3-Phase systems

Day 29

Symmetrical components

ILOs – Day 29

- Explain the use of **symmetrical components** in analyzing unbalanced signals
- Describe the ‘α’ operator
- Derive the synthesis equations for relating unbalanced signals to their symmetrical components
- Derive the analysis equations for relating symmetrical components to the original unbalanced set of signals

Symmetrical Components

- Analysis of unbalanced systems is not straightforward as in the case of power system with faults or electric machines connected
- Using the concept of **symmetrical components**, an unbalanced 3-phase system can be equivalently represented by a combination of three individually balanced set of 3-phase systems
- Thus, though the original system is unbalanced
- Their equivalent symmetrical components are balanced
- Thus analysis becomes easier
- Overall performance of the unsymmetrical system is superposition of performances of the individual symmetrical component systems

Fortescue Theorem

- A set of '*n*' unbalanced phasors can be represented by *(n – 1)* balanced set of *n*-phase systems of *different phase-sequence* and one *zero phase-sequence* system

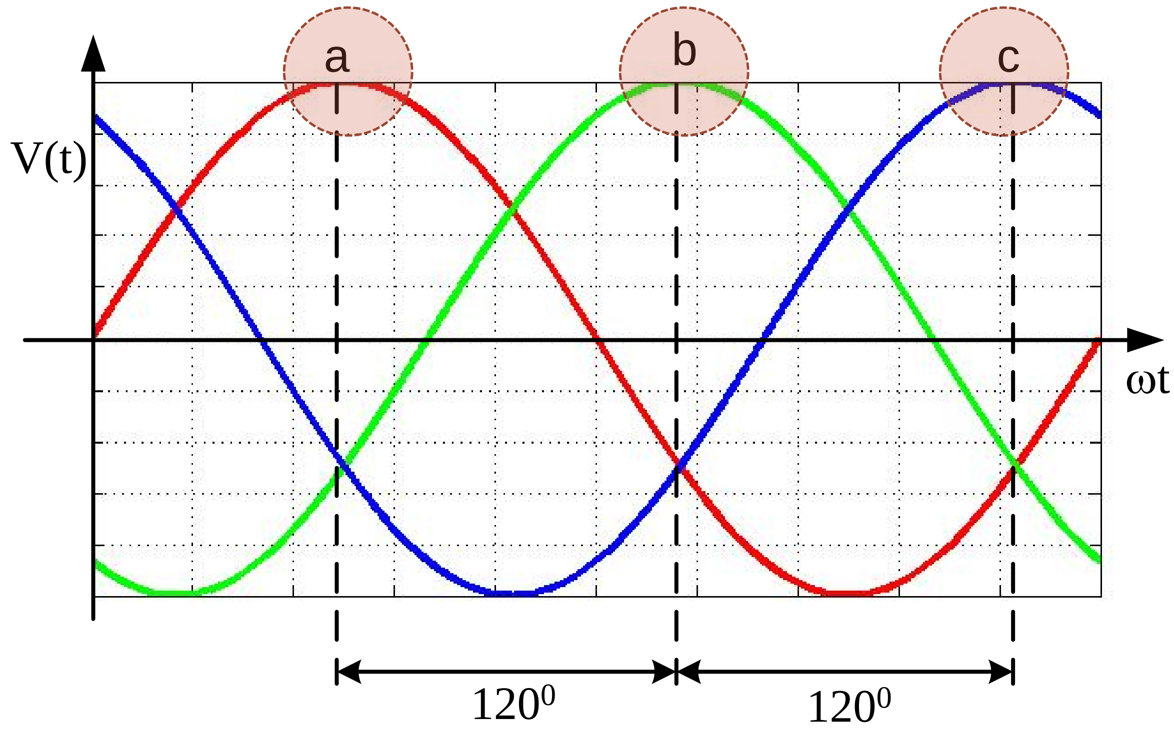
- In terms of an unbalanced 3-phase system, the Fortescue Theorem can thus be reframed as:
  - An unbalanced 3-phase system
  - can be represented by 2 balanced set of 3-phase systems
  - of different phase sequence
  - and one zero-sequence system

Phase sequence

- Set of 3-phase signals

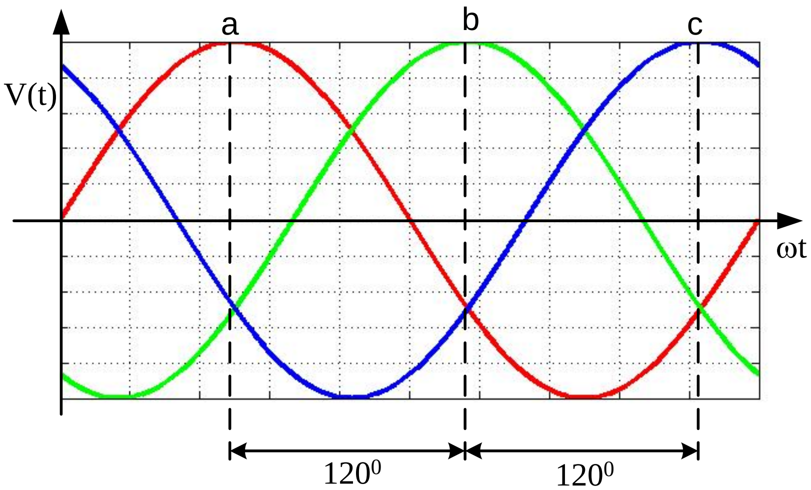
Phase sequence

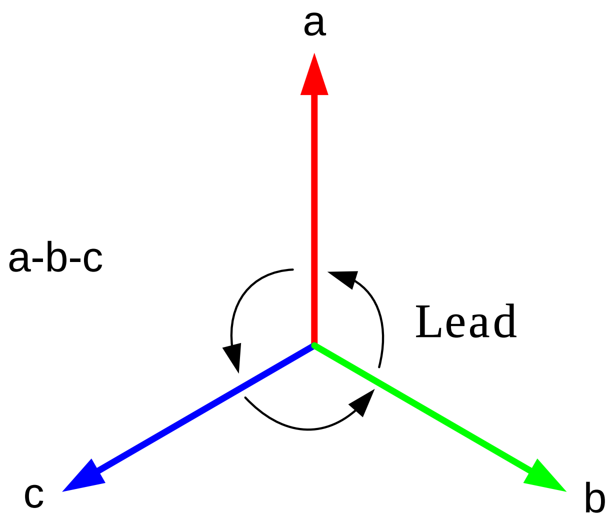
a-b-c



- Phase sequence of **a-b-c** indicates that phase ‘a’ attains its peak first in time followed by ‘b’ and then ‘c’
- a leads b, b leads c

