3-Phase systems

Day 28

Unbalanced 3-phase systems
Tutorial 1

ILOs – Day 28

- Define and characterize unbalanced 3-phase systems
- Calculate for an unbalanced 3-phase system
  - Phase currents
  - Line currents
  - Apparent power
  - Active power
  - Reactive power
  - Power factor

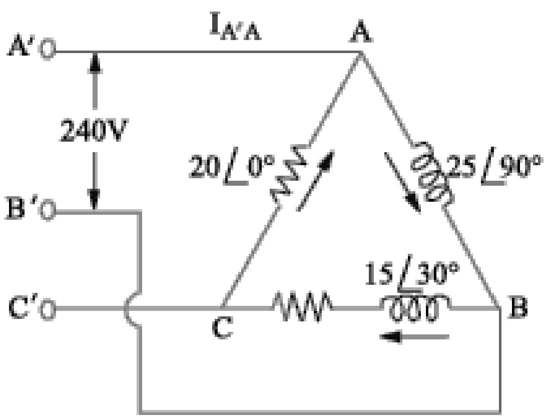
Unbalanced 3-phase system

- A 3-phase system is said to be unbalanced when:
  - The three voltages are unbalanced
    - Magnitudes may be different
    - Phase angle difference may not be 120^0
  - The three currents are unbalanced
    - Magnitudes may be different
    - Phase angle difference may not be 120^0
- By definition, an unbalanced circuit has at least one phase current that is not equal to the other phase currents
- Of course, all three phase currents could be of unequal magnitude
- Such unbalances may be in the
  - Supply system
  - Load
- Unbalances may be due to
  - Naturally occurring unbalances in load
  - Faults and disturbances in the power system

Unbalanced 3-phase system

Let us analyze such an unbalanced system using an example

A 3-phase, 3-wire, 240 volt, CBA system supplies a delta-connected load in which $Z_{AB} = 25 \angle 90^0$, $Z_{BC} = 15 \angle 30°$, $Z_{CA} = 20 \angle 0^0$ ohms. Find the line currents and total power.



In C-B-A phase sequence (which can be written as C-B-A-C-B-A), if we take V_{AB} as the reference, then V_{CA} lags behind V_{AB} by 120^0, and V_{BC} lags behind V_{AB} by 240^0.

Thus, $V_{AB} = V \angle 0^0$, $V_{CA} = V \angle -120^0$, $V_{BC} = V\angle -240^0 = V\angle +120^0$

The phase currents are calculated as:

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{240\angle 0^0}{25\angle 90^0} = 9.6\angle -90^0 = 0 - j9\ A$$

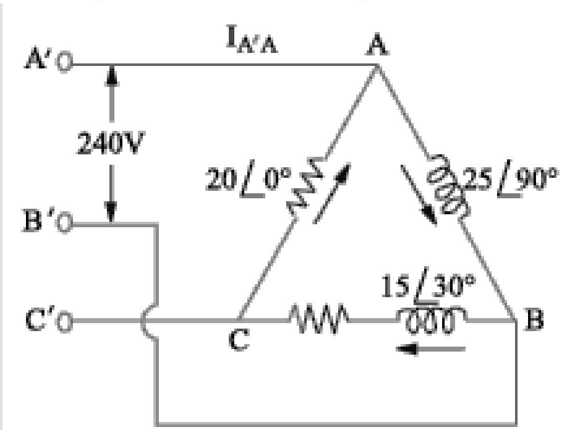
$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{240\angle 120^0}{15\angle 30^0} = 16\angle 90^0 = 0 + j16\ A$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{240\angle -120^0}{20\angle 0^0} = 12\angle -120^0 = -6 - j10.4\ A$$

