Chapter 6
Non-sinusoidal periodic waves

Day 41

Fourier Series:
Analysis of non-sinusoidal periodic signal

ILOs – Day 41

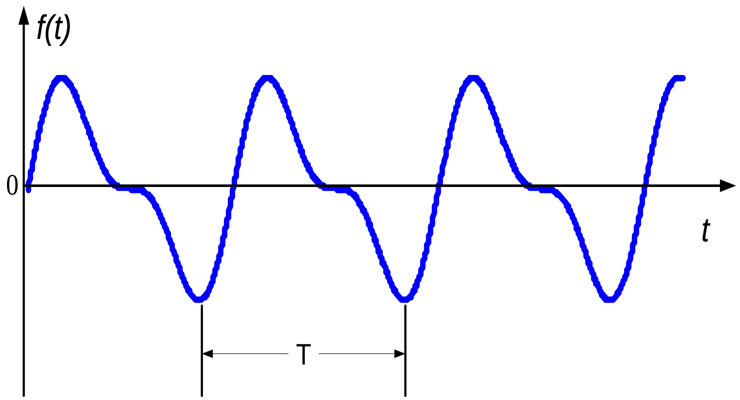
- Understand the concept of Fourier series expansion for analysis of non-sinusoidal periodic signals
- Apply Fourier series for analysis of non-sinusoidal periodic signals

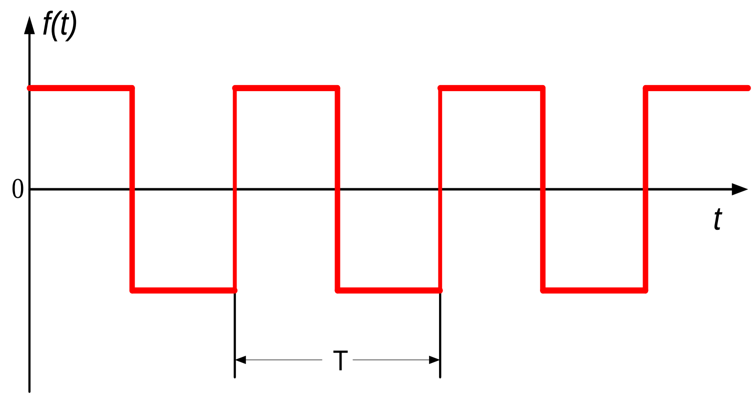
Periodic signals

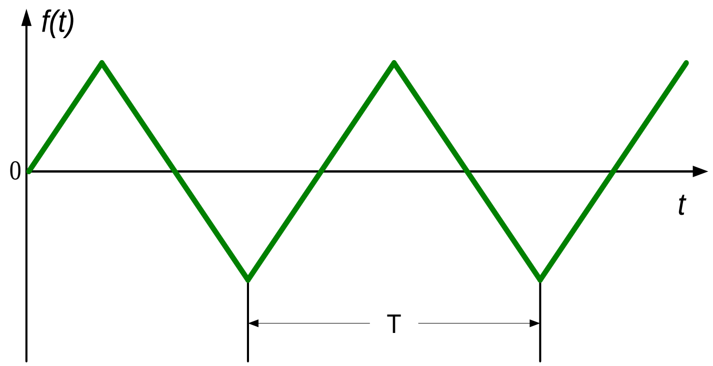
- A periodic signal is a kind of alternating signal that repeats itself after a fixed time period
- Whereas a sinusoidal periodic signal can be analyzed with ease, in real life most signals will contain disturbances and thus become son-sinusoidal
- The concept of Fourier series can be used effectively for analysis of such non-sinusoidal, yet periodic signals

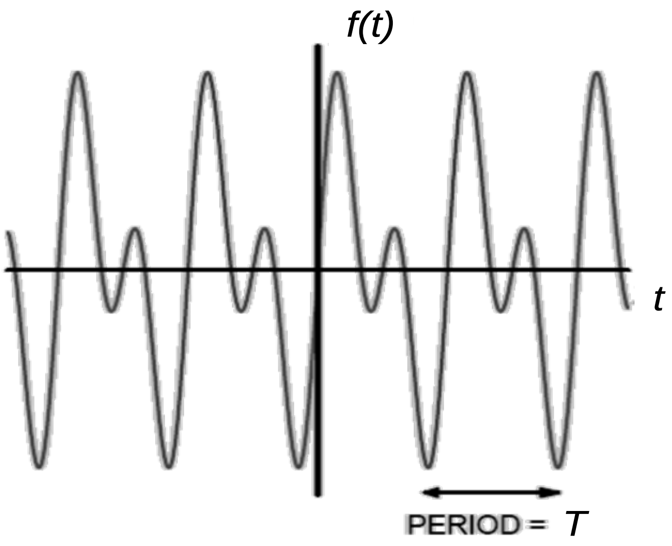
- Such a function $f(t)$ is periodic when it is defined for all real values of t, and if there is some positive real number T and any integer n such that $f(t + nT) = f(t)$, then T is the time period of the function

Examples of non-sinusoidal periodic signals









Fourier Series

- According to **Fourier Series**, such a periodic non-sinusoidal signal can be effectively represented by a suitable sum that involves:
  - a fundamental sine-wave plus
  - a combination of harmonics
  - These harmonics have different frequencies and different amplitudes

- The mathematical expression for such an infinite trigonometric Fourier Series expansion is:

