AC Fundamentals

Day 11

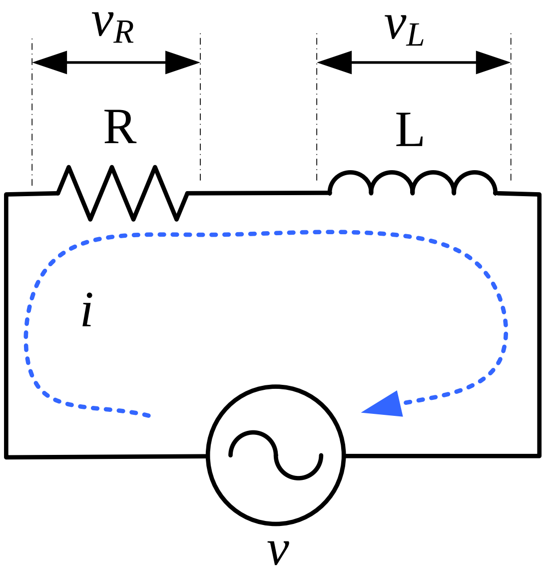
RL Circuit

ILOs – Day 11

- For a resistive + inductive circuit with AC supply:
  - Derive the expression for current and power
  - Draw phasor diagram

AC circuit operation with resistance and inductance together

AC circuit operation with R + L



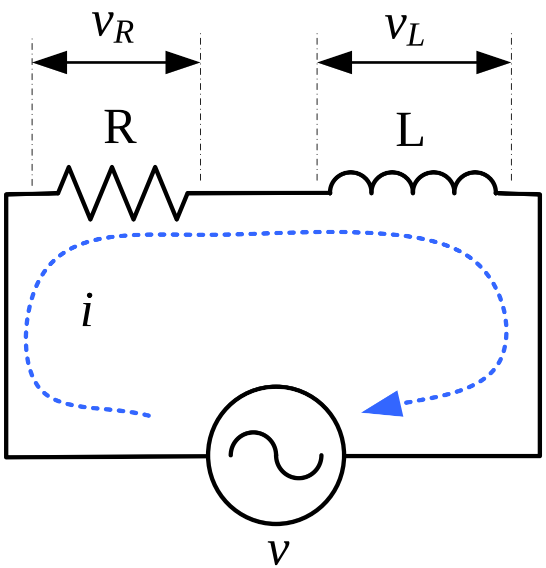
$$v = V_m \sin \omega t$$

Phasor diagram

- To draw the phasor diagram, we consider RMS values of the different signals
- These signals are: V_{RMS}, I_{RMS}, V_R, V_L
- The RMS value of current I_{RMS} is considered as the reference phasor and it is thus drawn along the X-axis

- *The general convention is to take the signal which is common to most part of the circuit as the reference phasor.*
- *(In a series circuit, current is common to all the elements)*

AC circuit operation with R + L

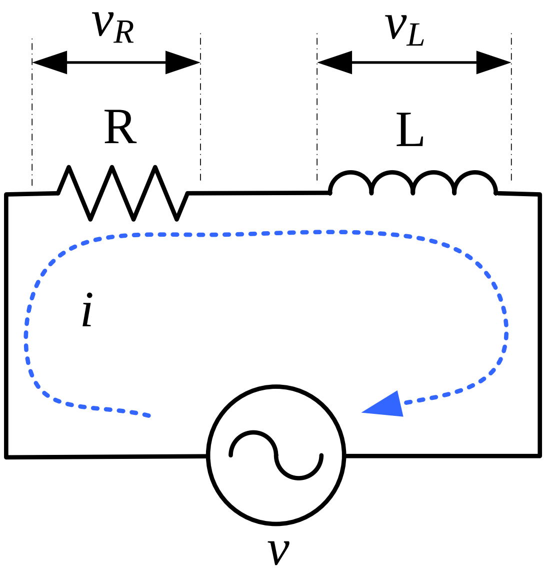


$$v = V_m \sin \omega t$$

<u>Phasor diagram</u>

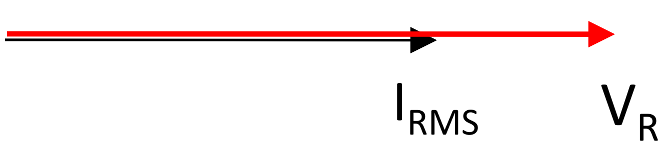
- The RMS value of current I_{RMS} is drawn along the X-axis

AC circuit operation with R + L

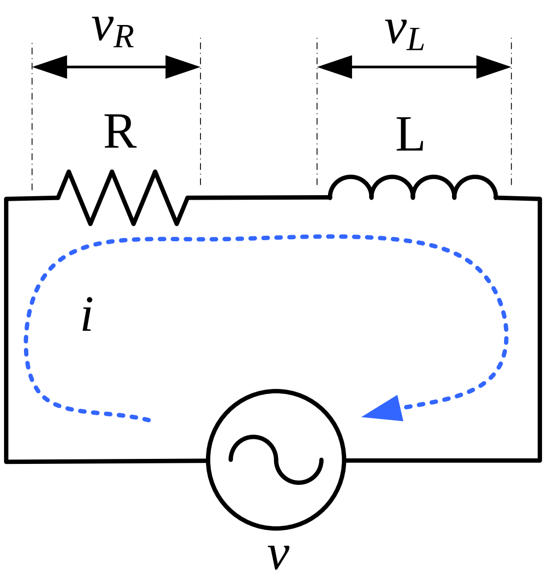


Phasor diagram

- Voltage drop across the resistance is
$$V_R = I_{RMS}R$$
- V_R phasor is drawn in the same direction as the current phasor
- (*remember, the current and voltage in a resistance are in the same phase*)



