Electromagnetism

Day 21

ILOs – Day 21

- State and explain the laws of electromagnetic induction
- Explain self-induced EMF and derive expression for self-inductance
- Explain mutually-induced EMF and derive expression for mutual-inductance
- Derive mathematical relations for magnetically coupled circuits

Electromagnetic Induction

- Changing flux linking with an electric circuit
- Basic phenomena observed in all electric machines

Laws of Electromagnetic Induction

Faraday's first law:

- **Whenever there is change in magnetic flux linking with a conductor, an EMF is induced in the conductor**

  - If the circuit is closed, then the induced EMF will circulate current through the closed circuit
  - This phenomenon is known as electromagnetic induction

The first law of electromagnetic induction gives a qualitative description of the EMI process

Laws of Electromagnetic Induction

Faraday's second law:

- **The amount of EMF induced in the conductor is proportional to the rate of change of flux linking with the conductor**

If, e = induced EMF

ϕ = magnetic flux

N = Number of turns in the coil

ψ = Flux linkage = $N\phi$

t = Time

$$e \infty \frac{d\psi}{dt}$$

$$e \infty \frac{d\left(N\phi\right)}{dt}$$

$$\boxed{e \infty N \frac{d\phi}{dt}}$$

Laws of Electromagnetic Induction

Faraday's second law:

$$\boxed{e \propto N \frac{d\phi}{dt}}$$

- Direction (polarity) of the induced EMF is given by the third law of electromagnetic induction or to be more precise, by the **Lenz's law.**

Laws of Electromagnetic Induction

$$\boxed{e \propto N \frac{d\phi}{dt}}$$

Lenz's law:

- **The direction of induced EMF is such that the current set up by it produces a magnetic field that opposes the change in original flux**

  - In general, Lenz's law states that *the effect always opposes the cause*.
  - The phenomena explained by Lenz's law can be incorporated by making (-1) as the constant of proportionality.
  - Thus final expression of EMF induced due to electromagnetic induction can be written as:

$$\boxed{e = -N \frac{d\phi}{dt}}$$

Laws of Electromagnetic Induction

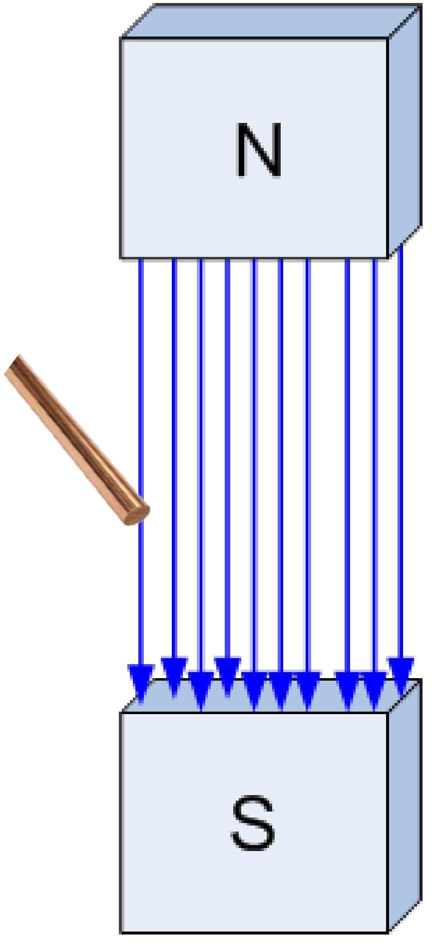
$$\boxed{e = -N\frac{d\phi}{dt}}$$

- Thus, whenever there is a relative change between magnetic flux and a coil, EMF is induced in the coil.

- There are several ways in which such relative change can be achieved:
  - Dynamically induced EMF
  - Statically induced EMF

Dynamically induced EMF

- When the magnetic field is stationery and constant
- But the conductor physically moves in the magnetic field
- Then EMF induced in the conductor due to change in flux linkage is called ***dynamically induced EMF***

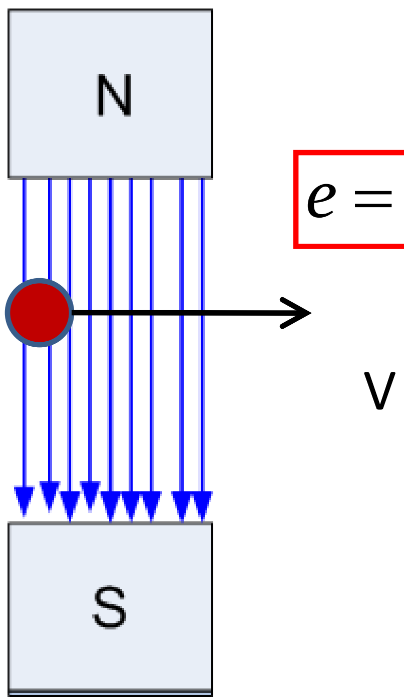


Dynamically induced EMF

- Flux density = B
- Length of the conductor = L
- Velocity of the conductor = V

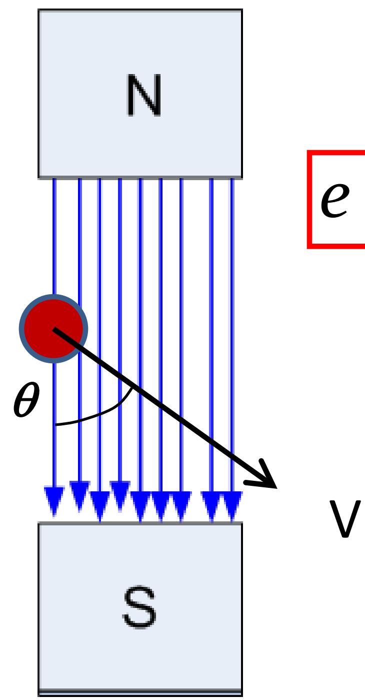
- If the conductor moves perpendicular to the flux lines, then dynamically induced EMF:

$$e = BLV$$



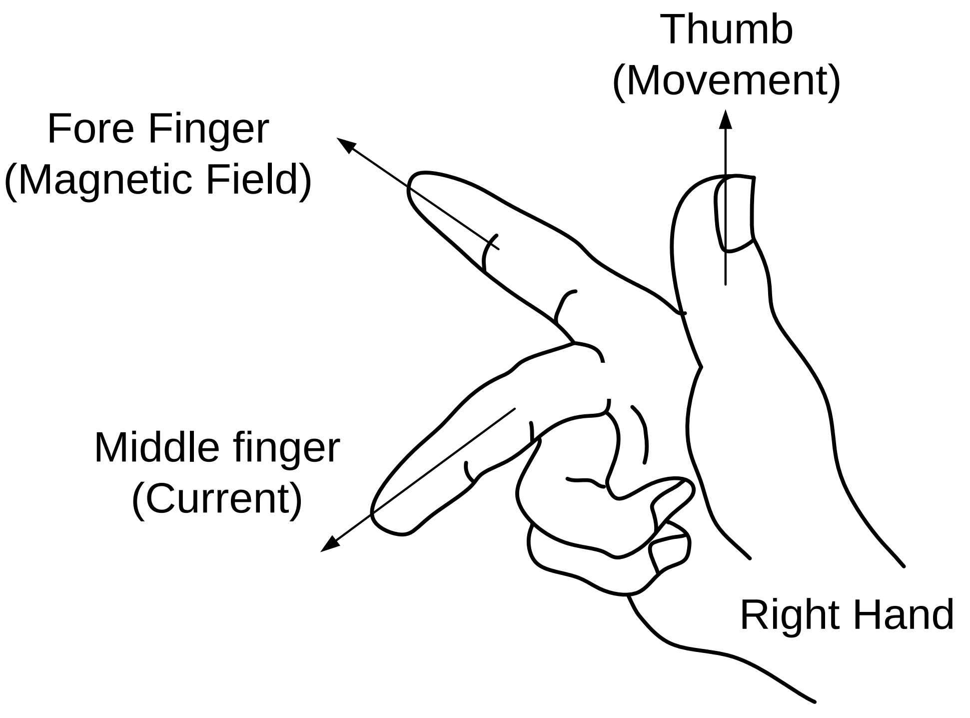
- If the conductor moves at an angle to the flux lines, then dynamically induced EMF:

$$e = BLV \sin \theta$$

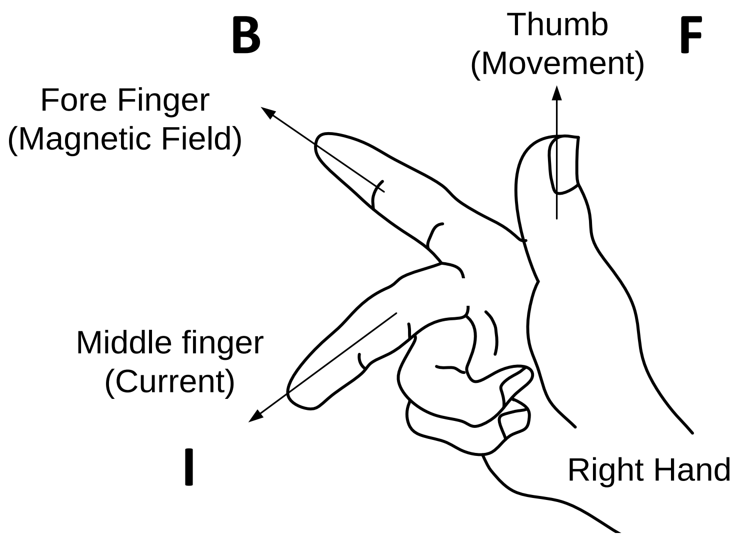


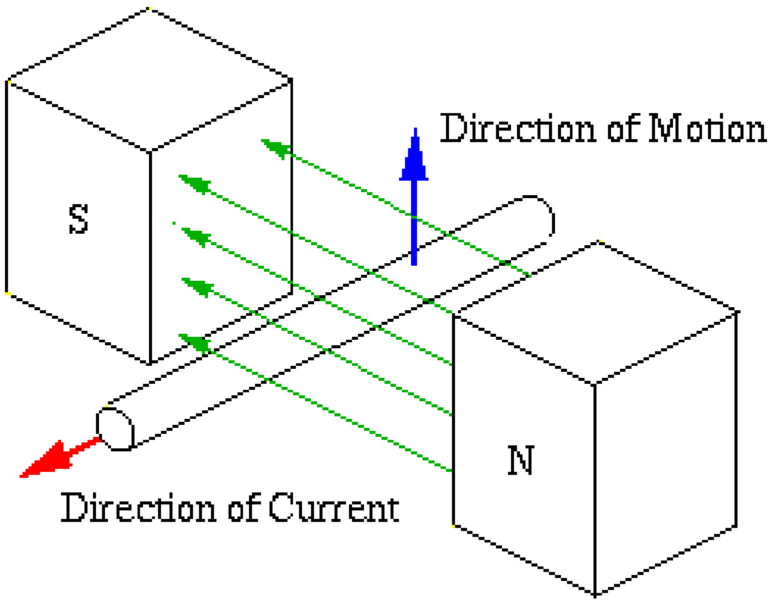
Dynamically induced EMF

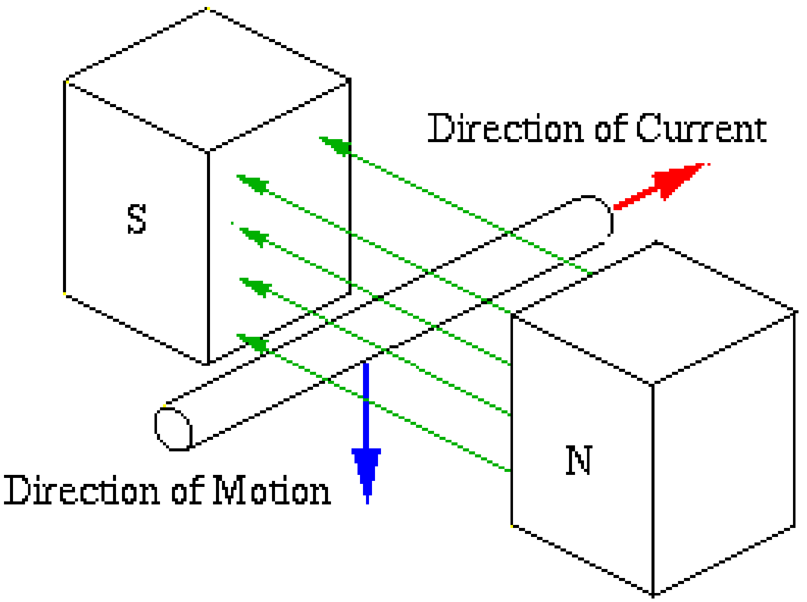
- Direction (polarity) of the induced EMF can be found out using Fleming’s right hand rule.



Dynamically induced EMF



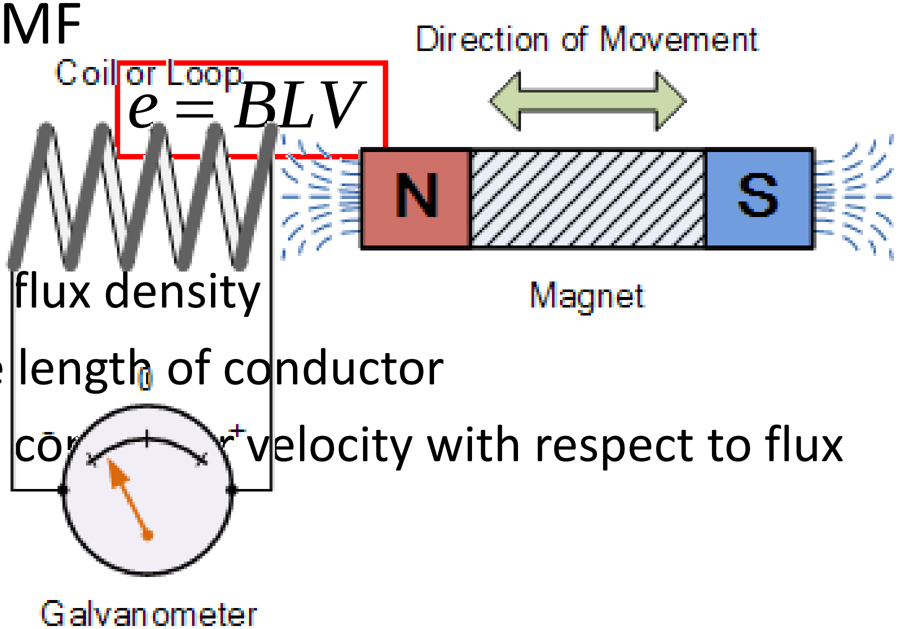




Dynamically induced EMF

- The second way of dynamically induced EMF is:
  - The conductor remains stationary, but the magnet (flux) moves physically
  - Then there will be relative motion between the conductor and magnetic flux
  - Thus, EMF will be induce in the conductor
  - Same mathematical relation holds good for dynamically induced EMF

