AC Fundamentals

Day 14

Series Resonance

ILOs – Day 14

- Investigate resonance condition in series RLC circuit
  - Determine the condition for series resonance
  - Identify the circuit conditions under series resonance
  - Plot the impedance and current variation profile under series resonance
  - Obtain expression for Quality factor of a series resonating circuit

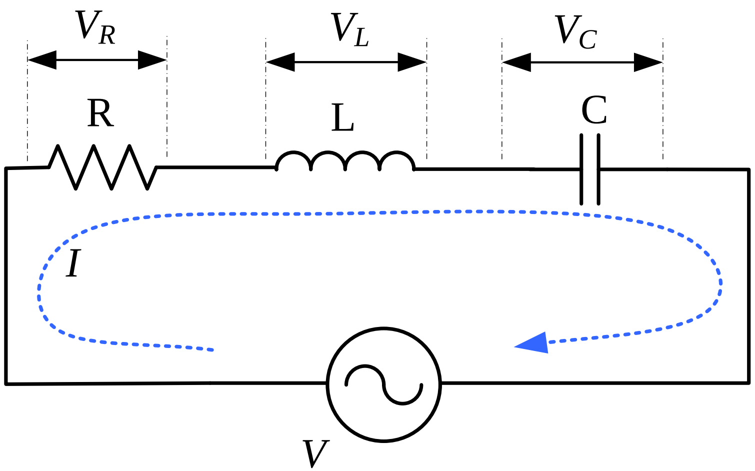
Resonance

- Resonance in electrical circuits is a particular condition of the circuit when
  - **The circuit impedance become maximum or minimum**
  - **The current in the circuit is minimum or maximum**
  - **The effective power factor of the circuit becomes unity**

- The phenomenon of resonance is observed in both series and parallel AC circuits comprising of R, L, and C

Series Resonance

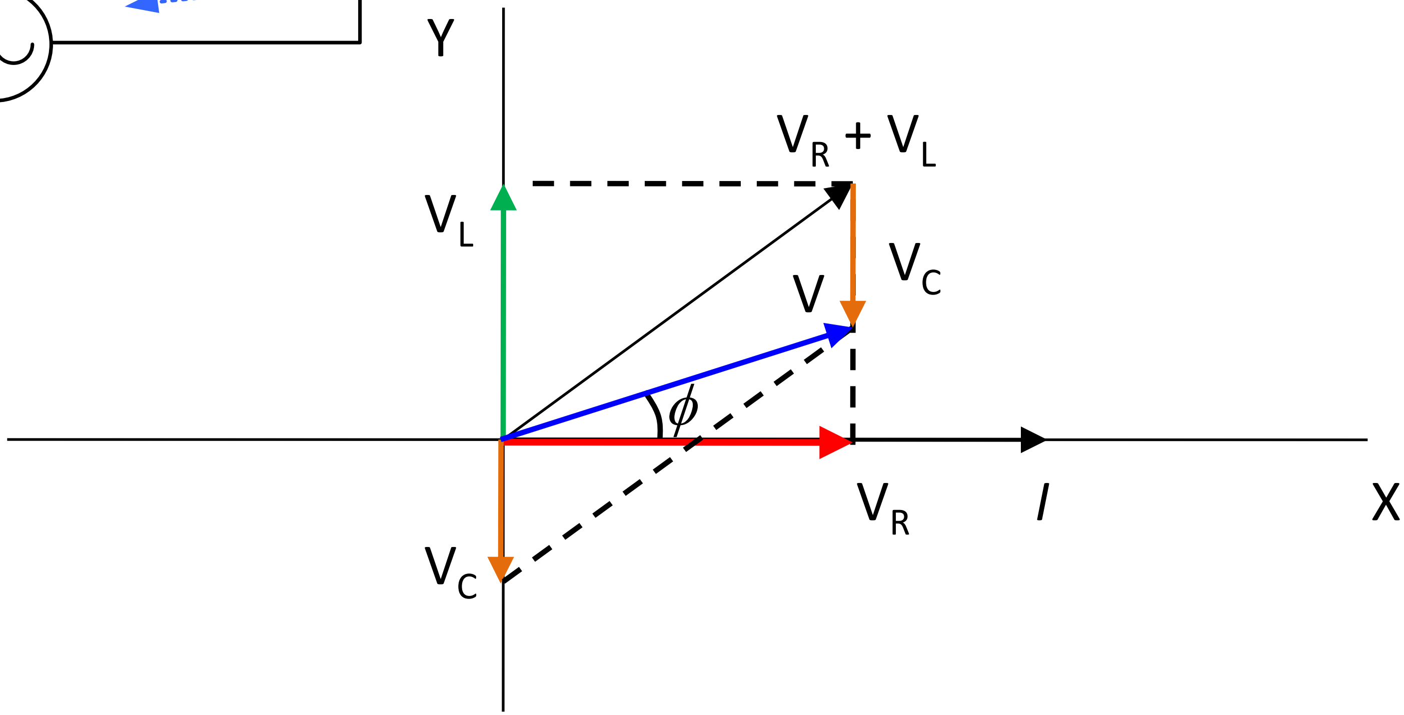
- Series resonance condition can occur in an AC circuit containing R, L, and C in series across an AC source



