Tutorial 1

Day 8a: AC signals

ILOs – Day 8a (Tutorial 1)

- Solve numerical problems related to AC signal parameters

#1) The equation of an AC current is $i = 62.35 \sin 323t$

Determine

a) **Its maximum value**

b) **Its frequency**

c) **Its RMS value**

d) **Its average value**

e) **Its form factor**

$$i = I_m \sin \frac{2\pi}{T} t$$

Comparing the signal expression with the standard form of a sinusoidal signal :

$$\boxed{e = E_m \sin \omega t = E_m \sin 2\pi f t = E_m \sin \frac{2\pi}{T} t}$$

a) Peak value $I_m = 62.35\ A$

b) $\frac{2\pi}{T} = 323$ $\therefore$ Frequency: $f = \frac{1}{T} = \frac{323}{2\pi} = 51.4\ Hz$

c) Since the current signal is a pure sine wave, its RMS value is:

$$i_{RMS} = \frac{I_m}{\sqrt{2}} = \frac{62.35}{\sqrt{2}} = 44.08\ A$$

#1) The equation of an AC current is $i = 62.35 \sin 323t$

Determine

a) **Its maximum value**

$$i = I_m \sin \frac{2\pi}{T} t$$

b) **Its frequency**

c) **Its RMS value**

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d) Average value: $i_{av} = \frac{2I_m}{\pi} = \frac{2 \times 62.35}{\pi} = 39.69\ A$

e) Form factor:

$$K_f = \frac{\text{RMS value of the signal}}{\text{Average value of the signal}} = \frac{44.08}{39.69} = 1.11$$

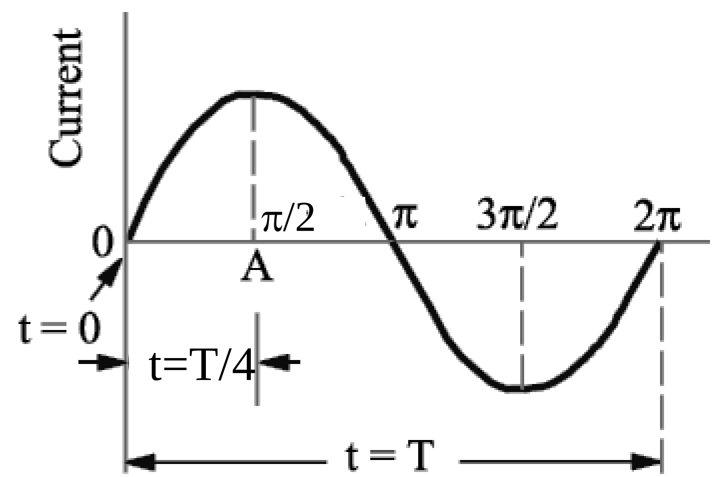
#2) An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value **(a) 0.0025 second (b) 0.0125 second after passing through a positive maximum value. (c)At what time,** measured from a positive maximum value, will the instantaneous current be 14.14 A ?

Peak value, $I_m = 20\sqrt{2} = 28.2\ A$

Angular frequency, $\omega = 2\pi \times 50 = 100\pi\ rad/s$

The equation of the sinusoidal current wave starting at the origin (without any phase shift) is: $i = 28.2 \sin 100\pi t$

Time period, $T = \frac{1}{f} = \frac{1}{50} = 0.02\ \text{sec}$



The signal reaches positive maximum value at point A at time T/4 = 0.005 s

We need to find instantaneous values at

a) $t_1 = 0.005 + 0.0025 = 0.0075$ s and

b) $t_2 = 0.005 + 0.0125 = 0.0175$ s

#2) An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value **(a) 0.0025 second (b) 0.0125 second after passing through a positive maximum value. (c)At what time,** measured from a positive maximum value, will the instantaneous current be 14.14 A ?

