AC Fundamentals

Day 12

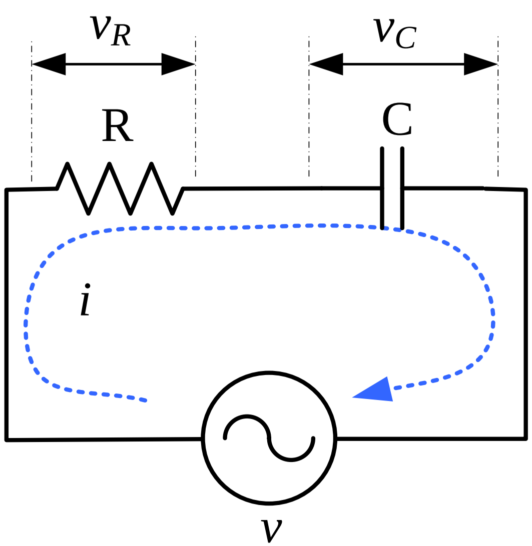
RC circuit

ILOs – Day 12

- For a resistive + capacitive circuit with AC supply:
  - Derive the expression for current and power
  - Draw phasor diagram
- Define active, reactive, and apparent power in AC circuits and obtain their expressions

AC circuit operation with resistance and capacitance together

AC circuit operation with R + C

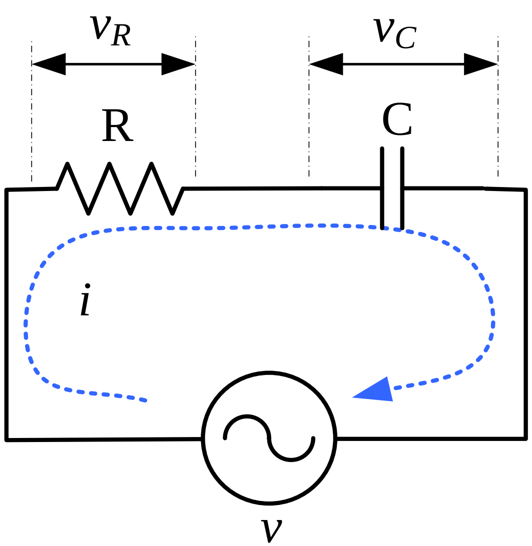


$$v = V_m \sin \omega t$$

Phasor diagram

- To draw the phasor diagram, we take the signals V_{RMS}, I_{RMS}, V_R, V_C
- The RMS value of current I_{RMS} is considered as the reference phasor and it is thus drawn along the X-axis
- *In series circuit, current is common to all the elements, so take current as reference*