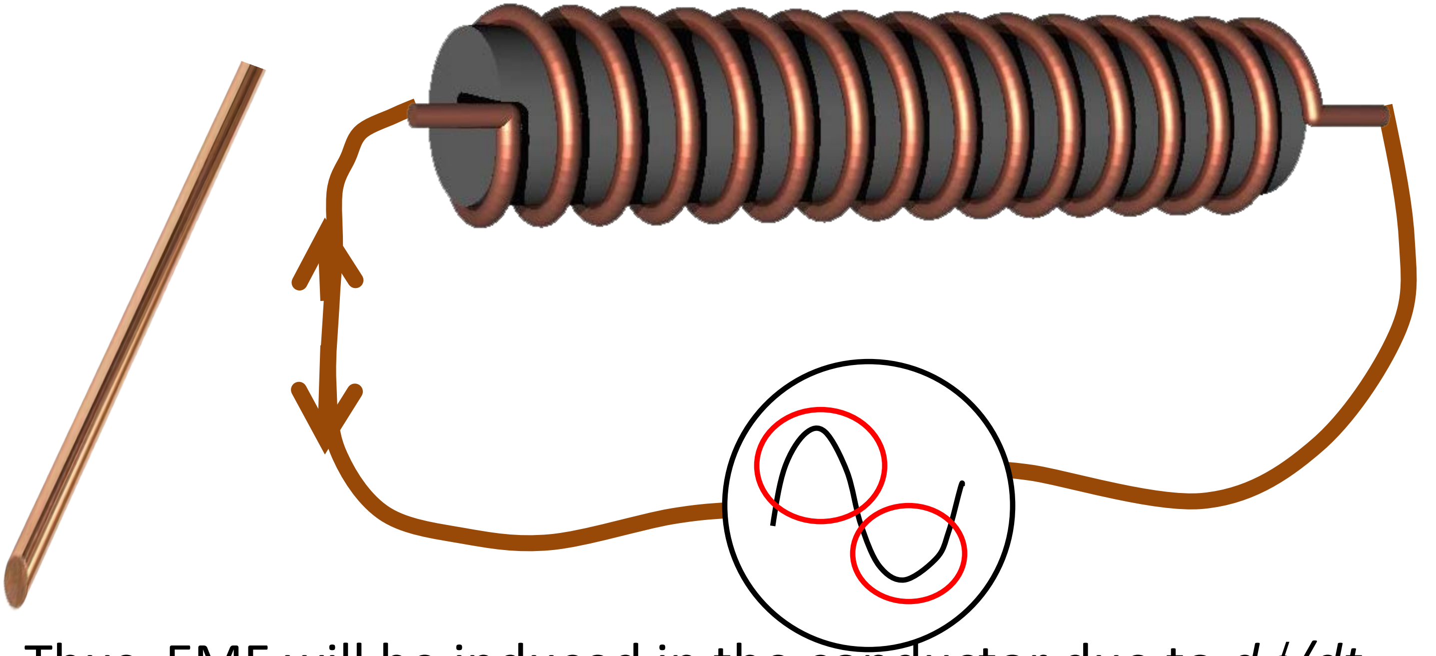
$$e = BLV$$



- Where:
  - *B* is the flux density
  - *L* is the length of conductor
  - *V* is the conductor velocity with respect to flux

Statically induced EMF

- There is a way when an EMF can be induced in a conductor while both the magnet as well as the magnet is stationary, but the supply current is alternating:
  - In such a case, there will still be change in the flux linking with the conductor due to the time varying flux

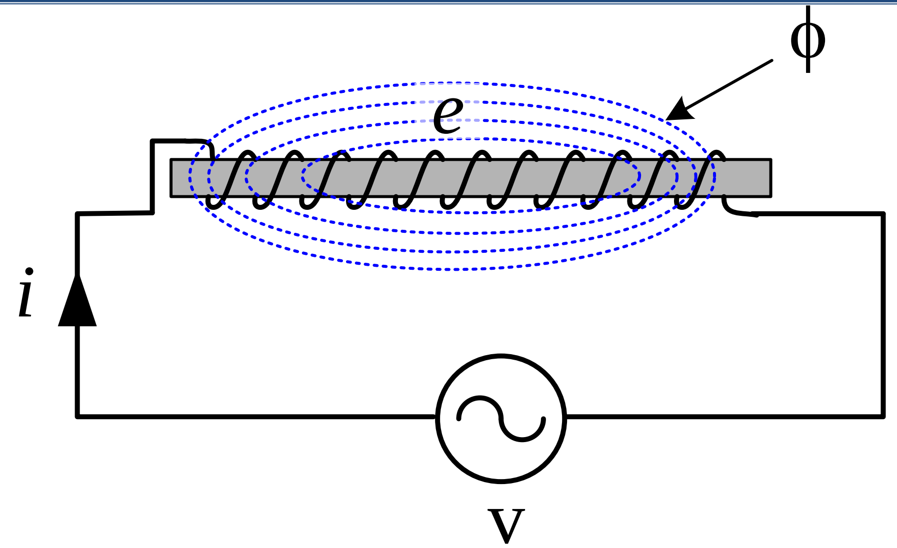


Thus, EMF will be induced in the conductor due to $d\phi/dt$

a) self-induced EMF

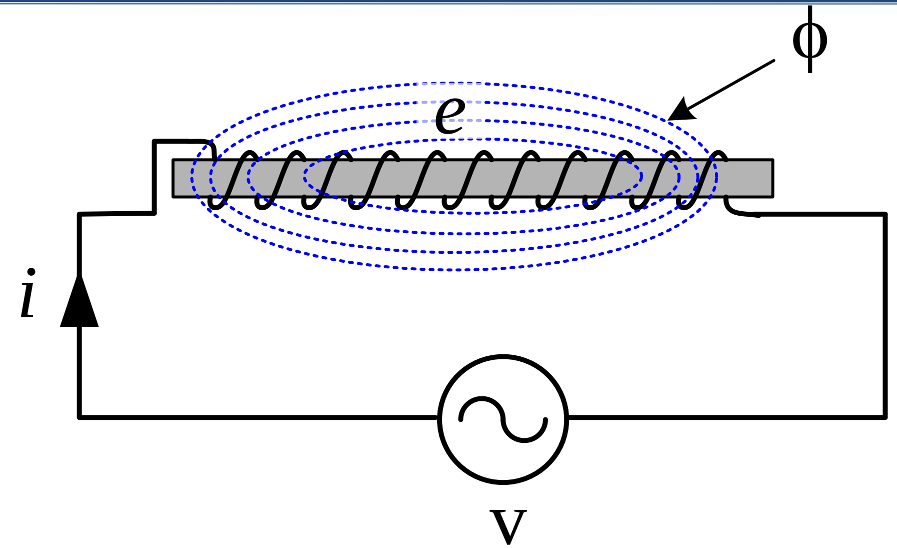
b) mutually induced EMF

Self induced EMF



- A coil is supplied from an AC voltage source
- Magnetic flux produced by the coil thus varies with time
- This flux links with the coil also
- Since this flux is varying with time, an EMF will be induced in the coil itself
- This EMF is called *self-induced EMF*
- Direction of this EMF will be such that it opposes the supply voltage (Lenz's law)

