Chapter 5
3-Phase systems

Day 26

Configurations

ILOs – Day 26

- Understand the utility of 3-phase system in electric power supply
- Visualize how 3-phase signal is generated
- Draw delta connected 3-phase system and derive it voltage and current relations
- Draw star connected 3-phase systems and derive it voltage and current relations

3-phase system

- Household electricity supply is single phase
  - One live conductor
  - One neutral conductor
  - (One optional earth conductor)

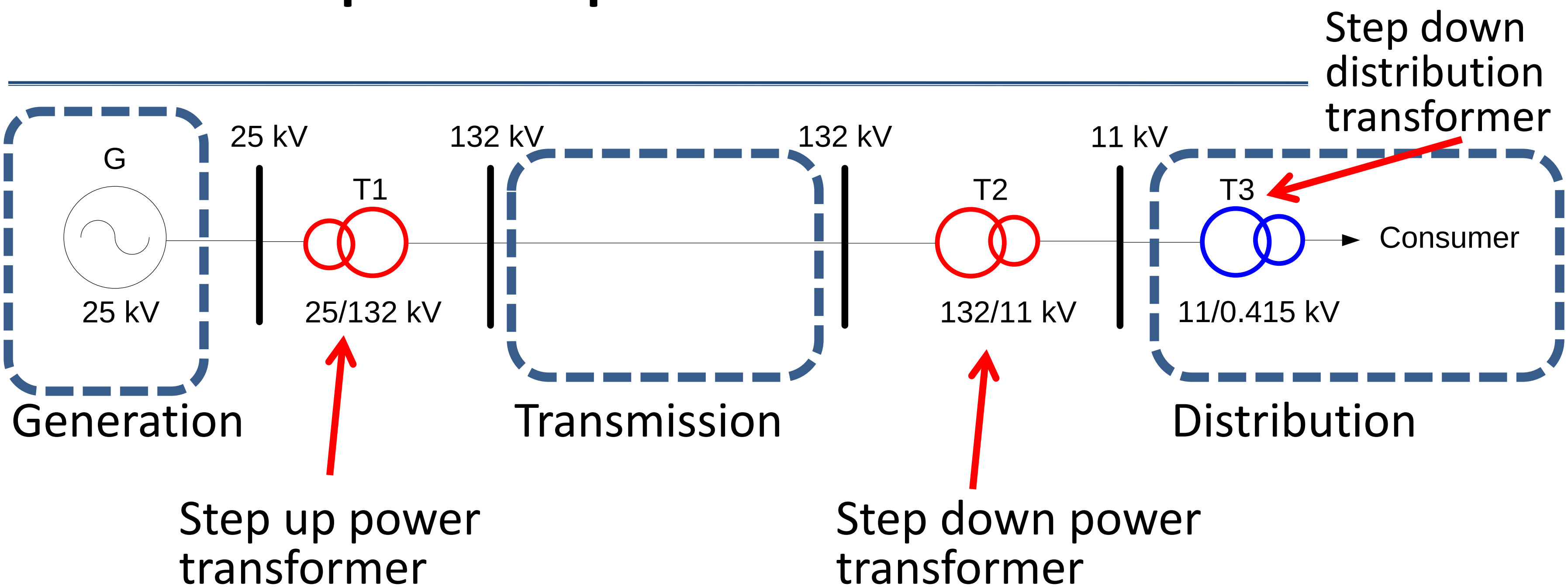
- But, larger power supply systems are 3-phase
  - Generating stations
  - Power transmission & distribution
  - Industries
  - Large buildings, large motors

3 phase system

- Why three-phase?
  - Large power transfer
  - Current very high at a given voltage
  - If single phase, then conductor too thick
  - Difficult to handle

- Divide power in three lines (3-phase)
  - One third power shared by each phase
  - Conductor size less
    - Easy to handle
  - Increased reliability

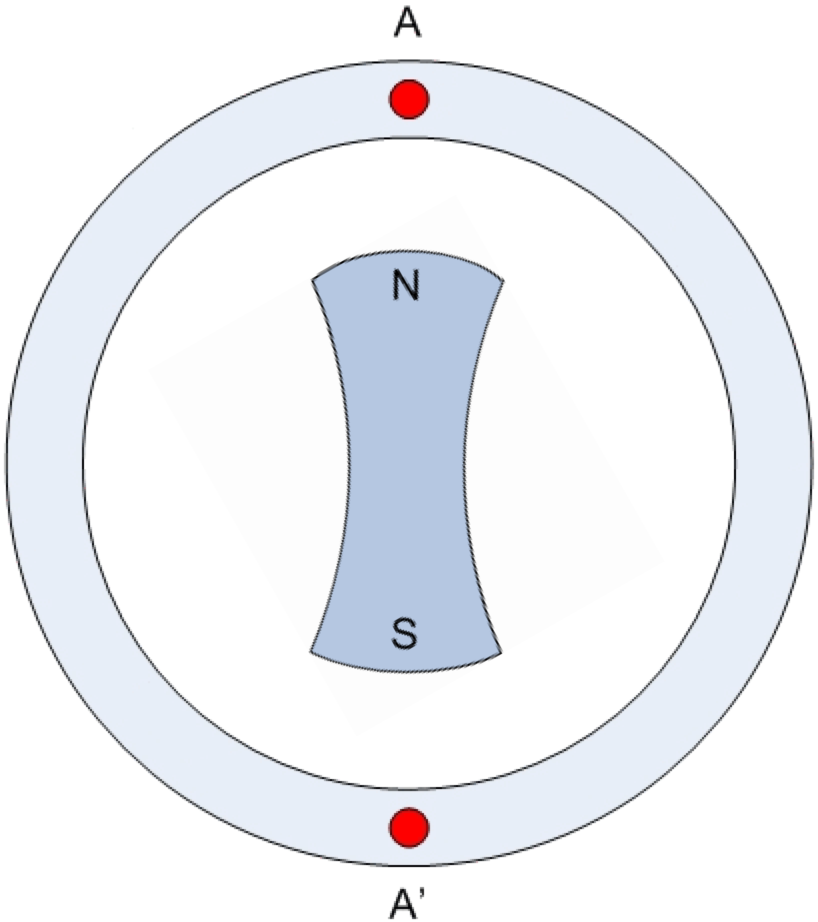
Three phase power transfer scheme



- Transmission at high voltage to reduce power loss during transmission
  - Higher voltage means lower current for a given power
  - Thus, I^2R losses are less in transmission line
  - Also, voltage drop (IZ) is less in transmission line

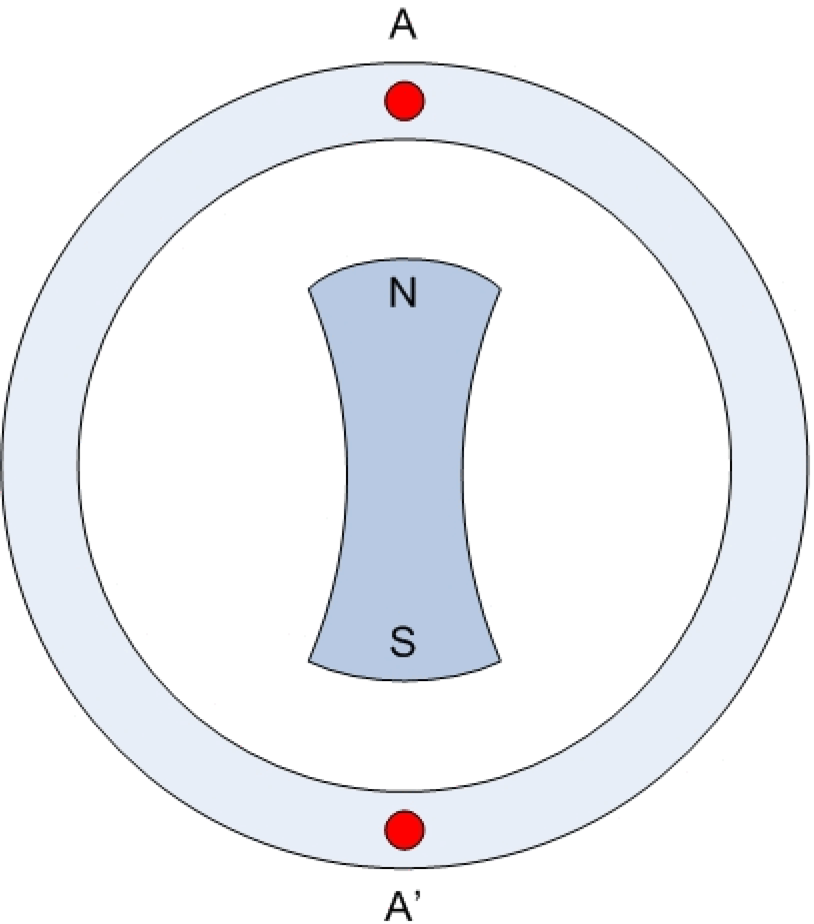
Generation of 3-phase signal

- One coil in the stator A-A’
- Pair of permanent magnets in rotor
- Rotor rotated by external mechanical force (prime mover)



