3-Phase systems

Day 27

3-Phase power

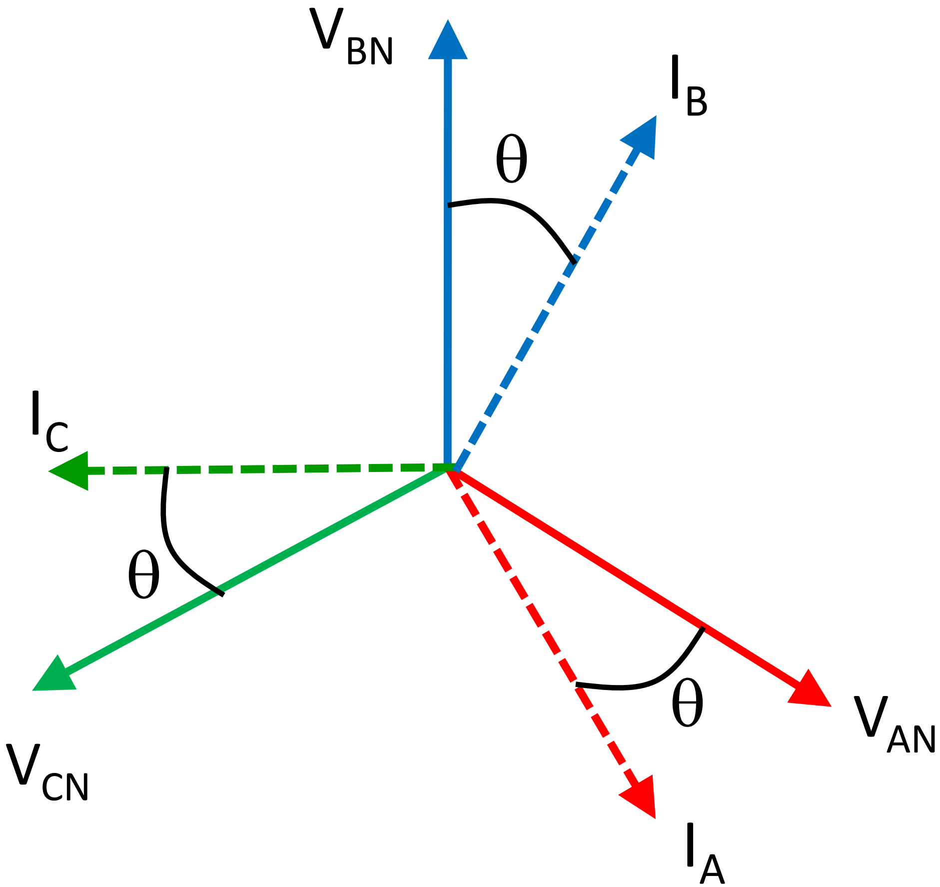
ILOs – Day 27

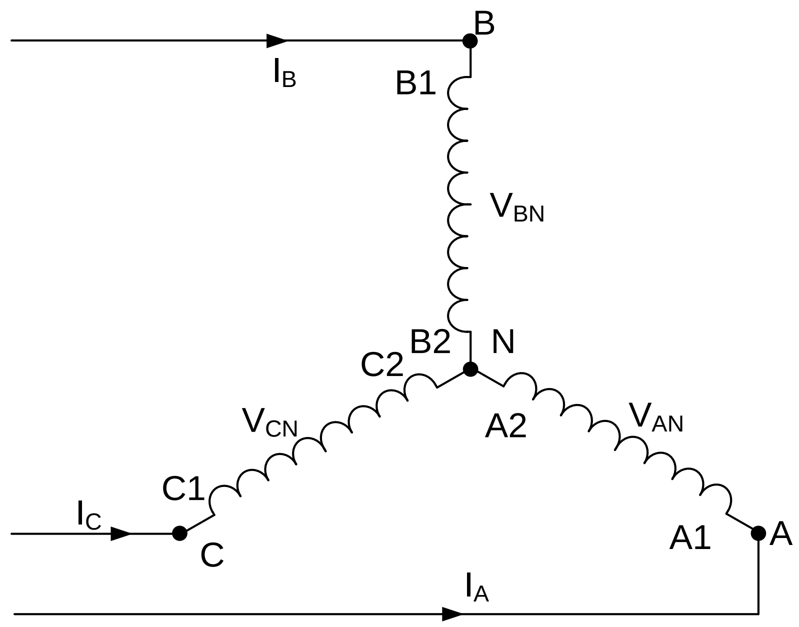
- Derive the power equations in 3-phase balanced system
- Explain and apply the 2-wattmeter method for measurement of 3-phase power
- Understand the effect of power factor on wattmeter readings in 2-wattmeter method for measurement of 3-phase power

Three phase power in Star connected system

Consider the system to be balanced

- *Phase voltages are equal and 120^0 apart*
- *Phase currents are also equal and 120^0 apart*





Total 3-phase active power (Watt)