Self induced EMF

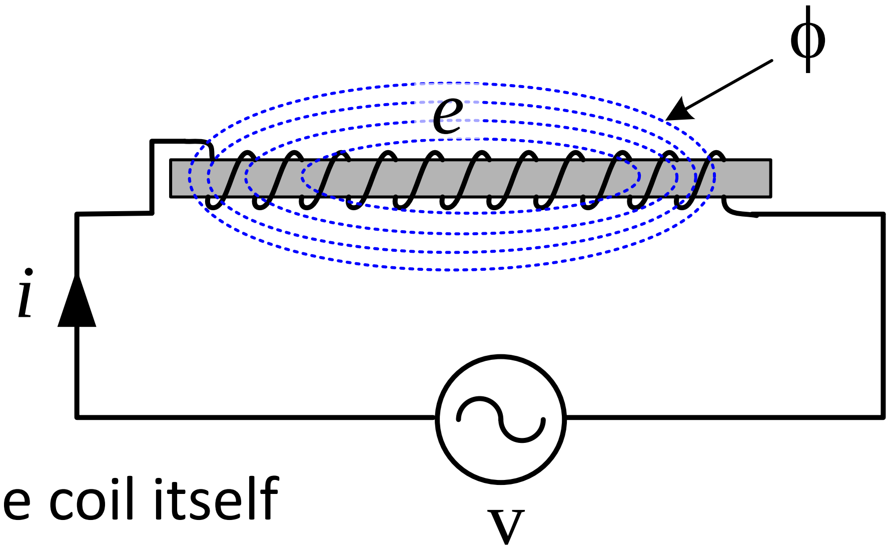


- Let number of turns in the coil is *N*
- The length of flux path is l
- Area of cross section of coil is a
- When the coil carries a current i , the magnetic flux produced by the coil is given by:

$$\phi = \frac{MMF}{\text{Reluctance}} = \frac{Ni}{S} = \frac{Ni}{\frac{1}{\mu_0 \mu_r}\frac{l}{a}} = \frac{Ni\mu_0\mu_r a}{l}$$

Self induced EMF

$$\boxed{\phi = \frac{Ni\mu_0\mu_r a}{l}}$$

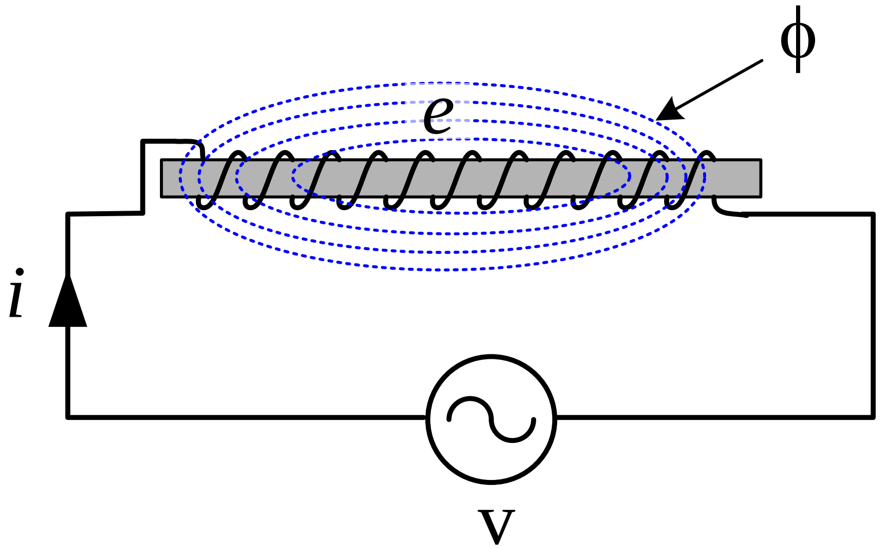


- This flux also links with the coil itself
- When the current *i* flowing through the coil is changed, the flux changes proportionately
- Thus, an EMF is induced in the coil due to its link with the changing flux
- This self-induced EMF is expressed as:

$$e = -\frac{Nd\phi}{dt} = -N\frac{d}{dt}\left(\frac{Ni\mu_0\mu_r a}{l}\right) = -\frac{N^2\mu_0\mu_r a}{l}\left(\frac{di}{dt}\right)$$

Self induced EMF

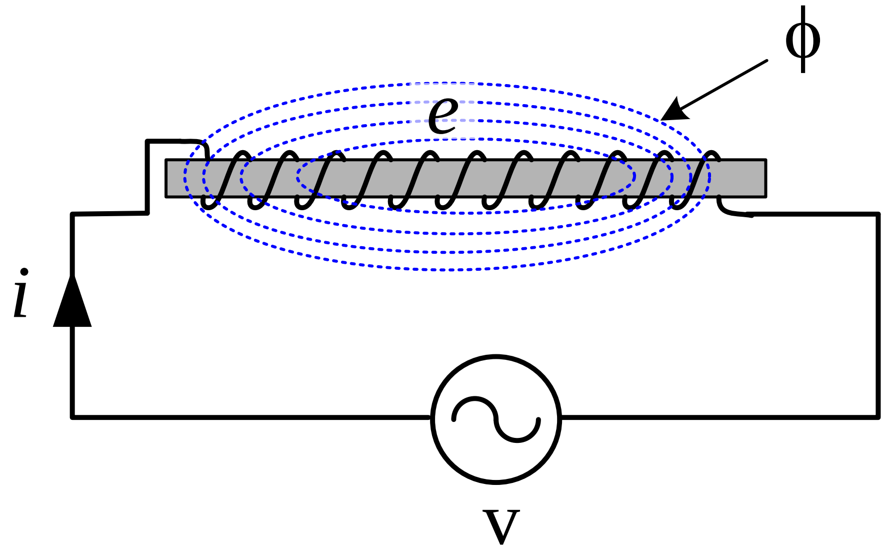
$$e = -\frac{N^2 \mu_0 \mu_r a}{l}\left(\frac{di}{dt}\right)$$



- Self-induced EMF in a coil is thus proportional to the rate of change of current in it
- The proportionality constant is $\dfrac{N^2 \mu_0 \mu_r a}{l}$
- This constant is fixed for a given coil
- It is called the **coefficient of self-induction** or simply **self-inductance** of the coil
- It is symbolized by the letter *L* and unit is Henry (H)

Self induced EMF

$$e = -\frac{N^2 \mu_0 \mu_r a}{l}\left(\frac{di}{dt}\right)$$



- Expression for self-inductance of a coil is thus:

$$L = \frac{N^2 \mu_0 \mu_r a}{l} = \frac{N^2}{\dfrac{l}{\mu_0 \mu_r a}} = \frac{N^2}{S}$$

- Self-induced EMF in a coil is thus: $e = -L\dfrac{di}{dt}$

Different expressions for Self inductance

$$\boxed{L=\frac{N^2\mu_0\mu_r a}{l}=\frac{N^2}{\dfrac{l}{\mu_0\mu_r a}}=\frac{N^2}{S}}\qquad \boxed{e=-L\frac{di}{dt}}$$