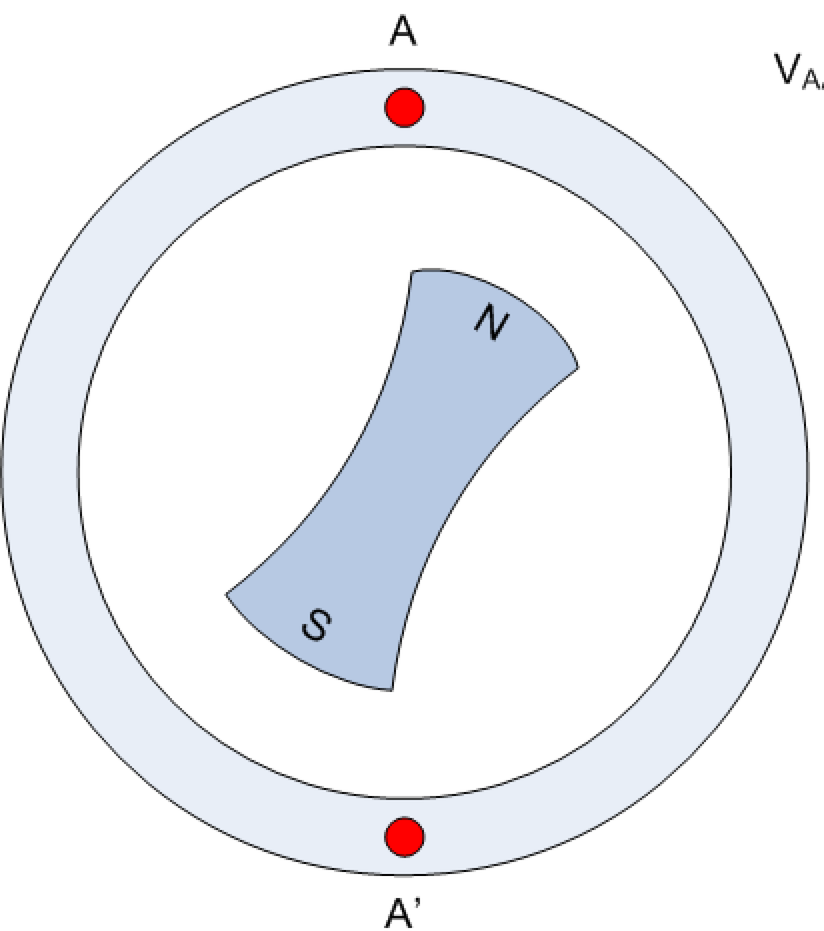
Generation of 3-phase signal

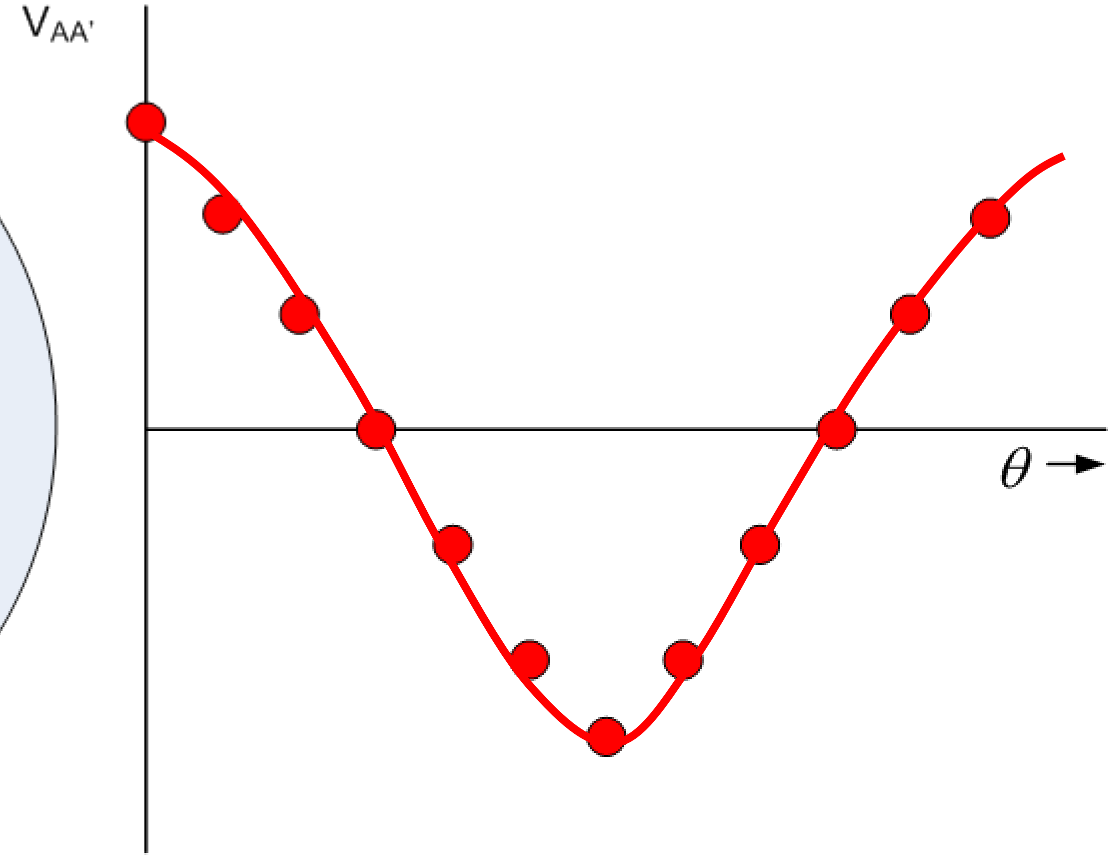
- EMF is induced in stator coil ($d\phi/dt$)



Generation of 3-phase signal

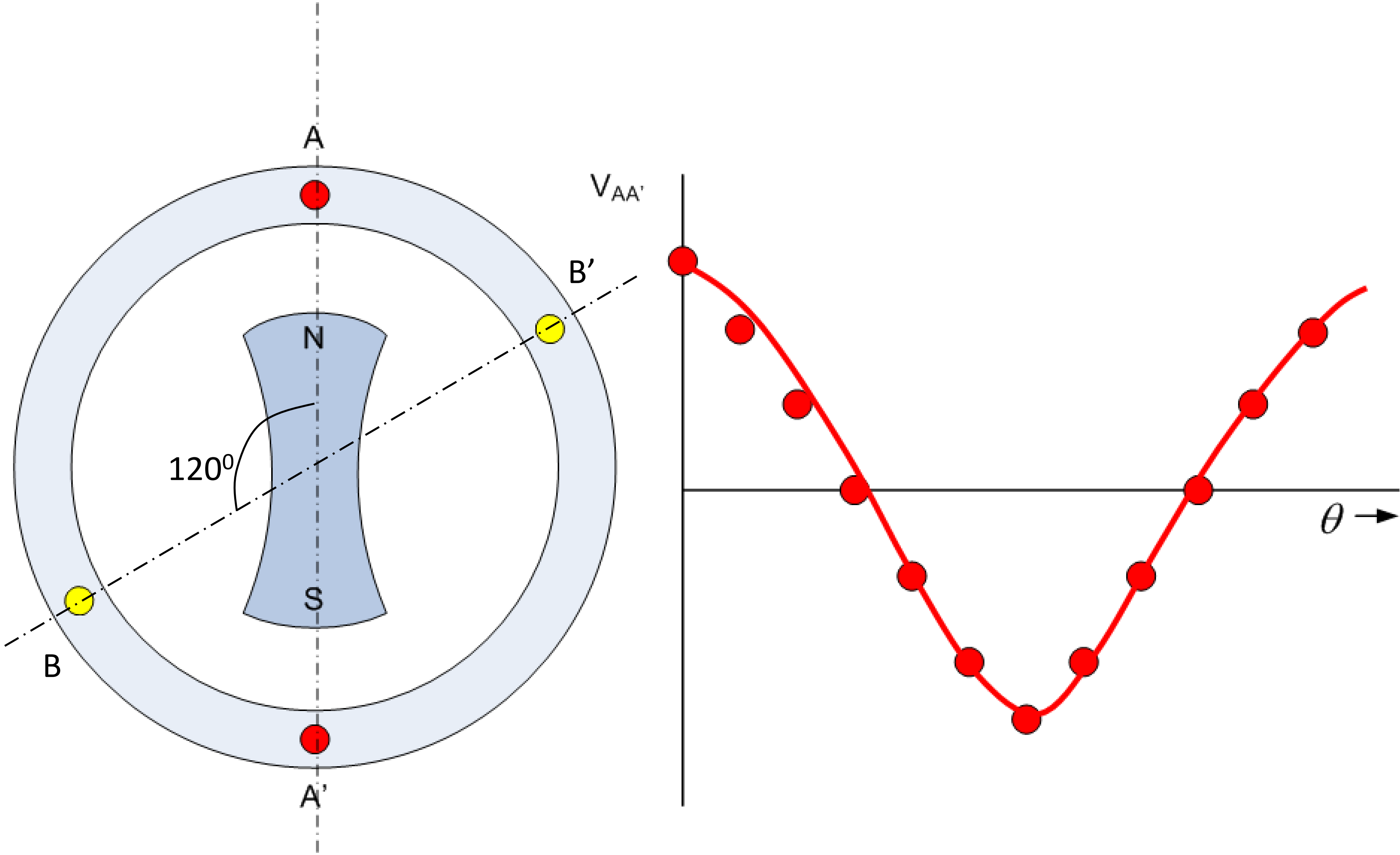
- EMF is sinusoidal $e_A = E_m \sin\theta = E_m \sin\omega t$





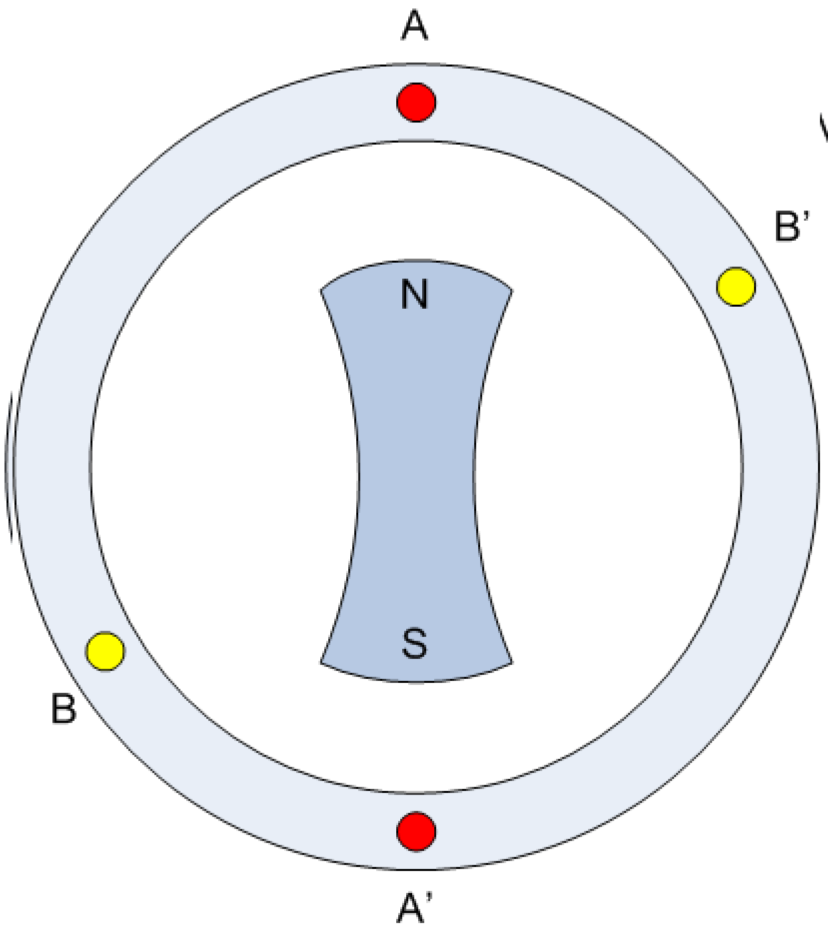
Generation of 3-phase signal

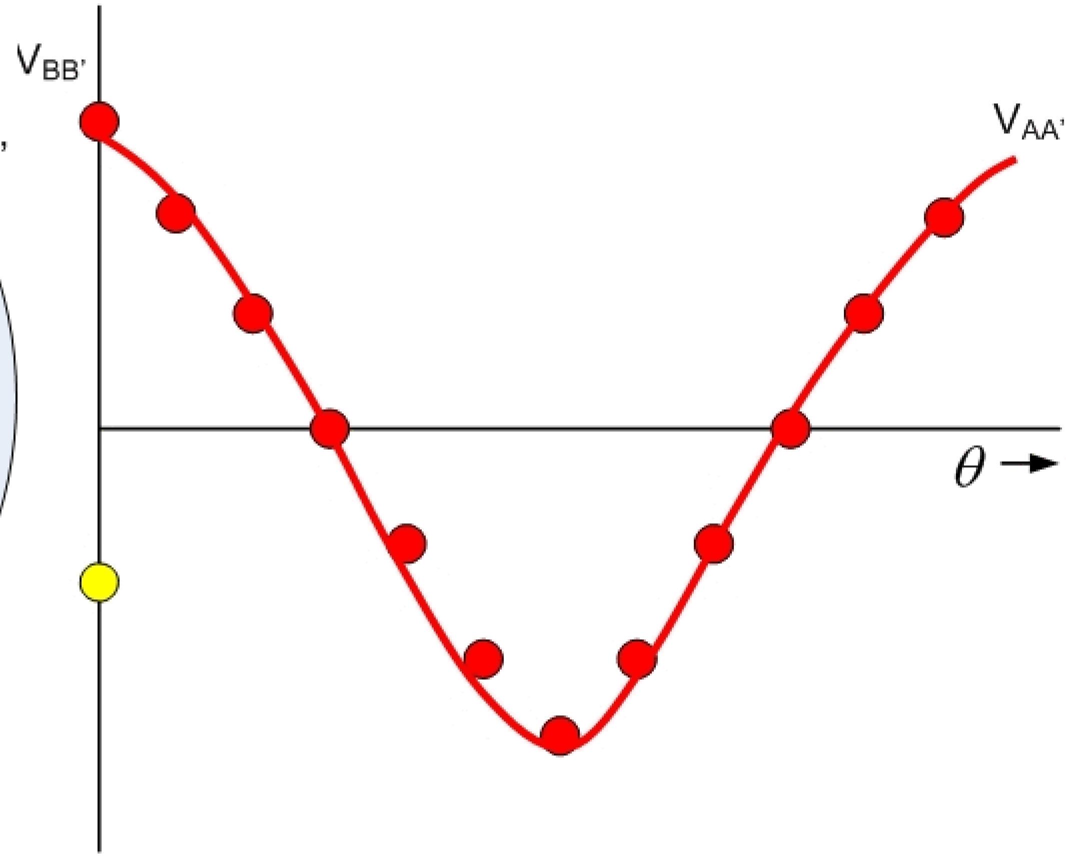
- Put a second coil B-B' after 120^0



Generation of 3-phase signal

- B-B' will also have induced EMF $V_{BB'}$
- But $V_{BB'}$ will come after 120^0 of $V_{AA'}$(phase difference of 120^0 from $V_{AA'}$)



