AC Fundamentals

Day 10

Phasor diagram

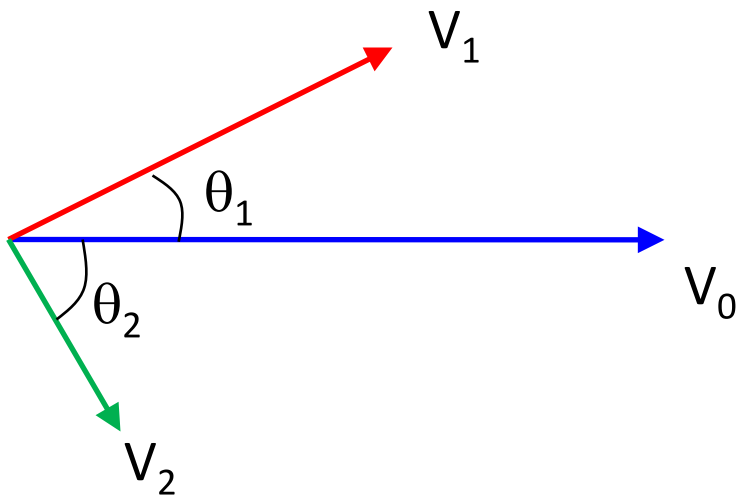
ILOs – Day 10

- Understand the basic concept of phasor diagram for AC circuits
- For a purely resistive circuit with AC supply:
    - Derive the expression for current and power
    - Draw phasor diagram
- For a purely inductive circuit with AC supply:
    - Derive the expression for current and power
    - Draw phasor diagram
- For a purely capacitive circuit with AC supply:
    - Derive the expressions for current and power
    - Draw phasor diagram

Phasor diagram

- AC signals are popularly represented by the use of phasor diagrams
- In AC circuits, different signals that are of the same frequency can be represented by phasor diagrams
- A phasor is a two-dimensional vector:
  - whose length is proportional to the RMS value of the signal
  - and angle is equal to the phase angle difference between that signal and a reference phasor
- The reference phasor is normally drawn along the X-axis
- If the signal leads the reference signals, then its phasor is drawn in the anticlockwise direction
- If the signal lags the reference signal, its phasor is drawn in the clockwise direction

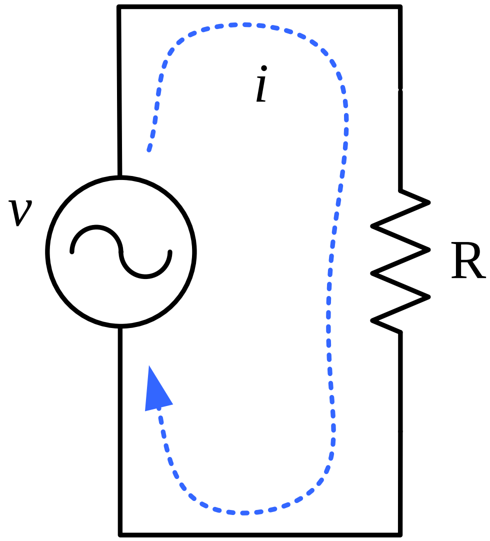
Phasor diagram



- Phasor diagram of three voltages with RMS values $V_0 > V_1 > V_2$
- Lengths of the phasors are proportional to their RMS values
- V_0 is the reference phasor, drawn along the X-axis
- Say, the phasor V_1 leads the reference phasor V_0 by θ_1
- Say, the phasor V_2 lags the reference phasor V_0 by θ_2
- The phasor V_1 leads the phasor V_2 by $(\theta_1 + \theta_2)$
- The phasor V_2 lags the phasor V_1 by $(\theta_1 + \theta_2)$

