Circuit Elements

- ILO – Day2
  - Combine circuit elements in series and parallel

Circuit Elements

- Resistance (Ohm)

$$\boxed{V = IR}$$

- Inductance (Henry)

$$\boxed{v = L\frac{di}{dt}}$$

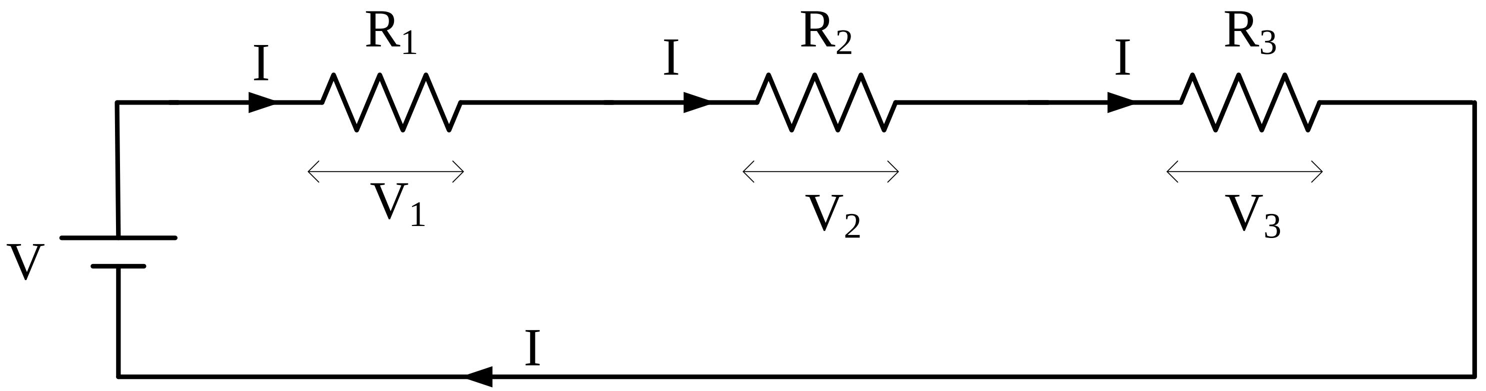
- Capacitance (Farad)

$$\boxed{i = C\frac{dv}{dt}} \qquad \boxed{V = \frac{1}{C}\int_0^t i\,dt}$$

Series and parallel combination of R, L, and C

- Series combination
  - When more than one elements are connected in series, it means that:
    - The same current passes through all the elements
    - But the total supply voltage gets divided among the elements.

Series combination of resistances



$V_1 = IR_1$ $\quad$ $V_2 = IR_2$ $\quad$ $V_3 = IR_3$

Since the supply voltage is shared between the three resistances, we can write:

$$V = V_1 + V_2 + V_3$$

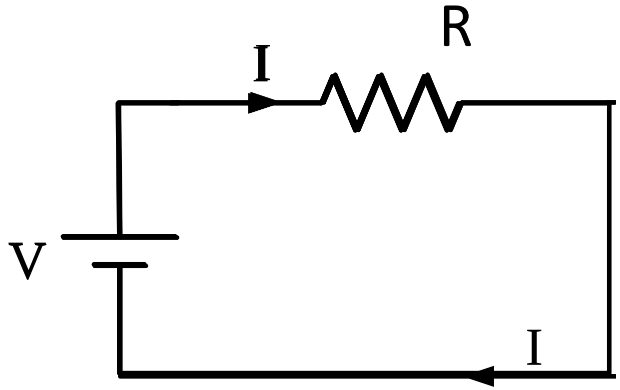
$$or, V = IR_1 + IR_2 + IR_3$$

$$or, V = I\left(R_1 + R_2 + R_3\right)$$

Series combination of resistances

$$V = I\left(R_1 + R_2 + R_3\right)$$

The series combination can be **equivalently** represented by a single resistance with same voltage V and same current I:



In this simple equivalent circuit, we can write the relation:

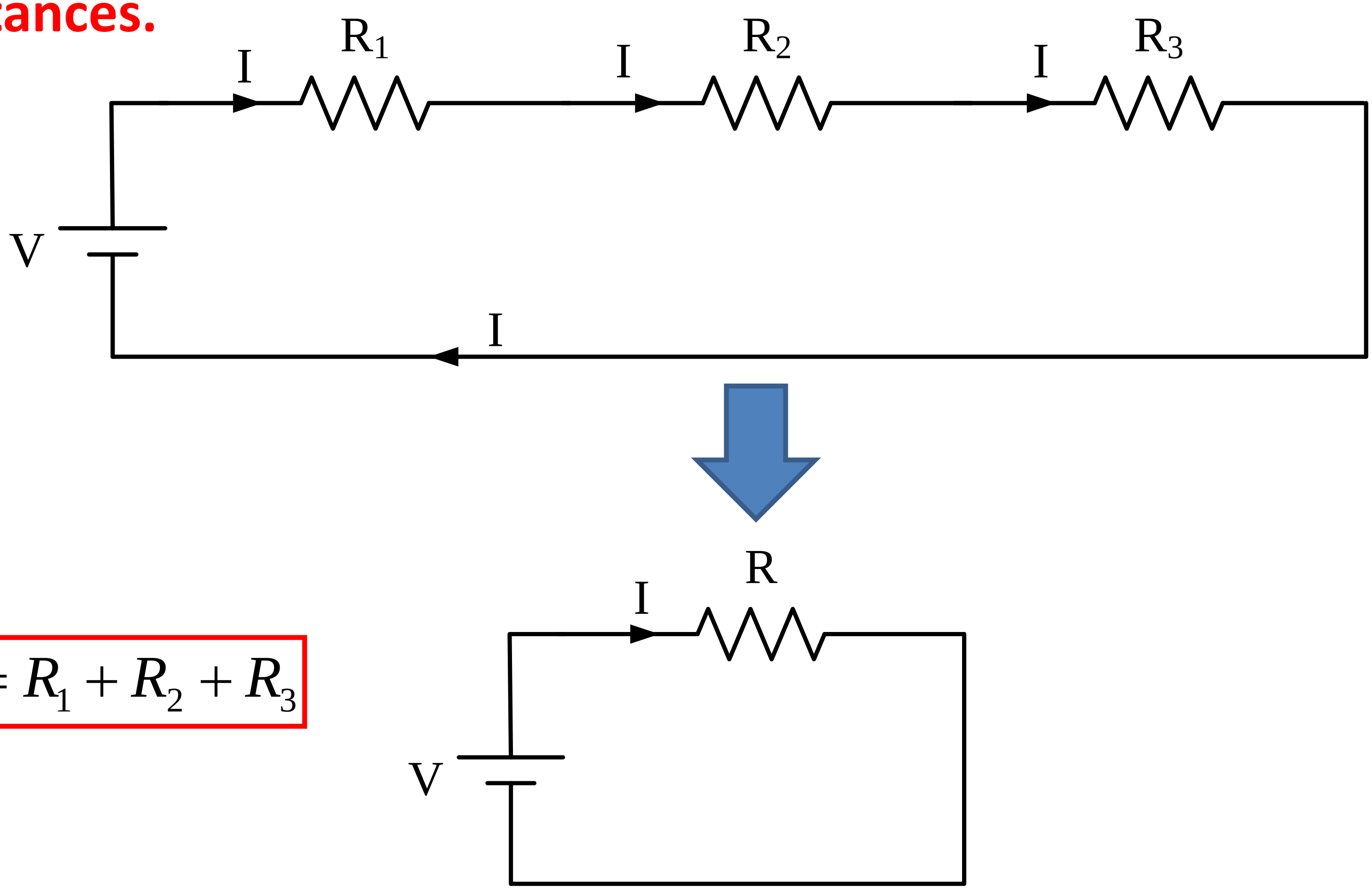
$$V = IR$$

Comparing the original equation with the equation of the simple equivalent circuit, we have the relation:

$$\boxed{R = R_1 + R_2 + R_3}$$

Series combination of resistances

Thus, when a number of resistances are connected in series, **their equivalent resistance is simply summation of all the individual resistances.**



$$\boxed{R = R_1 + R_2 + R_3}$$

Parallel combination of resistances

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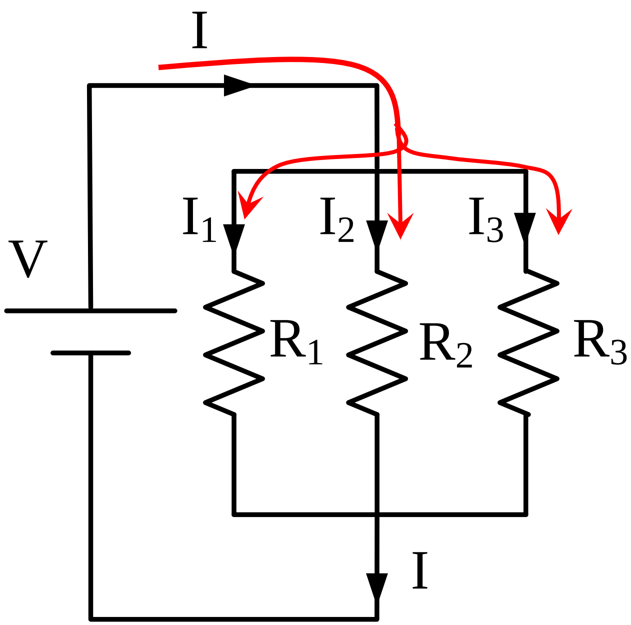
- Parallel combination
  - When more than one elements are connected in parallel, it means:
    - The same voltage is impressed across all the elements
    - But, the total supply current is divided among the elements.

Parallel combination of resistances



$$I_1 = \frac{V}{R_1} \qquad I_2 = \frac{V}{R_2} \qquad I_3 = \frac{V}{R_3}$$

Since the supply current is shared between the three resistances, we can write:

$$I = I_1 + I_2 + I_3$$

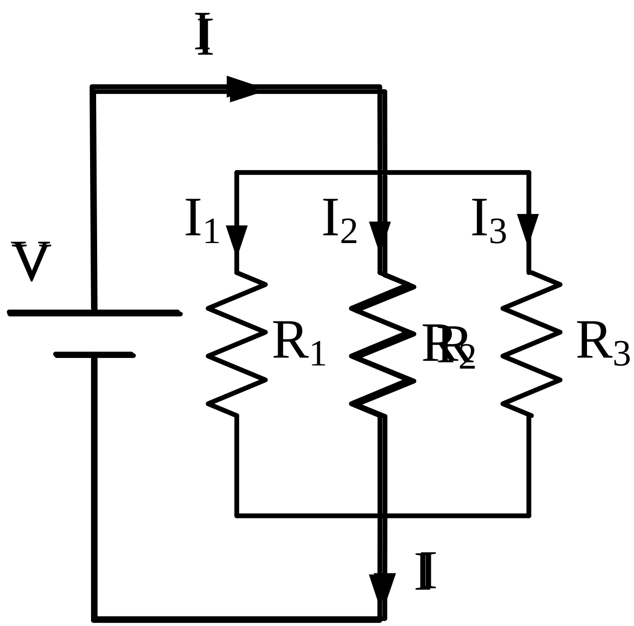
$$or, \quad I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$or, \quad I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Parallel combination of resistances

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

The parallel combination can be **equivalently** represented by a single resistance with same voltage V and same current I:



In this simple equivalent circuit, we can write the relation:

$$V = IR \qquad \Rightarrow I = \frac{V}{R}$$

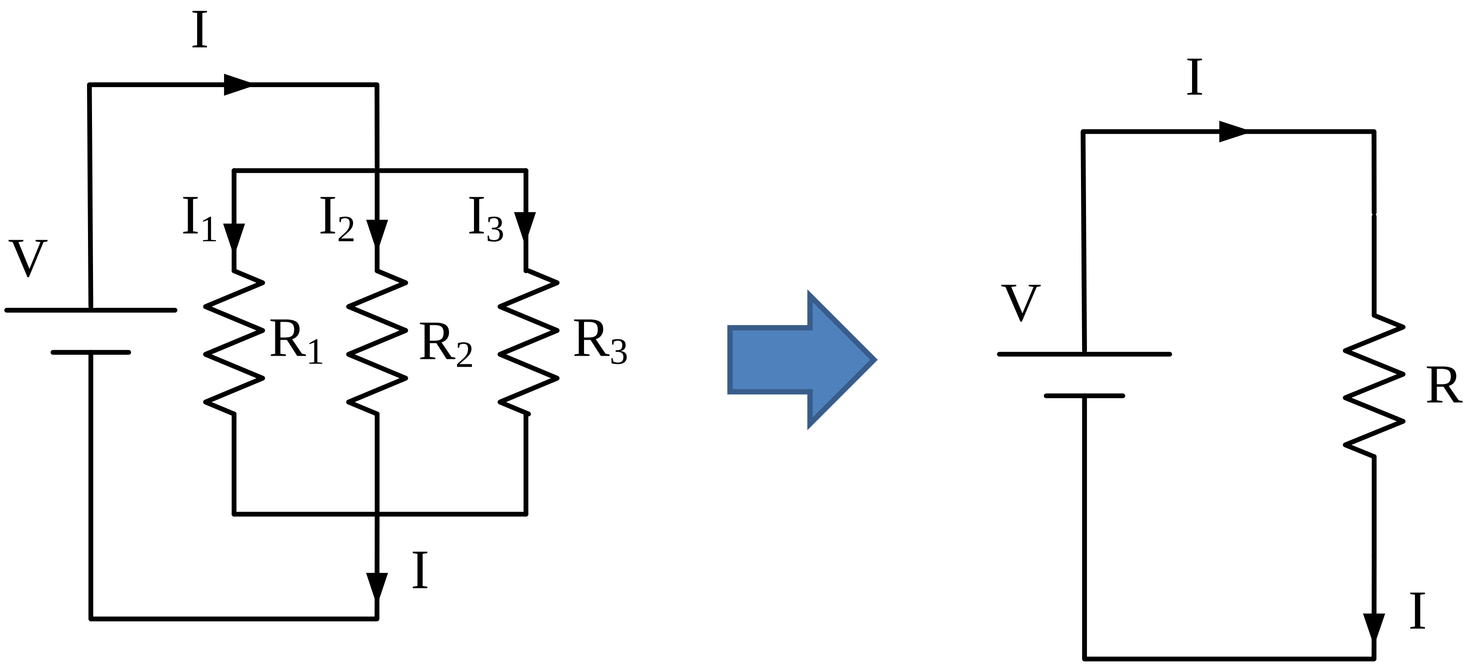
Comparing the original equation with the equation of the simple equivalent circuit, we have the relation:

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Parallel combination of resistances

$$\boxed{\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

Thus, when a number of resistances are connected in parallel, their **equivalent conductance** is simply summation of all the individual conductances.



Parallel combination of resistances

Special case:

When two resistances R_1 and R_2 connected in parallel, their equivalent resistance is:

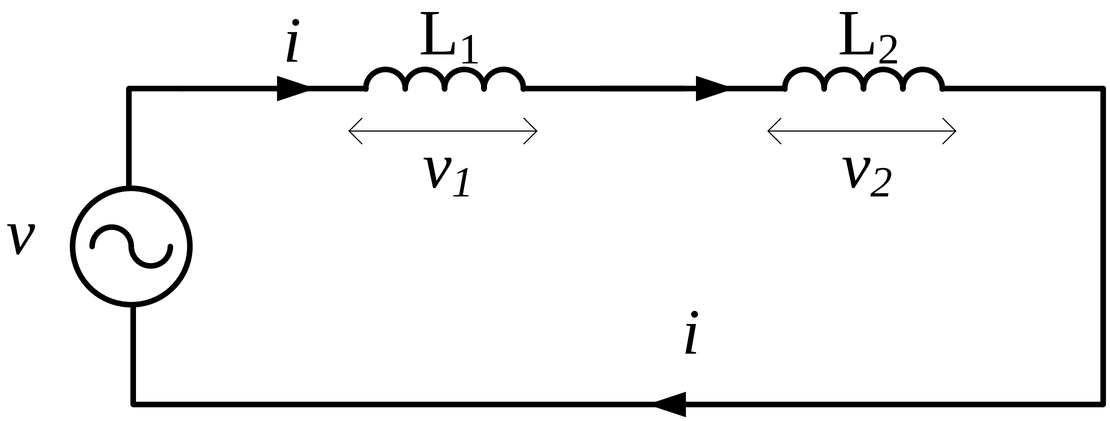
$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\frac{1}{R} = \left(\frac{R_1 + R_2}{R_1 R_2} \right)$$

$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

Series combination of inductances

The two inductors carry the same current, but the total supply voltage is divided among the two:



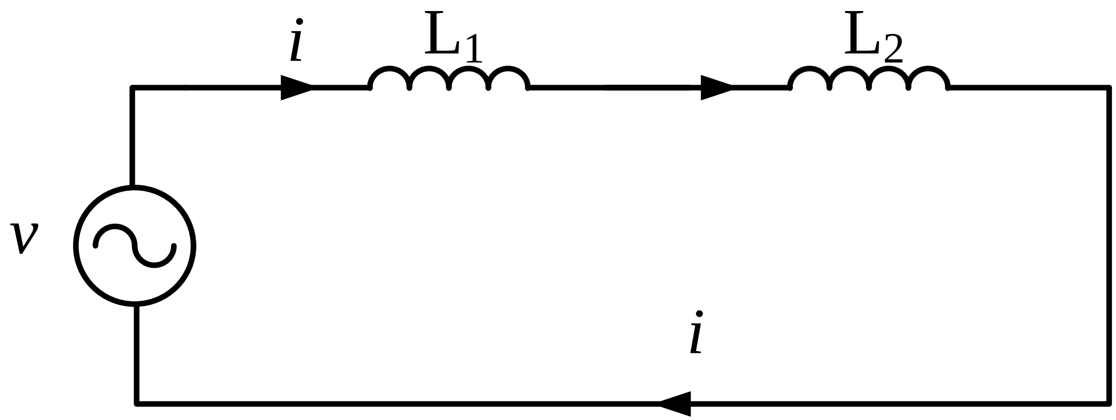
$$v_1 = L_1 \frac{di}{dt}$$

$$v_2 = L_2 \frac{di}{dt}$$

Since the supply voltage is shared between the two inductances, we can write:

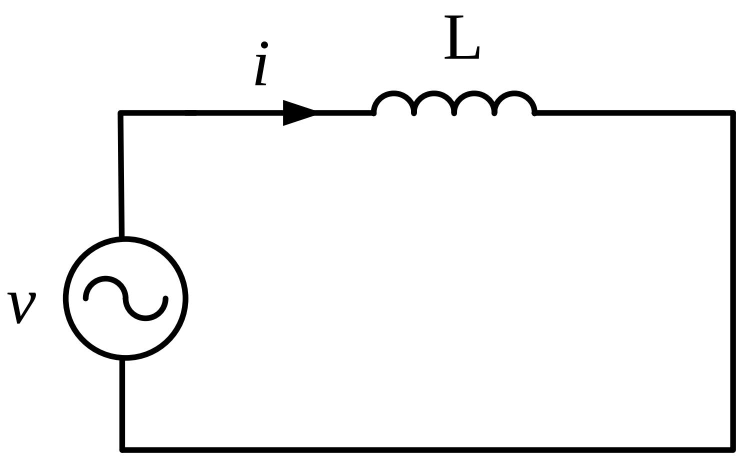
$$v = v_1 + v_2 \quad \Rightarrow v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} \qquad \boxed{\Rightarrow v = \left(L_1 + L_2\right)\frac{di}{dt}}$$

Series combination of inductances



$$v = (L_1 + L_2)\frac{di}{dt}$$

The series combination can be **equivalently** represented by a single inductor with same voltage *v* and same current *i*:



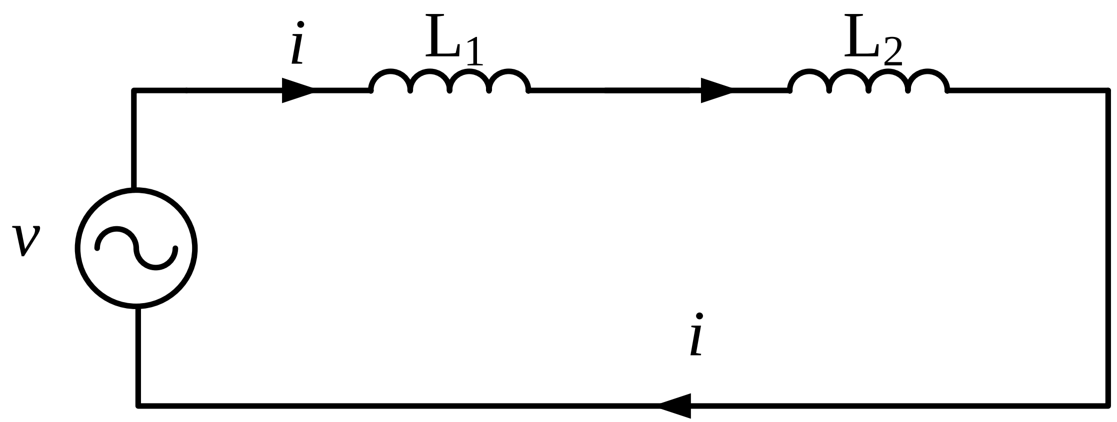
In this simple equivalent circuit, we can write the relation:

$$v = L\frac{di}{dt}$$

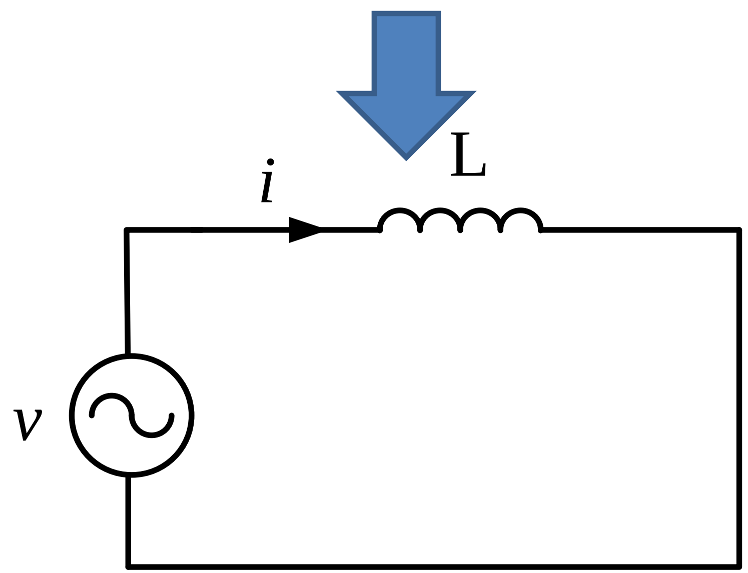
Comparing the original equation with the equation of the simple equivalent circuit, we have the relation: $\boxed{L = L_1 + L_2}$

Series combination of inductances

Thus, like resistances, inductances connected in series can be equivalently represented by **summation of the individual inductance values**.