According to KVL

$$\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_C}$$

Phasor diagram



Series Resonance

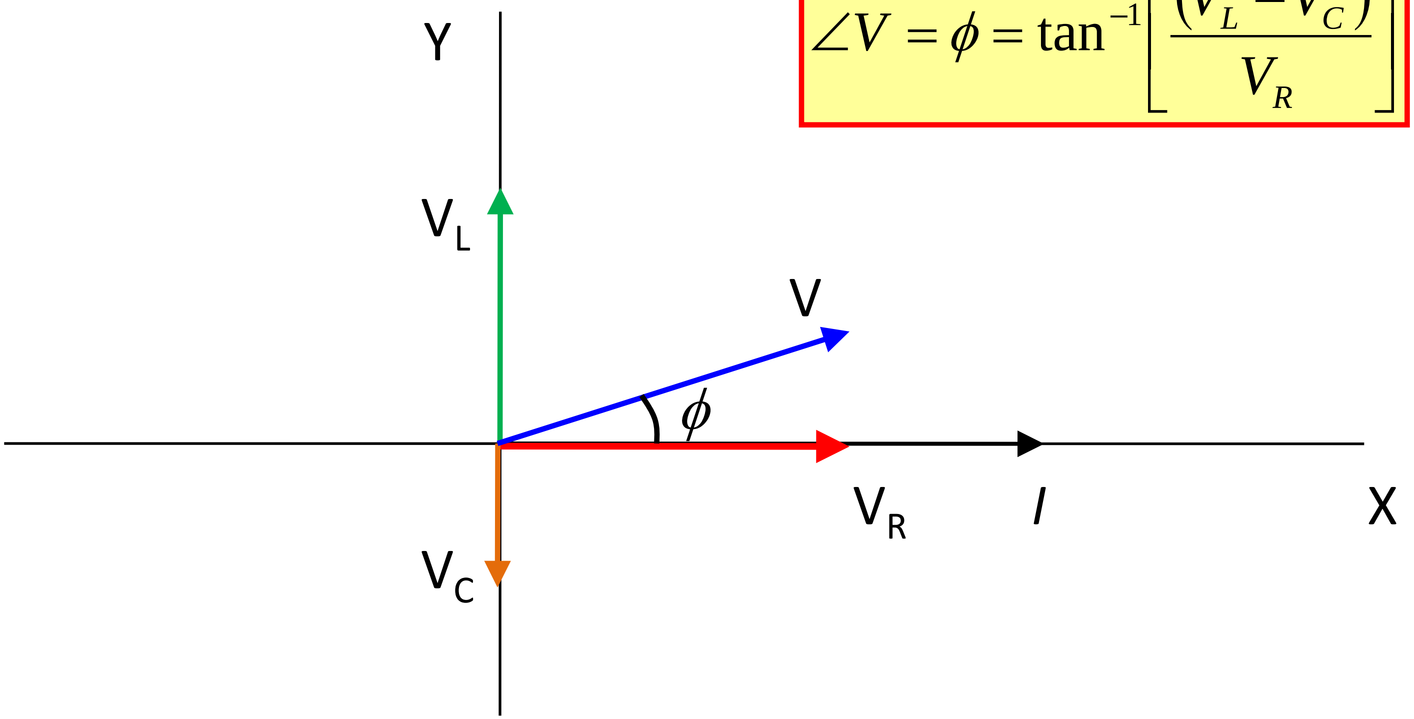
According to KVL $\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_C}$

In complex notations $\overline{V} = V_R + jV_L - jV_C$

$$\overline{V} = V_R + j(V_L - V_C)$$

$$\boxed{|V| = \sqrt{V_R^{\,2} + (V_L - V_C)^2}}$$

$$\boxed{\angle V = \phi = \tan^{-1}\left[\frac{(V_L - V_C)}{V_R}\right]}$$



Series Resonance

$$\overline{V} = V_R + j(V_L - V_C)$$

$$\overline{V} = \overline{I}R + j(\overline{I}X_L - \overline{I}X_C)$$

$$\overline{V} = \overline{I}R + j\overline{I}(X_L - X_C)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X^2}$$

Where, $X = (X_L - X_C)$

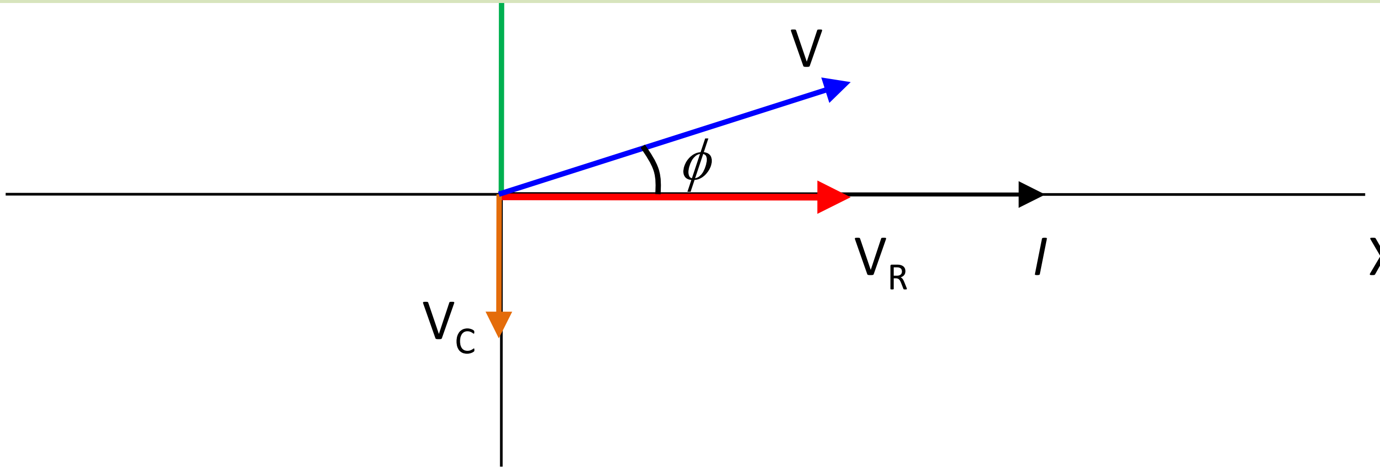
is the total effective reactance of the circuit

Thus, equivalent impedance of the circuit is:

$$Z = \frac{\overline{V}}{\overline{I}} = R + j(X_L - X_C)$$

$$\angle Z = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right] = \phi$$

Note that angle of the impedance is same as the power factor angle ϕ



Case I: $X_L > X_C$, i.e. $V_L > V_C$

$$\overline{V} = V_R + j(V_L - V_C)$$

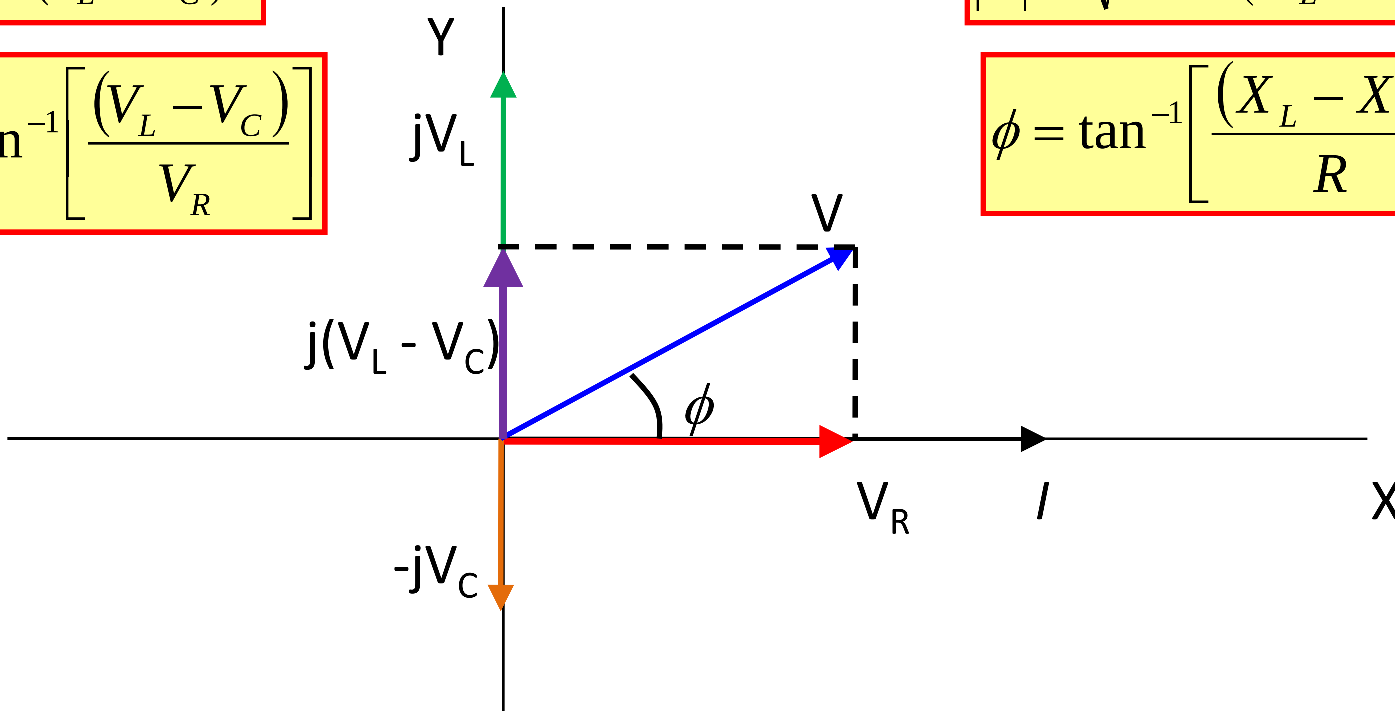
$$|V| = \sqrt{V_R^{\ 2} + (V_L - V_C)^2}$$

$$\angle V = \phi = \tan^{-1}\left[\frac{(V_L - V_C)}{V_R}\right]$$

$$Z = R + j(X_L - X_C)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right]$$



The supply voltage *V* leads the supply current *I* when $V_L > V_C$

The effect of inductance is dominant

Case II: $X_L < X_C$, i.e. $V_L < V_C$

$$\overline{V} = V_R + j(V_L - V_C) = V_R - j(V_C - V_L)$$

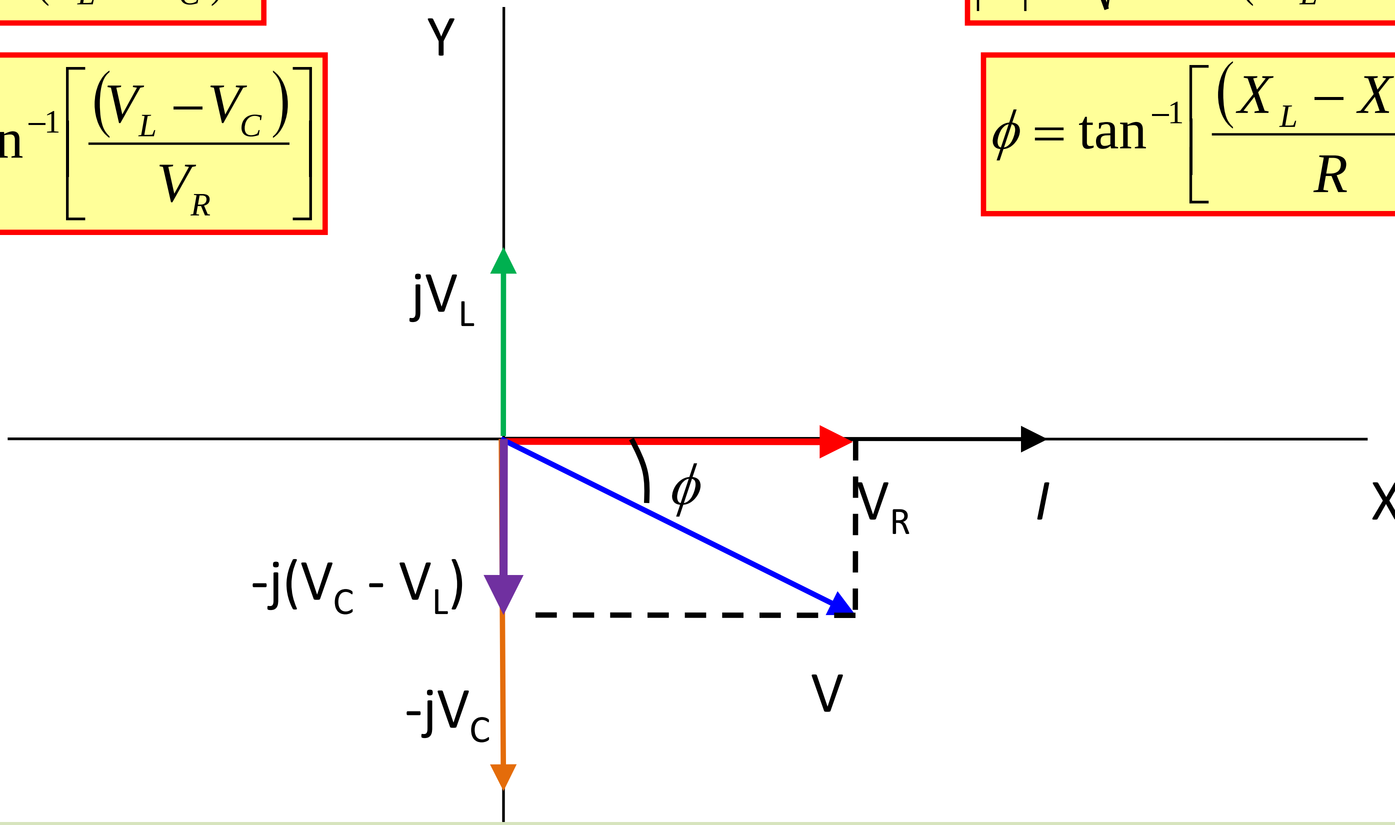
$$|V| = \sqrt{V_R^{\ 2} + (V_L - V_C)^2}$$

