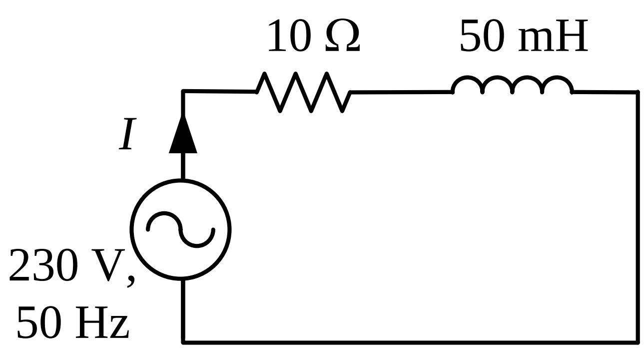
Tutorial 2

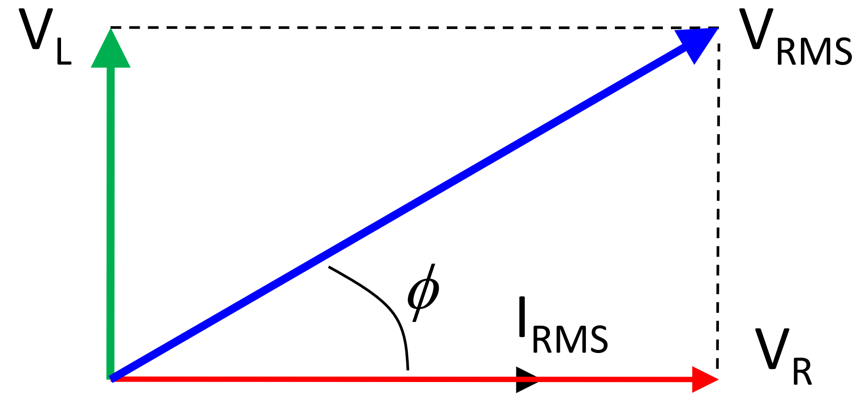
Day 13a: AC Circuit solutions

ILOs – Day 13a (Tutorial 2)

- Solve numerical problems related to AC circuits

#1) A resistance of 10 Ω is connected in series with a 50 mH inductance across a 230 V, 50 Hz supply. Calculate (a) current flowing in the circuit (b) phase angle of the current (c) draw phasor diagram.





$$X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 50 \times 10^{-3} = 15.71\,\Omega$$

$\therefore$ Equivalent impedance of the R-L circuit:

$$Z_1 = \sqrt{R^2 + X_L^{\ 2}} = \sqrt{10^2 + 15.71^2} = 18.62\,\Omega$$

(a) Current in the circuit:

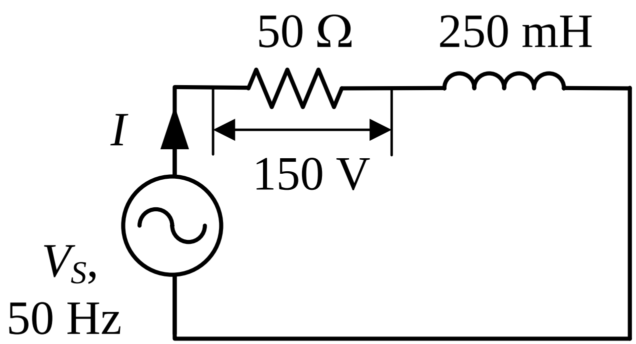
$$I_{RMS} = \frac{V_{RMS}}{Z} = \frac{230}{18.62} = 12.35\ A$$

