# AC Fundamentals

Day 15

## Parallel Resonance

# ILOs – Day 15

- Investigate resonance condition in parallel RLC circuit
  - Determine the condition for parallel resonance
  - Identify the circuit conditions under parallel resonance
  - Plot the impedance and current variation profile under parallel resonance
  - Obtain expression for Quality factor of a parallel resonating circuit

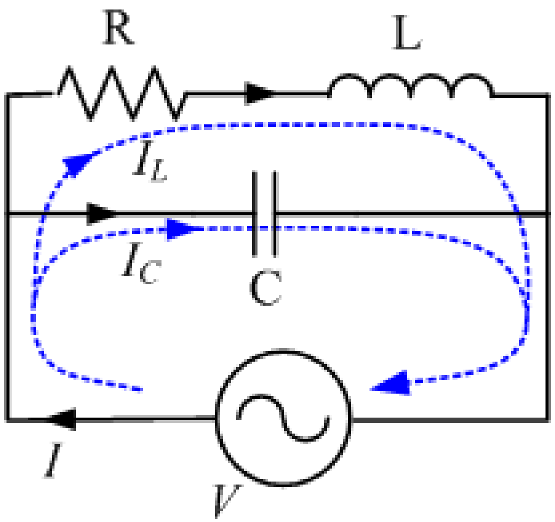
# Resonance

- Resonance in electrical circuits is a particular condition of the circuit when
  - **The circuit impedance become maximum or minimum**
  - **The current in the circuit is minimum or maximum**
  - **The effective power factor of the circuit becomes unity**

- The phenomenon of resonance is observed in both series and parallel AC circuits comprising of R, L, and C

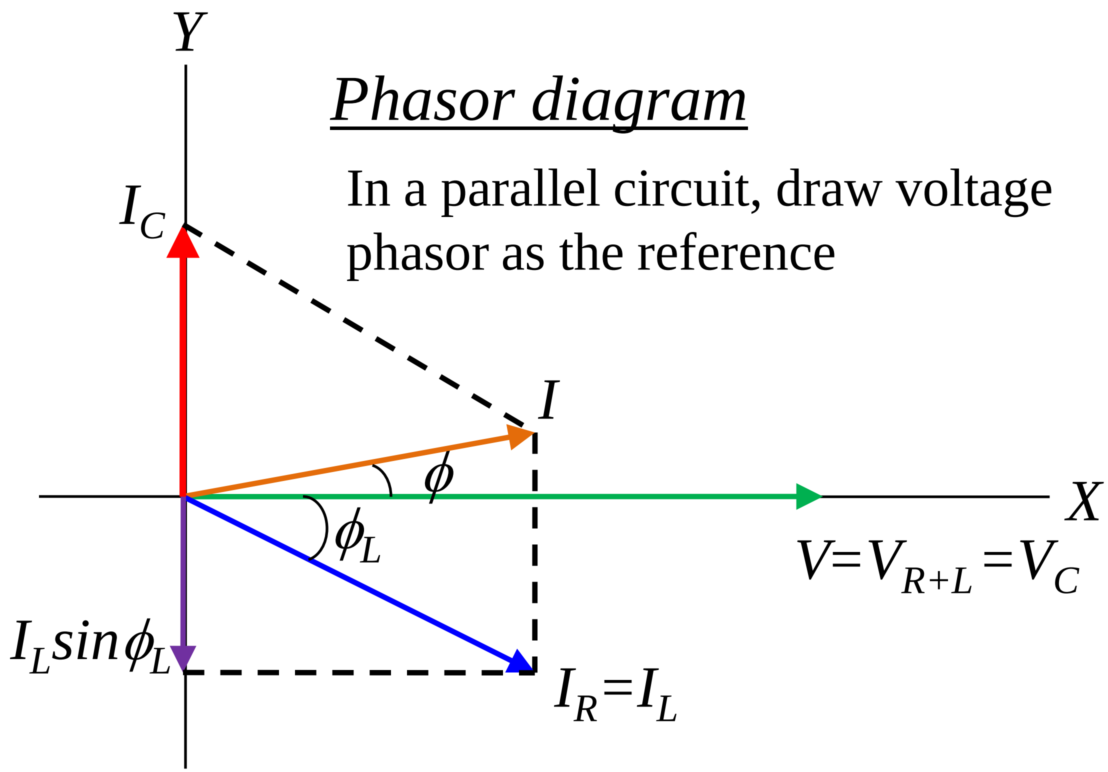
# Parallel Resonance

- Parallel resonance condition can occur in an AC circuit containing a practical coil (*L* in series with its inherent *R*), in parallel with a *C* and connected across an AC source