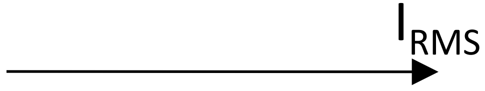
AC circuit operation with R + C



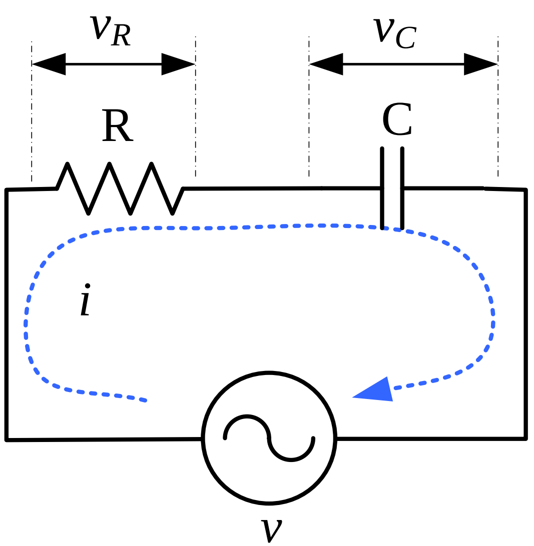
$$v = V_m \sin \omega t$$

Phasor diagram

- The RMS value of current I_{RMS} is drawn along the X-axis



AC circuit operation with R + C



$$v = V_m \sin \omega t$$

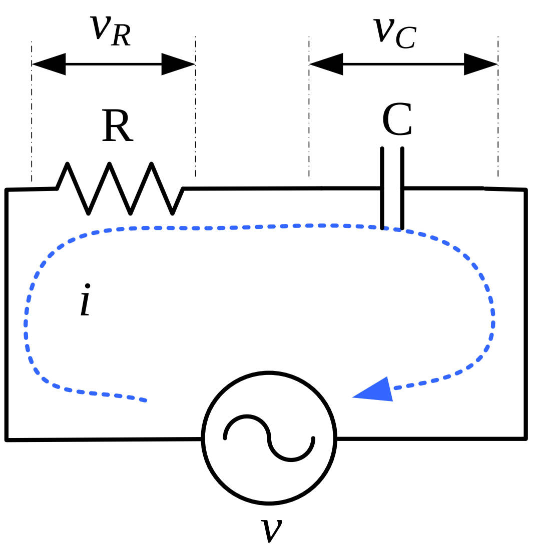
Phasor diagram

- Voltage drop across the resistance is

$$V_R = I_{RMS} R$$

- V_R phasor is drawn in the same direction as the current phasor

AC circuit operation with R + C



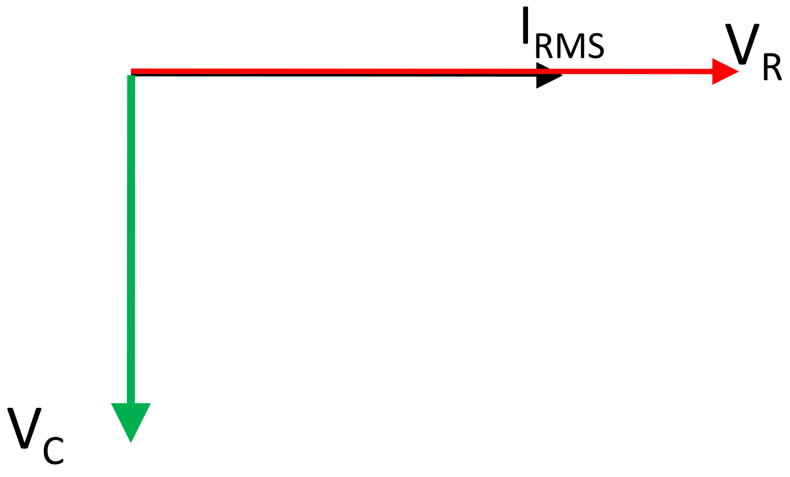
$$v = V_m \sin \omega t$$

Phasor diagram

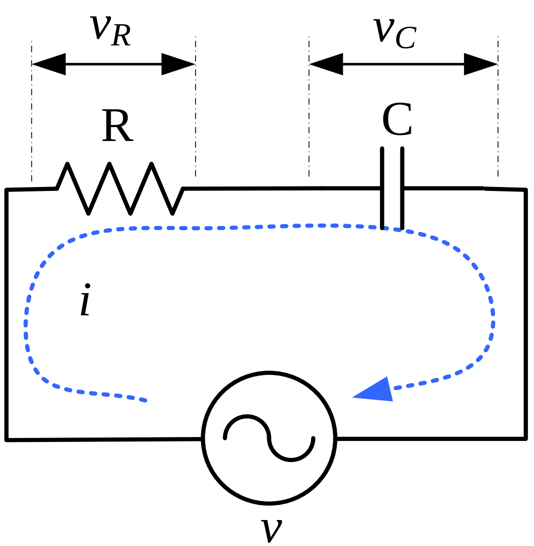
- Voltage drop across the capacitor is

$$V_C = I_{RMS} X_C$$

- V_C phasor is drawn 90^0 lagging to the current phasor
- (*remember, voltage across a capacitor lags the current through it by 90^0*)



AC circuit operation with R + C

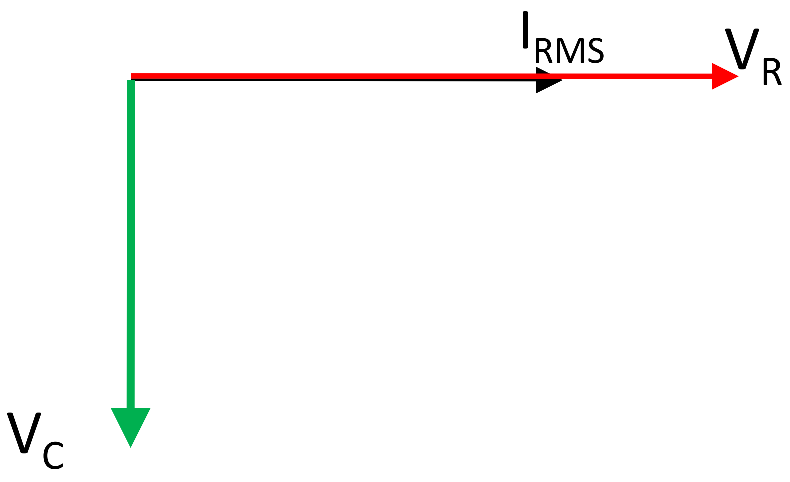


$$v = V_m \sin \omega t$$

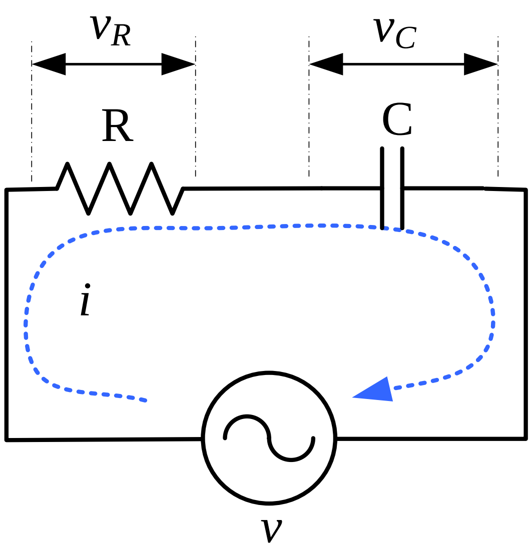
Phasor diagram

- According to KVL, the supply voltage must equal summation of the two voltage drops, one across the resistance and the other across the capacitance

$$\overline{V_{RMS}} = \overline{V_R} + \overline{V_C}$$



AC circuit operation with R + C

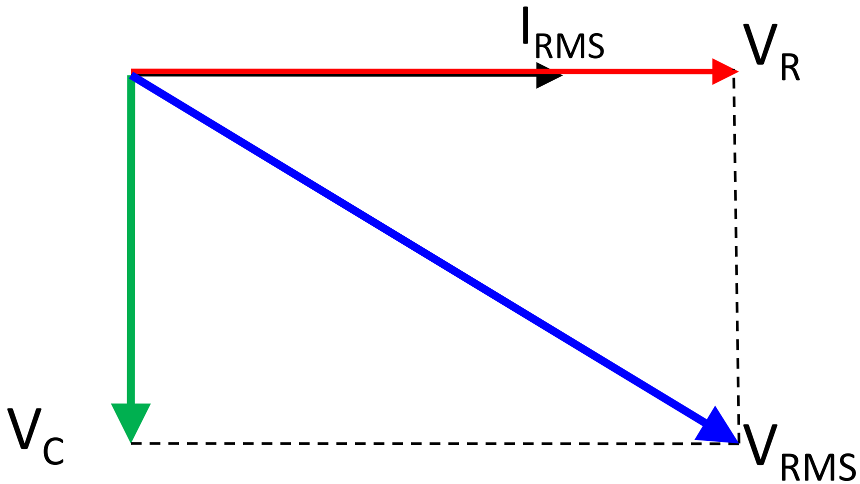


$$v = V_m \sin \omega t$$

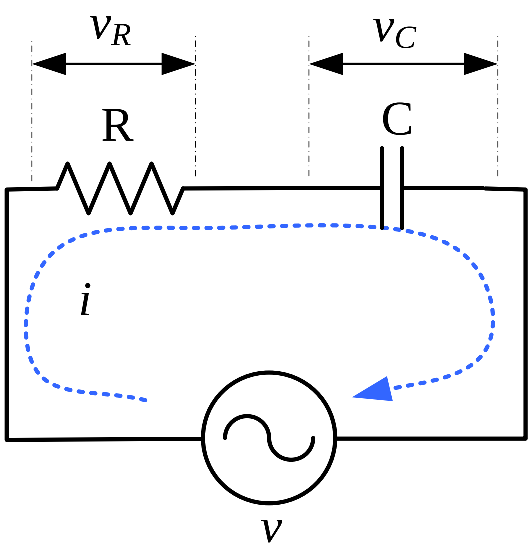
Phasor diagram

$$\overline{V_{RMS}} = \overline{V_R} + \overline{V_C}$$

- Thus, the supply voltage phasor V_{RMS} is drawn as the vector addition (resultant) of the two phasors V_R and V_C