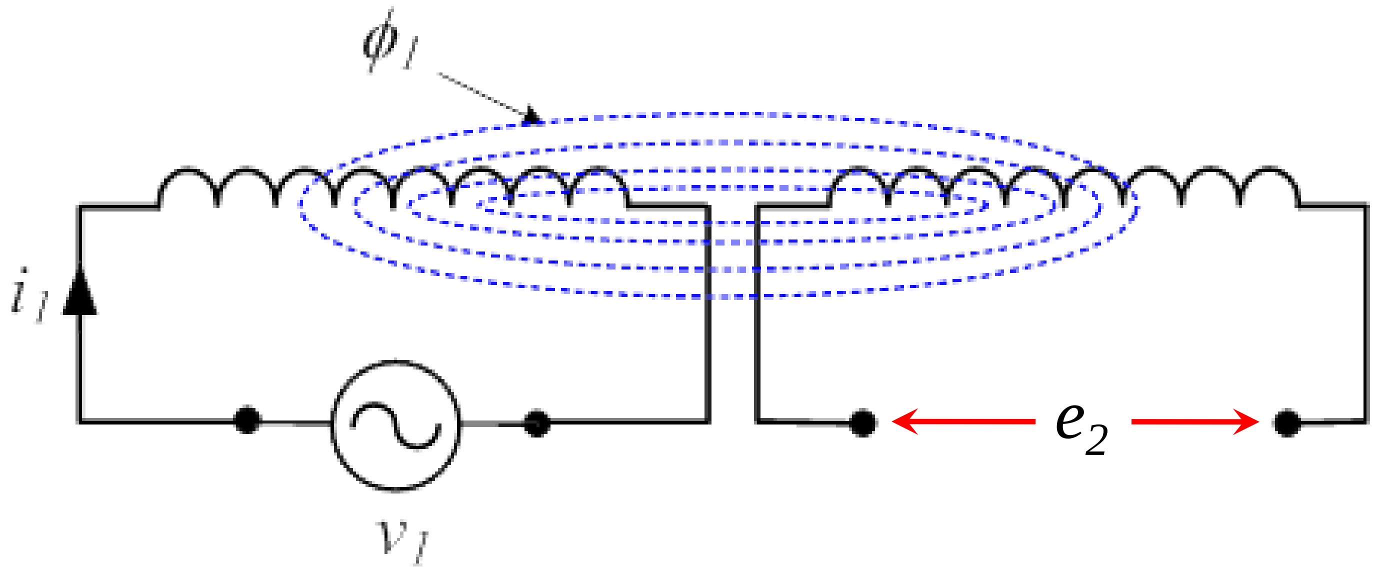
- Alternate expression:

$$e=-\frac{Nd\phi}{dt}=-L\frac{di}{dt}\Rightarrow\frac{Nd\phi}{dt}=L\frac{di}{dt}\Rightarrow\boxed{L=N\frac{d\phi}{di}}$$

- Another expression:

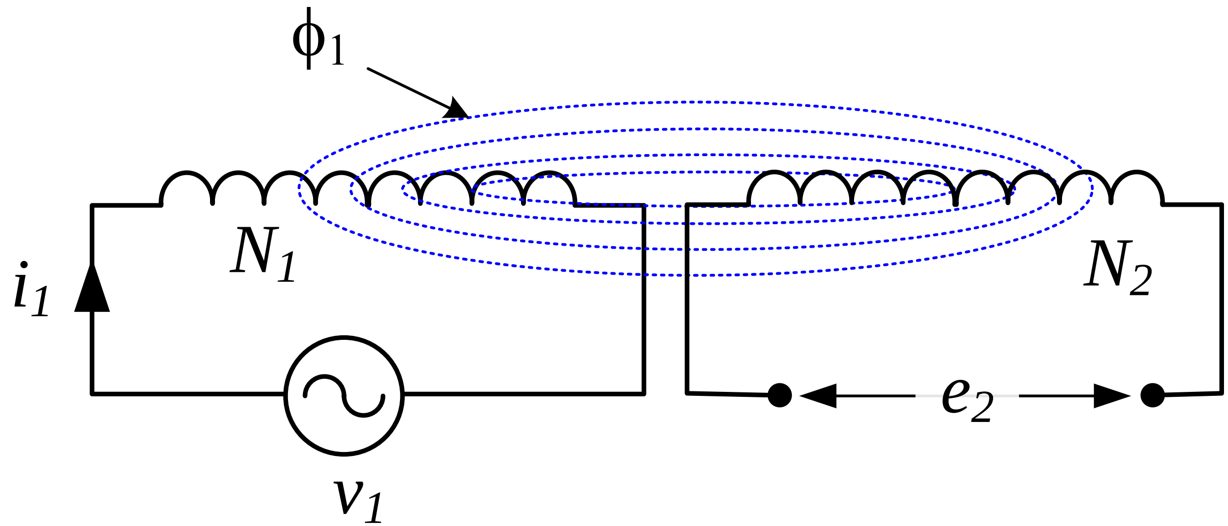
$$\phi=\frac{Ni}{S}=\frac{i}{N}\times\frac{N^2}{S}=\frac{i}{N}L\Rightarrow\boxed{L=N\frac{\phi}{i}}$$

Mutually induced EMF



- Two coils are placed close to each other
- An alternating voltage is applied to the first coil
- Then an alternating flux is produced by the current in the first coil
- Since this time varying flux links with the second coil, an EMF is induced in the second coil
- This EMF is called mutually induced EMF

Mutually induced EMF

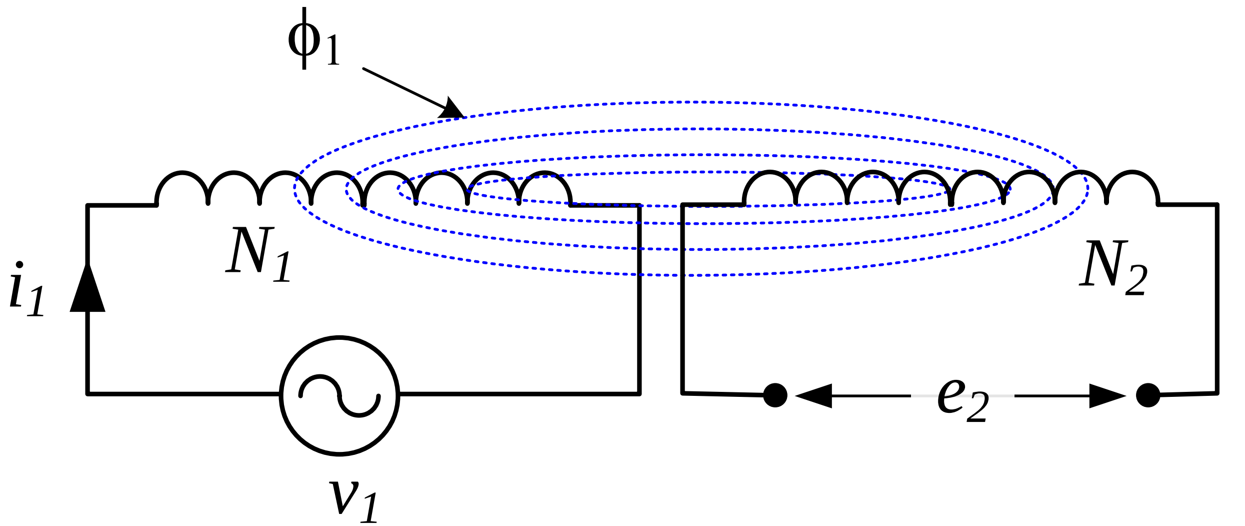


- Let number of turns in the first coil is N_1
- The length of flux path is l and area of cross section a
- When this coil carries a current i_1 , the magnetic flux produced by it is given by:

$$\phi_1 = \frac{MMF}{\text{Reluctance}} = \frac{N_1 i_1}{S} = \frac{N_1 i_1}{\frac{1}{\mu_0 \mu_r} \frac{l}{a}} = \frac{N_1 i_1 \mu_0 \mu_r a}{l}$$