<u>According to KCL</u>

$$\overline{I} = \overline{I_L} + \overline{I_C}$$



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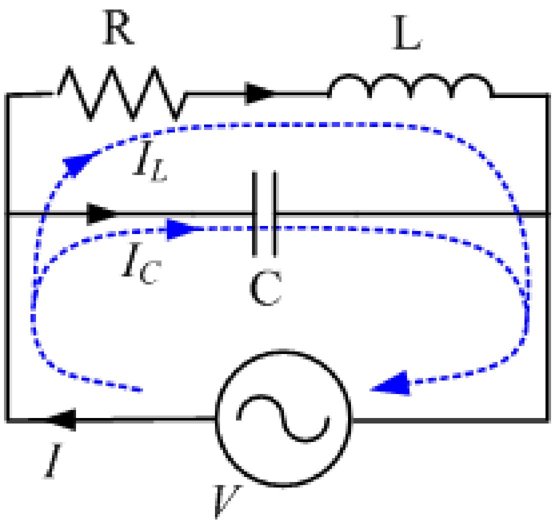
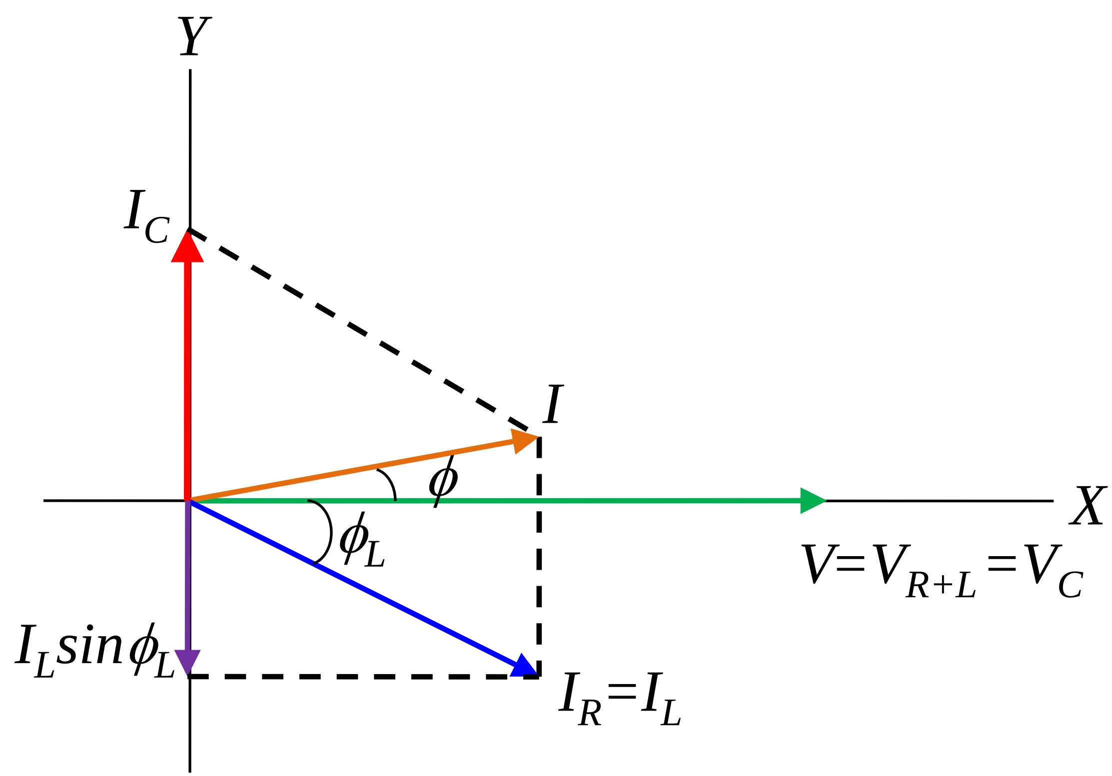



