Network Theorems

- ILOs – Day7
  - State and explain DC network theorems
    - Norton’s theorem
    - Maximum power transfer theorem

Network Theorems

- **Norton's theorem**
- Maximum power transfer theorem

Norton's Theorem

- Used for solution of complicated circuits
- It allows to replace a complicated circuit with a simple equivalent circuit containing only a **single current source in parallel with a single resistor**

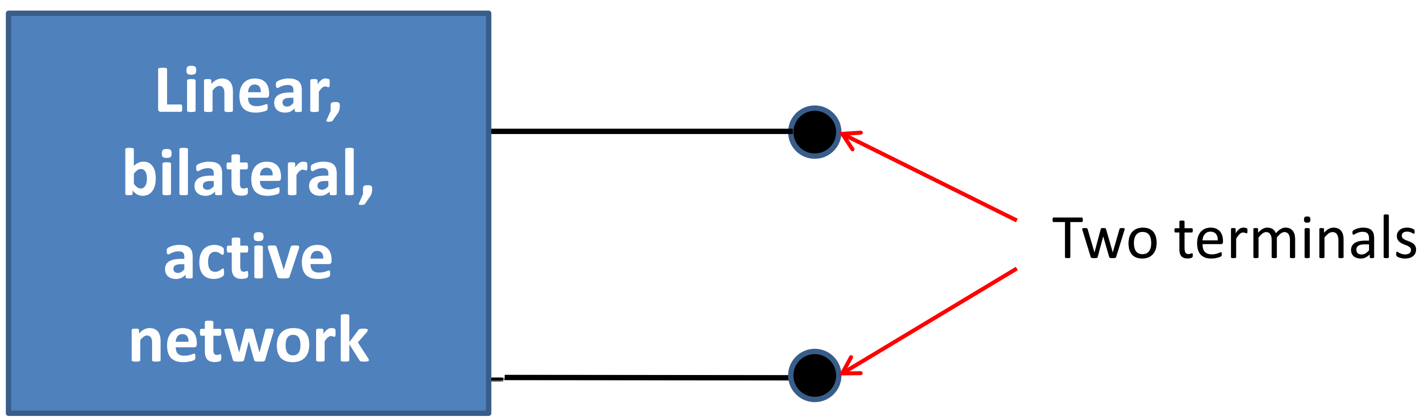
Norton’s Theorem

- *The Theorem states that:*

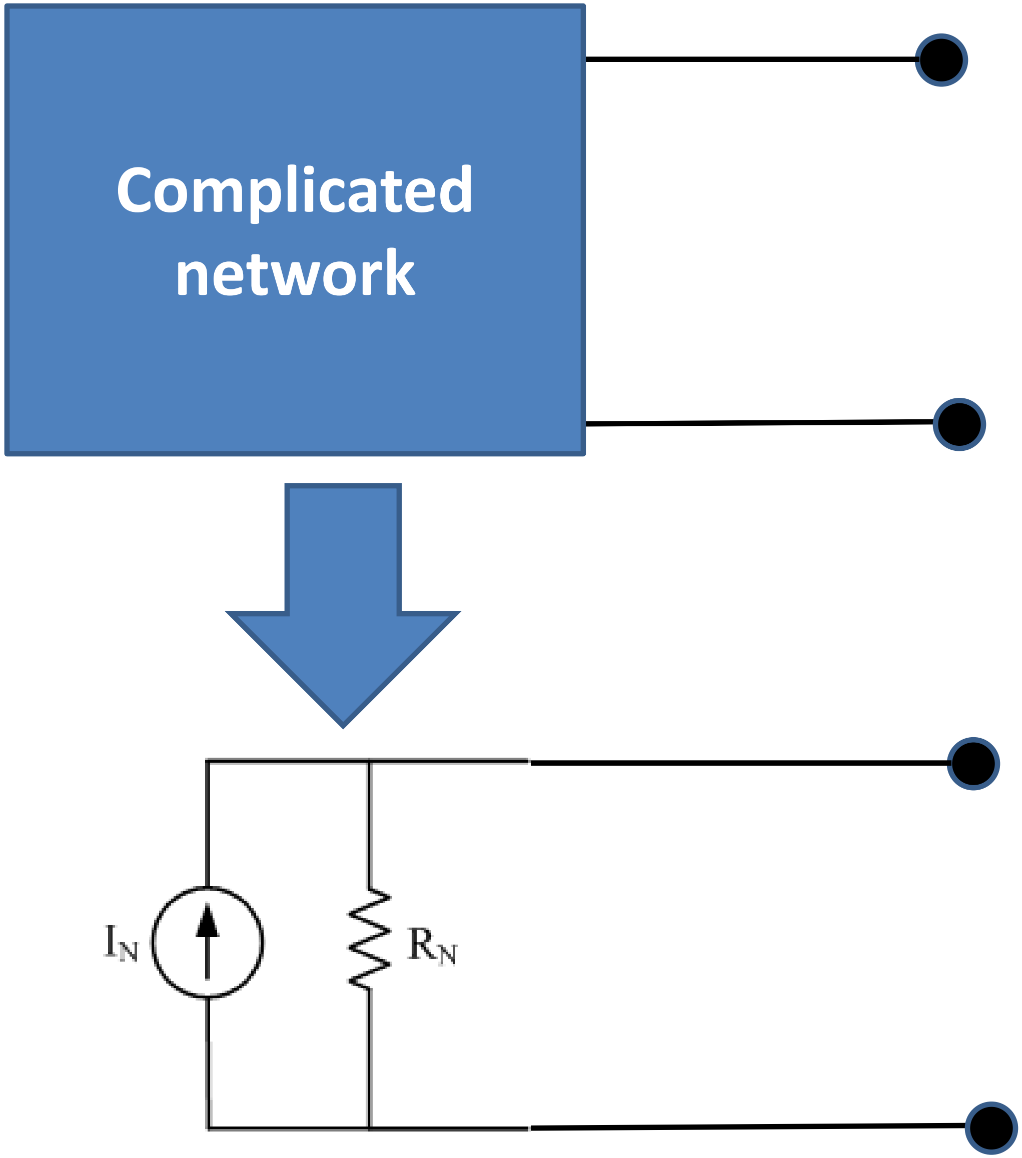
- Any linear, bilateral, active network **between two terminals** can be replaced by an equivalent circuit consisting of a single current source (**called the Norton’s current source, I_N**) in parallel with a single resistance (**called the Norton’s resistance, R_N**).

Norton's Theorem

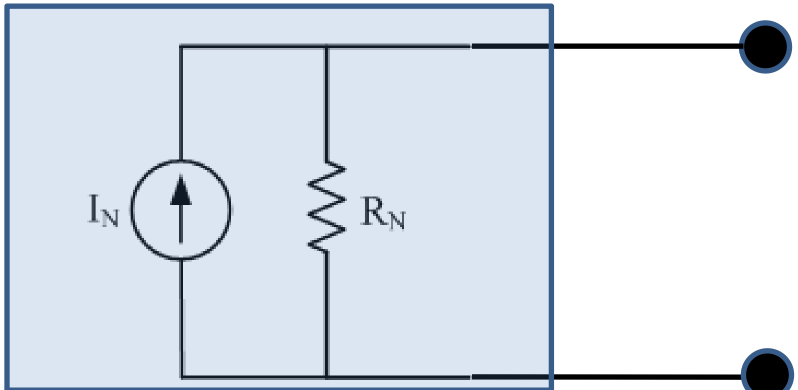
- *The Theorem states that:*
- Any linear, bilateral, active network
- **Between two terminals**
- Can be replaced by an equivalent circuit consisting of
- A single current source (**called the Norton's current source, I_N**)
- In parallel with a single resistance (**called the Norton's resistance, R_N**).



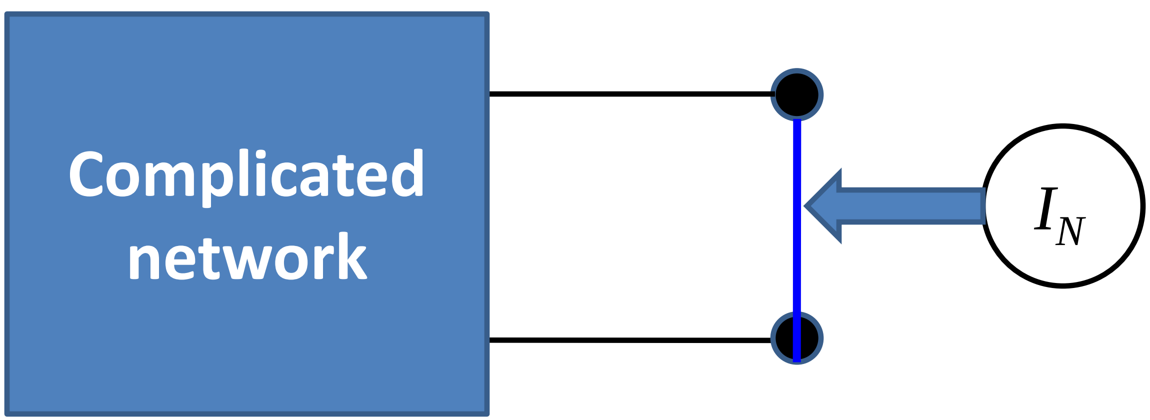
Norton's Theorem



Norton's Theorem- I_N

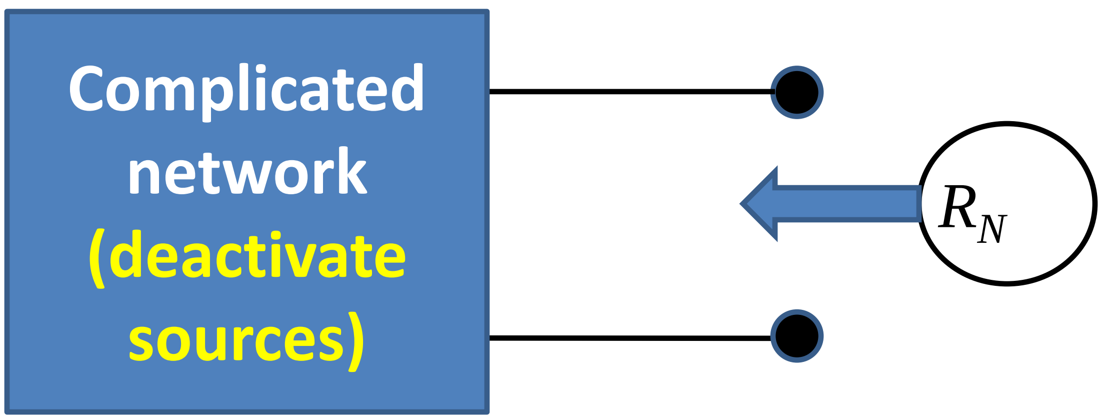


- Value of this current source I_N is obtained as the **short circuit current** between these two terminals.
- Make the two terminals **SHORTED**
- Measure the current through it using an ammeter