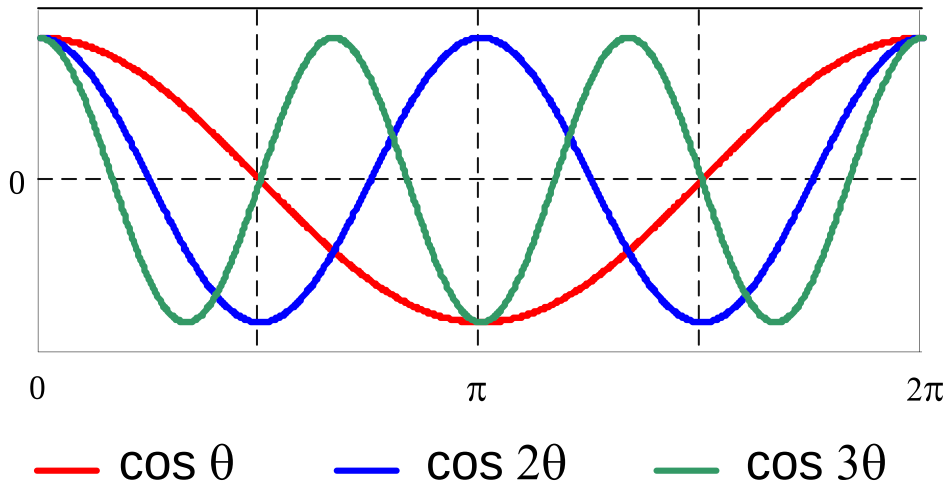
$$f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots\ldots\ldots + a_n \cos n\omega t + \ldots\ldots$$
$$+ b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots\ldots\ldots\ldots + b_n \sin n\omega t + \ldots\ldots$$

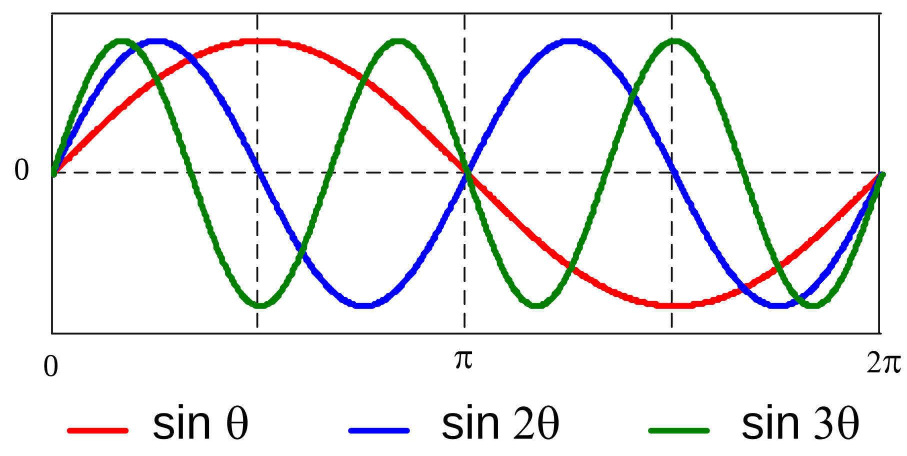
or, $$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right)$$

Fourier Series

$$f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots\ldots. + a_n \cos n\omega t + \ldots\ldots$$
$$+ b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots\ldots\ldots + b_n \sin n\omega t + \ldots\ldots$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right)$$



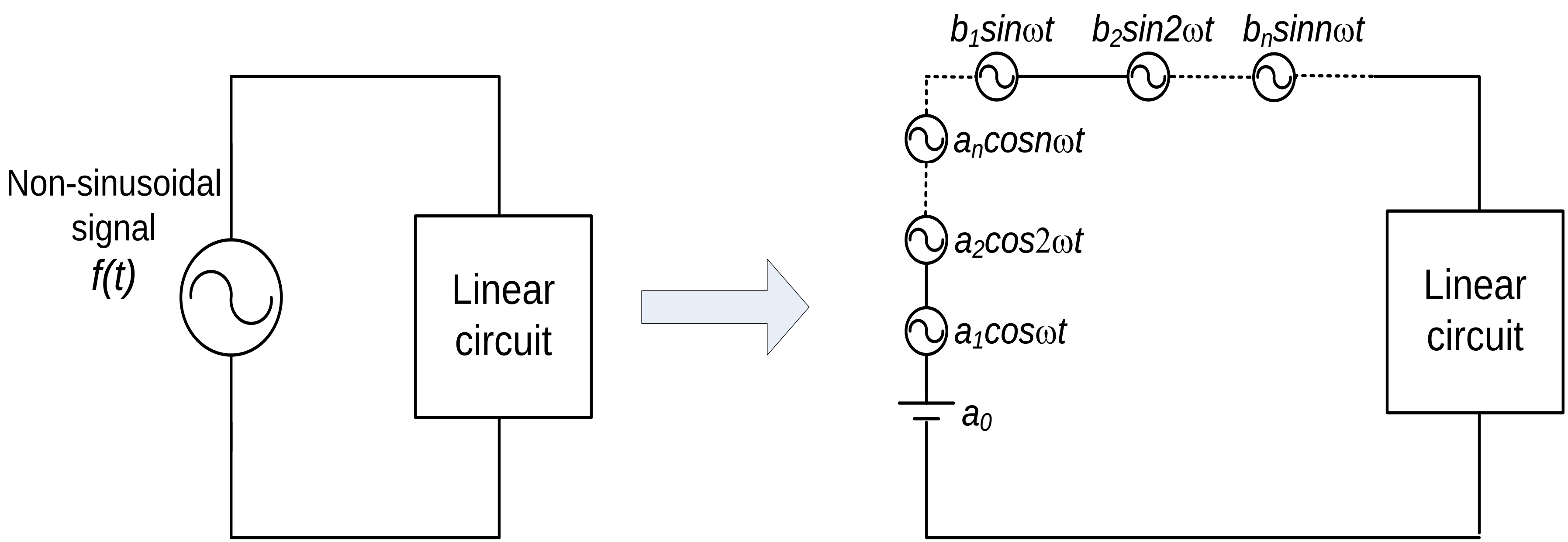


Fourier Series

$$f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \dots\dots + a_n \cos n\omega t + \dots\dots$$
$$+ b_1 \sin \omega t + b_2 \sin 2\omega t + \dots\dots + b_n \sin n\omega t + \dots\dots$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right)$$