Phase sequence – phasor diagram





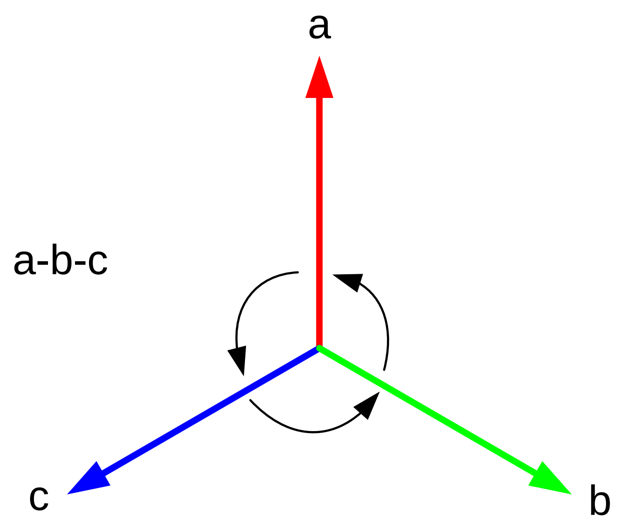
- In phasor diagram, the a-b-c phase sequence is denoted by phasors a-b-c coming in ANTICLOCKWISE sequence as per convention

- 'a' phasor leads 'b' by 120^0 and 'c' by 240^0 in the anticlockwise direction

Phase sequence – phasor diagram

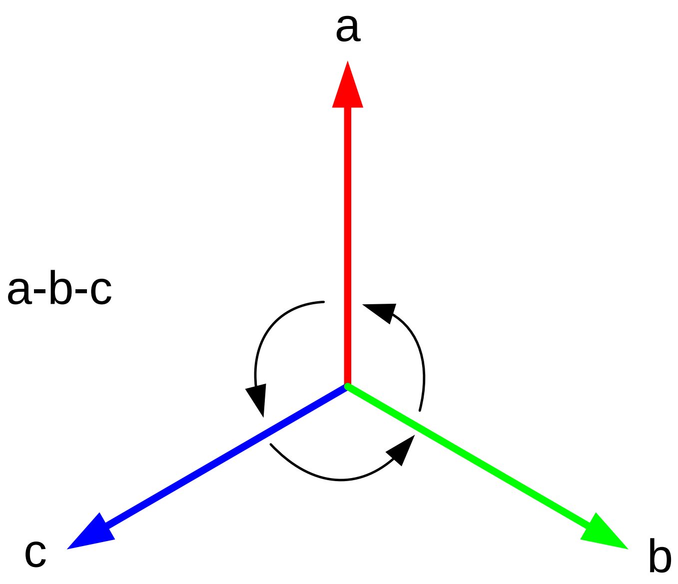
- This sequence is continued in a cyclic fashion a-b-c-a-b-c-a-........
- Thus a phase sequence of “a-b-c” is basically same as:
- ‘b-c-a’ or ‘c-a-b’

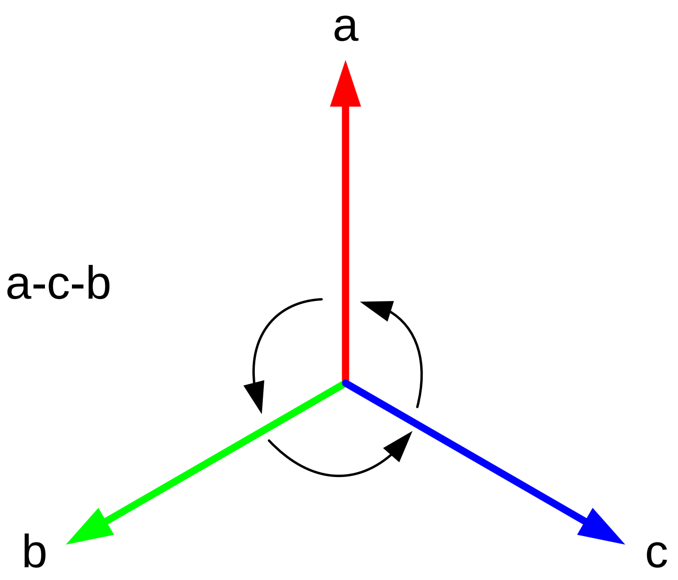
a – b – c – a – b – c – a – b – c



Negative phase sequence

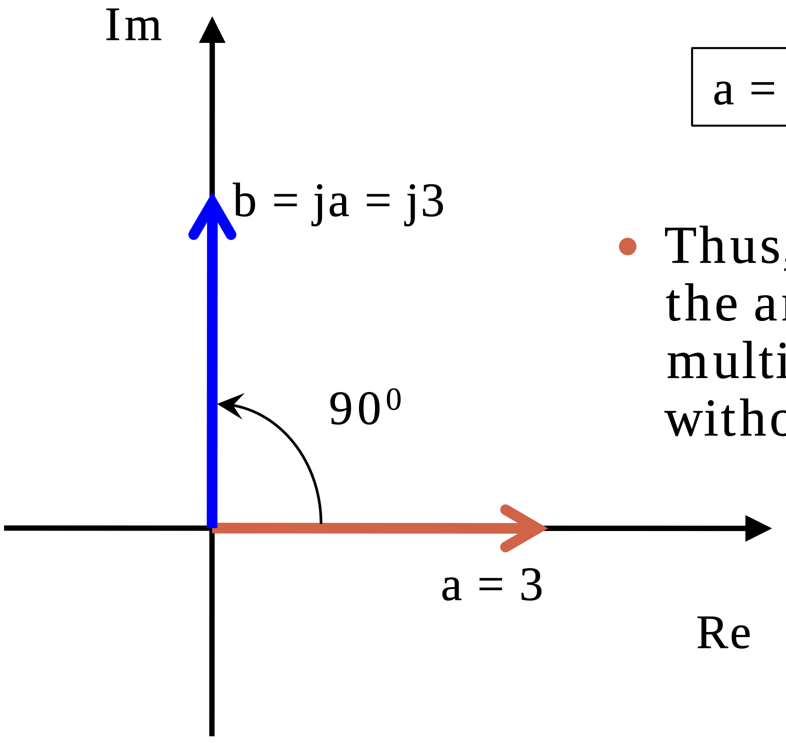
- Positive phase sequence a-b-c
- An opposite phase sequence will be indicated by ‘a-c-b’
- ‘a-c-b’ is called ***negative phase sequence***
- In negative sequence, the phasor ‘a’ leads ‘c’ by 120^0 and ‘b’ by 240^0