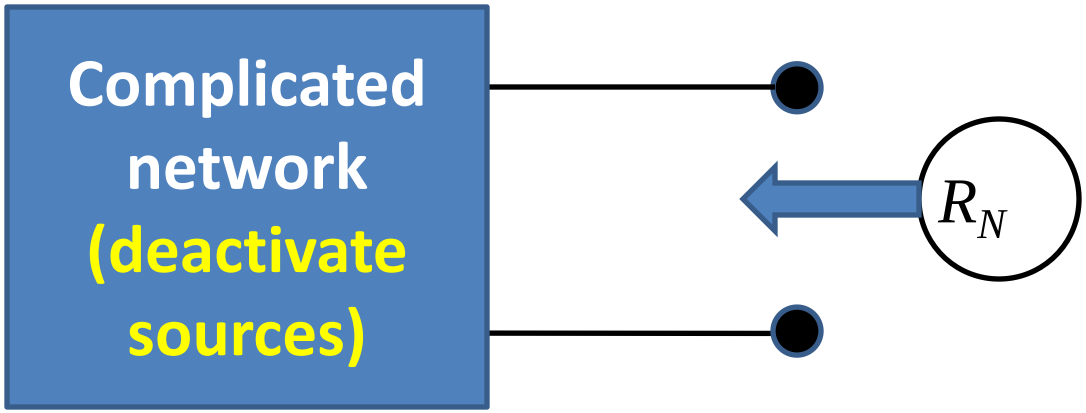
Norton's Theorem- R_N

- The Norton's resistance R_N is calculated as
- The **<u>equivalent resistance of the network measured between the two open circuited terminals</u>**
- By deactivating all sources in the circuit.



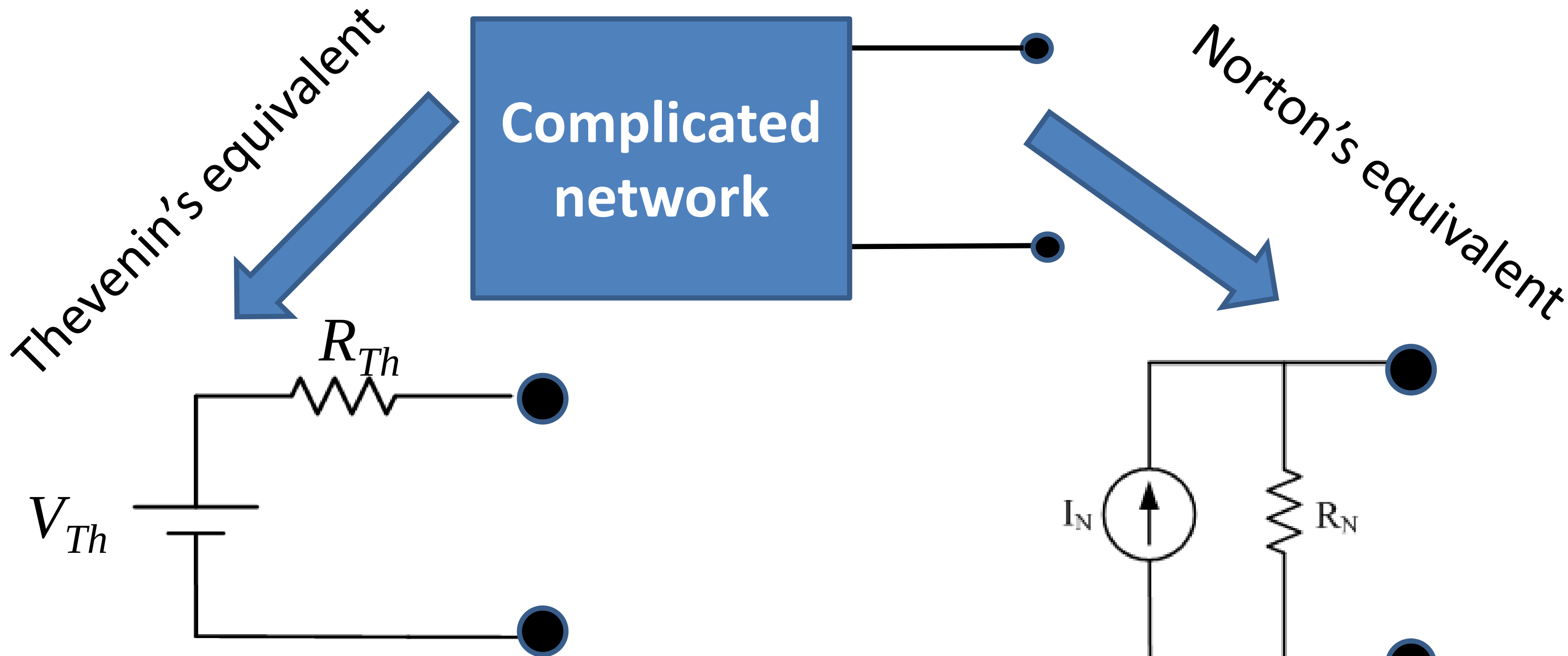
Norton's Theorem- R_N

- To make a voltage source inactive, it must be replaced by its internal resistance (**short circuit for an ideal voltage source**)
- To make a current source inactive, it must be replaced by its internal resistance (**open-circuited for an ideal current source**)

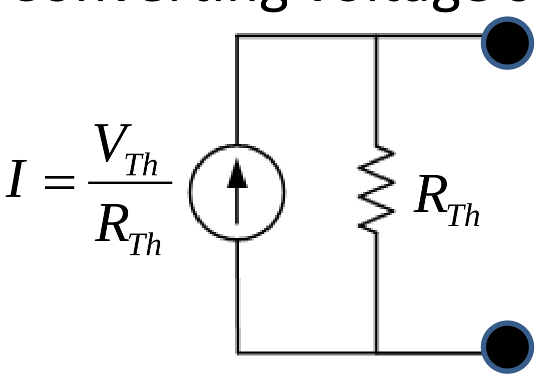


Practically, the definition of R_{Th} and R_N are the same. Thus for a given circuit, the values of R_{Th} and R_N are same.

Relationships between Thevenin’s and Norton’s equivalent parameters



Converting voltage source to current source

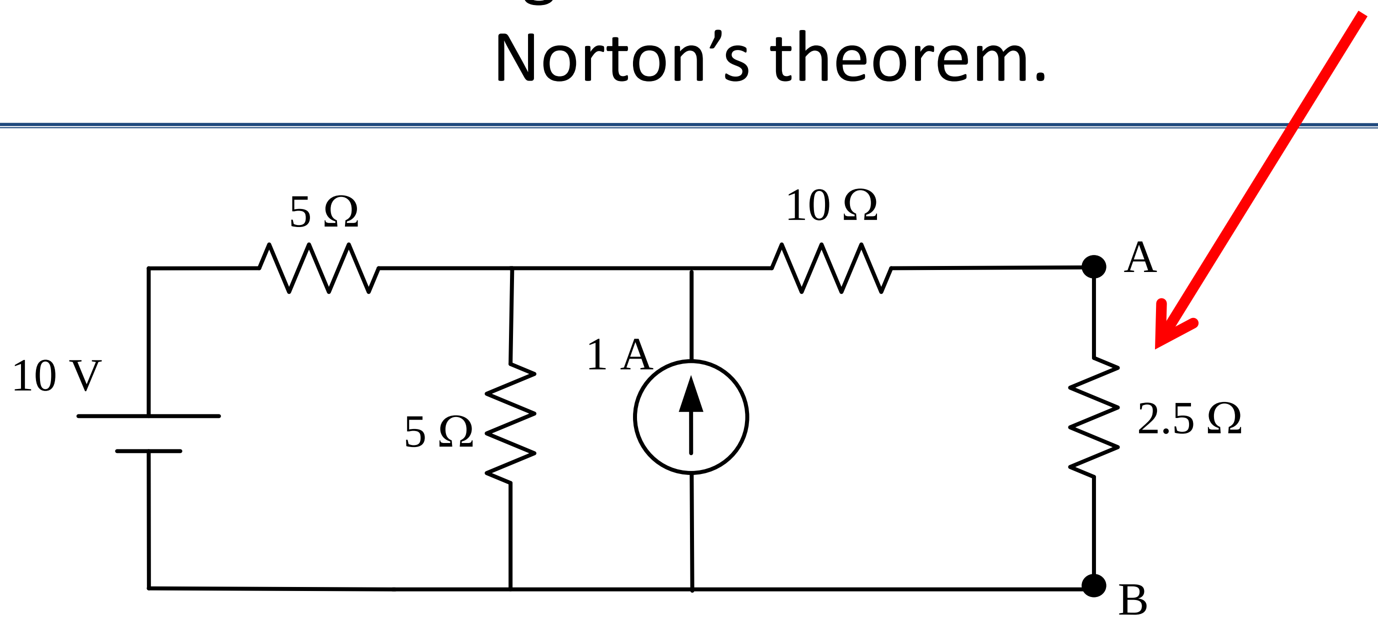


$$\boxed{R_{Th} = R_N}$$

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}} = \frac{V_{Th}}{R_N}}$$

$$\boxed{V_{Th} = I_N \times R_{Th} = I_N \times R_N}$$

Find current through the load resistance of 2.5 Ω using Norton's theorem.