$$i = 28.2 \sin 100\pi t$$

(a) $t_1 = 0.0075$ s (b) $t_2 = 0.0175$ s

(a) $i_1 = 28.2 \sin 100\pi(0.0075) = 19.94\ A$

Note that these angles are in radian

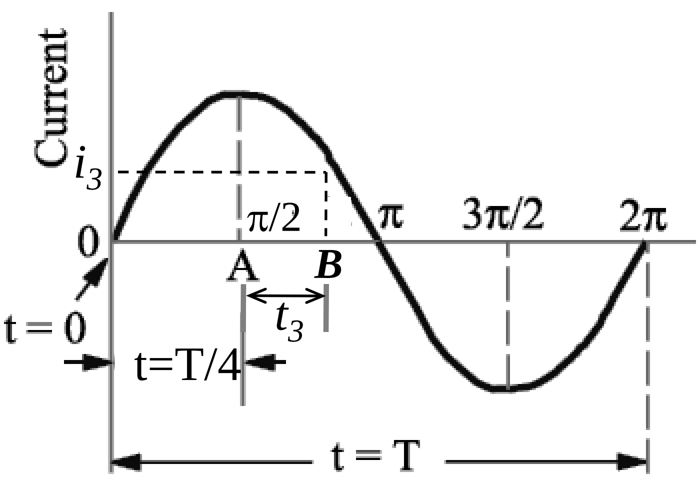
(b) $i_2 = 28.2 \sin 100\pi(0.0175) = -20.36\ A$

(c) Here at point *B*, i_3 = 14.14 A

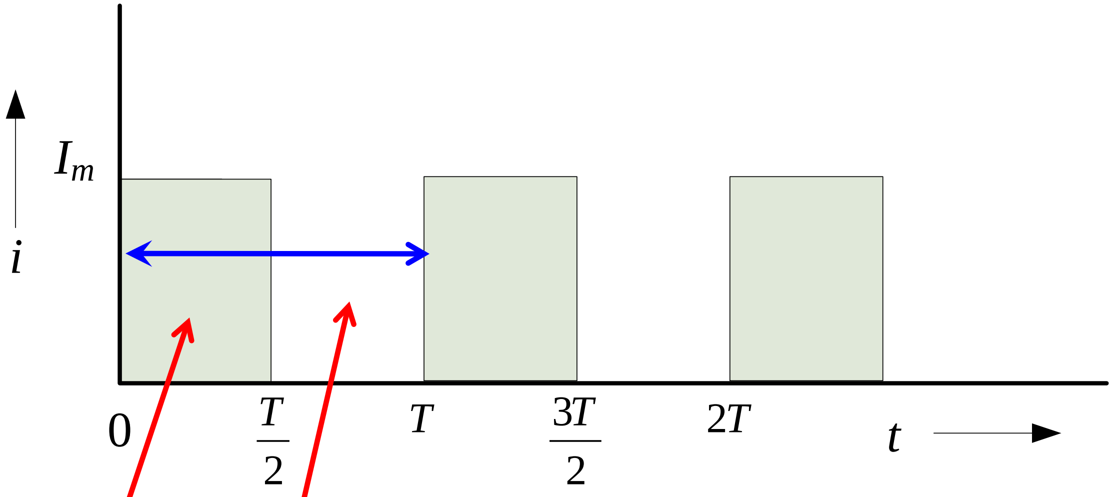
$$\Rightarrow 14.14 = 28.2 \sin\left(\frac{\pi}{2} + 100\pi t_3\right)$$

$$\Rightarrow 14.14 = 28.2 \cos(100\pi t_3)$$

$$\Rightarrow t_3 = 0.0033\ s$$



#3) Find RMS value and average value of the following signal:



RMS value

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

$$I_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{T/2} i^2 dt + \int_{T/2}^{T} i^2 dt\right]}$$

$$I_{rms} = \sqrt{\frac{1}{T}\left[\int_0^{T/2} I_m{}^2 dt + \int_{T/2}^{T} 0^2 dt\right]}$$