The j - operator

- The complex operator 'j' rotates a phasor by 90^0 in the anticlockwise direction without changing its magnitude



$a = 3\angle 0^0$

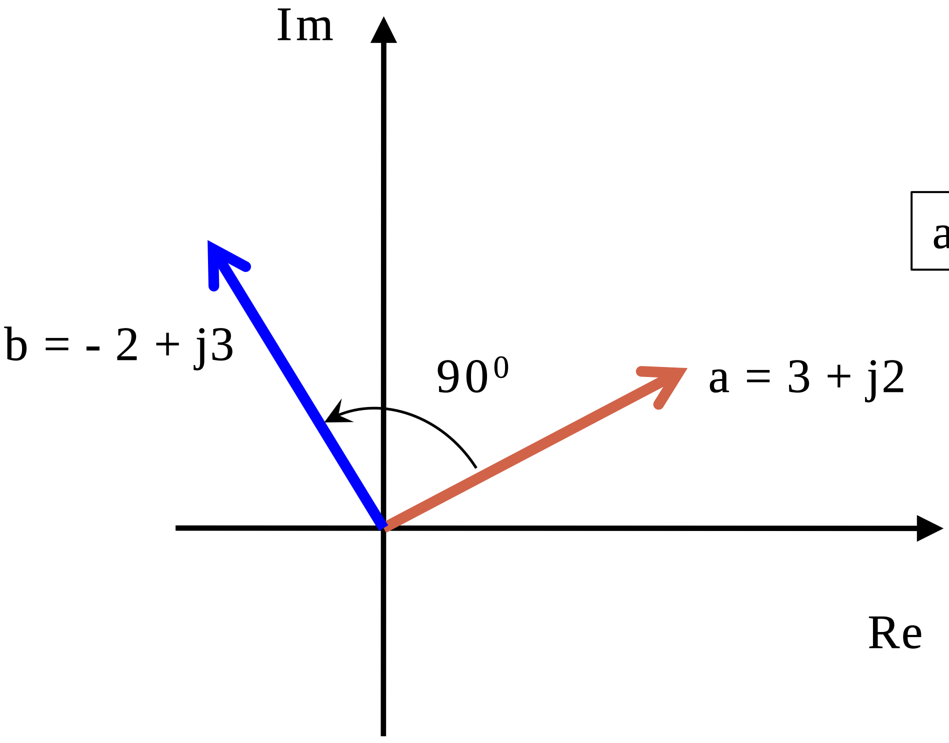
$b = ja$ | $= j3$ | $= 3\angle 90^0$

- Thus, the phasor 'a' is rotated 90^0 in the anticlockwise direction when multiplied by the operator 'j', without any change in its magnitude

$j = 1\angle 90^0$

The j - operator

- The complex operator ‘j’ rotates a phasor by 90^0 in the anticlockwise direction without changing its magnitude



$\boxed{a = 3+j2}$ $\boxed{b = ja = j(3+j2)}$

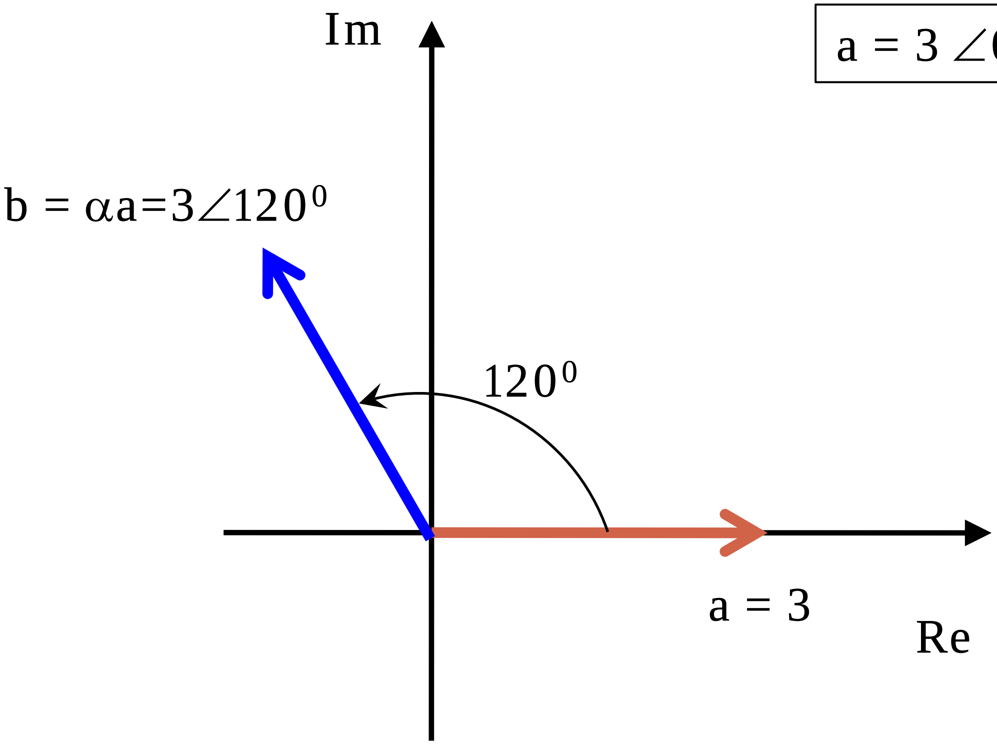
$\boxed{b = j3+j^2 2}$

$\boxed{b = -2 + j3}$

The α - operator

- Similarly, the 'α' operator rotates a phasor by **120^0** in the **anticlockwise direction** without changing its magnitude