In this case $I_C > I_L \sin\phi_L$

The circuit is predominantly capacitive

Supply current *I* leads supply voltage *V*



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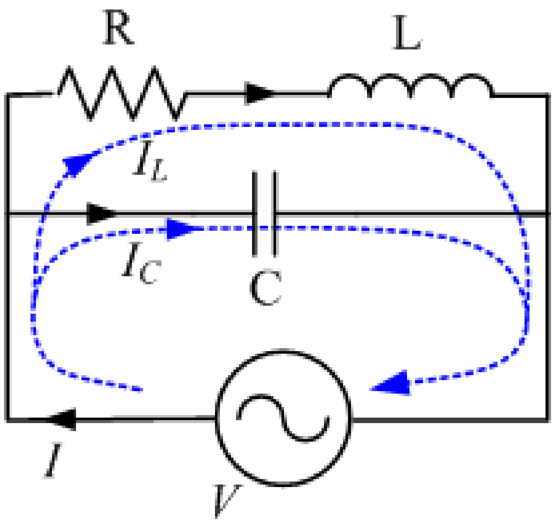
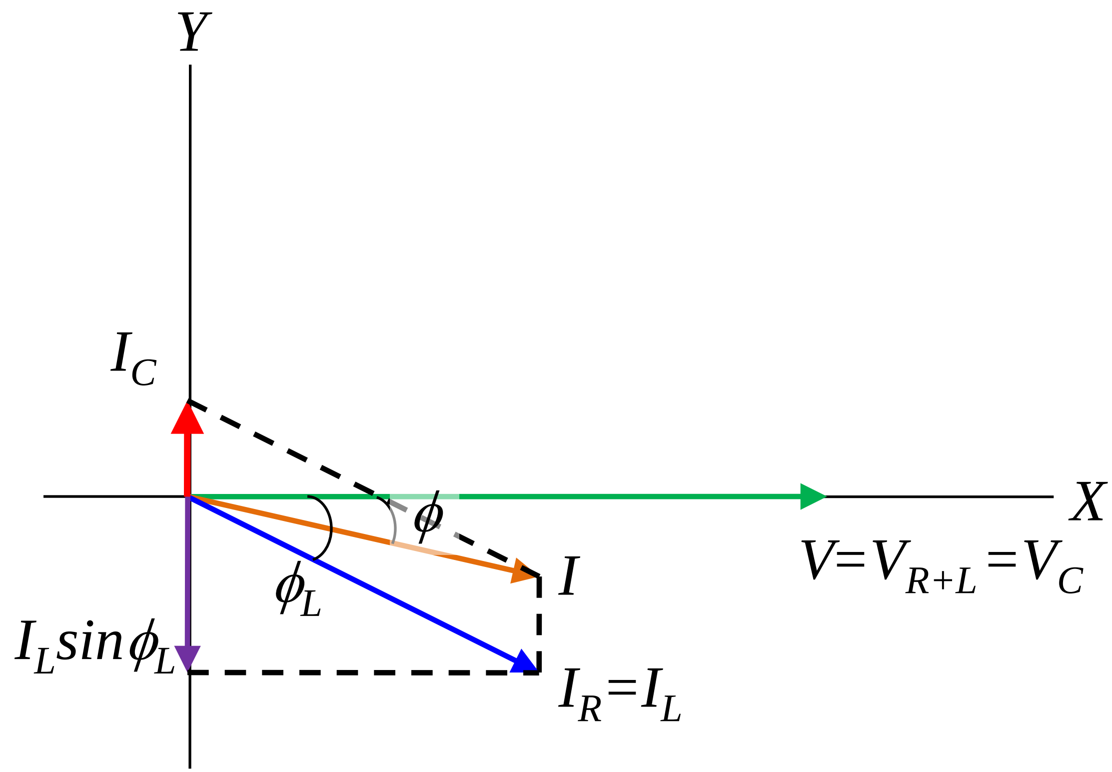



In the case when $I_C < I_L \sin\phi_L$
The circuit is predominantly inductive
Supply current $I$ lags supply voltage $V$



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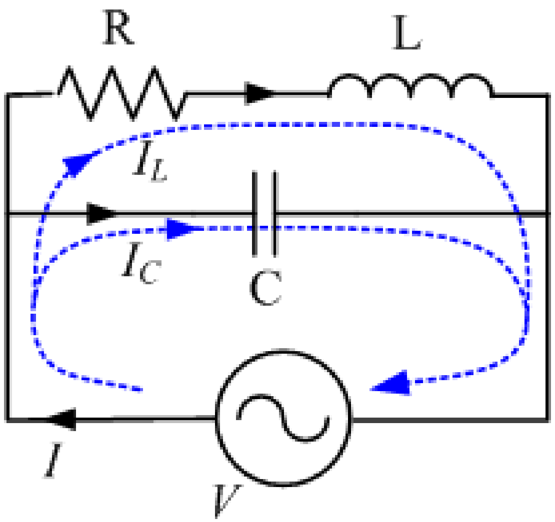
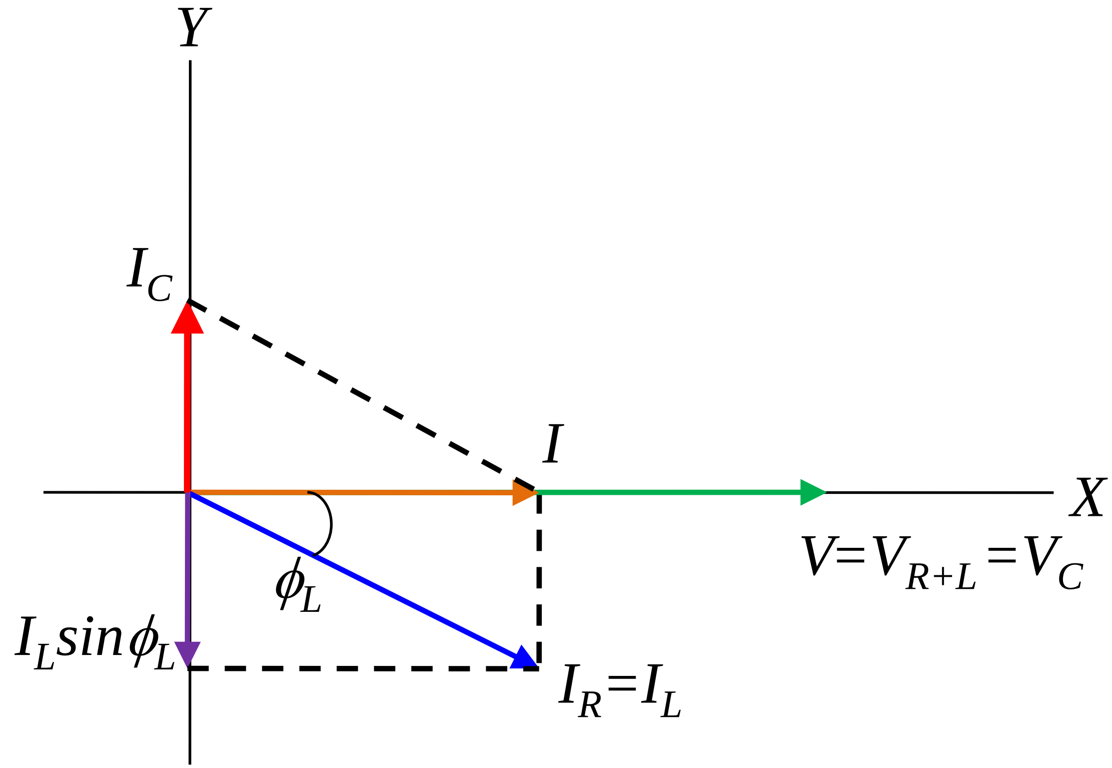