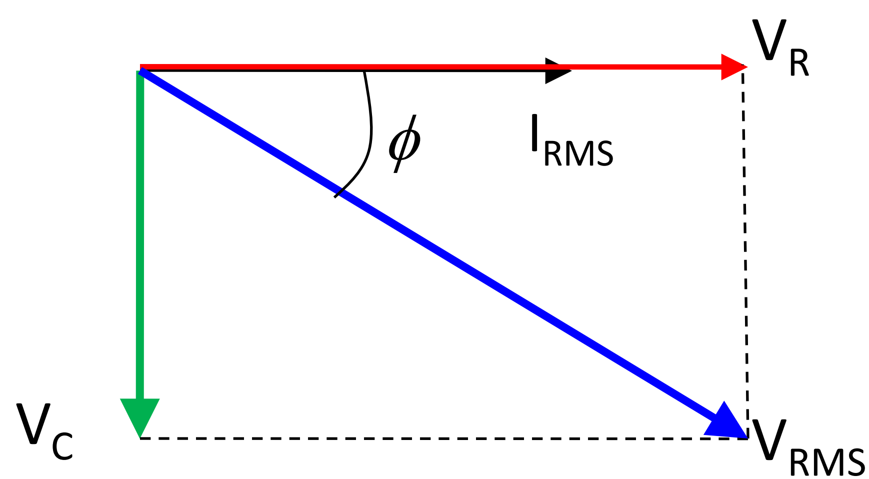
AC circuit operation with R + C



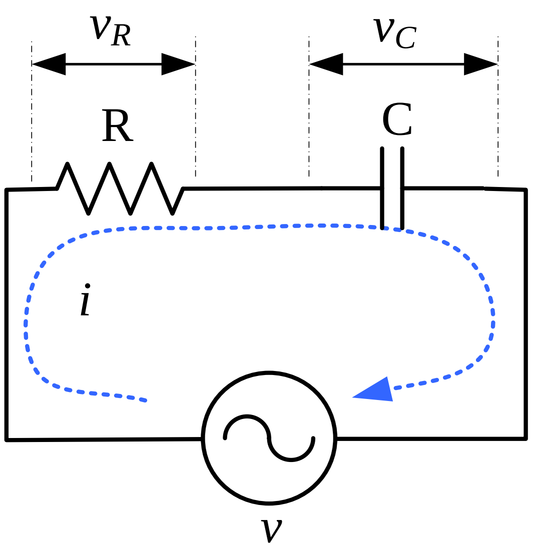
$$v = V_m \sin \omega t$$

Phasor diagram

- The angle between the supply voltage V_{RMS} phasor and the supply current I_{RMS} phasor is known as the **power factor angle** ϕ



AC circuit operation with R + C



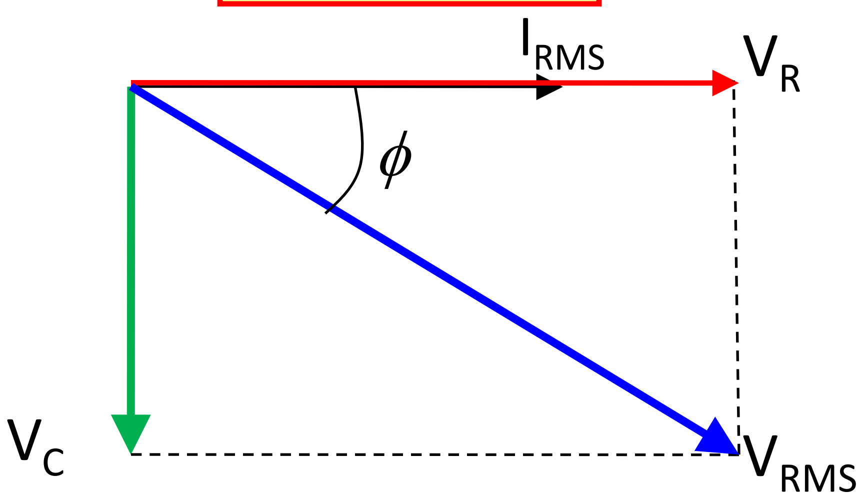
$$v = V_m \sin \omega t$$

Phasor diagram

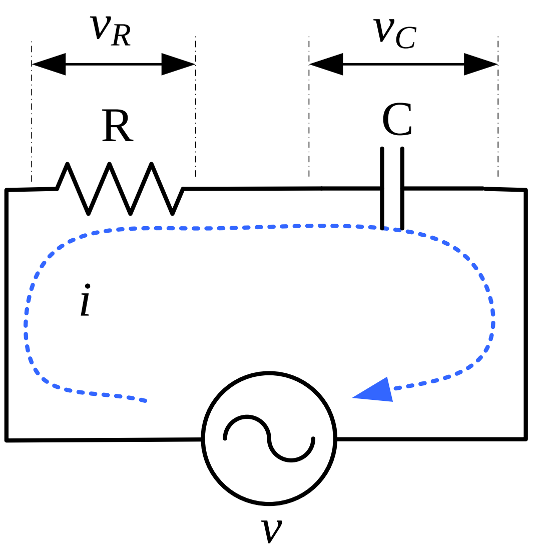
Value of the power factor angle can be expressed from the phasor diagram as:

$$\phi = \tan^{-1}\left(\frac{V_C}{V_R}\right) = \tan^{-1}\left(\frac{I_{RMS} \times X_C}{I_{RMS} \times R}\right) = \tan^{-1}\left(\frac{X_C}{R}\right)$$