AC circuit operation with R + L

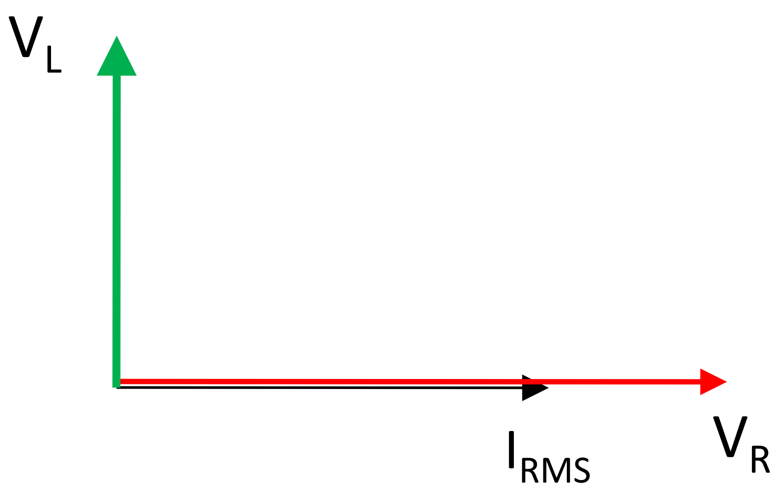


Phasor diagram

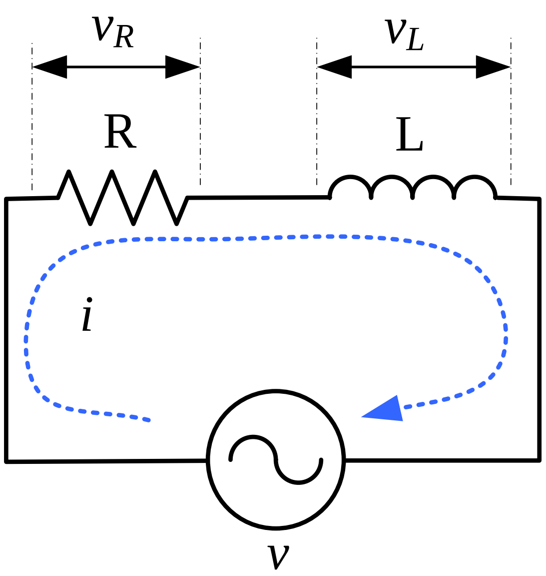
- Voltage drop across the inductor is

$$V_L = I_{RMS} X_L$$

- V_L phasor is drawn 90^0 leading to the current phasor
- (*remember, voltage across an inductor leads the current through it by 90^0*)



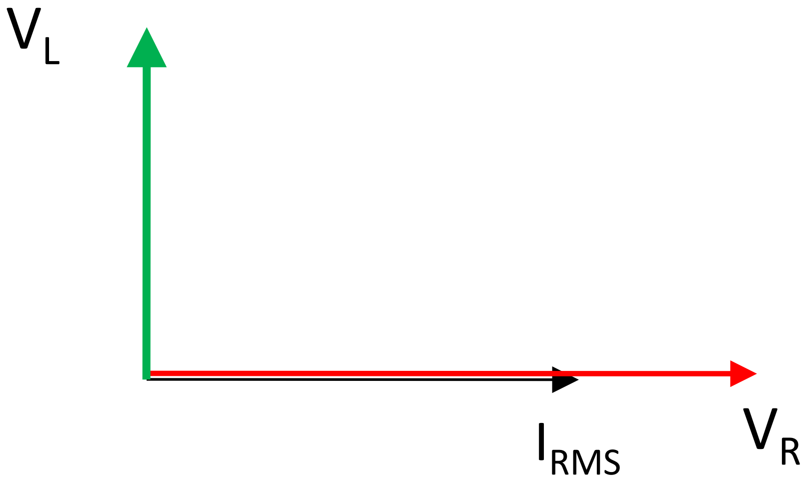
AC circuit operation with R + L



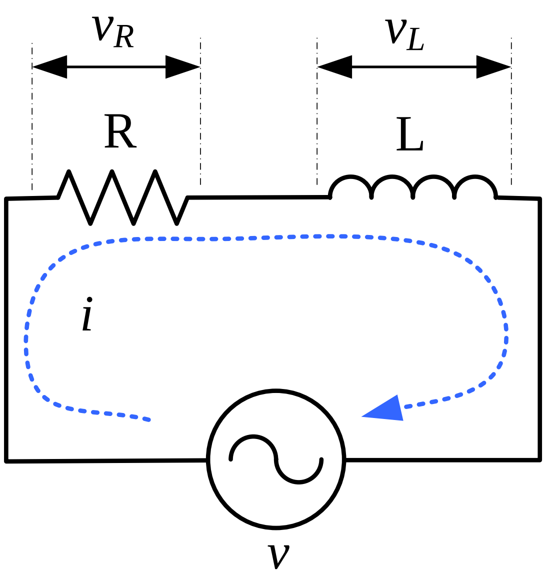
Phasor diagram

- According to KVL, the supply voltage must equal summation of the two voltage drops, one across the resistance and the other across the inductance