In the case when $I_C = I_L \sin\phi_L$
Supply current $I$ is in the same phase as the supply voltage $V$



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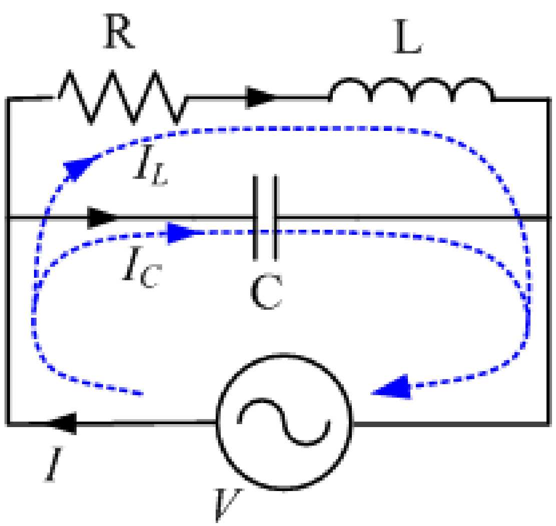
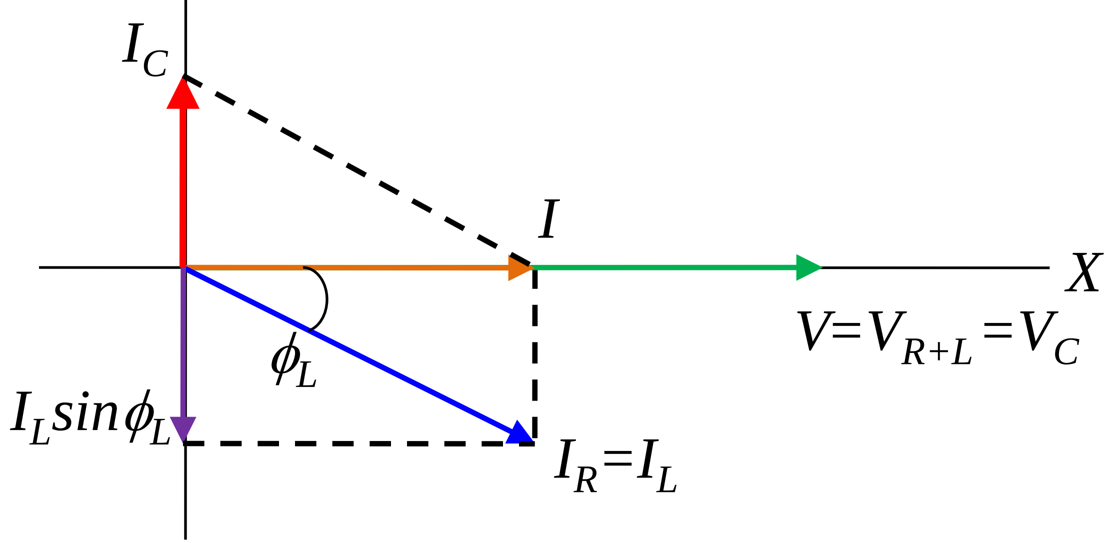



In the case when $I_C = I_L \sin\phi_L$
Supply current $I$ is in the same phase as the supply voltage $V$

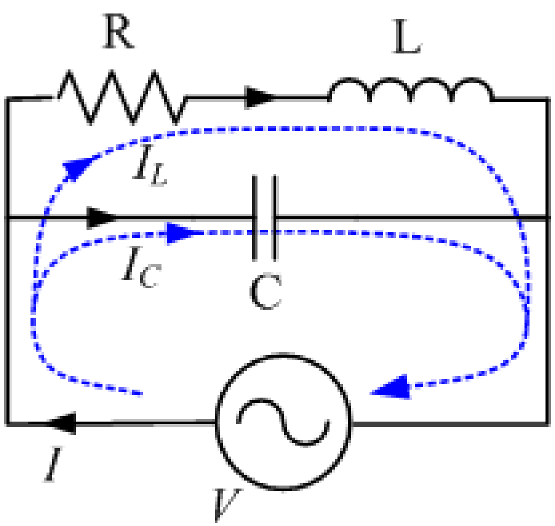
**This is parallel resonance condition**

- **Supply voltage & current in same phase**
- **Reactive power is zero**
- **Only active power**