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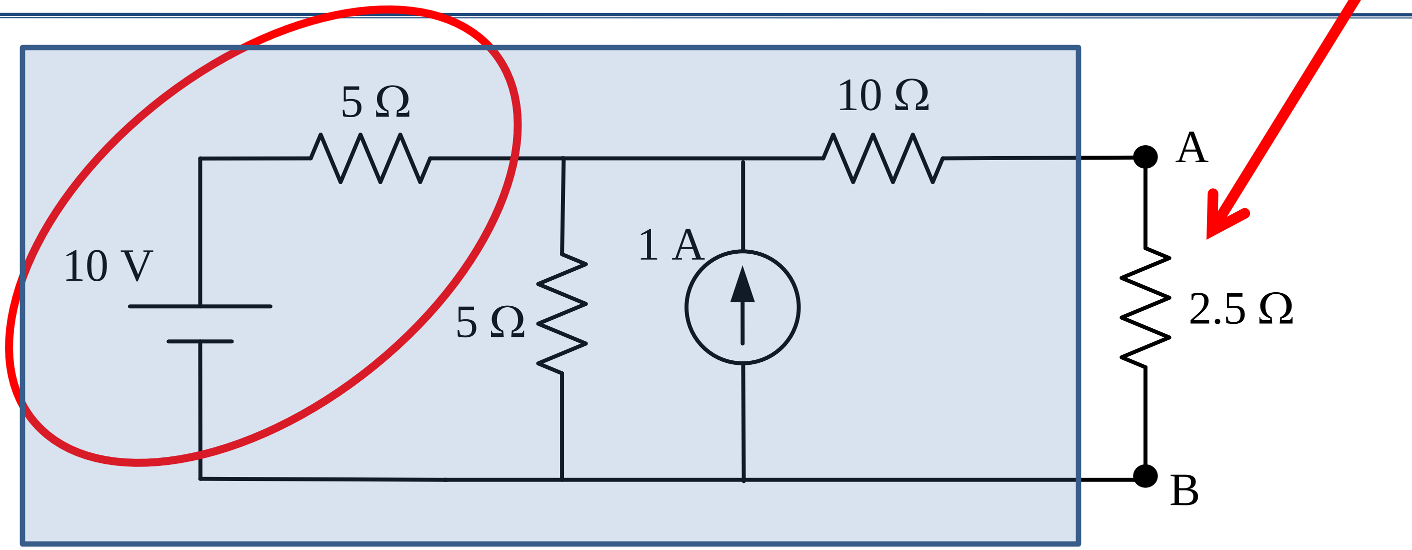
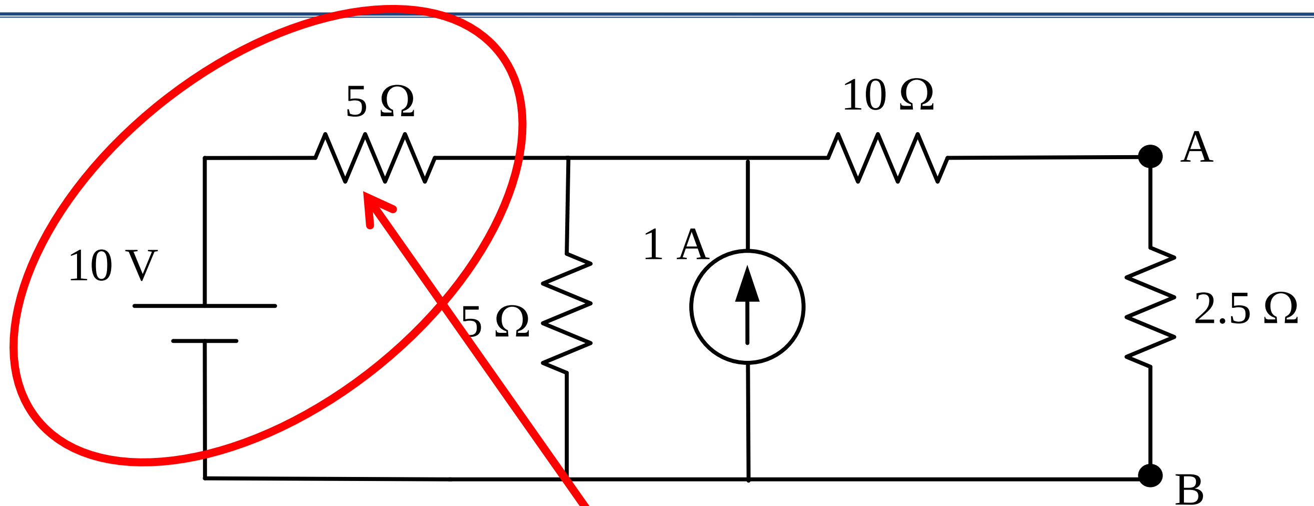


- We are to find out the Norton's equivalent circuit of the network across the 2.5 Ω resistance (i.e. between the terminals A and B).

Step 1:

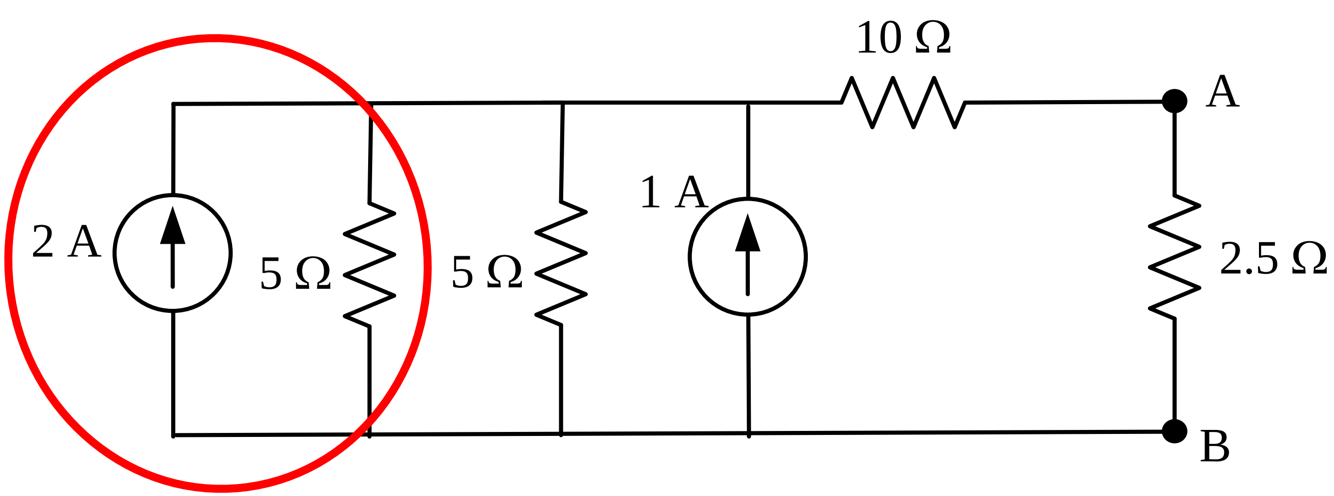
Convert the 10 V source with 5Ω series resistance into an equivalent current source in parallel with the resistance 5 Ω:

Find current through the load resistance of 2.5 Ω using Norton's theorem.

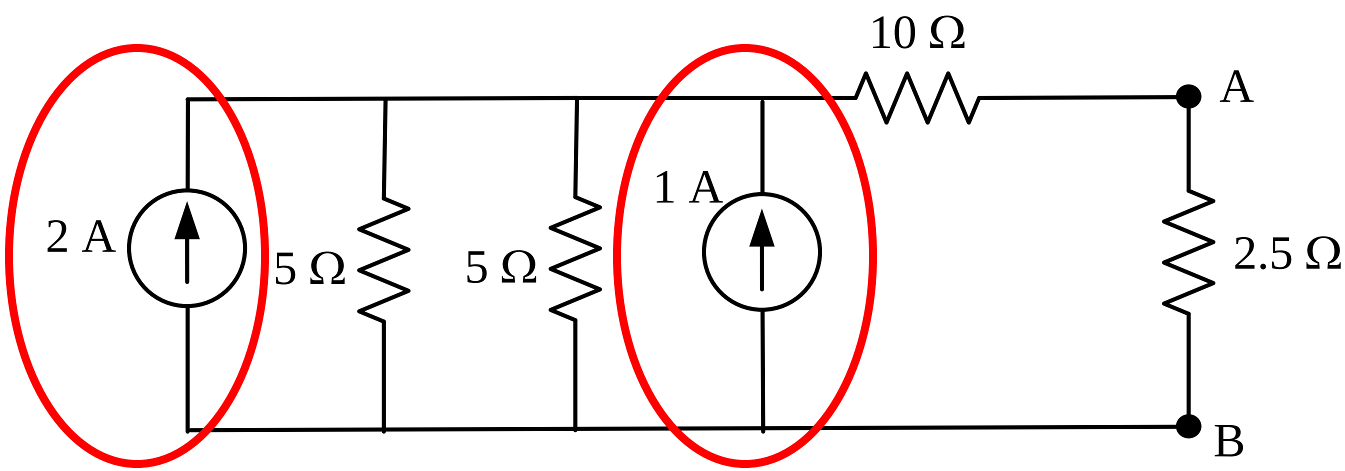


Value of the equivalent current source: $I = \dfrac{V}{R} = \dfrac{10}{5} = 2\ A$

