Electromagnetism

Day 11
Tutorial on magnetic circuits

ILOs – Day 11 (Tutorial 1)

Solve numerical problems related to magnetic circuits

#1) An iron ring has cross section area of 100 mm² and mean diameter of 10 cm. Calculate the MMF required to produce a flux of 200 μ Wb. Assume relative permeability of iron as 800.

Length of flux path = mean length of the ring $l = \pi d = \pi \times 10 \times 10^{-2} = 0.314$ m Area of cross section of the ring, $A = 100 \times 10^{-6}$ m²

Relative permeability of iron, $\mu_r = 800$

$$\therefore \text{ Reluctance} = \frac{l}{\mu_0 \mu_r A} = \frac{0.314}{4\pi \times 10^{-7} \times 800 \times 100 \times 10^{-6}} = 3.123 \times 10^6$$

$$\phi = \frac{MMF}{\text{Reluctance}} \implies MMF = \phi \times \text{Reluctance}$$
$$= 200 \times 10^{-6} \times 3.123 \times 10^{6} = 624.6 \text{ AT}$$

#2) An iron ring of mean length 30 cm has an air gap of 2 mm and a coil with 200 turns on it. If relative permeability of iron is 300 when a current of 1 A flows through the coil; find the flux density.

Length of flux path = mean length of the ring l = 30 cm = 0.3 m

Relative permeability

$$\mu_r = 300$$

Length of air gap

$$l_1 = 2 \text{ mm} = 0.002 \text{ m}$$

Number of turns in coil

$$N = 200$$

Current through coil

$$I = 1 A$$

$$\therefore MMF = NI = 200 \text{ AT}$$

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Length of flux path = mean length of the ring l = 30 cm = 0.3 mRelative permeability $\mu_r = 300$ Length of air gap $l_1 = 2 \text{ mm} = 0.002 \text{ m}$

Let, cross sectional area of the ring is $A \text{ m}^2$.

∴ Resultant flux
$$\phi = \frac{MMF}{\text{Reluctance}} = \frac{NI}{S} = \frac{NI}{\frac{l_1}{\mu_0 \mu_{r_1} A_1} + \frac{l_2}{\mu_0 \mu_{r_2} A_2}}$$

Putting $l_1 = 0.002$ m, $l_2 = 0.298$ m, $\mu_{r1} = 1$, $\mu_{r2} = 300$, and $A_1 = A_2 = A$.

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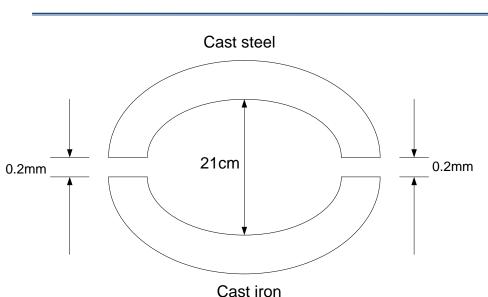
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$$l_1 = 0.002 \text{ m}, l_2 = 0.298 \text{ m}, \mu_{r1} = 1, \mu_{r2} = 300, \text{ and } A_1 = A_2 = A$$

$$\phi = \frac{200}{0.002} + \frac{0.298}{4\pi \times 10^{-7} \times 1 \times A} + \frac{0.298}{4\pi \times 10^{-7} \times 300 \times A} = 0.085A \text{ Wb}$$

$$\therefore \text{Flux density } B = \frac{\phi}{A} = \frac{0.085A}{A} = 0.085 \text{ Wb/m}^2$$

#3) A ring having a mean diameter of 21cm, and a cross section of 10cm² is made of two semi circular sections of cast-iron and cast-steel respectively with each joint having reluctance equal to air-gap of 0.2mm as shown in figure. Determine the ampere-turns required to produce a flux of 0.8mWb. The relative permeabilities of cast-iron and cast-steel are 166 and 800 respectively. Neglect fringing and leakage effects.



$$\phi = \frac{MMF}{\text{Reluctance}} = \frac{NI}{S} = \frac{AT}{S}$$

$$\therefore AT = \phi \times S$$

For a series magnetic circuit, the total AT required will be summation of ATs required for steel, iron, and air separately:

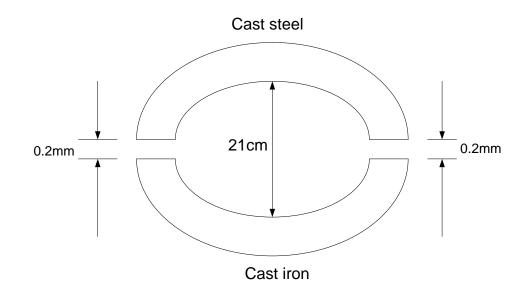
$$AT = \phi \times [S_{St} + S_{Fe} + S_{air}] = \phi \times \left[\frac{l_1}{\mu_0 \mu_{r_1} A_1} + \frac{l_2}{\mu_0 \mu_{r_2} A_2} + \frac{l_3}{\mu_0 \mu_{r_3} A_3} \right]$$