$$P = 3 \times \left(V_{ph} \times I_{ph} \times \cos\theta\right)$$

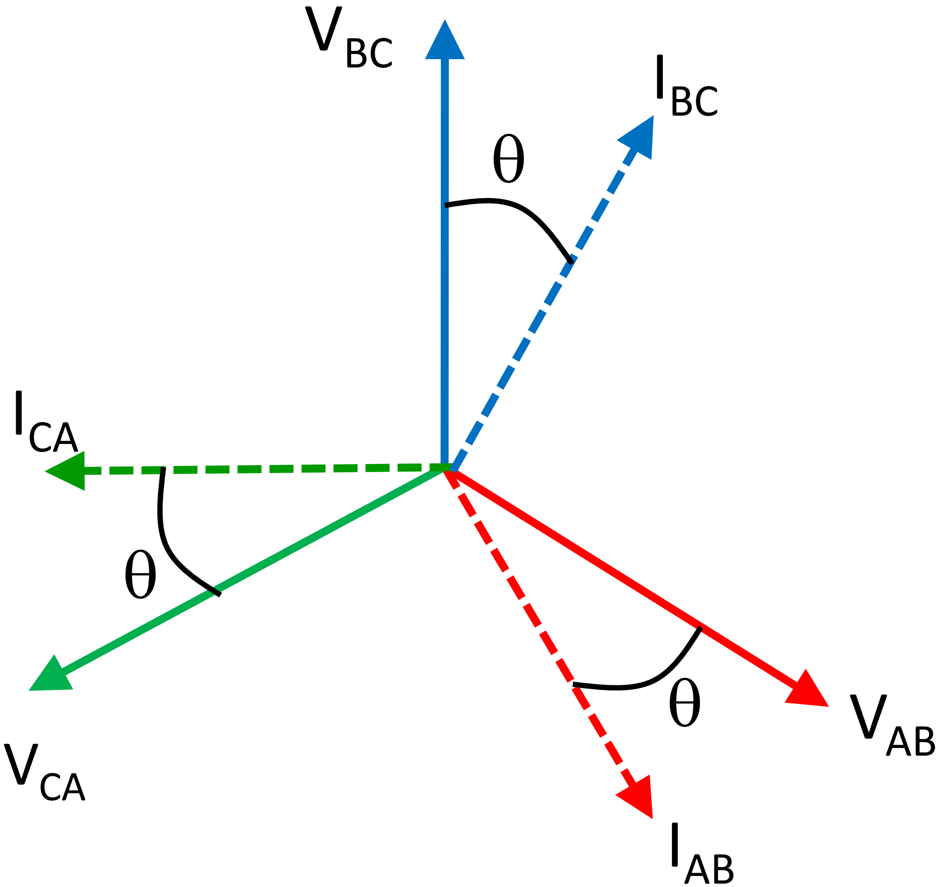
$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\theta$$

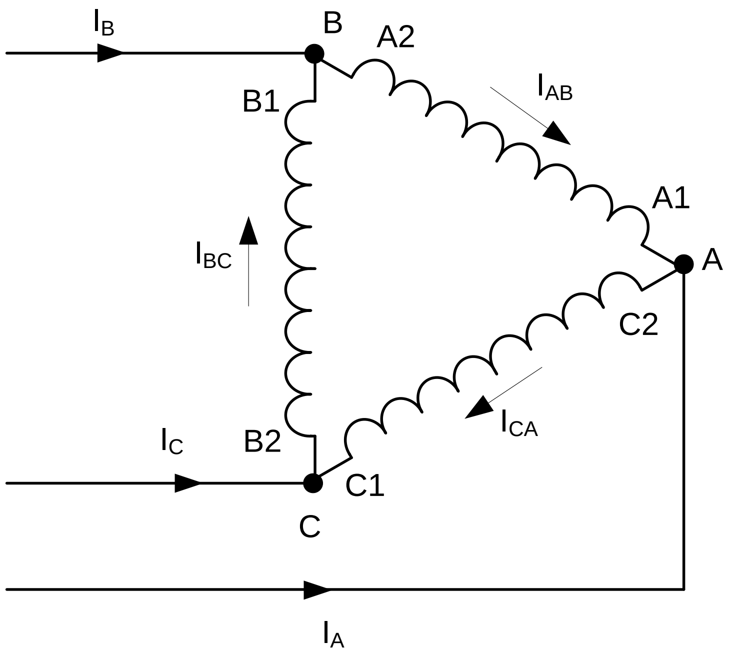
$$= \sqrt{3} V_L I_L \cos\theta$$

Three phase power in Delta connected system

Consider the system to be balanced

- *Phase voltages are equal and 120^0 apart*
- *Phase currents are also equal and 120^0 apart*





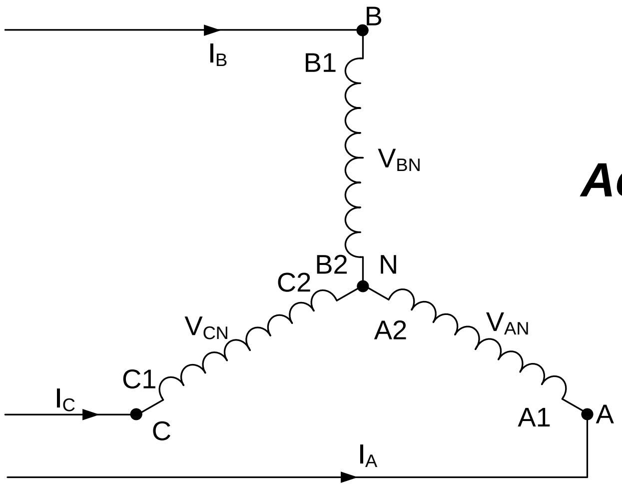
Total 3-phase active power (Watt)

$$P = 3\times\left(V_{ph}\times I_{ph}\times\cos\theta\right)$$

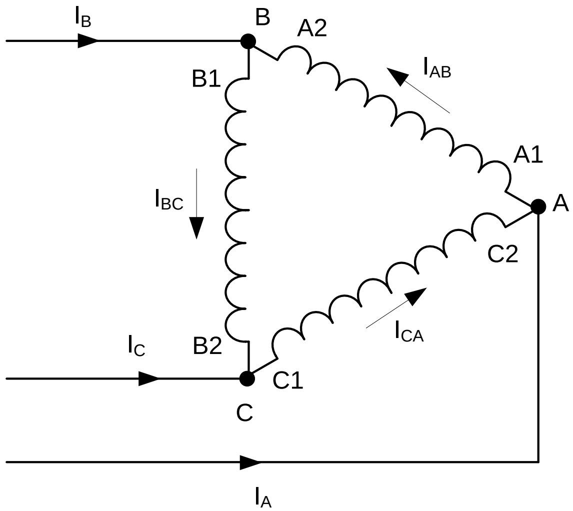
$$= 3\times V_L\times\frac{I_L}{\sqrt{3}}\times\cos\theta$$

$$= \sqrt{3}V_L I_L\cos\theta$$

Three phase power



Active power (Watt)