AC circuit operation with resistance

AC circuit operation with R



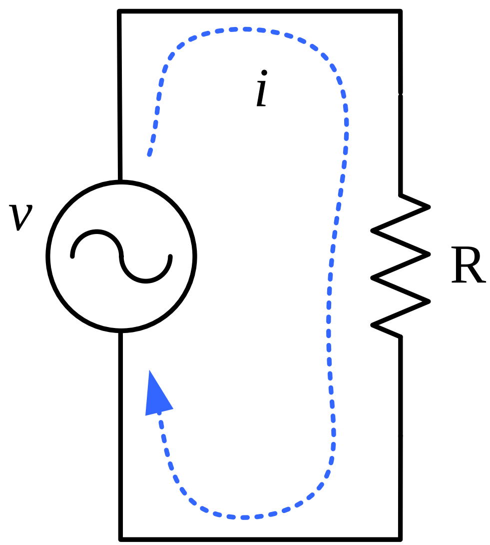
$$v = V_m \sin \omega t$$

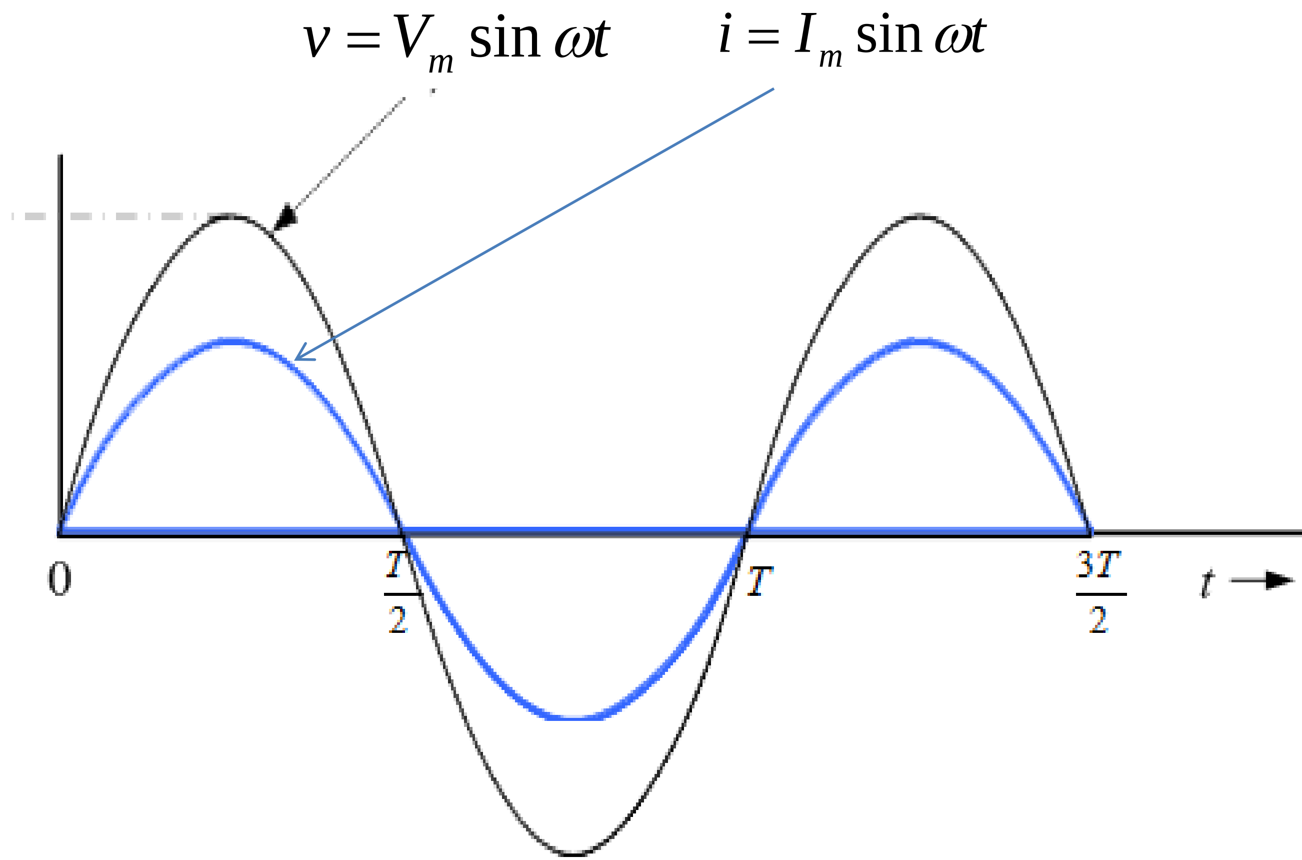
$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = \boxed{I_m \sin \omega t}$$