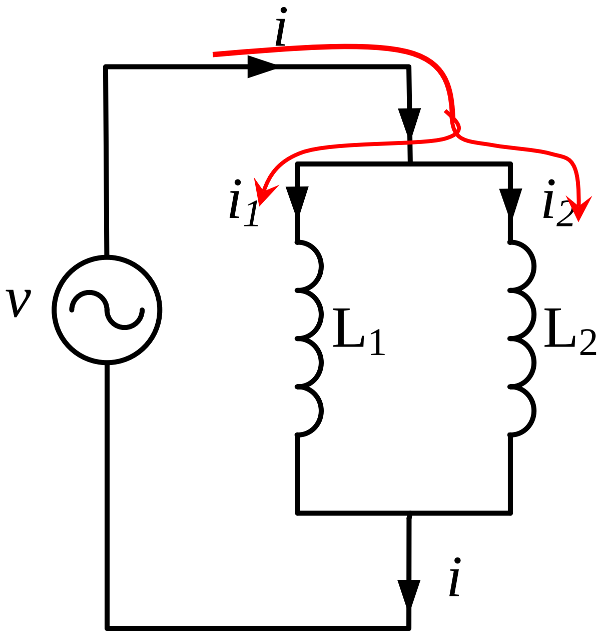


$$\boxed{L = L_1 + L_2}$$



Parallel combination of inductances

The two inductors L_1 and L_2 connected in parallel, have the same voltage across them, but the total supply current is divided among the two:



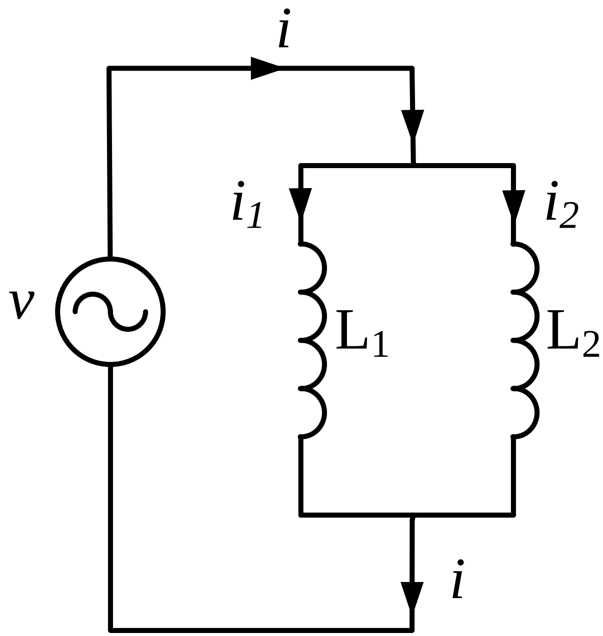
$$v = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$$

$$i_1 = \frac{1}{L_1} \int v dt \qquad i_2 = \frac{1}{L_2} \int v dt$$

The supply current is summation of these two branch currents:

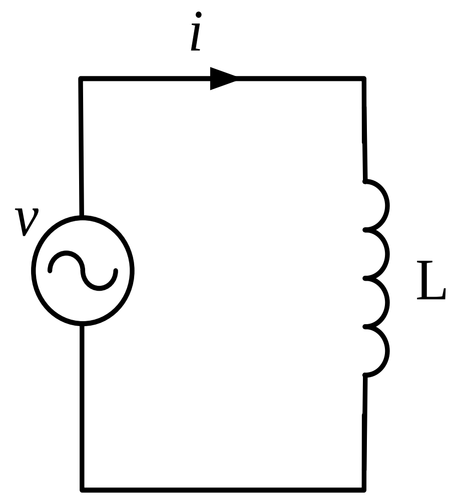
$$i = i_1 + i_2 = \frac{1}{L_1} \int v dt + \frac{1}{L_2} \int v dt = \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \int v dt$$

Parallel combination of inductances



$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} \right) \int v dt$$

The parallel combination can be **equivalently** represented by a single inductance with same voltage V and same current I:



In this simple equivalent circuit, we can write the relation:

$$v = L \frac{di}{dt}$$

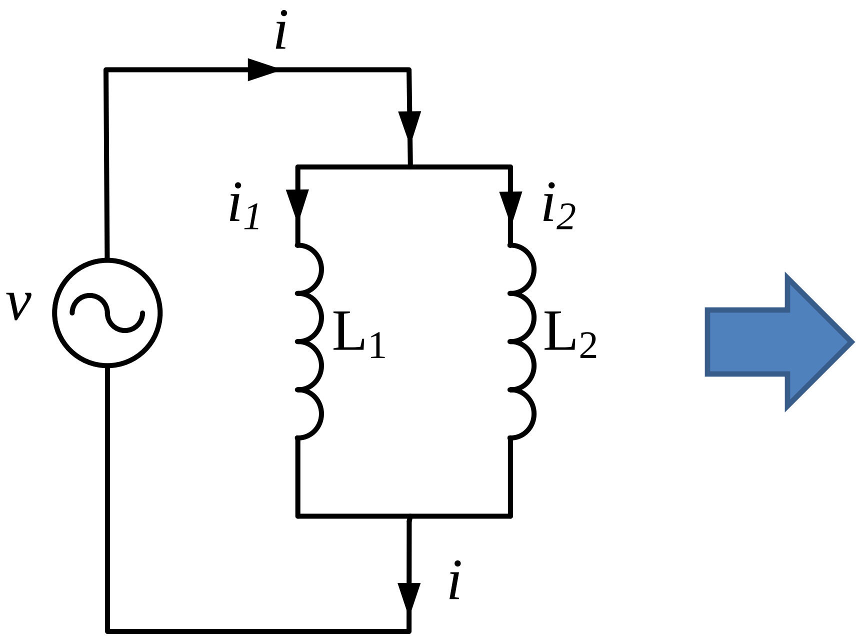
$$\Rightarrow i = \frac{1}{L} \int v dt$$