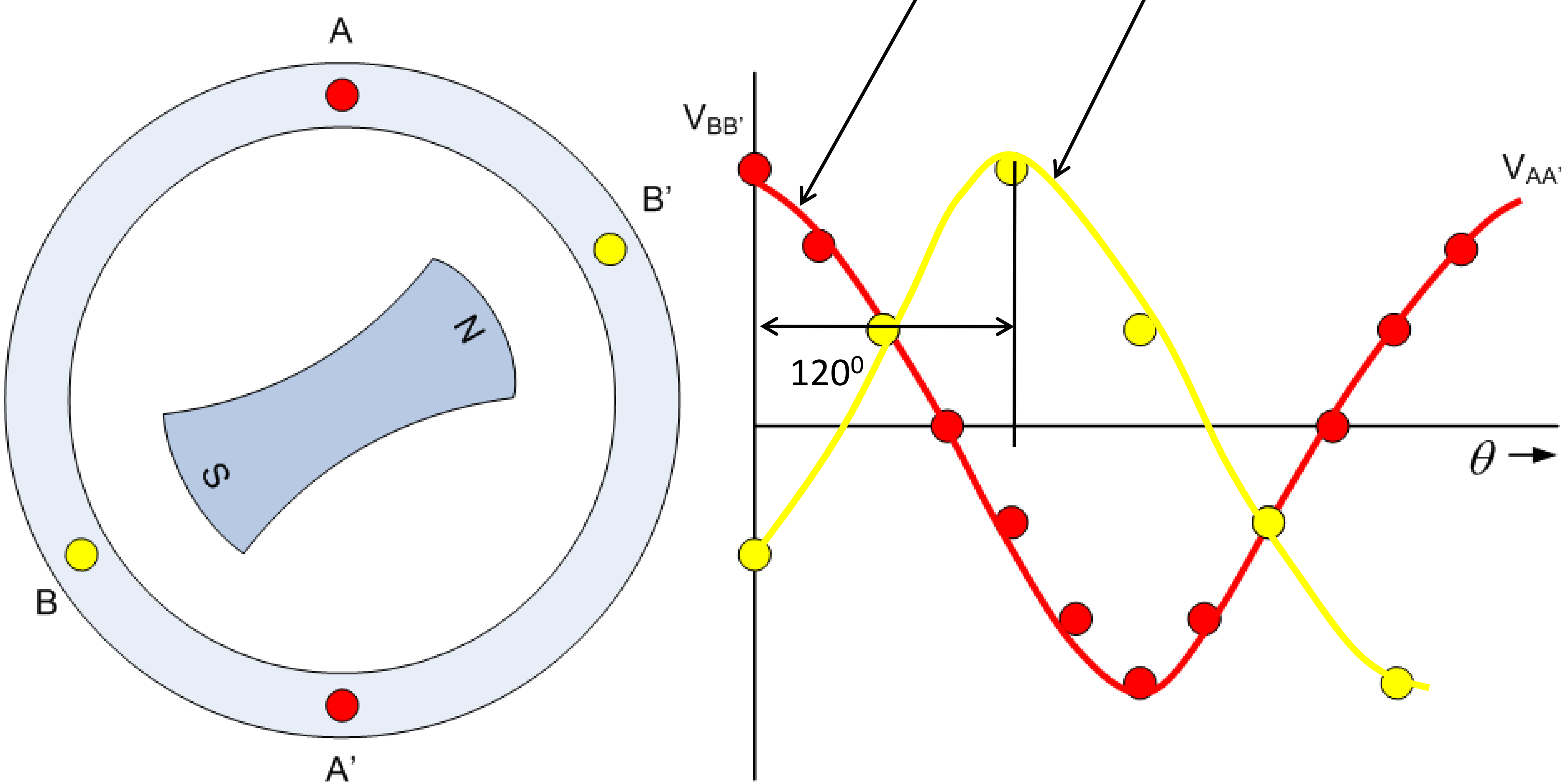


Generation of 3-phase signal

- B-B' will also have induced EMF $V_{BB'}$
- $V_{BB'}$ at phase difference of 120^0 from $V_{AA'}$

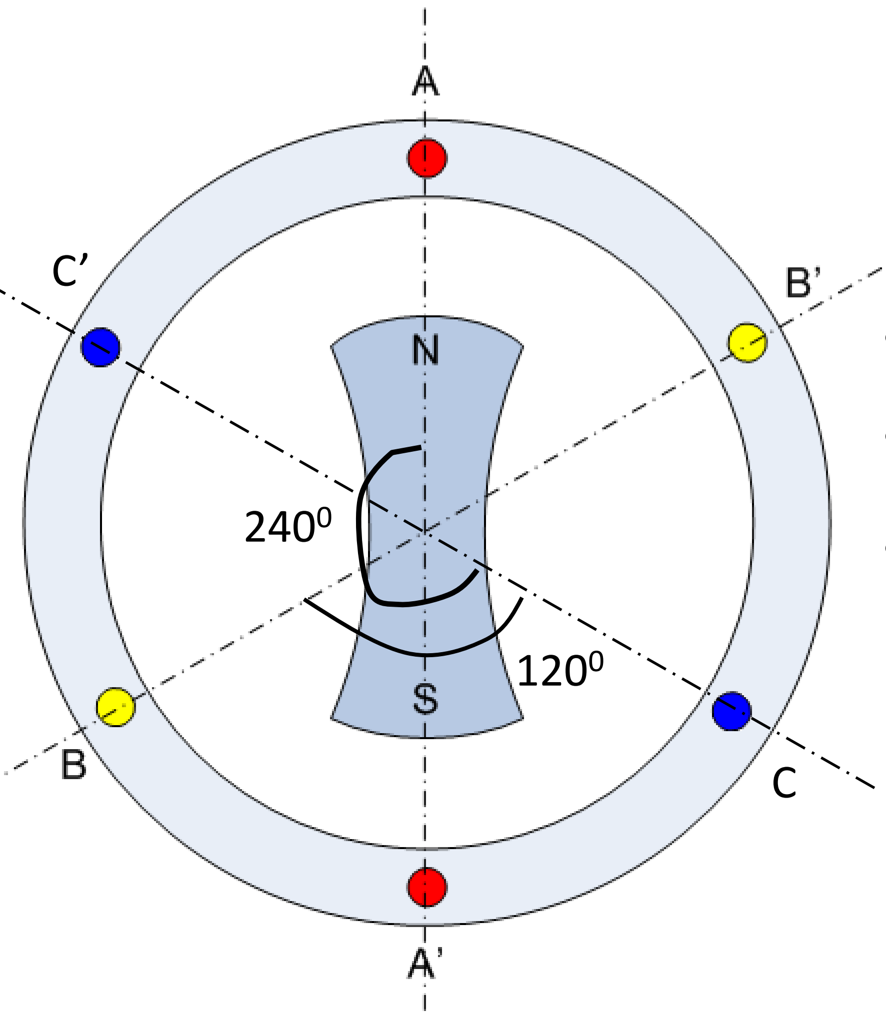
$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin\left(\omega t - 120^0\right)$$



Generation of 3-phase signal

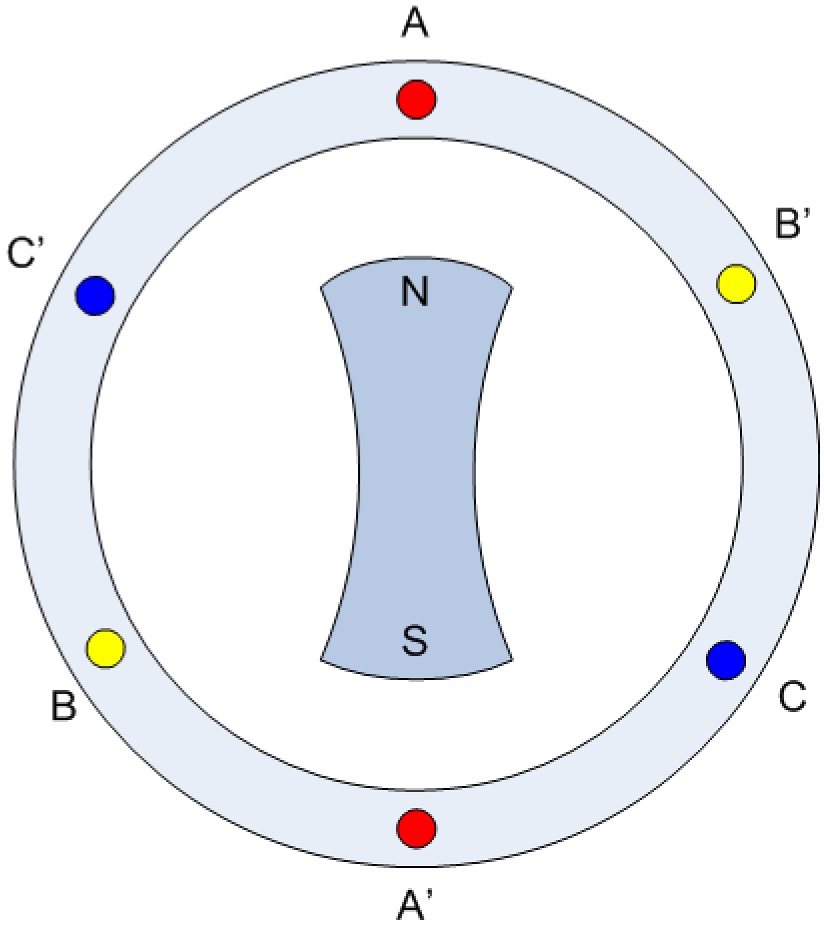
- Now have a 3rd Coil C-C’ that will be 120^0 after B-B’