Where $I_m = \frac{V_m}{R}$ is peak value of the current

- Thus current flowing through the circuit is also sinusoidal
- Current has the same frequency (ω)as the input voltage signal
- Magnitude of current depends on value of R

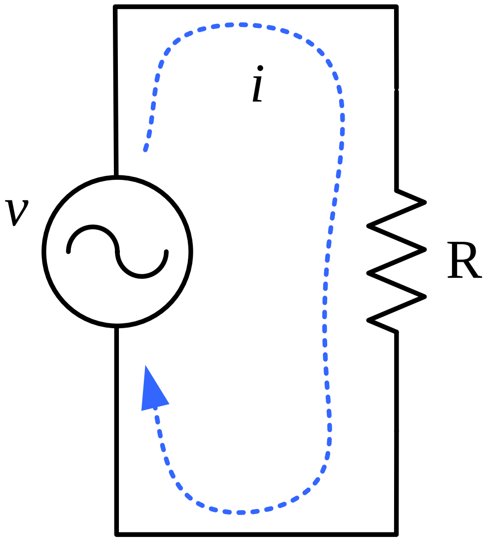
AC circuit operation with R





- Voltage and current signals are in the same phase
- They do not have any phase angle difference
- They have same frequency

AC circuit operation with R



$$v = V_m \sin \omega t \qquad i = I_m \sin \omega t$$

- Voltage and current signals are in the same phase
- They do not have any phase angle difference
- They have same frequency

<u>Phasor diagram</u>

Reference phasor V_{RMS}

Current phasor I_{RMS}

Thus, **in a resistive circuit, the voltage and current are always in the same phase**.

AC circuit operation with R

$$v = V_m \sin \omega t \qquad i = I_m \sin \omega t$$

Instantaneous power

$$p = v \times i$$

$$p = V_m \sin \omega t \times I_m \sin \omega t$$

$$p = V_m I_m \sin^2 \omega t$$

$$\boxed{p = \frac{V_m I_m}{2}\left(1 - \cos 2\omega t\right)}$$

Thus, in a resistive circuit, the instantaneous power is also an alternating quantity, but it varies at twice the frequency of the input voltage signal (*note the 2ω term*).

AC circuit operation with R

Average power

$$P = \frac{1}{T}\int_0^T p\,dt$$

$$P = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\left(1-\cos 2\omega t\right)dt$$

$$P = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\left(1-\cos 2\frac{2\pi}{T}t\right)dt$$

$$P = \frac{V_m I_m}{2T}\int_0^T \left(1-\cos\frac{4\pi}{T}t\right)dt$$

$$P = \frac{V_m I_m}{2T}\left(t-\frac{T}{4\pi}\sin\frac{4\pi}{T}t\right)\Bigg|_0^T$$

$$\boxed{p = \frac{V_m I_m}{2}\left(1-\cos 2\omega t\right)}$$

$$P = \frac{V_m I_m}{2T}\left[\left(T-\frac{T}{4\pi}\sin\frac{4\pi}{T}T\right)-\left(0-\frac{T}{4\pi}\sin\frac{4\pi}{T}0\right)\right]$$

$$P = \frac{V_m I_m}{2T}\left[\left(T-\frac{T}{4\pi}\sin 4\pi\right)-\left(0-0\right)\right]$$