$$\overline{V_{RMS}} = \overline{V_R} + \overline{V_L}$$



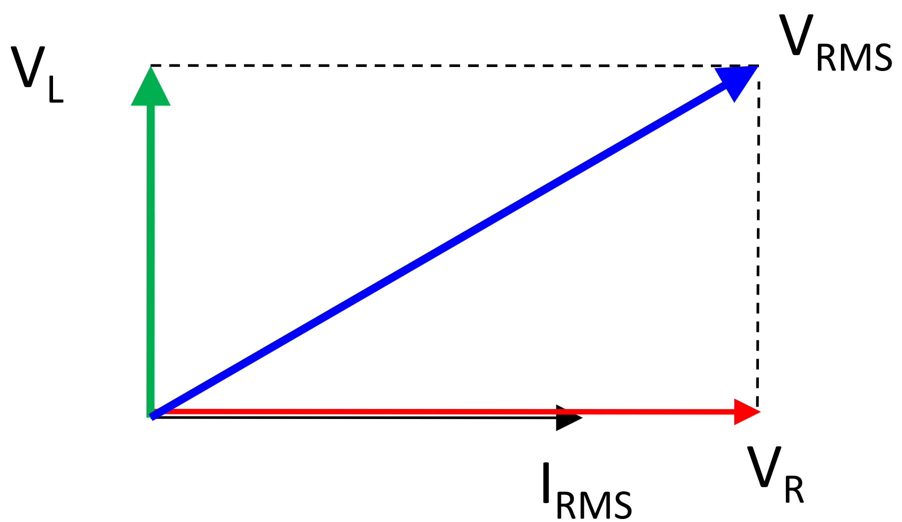
AC circuit operation with R + L



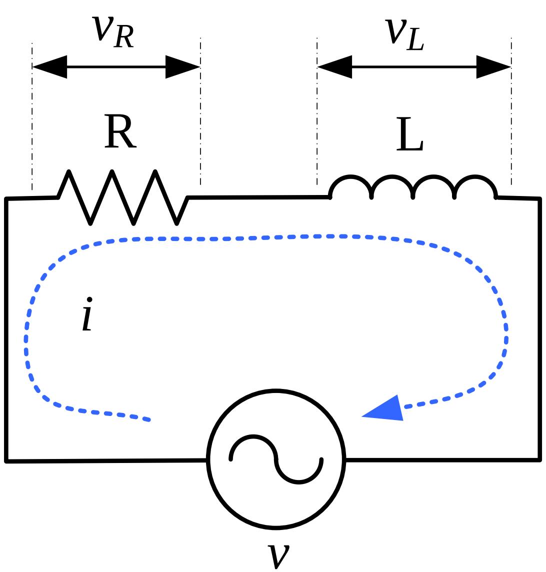
Phasor diagram

$$\overline{V_{RMS}} = \overline{V_R} + \overline{V_L}$$

- Thus, the supply voltage phasor V_{RMS} is drawn as the vector addition (resultant) of the two phasors V_R and V_L

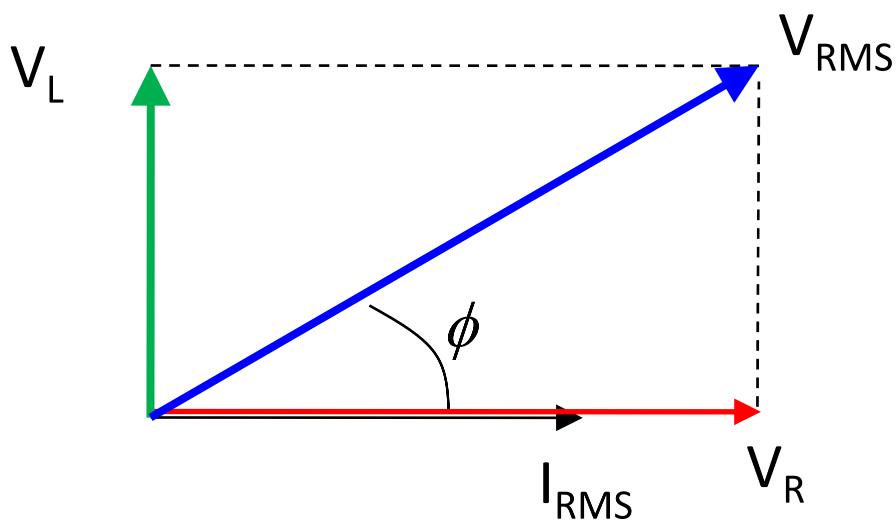


AC circuit operation with R + L

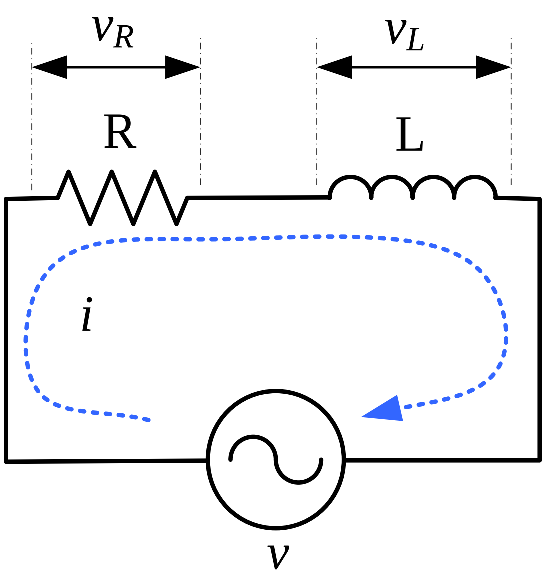


Phasor diagram

- The angle between the supply voltage V_{RMS} phasor and the supply current I_{RMS} phasor is known as the **power factor angle** ϕ



AC circuit operation with R + L

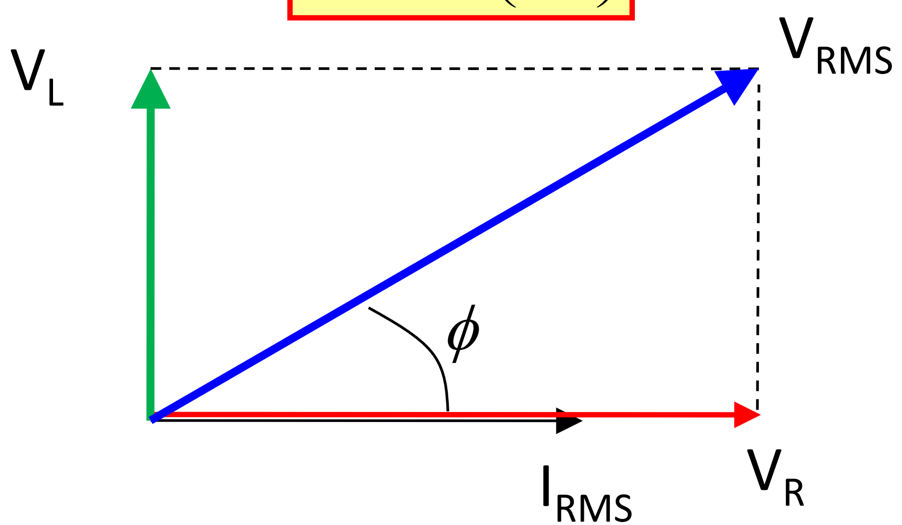


Phasor diagram

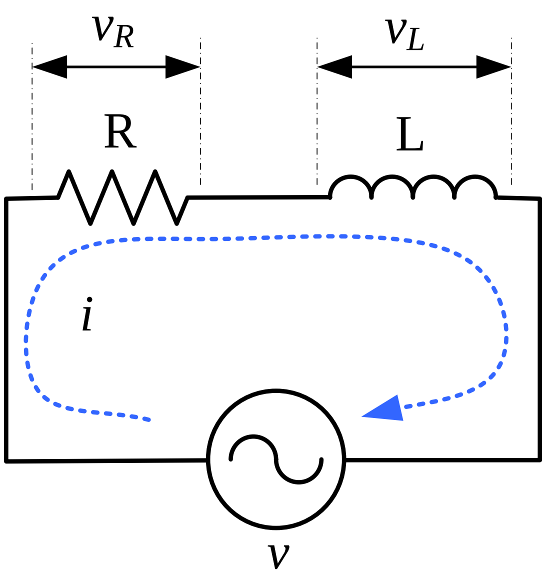
Value of the power factor angle can be expressed from the phasor diagram as:

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{I_{RMS} \times X_L}{I_{RMS} \times R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\boxed{\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)}$$



AC circuit operation with R + L

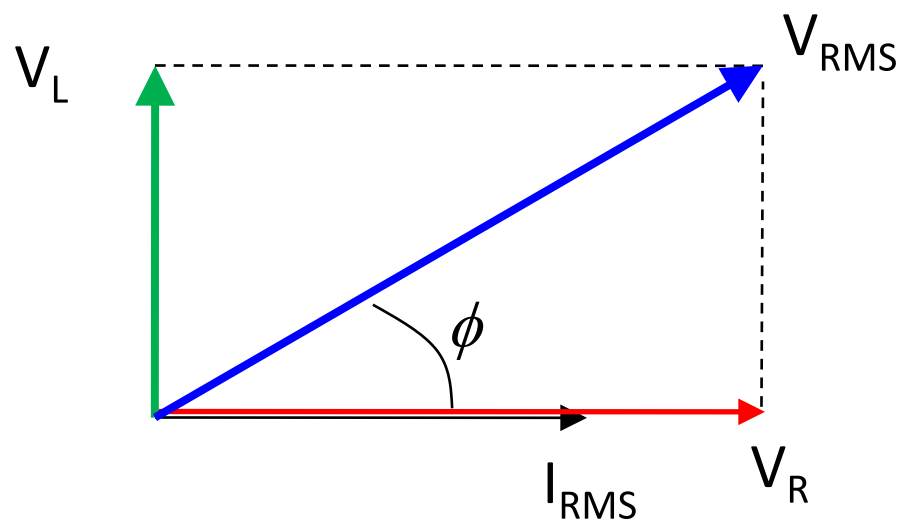


Phasor diagram

Drawing only the voltage phasors from the phasor diagram we obtain the so-called **voltage triangle** of a series R-L circuit:

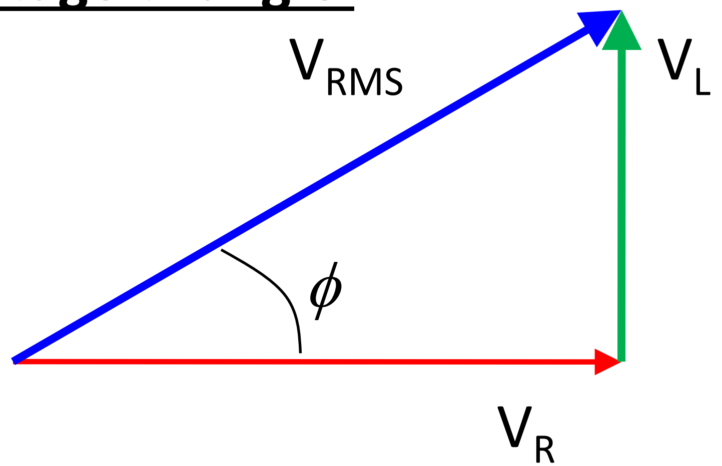
V_{RMS} V_L

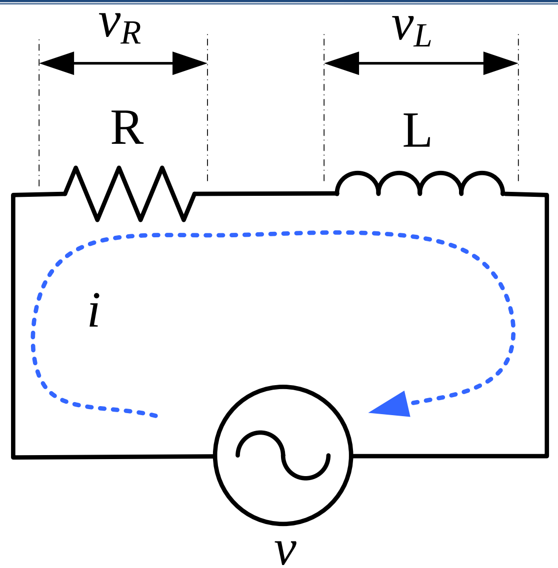
V_R



AC circuit operation with R + L

Voltage triangle





From the voltage triangle we have the relation: $\boxed{V_{RMS} = \sqrt{V_R{}^2 + V_L{}^2}}$

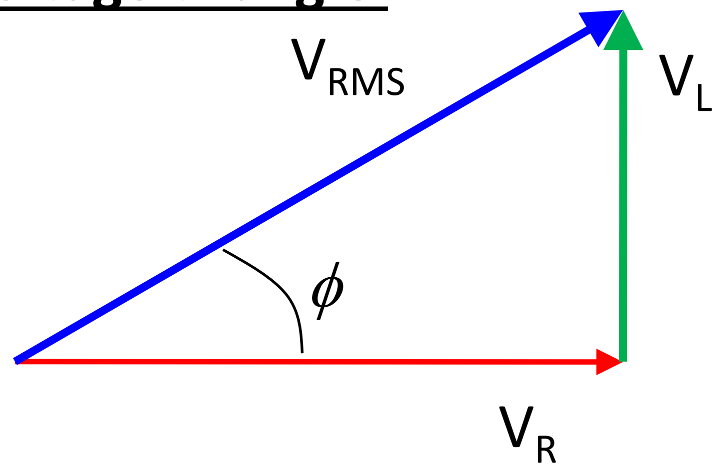
$$V_{RMS} = \sqrt{(I_{RMS}R)^2 + (I_{RMS}X_L)^2}$$