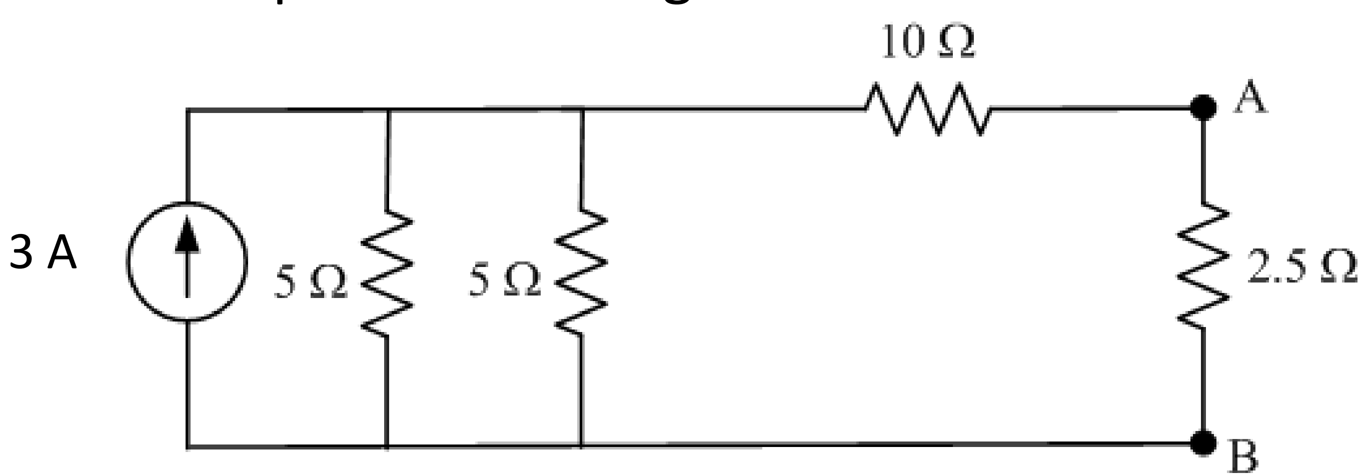
In parallel with the same resistance 5Ω



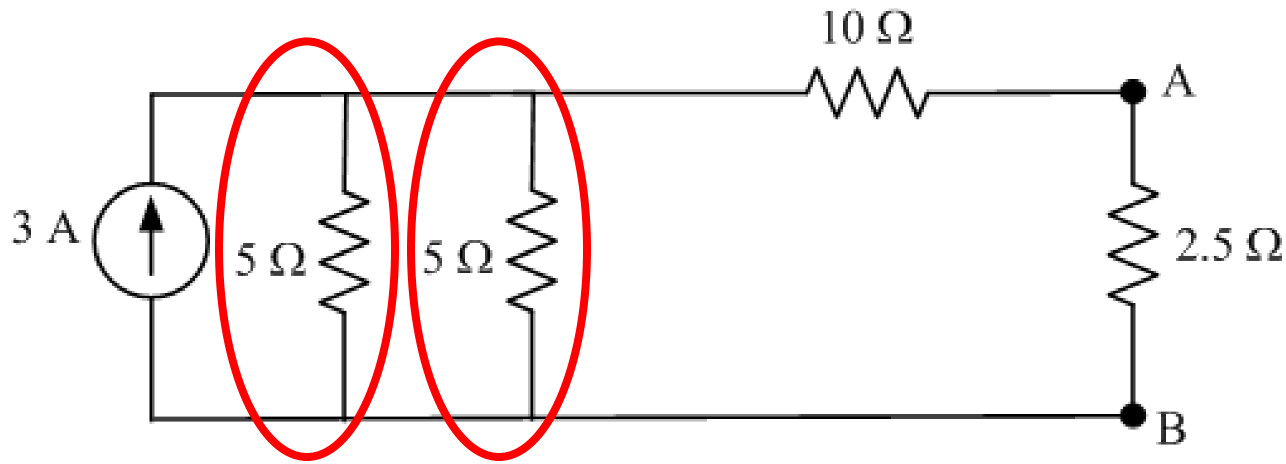
Find current through the load resistance of 2.5 Ω using Norton's theorem.



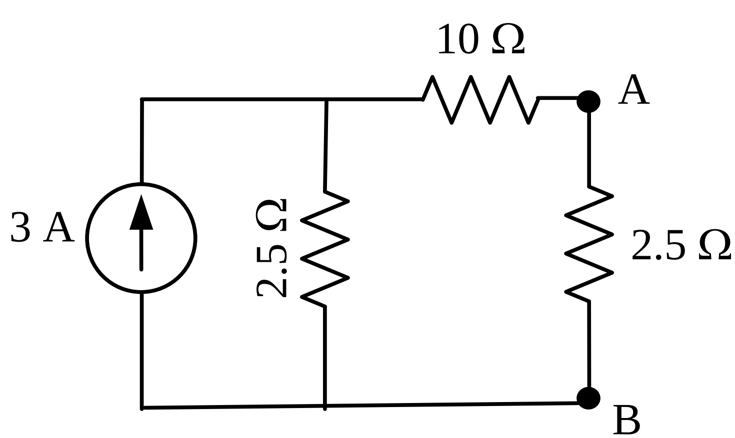
The two current sources, 2 A and 1 A are in parallel, so they can be added up to make a single current source of value 2 + 1 = 3 A.



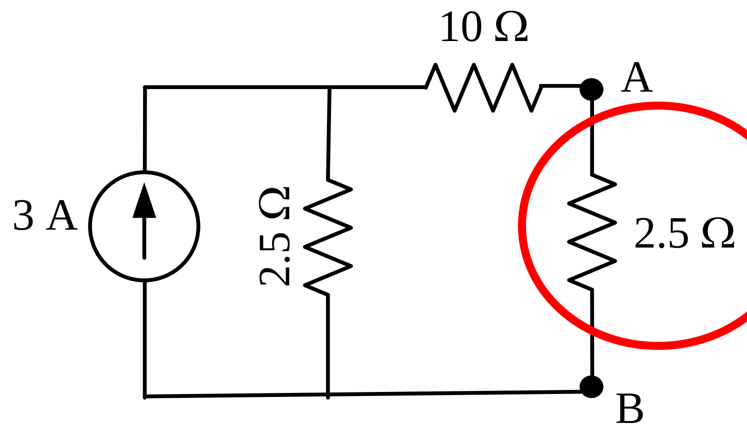
Find current through the load resistance of 2.5 Ω using Norton's theorem.



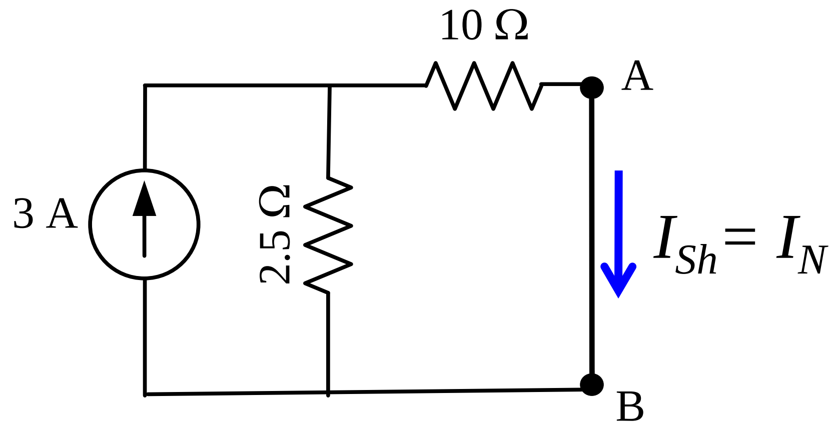
The two 5 Ω resistances in parallel can be combined into a single resistance of value 2.5 Ω:



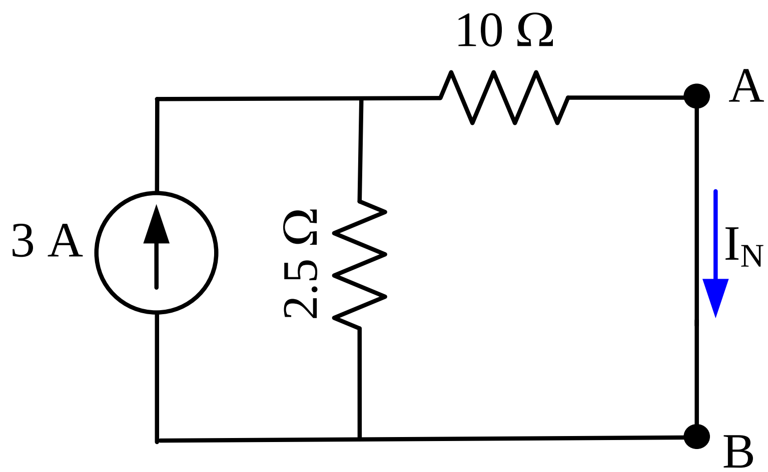
Find current through the load resistance of 2.5 Ω using Norton's theorem.



To find the Norton's current, remove the 2.5 Ω resistance and short circuit the terminals A-B. Then find out current through the shorted path A-B:



Find current through the load resistance of 2.5 Ω using Norton's theorem.



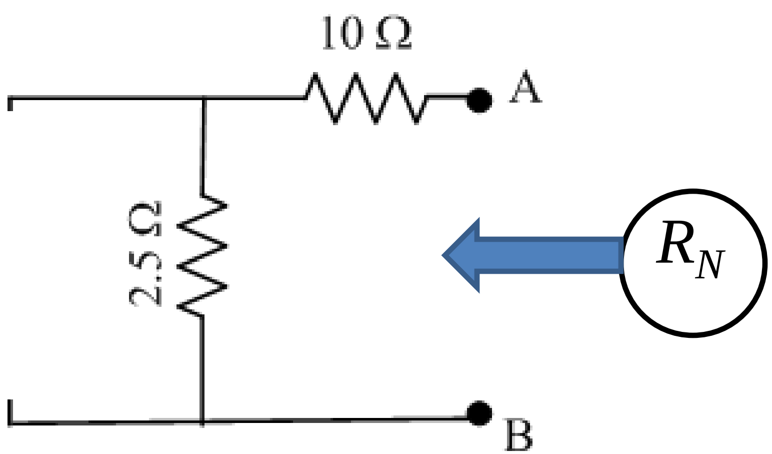
Use current division rule to find out current through the shorted path A-B:

$$I_N = I_{Sh} = 3 \times \frac{2.5}{2.5+10} = 3 \times \frac{2.5}{12.5} = 0.6\ A$$

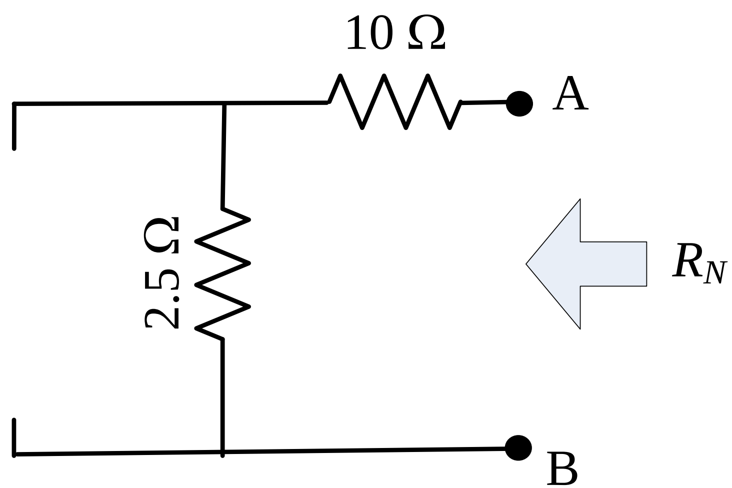
Find current through the load resistance of 2.5 Ω using Norton's theorem.