$$\angle V = \phi = \tan^{-1}\left[\frac{(V_L - V_C)}{V_R}\right]$$

$$Z = R + j(X_L - X_C)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

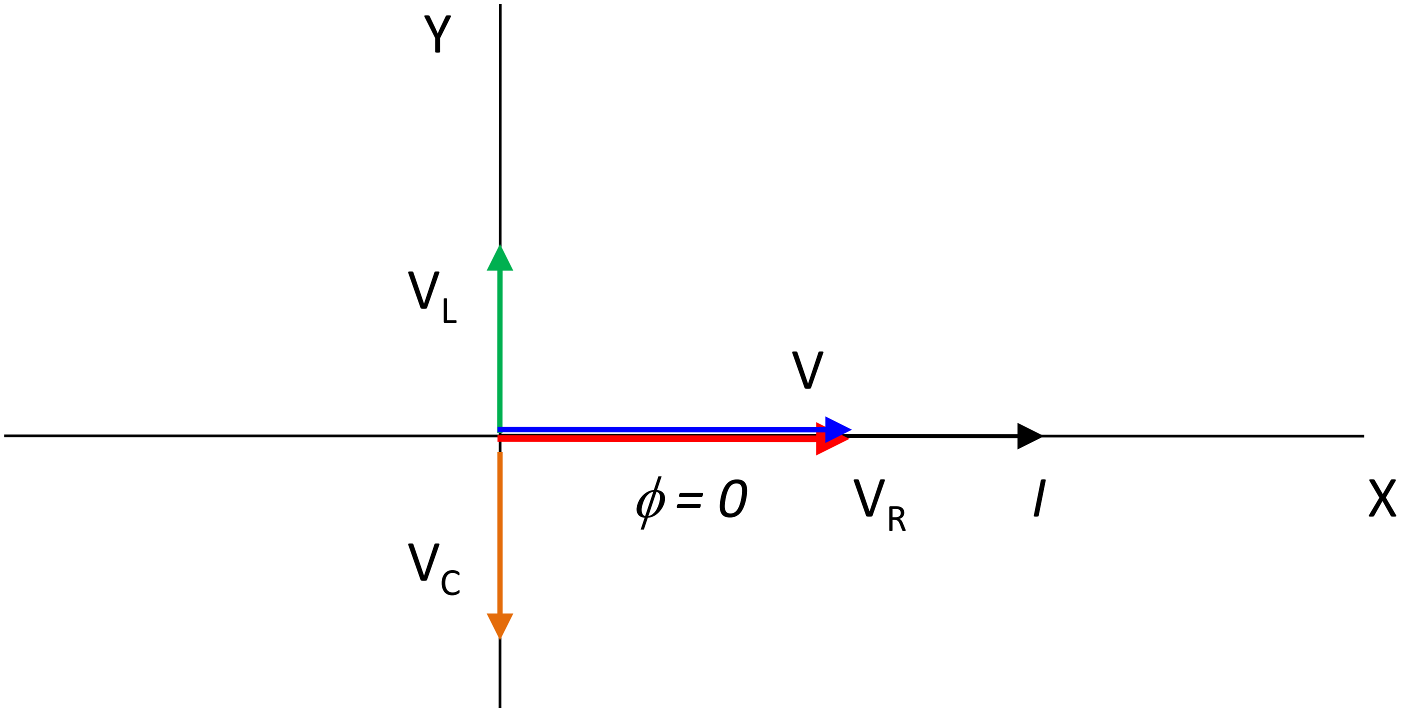
$$\phi = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right]$$