$$P = 3\times\left(V_{ph}\times I_{ph}\times\cos\theta\right)$$

$$= 3\times\frac{V_L}{\sqrt{3}}\times I_L\times\cos\theta$$

$$= \sqrt{3}V_L I_L\cos\theta$$

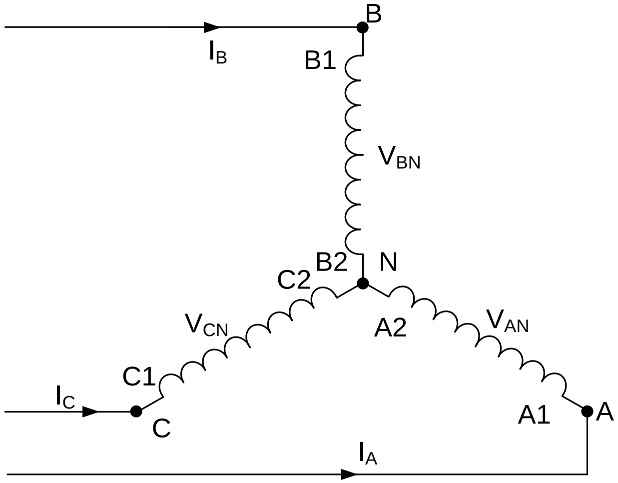
$$P = 3\times\left(V_{ph}\times I_{ph}\times\cos\theta\right)$$

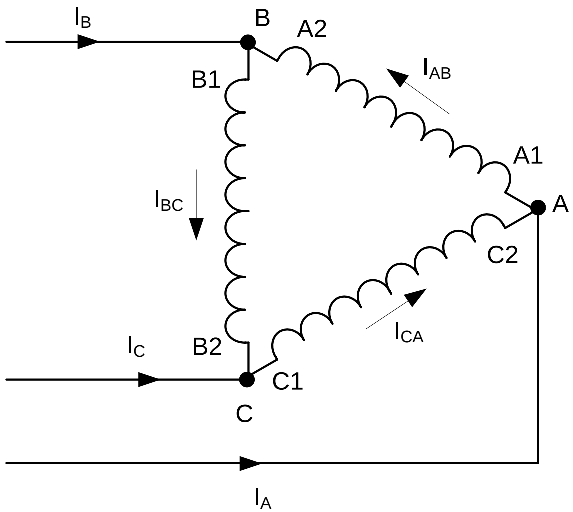
$$= 3\times V_L\times\frac{I_L}{\sqrt{3}}\times\cos\theta$$

$$= \sqrt{3}V_L I_L\cos\theta$$

Irrespective of the type of connection, star or delta, the power delivered (or consumed) remains same for a given three phase system

Three phase powers





$$P = \sqrt{3} V_L I_L \cos\theta$$ ***Active power (W)*** $$P = \sqrt{3} V_L I_L \cos\theta$$

$$Q = \sqrt{3} V_L I_L \sin\theta$$ ***Reactive power (VAr)*** $$Q = \sqrt{3} V_L I_L \sin\theta$$

$$S = \sqrt{3} V_L I_L$$ ***Apparent (total) power (VA)*** $$S = \sqrt{3} V_L I_L$$

$$= P + jQ$$

$$= \sqrt{P^2 + Q^2}, \angle \tan^{-1}\frac{Q}{P}$$

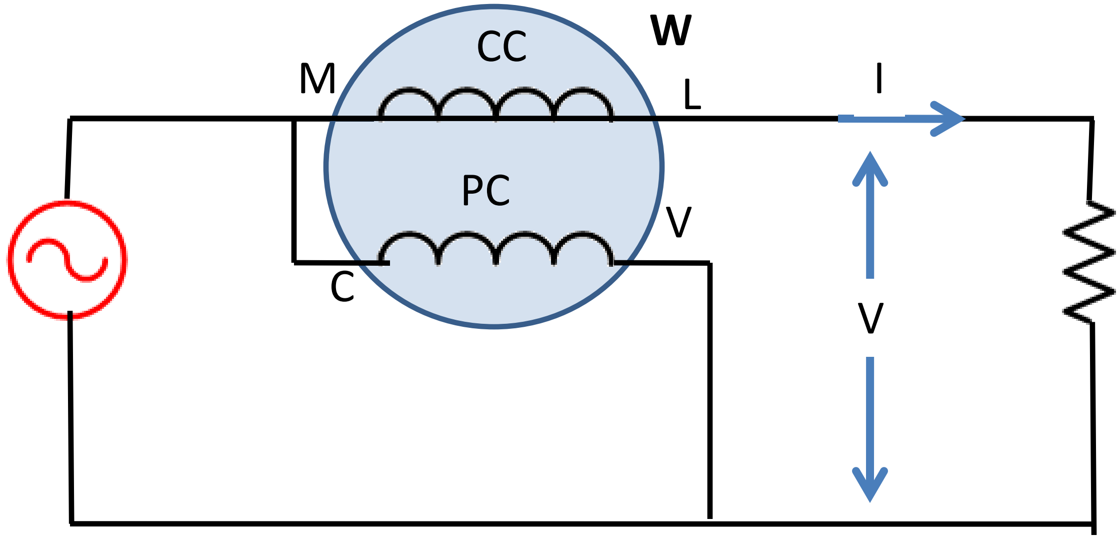
$$= P + jQ$$

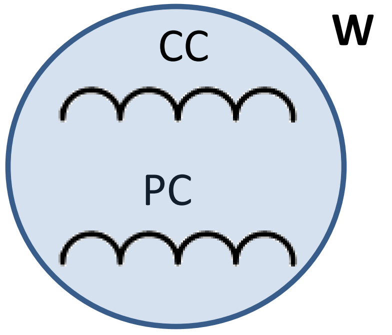
$$= \sqrt{P^2 + Q^2}, \angle \tan^{-1}\frac{Q}{P}$$

Power factor, $$\cos\theta = \frac{P}{S} = \frac{\text{Active power}}{\text{Appartent power}}$$

Measurement of power

- A wattmeter is used to measure active power
- Thus, it has to measure voltage and current both at the same time
- Thus, a wattmeter has two measuring coils
- One is current coil (CC) connected in series with the line to measure current
- The other is the voltage coil or pressure coil (PC) connected across the line (parallel) to measure the voltage