Express ω as ω = 2π/T

$$P = \frac{V_m I_m}{2}$$

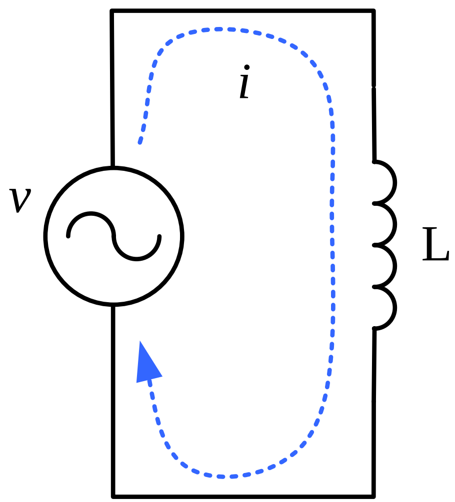
$$P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$\boxed{P = V_{RMS} \times I_{RMS}}$$

Thus, average power in a purely resistive circuit is equal to the product of the RMS values of voltage and current

AC circuit operation with inductance

AC circuit operation with L



$$v = L\frac{di}{dt} \Rightarrow di = \frac{v}{L}dt$$

$$v = V_m \sin \omega t$$

$$i = \int di = \int \frac{v}{L}dt$$

$$i = \frac{1}{L}\int V_m \sin \omega t dt$$

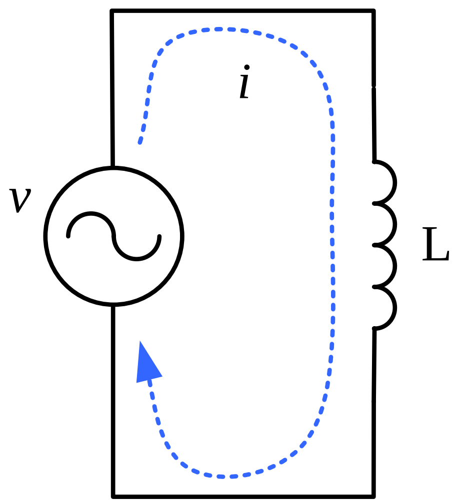
$$i = \frac{V_m}{\omega L}\left(-\cos \omega t\right)$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$

$I_m = \dfrac{V_m}{\omega L}$ is peak value of the current

AC circuit operation with L



$$\boxed{v = V_m \sin \omega t}$$

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$

$$I_m = \frac{V_m}{\omega L}$$

- Thus, the current signal is also a sinusoidal quantity
- Current signal has the same frequency (ω) as the voltage signal
- But as compared to voltage signal, the **current signal is lagging behind** by a phase angle of $\pi/2$
- Magnitude of the current is determined by the quantity ωL