The supply voltage V lags the supply current I when $V_L < V_C$

The effect of capacitance is dominant

Case III: $X_L = X_C$, i.e. $V_L = V_C$



V_L and V_C cancel out each other, so the supply voltage $V = V_R$

Case III: $X_L = X_C$, i.e. $V_L = V_C$

$$\overline{V} = V_R + j(V_L - V_C)$$

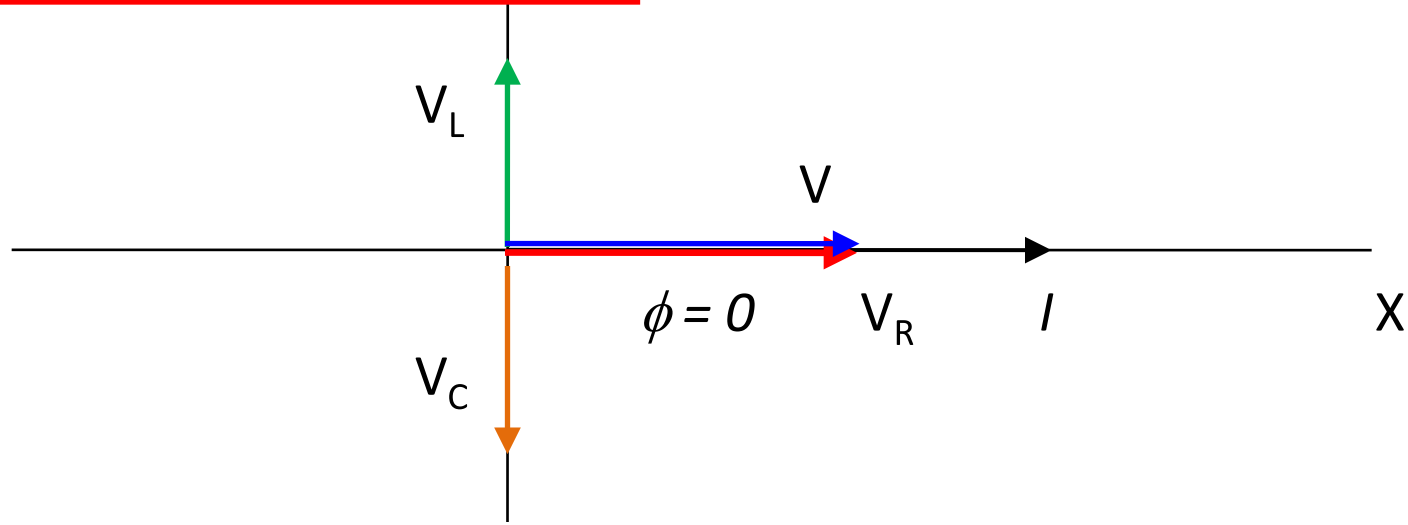
$$\overline{V} = V_R$$

$$\angle V = \phi = \tan^{-1}\left[\frac{(V_L - V_C)}{V_R}\right] = \tan^{-1}\left[\frac{0}{V_R}\right] = 0^0$$

$$Z = R + j(X_L - X_C) = R$$

$$|Z| = R$$

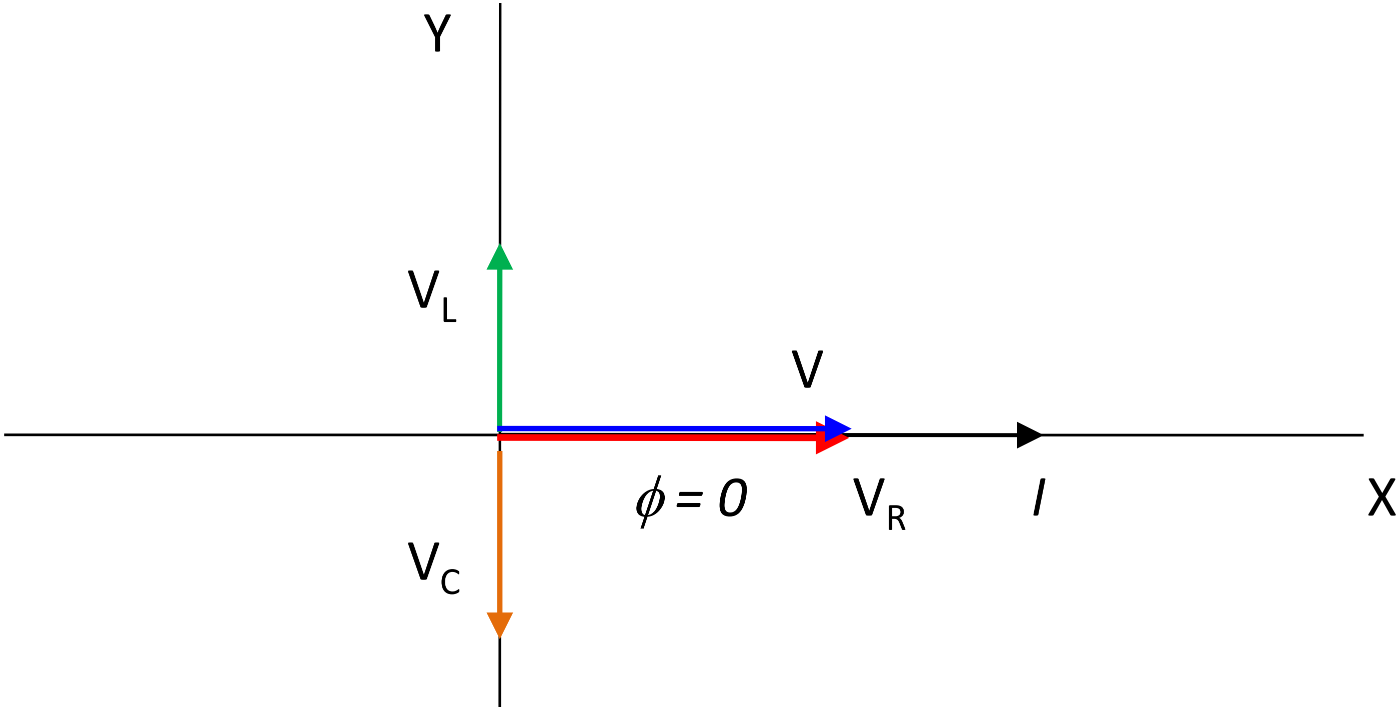
$$\phi = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right] = \tan^{-1}\left[\frac{0}{R}\right] = 0^0$$