(b) Phase angle of current, i.e. power factor angle of the circuit

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{15.71}{10}\right)$$

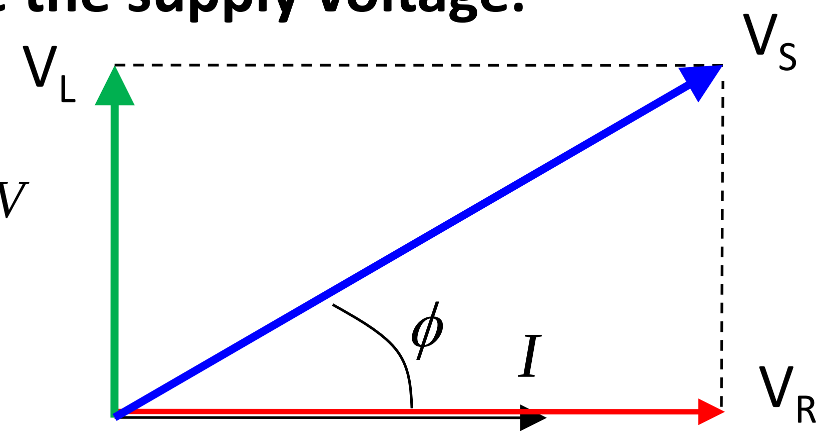
$$= 57.52^0 \text{ (lagging)}$$

#2) A resistance of 50 Ω is connected in series with a 250 mH inductance across a 50 Hz sinusoidal supply. If the voltage across the resistance is 150 V, calculate the supply voltage.



$$V_R = 150\,V$$

$$V_S = ?$$



$$X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 250 \times 10^{-3} = 78.54\,\Omega$$

Phase angle of the circuit:

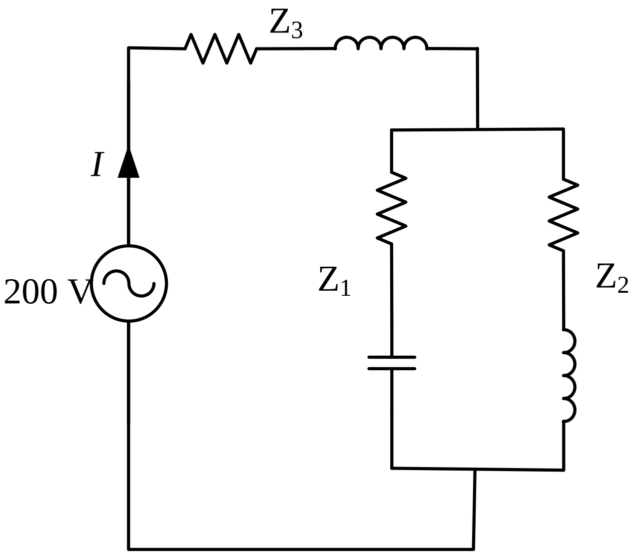
$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{78.54}{50}\right) = 57.52^0$$

Since we have the relation from voltage triangle: $V_R = V_S \cos\phi$

$$\Rightarrow 150 = V_S \cos\left(57.52^0\right)$$

$$\Rightarrow V_S = \frac{150}{\cos\left(57.52^0\right)} = 279.3\,V$$

#3) Find RMS value of source current in the flowing circuit



$$Z_1 = 4 - j10$$

$$Z_2 = 20 + j25$$

$$Z_3 = 10 + j5$$

Phasor addition (and subtraction):
Rectangular form

Phasor multiplication (and division):
Polar form

Equivalent impedance of the circuit is:

$$Z = (Z_1 // Z_2) + Z_3 = \frac{(4 - j10)(20 + j25)}{(4 - j10) + (20 + j25)} + (10 + j5)$$

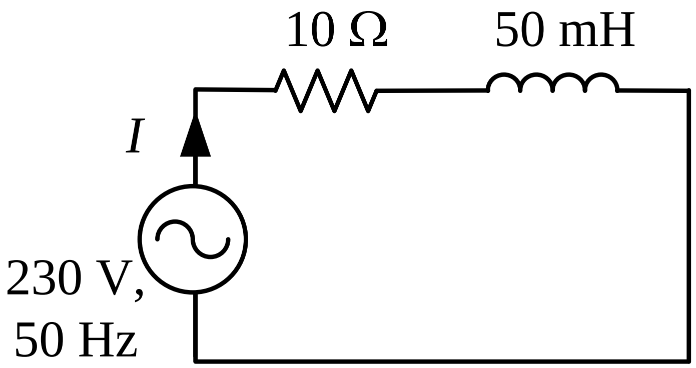
$$Z = (18.02 - j4.18)\,\Omega$$

$$Z = 18.49\angle -13.05^0\ \Omega$$

Thus, current in the circuit is

$$I = \frac{V}{Z} = \frac{200}{18.49} = 10.81\,A$$

#4) A resistance of 10 Ω is connected in series with a 50 mH inductance across a 230 V, 50 Hz supply. Calculate (a) current flowing in the circuit (b) phase angle of the current