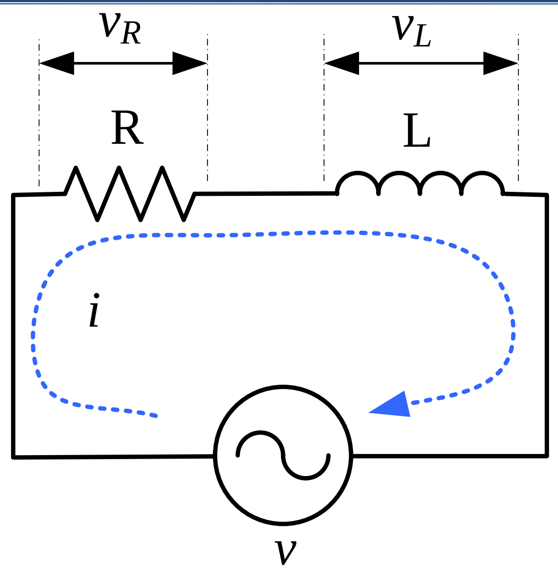
$$V_{RMS} = I_{RMS}\sqrt{R^2 + X_L{}^2}$$

$$\frac{V_{RMS}}{I_{RMS}} = \sqrt{R^2 + X_L{}^2}$$

AC circuit operation with R + L

Voltage triangle





$$\frac{V_{RMS}}{I_{RMS}} = \sqrt{R^2 + X_L{}^2}$$

$$\frac{V_{RMS}}{I_{RMS}} = \sqrt{R^2 + X_L{}^2} = Z$$

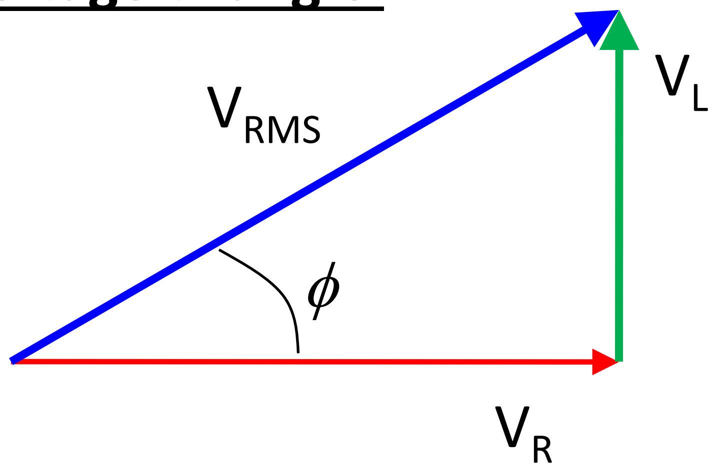
This ratio of V_{RMS} and I_{RMS} in a series R-L circuit is called **impedance** of the circuit that has a magnitude

$$Z = \sqrt{R^2 + X_L{}^2}$$

The unit of impedance is also “Ohm” (Ω).

AC circuit operation with R + L

Voltage triangle



$$V_R = I_{RMS} \times R$$

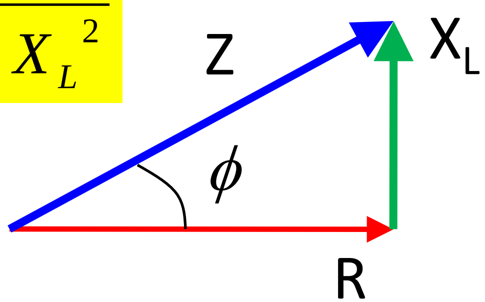
$$V_L = I_{RMS} \times X_L$$

$$V_{RMS} = I_{RMS} \times Z$$

- Redraw the same triangle in terms of the resistance, reactance, and impedance only
- (by eliminating the common quantity I_{RMS})

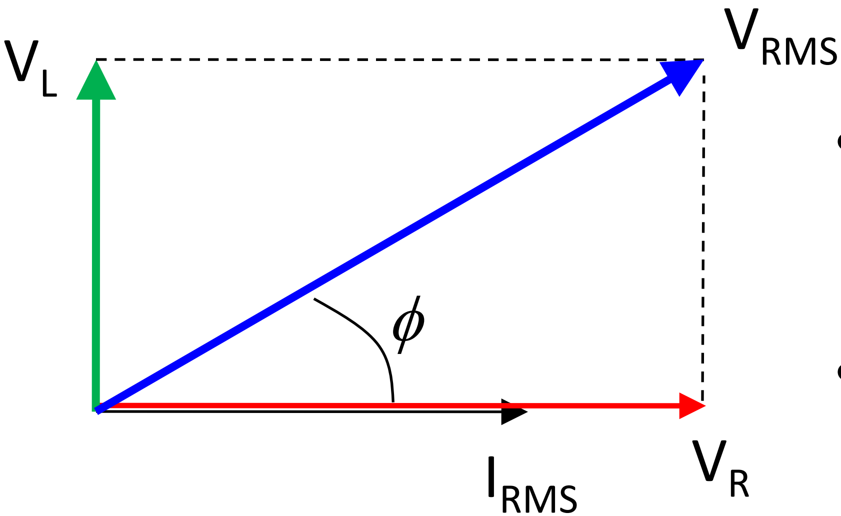
Impedance triangle

$$Z = \sqrt{R^2 + X_L{}^2}$$



Hypotenuse of the impedance triangle is the impedance *Z*

AC circuit operation with R + L

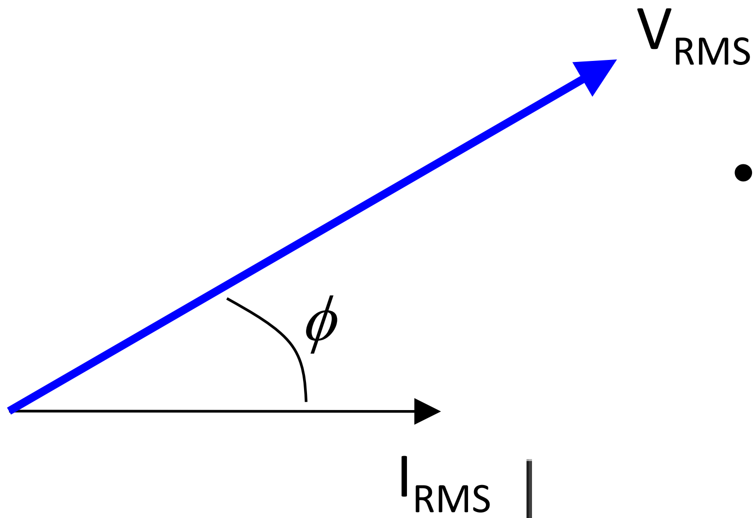


More on phase angle

- The supply voltage signal leads the current by a phase angle ϕ which is less than 90^0
- This is expected because it is a combination of R and L circuit