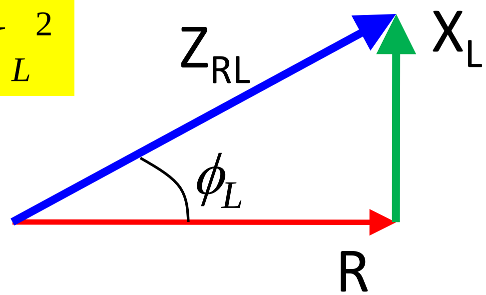


# Parallel Resonance

- Parallel resonance condition $I_C = I_L \sin\phi_L$
- Draw the impedance triangle of the R-L branch




$$Z_{RL} = \sqrt{R^2 + X_L^{\ 2}}$$




$$I_L = \frac{V}{Z_{RL}} \qquad \sin\phi_L = \frac{X_L}{Z_{RL}} \qquad I_C = \frac{V}{X_C}$$

Hence, condition for resonance becomes

$$\frac{V}{X_C} = \frac{V}{Z_{RL}} \times \frac{X_L}{Z_{RL}} \Rightarrow Z_{RL}^{\ 2} = X_L \times X_C$$

Now, $X_L = \omega L \qquad X_C = \dfrac{1}{\omega C} \qquad \Rightarrow Z_{RL}^{\ 2} = \omega L \times \dfrac{1}{\omega C} \;\; \Rightarrow Z_{RL}^{\ 2} = \dfrac{L}{C}$

# Parallel Resonance – resonant Frequency

$$Z_{RL}{}^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + X_L{}^2 = \frac{L}{C} \qquad \Rightarrow R^2 + (\omega_0 L)^2 = \frac{L}{C}$$

$$\Rightarrow (\omega_0 L)^2 = \frac{L}{C} - R^2 \quad \Rightarrow \omega_0{}^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\Rightarrow \omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If the resistance R is negligible, then the resonant frequency becomes:

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}\ rad/s}$$

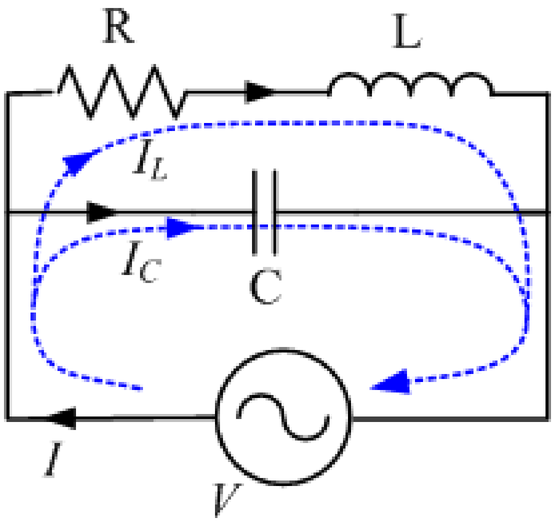
$$\boxed{f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}\ Hz}$$

**Note that this expression is same as that for series resonance**

# Parallel Resonance - Mathematically

It is convenient to express admittances in a parallel circuit:

$$\bar{Y}_{RL} = \frac{1}{\bar{Z}_{RL}} = \frac{1}{R + jX_L} = \frac{1}{R + jX_L} \times \frac{R - jX_L}{R - jX_L} = \frac{R}{R^2 + X_L{}^2} - j\frac{X_L}{R^2 + X_L{}^2}$$




$$\bar{Y}_C = \frac{1}{\bar{Z}_C} = \frac{1}{-jX_C} = \frac{j}{X_C}$$

$$\bar{Y} = \bar{Y}_{RL} + \bar{Y}_C = \frac{R}{R^2 + X_L{}^2} - j\frac{X_L}{R^2 + X_L{}^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2 + X_L{}^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L{}^2}\right)$$

Now, the circuit would be in resonance when the effective total impedance is purely resistive, i.e. *j-component of the complex admittance is zero i.e.:*

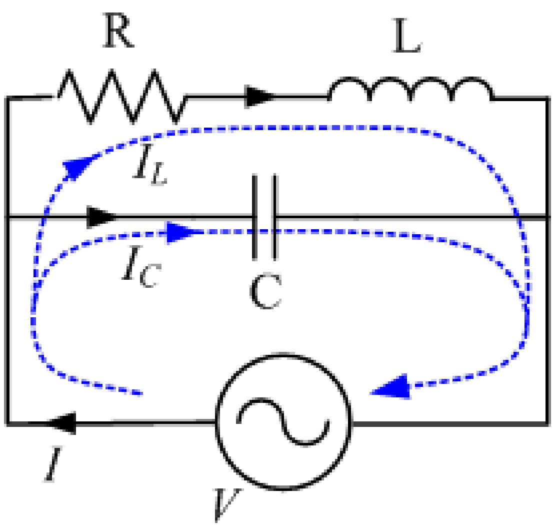
$$j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L{}^2}\right) = 0 \;\Rightarrow \frac{1}{X_C} = \frac{X_L}{R^2 + X_L{}^2} \;\Rightarrow X_L \times X_C = R^2 + X_L{}^2 = Z_{RL}{}^2$$

**Note that by this method also we are getting the same condition.**

# Parallel Resonance - Mathematically

In terms of susceptance:

$$\bar{Y}_{RL} = \frac{1}{\bar{Z}_{RL}} = \frac{1}{R + jX_L} = \frac{1}{R + jX_L} \times \frac{R - jX_L}{R - jX_L} = \frac{R}{R^2 + X_L^{\ 2}} - j\frac{X_L}{R^2 + X_L^{\ 2}} = G_{RL} - jB_{RL}$$




$$\bar{Y}_C = \frac{1}{\bar{Z}_C} = \frac{1}{-jX_C} = \frac{j}{X_C} = G_C + jB_C$$