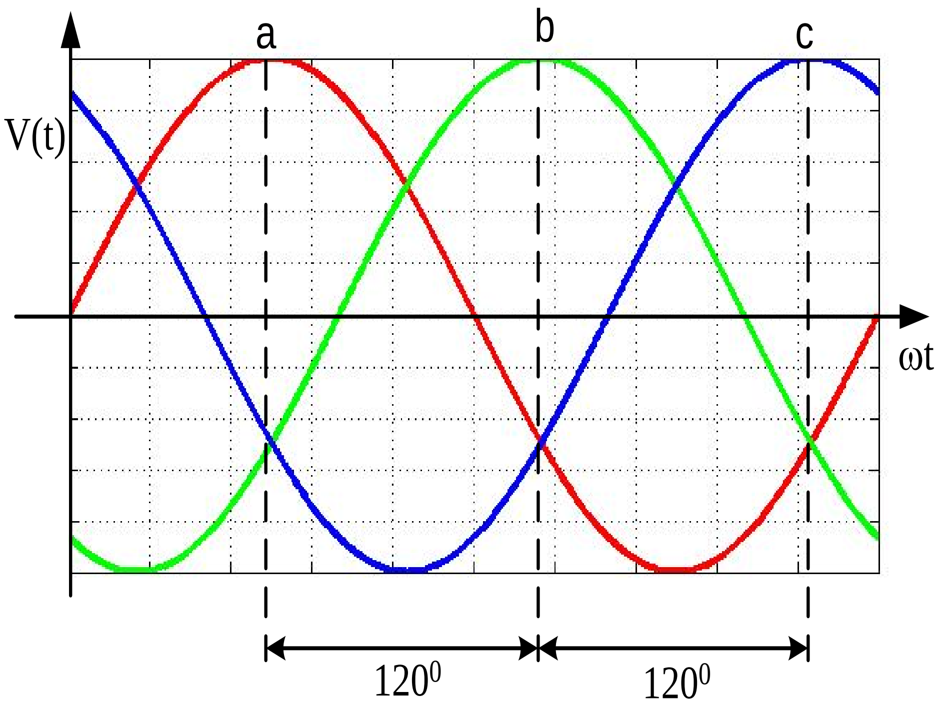


- C-C’ will also have similar induced EMF
- With phase difference of 120^0 from $V_{BB'}$
- i.e. phase difference of 240^0 from $V_{AA'}$

Generation of 3-phase signal

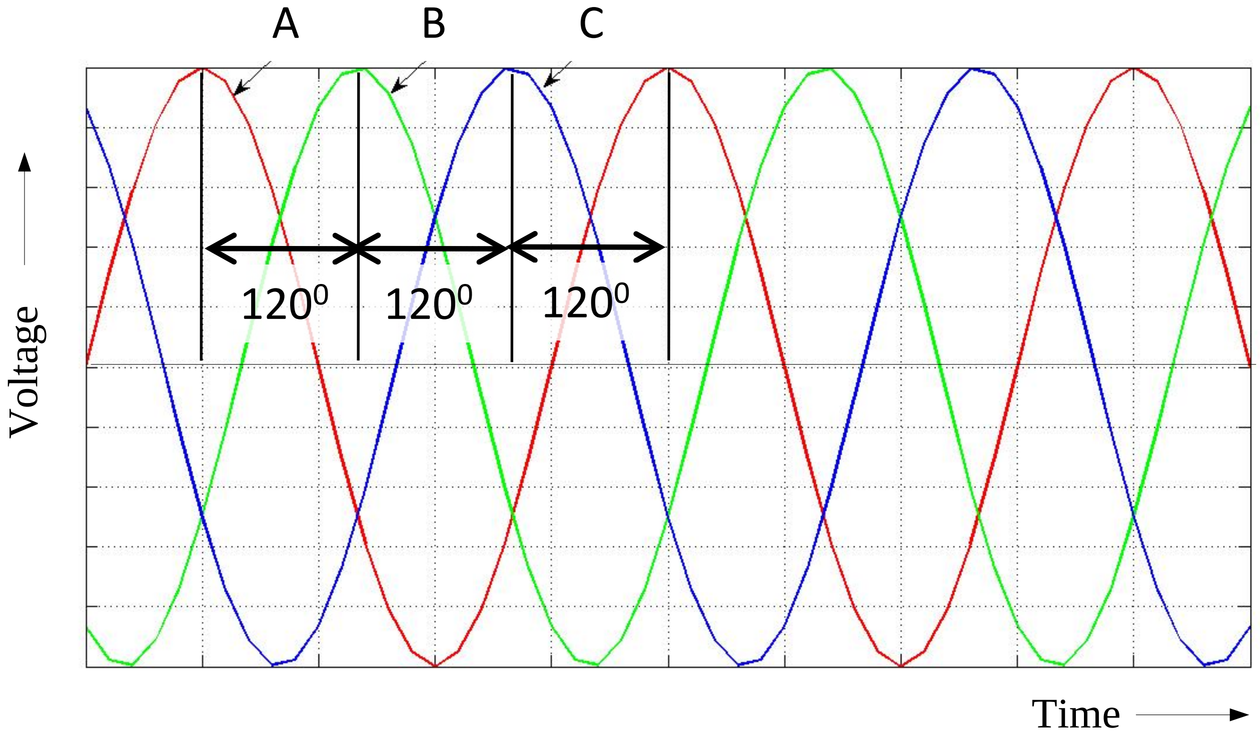
$$e_A = E_m \sin \omega t \qquad e_B = E_m \sin\left(\omega t - 120^0\right) \qquad e_C = E_m \sin\left(\omega t - 240^0\right)$$





Three phase signal

- 3-phase balanced signal (voltage or current)
- Same magnitude in all three phases
- Same frequency of all three signals
- Phase angle 120^0 between them

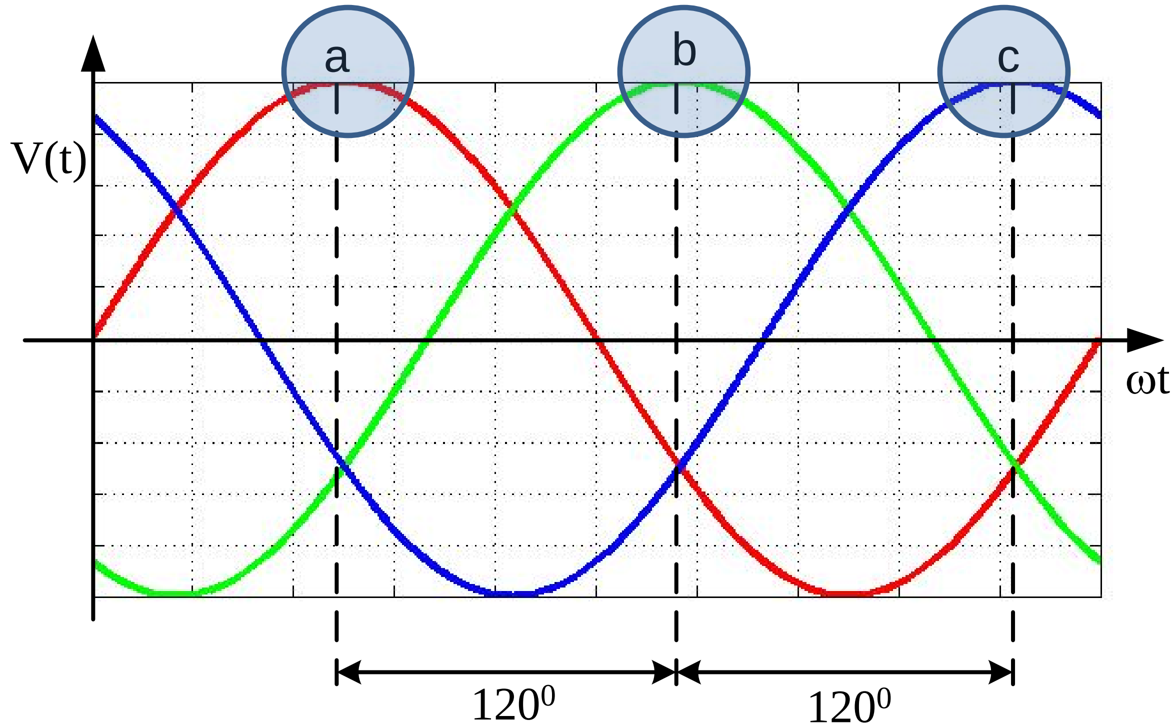


Phase sequence

- Set of 3-phase signals

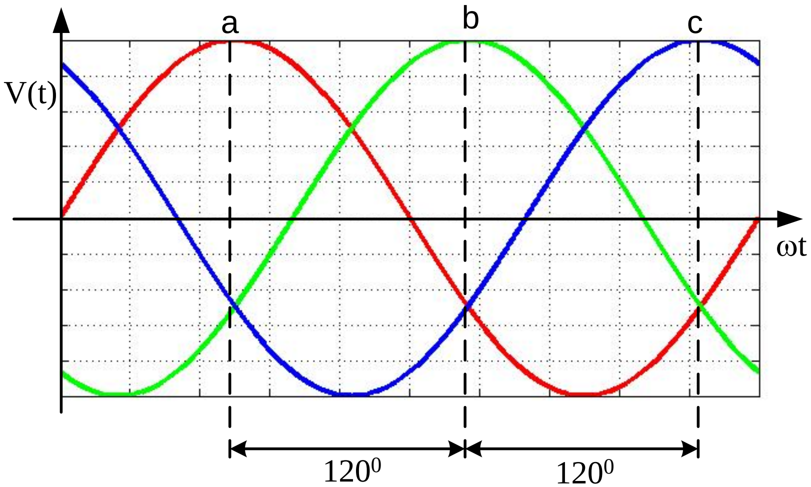
Phase sequence

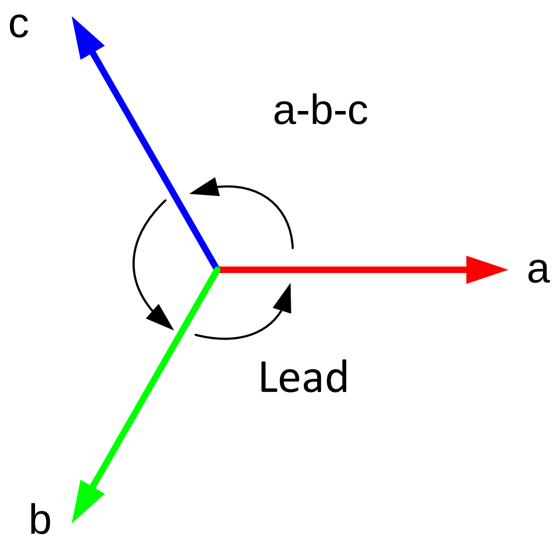
a-b-c



- Phase sequence of **a-b-c** indicates that phase ‘a’ attains its peak first in time followed by ‘b’ and then ‘c’
- a leads b, b leads c

Phase sequence – phasor diagram





- In phasor diagram, the a-b-c phase sequence is denoted by phasors a-b-c coming in CLOCKWISE sequence as per convention