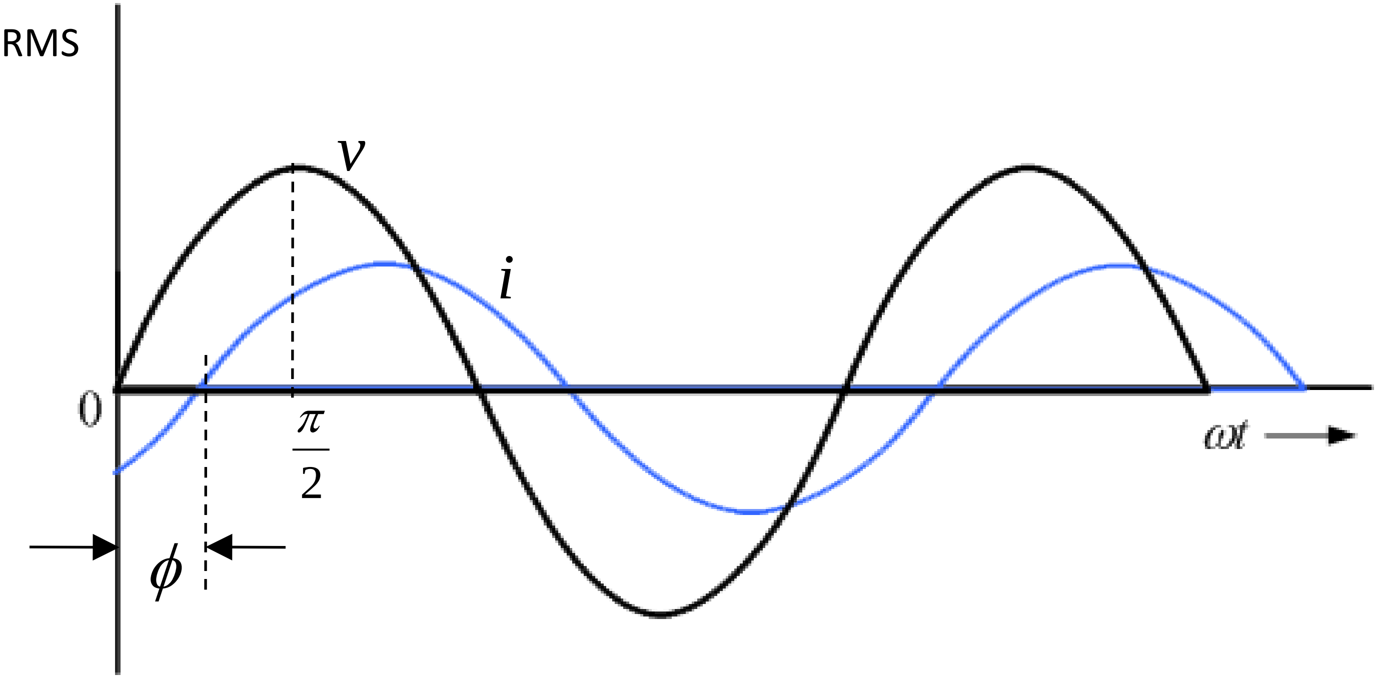
- In an R + L circuit, the angle between voltage and current should lie somewhere between 0^0 (pure resistance) and 90^0 (pure inductance)
- When $R > X_L$, the phase angle $\phi < 45^0$
- When $X_L > R$, then the phase angle $\phi > 45^0$
- When $X_L = R$, the phase angle $\phi = 45^0$

AC circuit operation with R + L

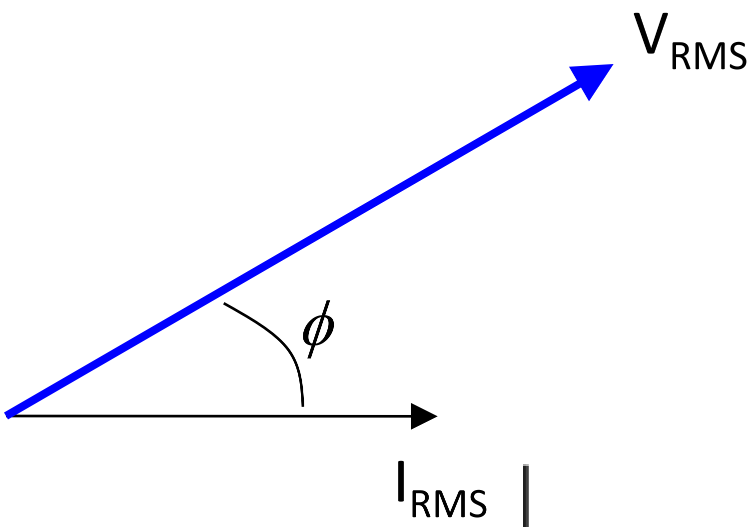


Instantaneous values

- The current is drawn lagging behind the voltage by an angle ϕ that is less than 90^0