$a = 3 \angle 0^0$ | $b = \alpha a = 3 \angle 120^0$



$$\alpha = 1 \angle 120^0$$

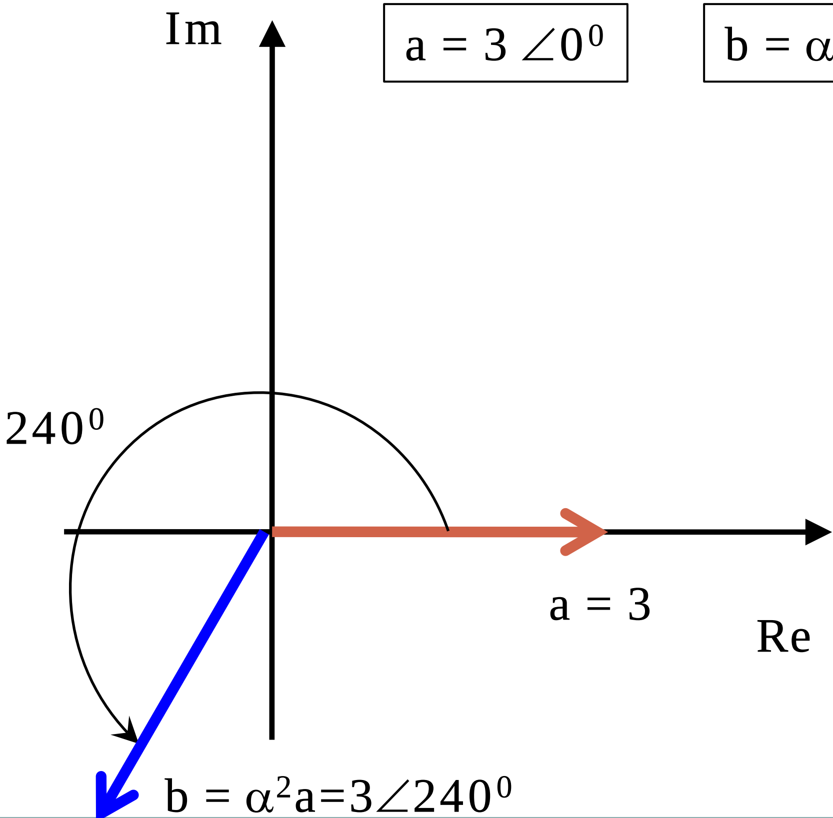
$$\alpha = 1 e^{j120^0}$$

$$= \cos 120^0 + j \sin 120^0$$

$$= -0.5 + j0.866$$

The α - operator

- The 'α^2' operator rotates a phasor by **240^0** in the **anticlockwise direction** without changing its magnitude



$a = 3\angle 0^0$

$b = \alpha^2 a$ | $=3\times 1\angle 120^0 \times 1\angle 120^0$ | $=3\angle 240^0$

$$\alpha^2 = 1\angle 240^0$$

$$\alpha^2 = 1e^{j240^0}$$

$$= \cos 240^0 + j\sin 240^0$$

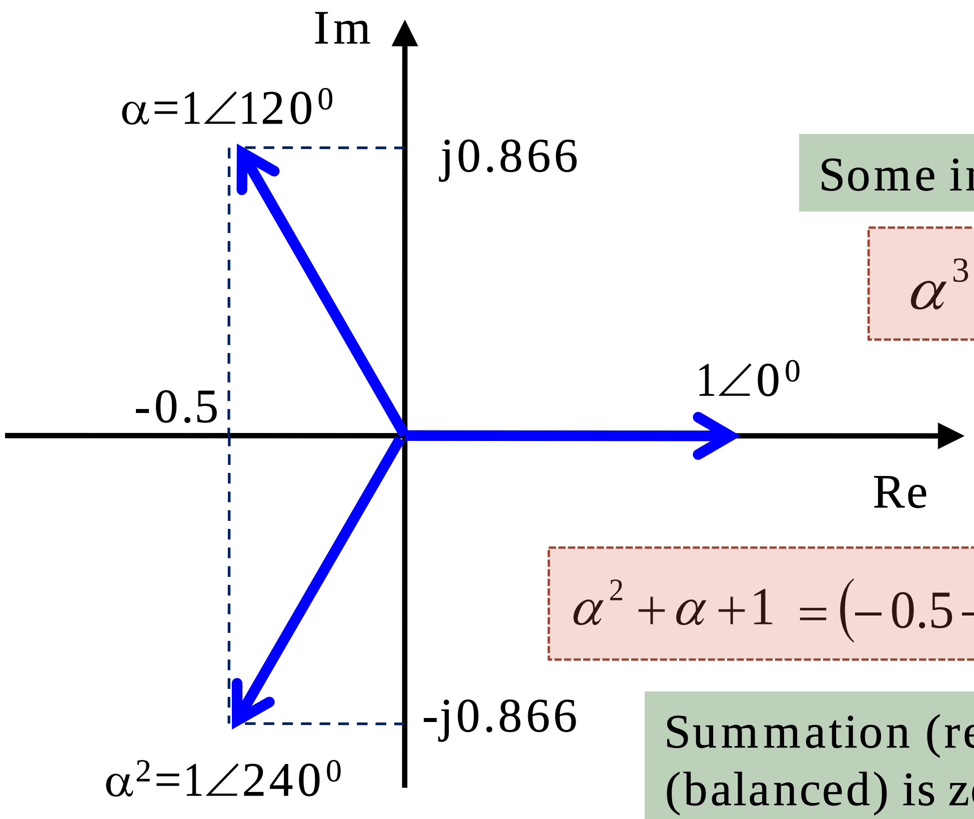
$$= -0.5 - j0.866$$

The α - operator

$$1\angle 0^0 = 1 + j0$$

$$\alpha = 1\angle 120^0 \;= -0.5 + j0.866$$

$$\alpha^2 = 1\angle 240^0 = -0.5 - j0.866$$



Some important properties of α operator

$$\alpha^3 = 1\angle 360^0 \;= 1$$

$$\alpha^4 = 1\angle 480^0 = 1\angle 120^0 = \alpha$$

$$\alpha^2 + \alpha + 1 \;= \left(-0.5 - j0.866\right) + -\left(0.5 + j0.866\right) + \left(1\right) = 0$$