AC circuit operation with L

$$\boxed{v = V_m \sin \omega t}$$

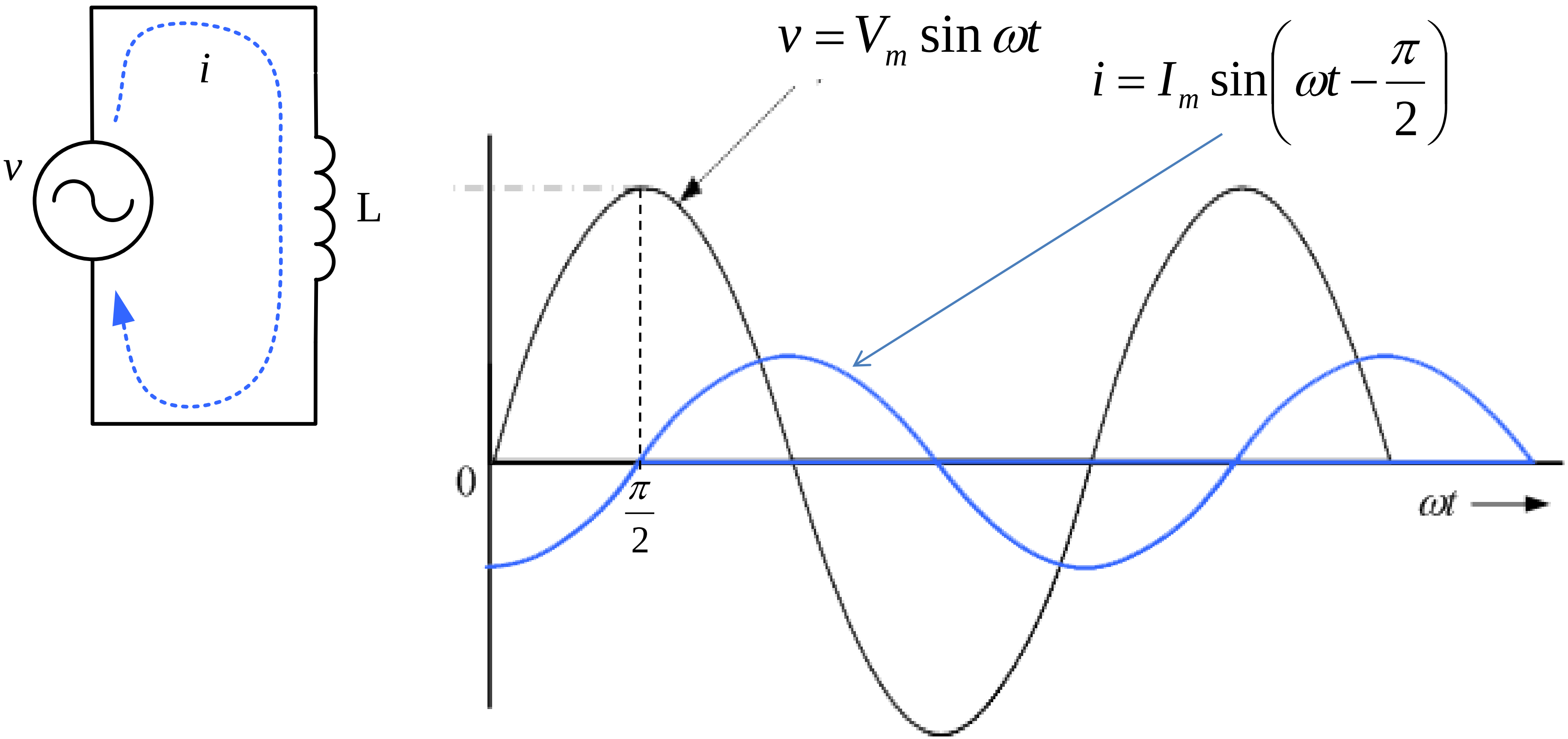
$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$

$$I_m = \frac{V_m}{\omega L} \Rightarrow \omega L = \frac{V_m}{I_m} = \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{V_{RMS}}{I_{RMS}}$$

- The quantity ωL in an inductive circuit is the ratio of the RMS values of voltage and current
- It is called **inductive reactance** of the circuit
- Inductive reactance is denoted by the symbol $\mathbf{X_L}$
- Its unit is as usual 'ohm' (Ω)

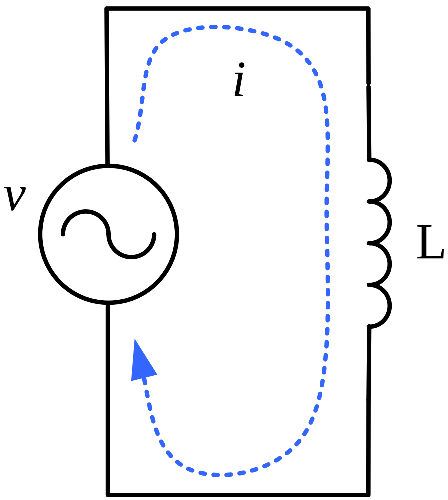
$$\boxed{X_L = \omega L = 2\pi f L}$$

AC circuit operation with L



- The current lags behind the voltage by a phase angle of 90^0
- They have same frequency

AC circuit operation with L



$$v = V_m \sin \omega t \qquad i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

- The current lags behind the voltage by a phase angle of 90^0

Phasor diagram

Reference phasor V_{RMS}

Current phasor I_{RMS}

Thus, **in a purely inductive circuit, the current lags behind the voltage by a phase angle of 90^0, or in other words, the voltage leads ahead of the current by a phase angle of 90^0**.