$$I_{rms} = \sqrt{\frac{1}{T}\left[I_m{}^2 \int_0^{T/2} dt + 0\right]}$$

$$I_{rms} = \sqrt{\frac{I_m{}^2}{T}\left[t\Big|_0^{T/2}\right]}$$

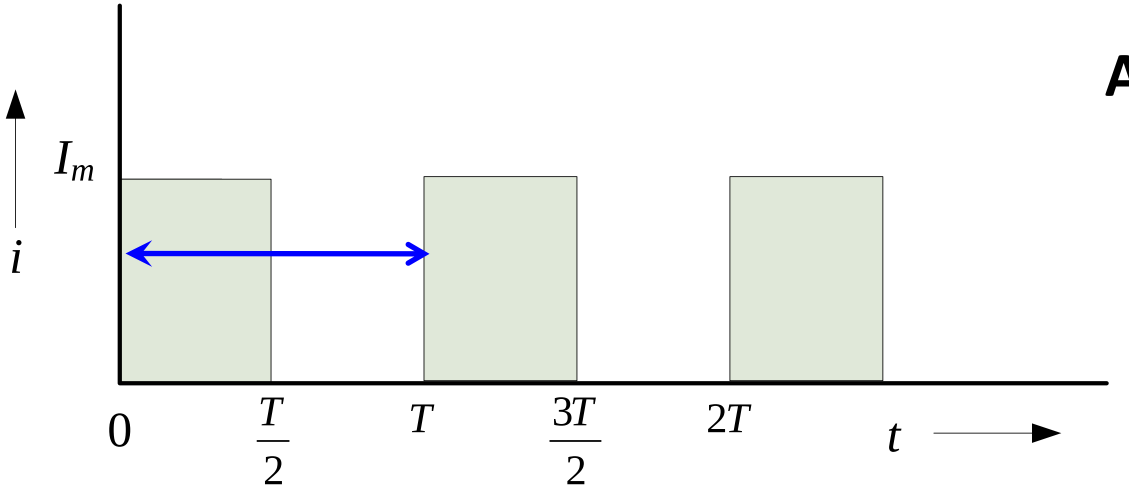
$$I_{rms} = \sqrt{\frac{I_m{}^2}{T}\left[\frac{T}{2} - 0\right]}$$

$$I_{rms} = \sqrt{\frac{I_m{}^2}{T} \times \frac{T}{2}}$$

$$I_{rms} = \sqrt{\frac{I_m{}^2}{2}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

Find RMS value and average value of the following signal:



Average value

$$I_{av} = \frac{1}{T}\int_0^T i\,dt$$

$$I_{av} = \frac{1}{T}\left[\int_0^{T/2} i\,dt + \int_{T/2}^{T} i\,dt\right]$$

$$I_{av} = \frac{1}{T}\left[\int_0^{T/2} I_m\,dt + \int_{T/2}^{T} 0\,dt\right]$$

$$I_{av} = \frac{1}{T}\left[I_m \int_0^{T/2} dt + 0\right]$$

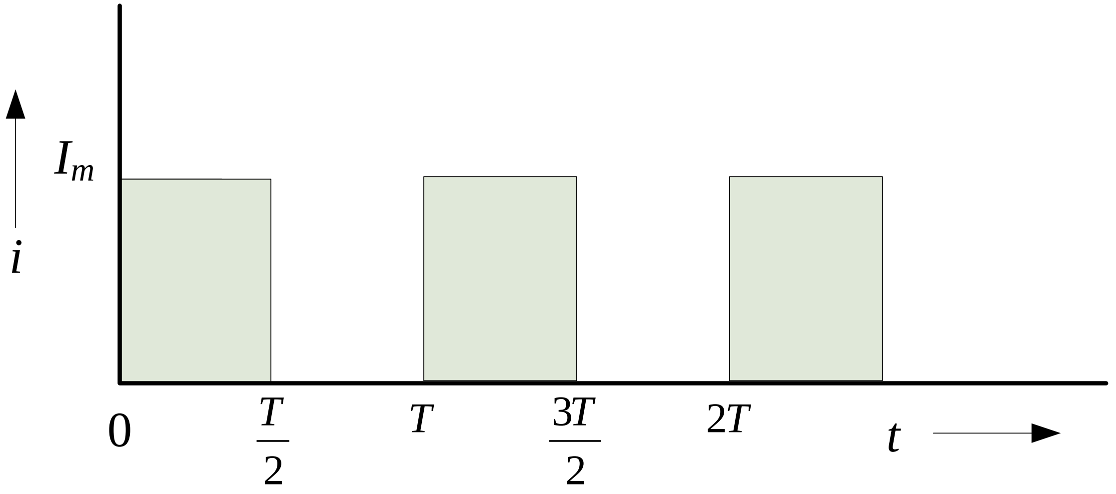
$$I_{av} = \frac{1}{T}\left[I_m\left[t\Big|_0^{T/2}\right]\right]$$

$$I_{av} = \frac{1}{T}\left[I_m\left(\frac{T}{2} - 0\right)\right]$$

$$I_{av} = \frac{1}{T} \times I_m \times \frac{T}{2}$$

$$\boxed{I_{av} = \frac{I_m}{2}}$$

#4) Find peak factor and form factor of the following signal:



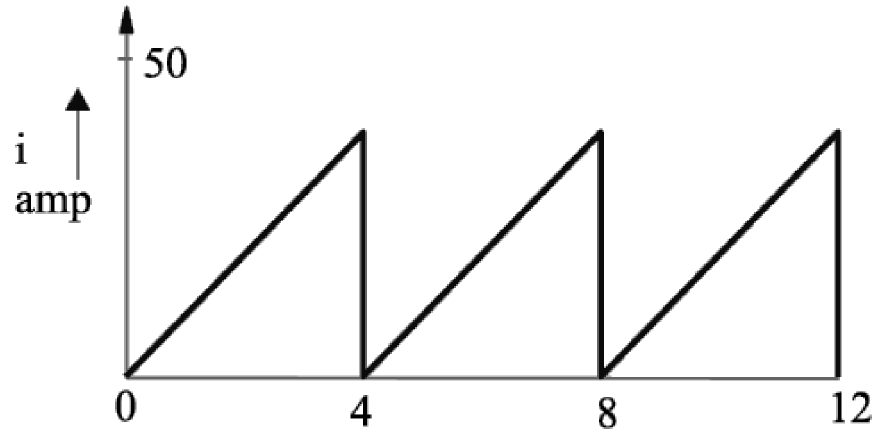
$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$I_{av} = \frac{I_m}{2}$$

$$K_p = \frac{\text{Maximum value of the signal}}{\text{RMS value of the signal}} = \frac{I_m}{I_m / \sqrt{2}} = \sqrt{2}$$

$$K_f = \frac{\text{RMS value of the signal}}{\text{Average value of the signal}} = \frac{I_m / \sqrt{2}}{I_m / 2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

#5) Find form factor of the following signal:



$$K_f = \frac{\text{RMS value of the signal}}{\text{Average value of the signal}}$$

Time period of the current T = 4 s

Equation of the current signal in the interval (0 – T) is: $i(t) = \frac{50}{4} t$

Average value

$$I_{av} = \frac{1}{T}\left[\int_0^T i(t)dt\right]$$

$$I_{av} = \frac{1}{4}\left[\int_0^4 \frac{50}{4} tdt\right]$$

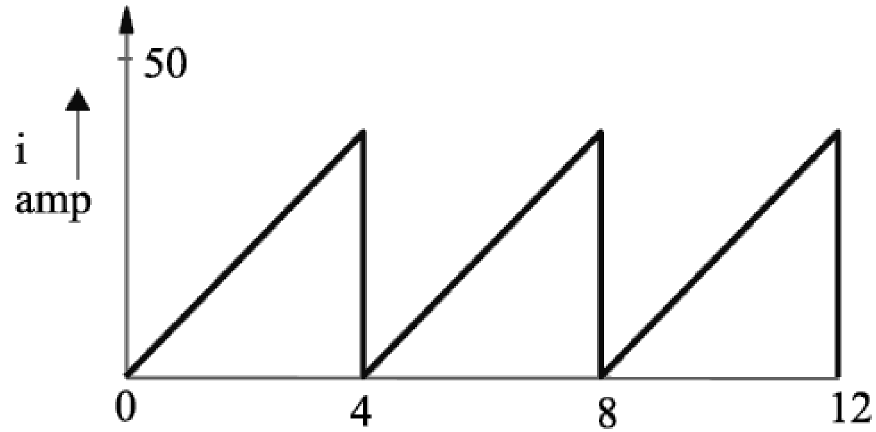
$$I_{av} = \frac{50}{16}\left[\int_0^4 tdt\right]$$

$$I_{av} = \frac{50}{16}\left[\frac{1}{2}t^2\Big|_0^4\right]$$

$$I_{av} = \frac{50}{32}[16-0]$$

$$I_{av} = 25\ A$$

#5) Find form factor of the following signal:



$$K_f = \frac{\text{RMS value of the signal}}{\text{Average value of the signal}}$$

$$K_f = \frac{28.87}{25} = 1.155$$

Time period of the current T = 4 s

Equation of the current signal in the interval (0 – T) is: $i(t) = \frac{50}{4}t$

RMS value $$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2 dt}$$

$$I_{rms} = \sqrt{\frac{1}{4}\int_0^4 \left[\frac{50}{4}t\right]^2 dt}$$

$$I_{rms} = \sqrt{\frac{2500}{64}\int_0^4 t^2 dt}$$

$$I_{rms} = \sqrt{\frac{2500}{64}\left[\frac{1}{3}t^3\Big|_0^4\right]}$$

$$I_{rms} = \sqrt{\frac{2500}{64}\left[\frac{1}{3}(64-0)\right]}$$

$$I_{rms} = 28.87\ A$$