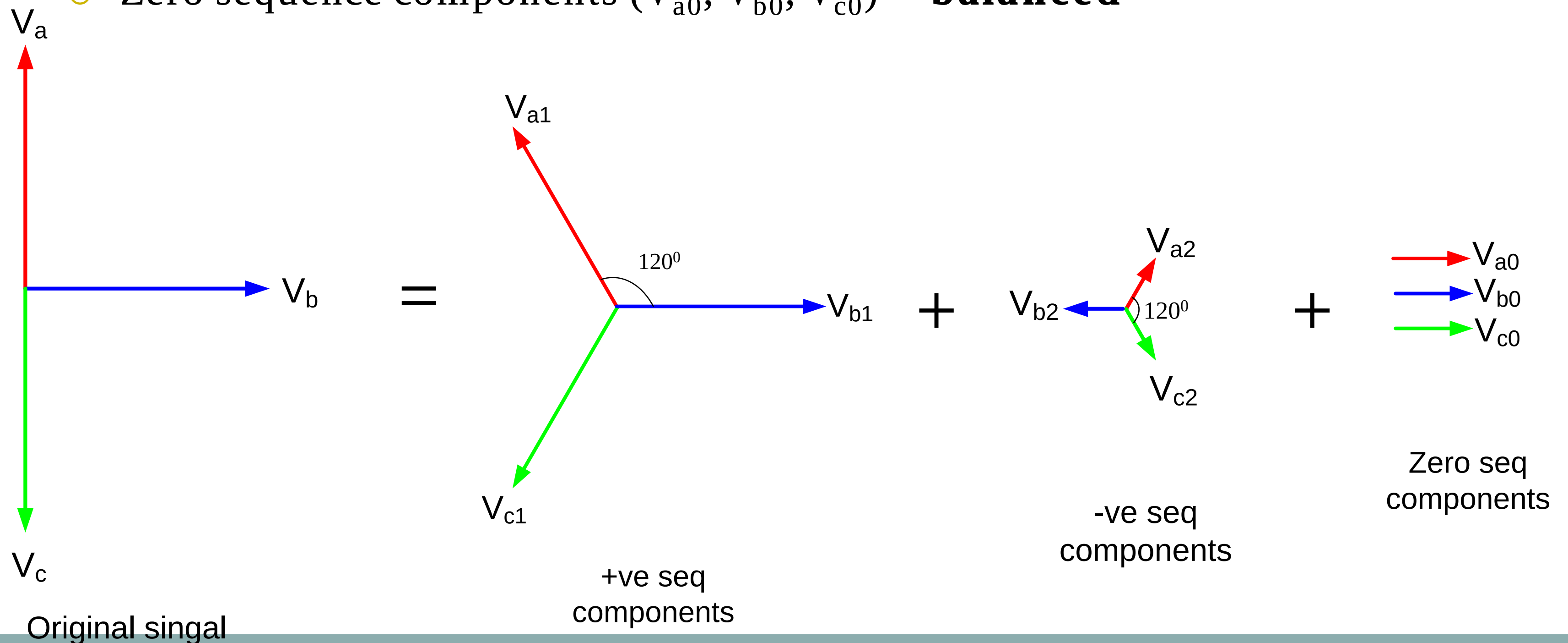
Summation (resultant) of the three phasors (balanced) is zero

Representation of unbalanced 3-phase system by symmetrical components

- As per Fortescue theorem, 3 unsymmetrical and unbalanced phasors (voltage or current) of a 3-phase system can be resolved (broken) into 3 balanced set of phasors those have the following features:
- **Positive sequence components**:
  - A balanced set of 3 phasors of equal magnitude, apart by 120^0 and **having *same* phase sequence as the original unbalanced phasors**
- **Negative sequence components**:
  - A balanced set of 3 phasors of equal magnitude, apart by 120^0 and **having *opposite* phase sequence as the original unbalanced phasors**.
- **Zero sequence components**:
  - A balanced set of 3 phasors of equal magnitude, but zero phase displacement among themselves
  - The three zero sequence component signals are thus co-phasal

Representation of unbalanced 3-phase system by symmetrical components

- For example, take the set of three **unbalanced** phasors (V_a, V_b, V_c)
- These 3 voltages can be resolved into:
  - Positive sequence components (V_{a1}, V_{b1}, V_{c1}) – **balanced**
  - Negative sequence components (V_{a2}, V_{b2}, V_{c2}) – **balanced**
  - Zero sequence components (V_{a0}, V_{b0}, V_{c0}) – **balanced**

