3-Phase systems

Day 30

Symmetrical components
Tutorial 2

ILOs – Day 30

- Solve numerical problems related to use of symmetrical components for analysis of unbalanced signals

Example #1

A 3-phase, 4-wire system has line currents as

$$I_a = 100\angle 30^0 \qquad I_b = 50\angle 300^0 \qquad I_c = 30\angle 180^0$$

Calculate the positive, negative, and zero sequence currents in 'a' line and also calculate current in the neutral (4th) wire.

Original 3-phase currents are unbalanced

Original set of unbalanced currents

$$I_a = 100\angle 30^0$$

$$I_b = 50\angle 300^0$$

$$I_c = 30\angle 180^0$$

- We have to calculate the symmetrical components
- So left hand side of the equations must have the symmetrical components
- i.e. I_{a0}, I_{a1}, and I_{a2}
- Thus, we have to make use of the analysis equation

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

$$I_{a1} = \frac{1}{3}\left(I_a + \alpha I_b + \alpha^2 I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + \alpha^2 I_b + \alpha I_c\right)$$

Original set of unbalanced currents

$$I_a = 100\angle 30^0$$

$$I_b = 50\angle 300^0$$

$$I_c = 30\angle 180^0$$

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

$$I_{a1} = \frac{1}{3}\left(I_a + \alpha I_b + \alpha^2 I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + \alpha^2 I_b + \alpha I_c\right)$$

- Zero sequence current in line "a"

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

$$= \frac{1}{3}\left(100\angle 30^0 + 50\angle 300^0 + 30\angle 180^0\right)$$

$$= \frac{1}{3}\left[\left(86.6 + j50\right) + \left(25 - j43.3\right) + \left(-30 + j0\right)\right]$$

$$= 27.17 + j2.23$$

$$= 27.26\angle 4.69^0\ A$$

Original set of unbalanced currents

$$I_a = 100\angle 30^0$$

$$I_b = 50\angle 300^0$$

$$I_c = 30\angle 180^0$$

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

$$I_{a1} = \frac{1}{3}\left(I_a + \alpha I_b + \alpha^2 I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + \alpha^2 I_b + \alpha I_c\right)$$

- Positive sequence current in line “a”

$$I_{a1} = \frac{1}{3}\left(I_a + \alpha I_b + \alpha^2 I_c\right)$$

$$= \frac{1}{3}\left(100\angle 30^0 + 1\angle 120^0 \times 50\angle 300^0 + 1\angle 240^0 \times 30\angle 180^0\right)$$

$$= \frac{1}{3}\left(100\angle 30^0 + 50\angle 420^0 + 30\angle 420^0\right)$$

$$= \frac{1}{3}\left[(86.6 + j50) + (25 + j43.3) + (15 + j25.98)\right]$$

$$= 42.2 + j39.76$$

$$= 57.98\angle 43.3^0 \ A$$

Original set of unbalanced currents

$$I_a = 100\angle 30^0$$

$$I_b = 50\angle 300^0$$

$$I_c = 30\angle 180^0$$

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

$$I_{a1} = \frac{1}{3}\left(I_a + \alpha I_b + \alpha^2 I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + \alpha^2 I_b + \alpha I_c\right)$$

- Negative sequence current in line "a"

$$I_{a2} = \frac{1}{3}\left(I_a + \alpha^2 I_b + \alpha I_c\right)$$

$$= \frac{1}{3}\left(100\angle 30^0 + 1\angle 240^0 \times 50\angle 300^0 + 1\angle 120^0 \times 30\angle 180^0\right)$$

$$= \frac{1}{3}\left(100\angle 30^0 + 50\angle 540^0 + 30\angle 300^0\right)$$

$$= \frac{1}{3}\left[\left(86.6 + j50\right) + \left(-50 + j0\right) + \left(15 - j25.98\right)\right]$$