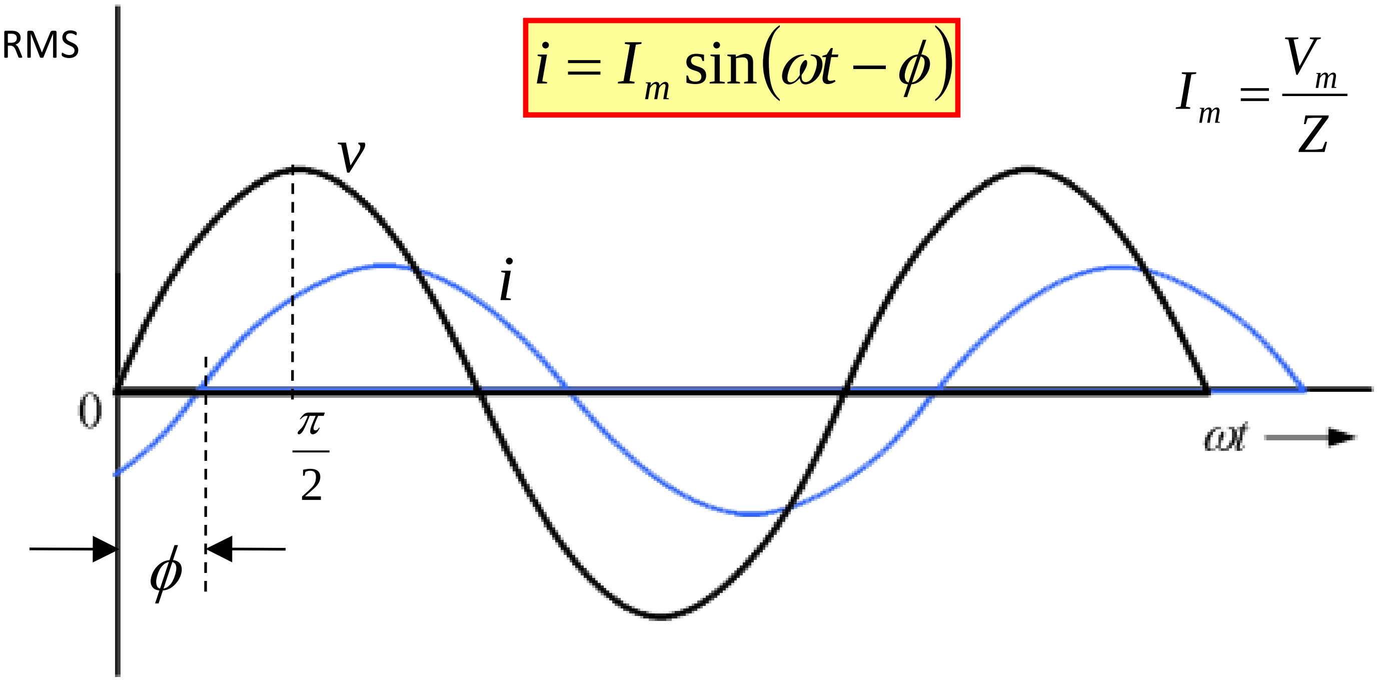
AC circuit operation with R + L



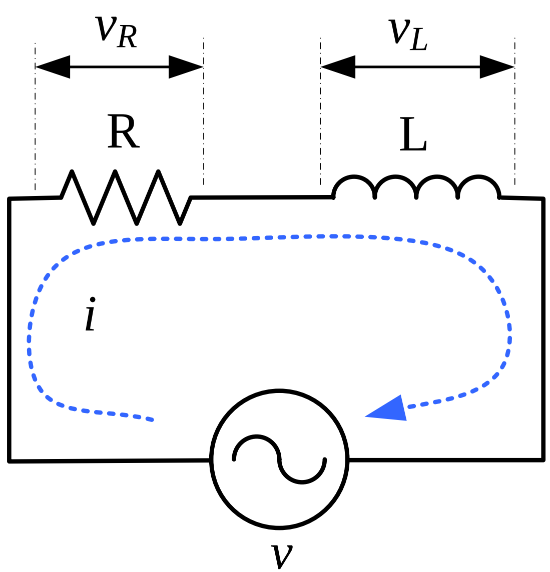
- If the voltage signal is given by the expression $$\boxed{v = V_m \sin \omega t}$$
- then the current signal that lags behind the voltage by an angle ϕ, will be given by the expression

$$\boxed{i = I_m \sin(\omega t - \phi)}$$

$$I_m = \frac{V_m}{Z}$$



AC circuit operation with R + L



$$\boxed{v = V_m \sin \omega t}$$

$$\boxed{i = I_m \sin(\omega t - \phi)}$$

Instantaneous power

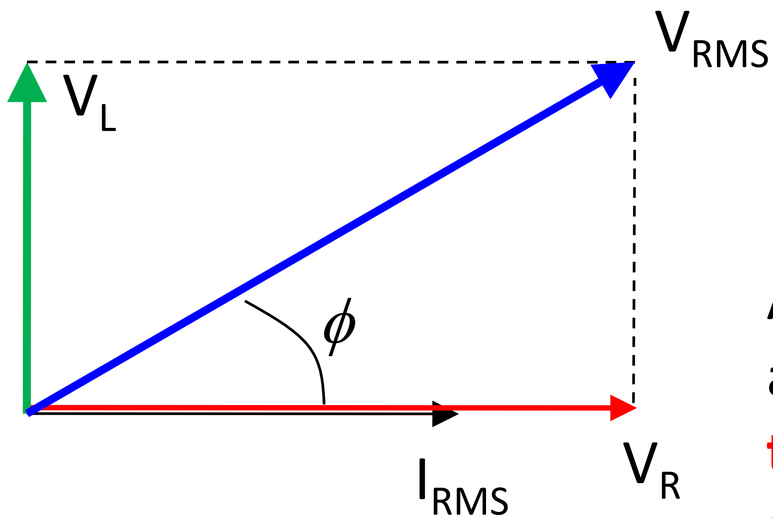
$$p = v \times i$$

$$p = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$$

$$p = V_m I_m \sin \omega t \sin(\omega t - \phi)$$

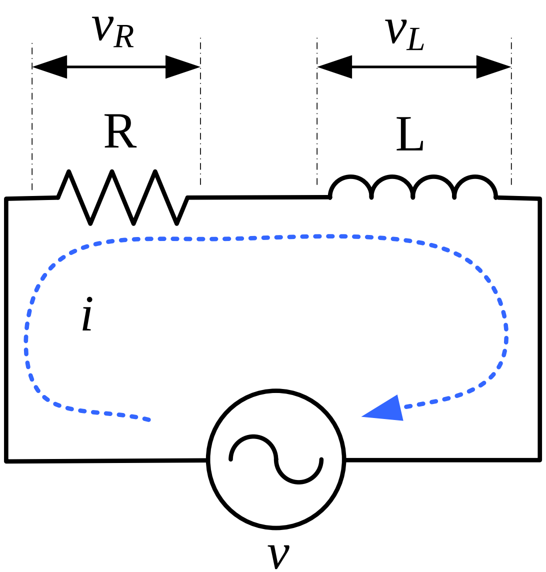
$$p = \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)\right]$$

$$\boxed{p = \frac{V_m I_m}{2}\left[\cos(\phi) - \cos(2\omega t - \phi)\right]}$$



