Solutions to Numerical Questions on Ch3: AC Fundamentals

1. At t=0, the instantaneous value of a 50Hz, sinusoidal current is 5 A and increases in magnitude further. Its R.M.S. value is 10 A.
   a) Write the expression for its instantaneous value.
   b) Find the current at t=0.01 and t=0.015 sec
   c) Sketch the waveform indicating these values

[Ans: (a) i = 10√2sin(314t+0.361) (b) -5 A, -13.2 A]

$i = I_m \sin(\omega t + \phi)$, ω=2π50

$$5 = 10\sqrt{2}\sin(0+\phi) \Rightarrow \phi = \sin^{-1}\frac{5}{10\sqrt{2}} = 20.7^0 = 0.361\, rad$$

$$\therefore i = 10\sqrt{2}\sin(\omega t + 0.361)$$

At, t = 0.01 s, $i = 10\sqrt{2}\sin(2\pi 50 \times 0.01 + 0.361) = -5\ A$

At, t = 0.015 s, $i = 10\sqrt{2}\sin(2\pi 50 \times 0.015 + 0.361) = -13.2\ A$

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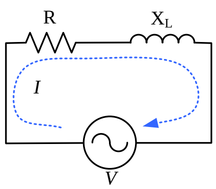
2. A coil takes a current of 2A when connected to a 240 V, 50 Hz sinusoidal supply and consumes 200 W. Calculate the resistance, impedance and inductance of the coil.

[Ans: R = 50 Ω, Z = 120 Ω, L = 0.347 H]



I = 2 A, I^2R = 200 W, ⇒ 2^2R = 200 ⇒ R = 50 Ω

$$Z = \frac{V}{I} = \frac{240}{2} = 120\Omega$$

$$\therefore Z = \sqrt{R^2 + X_L^{\ 2}} = 120 \Rightarrow X_L = 109.1\Omega$$

$$\therefore X_L = 109.1 \Rightarrow 2\pi 50 \times L = 109.1 \Rightarrow L = 0.347\ H$$

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3. The equation of an alternating current is *i* =62.35 sin (323 t) A. Determine its
   i. Maximum value
   ii. Frequency
   iii. R.M.S. value
   iv. Average value, and
   v. Form factor

[Ans: (i) 62.35 A (ii) 51.4 Hz (iii) 44.1 A (iv) 39.7 A (v) 1.11]

Comparing the signal expression with the standard form of a sinusoidal signal $i = I_m \sin\frac{2\pi}{T}t$, we have:

i) Maximum value I_m = 62.35

ii) $\frac{2\pi}{T} = 323$

$\therefore$Frequency $f = \frac{1}{T} = \frac{323}{2\pi} = 51.4\ Hz$

iii) Since the current signal is a pure sine wave, its RMS value is:

$$i_{RMS} = \frac{I_m}{\sqrt{2}} = \frac{62.35}{\sqrt{2}} = 44.1\ A$$

iv) Average value $i_{av} = \frac{2I_m}{\pi} = \frac{2 \times 62.35}{\pi} = 39.7\ A$

v) Form factor of a sinusoidal signal, $FF = \frac{I_{RMS}}{I_{av}} = \frac{44.1}{39.7} = 1.11$

4. A circuit takes a current of 3 A at a power factor of 0.6 lagging when connected to a 115 V, 50 Hz supply. Another one circuit takes a current of 5 A at a power factor of 0.707 leading when connected to same supply after the first circuit is removed. If the two circuits are connected in series across a 230 V, 50 Hz supply, Calculate
   a) The current drawn from the source
   b) The power consumed
   c) The power factor of the circuit

[Ans: (a) 5.5 A (b) 1.1890 kW (c) 0.94 (lag)]

Circuit 1:

$$I = 3 \Rightarrow \frac{V}{Z} = 3 \Rightarrow \frac{115}{Z} = 3 \Rightarrow Z = \frac{115}{3} = 38.3\Omega \Rightarrow \sqrt{R^2 + X_L{}^2} = 38.3$$