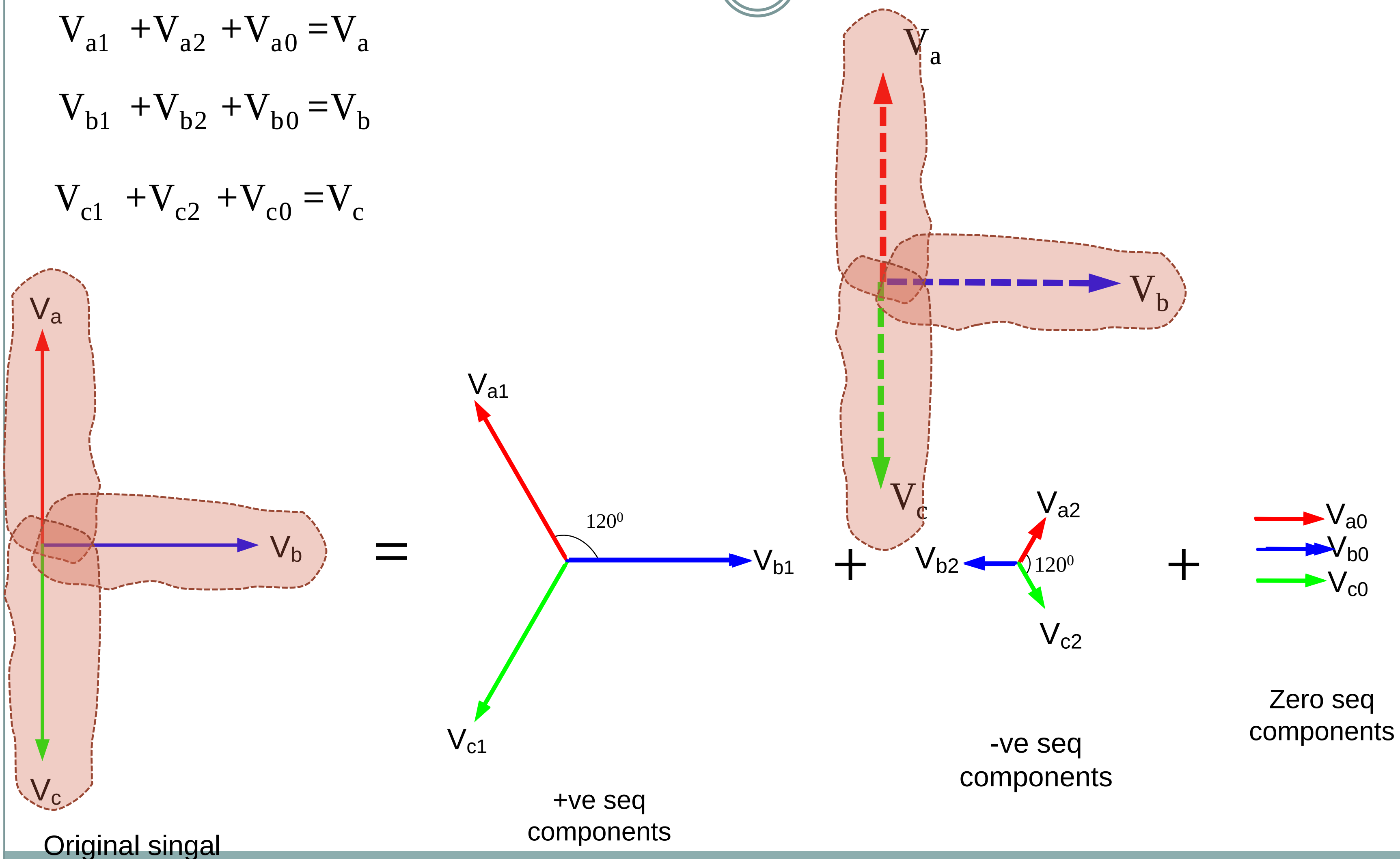


Representation of unbalanced 3-phase system by symmetrical components

$$V_{a1} + V_{a2} + V_{a0} = V_a$$

$$V_{b1} + V_{b2} + V_{b0} = V_b$$

$$V_{c1} + V_{c2} + V_{c0} = V_c$$



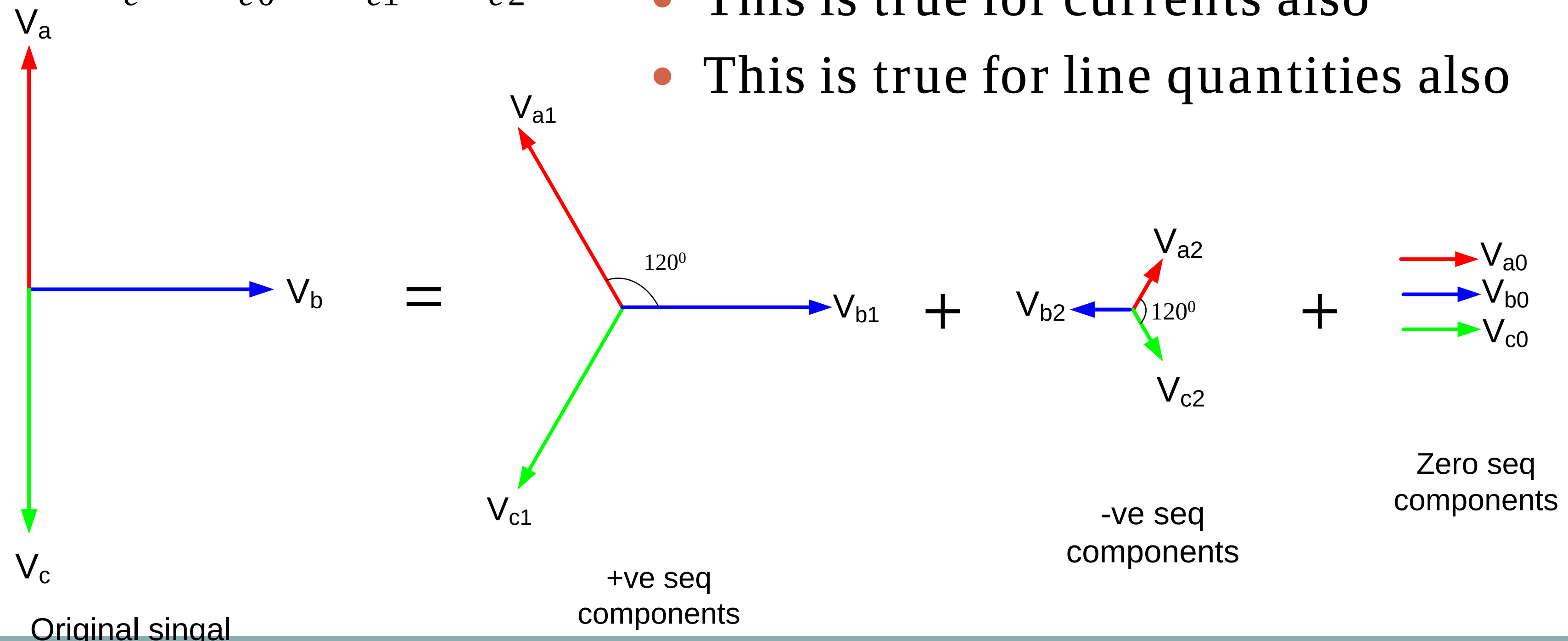
Mathematical relations

$$\overrightarrow{V_a} = \overrightarrow{V_{a0}} + \overrightarrow{V_{a1}} + \overrightarrow{V_{a2}}$$

$$\overrightarrow{V_b} = \overrightarrow{V_{b0}} + \overrightarrow{V_{b1}} + \overrightarrow{V_{b2}}$$

$$\overrightarrow{V_c} = \overrightarrow{V_{c0}} + \overrightarrow{V_{c1}} + \overrightarrow{V_{c2}}$$

- Voltage in each phase is equal to the **vector** (**phasor**) sum of positive, negative, and zero sequence voltage in that phase
- This is true for currents also
- This is true for line quantities also

