AC Fundamentals

Day 8

Basics

ILOs – Day 8

- Define AC voltage and current
- Describe the basic process of AC signal generation
- Derive mathematical expression for an AC signal
- Calculate average and RMS values of sinusoidal AC signal
- Define peak factor and form factor for AC signal

AC Fundamentals

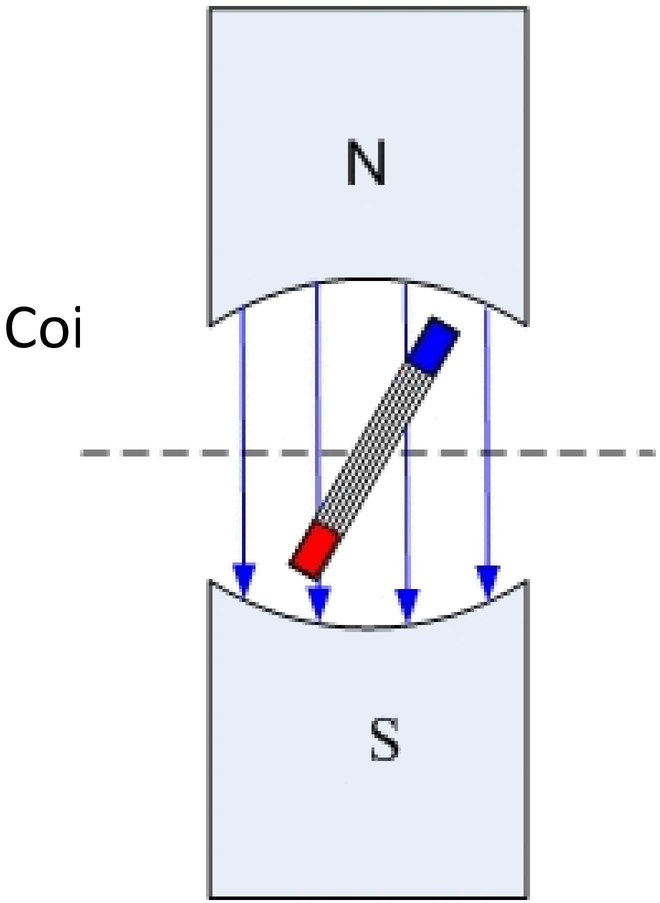
- In a given circuit, the current whose magnitude remains constant and unidirectional with time is called **Direct Current** or **DC**.
- A current that changes magnitude as well as direction periodically with time is called **Alternating Current** or **AC**
- The voltage that causes such current is also alternating
- The circuits in which alternating current flows are called AC circuits

AC Voltage generation

- According to Faraday's law of electromagnetic induction, an EMF gets induced in a coil whenever the magnetic flux linked with the coil changes.

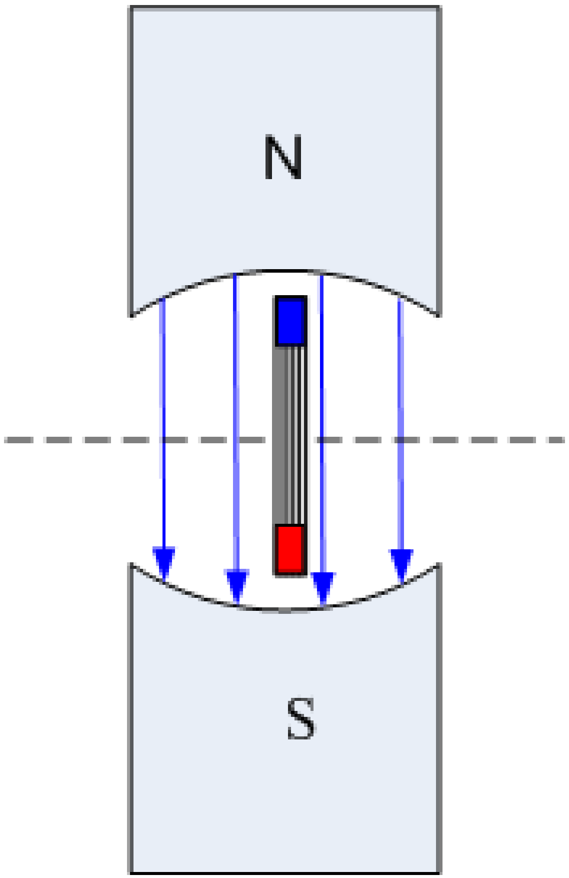
- If there is continuous relative motion between the coil and the magnet, there will be continuous generation of EMF in the coil