<u>Step 2</u>: Calculate R_N

- To calculate R_N we keep the terminals A and B open circuited
- Replace the 3 A current source by open circuit and
- Calculate the equivalent resistance of the circuit between the two points A and B:



Find current through the load resistance of 2.5 Ω using Norton's theorem.



Between A-B, the two resistances 10 and 2.5 are in series:

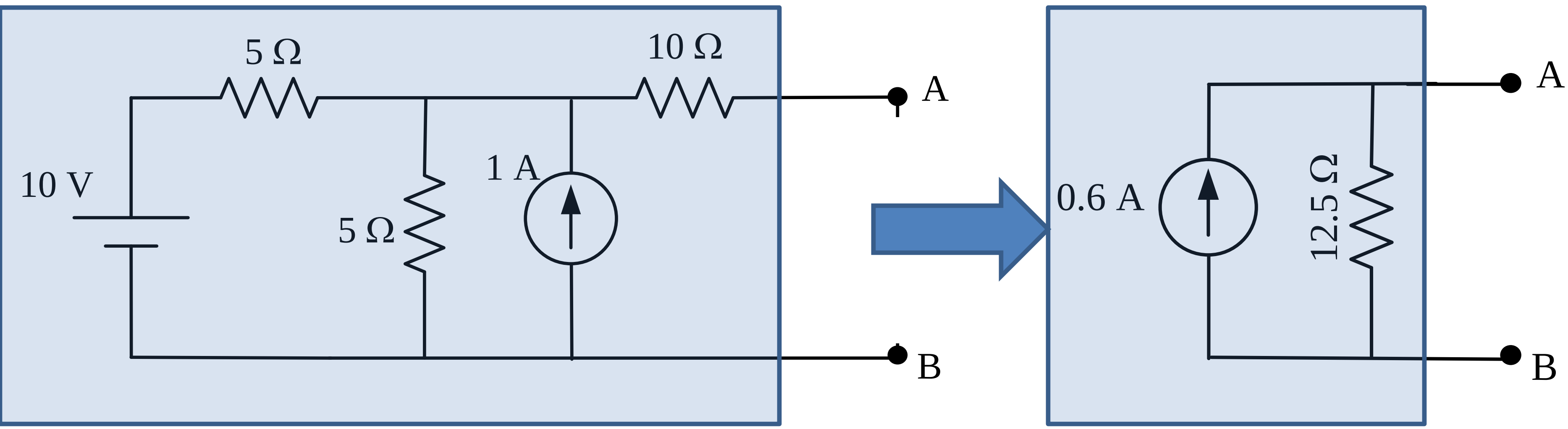
$$\therefore R_N = R_{AB} = 2.5 + 10 = 12.5\,\Omega$$

Find current through the load resistance of 2.5 Ω using Norton's theorem

$$\boxed{I_N = 0.6\ A}$$

$$\boxed{R_N = 12.5\,\Omega}$$

The Norton's equivalent circuit that can be drawn between terminals A and B is thus:

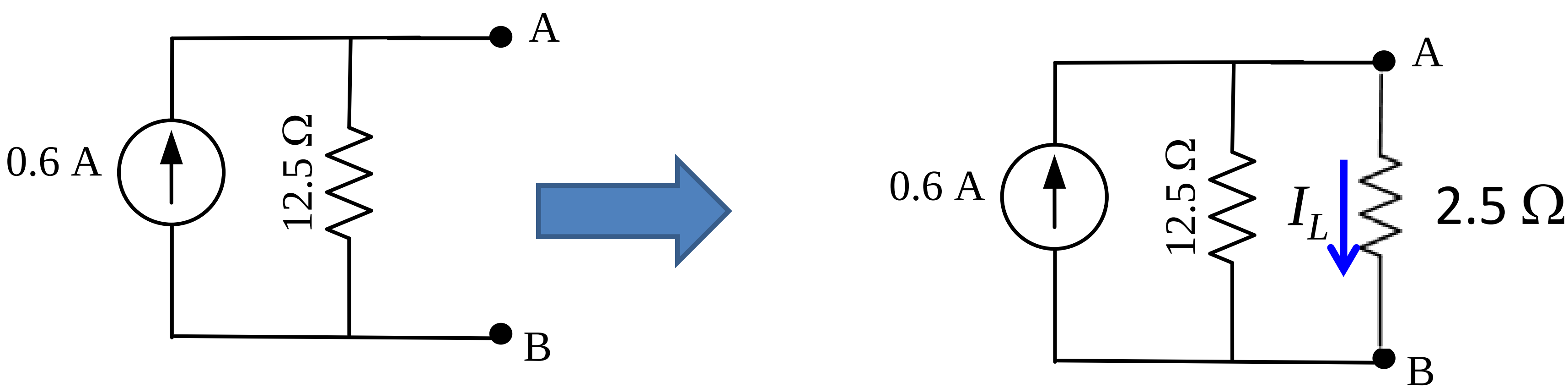


Original circuit

Norton's equivalent circuit

Find current through the load resistance of 2.5 Ω using Norton's theorem

To calculate the load current through the 2.5Ω resistance, connect it back between the output terminals A-B so that the total circuit becomes:



∴ The current (I_L) through the load resistance 2.5 Ω is:

$$\boxed{I_L = I_{AB} = 0.6 \times \frac{12.5}{12.5 + 2.5} = 0.6 \times \frac{12.5}{15} = 0.5\ A}$$

Network Theorems

- Norton's theorem
- **Maximum power transfer theorem**

Maximum Power Transfer Theorem

This theorem is used to find the value of load resistance in a circuit for which there would be maximum amount of power transfer from source to the load.

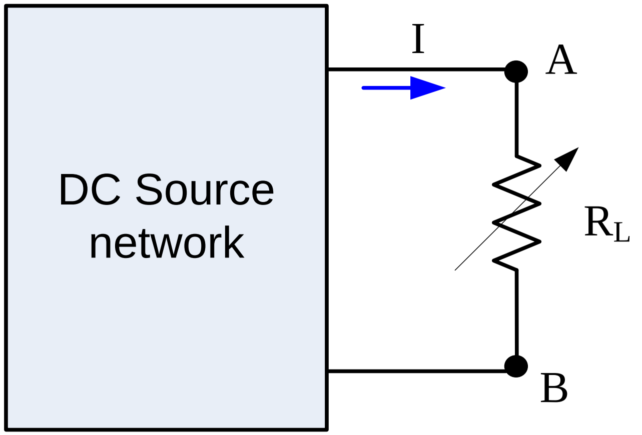
Maximum Power Transfer Theorem

- *The Theorem states that:*

- A resistive load, being connected to a DC network, receives maximum power when the load resistance is equal to the internal resistance of the source network as seen from the load terminals.

Maximum Power Transfer Theorem

- *The Theorem states that:*
- A resistive load (R_L)
- Connected to a DC network (source network)
- Will receive maximum power from the source network
- When the load resistance is equal to
- The internal resistance of the source network



Maximum Power Transfer Theorem

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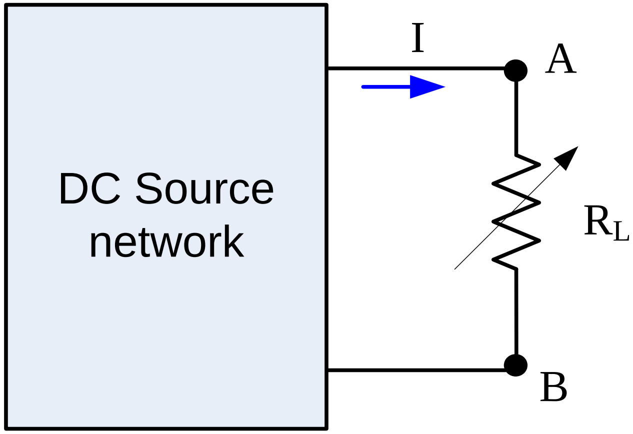
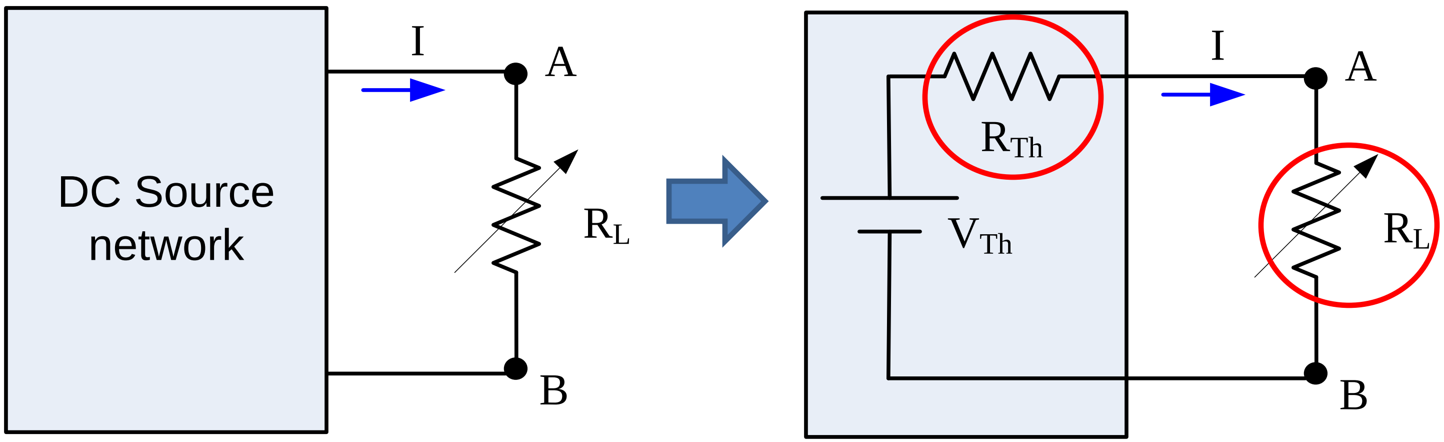


Maximum Power Transfer Theorem

- When the load resistance is equal to
- The internal resistance of the source network