The supply voltage *V* and supply current *I* are in the same phase

Case III: $X_L = X_C$, i.e. $V_L = V_C$

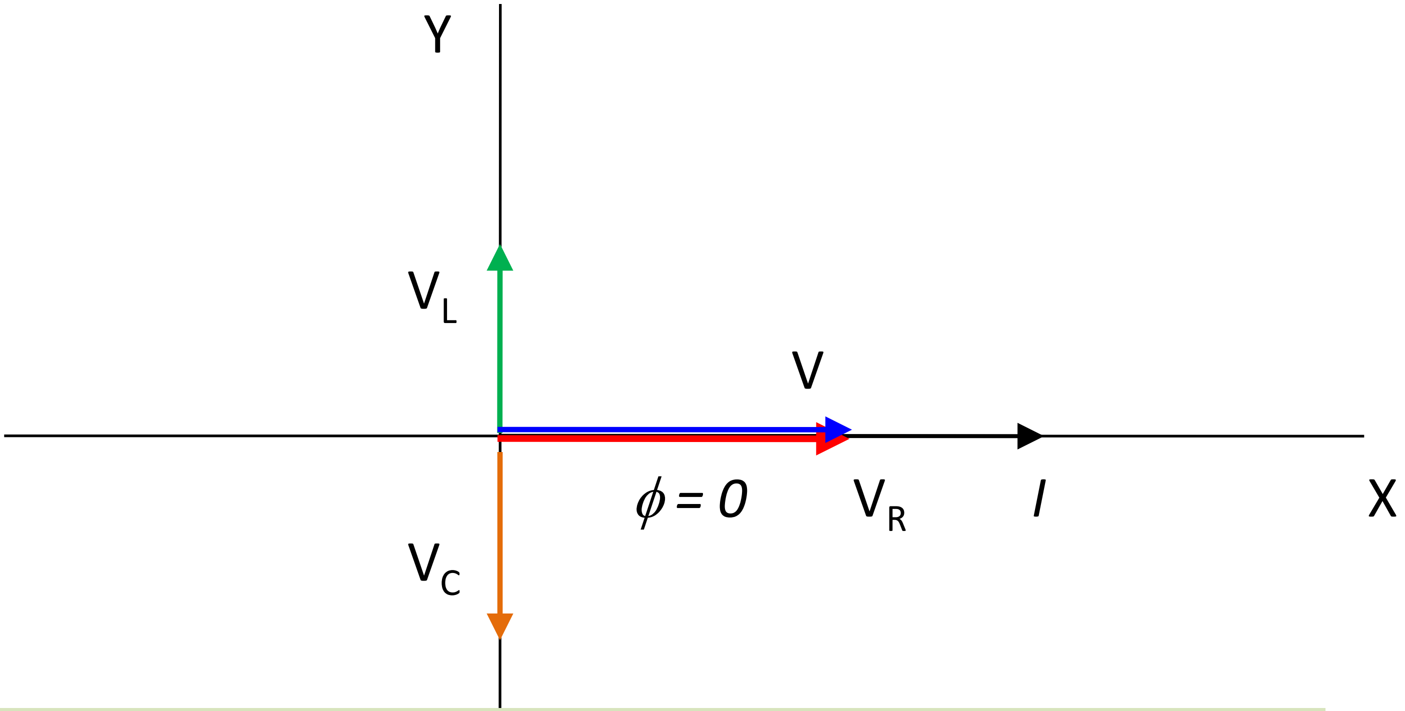
$$\angle V = \phi = \tan^{-1}\left[\frac{(V_L - V_C)}{V_R}\right] = \tan^{-1}\left[\frac{0}{V_R}\right] = 0$$



Phase angle difference between supply voltage and supply current is 0^0

Case III: $X_L = X_C$, i.e. $V_L = V_C$

$$\angle V = \phi = 0^0$$



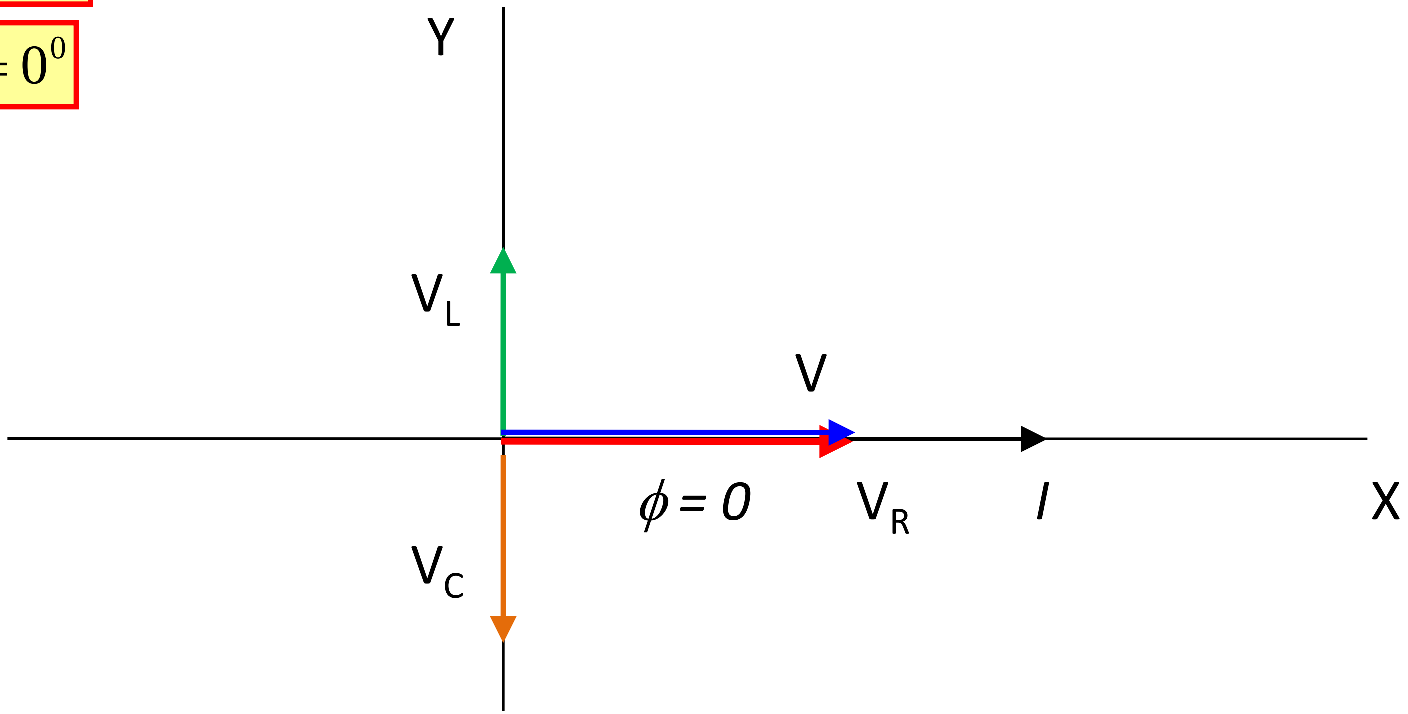
Hence the power factor of the circuit becomes $\cos 0^0 = 1$

Case III: $X_L = X_C$, i.e. $V_L = V_C$

$\boxed{\overline{V} = V_R}$

$\boxed{|Z| = R}$

$\boxed{\phi = 0^0}$



This is the phenomena of resonance in the series R-L-C circuit

Resonance in series RLC circuit

Relation	Remarks
$V_L = V_C$	Voltage drops across the inductance and capacitance are equal
$X_L = X_C$	**Inductive and capacitive reactances are equal**
$X = \left(X_L - X_C\right) = 0$	Effective reactance of the series circuit is zero
$\lvert Z \rvert = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + 0^2} = R$	The circuit behaves as a purely resistive circuit