- 'a' phasor leads 'b' by 120^0 and 'c' by 240^0 in the anticlockwise direction

- RMS values

$$E_A = V\angle 0^0$$

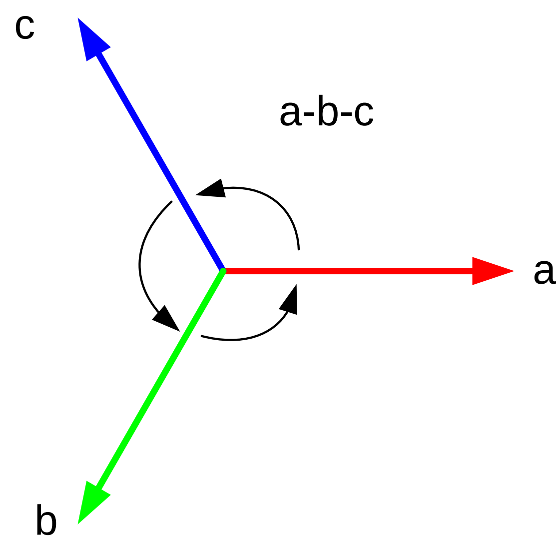
$$E_B = V\angle -120^0$$

$$E_C = V\angle -240^0$$

Phase sequence – phasor diagram

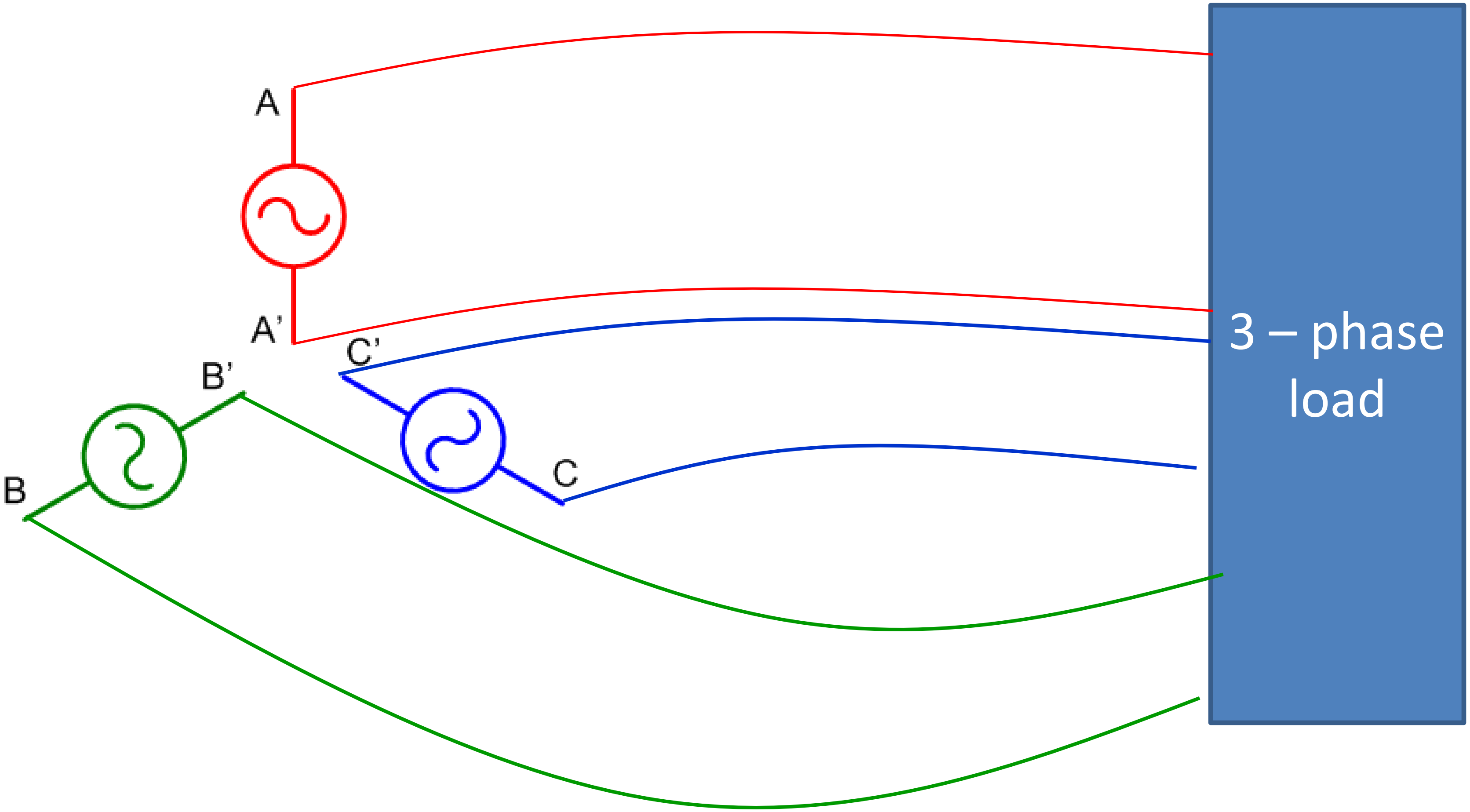
- This sequence is continued in a cyclic fashion a-b-c-a-b-c-a-........
- Thus a phase sequence of “a-b-c” is basically same as:
- ‘b-c-a’ or ‘c-a-b’

a – b – c – a – b – c – a – b – c



Three phase generator

- Three voltage sources (AA’ coil, BB’ coil, and CC’ coil)
  - Equal magnitude
  - 120^0 phase difference



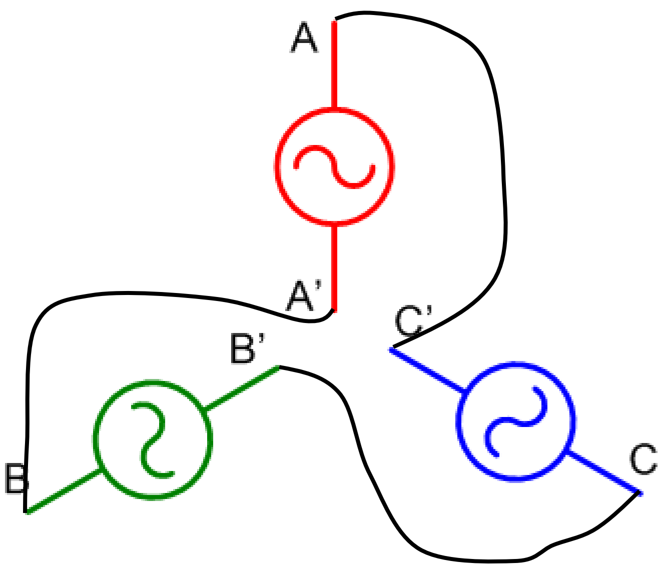
Quiz: How many conductors do we need to supply 3-phase power to load?

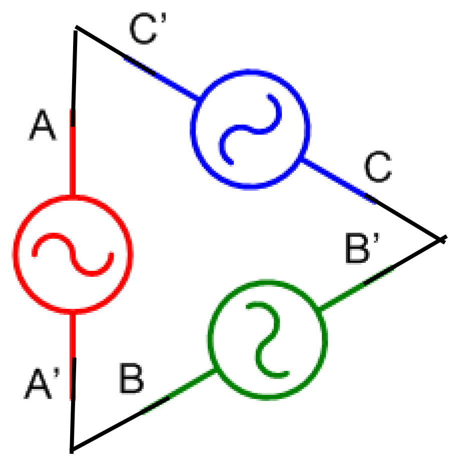
Can we reduce the number of conductors?

- From 6 conductors can we reduce to 3 or 4 conductors?
- Interconnect the three generator coils
  - Delta (3 conductors)
  - Star
    - with neutral (4 conductors)
    - without neutral (3 conductors)

Delta connection

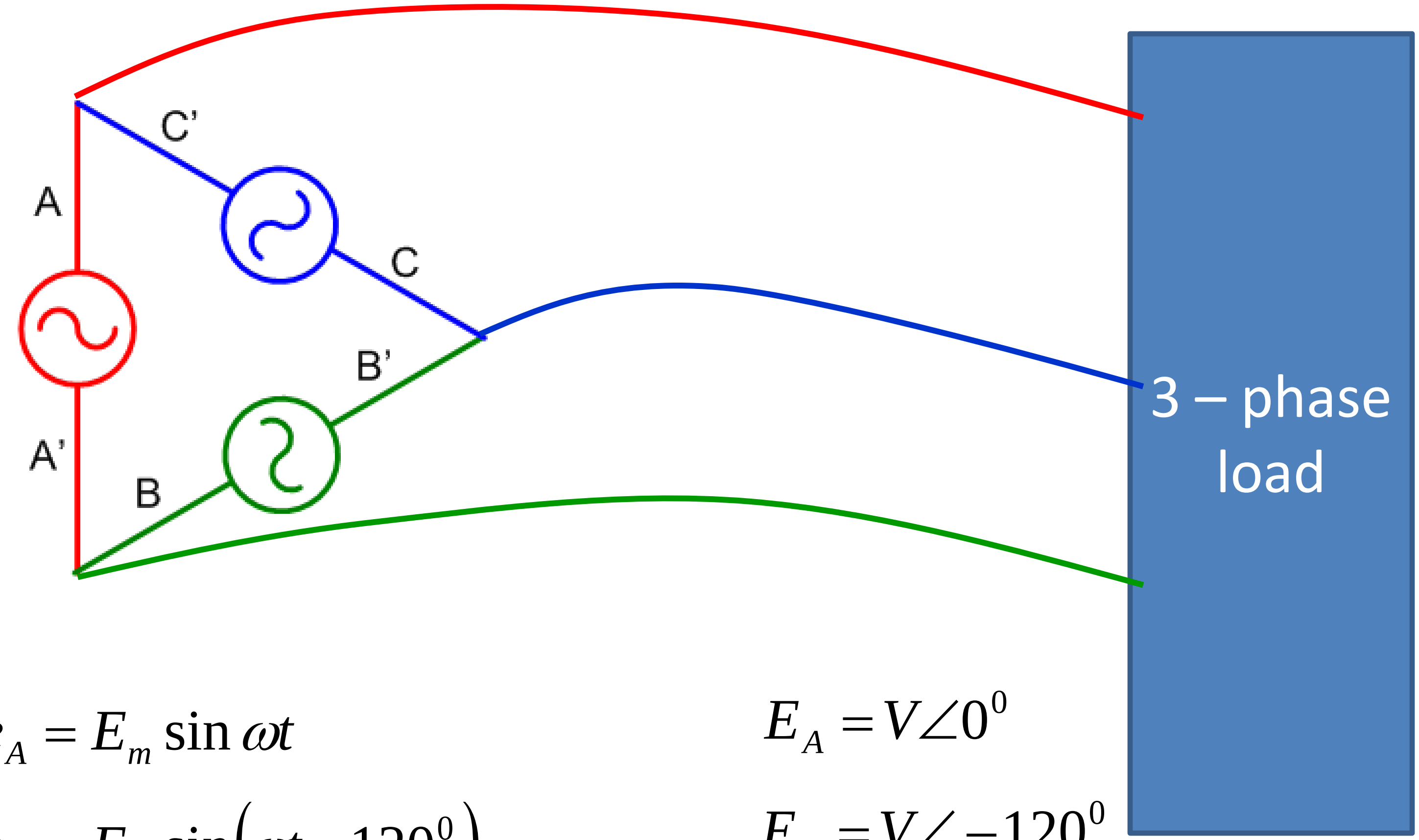
- The 3 generator coils are interconnected
- Finish of one coil connected to start of the next coil, and so on





Delta connection

- The output lines are connected to the junction points of delta



$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin\left(\omega t - 120^0\right)$$

$$e_C = E_m \sin\left(\omega t - 240^0\right)$$

$$E_A = V\angle 0^0$$

$$E_B = V\angle -120^0$$

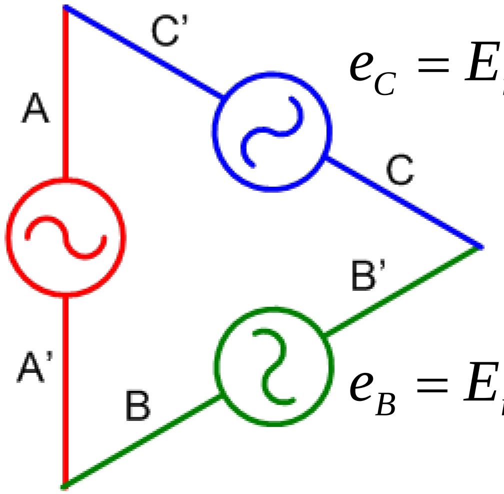
$$E_C = V\angle -240^0$$

Delta connection

- Summation of instantaneous voltages in a closed delta loop

$$e_C = E_m \sin\left(\omega t - 240^0\right)$$

$$e_A = E_m \sin \omega t$$



$$e_B = E_m \sin\left(\omega t - 120^0\right)$$

Thus, sum of INSTNATNEOUS values of three EMF’s is zero in a three-phase balanced AC circuit

$$e_A + e_B + e_C$$

$$= E_m \sin \omega t + E_m \sin\left(\omega t - 120^0\right) + E_m \sin\left(\omega t - 240^0\right)$$