Mutually induced EMF

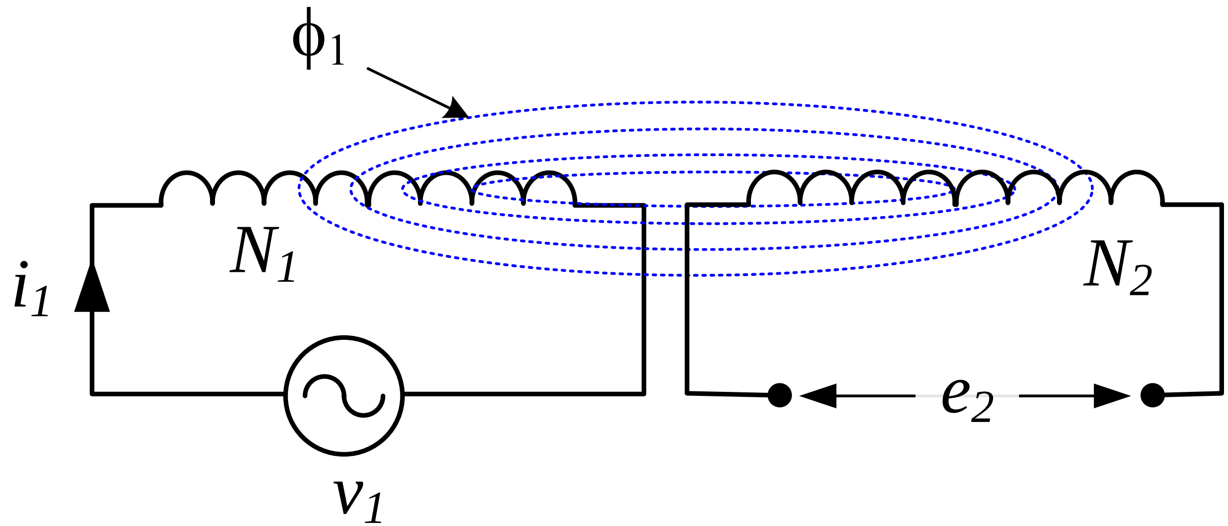
$$\boxed{\phi_1 = \frac{N_1 i_1 \mu_0 \mu_r a}{l}}$$



- We assume that whole of this flux produced by the first coil links with the second coil that is placed close enough to the first coil

Mutually induced EMF

$$\boxed{\phi_1 = \frac{N_1 i_1 \mu_0 \mu_r a}{l}}$$

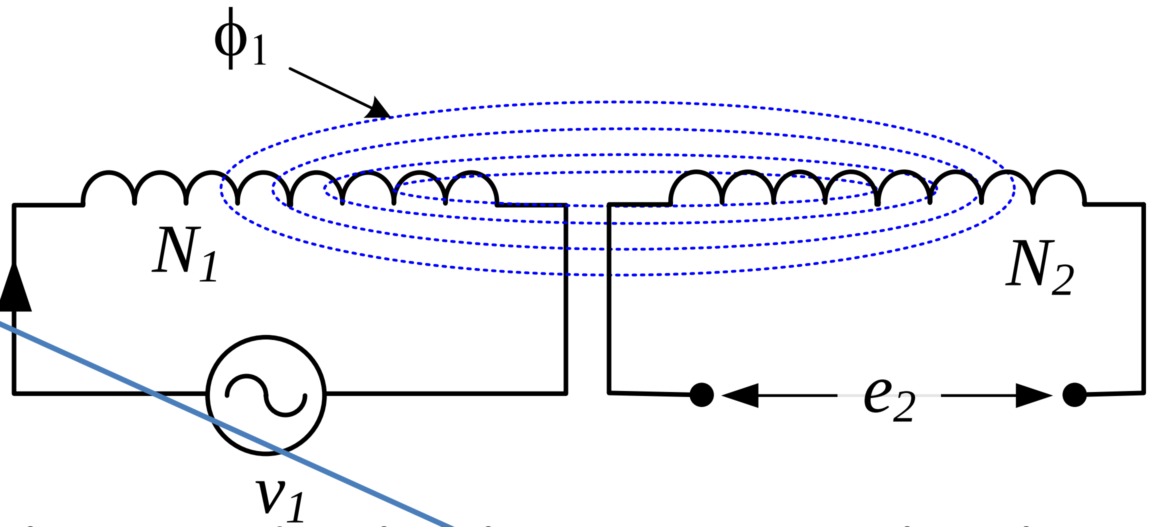


- When the current i_1 flowing through the first coil is changed, its flux changes proportionately
- Thus, an EMF is induced in the second coil due to its linkage with the changing flux from the first coil
- If the second coil has N_2 number of turns, then this ***mutually induced EMF*** is expressed as:

$$e_2 = -N_2 \frac{d\phi_1}{dt} = -N_2 \frac{d}{dt}\left(\frac{N_1 i_1 \mu_0 \mu_r a}{l}\right) = -\frac{N_1 N_2 \mu_0 \mu_r a}{l}\left(\frac{di_1}{dt}\right)$$

Mutually induced EMF

$$\boxed{e_2 = -\frac{N_1 N_2 \mu_0 \mu_r a}{l}\left(\frac{di_1}{dt}\right)}$$



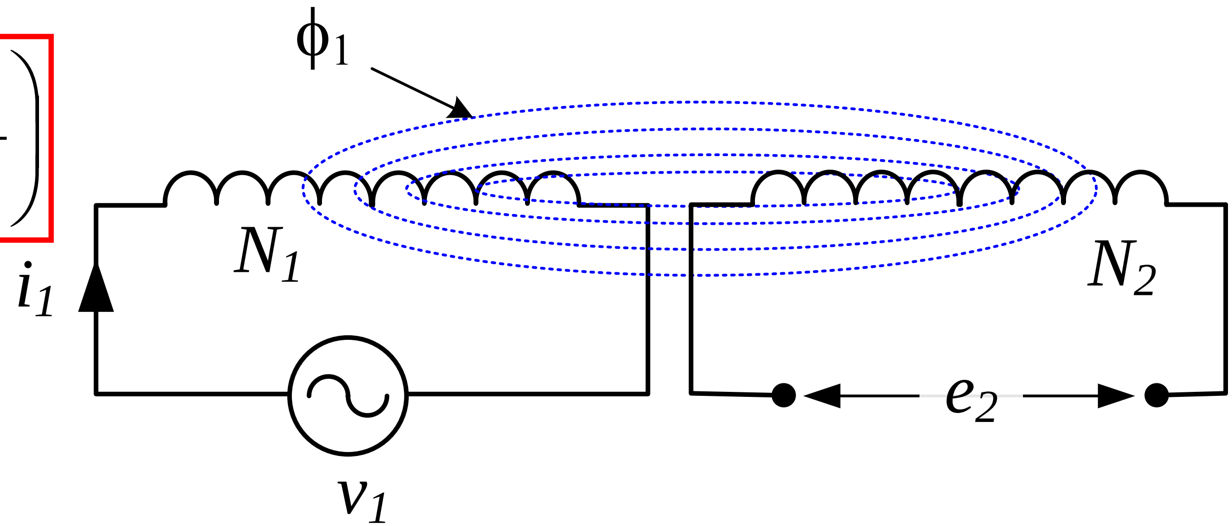
- Mutually induced EMF in the second coil is thus proportional to the rate of change of current in the first coil
- The proportionality constant is

$$\frac{N_1 N_2 \mu_0 \mu_r a}{l}$$

- This constant is fixed for a given pair of coils
- It is called the **coefficient of mutual-induction** or simply **mutual - inductance** between the two coils
- It is symbolized by the letter ***M*** and unit is Henry (H)

Mutually induced EMF

$$\boxed{e_2 = -\frac{N_1 N_2 \mu_0 \mu_r a}{l}\left(\frac{di_1}{dt}\right)}$$



- Expression for mutual-inductance between two coils is thus:

$$\boxed{M = \frac{N_1 N_2 \mu_0 \mu_r a}{l} = \frac{N_1 N_2}{\dfrac{l}{\mu_0 \mu_r a}} = \frac{N_1 N_2}{S}}$$

- Mutually-induced EMF in the second coil is thus: $\boxed{e_2 = -M\frac{di_1}{dt}}$