AC circuit operation with L

$$v = V_m \sin \omega t \qquad i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Instantaneous power

$$p = v \times i$$

$$p = V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$p = -V_m I_m \sin \omega t \cos \omega t$$

$$\boxed{p = -\frac{V_m I_m}{2} \sin 2\omega t}$$

Thus, in an inductive circuit also the instantaneous power is an alternating quantity, but it varies at twice the frequency of the input voltage signal (*note the 2ω term*).

AC circuit operation with L

$$\boxed{p = -\frac{V_m I_m}{2}\sin 2\omega t}$$

Average power

$$P = \frac{1}{T}\int_0^T p\,dt$$

$$P = \frac{1}{T}\int_0^T -\frac{V_m I_m}{2}\sin 2\omega t\,dt$$

$$P = -\frac{V_m I_m}{2T}\int_0^T \sin 2\frac{2\pi}{T}t\,dt$$

$$P = -\frac{V_m I_m}{2T}\times(-)\frac{T}{4\pi}\times\left(\cos\frac{4\pi}{T}t\right)\Bigg|_0^T$$

Express ω as ω = 2π/T

$$P = \frac{V_m I_m}{8\pi}\times\left(\cos\frac{4\pi}{T}T - \cos\frac{4\pi}{T}0\right)$$

$$P = \frac{V_m I_m}{8\pi}\times(\cos 4\pi - \cos 0)$$

$$P = \frac{V_m I_m}{8\pi}\times(1-1)$$

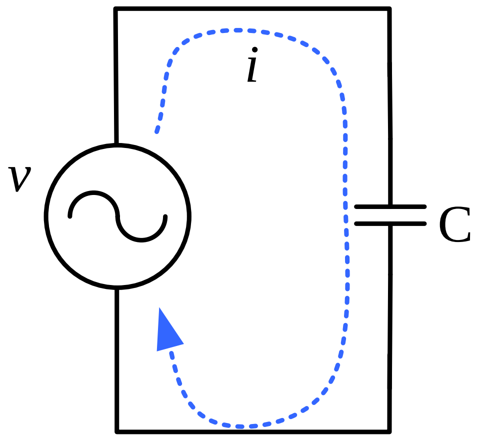
$$P = \frac{V_m I_m}{8\pi}\times 0$$

$$\boxed{P = 0}$$

Thus, **average active power consumed by a purely inductive circuit is zero**, i.e. a **purely inductive circuit does not take any active power**.

AC circuit operation with capacitance

AC circuit operation with C



$$i = C\frac{dv}{dt}$$

$$v = V_m \sin \omega t$$

$$i = C\frac{d}{dt}\left(V_m \sin \omega t\right)$$

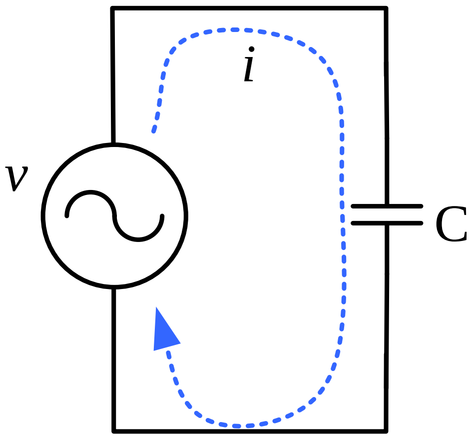
$$i = V_m \omega C \cos \omega t$$

$$i = V_m \omega C \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\boxed{i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)}$$