$$\boxed{\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)}$$

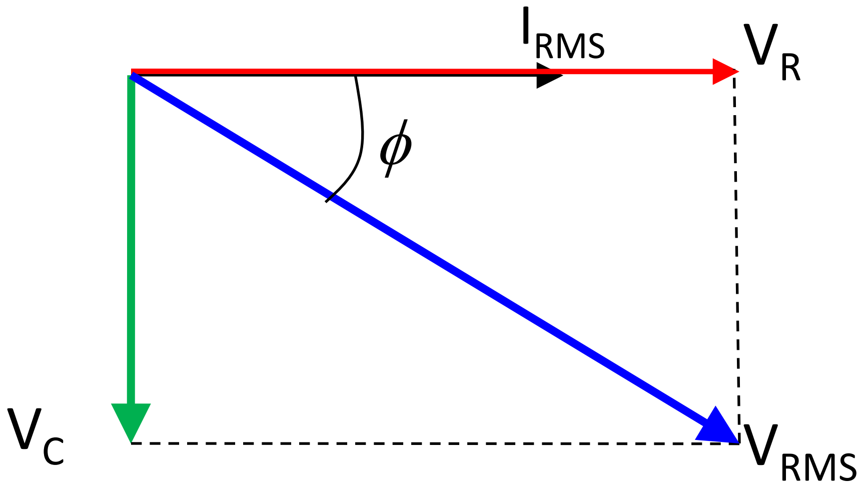


AC circuit operation with R + C



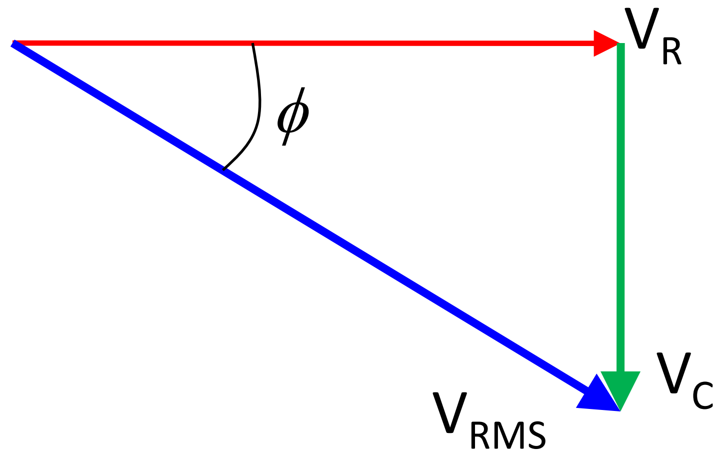
<u>Phasor diagram</u>

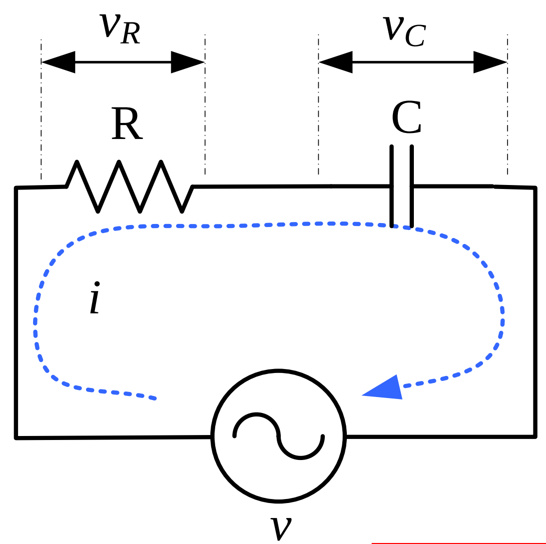
Drawing only the voltage phasors from the phasor diagram we obtain the so-called **voltage triangle** of a series R-C circuit:



AC circuit operation with R + C

Voltage triangle





From the voltage triangle we have the relation: $\boxed{V_{RMS} = \sqrt{V_R^{\ 2} + V_C^{\ 2}}}$

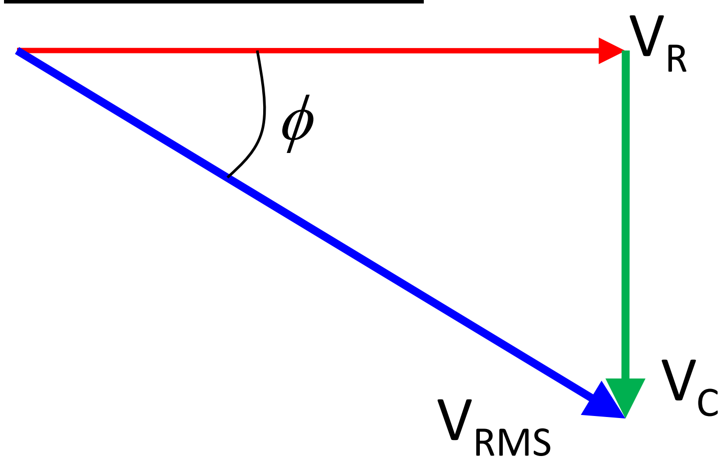
$$V_{RMS} = \sqrt{(I_{RMS}R)^2 + (I_{RMS}X_C)^2}$$

$$V_{RMS} = I_{RMS}\sqrt{R^2 + X_C^{\ 2}}$$

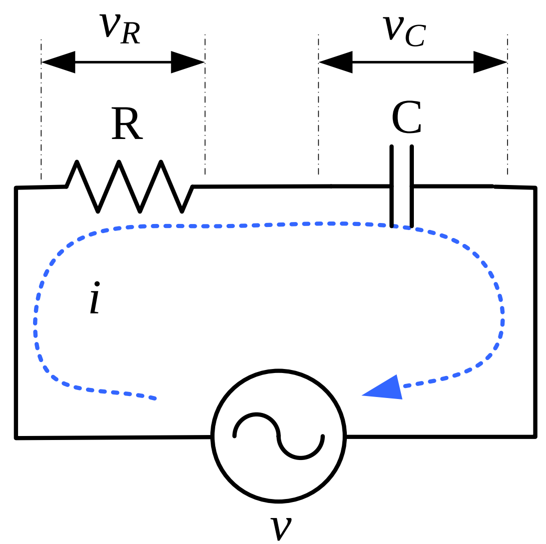
$$\frac{V_{RMS}}{I_{RMS}} = \sqrt{R^2 + X_C^{\ 2}}$$

AC circuit operation with R + C

Voltage triangle



$$\frac{V_{RMS}}{I_{RMS}} = \sqrt{R^2 + X_C{}^2}$$



$$\frac{V_{RMS}}{I_{RMS}} = \sqrt{R^2 + X_C{}^2} = Z$$

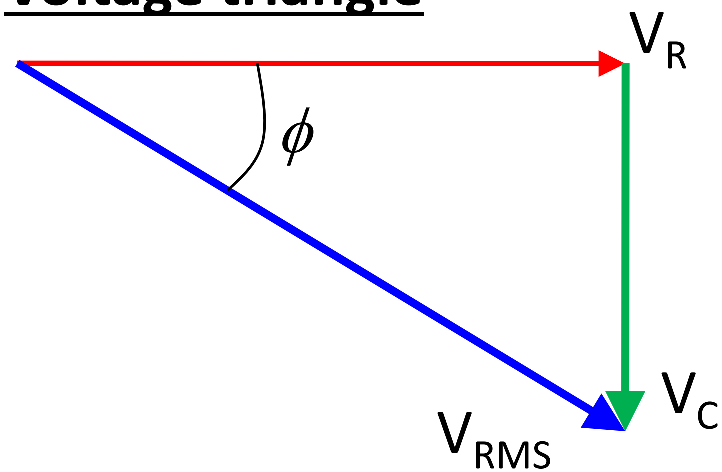
This ratio of V_{RMS} and I_{RMS} in a series R-C circuit is called **impedance** of the circuit that has a magnitude

$$Z = \sqrt{R^2 + X_C{}^2}$$

The unit of impedance is "Ohm" (Ω).