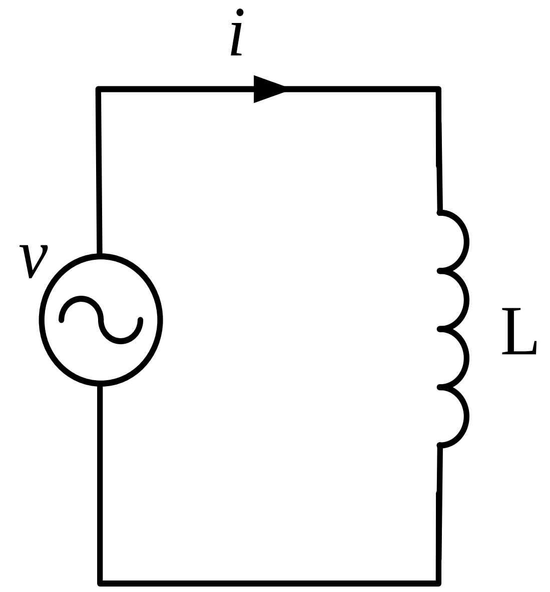
Comparing the original equation with the equation of the simple equivalent circuit, we have the relation: $\boxed{\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}}$

Parallel combination of inductances

$$\boxed{\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}}$$

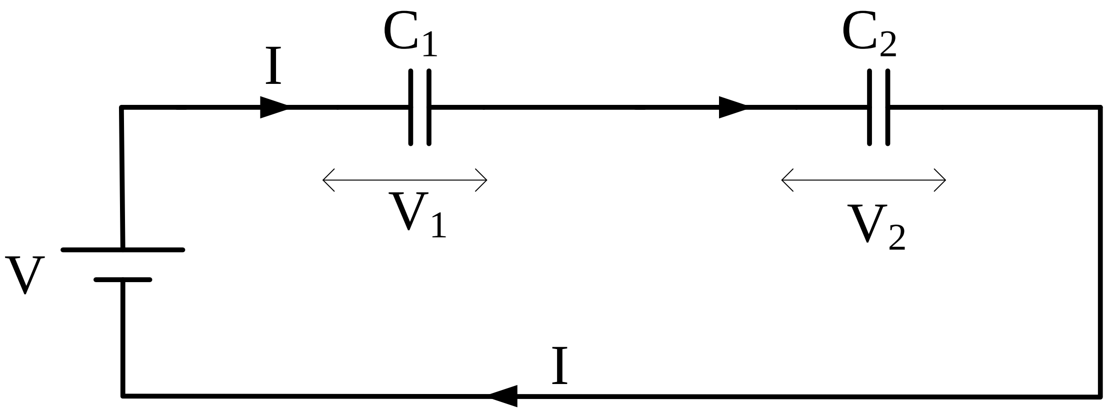
This also is very similar to the formula for resistances connected in parallel





Series combination of capacitances

The two capacitors carry the same current, but the total supply voltage is divided among the two:



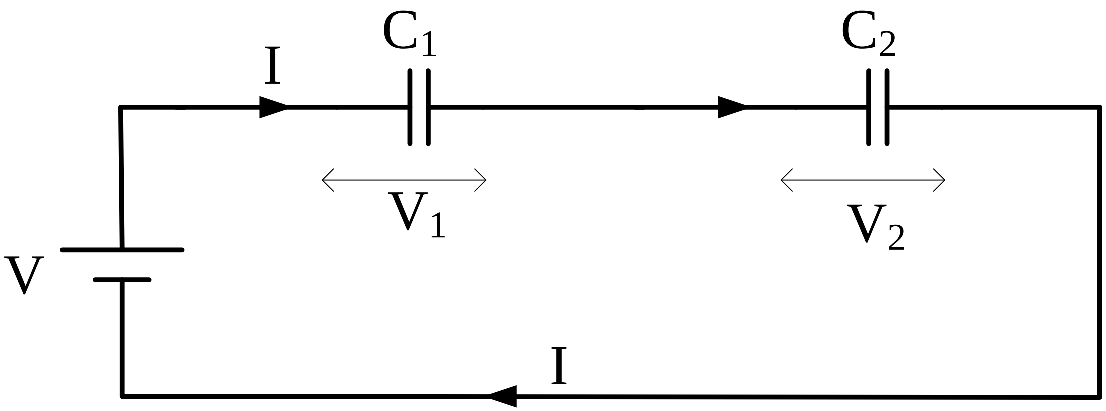
$$v_1 = \frac{1}{C_1}\int_0^t i\,dt$$

$$v_2 = \frac{1}{C_2}\int_0^t i\,dt$$

Since the supply voltage is shared between the two capacitances, we can write:

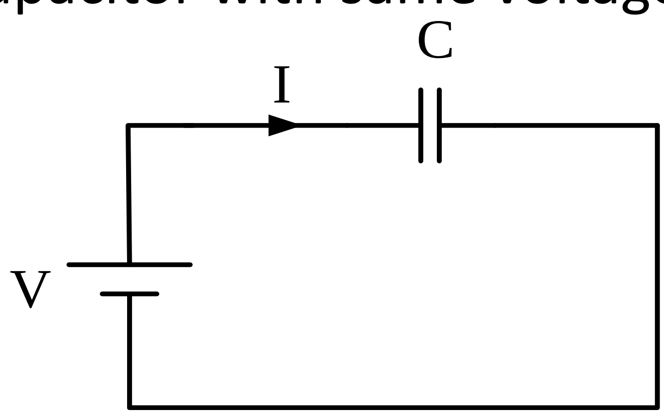
$$v = v_1 + v_2 = \frac{1}{C_1}\int_0^t i\,dt + \frac{1}{C_2}\int_0^t i\,dt \boxed{= \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dt}$$

Series combination of capacitances



$$v = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int i dt$$

The series combination can be **equivalently** represented by a single capacitor with same voltage *v* and same current i:

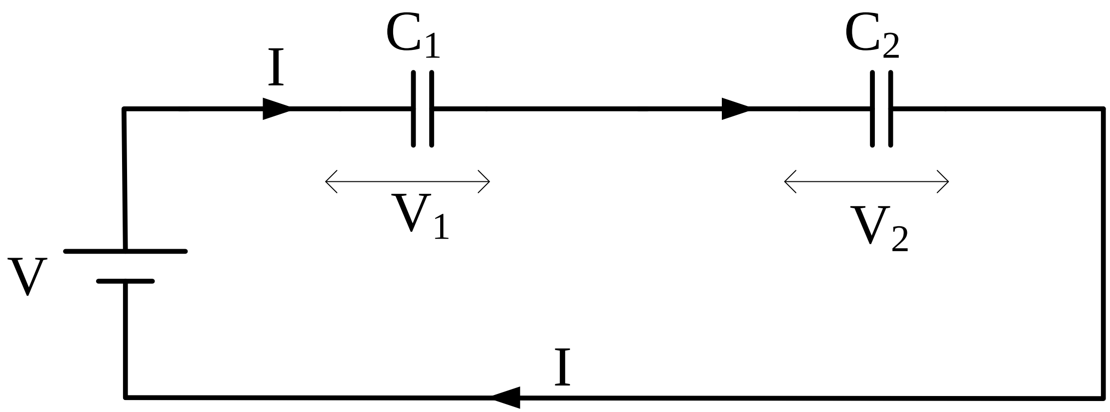


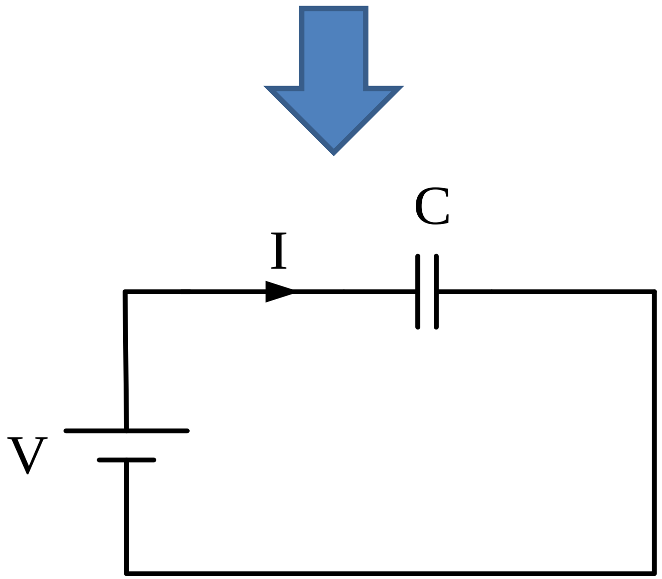
In this simple equivalent circuit, we can write the relation:

$$v = \frac{1}{C}\int i dt$$

Comparing the original equation with the equation of the simple equivalent circuit, we have the relation: $\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$

Series combination of capacitances



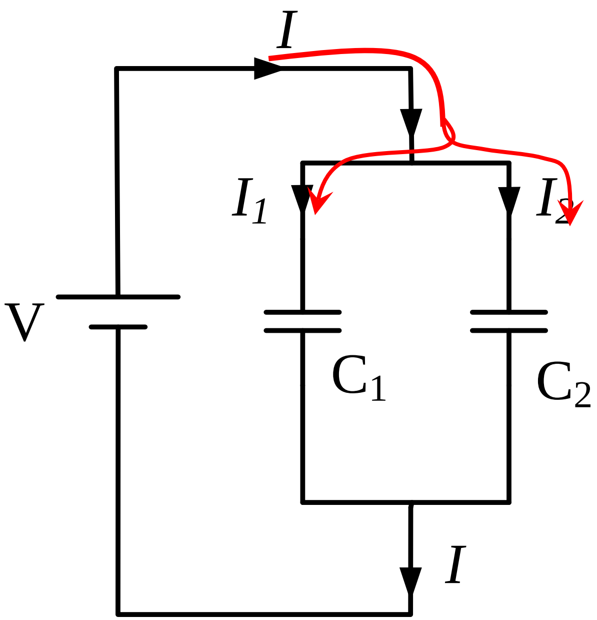


$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Note that the formula for series connection of capacitances is like the formula for ***parallel connection of resistances (or inductances)***

Parallel combination of capacitances

The two capacitors C_1 and C_2 connected in parallel, have the same voltage across them, but the total supply current is divided among the two:

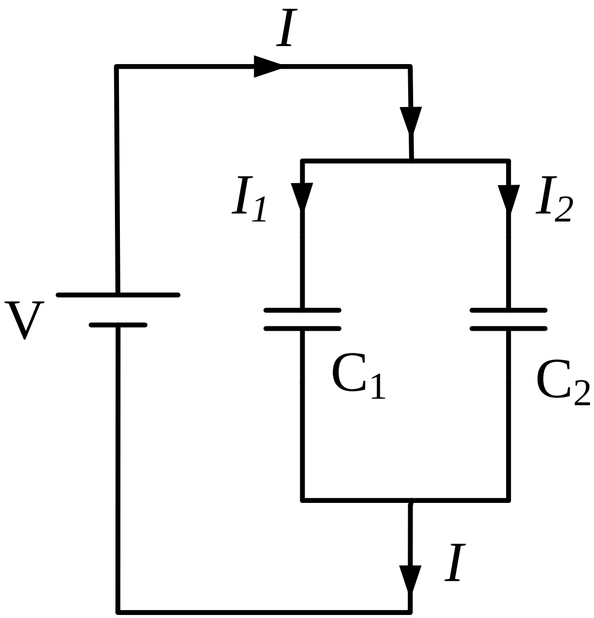


$$I_1 = C_1 \frac{dV}{dt} \qquad I_2 = C_2 \frac{dV}{dt}$$

The supply current is summation of these two branch currents:

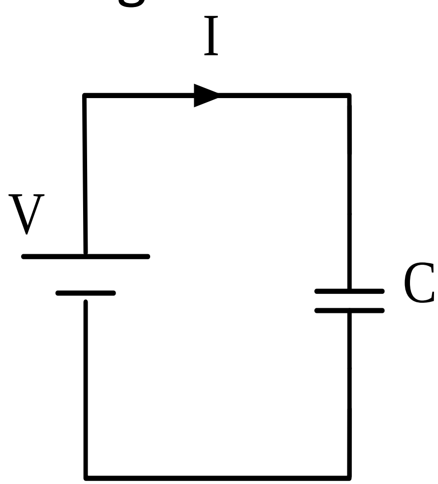
$$\begin{aligned} I &= I_1 + I_2 \\ &= C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} \\ &= (C_1 + C_2) \frac{dV}{dt} \end{aligned}$$

Parallel combination of capacitances



$$I = (C_1 + C_2)\frac{dV}{dt}$$

The parallel combination can be **equivalently** represented by a single capacitance with same voltage V and same current I:

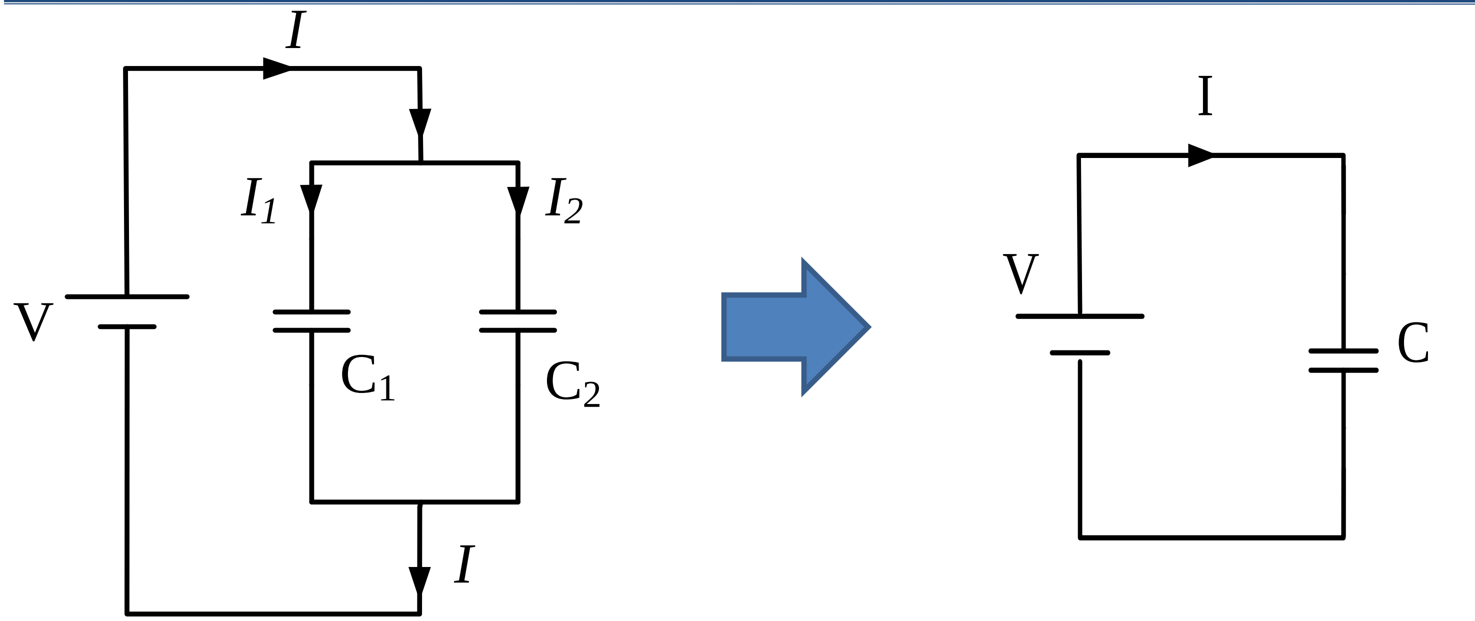


In this simple equivalent circuit, we can write the relation:

$$I = C\frac{dV}{dt}$$

Comparing the original equation with the equation of the simple equivalent circuit, we have the relation: $\boxed{C = C_1 + C_2}$

Parallel combination of capacitances



$$\boxed{C = C_1 + C_2}$$

Note that the formula for parallel connection of capacitances is like the formula for ***series connection of resistances (or inductances)***

Next class bring calculator

- CASIO fx-82MS