$I_m = V_m \omega C$ is peak value of the current

AC circuit operation with C



$$v = V_m \sin \omega t$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I_m = V_m \omega C$$

- Thus, the current signal is also a sinusoidal quantity
- Current signal has the same frequency (ω) as the voltage signal
- But as compared to voltage signal, the **current signal is leading ahead** by a phase angle of $\pi/2$
- Magnitude of the current is determined by the quantity ωC

AC circuit operation with C

$$\boxed{v = V_m \sin \omega t}$$

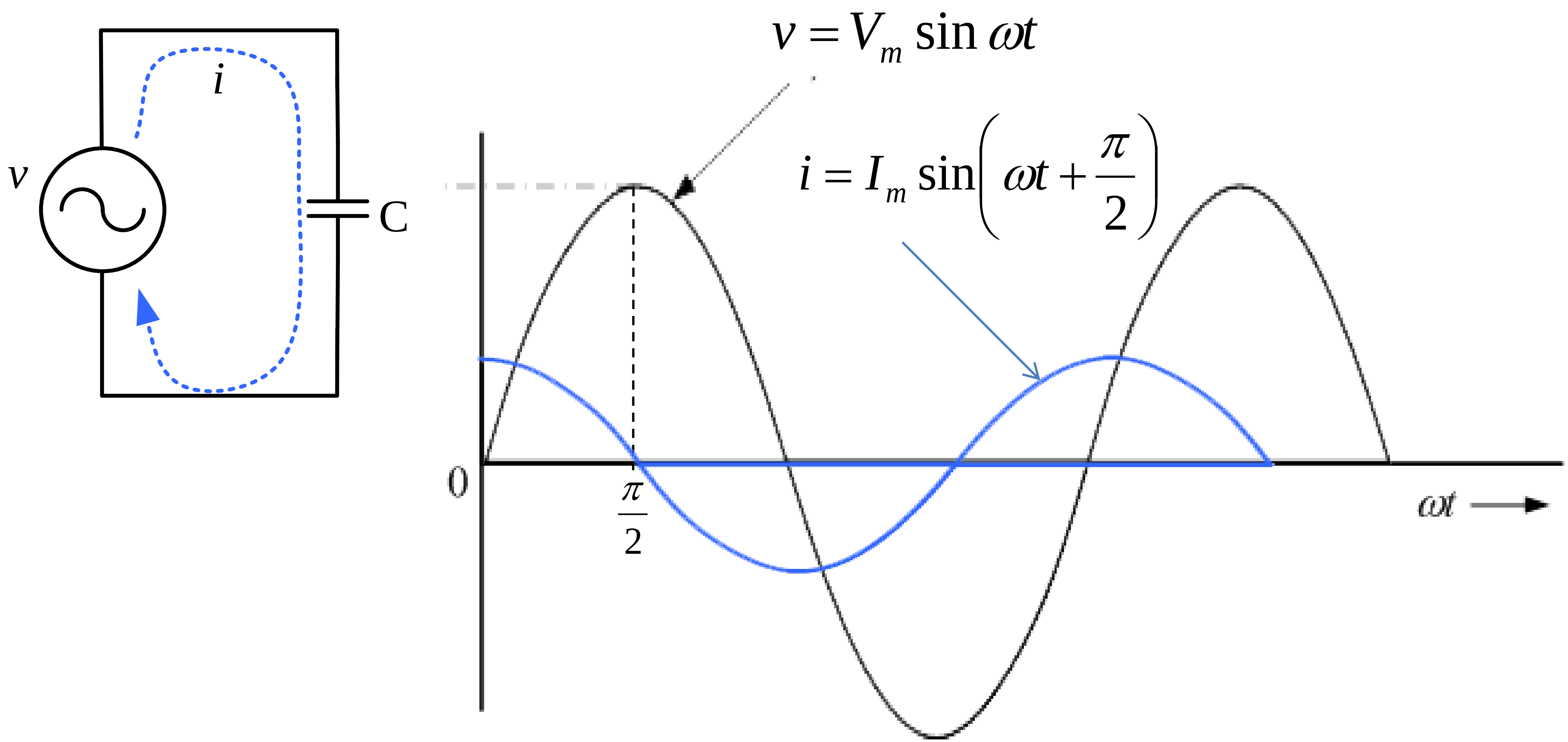
$$\boxed{i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)}$$