$$= 17.2 + j8$$

$$= 18.97\angle 24.94^0 \ A$$

Original set of unbalanced currents

$$I_a = 100\angle 30^0$$

$$I_b = 50\angle 300^0$$

$$I_c = 30\angle 180^0$$

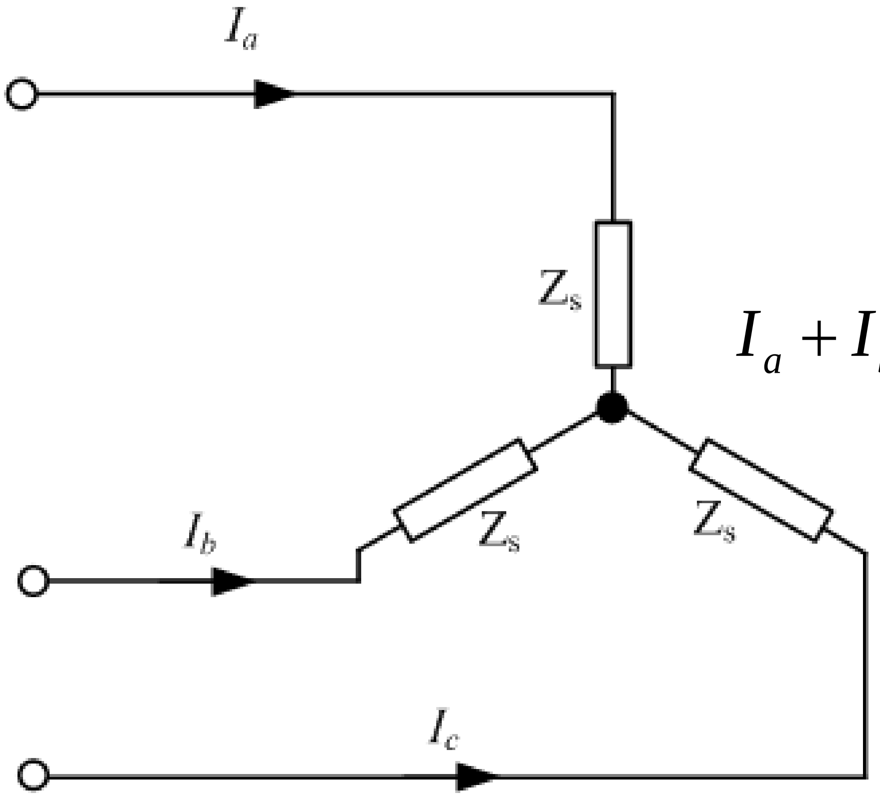
Symmetrical components

$$I_{a0} = 27.26\angle 4.69^0 \ A$$

$$I_{a1} = 57.98\angle 43.3^0 \ A$$

$$I_{a2} = 18.97\angle 24.94^0 \ A$$

- **Neutral current**
- In a 3-phase 3-wire system, summation of the three line currents must always be zero at the star point (KCL) since there is no return path.