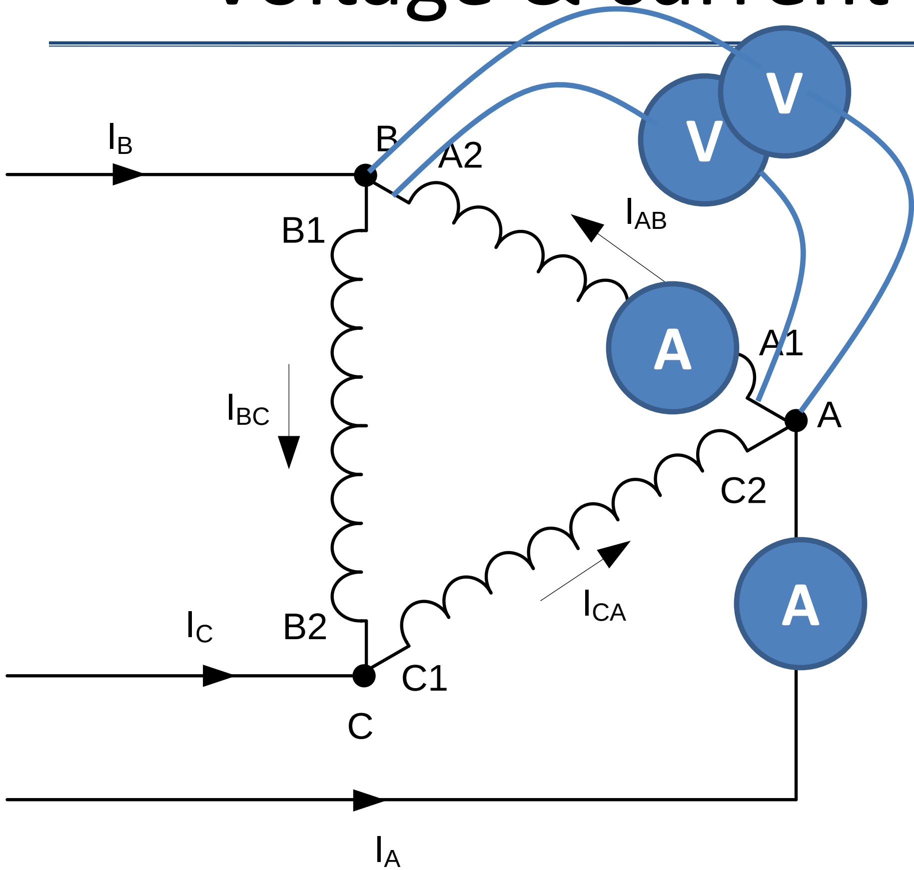
$$= E_m \left[\sin \omega t + \sin \omega t \cos 120^0 - \cos \omega t \sin 120^0 + \sin \omega t \cos 240^0 - \cos \omega t \sin 240^0\right]$$

$$= E_m \left[\sin \omega t - 0.5 \sin \omega t - 0.866 \cos \omega t - 0.5 \sin \omega t + 0.866 \cos \omega t\right]$$

$$= E_m \left[\sin \omega t - \sin \omega t\right]$$

$$= 0$$

Voltage & current in Delta connection



Phase voltage

$$\left|V_{A1A2}\right| = \left|V_{B1B2}\right| = \left|V_{C1C2}\right| = V_{ph}$$

Line voltage

$$\left|V_{AB}\right| = \left|V_{BC}\right| = \left|V_{CA}\right| = V_L$$

$$\boxed{V_L = V_{ph}}$$

Phase current

$$\left|I_{AB}\right| = \left|I_{BC}\right| = \left|I_{CA}\right| = I_{ph}$$

Line current

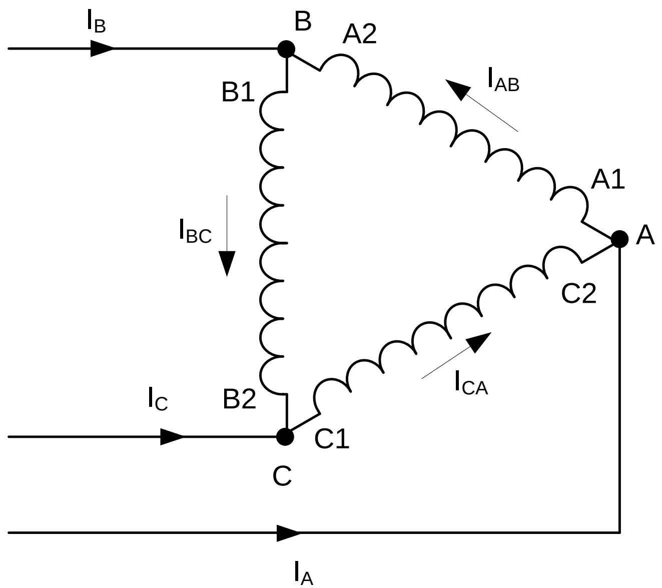
$$\left|I_A\right| = \left|I_B\right| = \left|I_C\right| = I_L$$

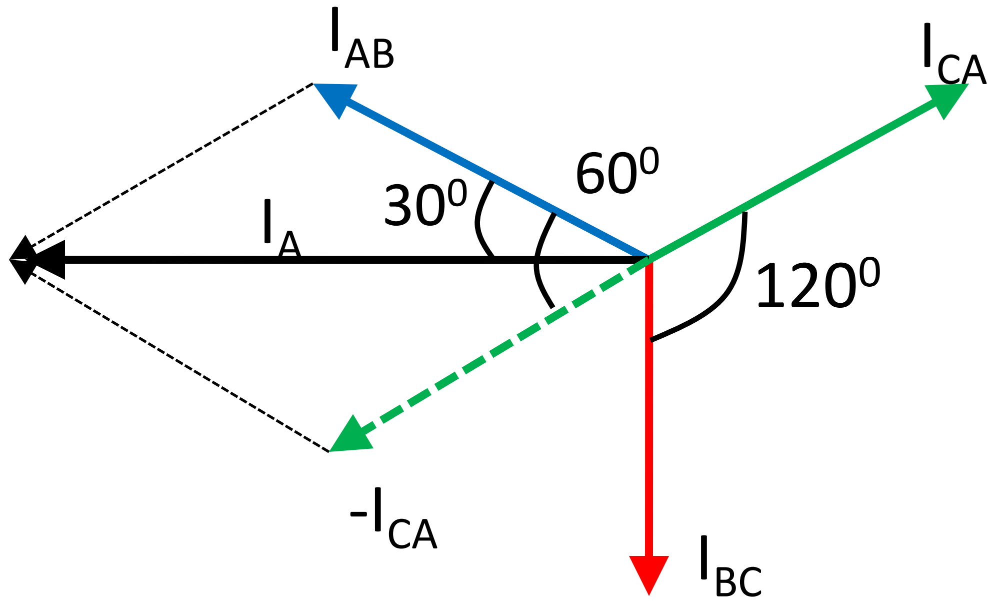
KCL at node A

$$\overline{I_A} = \overline{I_{AB}} - \overline{I_{CA}} = \overline{I_{AB}} + \left(\overline{-I_{CA}}\right)$$

$$\boxed{I_L \neq I_{ph}}$$

Delta connection





$$I_L = \overline{I_A} = \overline{I_{AB}} - \overline{I_{CA}} = \overline{I_{AB}} + \left(\overline{-I_{CA}}\right)$$

$$\boxed{I_L = \sqrt{3} I_{ph}}$$

$$\left|I_A\right| = \sqrt{\left|I_{CA}\right|^2 + \left|I_{AB}\right|^2 + 2 \times \left|I_{CA}\right| \times \left|I_{AB}\right| \cos 60^0}$$

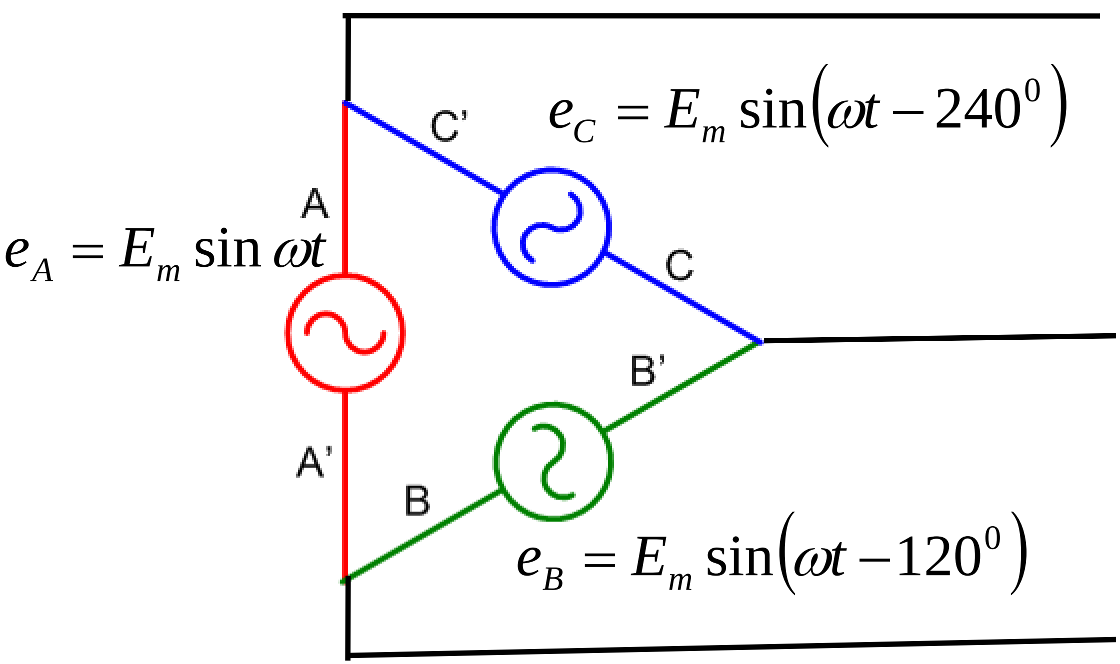
$$= \sqrt{I_{ph}^{\ 2} + I_{ph}^{\ 2} + 2 \times I_{ph} \times I_{ph} \times \frac{1}{2}} = \sqrt{I_{ph}^{\ 2} + I_{ph}^{\ 2} + I_{ph}^{\ 2}} = \sqrt{3 I_{ph}^{\ 2}} = \sqrt{3} I_{ph}$$

In a balanced delta connected system, line voltages are equal to phase voltages, but line currents are √3 times the phase currents.

