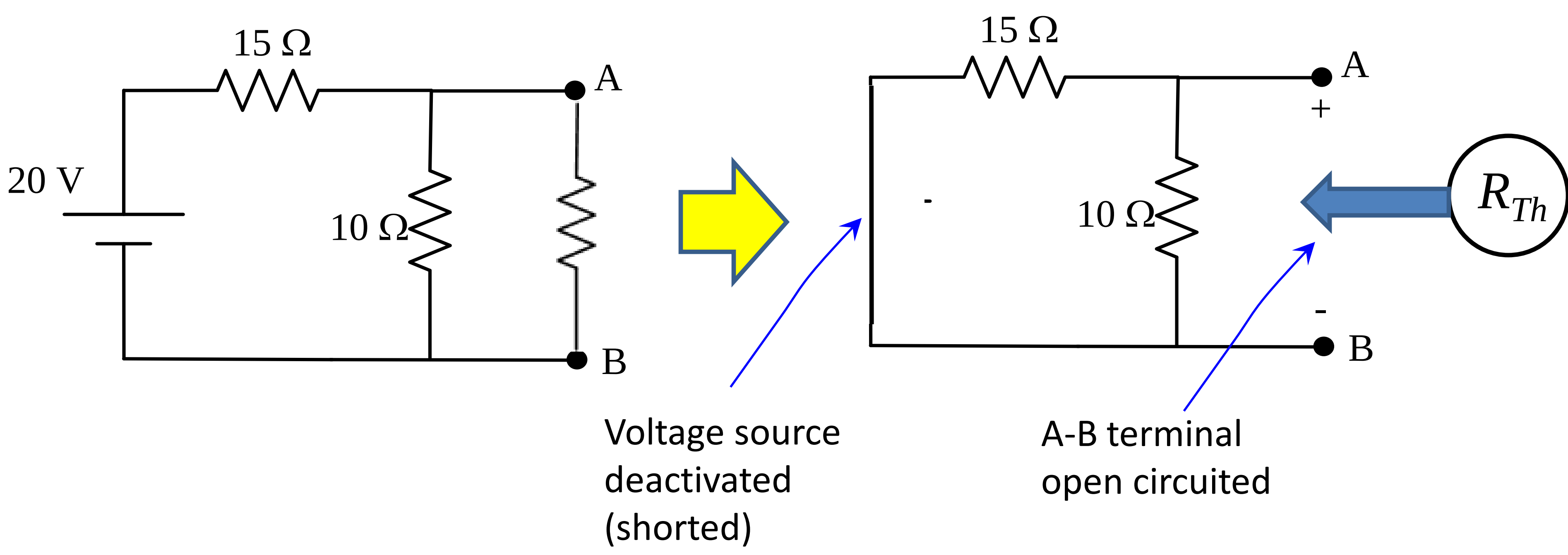
Thevenin's voltage is the open circuit voltage between A and B, and can be calculated using voltage division rule as:

$$V_{Th} = V_{AB} = 20 \times \frac{10}{15+10} = \frac{200}{25} = 8\,V$$

Find current through the 4Ω resistance using Thevenin's theorem

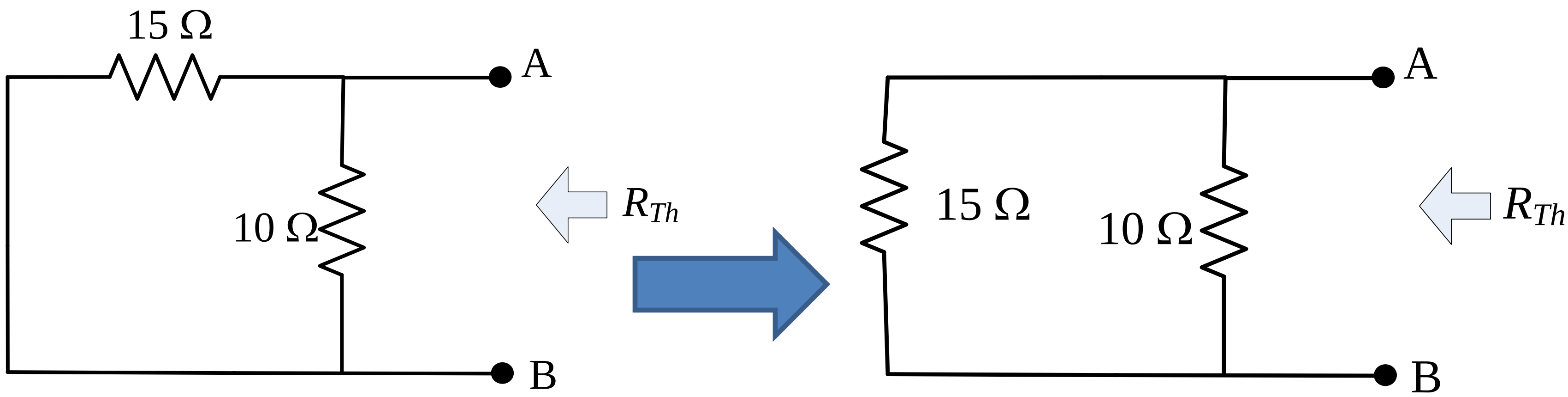
<u>Step 2</u>: Calculate R_{Th}

- To calculate R_{Th} we keep the terminals A and B open circuited
- Replace the 20 V voltage source by short circuit and
- Calculate the equivalent resistance of the circuit between the two points A and B:



Find current through the 4Ω resistance using Thevenin's theorem

- Calculate the equivalent resistance of the circuit between the two points A and B:



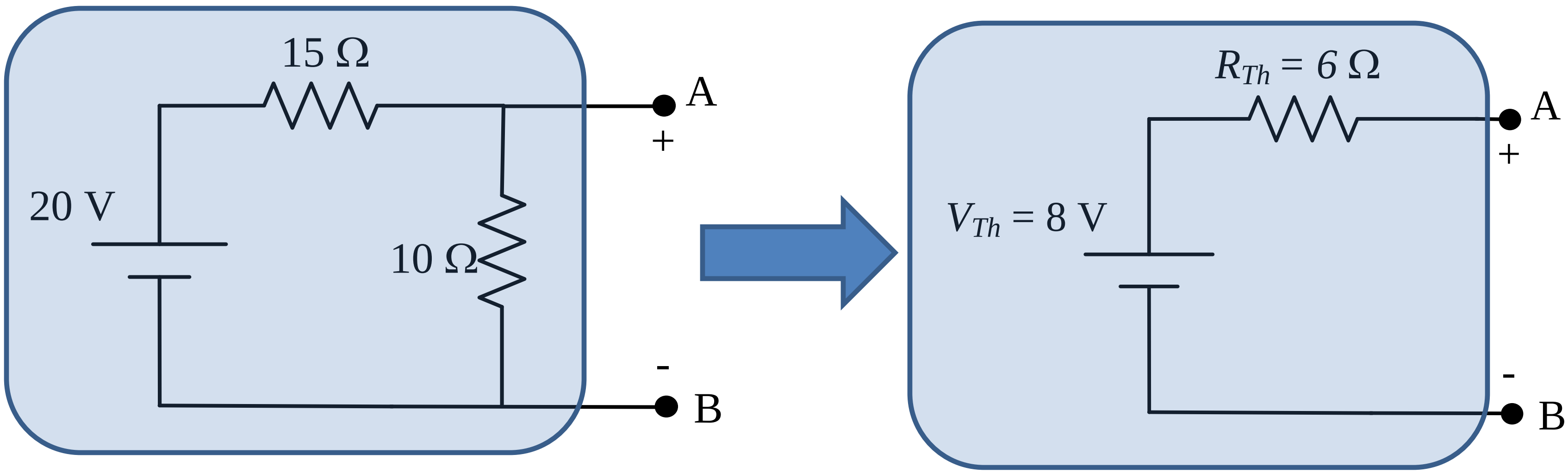
Thevenin's resistance is thus parallel combination of 15 Ω and 10 Ω between the terminals A and B:

$$R_{Th} = 15 // 10 = \frac{15 \times 10}{15 + 10} = 6\ \Omega$$

Find current through the 4Ω resistance using Thevenin's theorem

$$\boxed{V_{Th} = 8\,V} \qquad \boxed{R_{Th} = 6\,\Omega}$$

The Thevenin's equivalent circuit that can be drawn between terminals A and B is thus:

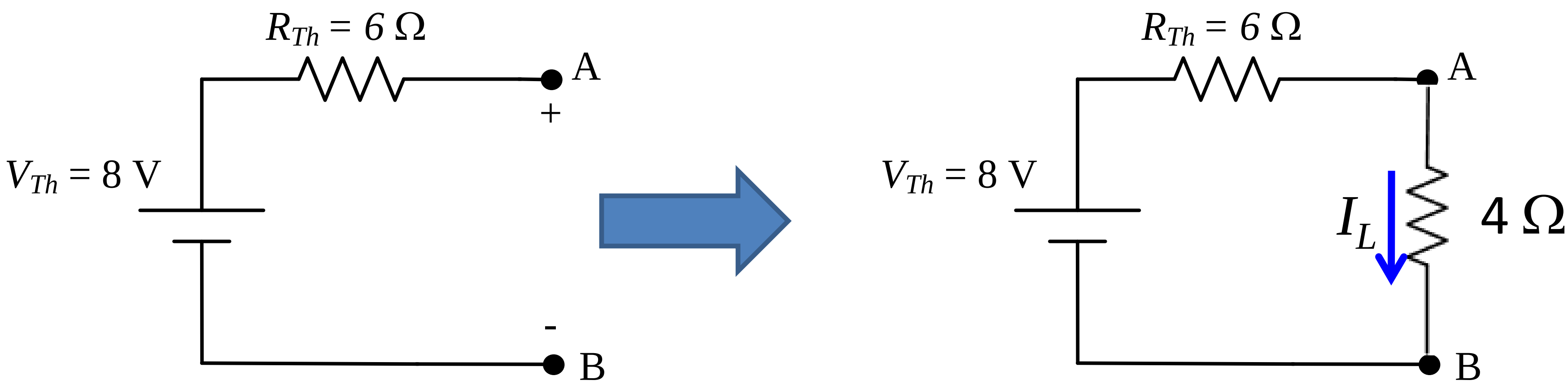


Original circuit

Thevenin's equivalent circuit

Find current through the 4Ω resistance using Thevenin's theorem

The 4Ω resistance can now be put back and the whole equivalent circuit is:



∴ The current (I_L) through the load resistance 4 Ω is:

$$\boxed{I_L = \frac{8}{6+4} = 0.8\ A}$$