$$I_m = V_m \omega C \Rightarrow \frac{1}{\omega C} = \frac{V_m}{I_m} = \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{V_{RMS}}{I_{RMS}}$$

- The quantity (1/ωC) in a capacitive circuit is the ratio of the RMS values of voltage and current
- It is called **Capacitive reactance** of the circuit
- Capacitive reactance is denoted by the symbol **X_C**
- Its unit is as usual 'ohm' (Ω)

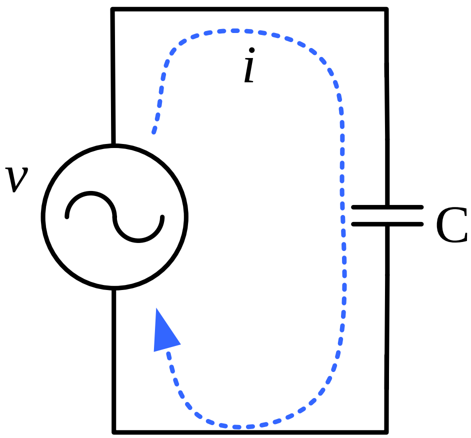
$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$

AC circuit operation with C



- The current leads ahead of the voltage by a phase angle of 90^0
- They have same frequency

AC circuit operation with C



$$v = V_m \sin \omega t \qquad i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

- The current leads ahead of the voltage by a phase angle of 90^0

Phasor diagram

Reference phasor V_{RMS}

Current phasor I_{RMS}

Thus, **in a purely capacitive circuit, the current leads ahead of the voltage by a phase angle of 90^0, or in other words, the voltage lags behind the current by a phase angle of 90^0**.

AC circuit operation with C

$$v = V_m \sin \omega t \qquad i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

Instantaneous power

$$p = v \times i$$

$$p = V_m \sin \omega t \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$p = V_m I_m \sin \omega t \cos \omega t$$

$$\boxed{p = \frac{V_m I_m}{2} \sin 2\omega t}$$

Thus, in a capacitive circuit also the instantaneous power is an alternating quantity, but it varies at twice the frequency of the input voltage signal (*note the 2ω term*).

AC circuit operation with C

Average power

$$\boxed{p = \frac{V_m I_m}{2} \sin 2\omega t}$$

$$P = \frac{1}{T}\int_0^T p dt$$

$$P = \frac{1}{T}\int_0^T \frac{V_m I_m}{2} \sin 2\omega t dt$$

$$P = \frac{V_m I_m}{2T}\int_0^T \sin 2\frac{2\pi}{T} t dt$$

$$P = \frac{V_m I_m}{2T} \times (-)\frac{T}{4\pi} \times \left(\cos\frac{4\pi}{T}t\right)\Bigg|_0^T$$

$$P = -\frac{V_m I_m}{8\pi} \times \left(\cos\frac{4\pi}{T}T - \cos\frac{4\pi}{T}0\right)$$

$$P = -\frac{V_m I_m}{8\pi} \times (\cos 4\pi - \cos 0)$$

$$P = -\frac{V_m I_m}{8\pi} \times (1-1)$$

$$P = -\frac{V_m I_m}{8\pi} \times 0$$

$$\boxed{P = 0}$$

Thus, like in a purely inductive circuit, the **average active power consumed by a purely capacitive circuit is also zero**, i.e. a **purely capacitive circuit does not take any active power**.