The angle between line and phase current is 30⁰

Delta connection

- Balanced Delta connected system

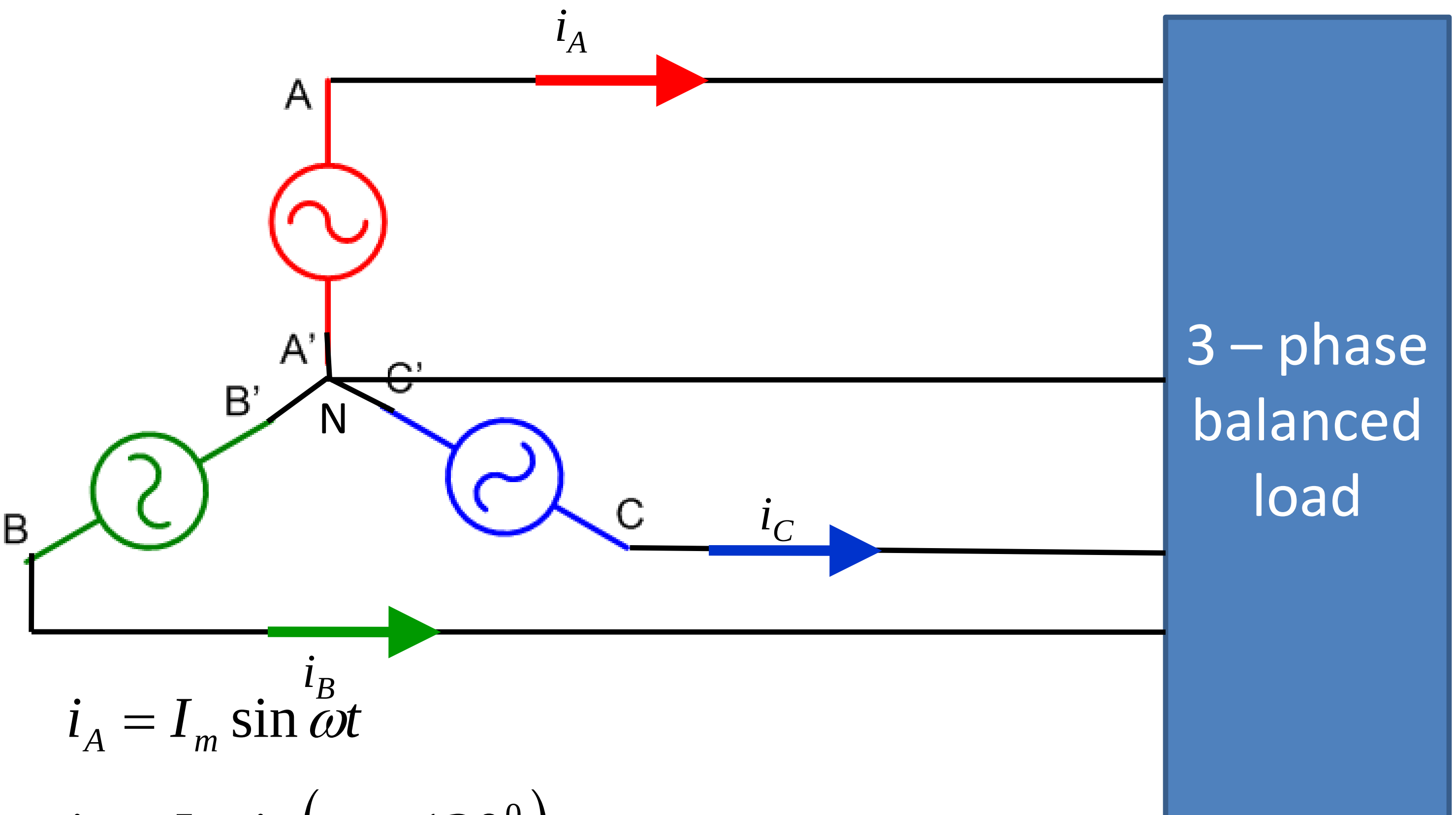


$$\boxed{V_L = V_{ph}}$$

$$\boxed{I_L = \sqrt{3} I_{ph}}$$

Star connection – 4 Wire system

- One terminal each of the 3 coils are joined together to the neutral point N
- 3 LIVE lines and 1 NEUTRAL line goes out

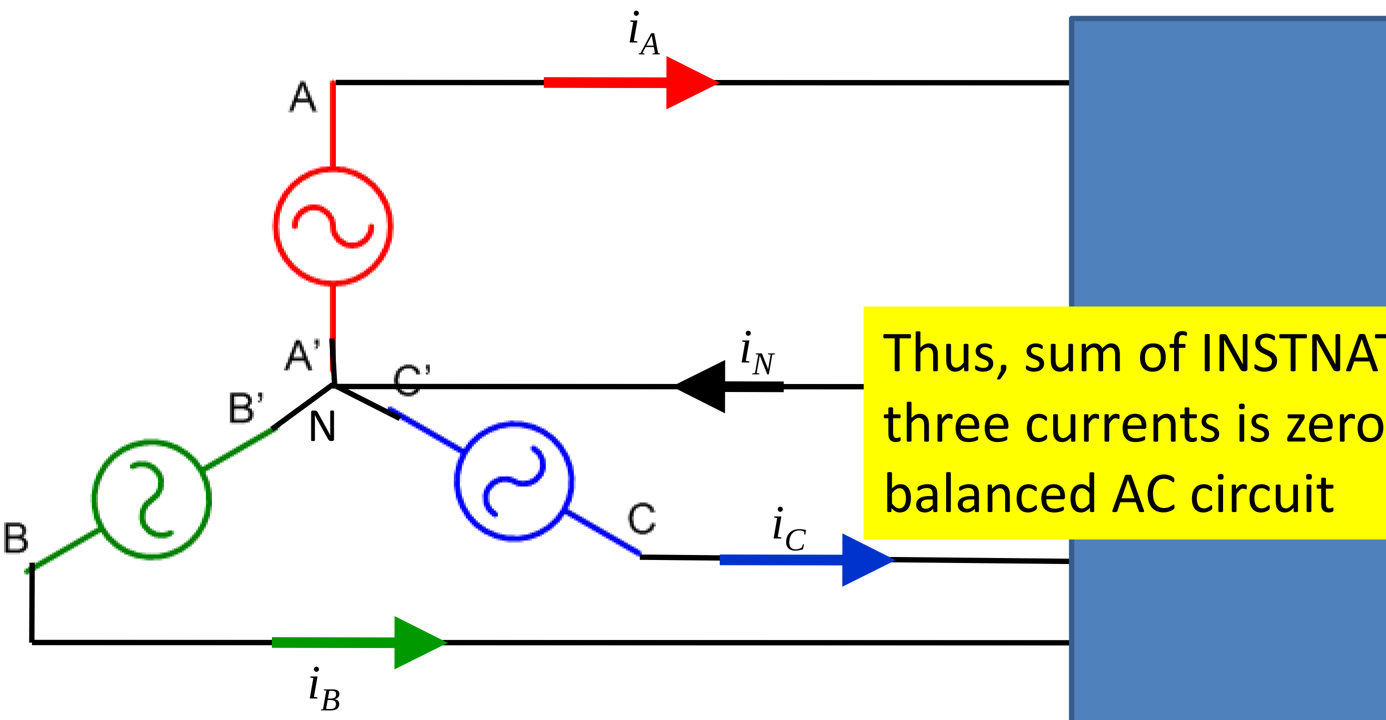


$$i_A = I_m \sin \omega t$$

$$i_B = I_m \sin\left(\omega t - 120^0\right)$$

$$i_C = I_m \sin\left(\omega t - 240^0\right)$$

Star connection



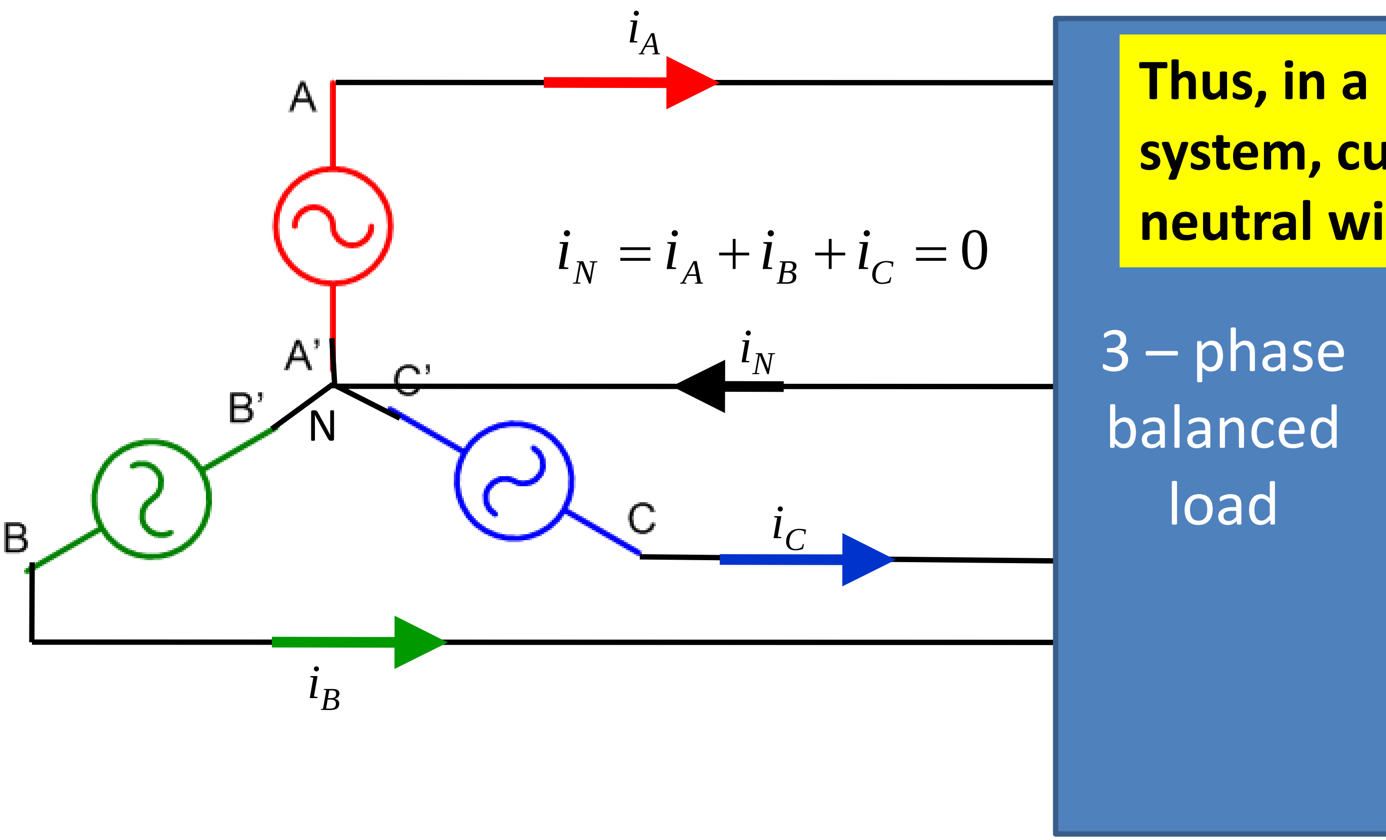
$$i_A = I_m \sin \omega t$$
$$i_B = I_m \sin\left(\omega t - 120^0\right)$$
$$i_C = I_m \sin\left(\omega t - 240^0\right)$$

Thus, sum of INSTNATNEOUS values of three currents is zero in a three-phase balanced AC circuit

- KCL at the star point (neutral point N)

$$\begin{aligned}
i_N &= i_A + i_B + i_C \\
&= I_m \sin \omega t + I_m \sin\left(\omega t - 120^0\right) + I_m \sin\left(\omega t - 240^0\right) \\
&= I_m\left[\sin \omega t + \sin \omega t \cos 120^0 - \cos \omega t \sin 120^0 + \sin \omega t \cos 240^0 - \cos \omega t \sin 240^0\right] \\
&= I_m\left[\sin \omega t - 0.5 \sin \omega t - 0.866 \cos \omega t - 0.5 \sin \omega t + 0.866 \cos \omega t\right] \\
&= I_m\left[\sin \omega t - \sin \omega t\right] \; = 0
\end{aligned}$$