Inductive susceptance: $B_{RL} = \dfrac{X_L}{R^2 + X_L^{\ 2}}$

Capacitive susceptance: $B_C = \dfrac{1}{X_C}$

Total admittance:

$$\bar{Y} = \bar{Y}_{RL} + \bar{Y}_C = \frac{R}{R^2 + X_L^{\ 2}} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^{\ 2}}\right) = G_{RL} + j\left(B_C - B_{RL}\right) = G + jB$$

The parallel circuit is said to be in resonance when imaginary part of total admittance is zero, i.e. $B = 0$

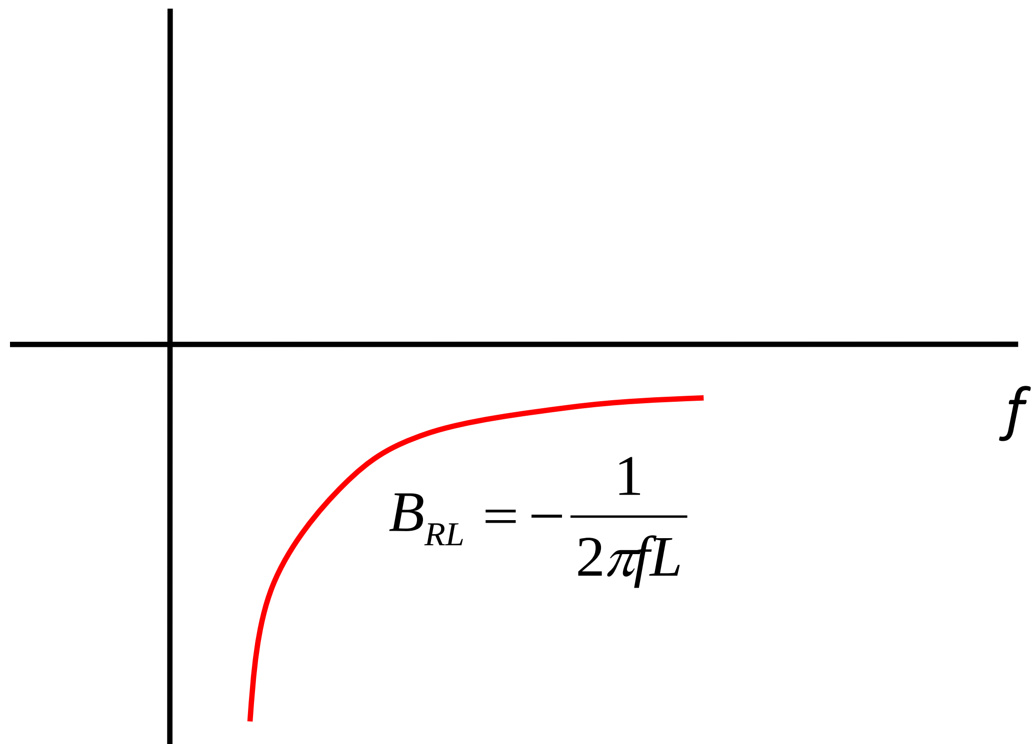
**i.e. inductive susceptance = capacitive susceptance**

# Characteristics of Parallel Resonance

Inductive susceptance: $B_{RL} = -\dfrac{X_L}{R^2 + X_L{}^2}$

Neglecting resistance: $B_{RL} = -\dfrac{1}{X_L} = -\dfrac{1}{2\pi f L}$

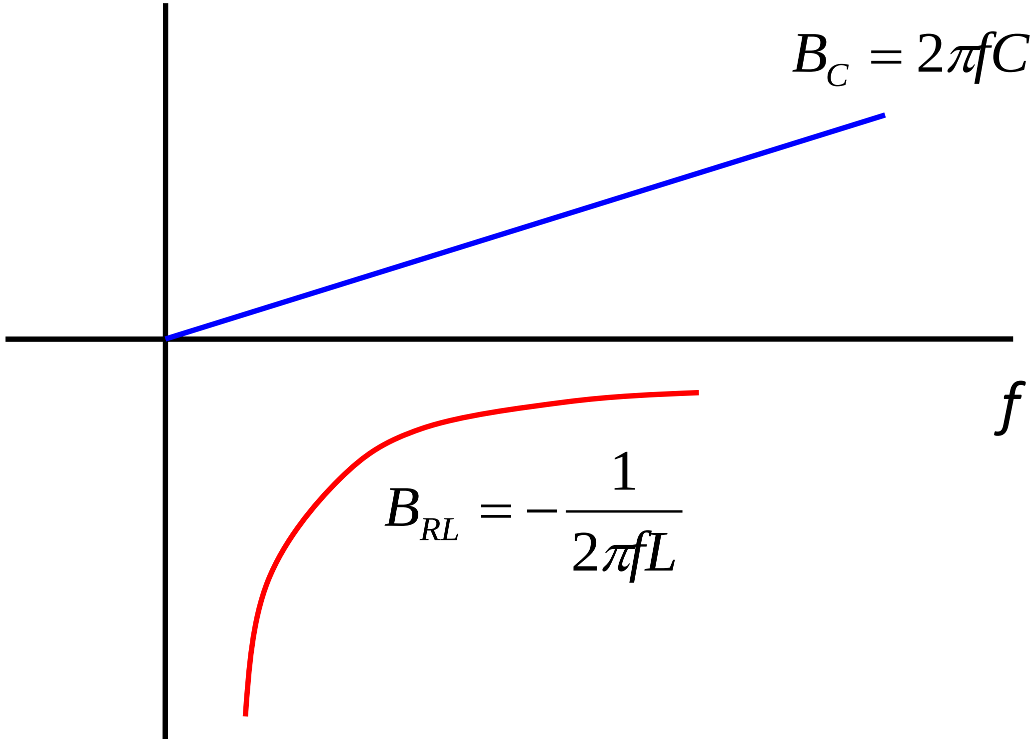
- Thus, it is inversely proportional to the frequency of the applied voltage
- Hence, it is represented by a rectangular hyperbola drawn in the fourth quadrant (it is assumed negative)