Equivalent impedance of the circuit is: $Z = R + jX_L = R + j2\pi fL$

$$Z = 10 + j2\pi 50 \times 50 \times 10^{-3} = 10 + j15.7 = 18.6\angle 57.51^0\ \Omega$$

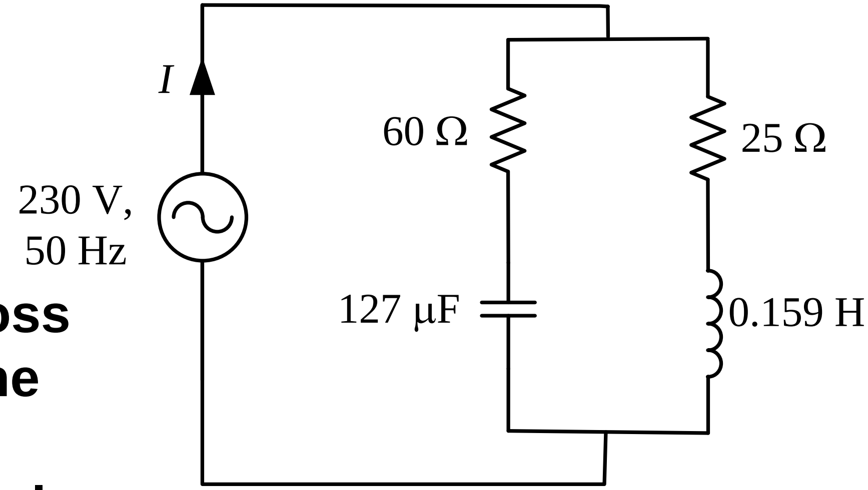
$\therefore$ Current in the circuit taking supply voltage as the reference signal (phase angle = 0^0) is:

$$I = \frac{V}{Z} = \frac{230\angle 0^0}{18.6\angle 57.51^0} = 12.37\angle -57.51^0\ A$$

$\therefore$ Phase angle of the current is -57.51^0

Note that phase angle of the current taking voltage as the reference phasor is same as the phase angle of the equivalent impedance, only with an opposite sign (for inductive circuit)

#5) A coil of resistance 25 Ω and inductance 0.159 H is connected in parallel with a circuit having a 60 Ω resistor and a 127 μF capacitor. This whole combination is connected across a 230 V, 50 Hz supply. Calculate (a) the equivalent circuit impedance, resistance, and reactance (b) the supply current and its phase angle.



Phasor addition:
Rectangular form

Phasor multiplication:
Polar form

Impedance of the R-L path:

$$Z_1 = R + jX_L = R + j2\pi fL = 25 + j2\pi 50 \times 0.159 = 25 + j50\ \Omega$$

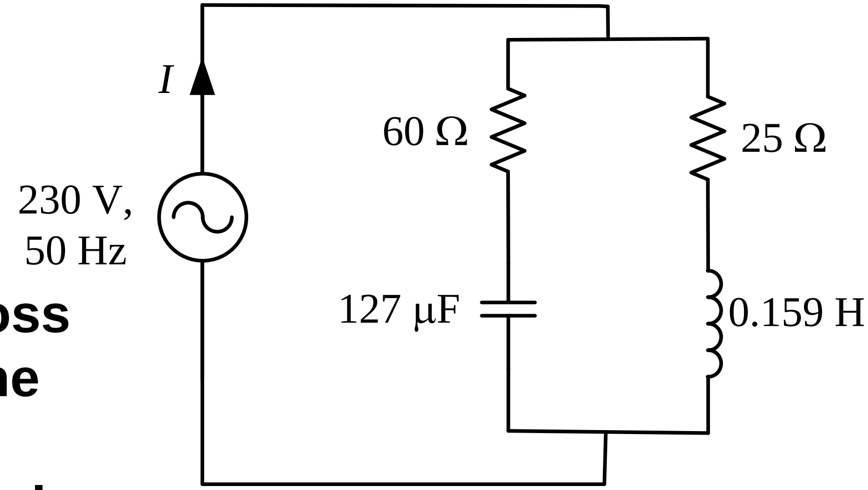
Impedance of the R-C path:

$$Z_2 = R - jX_C = R - j\frac{1}{2\pi fC} = 60 - j\frac{1}{2\pi 50 \times 127 \times 10^{-6}} = 60 - j25\ \Omega$$

These two impedances are in parallel. Thus their equivalent impedance is:

$$Z = Z_1 // Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(25 + j50) \times (60 - j25)}{(25 + j50) + (60 - j25)} = 37.34 + j16.96 = 41\angle 24.43^0\ \Omega$$

A coil of resistance 25 Ω and inductance 0.159 H is connected in parallel with a circuit having a 60 Ω resistor and a 127 μF capacitor. This whole combination is connected across a 230 V, 50 Hz supply. Calculate (a) the equivalent circuit impedance, resistance, and reactance (b) the supply current and its phase angle.



$$Z = 41\angle 24.43^0\ \Omega \qquad Z = R + jX$$

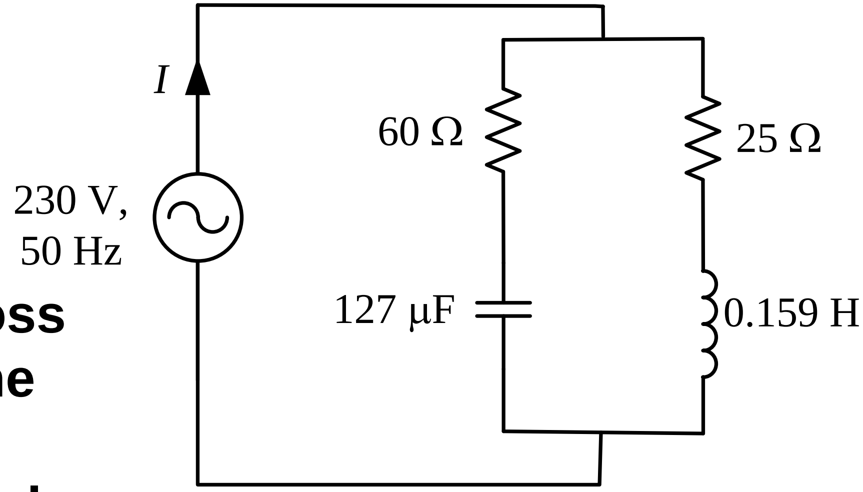
Equivalent resistance is the real part of Z, i.e.

$$R_{eq} = \mathrm{Real}(Z) = \mathrm{Real}(37.34 + j16.96) = 37.34\Omega$$

Equivalent reactance is the imaginary part of Z, i.e.

$$X_{eq} = \mathrm{Imag}(Z) = \mathrm{Imag}(37.34 + j16.96) = 16.96\Omega$$

A coil of resistance 25 Ω and inductance 0.159 H is connected in parallel with a circuit having a 60 Ω resistor and a 127 μF capacitor. This whole combination is connected across a 230 V, 50 Hz supply. Calculate (a) the equivalent circuit impedance, resistance, and reactance (b) the supply current and its phase angle.



$$Z = 41\angle 24.43^0\ \Omega \qquad\qquad Z = R + jX$$

Current in the circuit taking supply voltage as the reference signal (phase angle = 0^0) is:

$$I = \frac{V}{Z} = \frac{230\angle 0^0}{41\angle 24.43^0} = 5.61\angle -24.43^0\ A$$

Phase angle of the current is: -24.43^0