Star connection – 3 Wire system



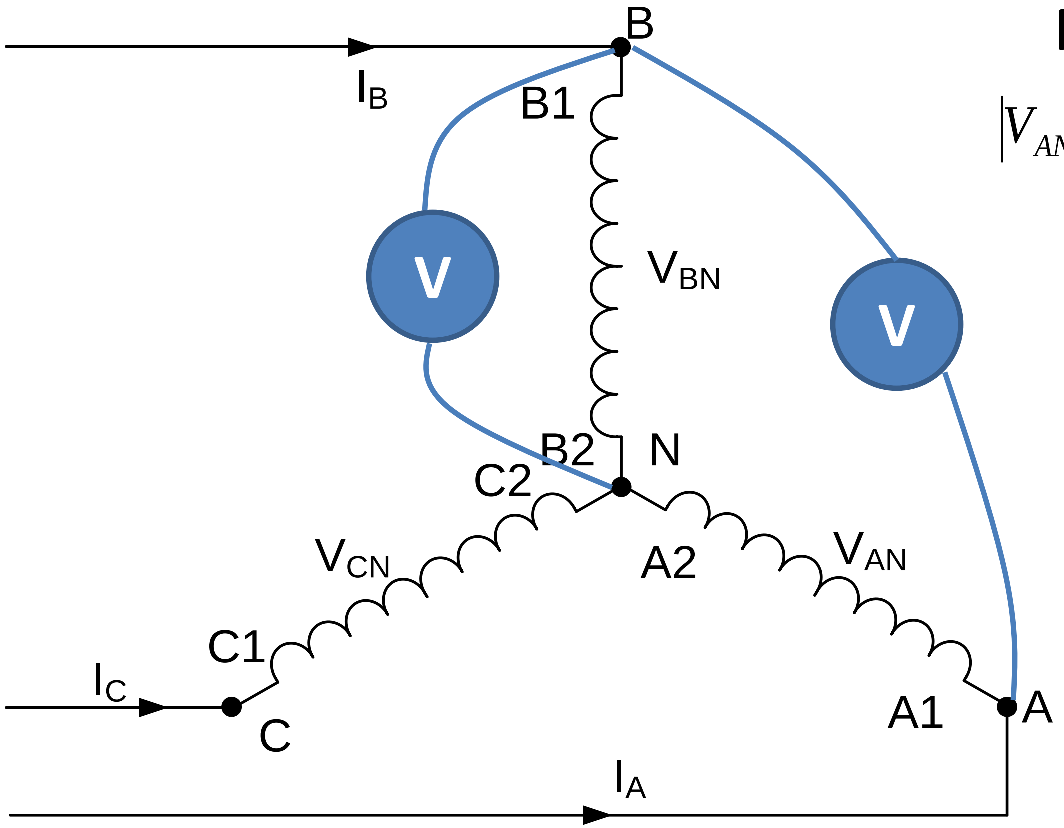
$$i_N = i_A + i_B + i_C = 0$$

Thus, in a balanced 3-phase system, current through the neutral wire is ZERO

Thus, since the neutral wire does not carry any current in a balanced 3-phase system, often the neutral wire is not used

Thus, we have the 3-phase 3-wire system

Voltage & current in STAR connection



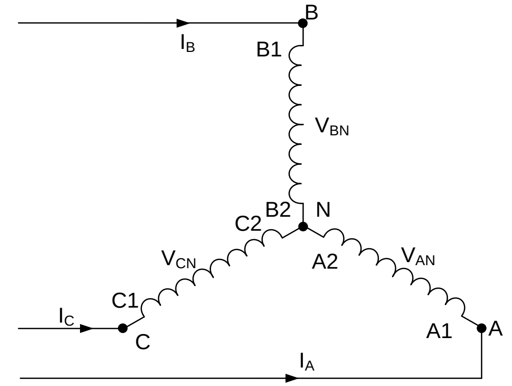
Phase voltage

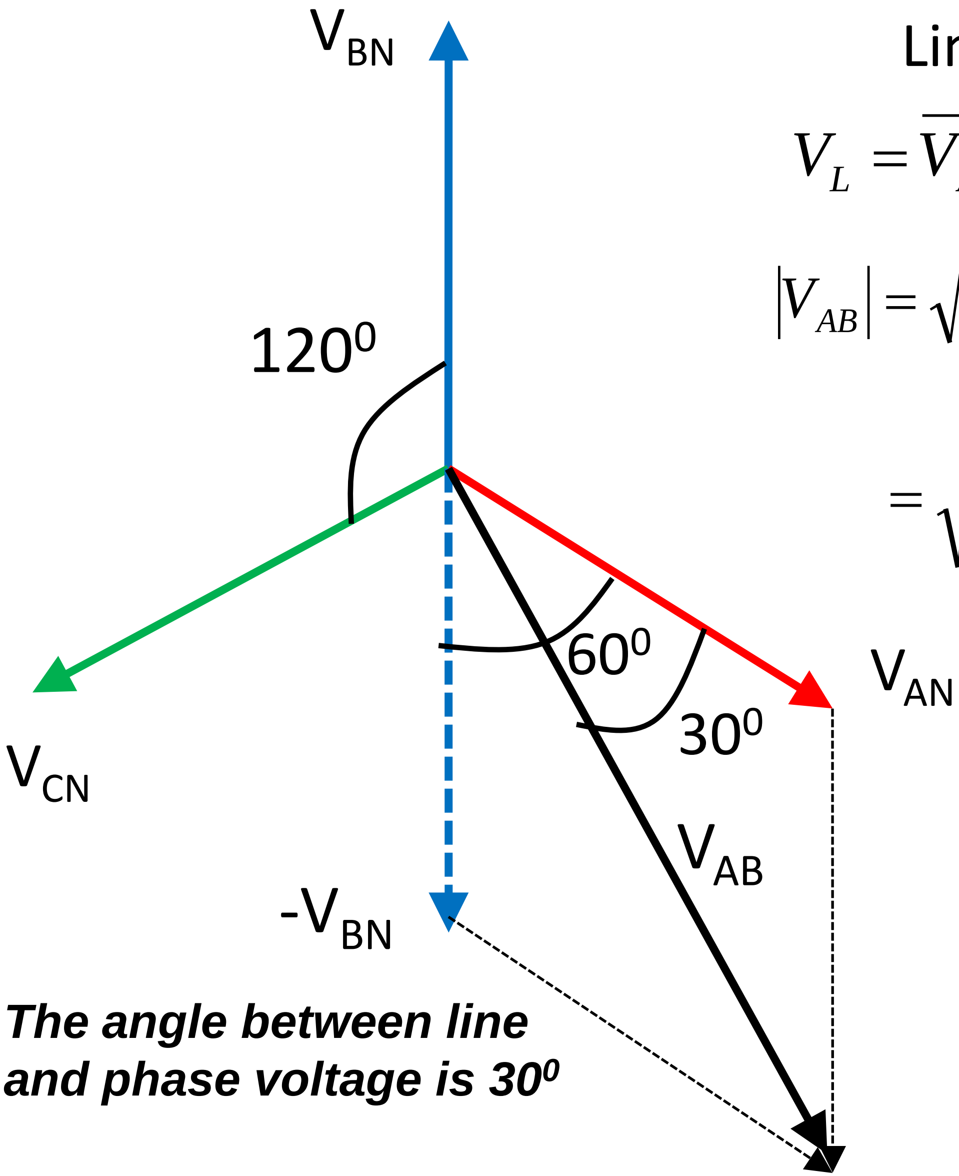
$$|V_{AN}| = |V_{BN}| = |V_{CN}| = V_{ph}$$

Line voltage

$$\overline{V_{AB}} = V_A - V_B$$
$$= V_A - V_N - V_B + V_N$$
$$= (V_A - V_N) - (V_B - V_N)$$
$$= \overline{V_{AN}} - \overline{V_{BN}}$$

STAR connection





Line voltage

$$V_L = \overline{V_{AB}} = \overline{V_{AN}} - \overline{V_{BN}}$$

$$|V_{AB}| = \sqrt{|V_{AN}|^2 + |V_{BN}|^2 + 2 \times |V_{AN}| \times |V_{BN}| \cos 60^0}$$

$$= \sqrt{V_{ph}{}^2 + V_{ph}{}^2 + 2 \times V_{ph} \times V_{ph} \times \frac{1}{2}}$$

$$= \sqrt{V_{ph}{}^2 + V_{ph}{}^2 + V_{ph}{}^2}$$

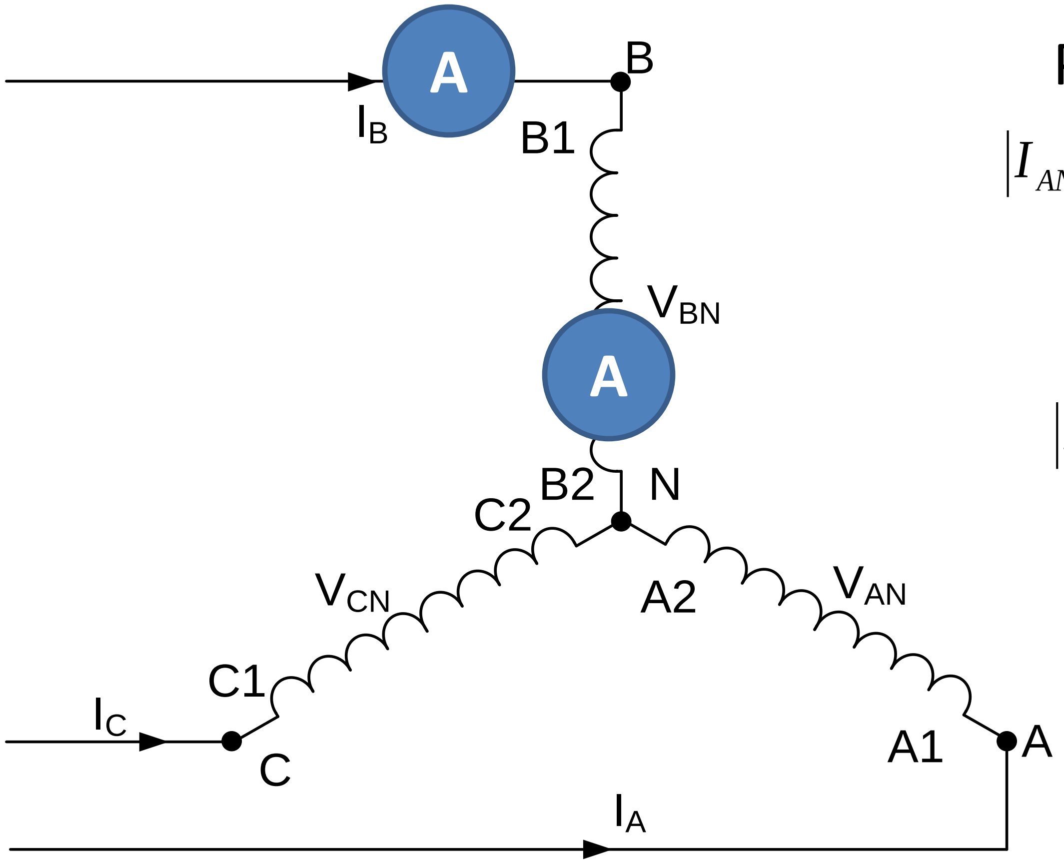
$$= \sqrt{3V_{ph}{}^2}$$

$$= \sqrt{3}V_{ph}$$

$$\boxed{V_L = \sqrt{3}V_{ph}}$$ For balanced system

The angle between line and phase voltage is 30⁰

STAR connection



Phase current

$$|I_{AN}| = |I_{BN}| = |I_{CN}| = I_{ph}$$

Line current

$$|I_A| = |I_B| = |I_C| = I_L$$

$$\boxed{I_L = I_{ph}}$$

In a balanced star connected system, line currents are equal to phase currents, but line voltages are √3 times the phase voltages.