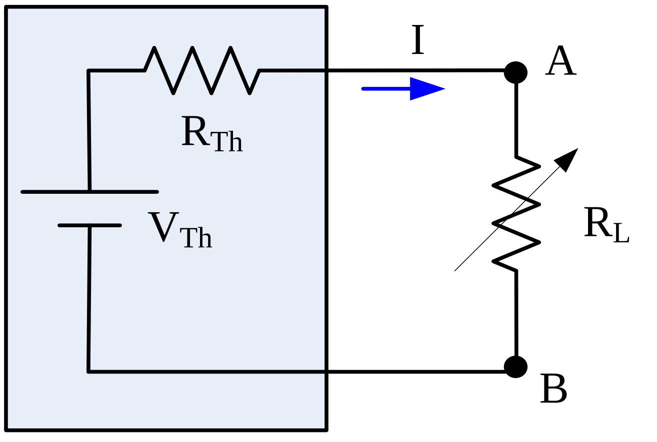
Internal resistance of the source network is nothing but its Thevenin's equivalent resistance between the terminals A- B



For maximum power transfer, $R_L = R_{Th}$

Maximum Power Transfer Theorem

Proof:



In the circuit:

$$I = \frac{V_{Th}}{R_{Th} + R_L}$$

Power received by the load resistance is:

$$P_L = I^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

For maximum power transfer to load, we set:

$$\frac{dP_L}{dR_L} = 0$$

Maximum Power Transfer Theorem

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L\right] = 0$$

$$V_{Th}{}^2 \frac{\left[(R_{Th} + R_L)^2 \times 1 - 2(R_{Th} + R_L)R_L\right]}{(R_{Th} + R_L)^4} = 0$$

$$R_{Th}{}^2 + R_L{}^2 + 2R_{Th}R_L - 2R_{Th}R_L - 2R_L{}^2 = 0$$

$$R_L{}^2 = R_{Th}{}^2$$

$$\boxed{R_L = R_{Th}}$$

Thus, it can be proved that power transfer from a DC source network to a resistive load is maximum when the load resistance is equal to the internal resistance of the DC source network.

Maximum Power Transfer Theorem

Value of this maximum power can be calculated (with $R_L = R_{Th}$) as:

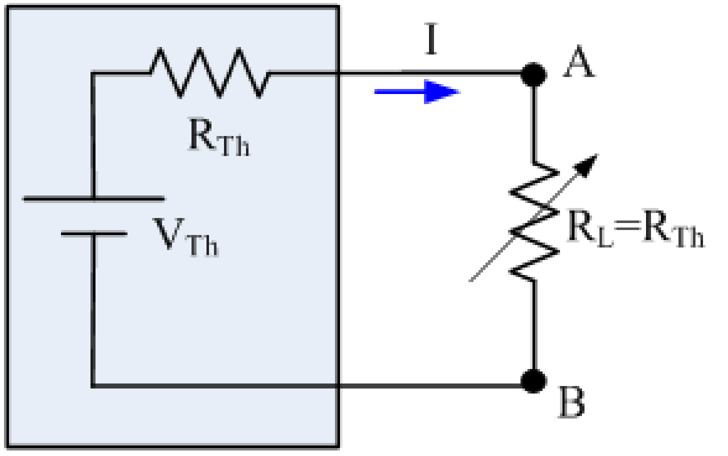
$$P_{L\max} = P_L\Big|_{R_L = R_{Th}} = I^2 R_L$$

$$P_{L\max} = I^2 R_{Th}$$

$$P_{L\max} = \left(\frac{V_{Th}}{R_{Th} + R_{Th}}\right)^2 R_{Th}$$

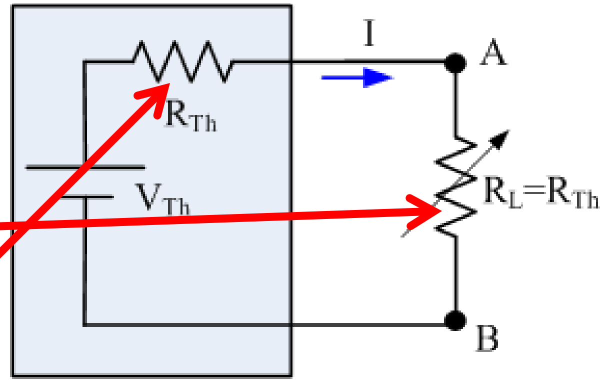
$$P_{L\max} = \left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th}$$

$$\boxed{P_{L\max} = \frac{V_{Th}^{\ 2}}{4R_{Th}} = \frac{V_{Th}^{\ 2}}{4R_L}}$$



Maximum Power Transfer Theorem

$$\boxed{P_{L\max} = \frac{V_{Th}^{2}}{4R_{Th}} = \frac{V_{Th}^{2}}{4R_L}}$$



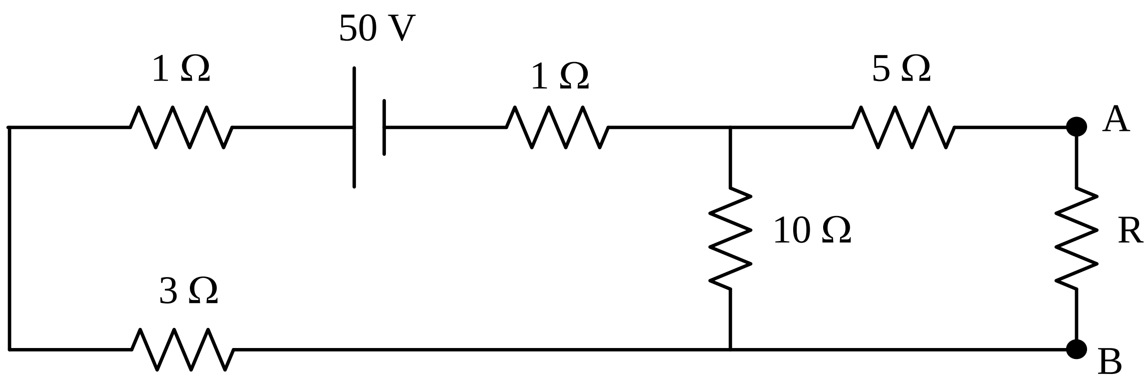
- Note that this power is consumed by the load.
- In fact this same power is consumed by the internal resistance (R_{Th}) of the source also.
- Thus, total power delivered by the source is:

$$P_T = \frac{V_{Th}^{2}}{4R_{Th}} + \frac{V_{Th}^{2}}{4R_{Th}} = \frac{V_{Th}^{2}}{2R_{Th}}$$

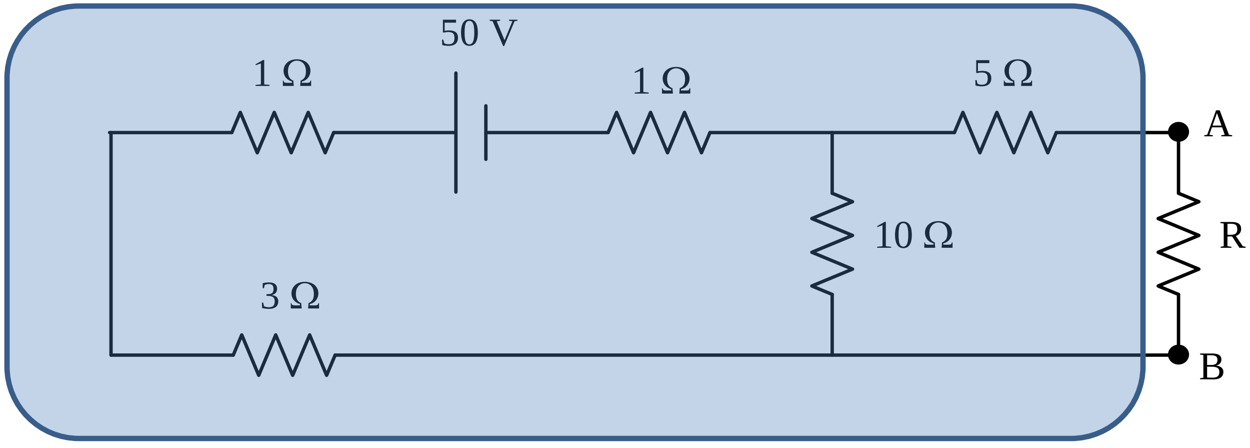
Thus, during maximum power transfer condition, the efficiency of power transfer becomes:

$$\eta = \frac{P_{L\max}}{P_T} \times 100\% = \frac{\dfrac{V_{Th}^{2}}{4R_L}}{\dfrac{V_{Th}^{2}}{2R_L}} \times 100\% \boxed{= 50\%}$$

In the following circuit, find the value of R to obtain maximum power in it. Also calculate value of the maximum power.

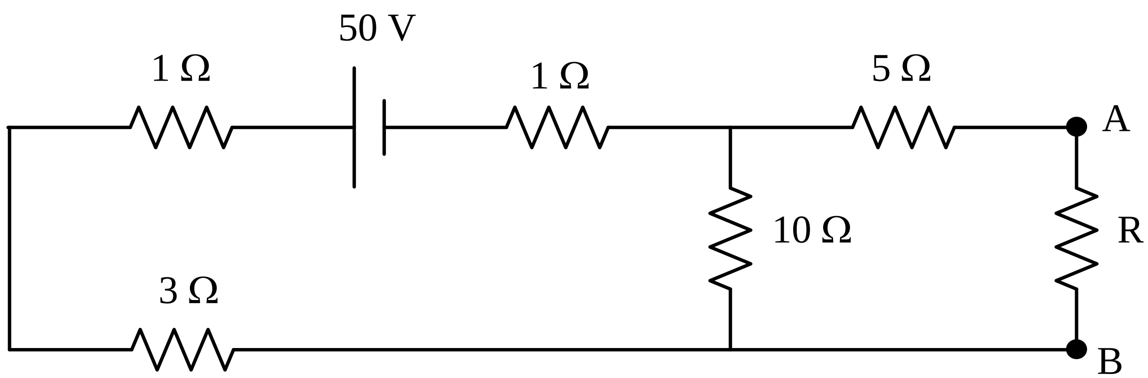


In the following circuit, find the value of R to obtain maximum power in it. Also calculate value of the maximum power.