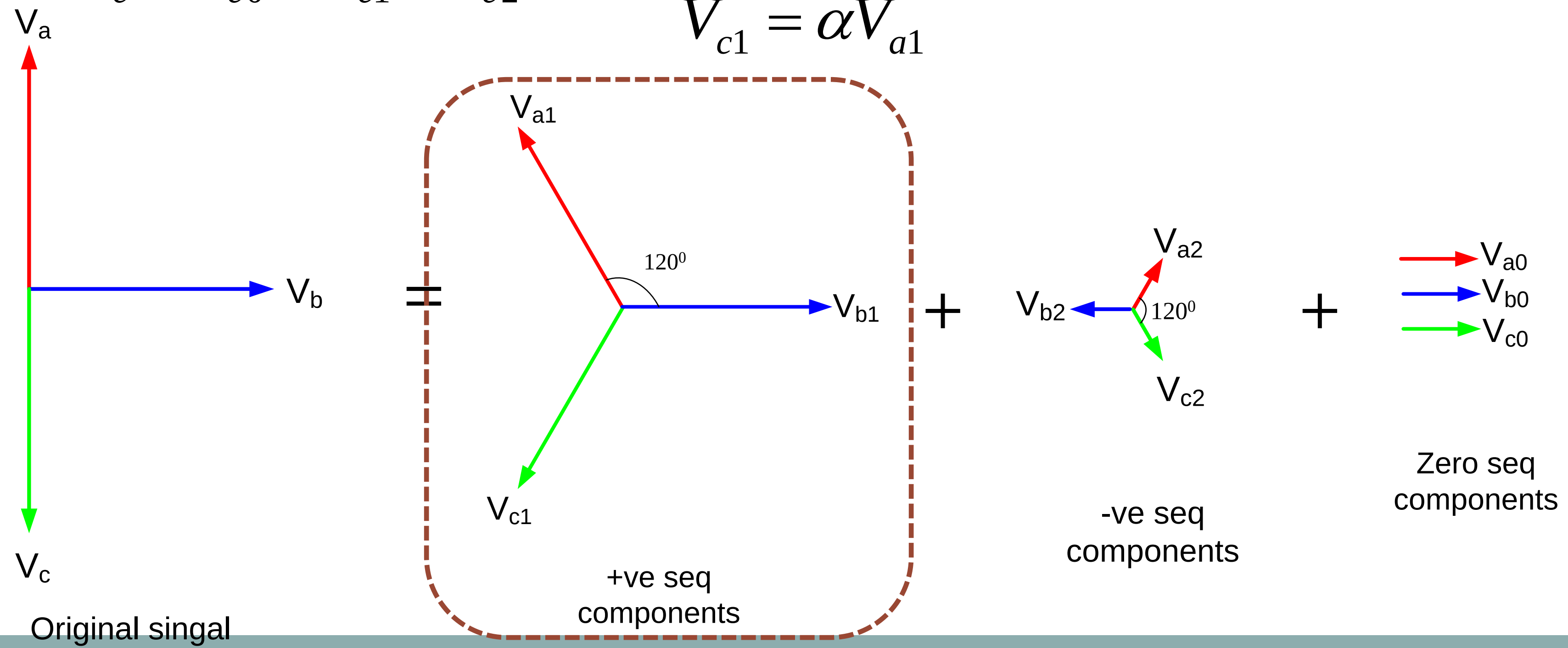


Mathematical relations

$$\overrightarrow{V_a} = \overrightarrow{V_{a0}} + \overrightarrow{V_{a1}} + \overrightarrow{V_{a2}}$$
$$\overrightarrow{V_b} = \overrightarrow{V_{b0}} + \overrightarrow{V_{b1}} + \overrightarrow{V_{b2}}$$
$$\overrightarrow{V_c} = \overrightarrow{V_{c0}} + \overrightarrow{V_{c1}} + \overrightarrow{V_{c2}}$$

- The positive sequence components are interrelated as:

$$V_{b1} = \alpha^2 V_{a1}$$
$$V_{c1} = \alpha V_{a1}$$

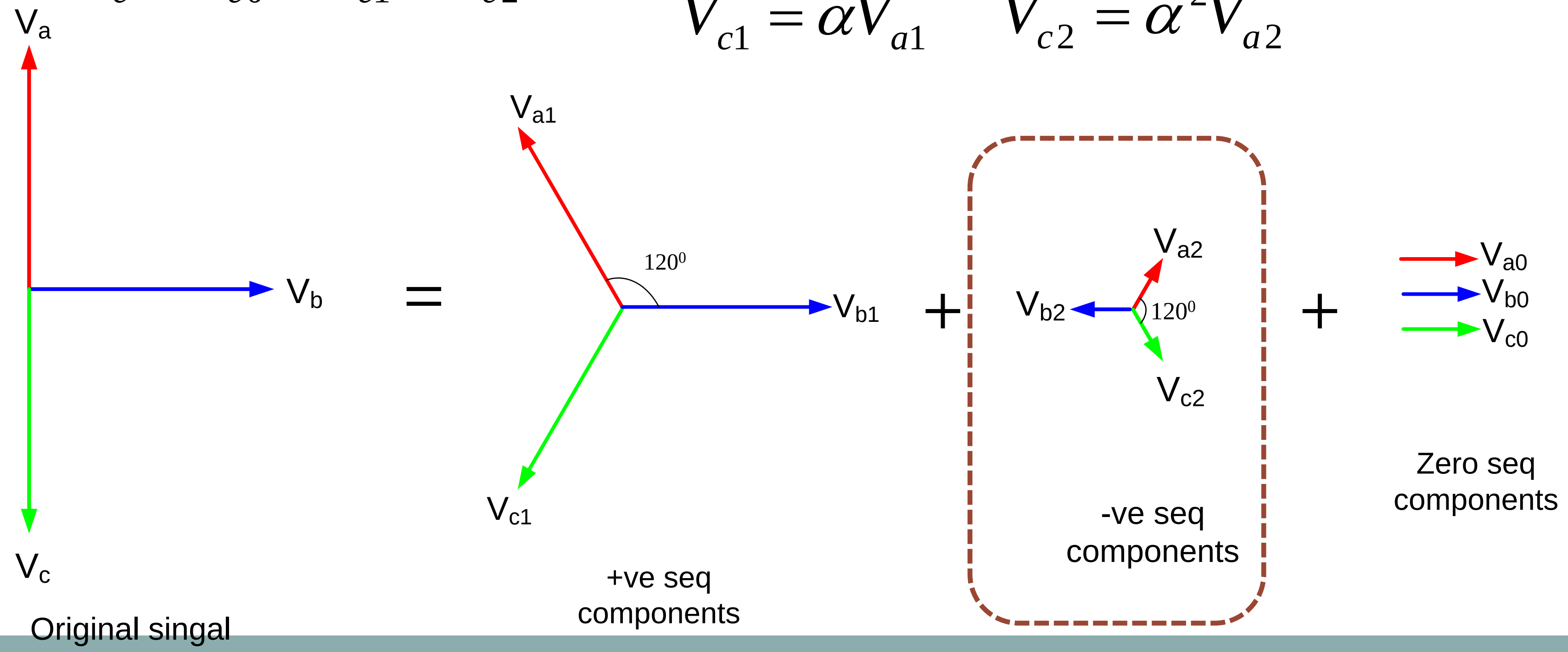


Mathematical relations

$$\overrightarrow{V_a} = \overrightarrow{V_{a0}} + \overrightarrow{V_{a1}} + \overrightarrow{V_{a2}}$$
$$\overrightarrow{V_b} = \overrightarrow{V_{b0}} + \overrightarrow{V_{b1}} + \overrightarrow{V_{b2}}$$
$$\overrightarrow{V_c} = \overrightarrow{V_{c0}} + \overrightarrow{V_{c1}} + \overrightarrow{V_{c2}}$$

- The negative sequence components are interrelated as:

$$V_{b1} = \alpha^2 V_{a1} \quad V_{b2} = \alpha V_{a2}$$
$$V_{c1} = \alpha V_{a1} \quad V_{c2} = \alpha^2 V_{a2}$$



