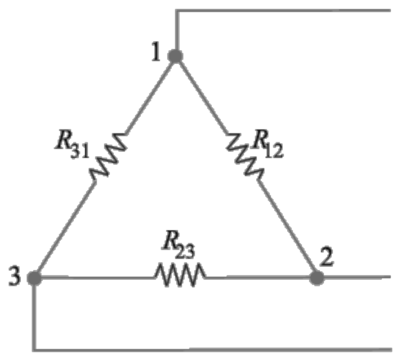
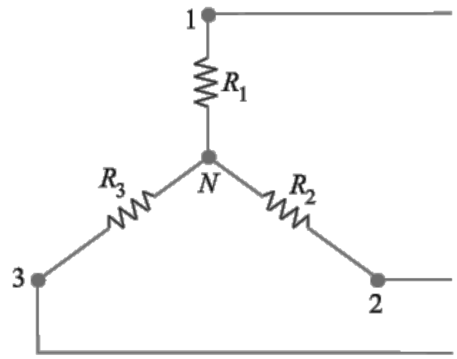
Delta/Star Transformation

Suppose we are given three resistances R12, R23 and R31 connected in delta fashion between terminals 1, 2 and 3. So far as the respective terminals are concerned, these three given resistances can be replaced by the three resistances R1, R2 and R3 connected in star. These two arrangements will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements.





First take delta connection between terminals 1 and 2, there are two parallel paths: one having a resistance R_{12} and the other having a resistance (R_{23}+R_{31}).

Hence, equivalent resistance between terminals 1 and 2 is $= \dfrac{R_{12}\times(R_{23}+R_{31})}{R_{12}+(R_{23}+R_{31})}$

Now take the star connection: The resistance between the same terminals 1 and 2 is (R_1+R_2).

Since the terminal resistances must remain same:

Thus, $R_1 + R_2 = \dfrac{R_{12}\times(R_{23}+R_{31})}{R_{12}+(R_{23}+R_{31})}$ (1)

Similarly, for terminals 2 and 3, by equivalence between the delta and star networks, we get:

$R_2 + R_3 = \dfrac{R_{23}\times(R_{31}+R_{12})}{R_{23}+(R_{31}+R_{12})}$ (2)

Also for terminals 3 and 1, we get:

$R_3 + R_1 = \dfrac{R_{31}\times(R_{12}+R_{23})}{R_{31}+(R_{12}+R_{23})}$ (3)

Now, from (1), (2) and (3), we solve for R_1:

(1) − (2) + (3) we get $R_1 + R_2 = \dfrac{R_{12}\times(R_{23}+R_{31})}{R_{12}+(R_{23}+R_{31})}$

\- $R_2 + R_3 = \dfrac{R_{23}\times(R_{31}+R_{12})}{R_{23}+(R_{31}+R_{12})}$

$$R_1 - R_3 = \frac{R_{12}\times(R_{23}+R_{31})}{R_{12}+(R_{23}+R_{31})} - \frac{R_{23}\times(R_{31}+R_{12})}{R_{23}+(R_{31}+R_{12})}$$

\+ $R_3 + R_1 = \dfrac{R_{31}\times(R_{12}+R_{23})}{R_{31}+(R_{12}+R_{23})}$

$$2R_1 = \frac{R_{12}\times(R_{23}+R_{31})}{R_{12}+(R_{23}+R_{31})} - \frac{R_{23}\times(R_{31}+R_{12})}{R_{23}+(R_{31}+R_{12})} + \frac{R_{31}\times(R_{12}+R_{23})}{R_{31}+(R_{12}+R_{23})}$$

$$2R_1 = \frac{R_{12}R_{23} + R_{12}R_{31} - R_{23}R_{31} - R_{23}R_{12} + R_{31}R_{12} + R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$$

$$2R_1 = \frac{2R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$\boxed{R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}} \tag{4}$$

Proceeding in similar manner, the other two resistances of the equivalent star network can be derived as:

$$\boxed{R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}} \tag{5}$$

$$\boxed{R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}} \tag{6}$$

How to remember?
Resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three delta resistances.

Star/Delta Transformation

Multiply (4) and (5):

$$R_1R_2 = \frac{R_{12}R_{31}R_{12}R_{23}}{(R_{12}+R_{23}+R_{31})^2} \tag{7}$$

Multiply (5) and (6):

$$R_2R_3 = \frac{R_{12}R_{23}R_{23}R_{31}}{(R_{12}+R_{23}+R_{31})^2} \tag{8}$$

Multiply (6) and (4):

$$R_3R_1 = \frac{R_{23}R_{31}R_{12}R_{31}}{(R_{12}+R_{23}+R_{31})^2} \tag{9}$$

Add (7) + (8)+(9):

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{31}R_{12}R_{23} + R_{12}R_{23}R_{23}R_{31} + R_{23}R_{31}R_{12}R_{31}}{(R_{12}+R_{23}+R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}}{R_{12}+R_{23} + R_{31}} \tag{10}$$

Comparing (10) with (6), we have:

$$R_1R_2 + R_2R_3 + R_3R_1 = R_{12}R_3$$

or, $\dfrac{R_1R_2}{R_3} + R_2 + R_1 = R_{12}$

or, $R_{12} = \dfrac{R_1R_2}{R_3} + R_2 + R_1$

or, $\boxed{R_{12} = \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}}$ (11)

Proceeding in similar manner, the other two resistances of the equivalent star network can be derived as:

$$\boxed{R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}} \tag{12}$$

$$\boxed{R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}} \tag{13}$$

How to remember?

Resistance of each arm of the delta is given by the sum of product of each pair of resistances of the star divided by the resistance of the star that is opposite to that particular delta arm.