- Schematically, the Fourier series expansion can be represented as:



Fourier Series

$$f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots\ldots\ldots + a_n \cos n\omega t + \ldots\ldots$$
$$+ b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots\ldots\ldots + b_n \sin n\omega t + \ldots\ldots$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right)$$

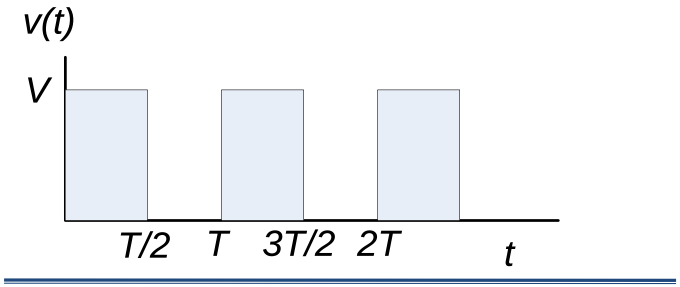
- The Fourier coefficients a_0, a_n, and b_n are:

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega t dt \qquad n = 1, 2, \cdots$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega t dt \qquad n = 1, 2, \cdots$$

Find Fourier series expansion for the following signal:



$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt \quad = \frac{V}{2}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

Define the signal mathematically:

$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

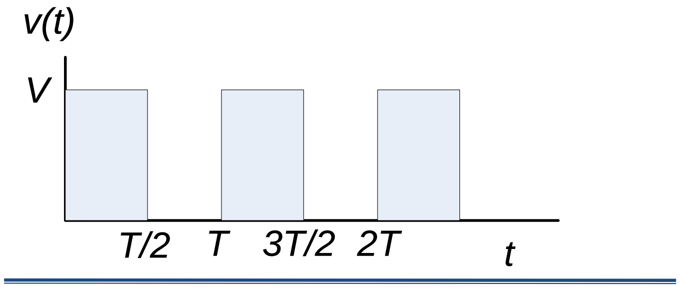
$$= 0 \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt \quad = \frac{1}{T}\left[\int_0^{\frac{T}{2}} f(t)dt + \int_{\frac{T}{2}}^{T} f(t)dt\right]$$

$$= \frac{1}{T}\left[\int_0^{\frac{T}{2}} V dt + \int_{\frac{T}{2}}^{T} 0 dt\right]$$

$$= \frac{1}{T}\left[Vt\right]_0^{\frac{T}{2}} \quad = \frac{1}{T} \times V \times \frac{T}{2} \quad = \frac{V}{2}$$

Find Fourier series expansion for the following signal:



$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt \quad = \frac{V}{2}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt$$

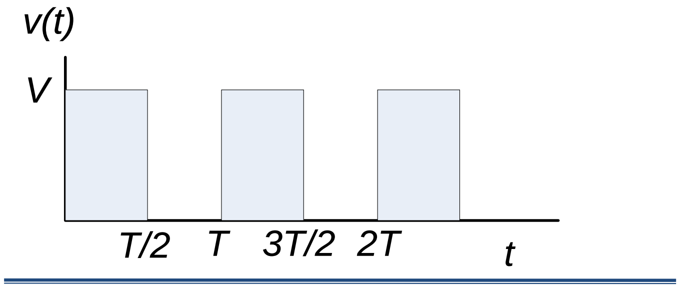
$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

Define the signal mathematically:

$$\begin{aligned} v(t) &= V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2} \\ &= 0 \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T \end{aligned}$$

$$\begin{aligned} a_n &= \frac{2}{T}\int_0^T f(t)\cos n\omega t dt \\ &= \frac{2}{T}\left[\int_0^{\frac{T}{2}} v(t)\cos n\omega t dt + \int_{\frac{T}{2}}^{T} v(t)\cos n\omega t dt\right] \\ &= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\cos n\omega t dt + \int_{\frac{T}{2}}^{T} 0\cos n\omega t dt\right] \end{aligned}$$

Find Fourier series expansion for the following signal:



$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt \quad = \frac{V}{2}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt$$

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Define the signal mathematically:

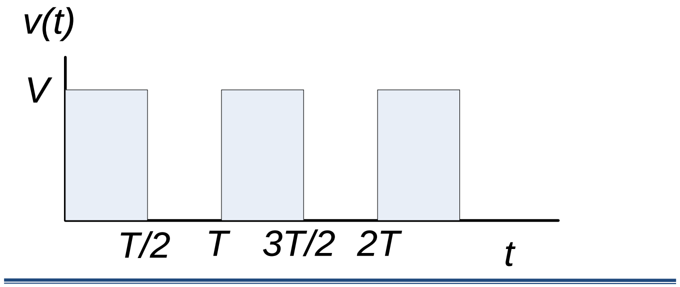
$$\begin{aligned} v(t) &= V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2} \\ &= 0 \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T \end{aligned}$$

$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\cos n\omega t dt + \int_{\frac{T}{2}}^{T} 0\cos n\omega t dt\right]$$

$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\cos n\omega t dt\right]$$

$$= \frac{2V}{T}\left[\int_0^{\frac{T}{2}} \cos n\frac{2\pi}{T} t dt\right] \qquad \because \omega = \frac{2\pi}{T}$$

Find Fourier series expansion for the following signal:



$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right)$$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt \quad = \frac{V}{2}$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega t dt \quad = 0$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega t dt$$