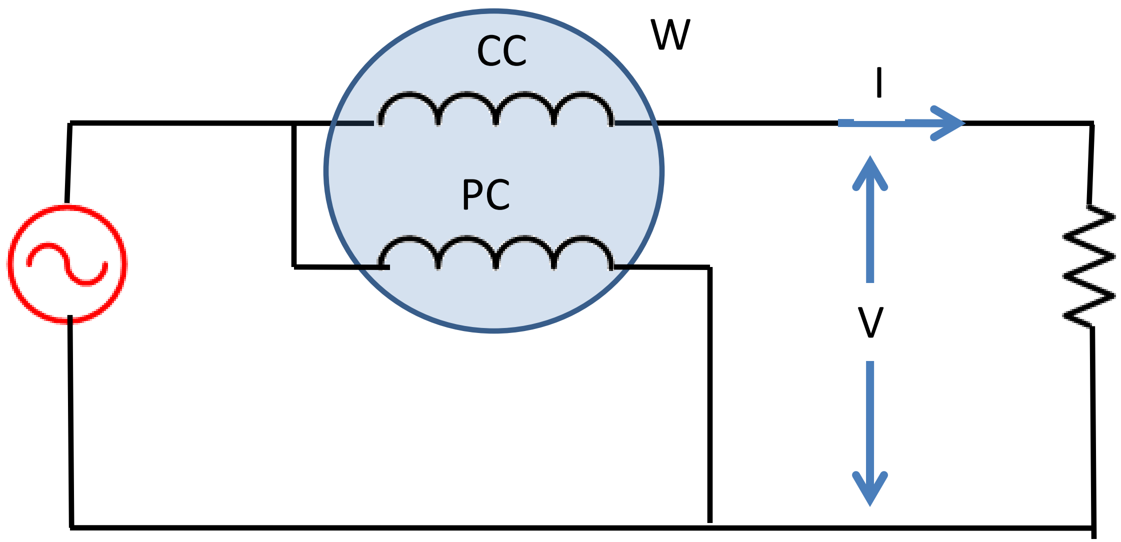




Measurement of power

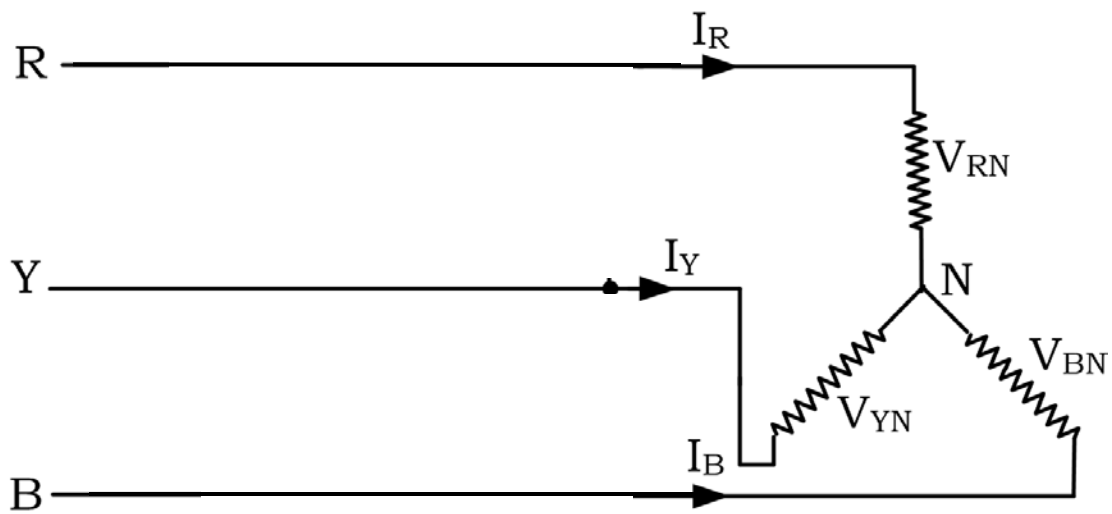
- Wattmeter reads **average** active power, $P = VI\cos\theta$
- P = (PC voltage)$\times$(CC current) $\times$(Cosine of angle between PC & CC)

$$V \qquad\qquad I \qquad\qquad\qquad\qquad \cos\theta$$



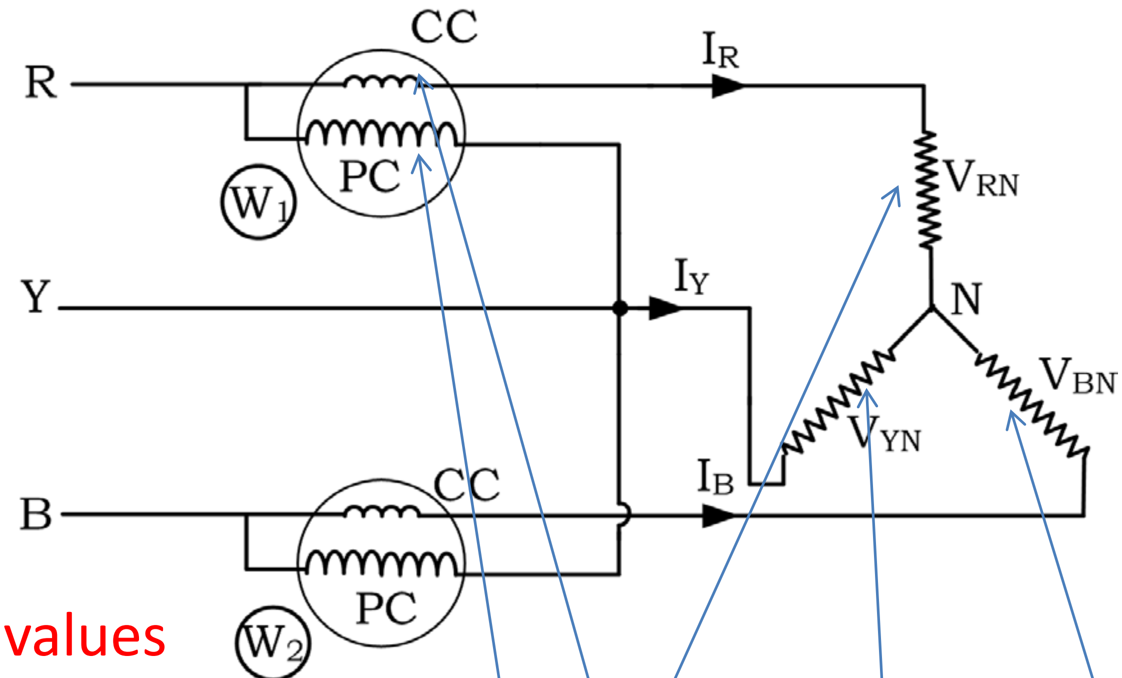
Measurement of three phase power

Two-wattmeter method in 3-phase 3-wire circuit
(the basic process is same for both delta or star connected system. The example is shown for a star connected system)