$$I_a + I_b + I_c = 0$$

Original set of unbalanced currents

$$I_a = 100\angle 30^0$$

$$I_b = 50\angle 300^0$$

$$I_c = 30\angle 180^0$$

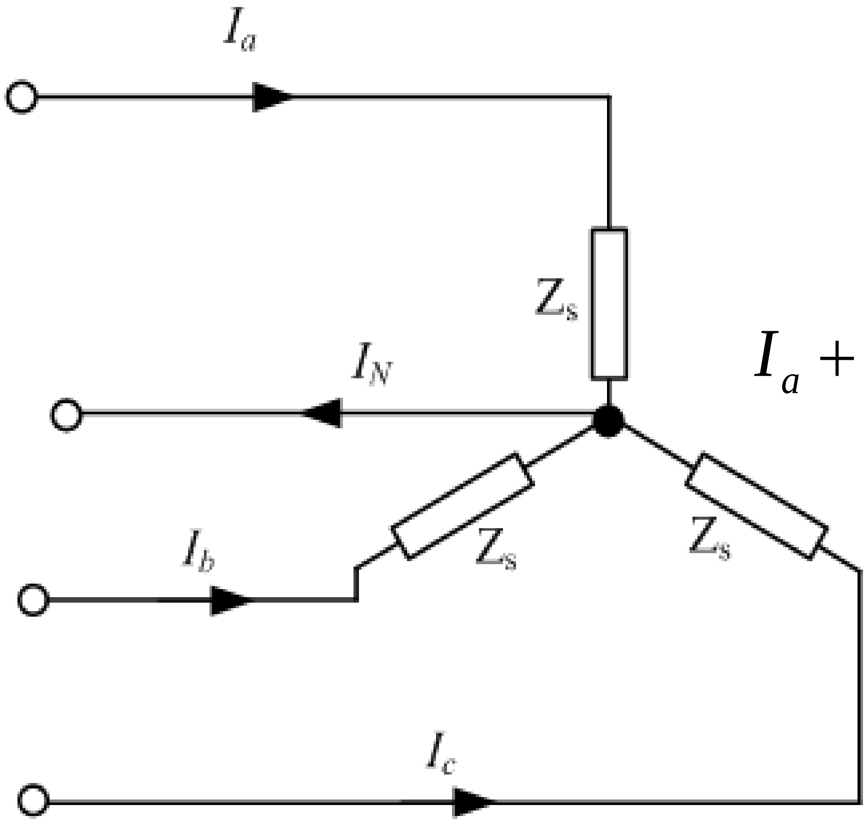
Symmetrical components

$$I_{a0} = 27.26\angle 4.69^0\ A$$

$$I_{a1} = 57.98\angle 43.3^0\ A$$

$$I_{a2} = 18.97\angle 24.94^0\ A$$

- **Neutral current**
- However, if there is a 4th wire (neutral wire), then any unbalance current can flow through the neutral return path



$$I_a + I_b + I_c = I_N$$

Original set of unbalanced currents

$$I_a = 100\angle 30^0$$

$$I_b = 50\angle 300^0$$

$$I_c = 30\angle 180^0$$

Symmetrical components

$$I_{a0} = 27.26\angle 4.69^0 \ A$$

$$I_{a1} = 57.98\angle 43.3^0 \ A$$

$$I_{a2} = 18.97\angle 24.94^0 \ A$$

- **Neutral current** $I_a + I_b + I_c = I_N$
- **Analysis equations:**

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

$$I_{a1} = \frac{1}{3}\left(I_a + \alpha I_b + \alpha^2 I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + \alpha^2 I_b + \alpha I_c\right)$$

- Now, from the analysis equations we have the relation:

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

Original set of unbalanced currents

$$I_a = 100\angle 30^0$$

$$I_b = 50\angle 300^0$$

$$I_c = 30\angle 180^0$$

Symmetrical components

$$I_{a0} = 27.26\angle 4.69^0 \ A$$

$$I_{a1} = 57.98\angle 43.3^0 \ A$$

$$I_{a2} = 18.97\angle 24.94^0 \ A$$

- **Neutral current** $I_a + I_b + I_c = I_N$
- Since from the analysis equations we have the relation:

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right)$$

- We get a very important relation in unsymmetrical systems

$$I_N = \left(I_a + I_b + I_c\right) = 3I_{a0}$$

- Thus, in this case:

$$I_N = 3I_{ao} = 3\times 27.26\angle 4.69^0 = 81.8\angle 4.69^0 \ A$$

Example #2

The sequence voltages in red phase of a 3-phase unbalanced system are as follows:

$V_{R0} = 100\,V$ **,** $V_{R1} = (200 - j100)\,V$ **,** $V_{R2} = -100\,V$

Find the phase voltages V_R **,** V_Y **, and** V_B **.**

Given:

$$V_{R0} = (100 + j0) = 100\angle 0^0\ V$$

$$V_{R1} = (200 - j100) = 223.6\angle -26.57^0\ V$$

$$V_{R2} = (-100 + j0) = 100\angle 180^0\ V$$

The sequence voltages in red phase of a 3-phase unbalanced system are as follows:

$V_{R0} = 100\,V$, $V_{R1} = (200 - j100)\,V$, $V_{R2} = -100\,V$

Find the phase voltages V_R **,** V_Y **, and** V_B **.**

From the synthesis equations we have:

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix}$$

$$\begin{aligned} V_R &= V_{R0} + V_{R1} + V_{R2} \\ &= (100 + j0) + (200 - j100) + (-100 + j0) \\ &= (200 - j100) \\ &= 223.6\angle -26.57^0\ V \end{aligned}$$

The sequence voltages in red phase of a 3-phase unbalanced system are as follows:

$V_{R0} = 100\,V$, $V_{R1} = (200 - j100)\,V$, $V_{R2} = -100\,V$

Find the phase voltages V_R **,** V_Y **, and** V_B **.**

$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix}$$

$$\begin{aligned} V_Y &= V_{R0} + \alpha^2 V_{R1} + \alpha V_{R2} \\ &= 100\angle 0^0 + 1\angle 240^0 \times 223.6\angle -26.57^0 + 1\angle 120^0 \times 100\angle 180^0 \\ &= 100\angle 0^0 + 223.6\angle 213.43^0 + 100\angle 300^0 \\ &= (100 + j0) + (-186.6 - j123.2) + (50 - j86.6) \\ &= -36.6 - j209.8 \\ &= 212.97\angle 99.9^0\ V \end{aligned}$$

The sequence voltages in red phase of a 3-phase unbalanced system are as follows:

$V_{R0} = 100\,V$, $V_{R1} = (200 - j100)\,V$, $V_{R2} = -100\,V$

Find the phase voltages V_R **,** V_Y **, and** V_B **.**

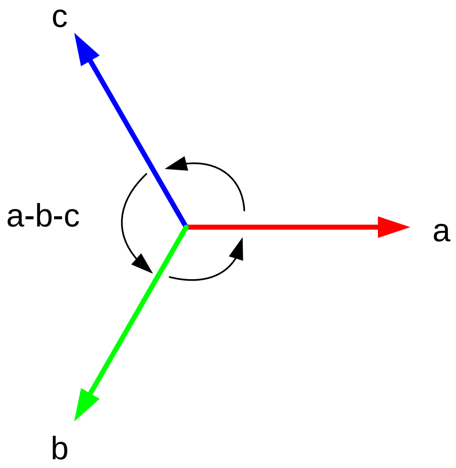
$$\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix}$$

$$\begin{aligned} V_B &= V_{R0} + \alpha V_{R1} + \alpha^2 V_{R2} \\ &= 100\angle 0^0 + 1\angle 120^0 \times 223.6\angle -26.57^0 + 1\angle 240^0 \times 100\angle 180^0 \\ &= 100\angle 0^0 + 223.6\angle 93.43^0 + 100\angle 420^0 \\ &= (100 + j0) + (-13.38 + j223.2) + (50 + j86.6) \\ &= 136.62 + j309.8 \\ &= 339.6\angle 66.2^0\ V \end{aligned}$$

Example #3

Show that a 3-phase balanced signal contain only the positive sequence components.

- In a balanced three-phase system with phase sequence a-b-c, the voltages are given as:

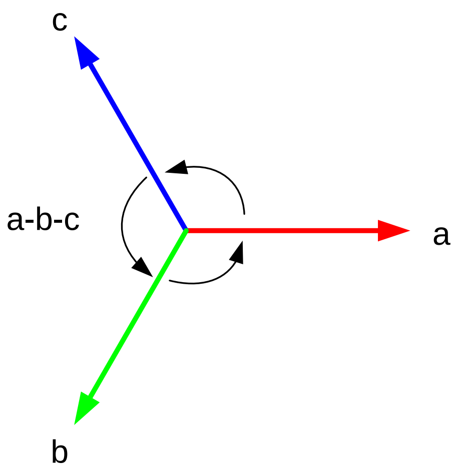


$$V_a = V\angle 0^0$$

$$V_b = V\angle 240^0 = \alpha^2 V_a$$

$$V_c = V\angle 120^0 = \alpha V_a$$

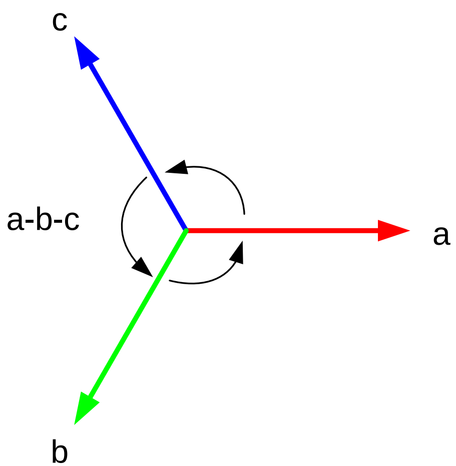
Show that a 3-phase balanced signal contain only the positive sequence components



$$V_a = V\angle 0^0 \qquad V_b = \alpha^2 V_a \qquad V_c = \alpha V_a$$

$$\begin{aligned}
V_{a0} &= \frac{1}{3}\left(V_a + V_b + V_c\right) \\
&= \frac{1}{3}\left(V_a + \alpha^2 V_a + \alpha V_a\right) \\
&= \frac{1}{3}\left[V_a\left(1 + \alpha + \alpha^2\right)\right] \\
&= \frac{1}{3}\left[V_a(0)\right] \\
&= 0
\end{aligned}$$

Show that a 3-phase balanced signal contain only the positive sequence components



$$V_a = V\angle 0^0 \qquad V_b = \alpha^2 V_a \qquad V_c = \alpha V_a$$

$$V_{a2} = \frac{1}{3}\left(V_a + \alpha^2 V_b + \alpha V_c\right)$$

$$= \frac{1}{3}\left(V_a + \alpha^2\left(\alpha^2 V_a\right) + \alpha\left(\alpha V_a\right)\right)$$

$$= \left(V_a + \alpha^4 V_a + \alpha^2 V_a\right)$$

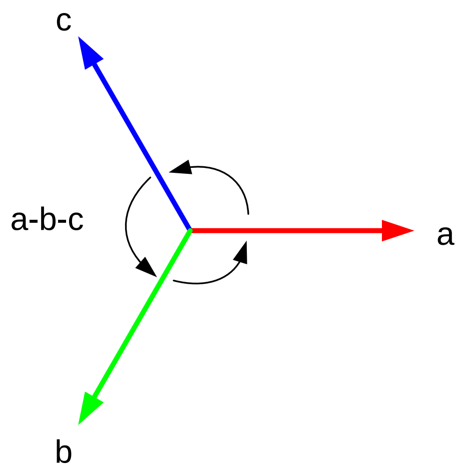
$$= \left(V_a + \alpha V_b + \alpha^2 V_c\right)$$

$$= \frac{1}{3}\left[V_a\left(1 + \alpha + \alpha^2\right)\right]$$

$$= \frac{1}{3}\left[V_a(0)\right]$$

$$= 0$$

Show that a 3-phase balanced signal contain only the positive sequence components



$$V_a = V\angle 0^0 \qquad V_b = \alpha^2 V_a \qquad V_c = \alpha V_a$$

$$V_{a1} = \frac{1}{3}\left(V_a + \alpha V_b + \alpha^2 V_c\right)$$

$$= \frac{1}{3}\left(V_a + \alpha\left(\alpha^2 V_a\right) + \alpha^2\left(\alpha V_a\right)\right)$$

$$= \left(V_a + \alpha^3 V_b + \alpha^3 V_c\right)$$

$$= \frac{1}{3}\left[V_a + V_a + V_a\right]$$

$$= \frac{1}{3}[3]$$

$$= V_a$$