As earlier, the instantaneous power is an alternating quantity, but it varies at **twice the frequency** of the input voltage signal (*note the 2ω term*).

AC circuit operation with R + L



$$\boxed{p = \frac{V_m I_m}{2}\left[\cos(\phi) - \cos(2\omega t - \phi)\right]}$$

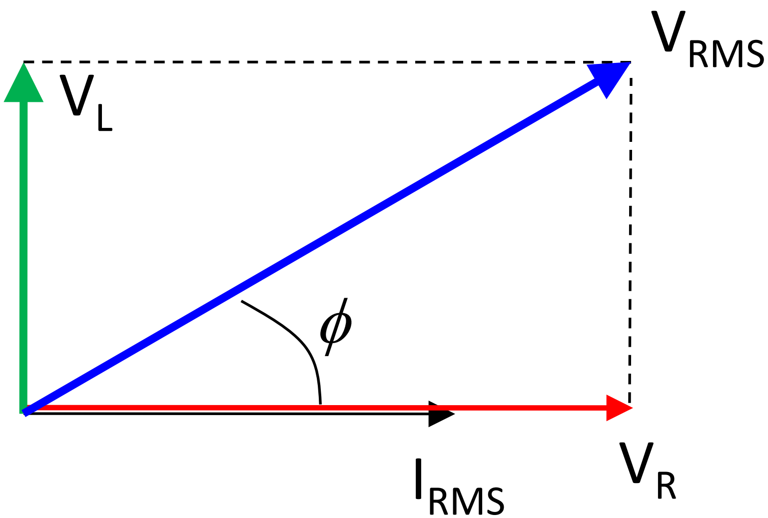
<u>**Average power**</u>

$$P = \frac{1}{T}\int_0^T p\,dt$$

$$P = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\left[\cos(\phi) - \cos(2\omega t - \phi)\right]dt$$

$$P = \frac{V_m I_m}{2T}\int_0^T \left[\cos\phi - \cos\left(2\frac{2\pi}{T}t - \phi\right)\right]dt$$

$$P = \frac{V_m I_m}{2T}\left[\cos\phi t - \frac{T}{4\pi}\sin\left(\frac{4\pi}{T}t - \phi\right)\right]\Bigg|_0^T$$

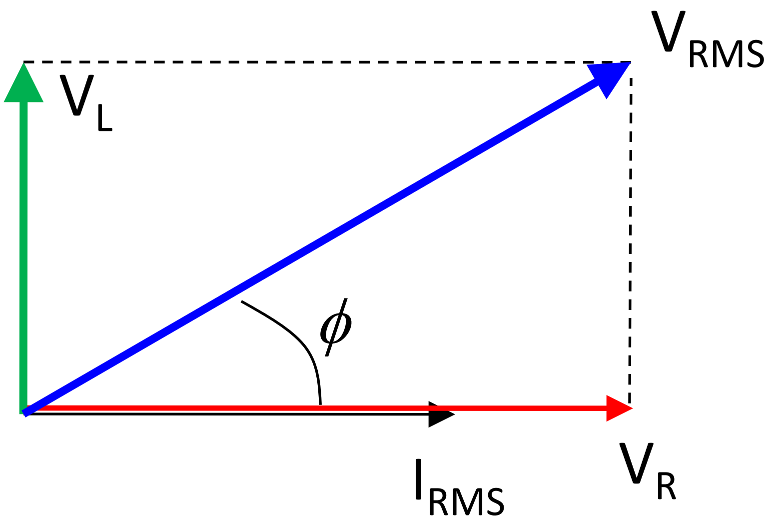


AC circuit operation with R + L

$$P=\frac{V_m I_m}{2T}\left[\cos\phi t-\frac{T}{4\pi}\sin\left(\frac{4\pi}{T}t-\phi\right)\right]\Bigg|_0^T$$

$$P=\frac{V_m I_m}{2T}\left[\left(\cos\phi\times T-\cos\phi\times 0\right)-\left\{\frac{T}{4\pi}\sin\left(\frac{4\pi}{T}T-\phi\right)-\frac{T}{4\pi}\sin\left(\frac{4\pi}{T}\times 0-\phi\right)\right\}\right]$$

$$P=\frac{V_m I_m}{2T}\left[\left(T\cos\phi\right)-\left\{\frac{T}{4\pi}\sin\left(4\pi-\phi\right)-\frac{T}{4\pi}\sin\left(-\phi\right)\right\}\right]$$



$$P=\frac{V_m I_m}{2T}\left[\left(T\cos\phi\right)-\left\{\frac{T}{4\pi}\sin\left(-\phi\right)-\frac{T}{4\pi}\sin s\left(-\phi\right)\right\}\right]$$

$$P=\frac{V_m I_m}{2T}\left(T\cos\phi\right)$$

$$P=\frac{V_m I_m}{2}\cos\phi$$

AC circuit operation with R + L

$$P = \frac{V_m I_m}{2} \cos\phi$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\boxed{P = V_{RMS} I_{RMS} \cos\phi}$$

Thus, the average active power consumed by a R-L series circuit is the product of RMS values of voltage and current and cosine of the angle between them.

The quantity *cos*ϕ is called **power factor** of the circuit

$$\boxed{\text{Power factor, } \cos\phi = \frac{P}{V_{RMS} I_{RMS}}}$$