AC circuit operation with R + C

Voltage triangle



- Redraw the same triangle in terms of the resistance, reactance, and impedance only

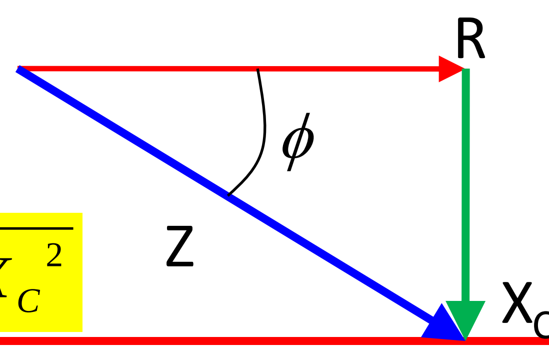
- (by eliminating the common quantity I_{RMS})

$$V_R = I_{RMS} \times R$$

$$V_L = I_{RMS} \times X_C$$

$$V_{RMS} = I_{RMS} \times Z$$

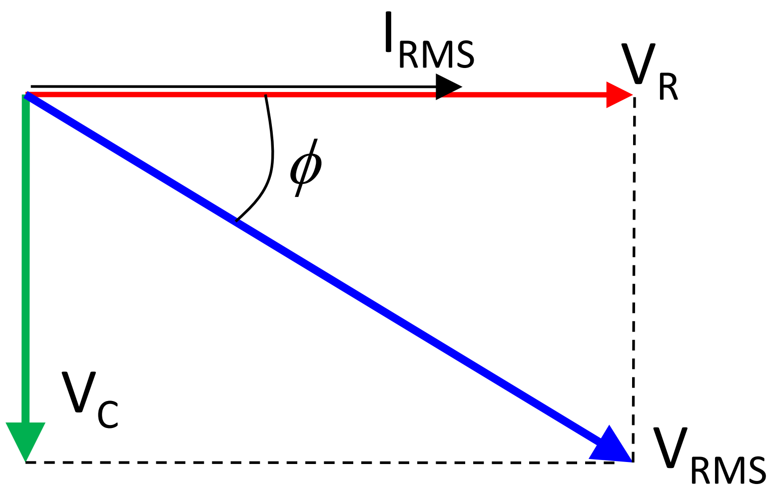
Impedance triangle



$$Z = \sqrt{R^2 + X_C{}^2}$$

Hypotenuse of the impedance triangle is the impedance Z
The angle between Z and R is the power factor angle ϕ

AC circuit operation with R + C

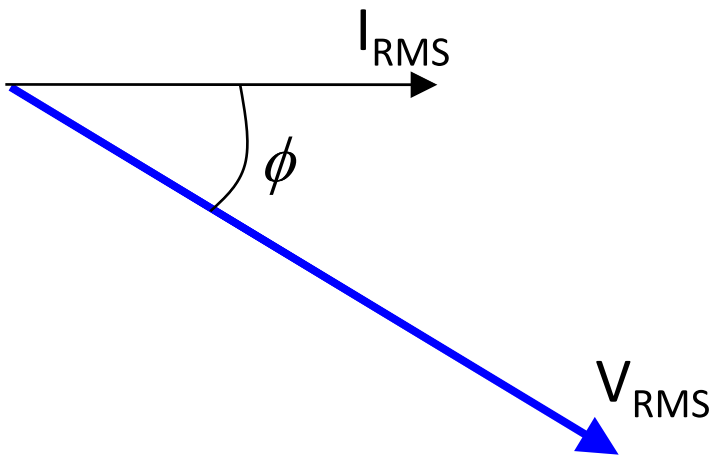


More on phase angle

- The supply voltage signal lags the current by a phase angle ϕ which is less than 90^0
- This is expected because it is a combination of R and C circuit

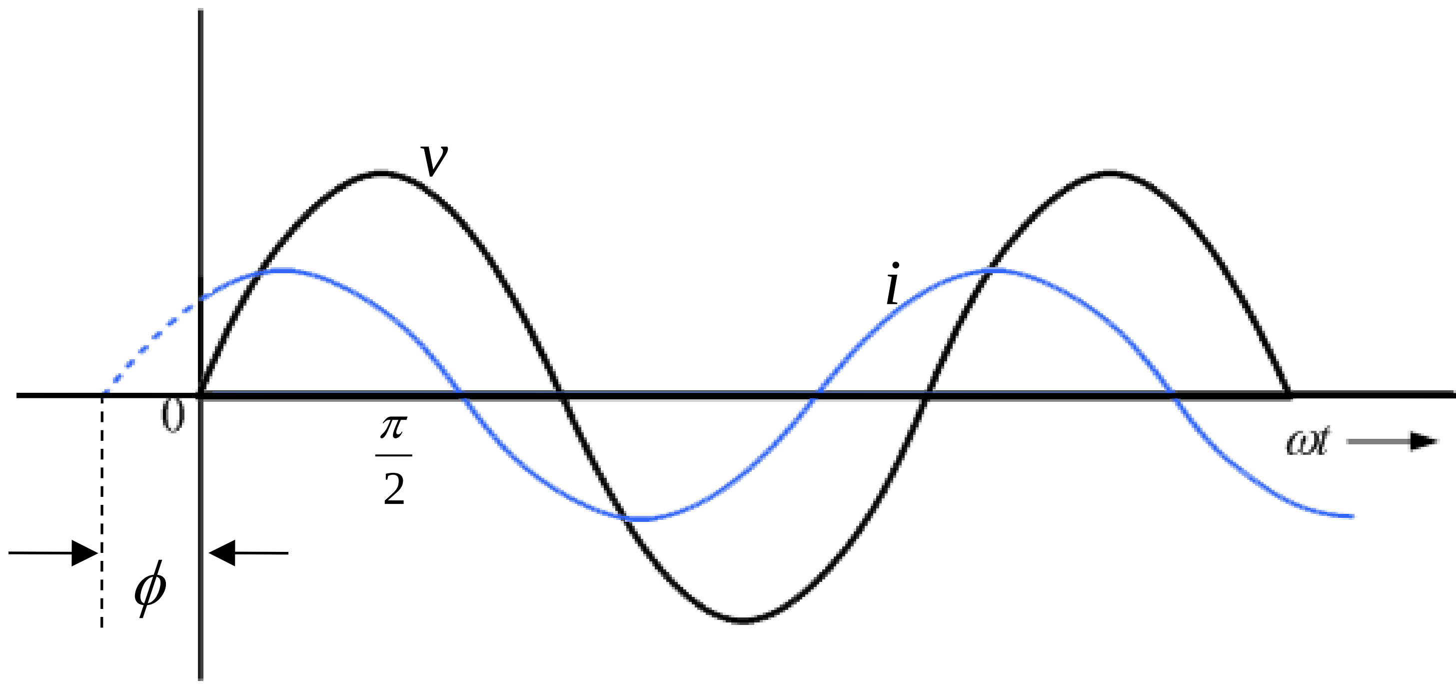
- In an R + C circuit, the angle between voltage and current should lie somewhere between 0^0 (pure resistance) and 90^0 (pure capacitance)
- When $X_C = R$, the phase angle $\phi = 45^0$
- When $R > X_C$, the phase angle $\phi < 45^0$
- When $X_C > R$, then the phase angle $\phi > 45^0$