Resonance in series RLC circuit

Relation	Remarks
$\phi = \angle Z = \tan^{-1}\frac{(X_L - X_C)}{R}$ $= \tan^{-1}\frac{0}{R}$ $= 0^0$	The phase angle between supply current and voltage is zero, i.e. current and voltage are in the same phase
$\cos\phi = \cos 0^0 = 1$	The overall circuit power factor is unity

Resonance in series RLC circuit

Under normal condition, the circuit impedance is:

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

Under series resonance condition, the circuit impedance is:

$$|Z| = R$$

Thus impedance of a series RLC circuit is minimum under resonance condition

As a result, the current under resonance condition $I = \frac{V}{|Z|} = \frac{V}{R}$

is more than the current under any other condition

$$I = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Resonance in series RLC circuit

Let the series resonance condition is achieved when the supply voltage has an angular frequency of ω_0

Since at series resonance condition we have: $X_L = X_C$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^{\ 2} = \frac{1}{LC}$$

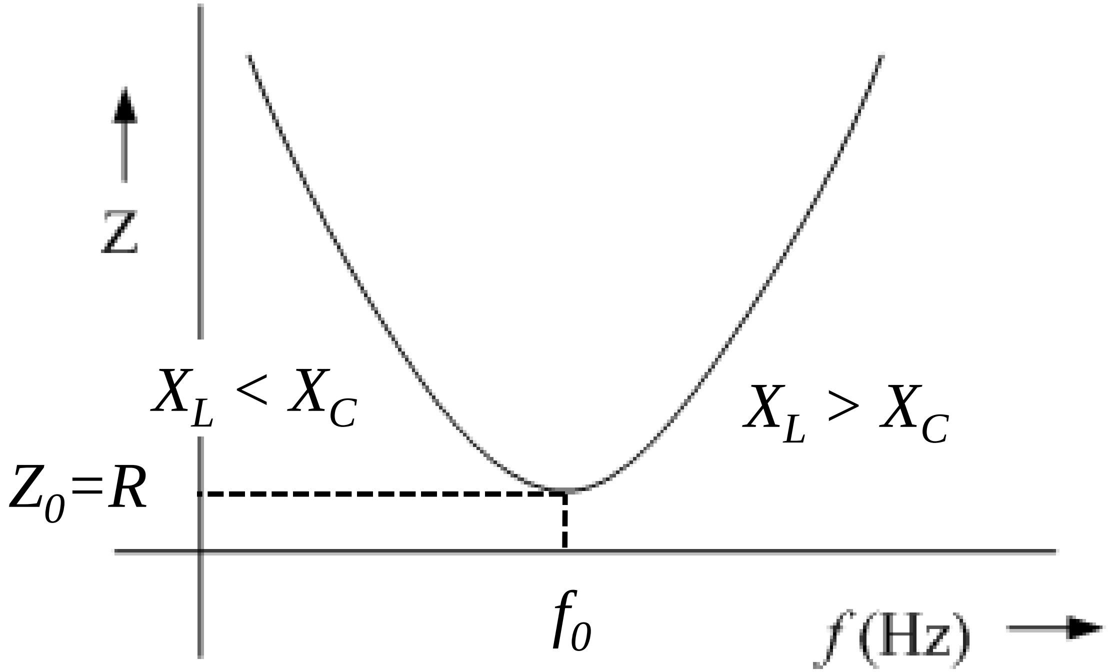
$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}\ rad/s}$$

$$\boxed{f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}\ Hz}$$

Resonance in series RLC circuit

$$\boxed{f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}\ Hz}$$

This is called the resonating frequency (or resonant frequency) of a series resonating circuit