- The current coils of the two wattmeters W_1 and W_2 are connected in lines R and B
- Voltage coil of W_1 is connected between lines R and Y
- Voltage coil of W_2 is connected between lines B and Y

Measurement of three phase power



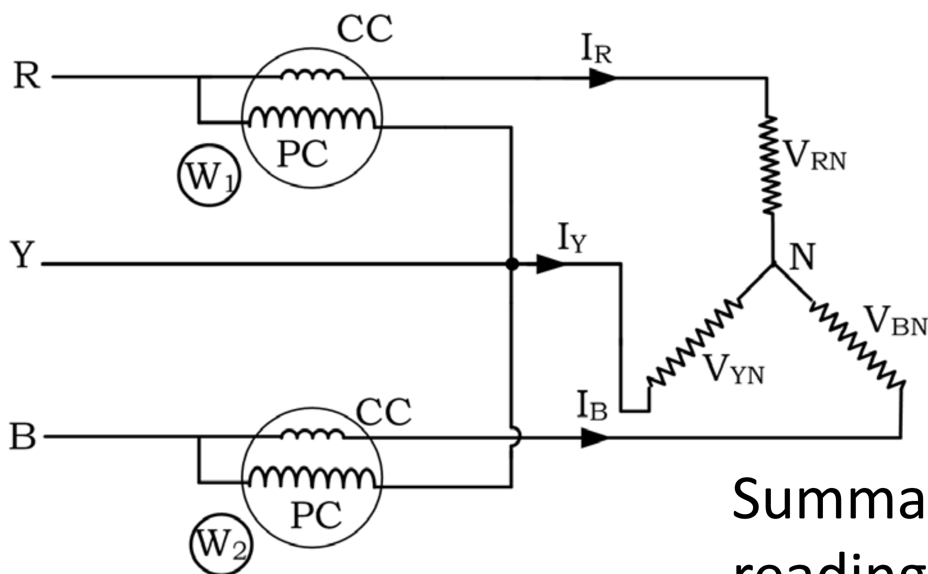
Instantaneous values

Power consumed by the load: $P = \boxed{V_{RN} \times I_R} + \boxed{V_{YN} \times I_Y} + \boxed{V_{BN} \times I_B}$

Reading of wattmeter W_1: $P_1 = \boxed{V_{RY}} \times \boxed{I_R} = (V_{RN} - V_{YN}) \times I_R$

Reading of wattmeter W_2: $P_2 = V_{BY} \times I_B = (V_{BN} - V_{YN}) \times I_B$

Measurement of three phase power



- It can thus, be concluded that, sum of the two wattmeter readings is equal to the total 3-phase power consumed by the load
- This is irrespective of fact whether the load is balanced or not.

Summation of the two wattmeter readings:

$$P_1 + P_2 = (V_{RN} - V_{YN}) \times I_R + (V_{BN} - V_{YN}) \times I_B$$

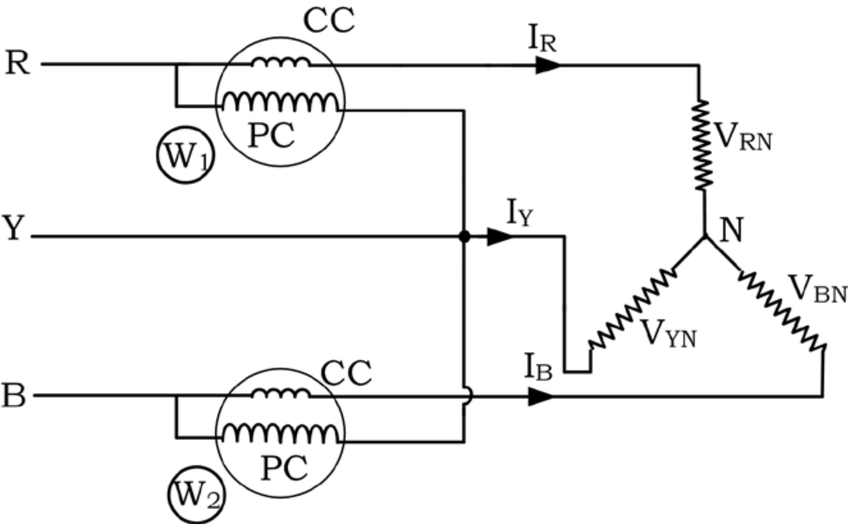
$$= V_{RN} \times I_R + V_{BN} \times I_B - V_{YN} \times (I_R + I_B)$$

Total 3-phase power $= V_{RN} \times I_R + V_{BN} \times I_B + V_{YN} \times I_Y$

From KCL, summation of currents at node N must be zero, i.e.