AC circuit operation with R + C

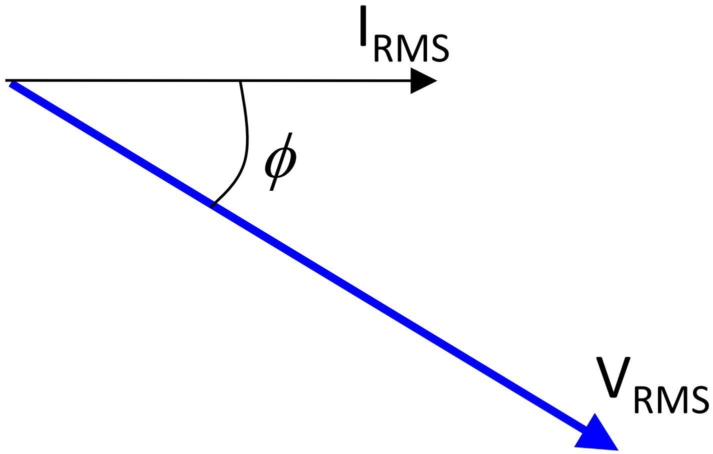


Instantaneous values

- The current is drawn leading ahead of the voltage by an angle ϕ that is less than 90^0



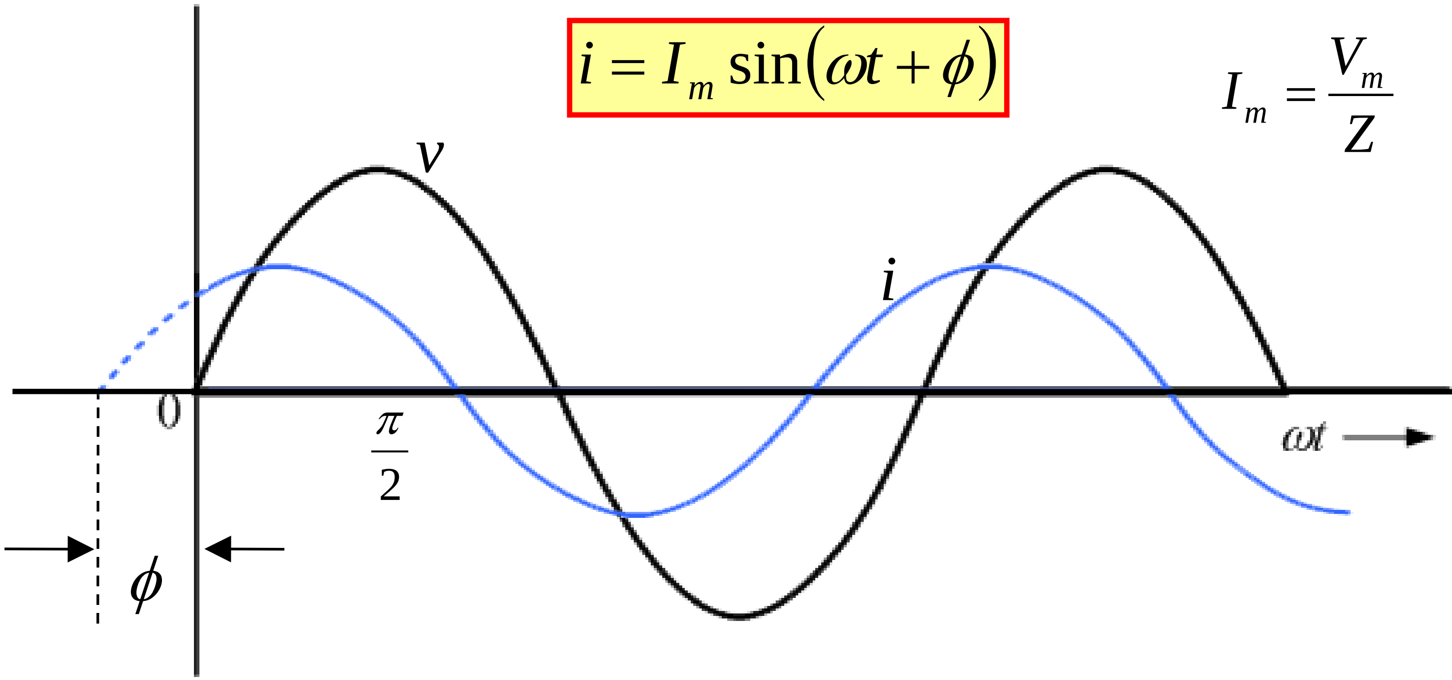
AC circuit operation with R + C



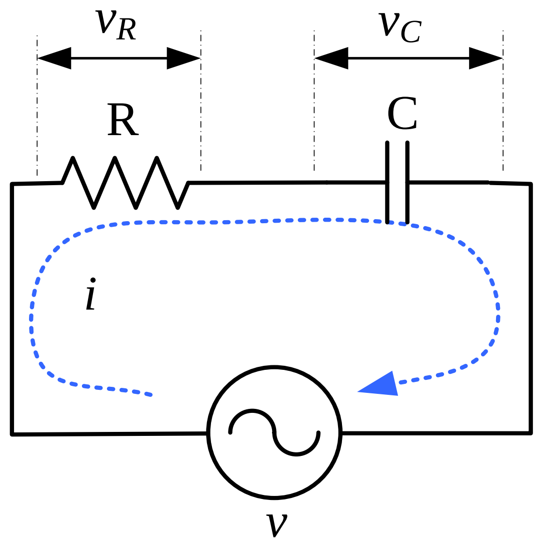
- If the voltage signal is given by the expression $$v = V_m \sin \omega t$$
- then the current signal that leads the voltage by an angle ϕ, will be given by the expression