# Characteristics of Parallel Resonance

Capacitive susceptance: $B_C = \dfrac{1}{X_C} = \dfrac{1}{\dfrac{1}{2\pi f C}} = 2\pi f C$

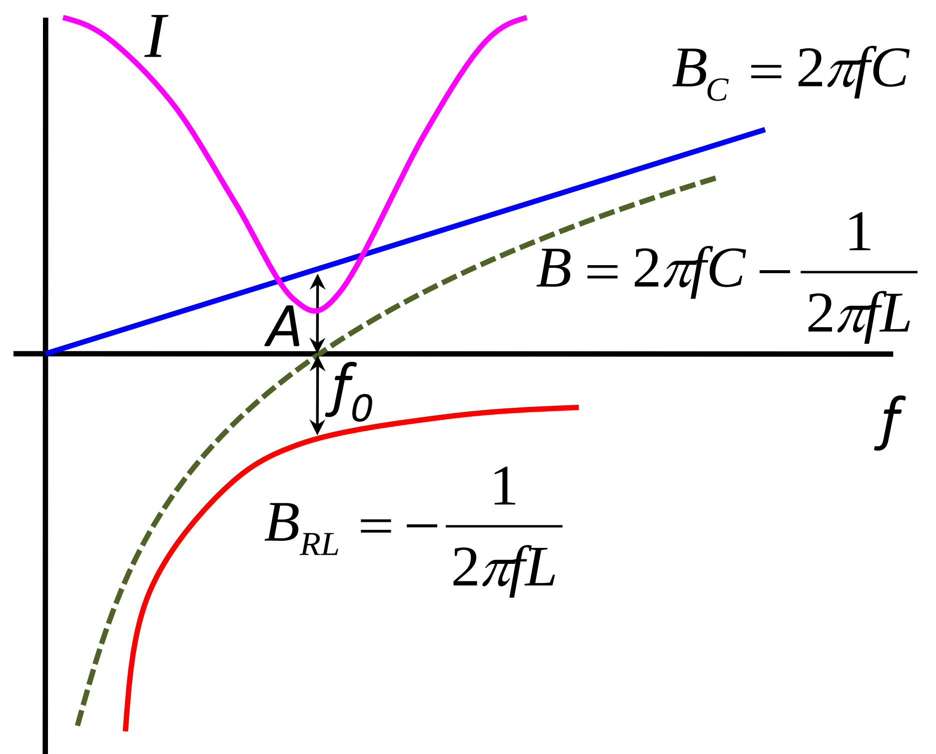
- Thus, it is directly proportional to the frequency of the applied voltage
- Hence, it is represented by a straight line drawn in the first quadrant passing through the origin



# Characteristics of Parallel Resonance

Total susceptance: $B = B_C + B_{RL} = 2\pi fC - \dfrac{1}{2\pi fL}$

- Total susceptance plot will thus be simple addition of the two individual susceptance plots
- It is represented by the dotted hyperbola
- At point *A*, where $B_C = B_{RL}$, the net susceptance *B* is zero
- Hence at *A*, the total admittance is minimum (and equal to the conductance G only: $Y = G + B = G + 0 = G$
- In other words, impedance is maximum
- **So at point A, line current *I* is minimum**
- **This is the resonant frequency $f_0$**




<u>Current magnitude at resonance:</u>

$$I = \frac{V}{Z} = V \times Y = V \times G = V \times \frac{R}{R^2 + X_L^{\ 2}} = V \times \frac{R}{Z_{RL}^{\ 2}}$$