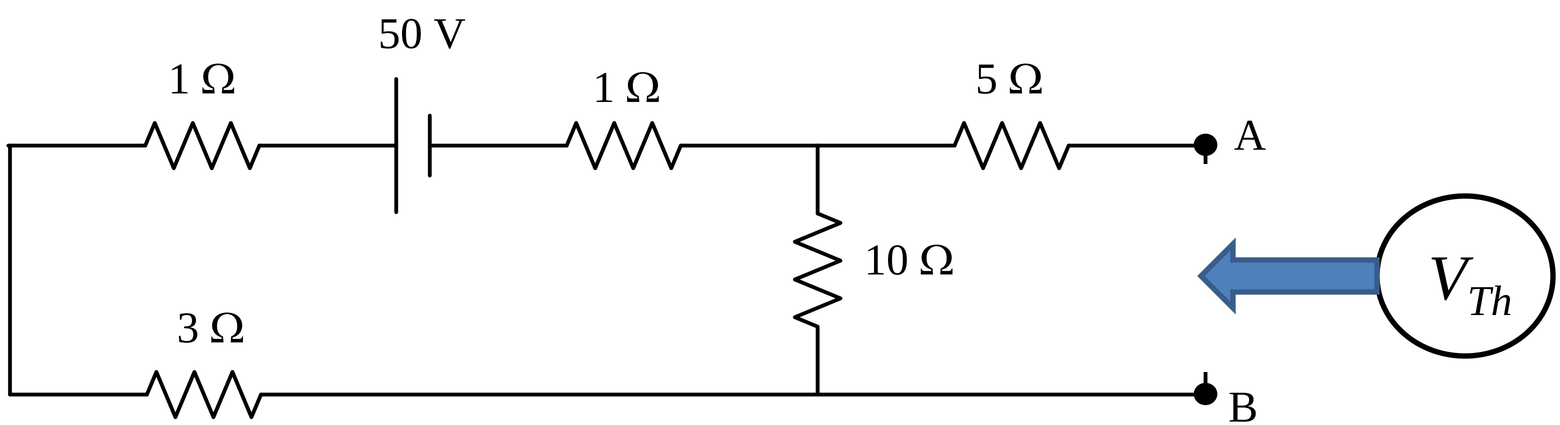


To calculate the value of load resistance R for obtaining maximum power in it, we need to convert the remaining part of the circuit, i.e. the circuit at the left hand side of the terminals A-B into its equivalent Thevenin's network.

In the following circuit, find the value of R to obtain maximum power in it. Also calculate value of the maximum power.

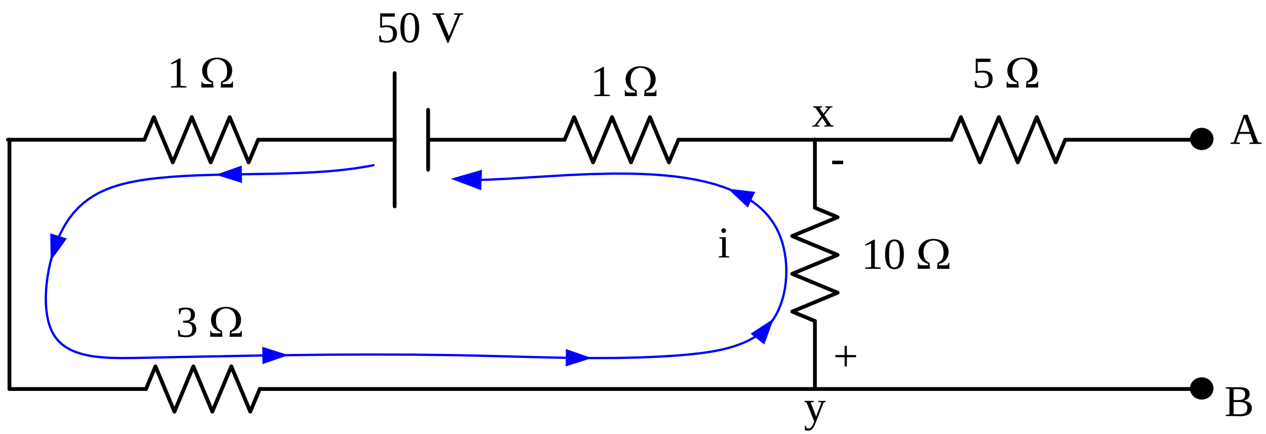


- <u>Step 1</u>: Calculate V_{Th}
- Open circuit the terminals A-B and calculate the voltage across it:



In the following circuit, find the value of R to obtain maximum power in it. Also calculate value of the maximum power.

- Take the current *i* in the loop as shown:

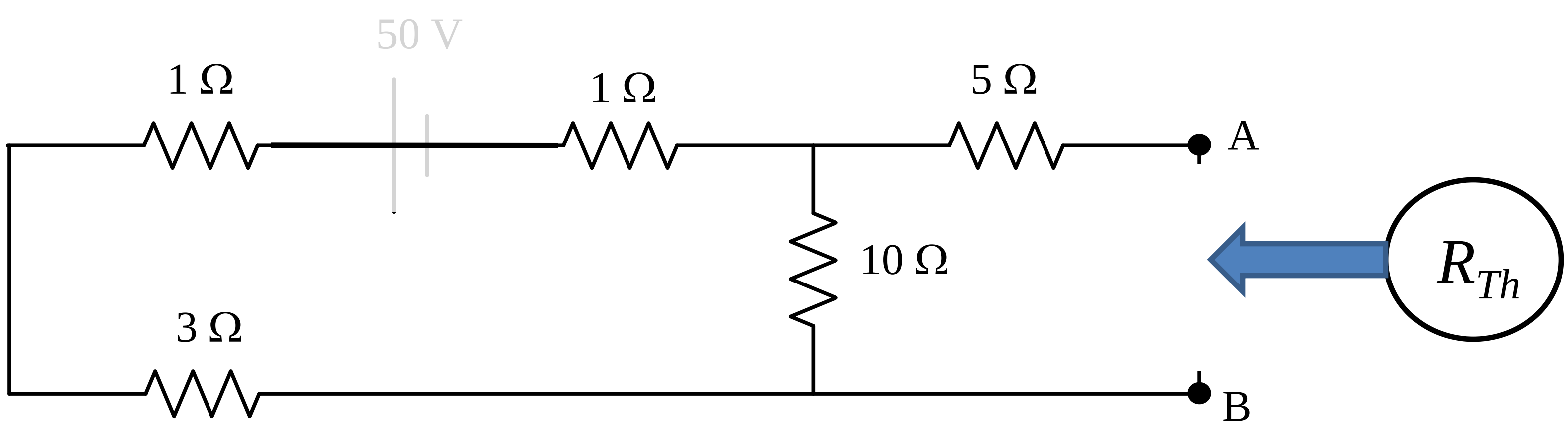


- Since A-B is open, thus no current will flow to the output
- Thus, no voltage will be dropped in the 5 Ω resistance
- Hence $V_{xy} = V_{AB}$

$$V_{Th} = V_{AB} = V_{xy} = 10 \times (-i) = -10 \times \frac{50}{1+3+10+1} = -10 \times \frac{50}{15} \boxed{= -\frac{100}{3} V}$$

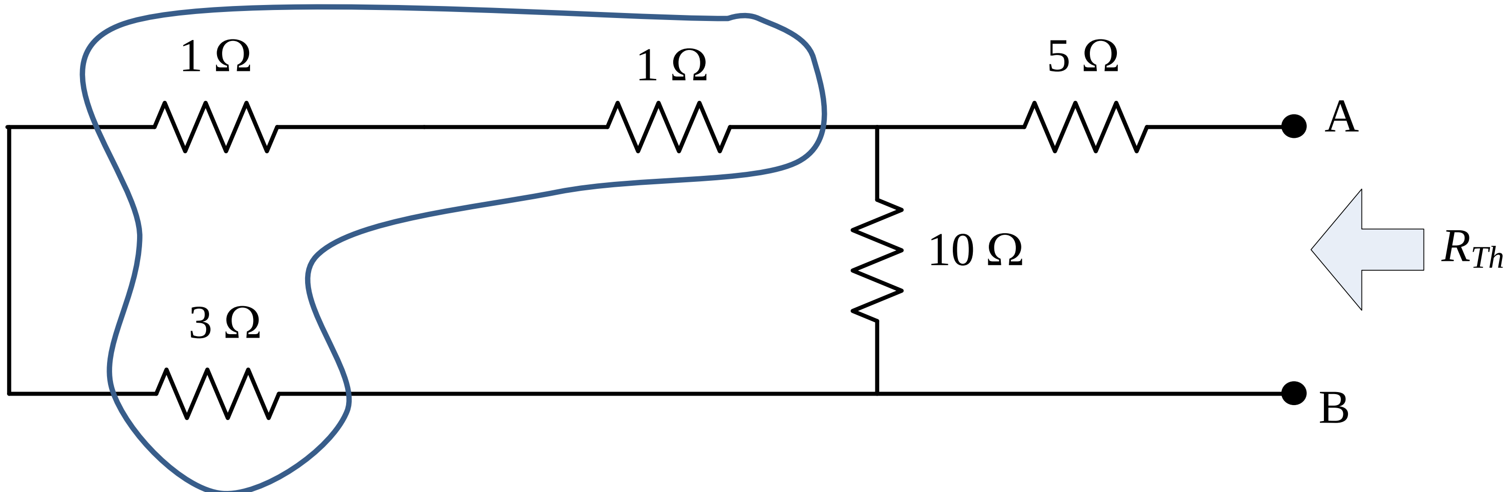
In the following circuit, find the value of R to obtain maximum power in it. Also calculate value of the maximum power.

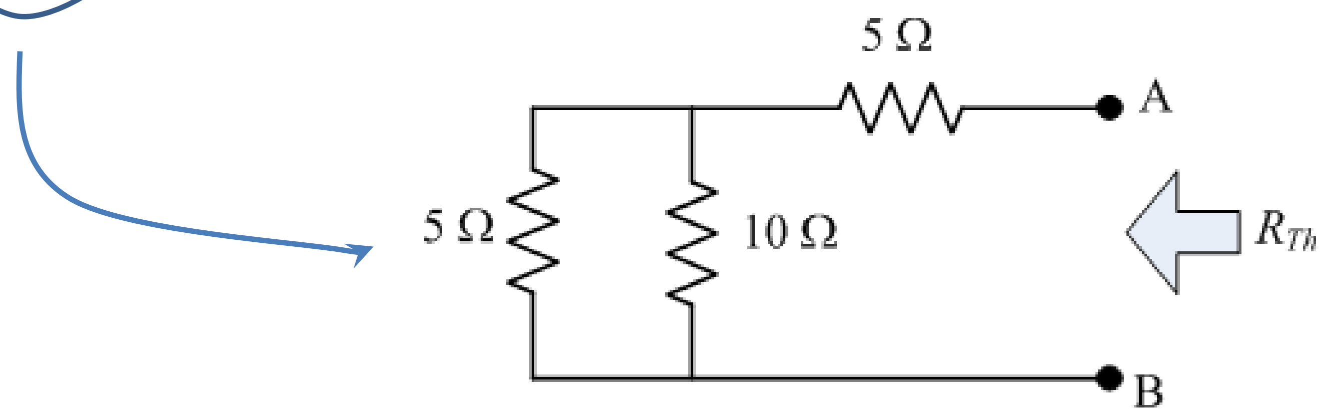
- <u>Step 2</u>: Calculate R_{Th}
- Open circuit the terminals A-B
- Short the 50 V source and
- Calculate the equivalent resistance between terminals A-B:



In the following circuit, find the value of R to obtain maximum power in it. Also calculate value of the maximum power.