$$i = I_m \sin(\omega t + \phi)$$

$$I_m = \frac{V_m}{Z}$$



AC circuit operation with R + C



$\boxed{v = V_m \sin \omega t}$ $\boxed{i = I_m \sin(\omega t + \phi)}$

Instantaneous power

$$p = v \times i$$

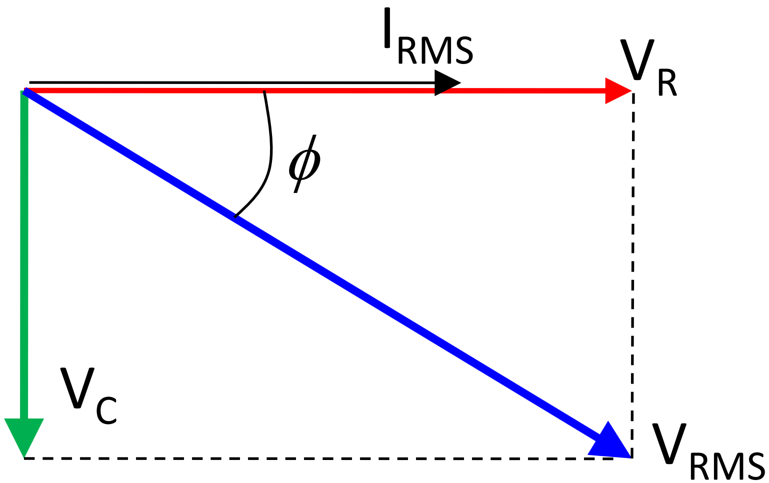
$$p = V_m \sin \omega t \times I_m \sin(\omega t + \phi)$$

$$p = V_m I_m \sin \omega t \sin(\omega t + \phi)$$

$$p = \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t - \phi) - \cos(\omega t + \omega t + \phi)\right]$$

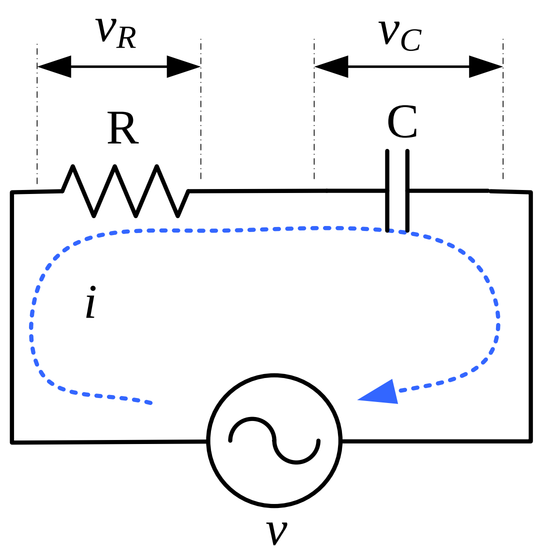
$$p = \frac{V_m I_m}{2}\left[\cos(-\phi) - \cos(2\omega t + \phi)\right]$$

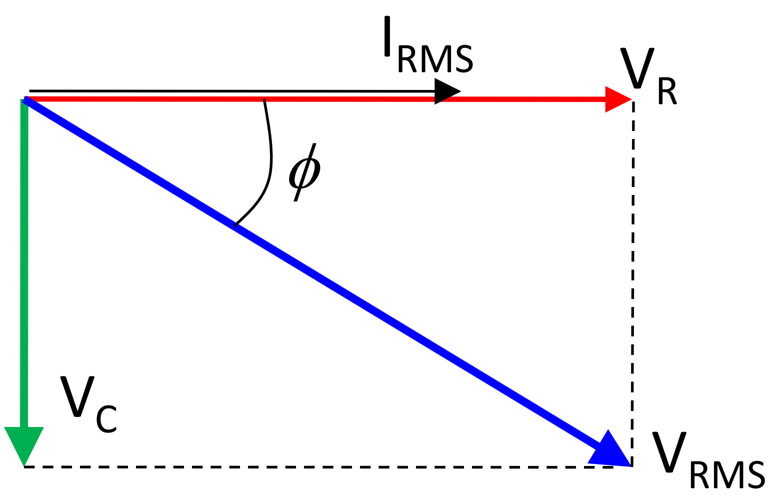
$$\boxed{p = \frac{V_m I_m}{2}\left[\cos(\phi) - \cos(2\omega t + \phi)\right]}$$



As earlier, the instantaneous power varies at **twice the frequency** of the input voltage signal.

AC circuit operation with R + C





<u>Average power</u>

$$\boxed{p = \frac{V_m I_m}{2}\left[\cos(\phi) - \cos(2\omega t + \phi)\right]}$$

$$P = \frac{1}{T}\int_0^T p\,dt$$

$$P = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\left[\cos(\phi) - \cos(2\omega t + \phi)\right]dt$$

……………………………

$$\boxed{P = V_{RMS} I_{RMS} \cos\phi}$$

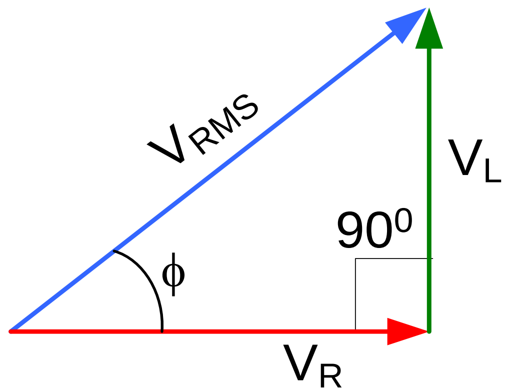
Same expression as for R+L circuit

Active power is the product of RMS values of voltage and current, and the power factor

Summary

Type of circuit	Effective impedance Z (Ω)	Current I_{RMS}	Phase angle (ϕ) between V & I	Power factor (cosϕ)	Active power
R	R	$\frac{V_{RMS}}{R}$	0^0	1	$V_{RMS} I_{RMS}$
L	$X_L = \omega L$	$\frac{V_{RMS}}{X_L}$	I_{RMS} lags V_{RMS} by 90^0	0	0
R-L	$Z = \sqrt{R^2 + X_L^{\;2}}$	$\frac{V_{RMS}}{Z}$	I_{RMS} lags V_{RMS} by ϕ $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$	(Lag) $\cos\phi$	$V_{RMS} I_{RMS} \cos\phi$
C	$X_C = \frac{1}{\omega C}$	$\frac{V_{RMS}}{X_C}$	I_{RMS} leads V_{RMS} by 90^0	0	0
R-C	$Z = \sqrt{R^2 + X_C^{\;2}}$	$\frac{V_{RMS}}{Z}$	I_{RMS} leads V_{RMS} by ϕ $\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$	(Lead) $\cos\phi$	$V_{RMS} I_{RMS} \cos\phi$