Define the signal mathematically:

$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

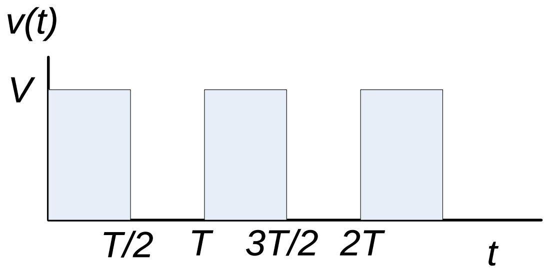
$$= 0 \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

$$= \frac{2V}{T} \left[\int_0^{\frac{T}{2}} \cos n \frac{2\pi}{T} t dt \right] \qquad \because \omega = \frac{2\pi}{T}$$

$$= \frac{2V}{T} \times \frac{T}{2n\pi} \left[\sin \frac{2n\pi}{T} t \right]_0^{\frac{T}{2}}$$

$$= \frac{V}{n\pi} [\sin n\pi - \sin 0] = \frac{V}{n\pi} [0 - 0] = 0$$

Find Fourier series expansion for the following signal:



$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right)$$

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$$a_0 = \frac{1}{T}\int_0^T f(t)dt \quad \boxed{= \frac{V}{2}}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt \quad \boxed{= 0}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$
$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} v(t)\sin n\omega t dt + \int_{\frac{T}{2}}^{T} v(t)\sin n\omega t dt\right]$$
$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\sin n\omega t dt + \int_{\frac{T}{2}}^{T} 0\sin n\omega t dt\right]$$

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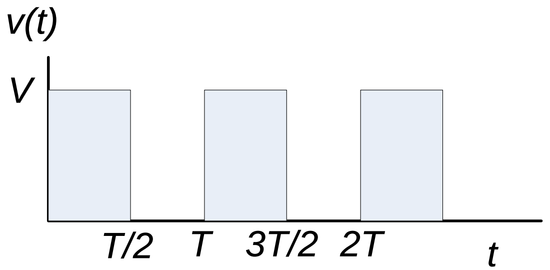

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right)$$

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$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V \sin n\omega t dt + \int_{\frac{T}{2}}^{T} 0 \sin n\omega t dt\right]$$

$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V \sin n\omega t dt\right]$$

$$= \frac{2V}{T}\left[\int_0^{\frac{T}{2}} \sin n\frac{2\pi}{T} t dt\right] \qquad \because \omega = \frac{2\pi}{T}$$

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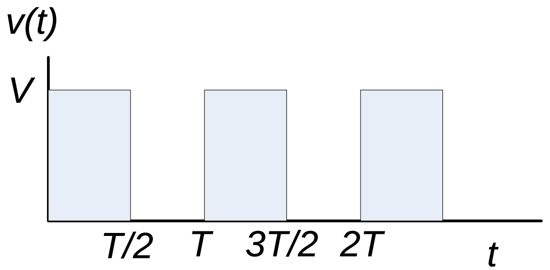


$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{V}{2}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

Define the signal mathematically:

$$v(t) = V \quad \text{for} \quad 0 < t < \frac{T}{2}$$

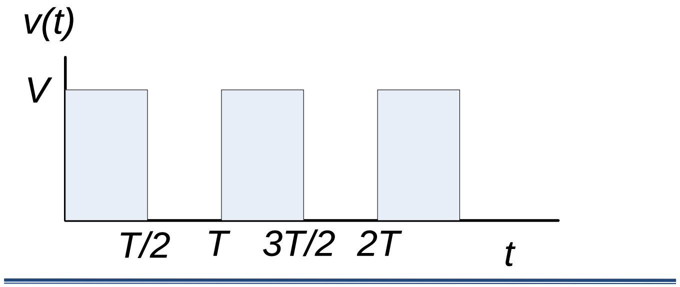
$$= 0 \quad \text{for} \quad \frac{T}{2} < t < T$$

$$= \frac{2V}{T}\left[\int_0^{\frac{T}{2}} \sin n\frac{2\pi}{T}t dt\right] \qquad \because \omega = \frac{2\pi}{T}$$

$$= -\frac{2V}{T} \times \frac{T}{2n\pi}\left[\cos\frac{2n\pi}{T}t\right]_0^{\frac{T}{2}}$$

$$= -\frac{V}{n\pi}[\cos n\pi - \cos 0]$$

Find Fourier series expansion for the following signal:



$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega t + b_n \sin n\omega t\right)$$

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$$a_0 = \frac{1}{T}\int_0^T f(t)dt \quad = \frac{V}{2}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt \quad = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

$$= -\frac{V}{n\pi}\left[\cos n\pi - \cos 0\right]$$

$$= \frac{V}{n\pi}\left[1 - \cos n\pi\right]$$

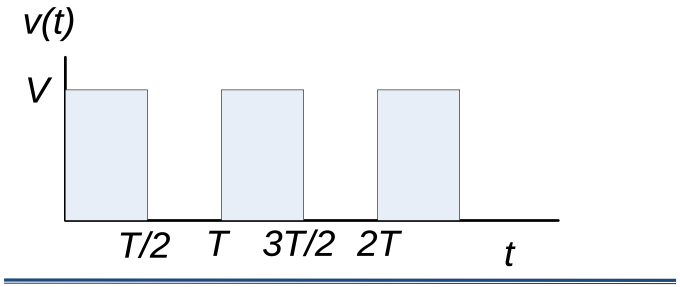
$$= 0 \qquad \text{for n} = \text{even}$$

$$= \frac{2V}{n\pi} \qquad \text{for n} = \text{odd}$$

$$= 0 \qquad \text{for n} = \text{even}$$

$$= \frac{2V}{n\pi} \qquad \text{for n} = \text{odd}$$

Find Fourier series expansion for the following signal:



$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right)$$

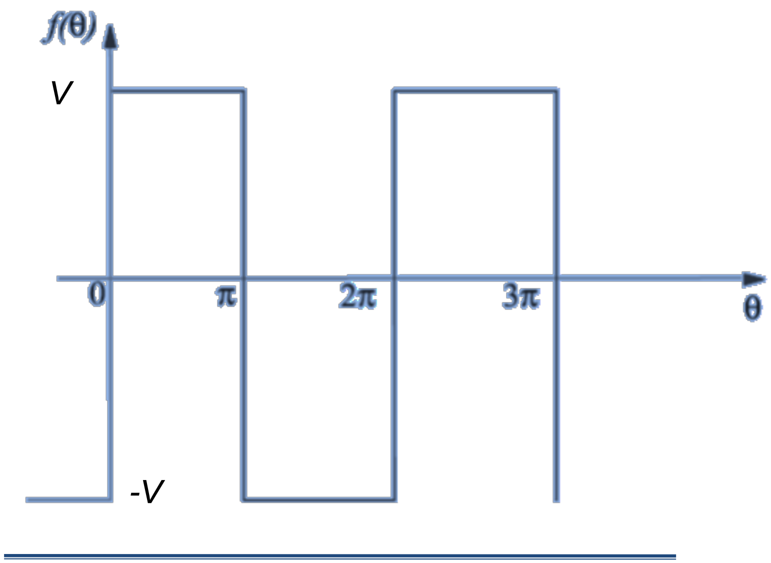
$$a_0 = \frac{V}{2} \qquad a_n = 0 \qquad b_n = \begin{cases} 0 & \text{for n = even} \\ \dfrac{2V}{n\pi} & \text{for n = odd} \end{cases}$$

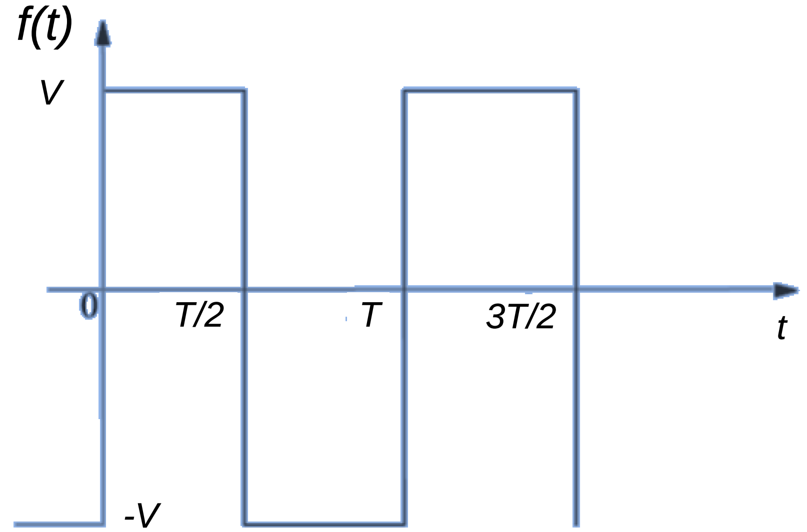
- Therefore, the corresponding Fourier series expansion is:

$$v(t) = \frac{V}{2} + \frac{2V}{\pi}\sin\omega t + \frac{2V}{3\pi}\sin 3\omega t + \frac{2V}{5\pi}\sin 5\omega t + \ldots\ldots\ldots\ldots$$

$$= V\left[\frac{1}{2} + \frac{2}{\pi}\sin\omega t + \frac{2}{3\pi}\sin 3\omega t + \frac{2}{5\pi}\sin 5\omega t + \ldots\ldots\ldots\ldots\right]$$

Find Fourier series expansion for the following signal:





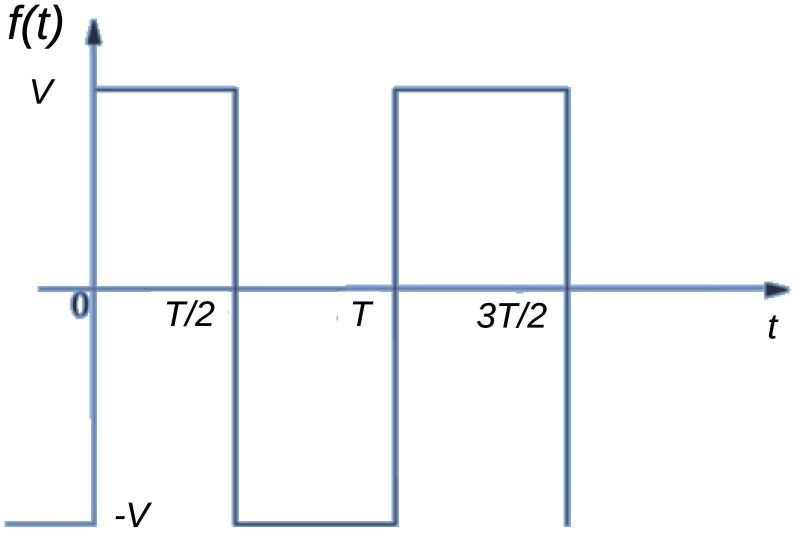
With the variable of integration being time (t), let us redraw the figure in terms of time along X-axis.