Alternate expression for Mutual inductance

$$\boxed{M = \frac{N_1 N_2 \mu_0 \mu_r a}{l} = \frac{N_1 N_2}{\dfrac{l}{\mu_0 \mu_r a}} = \frac{N_1 N_2}{S}}$$

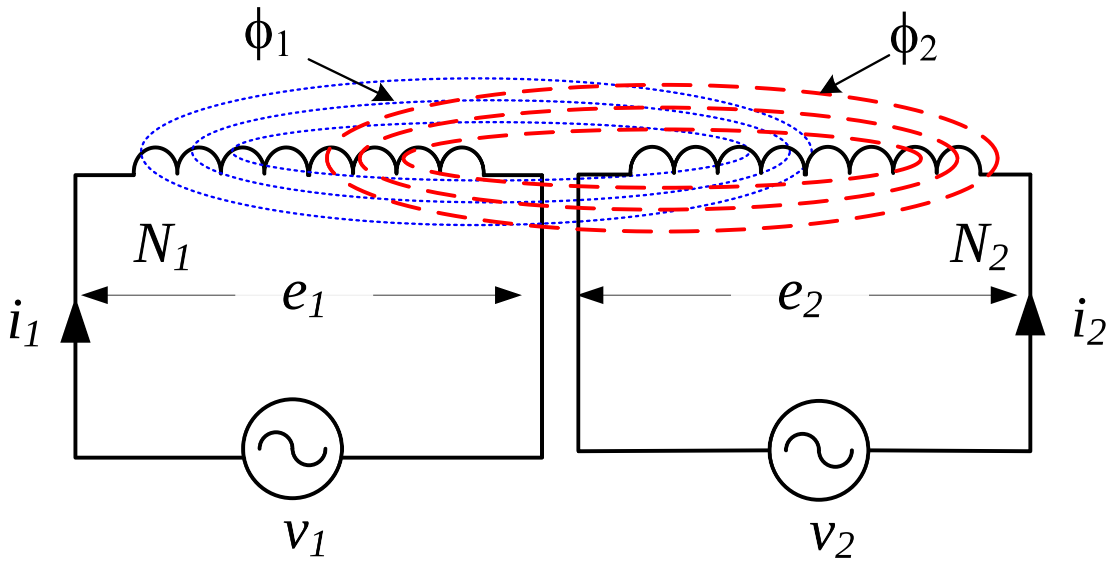
- Alternate expression:

$$\phi_1 = \frac{N_1 i_1}{S} = \frac{1}{N_2} \times \frac{N_2 N_1}{S} \times i_1 = \frac{i_1}{N_2} M \Rightarrow \boxed{M = N_2 \frac{\phi_1}{i_1}}$$

The action of mutual induction is reversible, i.e. any change in current in the second coil will also cause similar EMF to be induced in the first coil.

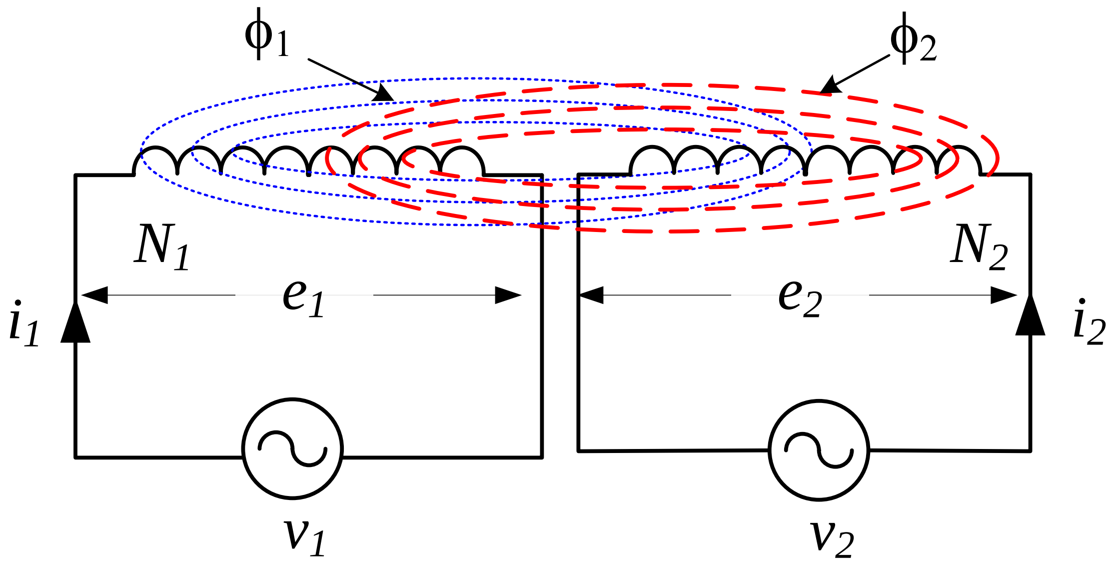
$$\boxed{M = N_1 \frac{\phi_2}{i_2}}$$

Coupled circuits



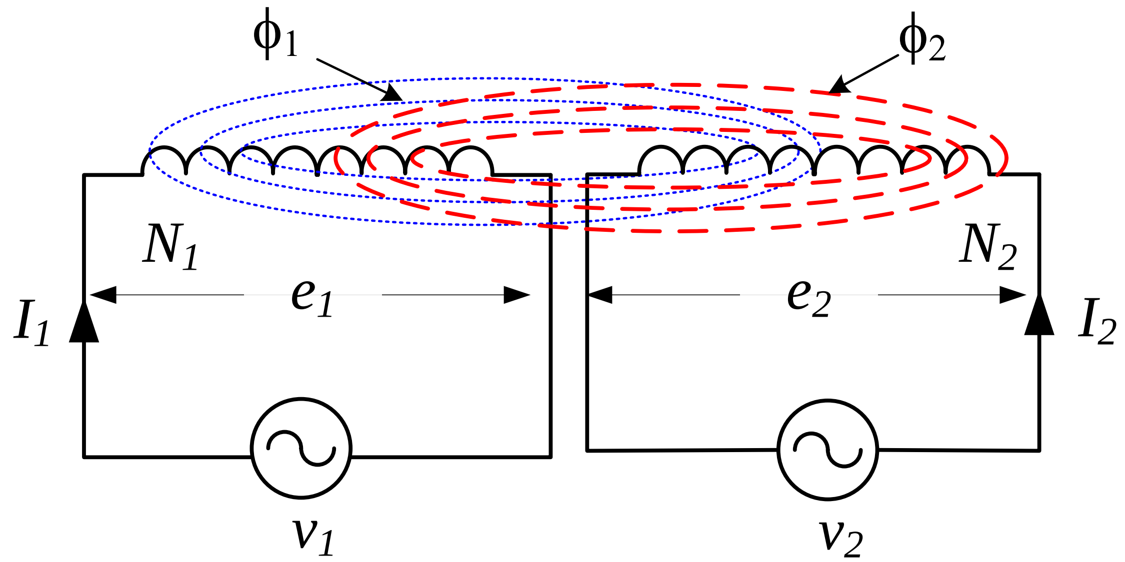
- When the second circuit is closed, so that current flows through it, the second coil produces another flux
- In general, if two or more coils are placed close to each other so that magnetic flux produced by each of them can link with the other
- Those inter-linked (magnetically) circuits are **called *coupled circuits***

Coupled circuits



- Flux produced by the first coil will not only link with its own turns, but will also link with turns of the second coil
- Similarly, flux produced by the second coil will link with its own turns and turns of the first coil as well
- Thus, each of the coils will have two types of EMF induced in them, one due to linkage with its own flux and the other due to linkage with flux of the other coil
- Thus, each of the coils will have both **self** as well as **mutually induced EMF** in them

Coupled circuits



- Total EMF induced in the two coils can hence be expressed respectively as:

$$e_1 = -\left(L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} \right)$$

$$e_2 = -\left(L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} \right)$$

Coefficient of coupling

- So far, we have assumed that flux produced by one coil completely links with the other coil
- What will happen if not the entire flux, but say a fraction *K* of the total flux coming out of the first coil links with the second coil?
- The quantity *K* is thus always less than unity, and is called the ***coefficient of coupling*** between the two coils.