$$I_R + I_Y + I_B = 0$$
$$I_R + I_B = -I_Y$$

Effect of power factor



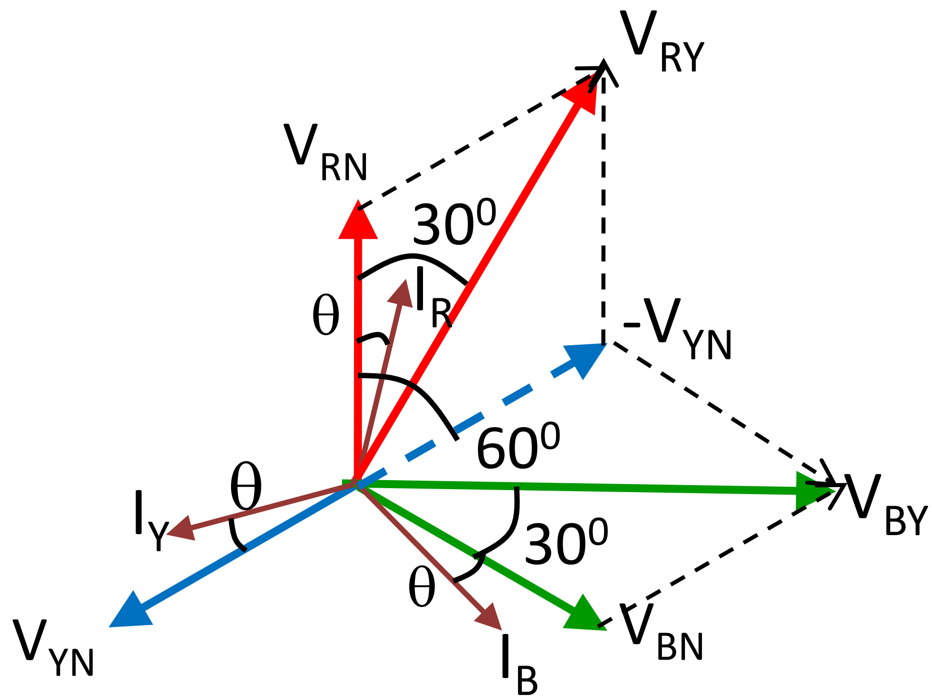
Phase voltages are 120^0 apart:

V_{RN}, V_{BN}, V_{YN}

Line voltages V_{RY} and V_{BY}

$$\overline{V}_{RY} = \overline{V}_{RN} - \overline{V}_{YN} \qquad \overline{V}_{BY} = \overline{V}_{BN} - \overline{V}_{YN}$$

Phasor diagram:

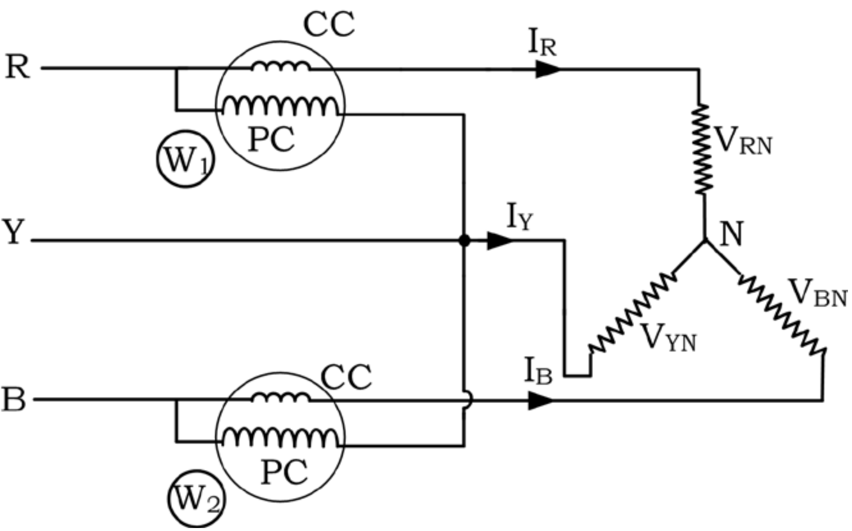


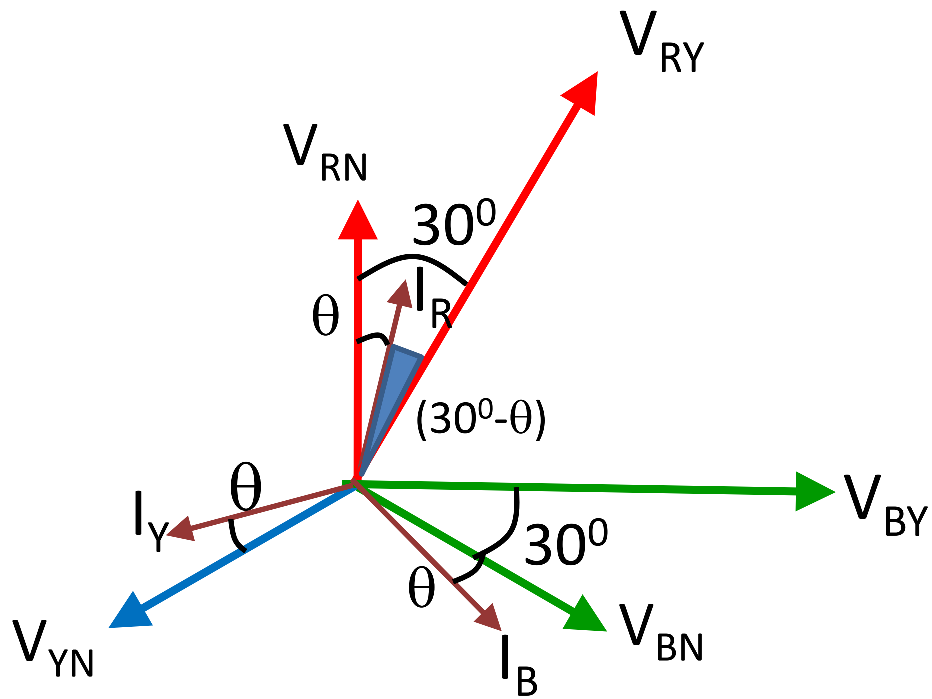
Let, power factor = cosθ

i.e. phase currents lag corresponding phase voltages by θ

Currents are also balanced and are 120^0 apart:

Effect of power factor





$V_{RN} = V_{BN} = V_{YN} = V$ (say)

$I_R = I_B = I_Y = I$ (say)