Define the signal mathematically:

$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

$$= -V \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

Find Fourier series expansion for the following signal:



$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

$$= -V \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt$$

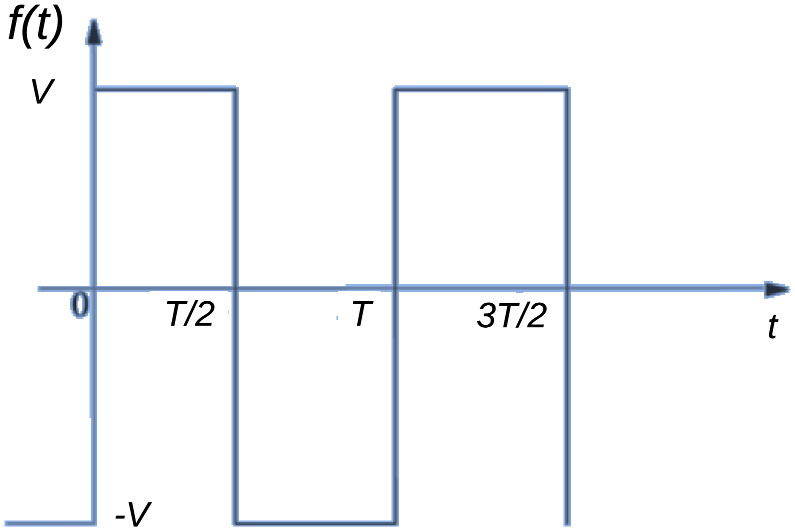
$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{\frac{T}{2}} f(t)\,dt + \int_{\frac{T}{2}}^{T} f(t)\,dt\right]$$

$$= \frac{1}{T}\left[\int_0^{\frac{T}{2}} V\,dt + \int_{\frac{T}{2}}^{T} -V\,dt\right]$$

Find Fourier series expansion for the following signal:



$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

$$= -V \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

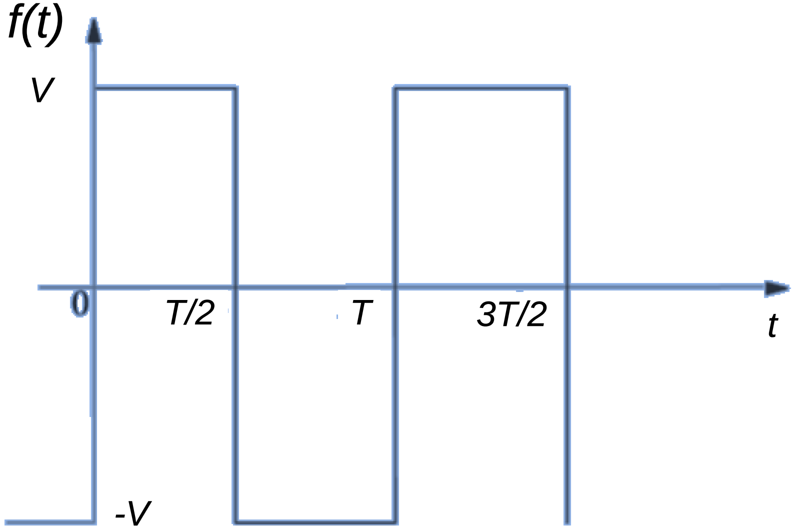
Note that a_0 is AVERAGE value of the signal

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T}\left[\int_0^{\frac{T}{2}} f(t)\,dt + \int_{\frac{T}{2}}^{T} f(t)\,dt\right]$$

$$= \frac{1}{T}\left[\int_0^{\frac{T}{2}} V\,dt + \int_{\frac{T}{2}}^{T} -V\,dt\right]$$

$$= \frac{1}{T}\left[(Vt)\Big|_0^{\frac{T}{2}} - (Vt)\Big|_{\frac{T}{2}}^{T}\right] = \frac{1}{T} \times V \times \frac{T}{2} - \frac{1}{T} \times V \times \frac{T}{2} = 0$$

Find Fourier series expansion for the following signal:



$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

$$= -V \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

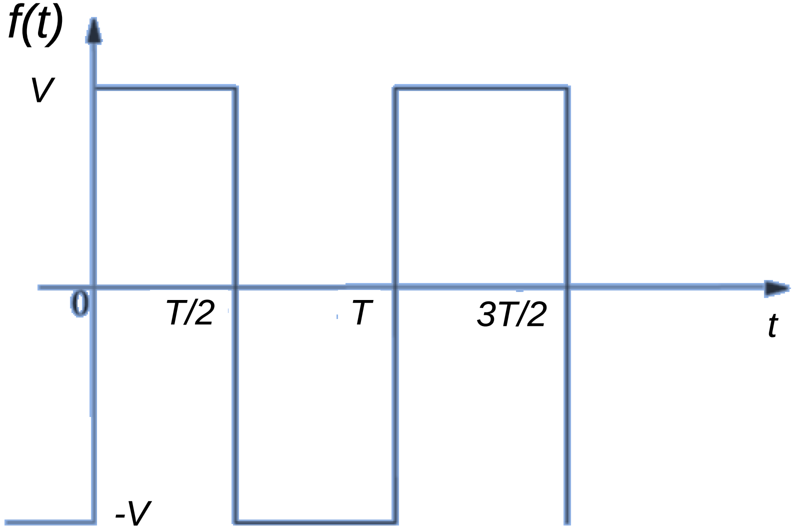
$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\, cos\, n\, \omega t dt = \frac{2}{T}\left[\int_0^{\frac{T}{2}} v(t)\, cos\, n\, \omega t dt + \int_{\frac{T}{2}}^{T} v(t)\, cos\, n\, \omega t dt\right]$$

$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\, cos\, n\, \omega t dt + \int_{\frac{T}{2}}^{T} (-V)\, cos\, n\, \omega t dt\right]$$

$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\, cos\, n\frac{2\pi}{T} t dt + \int_{\frac{T}{2}}^{T} (-V)\, cos\, n\frac{2\pi}{T} t dt\right]$$

Find Fourier series expansion for the following signal:



$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

$$= -V \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt$$

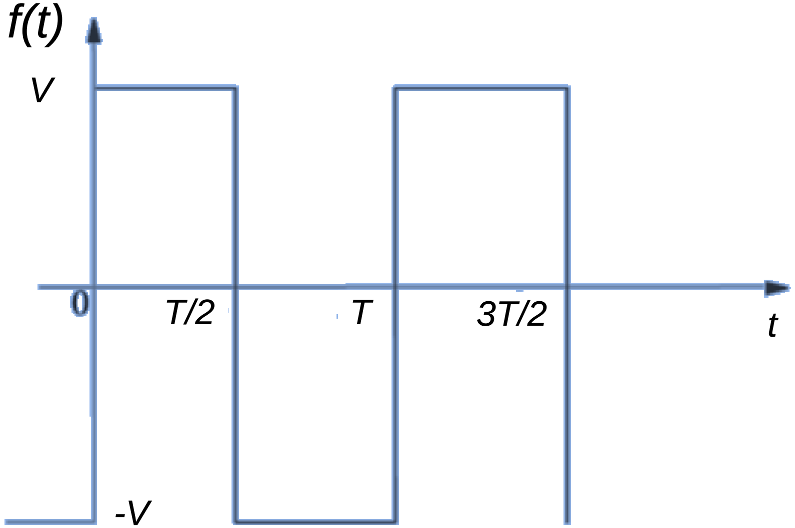
$$a_n = \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\, cos\, n\frac{2\pi}{T}tdt + \int_{\frac{T}{2}}^{T}(-V)\, cos\, n\frac{2\pi}{T}tdt\right]$$

$$= \frac{2V}{T}\times\frac{T}{2n\pi}\left\{\left[sin\frac{2n\pi}{T}t\right]_0^{\frac{T}{2}} - \left[sin\frac{2n\pi}{T}t\right]_{\frac{T}{2}}^{T}\right\}$$

$$= \frac{V}{n\pi}[sin\, n\, \pi - (sin2n\pi - sinn\pi)]$$

$$= 0$$

Find Fourier series expansion for the following signal:



$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

$$= -V \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

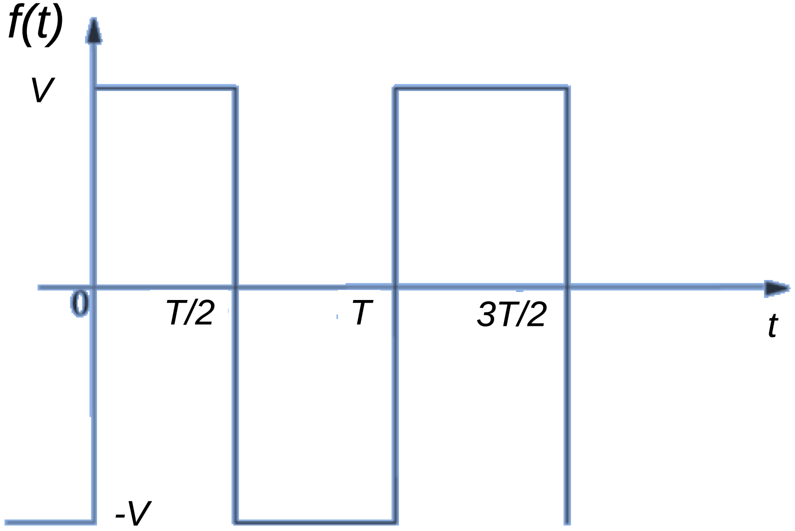
$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\, sin\, n\, \omega t dt \;= \frac{2}{T}\left[\int_0^{\frac{T}{2}} v(t)\, sin\, n\, \omega t dt + \int_{\frac{T}{2}}^{T} v(t)\, sin\, n\, \omega t dt\right]$$

$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\, sin\, n\, \omega t dt + \int_{\frac{T}{2}}^{T} -V\, sin\, n\, \omega t dt\right]$$

$$= \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\, sin\, n\frac{2\pi}{T} t dt + \int_{\frac{T}{2}}^{T} (-V)\, sin\, n\frac{2\pi}{T} t dt\right]$$

Find Fourier series expansion for the following signal:



$$v(t) = V \quad \text{for} \quad 0 < \text{t} < \frac{\text{T}}{2}$$

$$= -V \quad \text{for} \quad \frac{\text{T}}{2} < \text{t} < T$$

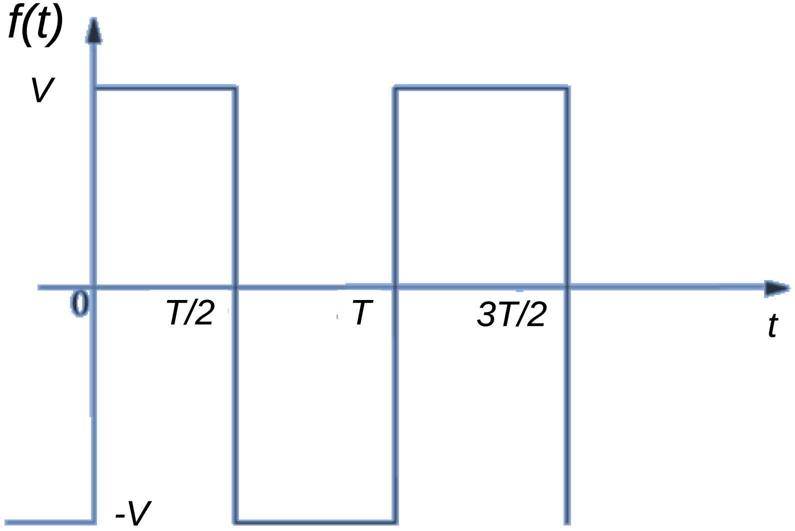
$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