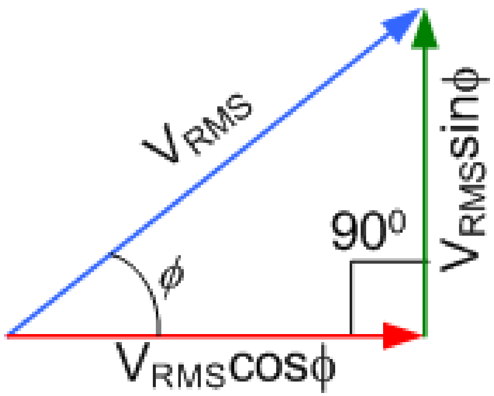
Power in AC circuits

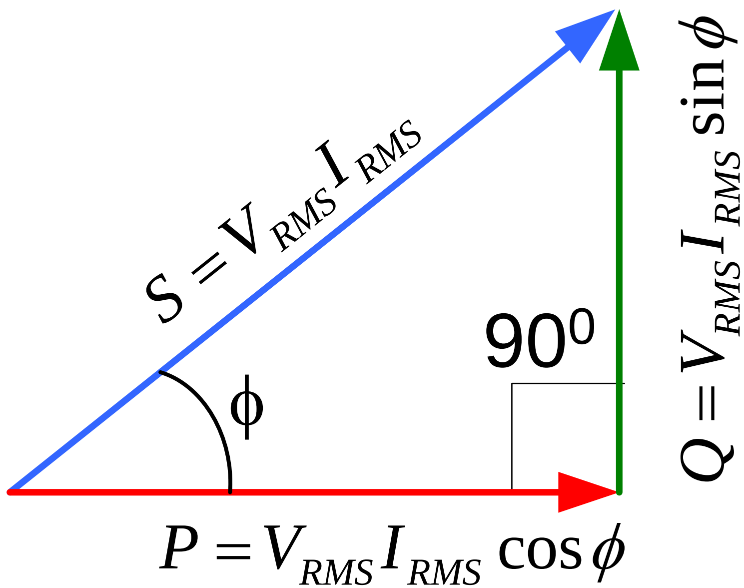


Voltage triangle of RL circuit

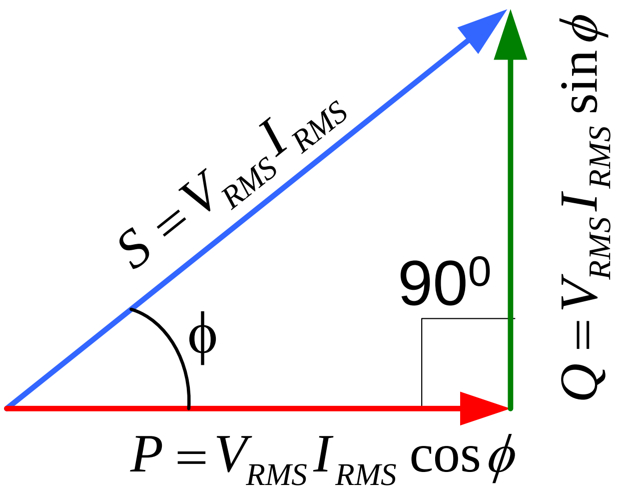
$$V_R = V_{RMS} \cos\phi \qquad V_L = V_{RMS} \sin\phi$$

Multiplying all three arms of the voltage triangle by the current I_{RMS} , we have the so called **power triangle:**





Power in AC circuits

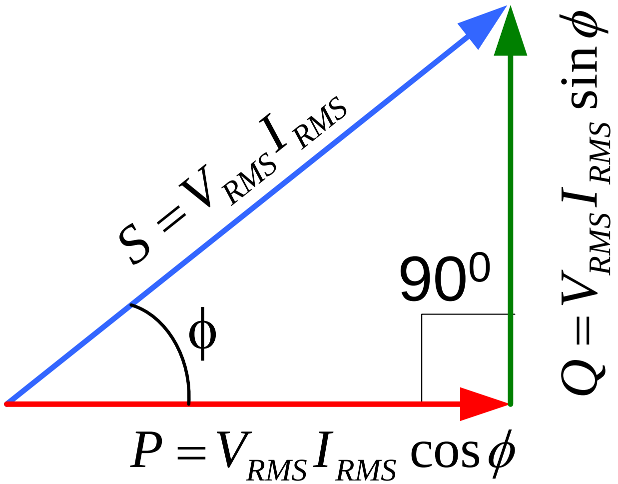


- The quantity $\boldsymbol{P = V_{RMS} I_{RMS} \cos\phi}$ is called the **active power**
- It is expressed in the unit of Watt (W)

- The quantity $\boldsymbol{Q = V_{RMS} I_{RMS} \sin\phi}$ is called the **reactive power**
- It is expressed in the unit of Volt-Ampere-Reactive (VA_R)

- The quantity $\boldsymbol{S = V_{RMS} I_{RMS}}$ is called the **apparent power (*Total power*)**
- It is expressed in the unit of Volt-Ampere (VA)

Power in AC circuits



- The relation among the three powers are:

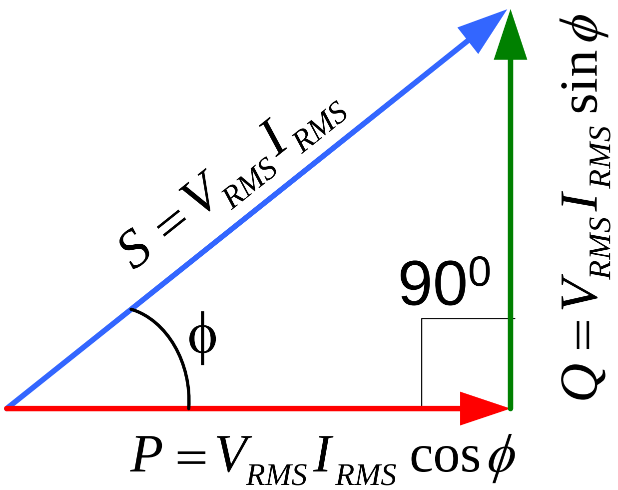
$$\boxed{S = \sqrt{P^2 + Q^2}}$$

- These mathematical relations are equally valid for R-C circuits also

- The practical definition of **power factor** can now be derived from the power triangle as:

$$\boxed{\cos\phi = \frac{P}{S} = \frac{\text{Active power}}{\text{Apparent power}}}$$

Power in AC circuits



$$\boxed{\cos\phi = \frac{P}{S} = \frac{\text{Active power}}{\text{Apparent power}}}$$

Thus, power factor can be defined as the fraction of total apparent power that is being used as active power

- The power factor is lagging when the current lags the applied voltage (inductive circuit, i.e. R-L circuit)
- The power factor is leading when the current leads the applied voltage (capacitive circuit, i.e. R-C circuit)
- For purely inductive or purely capacitive circuit, the power factor is $\cos 90^0 = 0$
- For purely resistive circuit, the power factor is $\cos 0^0 = 1$