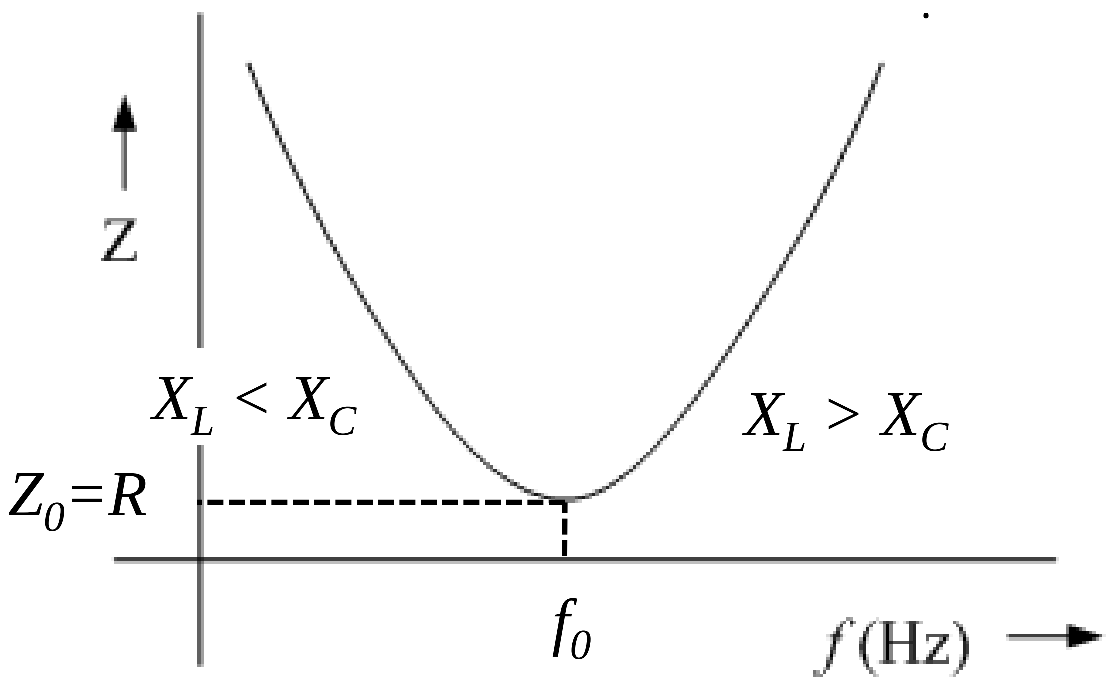


Resonance in series RLC circuit

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}\ Hz$$

Thus, the total impedance is minimum at this particular frequency only

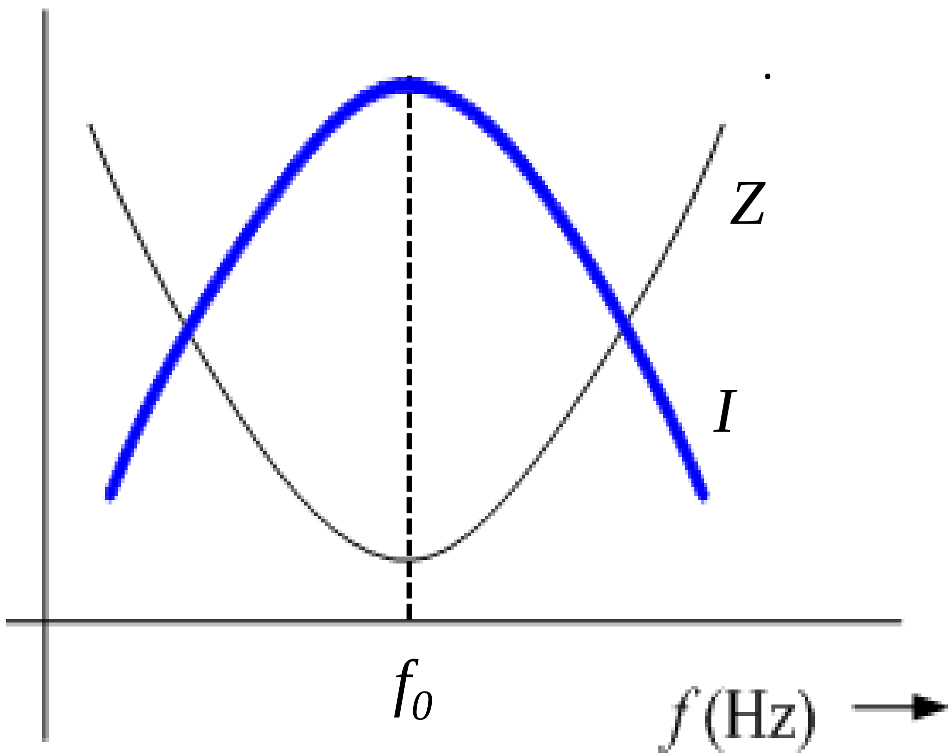
But the impedance is higher when the frequency is either higher or lower than this resonant frequency



Resonance in series RLC circuit

Current in the circuit will have an opposite nature of variation to that of the effective impedance

current will be maximum at the resonant frequency

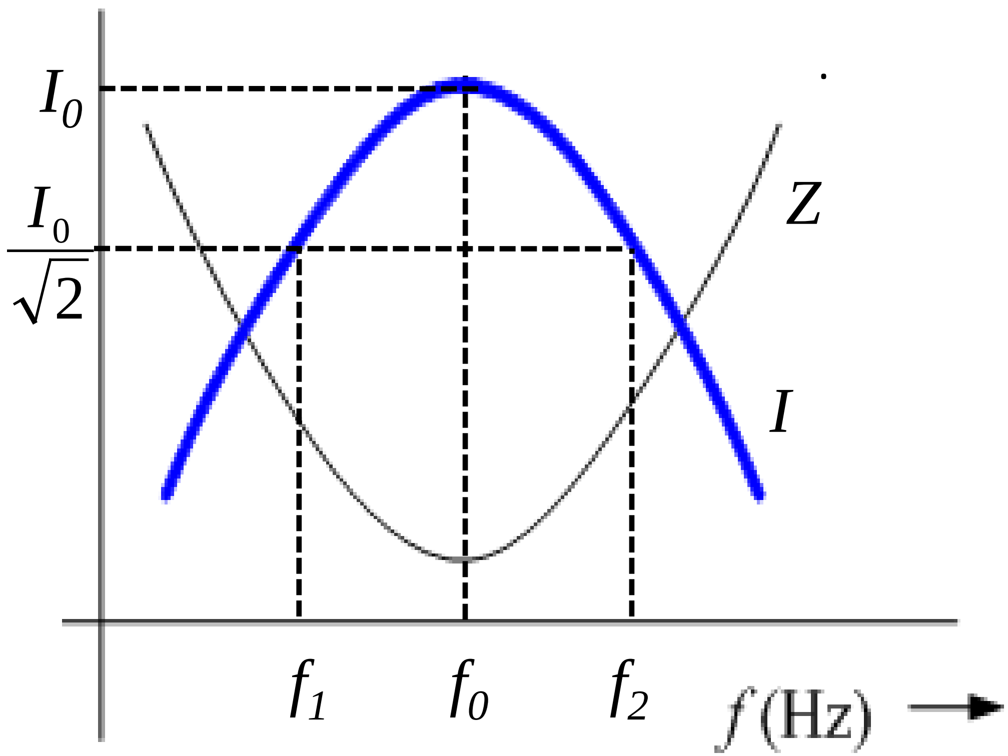


Resonance in series RLC circuit

Maximum current is I_0

When current is $I_0/\sqrt{2}$, the frequency are f_1, f_2

The difference ($f_2 - f_1$) is called **band-width** (BW) of the circuit



Q factor (Quality factor)

- In a series resonating circuit, the Q-factor is defined as the ratio of voltage across the inductor or the capacitor to the applied voltage
- Under resonating condition, the voltage drops across inductor and capacitor are same in a series R-L-C circuit
- Thus we have the expression for Q-factor as:

$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$

For the inductor: $$\boxed{Q = \frac{V_L}{V} = \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega_0 L}{R}}$$

For the capacitor: $$\boxed{Q = \frac{V_C}{V} = \frac{I_0 X_C}{I_0 R} = \frac{X_C}{R} = \frac{1}{\omega_0 C R}}$$

Q factor (Quality factor)

$$\boxed{Q = \frac{\omega_0 L}{R}}$$

$$\boxed{Q = \frac{1}{\omega_0 CR}}$$

Resonant frequency : $\omega_0 = \dfrac{1}{\sqrt{LC}}$

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$\boxed{Q = \frac{1}{\omega_0 CR} = \sqrt{LC} \times \frac{1}{CR} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

One more useful relation:

$$\boxed{Q = \frac{\text{Resonant Frequency}}{\text{Band width}} = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}}$$