Unbalanced 3-phase system

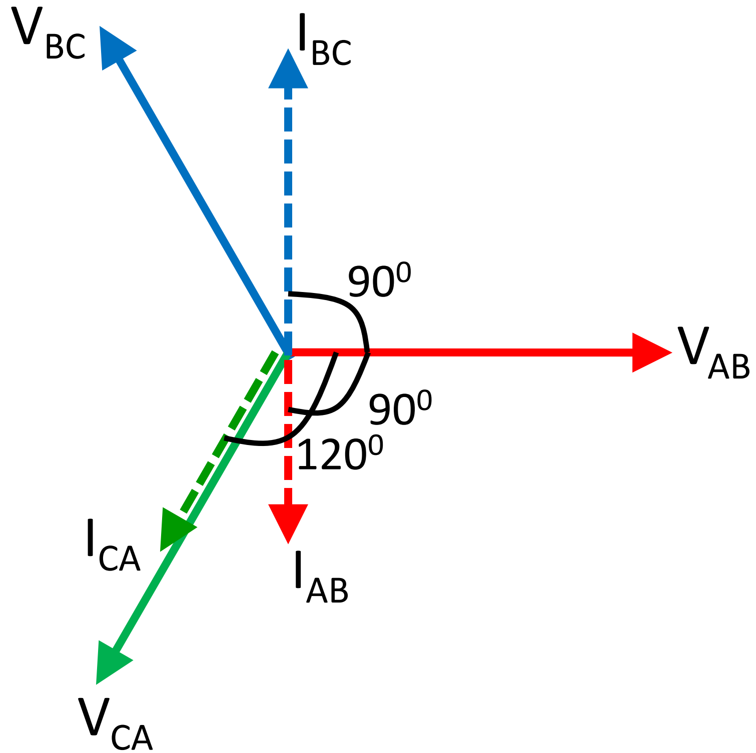
Let us analyze such an unbalanced system using an example

A 3-phase, 3-wire, 240 volt, CBA system supplies a delta-connected load in which $Z_{AB} = 25 \angle 90^0$, $Z_{BC} = 15 \angle 30°$, $Z_{CA} = 20 \angle 0^0$ ohms. Find the line currents and total power.



$V_{AB} = V \angle 0^0$, $V_{CA} = V \angle -120^0$, $V_{BC} = V\angle +120^0$

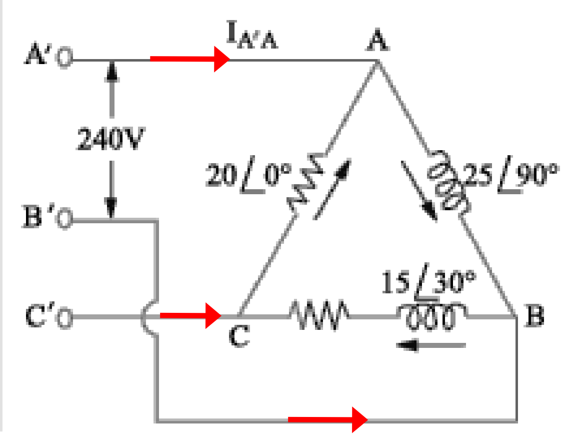
$$I_{AB} = 9.6\angle -90^0,\ I_{BC} = 16\angle 90^0,\ I_{CA} = 12\angle -120^0$$



Unbalanced 3-phase system

Let us analyze such an unbalanced system using an example

A 3-phase, 3-wire, 240 volt, CBA system supplies a delta-connected load in which $Z_{AB} = 25 \angle 90^0$, $Z_{BC} = 15 \angle 30°$, $Z_{CA} = 20 \angle 0^0$ ohms. Find the line currents and total power.



$$I_{AB} = -j9.6\ A \qquad I_{BC} = j16\ A \qquad I_{CA} = -6 - j10.4\ A$$

The LINE currents are hence calculated as:

$$I_{A'A} = I_{AB} - I_{CA} = -j9.6 - (-6 - j10.4) = 6 + j0.8\ A$$

$$I_{B'B} = I_{BC} - I_{AB} = j16 - (-j9.6) = j25.6\ A$$

$$I_{C'C} = I_{CA} - I_{BC} = (-6 - j10.4) - j16 = -6 - j26.4\ A$$

Total active power is the summation of active powers in three phases:

$$V_{AB} = 240\angle 0^0,\ I_{AB} = 9.6\angle -90^0 \Rightarrow P_{AB} = V_{AB} I_{AB} \cos(\angle \theta_{AB}) = 240 \times 9.6 \times \cos(\angle 0^0 + 90^0) = 2304 \cos(90^0) = 0$$

$$V_{BC} = 240\angle 120^0,\ I_{BC} = 16\angle 90^0 \ \Rightarrow P_{BC} = V_{BC} I_{BC} \cos(\angle \theta_{BC})$$

$$= 240 \times 16 \times \cos(\angle 120^0 - 90^0) = 3840 \cos(30^0) = 3326\, W$$

$$V_{CA} = 240\angle -120^0,\ I_{CA} = 12\angle -120^0 \Rightarrow P_{CA} = V_{CA} I_{CA} \cos(\angle \theta_{CA})$$

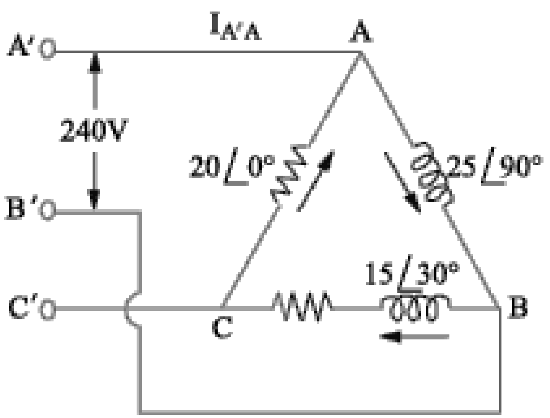
$$= 240 \times 12 \times \cos(\angle -120^0 + 120^0) = 2880 \cos(0^0) = 2880\, W$$

$$\Rightarrow P = P_{AB} + P_{BC} + P_{CA} = 0 + 3326 + 2880 = 6206\, W$$

Unbalanced 3-phase system

Let us analyze such an unbalanced system using an example

A 3-phase, 3-wire, 240 volt, CBA system supplies a delta-connected load in which $Z_{AB} = 25 \angle 90^0$, $Z_{BC} = 15 \angle 30°$, $Z_{CA} = 20 \angle 0^0$ ohms. Find the line currents and total power.



$V_{AB} = V \angle 0^0$, $V_{CA} = V \angle -120^0$, $V_{BC} = V \angle +120^0$

$I_{AB} = 9.6 \angle -90^0$, $I_{BC} = 16 \angle 90^0$, $I_{CA} = 12 \angle -120^0$

In case we are interested in finding out:

- Apparent power
- Reactive power
- Power factor

$V_{AB} = 240 \angle 0^0 = 240 + j0$, $V_{BC} = 240 \angle 120^0 = -120 + j207.8$, $V_{CA} = 240 \angle -120^0 = -120 - j207.8$

$I_{AB} = -j9.6$ $\quad I_{BC} = j16$ $\quad I_{CA} = -6 - j10.4$

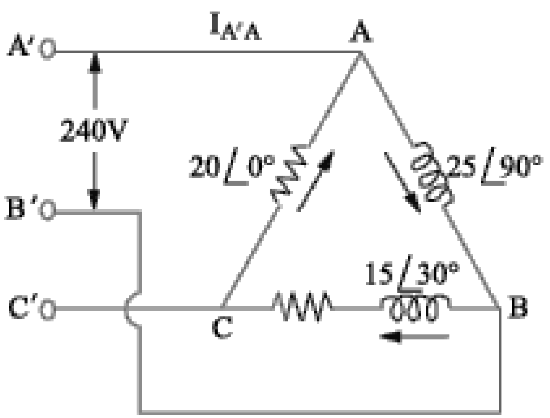
- As we are working with an unbalanced circuit we must calculate the power for each phase
- So let's calculate the apparent power for each phase in the complex form
- Apparent power in complex form is calculated by the product between the voltage and the **conjugate complex** of the current, i.e. $S = V \times I^*$

Conjugate is used to get the phase **difference** between voltage and current while calculating *S*, otherwise simple $V \times I$ would have added up their phase angles rather than taking their difference