$$b_n = \frac{2}{T}\left[\int_0^{\frac{T}{2}} V\, sin\, n\frac{2\pi}{T}tdt + \int_{\frac{T}{2}}^{T} (-V)\, sin\, n\frac{2\pi}{T}tdt\right]$$

$$= \frac{2V}{T} \times \frac{T}{2n\pi}\left\{\left[-cos\frac{2n\pi}{T}t\right]_0^{\frac{T}{2}} - \left[-cos\frac{2n\pi}{T}t\right]_{\frac{T}{2}}^{T}\right\}$$

$$= -\frac{V}{n\pi}\left\{\left[cos\frac{2n\pi}{T}t\right]_0^{\frac{T}{2}} - \left[cos\frac{2n\pi}{T}t\right]_{\frac{T}{2}}^{T}\right\}$$

Find Fourier series expansion for the following signal:



$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

$$= -V \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

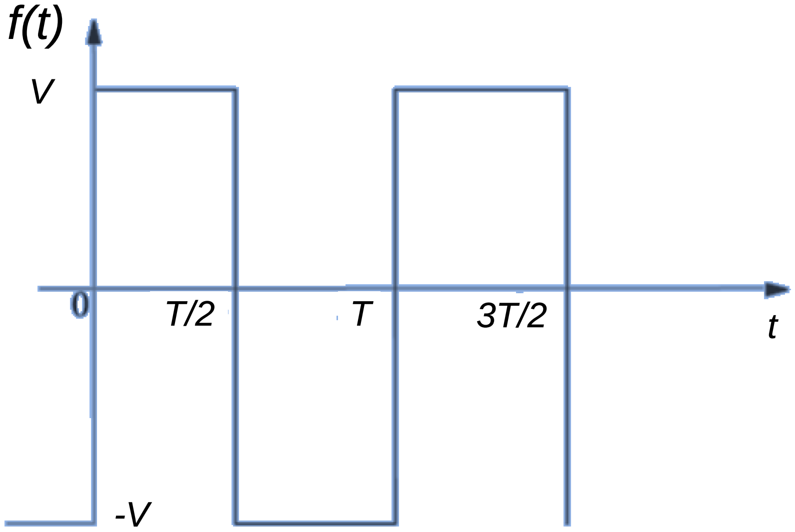
$$b_n = -\frac{V}{n\pi}\left\{\left[cos\frac{2n\pi}{T}t\right]_0^{\frac{T}{2}} - \left[cos\frac{2n\pi}{T}t\right]_{\frac{T}{2}}^{T}\right\}$$

$$= -\frac{V}{n\pi}\{[cosn\pi - cos0] - [cos2n\pi - cosn\pi]\}$$

$$= -\frac{V}{n\pi}\{[cosn\pi - 1] - [1 - cosn\pi]\}$$

$$= -\frac{2V}{n\pi}(cosn\pi - 1) = \frac{2V}{n\pi}(1 - cosn\pi)$$

Find Fourier series expansion for the following signal:



$$v(t) = V \quad \text{for} \quad 0 < \mathrm{t} < \frac{\mathrm{T}}{2}$$

$$= -V \quad \text{for} \quad \frac{\mathrm{T}}{2} < \mathrm{t} < T$$

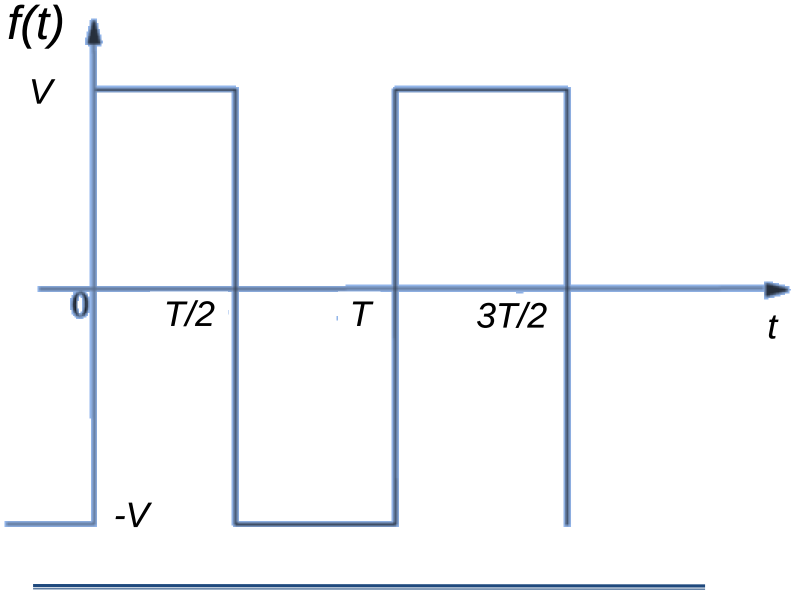
$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

$$b_n = \frac{2V}{n\pi}(1 - cos n\pi)$$

$$= 0 \; for \; n = Even$$

$$= \frac{4V}{n\pi} \; for \; n = Odd$$

Find Fourier series expansion for the following signal:



$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt \qquad = 0$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt \qquad = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

$$= 0 \; for \; n = Even$$

$$= \frac{4V}{n\pi} \; for \; n = Odd$$

Therefore, the corresponding Fourier series expansion is:

$$v(t) = \sum_{\substack{n=1 \\ n=odd}}^{\infty} \frac{4V}{n\pi} \sin n\omega\, t$$

$$v(t) = \frac{4V}{\pi}\left[\sin \omega\, t + \frac{1}{3}\sin 3\,\omega t + \frac{1}{5}\sin 5\,\omega t + \ldots\ldots\ldots\ldots\right]$$