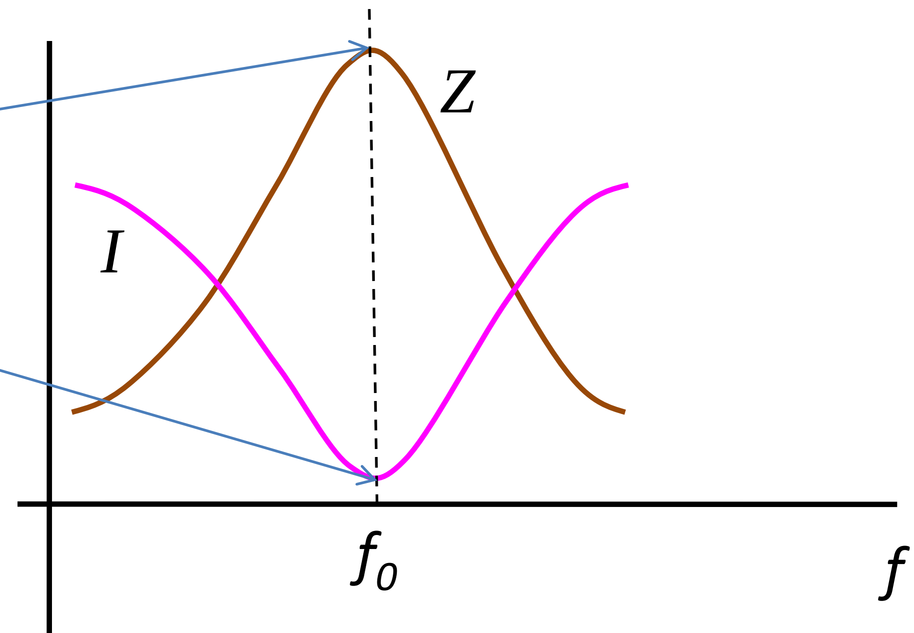
$$\text{Since, } Z_{RL}^{\ 2} = \frac{L}{C} \Rightarrow I = V \times \frac{R}{\frac{L}{C}} = \boxed{\frac{V}{L/CR}}$$

# Characteristics of Parallel Resonance

Current magnitude at resonance:

$$I = \frac{V}{L/CR}$$

- The denominator $L/CR$ *is known as the* ***dynamic impedance of the parallel circuit*** at resonance
- It should be noted that impedance is 'resistive' only
- Since current is minimum at resonance, $L/CR$ *must, therefore, represent the maximum impedance of the circuit*
- *In fact, parallel* resonance is a condition of maximum impedance or minimum admittance
- At resonant frequency, impedance is maximum and equals $L/CR$
- Consequently, current at resonance is minimum and is $I = V / (L/CR)$



# Resonance in parallel RLC circuit

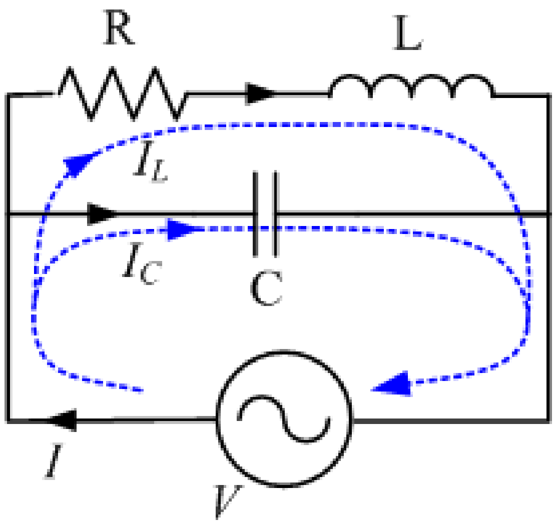
| Relation | Remarks |
|---|---|
| $B_{RL} = B_C$ | Inductive susceptance equals capacitive susceptance |
| $B = B_C - B_{RL} = 0$ | Net susceptance is zero |
| $Y = G + jB = G$ | Total admittance equals total conductance |
| $\lvert Y \rvert = \sqrt{G^2 + \left(B_C - B_{RL}\right)^2}$ <br> $= \sqrt{G^2 + 0^2}$ <br> $= G$ | The circuit behaves as a purely resistive circuit with no imaginary part of admittance (or impedance) being present at resonance |

# Resonance in parallel RLC circuit

| Relation | Remarks |
|---|---|
| $Z_0 = L/RC$ | Dynamic impedance at resonance. The impedance value is maximum at resonance |
| $I = \dfrac{V}{L/CR}$ | Line current at resonance is minimum and it is purely resistive current, no imaginary part |
| $\phi_0 = \angle Z_0 = \tan^{-1}\dfrac{\text{Imag}(Z_0)}{\text{Real}(Z_0)}$ <br> $= \tan^{-1}\dfrac{0}{L/RC}$ <br> $= 0^0$ | The phase angle between supply current and voltage is zero, i.e. current and voltage are in the same phase |
| $\cos\phi = \cos 0^0 = 1$ | The overall circuit power factor is unity |

# Q factor (Quality factor)

- In a parallel resonating circuit, the Q-factor is defined the current in any of the two parallel branches to the line current drawn from the supply
- Thus we have the expression for Q-factor as:




$$Q = \frac{I_C}{I}$$

$$\text{Now, } I_C = \frac{V}{X_C} = \frac{V}{1/\omega_0 C} = V\omega_0 C$$

$$\text{And, } I = \frac{V}{Z_0} = \frac{V}{L/RC} = \frac{VRC}{L}$$

$$\therefore \text{Q factor} = Q = \frac{I_C}{I} = \frac{V\omega_0 C}{\frac{VRC}{L}} = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R}$$

This expression is exactly same as in series resonating circuit

Remember the expression for resonating frequency with negligible resistance:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow Q = \frac{2\pi \frac{1}{2\pi\sqrt{LC}} L}{R} \quad \boxed{\Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$