For star connected system,

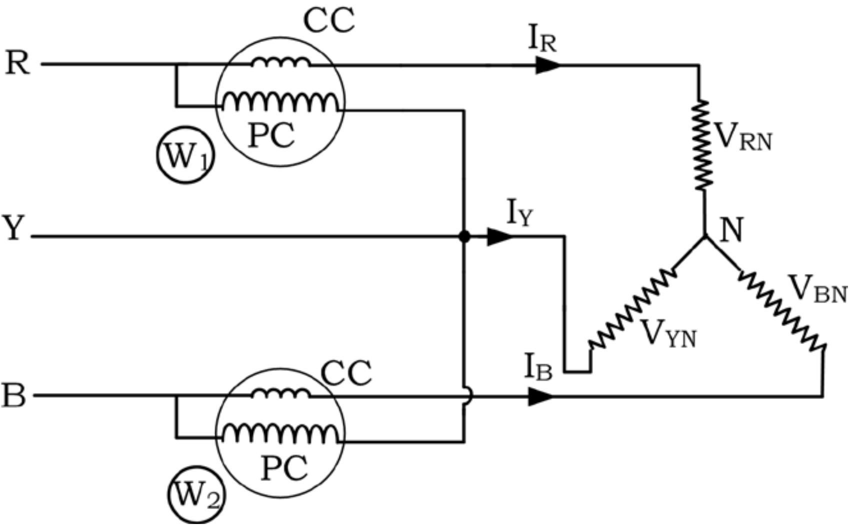
Line voltages $V_{RY} = V_{YB} = V_{BR} = \sqrt{3}V$

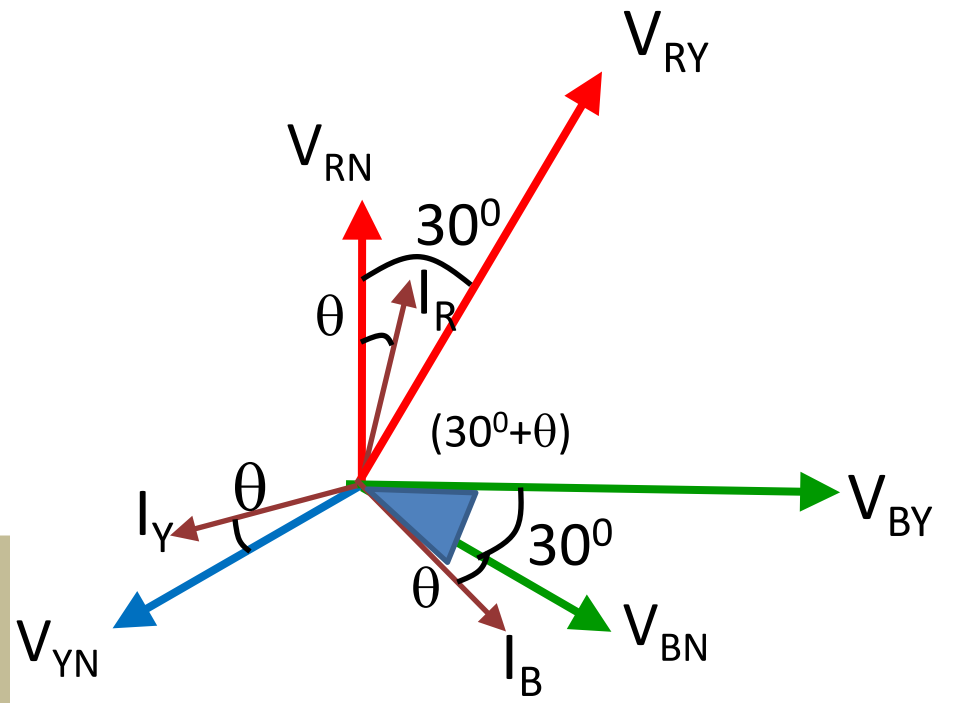
Line currents $I_R = I_B = I_Y = I$

- Current through the CC of wattmeter W_1 is I_R and voltage across its potential coil is V_{RY}
- The current I_R leads the voltage by V_{RY} an angle $(30^0-\theta)$

$\therefore$ Reading of wattmeter W_1 is: $P_1 = V_{RY} \times I_R \cos(30^0 - \theta) = \sqrt{3}VI\cos(30^0 - \theta)$

Effect of power factor

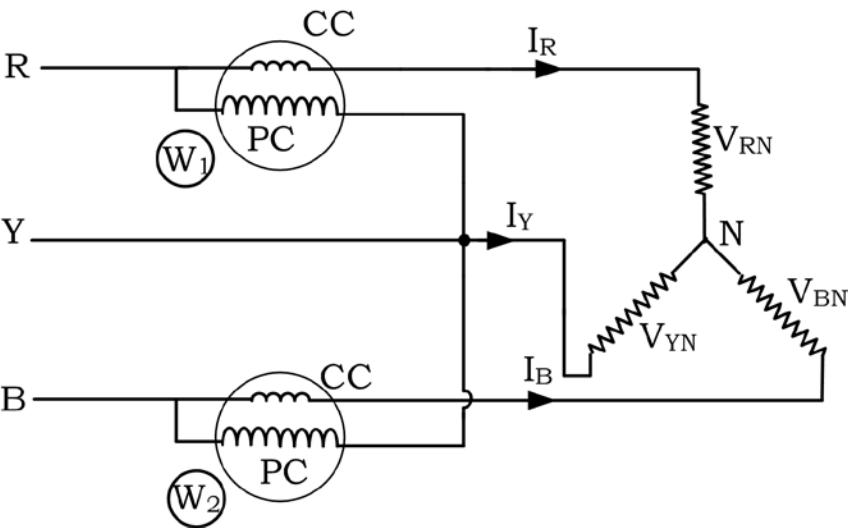




- Current through the CC of wattmeter W_2 is I_B and voltage across its potential coil is V_{BY}
- The current I_B lags the voltage by V_{BY} an angle $(30^0+\theta)$

$\therefore$ Reading of wattmeter W_2 is: $P_2 = V_{BY} \times I_B \cos(30^0 + \theta) = \sqrt{3}VI\cos(30^0 + \theta)$