Unbalanced 3-phase system

Let us analyze such an unbalanced system using an example

A 3-phase, 3-wire, 240 volt, CBA system supplies a delta-connected load in which $Z_{AB} = 25 \angle 90^0$, $Z_{BC} = 15 \angle 30°$, $Z_{CA} = 20 \angle 0^0$ ohms. Find the line currents and total power.



$$V_{AB} = 240 + j0 \,, V_{BC} = -120 + j207.8 \,, V_{CA} = -120 - j207.8$$

$$I_{AB} = -j9.6 \qquad I_{BC} = j16 \qquad I_{CA} = -6 - j10.4$$

Apparent power in complex form:

$$S_{AB} = V_{AB} \times I_{AB}{}^* = (240 + j0) \times (j9.6) = j2304$$

$$S_{BC} = V_{BC} \times I_{BC}{}^* = (-120 + j207.8) \times (-j16) = 3324.8 + j1920$$

$$S_{CA} = V_{CA} \times I_{CA}{}^* = (-120 - j207.8) \times (-6 + j10.4) = 2880 + j0$$

Total apparent power in complex form:

$$S = S_{AB} + S_{BC} + S_{CA} = j2304 + 3324.8 + j1920 + 2880 = 6204.8 + j4224$$

Total active power = real part of the complex apparent power: $P = \mathrm{Re}(S) = 6204.8\, W$

Total reactive power = imaginary part of the complex apparent power: $Q = \mathrm{Im}(S) = 4224\, VAr$

Total apparent power: $S = \sqrt{P^2 + Q^2} = \sqrt{6204.8^2 + 4224^2} = 7506\, VA$

Overall power factor: $\cos\theta = \dfrac{P}{S} = \dfrac{6204.8}{7506} = 0.826\,(\mathrm{lag})$

Unbalanced 3-phase system

Let us solve another example to make things clearer:

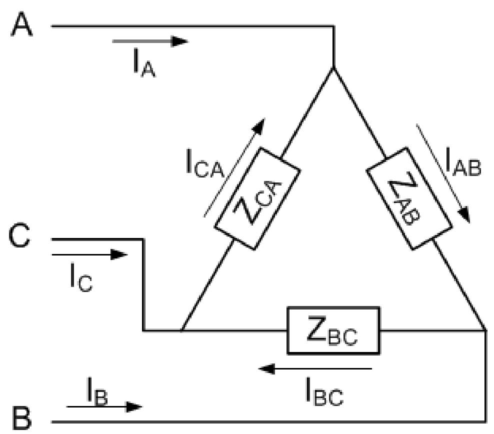
Let us have a circuit with load in the configuration delta, with the following values of voltage and impedances:

V_{AB} = 300∠30°, V_{BC} = 200∠-60°, V_{CA} = 150∠150°,

Z_{AB} =10∠30°, Z_{BC} =10∠45° and Z_{CA} =15∠-70°.

a) Calculate the phase and line currents.

b) Calculate the total apparent, real and reactive power and average power factor



a) The phase currents are calculated as:

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{300\angle 30^0}{10\angle 30^0} = 30\angle 0^0 = 30 + j0\ A$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{200\angle -60^0}{10\angle 45^0} = 20\angle -105^0 = -5.2 - j19.3\ A$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{150\angle 150^0}{15\angle -70^0} = 10\angle 220^0 = -7.7 - j6.4\ A$$

The LINE currents are hence calculated as:

$$I_A = I_{AB} - I_{CA} = 30 + j0 - (-7.7 - j6.4) = 37.7 + j6.4\ A = 38.2\angle 9.6^0\ A$$

$$I_B = I_{BC} - I_{AB} = -5.2 - j19.3 - (30 + j0) = -35.2 - j19.3\ A = 40.1\angle 208.7^0\ A$$

$$I_C = I_{CA} - I_{BC} = (-7.7 - j6.4) - (-5.2 - j19.3) = -2.5 + j12.9\ A = 13.1\angle 101^0\ A$$

Unbalanced 3-phase system

Let us solve another example to make things clearer:

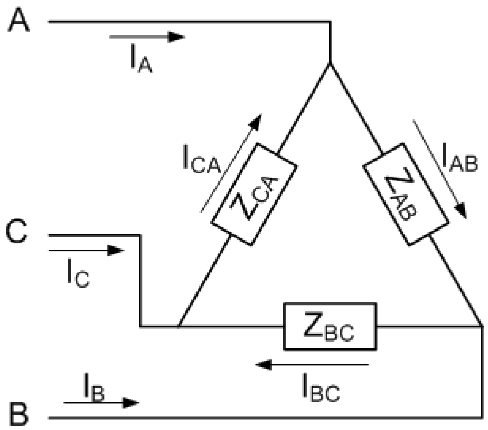
Let us have a circuit with load in the configuration delta, with the following values of voltage and impedances:

$V_{AB} = 300\angle 30°$, $V_{BC} = 200\angle -60°$, $V_{CA} = 150\angle 150°$,

$Z_{AB} = 10\angle 30°$, $Z_{BC} = 10\angle 45°$ and $Z_{CA} = 15\angle -70°$.

a) Calculate the phase and line currents.

b) Calculate the total apparent, real and reactive power and average power factor



b) Apparent power in complex form:

$$S_{AB} = V_{AB} \times I_{AB}{}^* = 300\angle 30^0 \times 30\angle -0^0 = 9000\angle 30^0 = 7794.2 + j4500\,VA$$

$$S_{BC} = V_{BC} \times I_{BC}{}^* = 200\angle -60^0 \times 20\angle +105^0 = 4000\angle 45^0$$
$$= 2828.4 + j2828.4\,VA$$

$$S_{CA} = V_{CA} \times I_{CA}{}^* = 150\angle 150^0 \times 10\angle -220^0 = 1500\angle -70^0$$
$$= 513 - j1409.5\,VA$$

Total apparent power in complex form:

$$S = S_{AB} + S_{BC} + S_{CA} = 7794.2 + j4500 + 2828.4 + j2828.4 + 513 - j1409.5$$
$$= 11135.6 + j5919.4 = 12611.1\angle 28^0\ VA$$

Unbalanced 3-phase system

Let us solve another example to make things clearer:

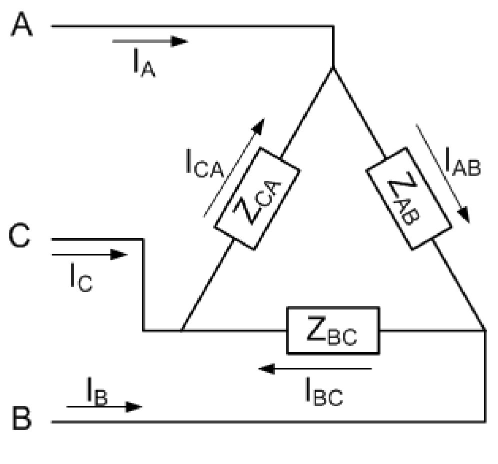
Let us have a circuit with load in the configuration delta, with the following values of voltage and impedances:

V_{AB} = 300∠30°, V_{BC} = 200∠-60°, V_{CA} = 150∠150°,

Z_{AB} =10∠30°, Z_{BC} =10∠45° and Z_{CA} =15∠-70°.

a) Calculate the phase and line currents.

b) Calculate the total apparent, real and reactive power and average power factor



Total apparent power in complex form:

$$S = 11135.6 + j5919.4 = 12611.1\angle 28^0\ VA$$

Total active power = real part of the complex apparent power:

$$P = \mathrm{Re}(S) = 11135.6\,W$$

Total reactive power = imaginary part of the complex apparent power:

$$Q = \mathrm{Im}(S) = 5919.4\,VAr$$

Total apparent power: $S = \sqrt{P^2 + Q^2} = \sqrt{11135.6^2 + 5919.4^2} = 12611.1\,VA$

Overall power factor: $\cos\theta = \frac{P}{S} = \frac{11135.6}{12611.1} = 0.88\,(\text{lag})$