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$$A_1 = A_2 = A_3 = 10 \text{ cm}^2 = 10 \text{x} 10^{-4} \text{ m}^2$$

 $l_3 = 2 \text{x} 0.2 \text{ mm} = 2 \text{x} 0.0002 \text{ m}$
 $l_1 = l_2 \approx \pi \text{x} 21 \text{x} 10^{-2} / 2 = 0.33 \text{ m}$



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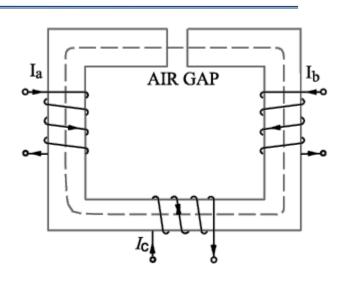
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$$\therefore AT = 0.8 \times 10^{-3} \times \left[\frac{0.33}{4\pi \times 10^{-7} \times 800 \times 10 \times 10^{-4}} + \frac{0.33}{4\pi \times 10^{-7} \times 166 \times 10 \times 10^{-4}} + \frac{2 \times 0.0002}{4\pi \times 10^{-7} \times 1 \times 10 \times 10^{-4}} \right]$$

$$AT = 0.8 \times 10^{-3} \times [328125 + 1581325 + 318310] = 1782.2$$

#4) A rectangular iron core has a mean length of magnetic path of 100 cm, cross-section of (2 cm \times 2 cm), relative permeability of 1400 and an air-gap of 5 mm cut in the core. The three coils carried by the core have number of turns $N_a = 335$, $N_b = 600$ and $N_c = 600$; and the respective currents are 1.6 A, 4 A and 3 A. The directions of the currents are as shown. Find the flux in the air-gap.

By applying the Right-Hand Thumb rule, it is found that fluxes produced by the current I_a and I_b are directed in the clockwise direction through the iron core whereas that produced by current I_c is directed in the anticlockwise direction through the core.



: Total MMF =
$$N_a I_a + N_b I_b - N_c I_c = 335 \times 1.6 + 600 \times 4 - 600 \times 3 = 1136 \text{ AT}$$

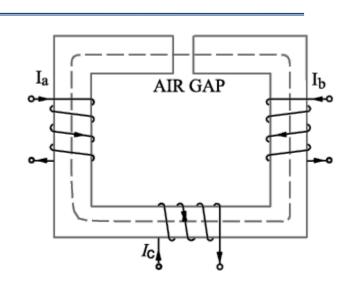
Reluctance of the air-gap =
$$\frac{l_G}{\mu_0 A} = \frac{5 \times 10^{-3}}{4 \pi \times 10^{-7} \times 4 \times 10^{-4}} = 9.95 \times 10^6 \text{ AT/Wb}$$

Reluctance of iron path =
$$\frac{l_i}{\mu_0 A} = \frac{100 \times 10^{-2} - 5 \times 10^{-3}}{4\pi \times 10^{-7} \times 1400 \times 4 \times 10^{-4}} = 1.414 \times 10^6 \text{ AT/Wb}$$

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Reluctance of the air-gap = 9.95×10^6 AT/Wb

Reluctance of iron path = 1.414×10^6 AT/Wb



: Total reluctance,
$$S = 9.95 \times 10^6 + 1.414 \times 10^6 = 11.36 \times 10^6 \text{ AT/Wb}$$

In the series circuit, flux in the air-gap is the same as in the iron core

∴ Air-gap flux,
$$\phi = \frac{MMF}{\text{Reluctance}} = \frac{1136}{11.36 \times 10^6} = 100 \times 10^{-6} \text{ Wb} = 100 \ \mu\text{Wb}$$

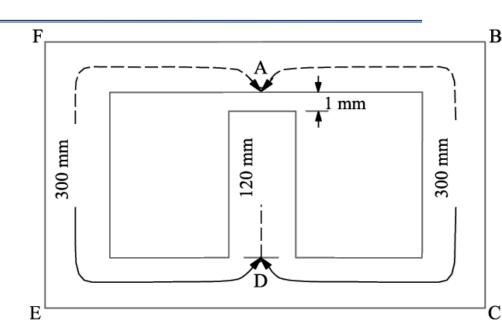
Central limb

Flux in the central limb, $\phi_c = 1.3 \times 10^{-3}$ Wb

Area of central limb, $A_c = 800 \times 10^{-6} \text{ m}^2$

∴ Flux density in central limb:

$$B_c = \frac{\phi_c}{A} = \frac{1.3 \times 10^{-3}}{800 \times 10^{-6}} = 1.625 \text{ T}$$



Corresponding value of H for this flux density is given as 3800 AT/m.