- Expressions for self-inductance of the two coils remain as usual:

$$L_1 = \frac{{N_1}^2}{S} \qquad L_2 = \frac{{N_2}^2}{S}$$

Coefficient of coupling

$$\boxed{L_1 = \frac{{N_1}^2}{S}} \qquad \boxed{L_2 = \frac{{N_2}^2}{S}}$$

- Since the amount of flux from first coil that links with the second coil is $K\phi_1$, then mutual inductance between the two coils can be written as

$$\boxed{M = N_2 \frac{K\phi_1}{I_1} = N_2 K \frac{N_1 I_1}{S} \frac{1}{I_1} = \frac{K N_1 N_2}{S}}$$

- We also have from the L_1 and L_2 expressions:

$$\boxed{L_1 L_2 = \frac{{N_1}^2 {N_2}^2}{S^2}}$$

- Combining these two, we have:

$$\boxed{M = \frac{K N_1 N_2}{S} = K\sqrt{L_1 L_2}}$$

- Thus, the coefficient of coupling:

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

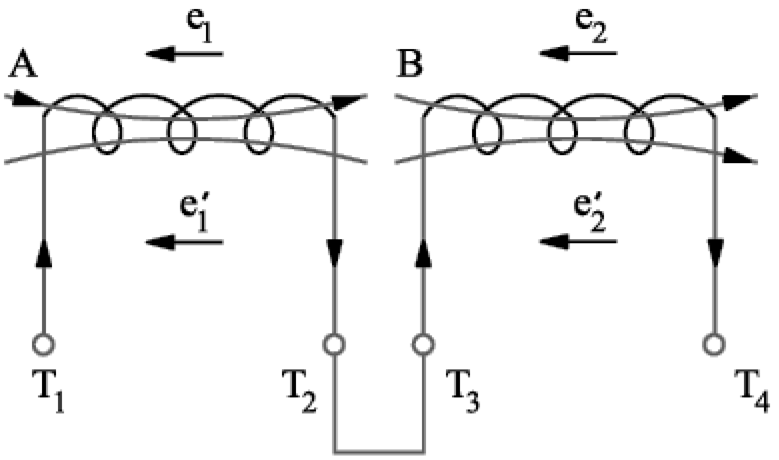
Inductances in series

Let the two coils be so joined in series that their fluxes (or MMFs) are additive *i.e., in the same direction*

Let M = Mutual inductance

L_1 = Self-inductance of 1st coil

L_2 = Self-inductance of 2nd coil



Since the two coils are in series, they carry the same current *i*

Then, self induced e.m.f. in *A is*: $e_1 = -L_1 \frac{di}{dt}$

Mutually-induced e.m.f. in *A due to change of current in B* is: $e_1' = -M \frac{di}{dt}$

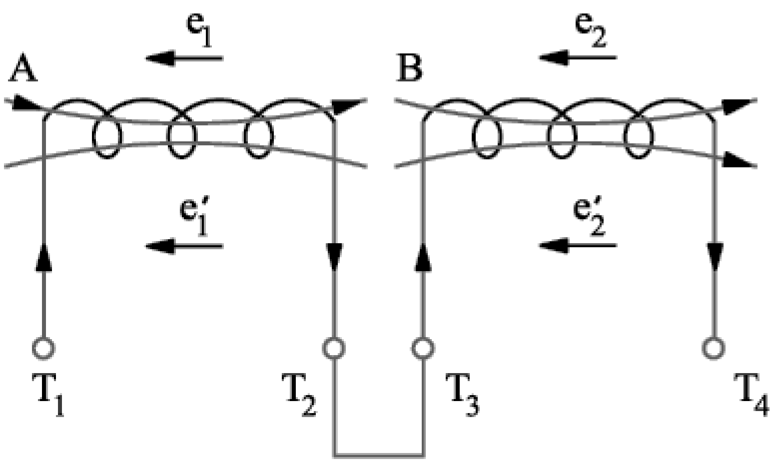
Self induced e.m.f. in *B is*: $e_2 = -L_2 \frac{di}{dt}$

Mutually-induced e.m.f. in *B due to change of current in A* is: $e_2' = -M \frac{di}{dt}$

Inductances in series

$$e_1 = -L_1 \frac{di}{dt} \qquad e_1' = -M \frac{di}{dt}$$

$$e_2 = -L_2 \frac{di}{dt} \qquad e_2' = -M \frac{di}{dt}$$



$\therefore$ Total induced e.m.f. in the combination

$$e_1 + e_1' + e_2 + e_2' = -\left(L_1 + M + L_2 + M\right)\frac{di}{dt}$$

$$= -\left(L_1 + L_2 + 2M\right)\frac{di}{dt}$$

If *L* is the equivalent inductance of the combination, then total induced e.m.f. in that single coil would have been:

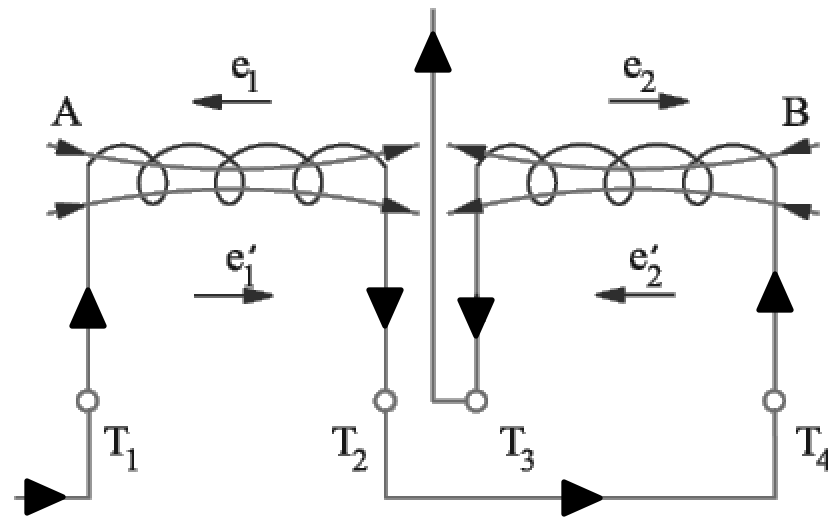
$$e = -L\frac{di}{dt}$$

Thus, we have the relation: $\boxed{L = L_1 + L_2 + 2M}$

Inductances in series (opposite)

When the coils are so joined that their *current directions and hence fluxes are in opposite directions*

Since the two coils are in series, they carry the same current *i*, but in opposite directions



As done earlier:

$$e_1 = -L_1 \frac{di}{dt} \qquad e_2 = -L_2 \frac{di}{dt}$$

The two self induced EMFs both oppose supply voltage and hence are directed to oppose the flow the current.

But since the fluxes in the two coils oppose each other, the mutually induced EMFs will be reversed w.r.t. the self-induced EMFs.

$$e_1' = M \frac{di}{dt} \qquad e_2' = M \frac{di}{dt}$$

Thus, equivalent inductance of the combination is: $\boxed{L = L_1 + L_2 - 2M}$