Also, for lagging power factor, $\cos\phi = \cos\left(\tan^{-1}\frac{X_L}{R}\right) = 0.6 \Rightarrow \frac{X_L}{R} = \tan\left(\cos^{-1} 0.6\right) = 1.33$

$$\therefore \sqrt{R^2 + (1.33R)^2} = 38.3 \Rightarrow R = 23\ \Omega$$

$$\therefore X_L = 1.33R \Rightarrow X_L = 30.6\ \Omega$$

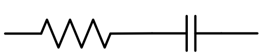
Circuit 2:

$$I = 5 \Rightarrow \frac{V}{Z} = 5 \Rightarrow \frac{115}{Z} = 5 \Rightarrow Z = \frac{115}{5} = 23\Omega \Rightarrow \sqrt{R^2 + X_C{}^2} = 23$$

Also, for leading power factor, $\cos\phi = \cos\left(\tan^{-1}\frac{X_L}{R}\right) = 0.707 \Rightarrow \frac{X_L}{R} = \tan\left(\cos^{-1} 0.707\right) = 1$

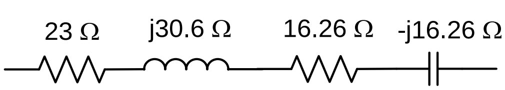
$$\therefore \sqrt{R^2 + R^2} = 23 \Rightarrow R = 16.26\ \Omega$$

$$\therefore X_C = R = 16.26\ \Omega$$



Circuit 3:

Two circuits connected in series:



$$Z = Z_1 + Z_2 = (R + jX_L) + (R - jX_C) = (23 + j30.6) + (16.26 - j16.26) = 39.26 + j14.34 = 41.8\angle 20^0$$

(a) Source current $I = \frac{V}{Z} = \frac{230\angle 0^0}{41.8\angle 20^0} = 5.5\angle -20^0$

(b) Power consumed $P = VI\cos\phi = 230\times 5.5\times \cos 20^0 = 1.189\ kW$

(c) Power factor, $\cos\phi = \cos 20^0 = 0.94$ (lagging)

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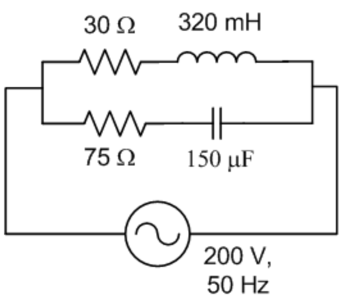
5. A coil of resistance 30Ω and inductance 320mH is connected in parallel to circuit consisting of 75Ω is series with 150μF capacitor. The circuit is connected to a 200V, 50Hz supply. Determine supply current and circuit power factor.

[Ans: 3.24 A, 0.935 (lag)]



$$Z_1 = R + jX_L = 30 + j2\pi 50\times 320\times 10^{-3} = 30 + j100.5$$

$$Z_2 = R - jX_C = 75 - j\frac{1}{2\pi 50\times 150\times 10^{-6}} = 75 - j21.22$$

$$Z = Z_1 // Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(30 + j100.5)(75 - j21.22)}{(30 + j100.5) + (75 - j21.22)}$$

$$= \frac{104.9\angle 73.4^0 \times 77.9\angle -15.8^0}{105 + j79.28}$$

$$= \frac{8171.7\angle 57.6^0}{132.29\angle 36.82^0} = 61.77\angle 20.78^0$$

Current, $I = \dfrac{V}{Z} = \dfrac{200\angle 0^0}{61.77\angle 20.78^0} = 3.24\angle -20.78^0\ A$

Power factor, $\cos\phi = \cos(-20.78^0) = 0.935$ (lagging)

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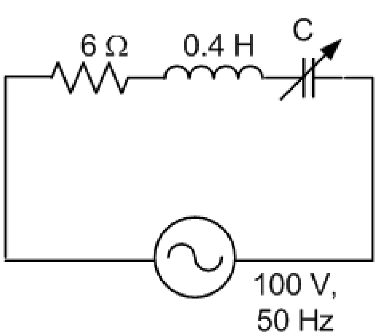
6. A circuit consisting of series combination of elements as resistance of 6Ω, inductance of 0.4H and a variable capacitor is connected across a 100V, 50Hz supply. Calculate (i) value of capacitance at resonance, (ii) voltage drop across capacitor and (iii) Q-factor of coil.