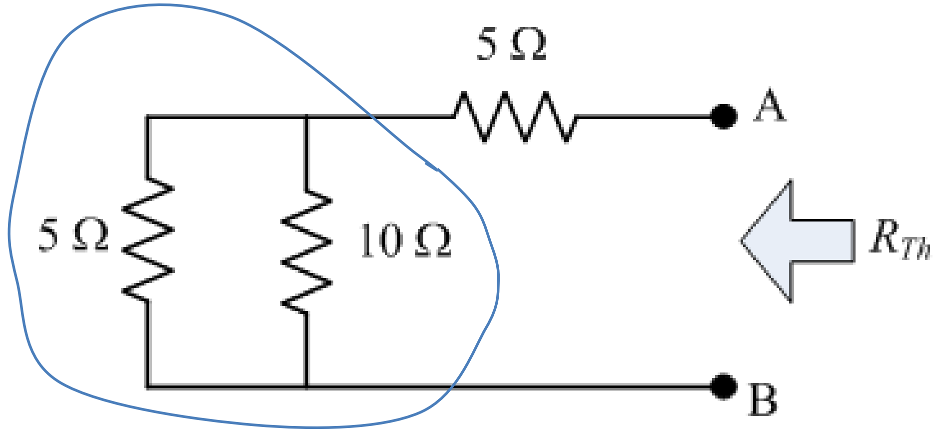
- Calculate R_{Th}
- 1 Ω, 1 Ω, and 3 Ω are in series (3 +1 + 1 = 5)



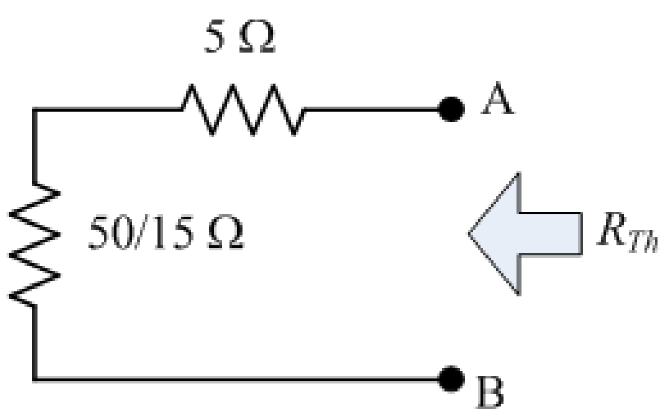


In the following circuit, find the value of R to obtain maximum power in it. Also calculate value of the maximum power.

- Calculate R_{Th}

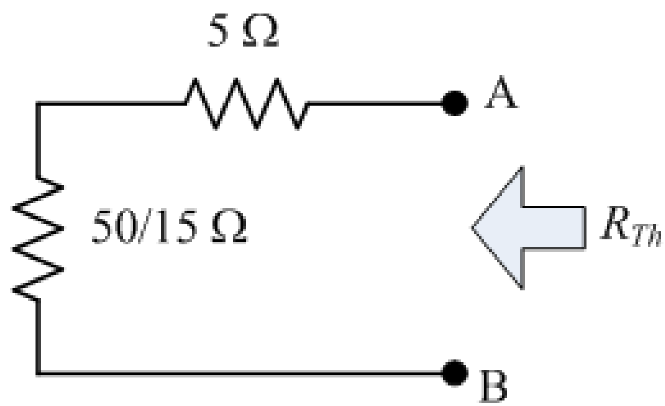


- This 5 Ω and 10 Ω are in parallel (equivalent = 50/15)

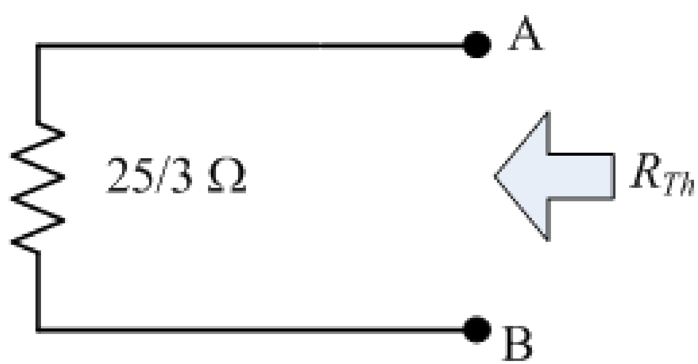


In the following circuit, find the value of R to obtain maximum power in it. Also calculate value of the maximum power.

- Calculate R_{Th}



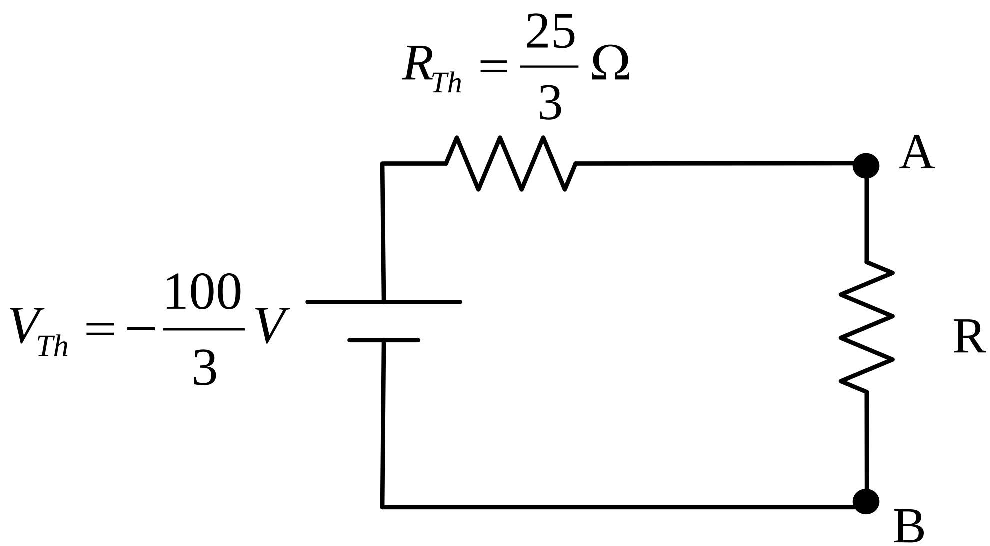
- Finally, 50/15 and 5 are in series between A-B (50/15 + 5 = 25/3 Ω)



$$\boxed{R_{Th} = \frac{25}{3}\,\Omega}$$

In the following circuit, find the value of R to obtain maximum power in it. Also calculate value of the maximum power.

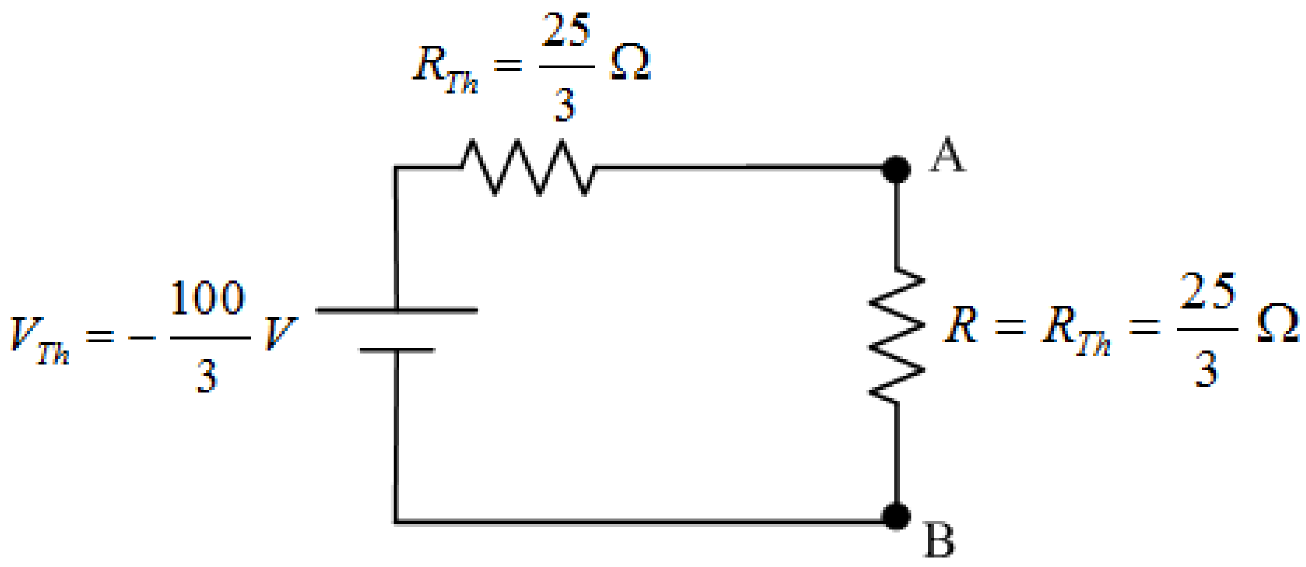
$\therefore$ The Thevenin's equivalent circuit is:



$\therefore$ According to maximum power transfer theorem, maximum power will be obtained by the resistance R when its value is equal to the Thevenin's equivalent resistance of the remaining part of the circuit, i.e. for

$$R = R_{Th} = \frac{25}{3}\,\Omega$$

In the following circuit, find the value of R to obtain maximum power in it. **Also calculate value of the maximum power.**



& Value of maximum power

$$P_{\max} = \frac{V_{Th}^{\;2}}{4R_{Th}} = \frac{\left(\frac{100}{3}\right)^2}{4\times\frac{25}{3}} = 33.33\,W$$