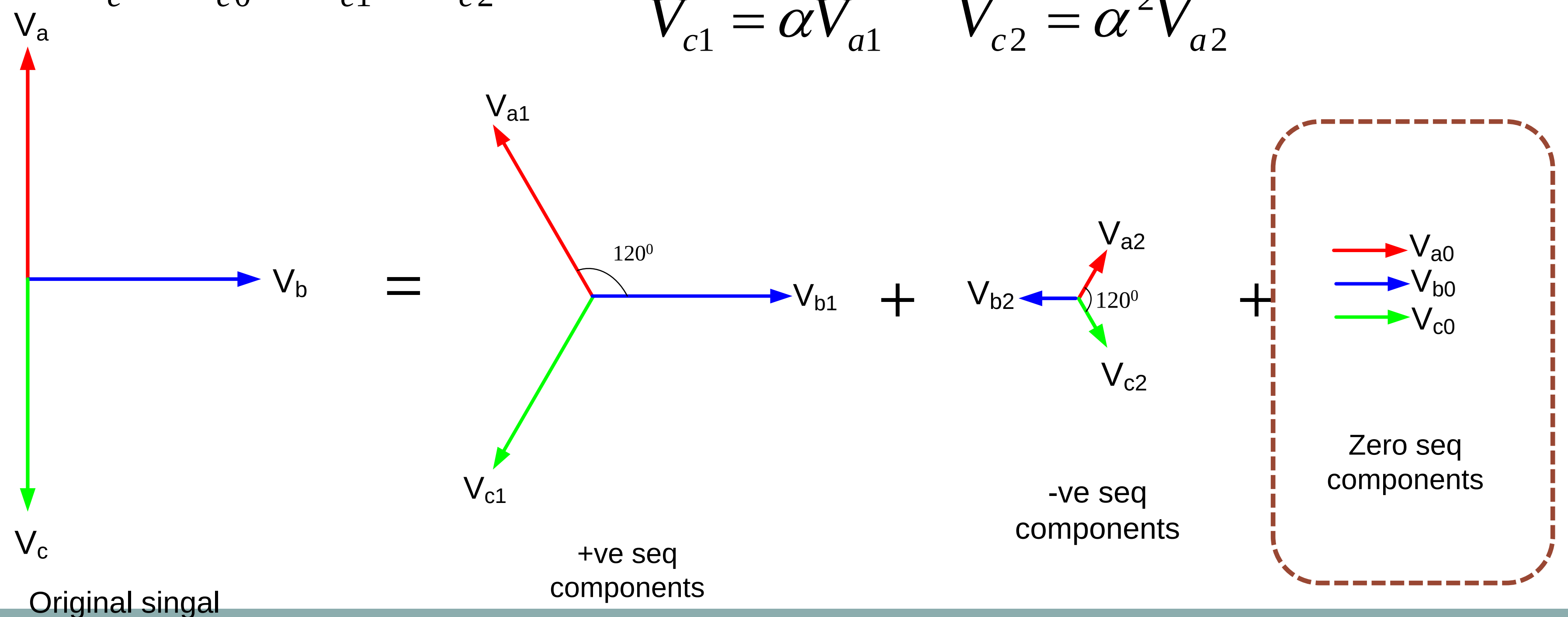
Mathematical relations

$$\overrightarrow{V_a} = \overrightarrow{V_{a0}} + \overrightarrow{V_{a1}} + \overrightarrow{V_{a2}}$$
$$\overrightarrow{V_b} = \overrightarrow{V_{b0}} + \overrightarrow{V_{b1}} + \overrightarrow{V_{b2}}$$
$$\overrightarrow{V_c} = \overrightarrow{V_{c0}} + \overrightarrow{V_{c1}} + \overrightarrow{V_{c2}}$$

- The zero sequence components are interrelated as:

$$V_{b1} = \alpha^2 V_{a1} \qquad V_{b2} = \alpha V_{a2}$$
$$V_{c1} = \alpha V_{a1} \qquad V_{c2} = \alpha^2 V_{a2}$$

$$V_{a0} = V_{b0} = V_{c0}$$



Mathematical relations

$$\vec{V_a} = \vec{V_{a0}} + \vec{V_{a1}} + \vec{V_{a2}}$$
$$\vec{V_b} = \vec{V_{b0}} + \vec{V_{b1}} + \vec{V_{b2}}$$
$$\vec{V_c} = \vec{V_{c0}} + \vec{V_{c1}} + \vec{V_{c2}}$$

$$V_{b1} = \alpha^2 V_{a1} \qquad V_{b2} = \alpha V_{a2} \qquad V_{b0} = V_{c0} = V_{a0}$$
$$V_{c1} = \alpha V_{a1} \qquad V_{c2} = \alpha^2 V_{a2}$$

- The **Synthesis Equations** are hence derived as:

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$$
$$V_c = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}$$

- All symmetrical components expressed w.r.t. "a" only *a0*, *a1*, *a2*

Synthesis Equation

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$$
$$V_c = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}$$

- In matrix form, the **Synthesis Equations** are written as:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$$
$$V_c = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}$$

Unbalanced phasors

short:

$$= [A][V_s]$$

Symmetrical components

Synthesis equation express the original unbalanced phasors in terms of symmetrical components

Synthesis Equation

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$[V_p] = [A][V_s]$$

$$[V_p] \equiv \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$ Array containing original unbalanced phasors

$$[V_s] \equiv \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$ Array containing symmetrical components

Synthesis Equation

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$[V_p] = [A][V_s]$$

$$[A] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

α-operator matrix

Analysis Equation

Analysis equation express symmetrical components in terms of original unbalanced phasors (reverse of synthesis equation)

Synthesis equation $[V_p] = [A][V_s]$ Rearrange it $[V_s] = [A]^{-1}[V_p]$

Where $[A]^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$

1's retain their positions

α And α^2 exchange their positions

Thus, the **analysis equation** in matrix form is written as:

$$\boxed{\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}} = \boxed{\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}}\boxed{\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}}$$

Analysis Equation

Expanded form of the **analysis equations** are thus:

$$V_{a0} = \frac{1}{3}\left(V_a + V_b + V_c\right)$$

$$V_{a1} = \frac{1}{3}\left(V_a + \alpha V_b + \alpha^2 V_c\right)$$

$$V_{a2} = \frac{1}{3}\left(V_a + \alpha^2 V_b + \alpha V_c\right)$$

Thus, the **analysis equation** in matrix form is written as:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$