Effect of power factor



$$P_1 = \sqrt{3}VI\cos\left(30^0 - \theta\right)$$

$$P_2 = \sqrt{3}VI\cos\left(30^0 + \theta\right)$$

Sum of these two wattmeter readings:

$$P_1 + P_2 = \sqrt{3}VI\cos\left(30^0 - \theta\right) + \sqrt{3}VI\cos\left(30^0 + \theta\right)$$

$$= \sqrt{3}VI\begin{bmatrix}\cos 30^0 \cos\theta + \sin 30^0 \sin\theta \\ + \cos 30^0 \cos\theta - \sin 30^0 \sin\theta\end{bmatrix}$$

$$= \sqrt{3}VI\left[2\cos 30^0 \cos\theta\right]$$

$$= \sqrt{3}VI\left[2\frac{\sqrt{3}}{2}\cos\theta\right]$$

$$= 3VI\cos\theta$$

This is the total active power consumed by the load, adding together power in the three individual phases

Thus, summation of the two wattmeters gives total 3-phase power

Effect of power factor

$$P_1 = \sqrt{3}VI\cos\left(30^0 - \theta\right)$$

$$P_2 = \sqrt{3}VI\cos\left(30^0 + \theta\right)$$

Sum of these two wattmeter readings:

$$P_1 + P_2 = 3VI\cos\theta$$

Difference of these two wattmeter readings:

$$P_1 - P_2 = \sqrt{3}VI\cos\left(30^0 - \theta\right) - \sqrt{3}VI\cos\left(30^0 + \theta\right) = \sqrt{3}VI\sin\theta$$

Taking the ratio :

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{\sqrt{3}VI\sin\theta}{3VI\cos\theta} = \frac{\tan\theta}{\sqrt{3}} \Rightarrow \theta = \tan^{-1}\left(\sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2}\right)$$

Thus, power factor

$$\cos\theta = \cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2}\right)\right]$$

Effect of power factor

$$P_1 = \sqrt{3}VI\cos(30^0 - \theta) \qquad P_2 = \sqrt{3}VI\cos(30^0 + \theta) \qquad \cos\theta = \cos\left[\tan^{-1}\left(\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}\right)\right]$$

With unity power factor: $\cos\theta = 1, \theta = 0$

Total power : $P = P_1 + P_2 = 3VI\cos\theta = 3VI$

Reading of wattmeter W_1 $P_1 = \sqrt{3}VI\cos(30^0 - \theta) = \sqrt{3}VI\cos 30^0 = \frac{3}{2}VI$

Reading of wattmeter W_2 $P_2 = \sqrt{3}VI\cos(30^0 + \theta) = \sqrt{3}VI\cos 30^0 = \frac{3}{2}VI$

Thus, summation of two wattmeter readings : $P_1 + P_2 = \frac{3}{2}VI + \frac{3}{2}VI = 3VI$

Which is same as the total power

- Thus, at unity power factor, readings of the two wattmeters are equal
- Each wattmeter reads half the total power

Effect of power factor

$$P_1 = \sqrt{3}VI\cos\left(30^0 - \theta\right) \qquad P_2 = \sqrt{3}VI\cos\left(30^0 + \theta\right) \qquad \cos\theta = \cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2}\right)\right]$$

With 0.5 power factor: $\cos\theta = 0.5,\ \theta = 60^0$

Total power : $P = P_1 + P_2 = 3VI\cos\theta = 3VI\cos 60^0 = \frac{3}{2}VI$

Reading of wattmeter W_1 $P_1 = \sqrt{3}VI\cos\left(30^0 - \theta\right) = \sqrt{3}VI\cos\left(30^0 - 60^0\right) = \sqrt{3}VI\cos\left(-30^0\right) = \frac{3}{2}VI$

Reading of wattmeter W_2 $P_2 = \sqrt{3}VI\cos\left(30^0 + \theta\right) = \sqrt{3}VI\cos\left(30^0 + 60^0\right) = \sqrt{3}VI\cos 90^0 = 0$

Thus, summation of two wattmeter readings : $P_1 + P_2 = \frac{3}{2}VI + 0 = \frac{3}{2}VI$

Which is same as the total power

- Thus, at 0.5 power factor, one of the two wattmeters reads zero
- The other wattmeter reads total power

Effect of power factor

$$P_1 = \sqrt{3}VI\cos\left(30^0 - \theta\right) \qquad P_2 = \sqrt{3}VI\cos\left(30^0 + \theta\right) \qquad \cos\theta = \cos\left[\tan^{-1}\left(\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}\right)\right]$$

With 0 power factor: $\cos\theta = 0$, $\theta = 90^0$

Total power : $P = P_1 + P_2 = 3VI\cos\theta = 3VI\cos 90^0 = 0$

Reading of wattmeter W_1 $P_1 = \sqrt{3}VI\cos\left(30^0 - \theta\right) = \sqrt{3}VI\cos\left(30^0 - 90^0\right) = \sqrt{3}VI\cos\left(-60^0\right) = \frac{\sqrt{3}}{2}VI$

Reading of wattmeter W_2 $P_2 = \sqrt{3}VI\cos\left(30^0 + \theta\right) = \sqrt{3}VI\cos\left(30^0 + 90^0\right) = \sqrt{3}VI\cos 120^0 = -\frac{\sqrt{3}}{2}VI$

Thus, summation of two wattmeter readings : $P_1 + P_2 = \frac{\sqrt{3}}{2}VI - \frac{\sqrt{3}}{2}VI = 0$

Which is same as the total power

- Thus, at zero power factor, readings of the two wattmeters are equal but of opposite sign
- At power factors below 0.5, one of the wattmeters tends to give negative readings

Effect of power factor

Pf (cosθ)	Pf angle (θ)	Remarks
1	0^0	P1 = P2 Total power = 2P1
1> pf > 0.5	$0^0 < \theta < 60^0$	P1 > P2 Total power = P1 + P2
0.5	60^0	P1 = +ve, P2 = 0 Total power = P
0.5 > pf > 0	$60^0 < \theta < 90^0$	P1 = +ve, P2 = -ve Total power = P1 – P2
0	90^0	P1 = - P2 Total power =0