Since the length of the central limb is (l_c = 120-1 = 119) mm, MMF required is:

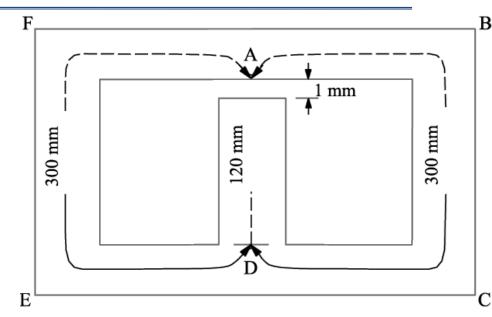
$$AT_c = H_c \times l_c = 3800 \times (119 \times 10^{-3}) = 452.2 \text{ AT}$$

Air gap

Since air gap is in the central limb, flux density in the air-gap is the same as that in the central limb, $B_{air} = 1.625 \text{ T}$

Corresponding value of *H* for this flux density is given by:

$$H_{air} = \frac{B_{air}}{40} = \frac{1.625}{4\pi \times 10^{-7}} = 0.1293 \times 10^7 \text{ AT/m}$$

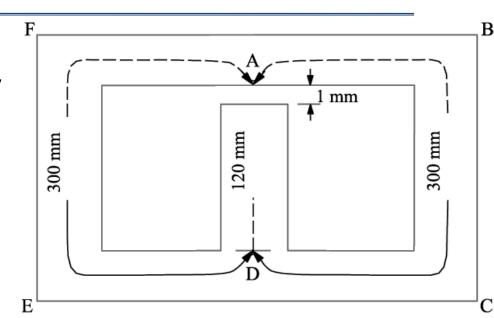


Since the length of air gap is $(l_{air} = 1)$ mm, MMF required for air gap is:

$$AT_{air} = H_{air} \times l_{air} = 0.1293 \times 10^7 \times 1 \times 10^{-3} = 1293$$
 AT

Outer limb

The flux of the central limb divides equally at point A in figure along the two parallel path *ABCD* and *AFED*. We may consider either path, say *ABCD* and calculate the MMF required for it. The same MMF will also send the flux through the other parallel path *AFED*.



Flux through *ABCD*, $\phi_0 = \phi_c/2 = 1.3 \times 10^{-3}/2 = 0.65 \times 10^{-3}$ Wb

Area of outer limb, $A_o = 500 \times 10^{-6} \text{ m}^2$

: Flux density in outer limb:
$$B_o = \frac{\phi_o}{A_o} = \frac{0.65 \times 10^{-3}}{500 \times 10^{-6}} = 1.3 \text{ T}$$

Outer limb

$$B_o = \frac{\phi_o}{A_o} = \frac{0.65 \times 10^{-3}}{500 \times 10^{-6}} = 1.3 \text{ T}$$

The corresponding value of H for this value of B_o is given as 850 AT/m

Since the length of the outer limb ABCD is (l_o = 300) mm, MMF required is:

$$AT_o = H_o \times l_o = 850 \times (300 \times 10^{-3}) = 255 \text{ AT}$$

300 mm 005 mm 000 mm 00

Since the two outer limbs are in parallel, this MMF will also send the same flux in the parallel path AFED also.

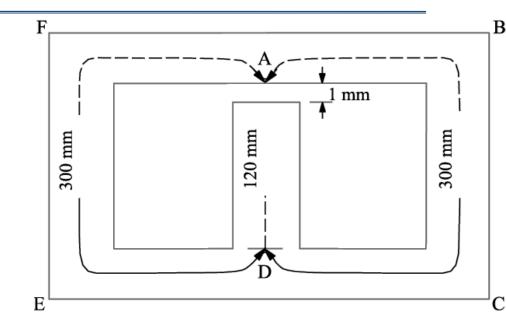
Total MMF

Central limb : $AT_c = 452.2$ AT

Air gap : $AT_{gir} = 1293$ AT

Outer limb : $AT_o = 255$ AT

= 452.2 + 1293 + 255 = 2000.2 AT



Since the number of turns is 500, current required in coil: $I = 2000.2/500 \approx 4A$