[Ans: (i) 25.33 μF (ii) 2094.4 V (iii) 20.9]



(i) At resonance,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow 50 = \frac{1}{2\pi\sqrt{0.4C}} \Rightarrow C = 25.33\ \mu F$$

(ii) Voltage drop across capacitor

$$V_C = I\times X_C = \frac{V}{R}\times\frac{1}{2\pi f_0 C} = \frac{100}{6}\times\frac{1}{2\pi 50\times 25.33\times 10^{-6}} = 2094.4\,V$$

(iii) Q factor of the coil:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{6}\sqrt{\frac{0.4}{25.33\times 10^{-6}}} = 20.94$$

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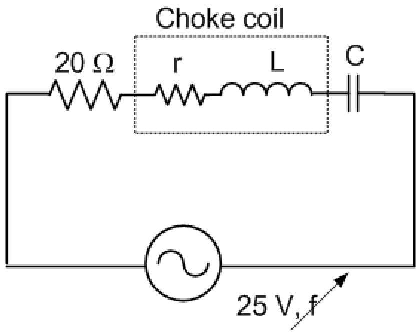
7. A 20 Ω resistor, a choke coil having some inductance and some resistance and a capacitor are connected in series across a 25V variable frequency source. When the frequency is 400 Hz, the current is maximum and its value is 0.5 A and the potential difference across the capacitor is 150 V. Calculate the resistance and the inductance of the choke and the capacitance of the capacitor.

[Ans: r = 30 Ω, L = 0.119 H, C = 1.33 μF]



At resonance, the total circuit impedance becomes purely resistive,

Thus, $Z = 20 + r$

$$\therefore I_0 = 0.5 \Rightarrow \frac{V}{Z} = 0.5 \Rightarrow \frac{25}{20+r} = 0.5 \Rightarrow r = 30\,\Omega$$

Resonance frequency is 400 Hz, and voltage across capacitance at resonance is 150 V

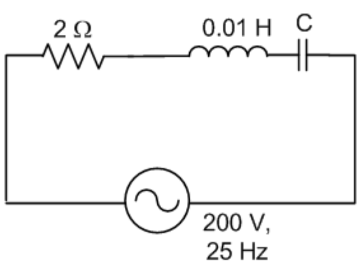
$$\therefore V_C = 150 \Rightarrow I_0 X_C = 150 \Rightarrow 0.5 \times \frac{1}{2\pi 400 \times C} = 150 \Rightarrow C = 1.33\,\mu F$$

From resonant frequency expression, we have:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow 400 = \frac{1}{2\pi\sqrt{L \times 1.33 \times 10^{-6}}} \Rightarrow L = 0.119\,H$$

8. A coil of resistance 2 ohms and inductance 0.01H is connected in series with a capacitor across 200V mains. What must be the capacitance for maximum current at 25Hz? Find also the current and voltage in the capacitor.

[Ans: C = 4.05 mF, Vc = 157 V]



From resonant frequency expression, we have:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow 25 = \frac{1}{2\pi\sqrt{0.01 \times C}} \Rightarrow C = 4.05 \times 10^{-3}\ F$$

$$I_0 = \frac{V}{R} = \frac{200}{2} = 100\,A$$

$$\therefore V_C = I_0 X_C = 100 \times \frac{1}{2\pi 25 \times 4.05 \times 10^{-3}} = 157\,V$$

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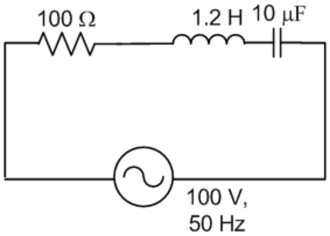
9. A resistance of 100 ohms is connected with an inductance of 1.2 Henry and capacitance of 10 microfarad in series. The combination is connected across 100 volts and 50 Hz supply. Find...
   a. Current in the resistance
   b. Voltage across the capacitor
   c. Power consumed
   d. Draw the phasor diagram.

[Ans: (a) I = 0.86 A, (b) Vc = 274 V, (c) P = 74.1 W]



$$Z = 100 + j\left(2\pi 50 \times 1.2 - \frac{1}{2\pi 50 \times 10 \times 10^{-6}}\right)$$
$$= 100 + j(377 - 318)$$
$$= 100 + j59$$
$$= 116\angle 30.5^0$$

a) $\therefore$ Current $I = \dfrac{V}{Z} = \dfrac{100\angle 0^0}{116\angle 30.5^0} = 0.86\angle -30.5^0 \ A$

b) Voltage across the capacitor: $V_C = IX_C = 0.86 \times 318 = 274\,V$

c) Power consumed: $P = VI\cos\phi = 100 \times 0.86\cos\left(-30.5^0\right) = 74.1\,W$