AC Voltage generation



- A pair of permanent magnets
- And a coil with *N* number turns
- Coil rotates in the space between the pair of magnets
- Thus, there is continuous change in flux linkage with the coil
- EMF is hence induced in the coil

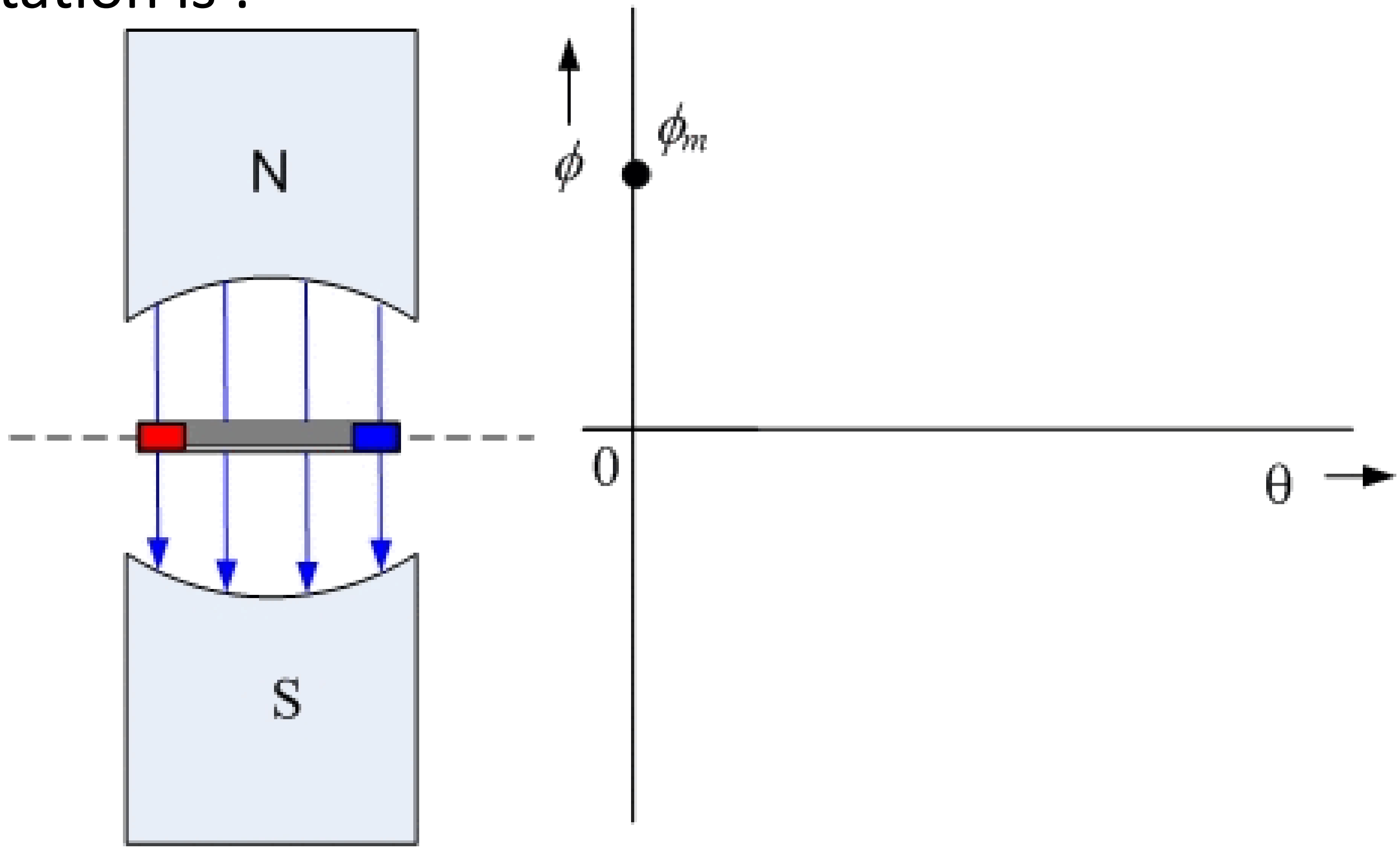
AC Voltage generation



- When the coil is horizontal, maximum flux links with it
- As it rotates, the amount of flux linking the coil gradually reduces
- Finally when the angle $\theta = 90^0$, i.e. the coil is vertical, the flux lines become parallel with the coil and thus flux linkage is zero

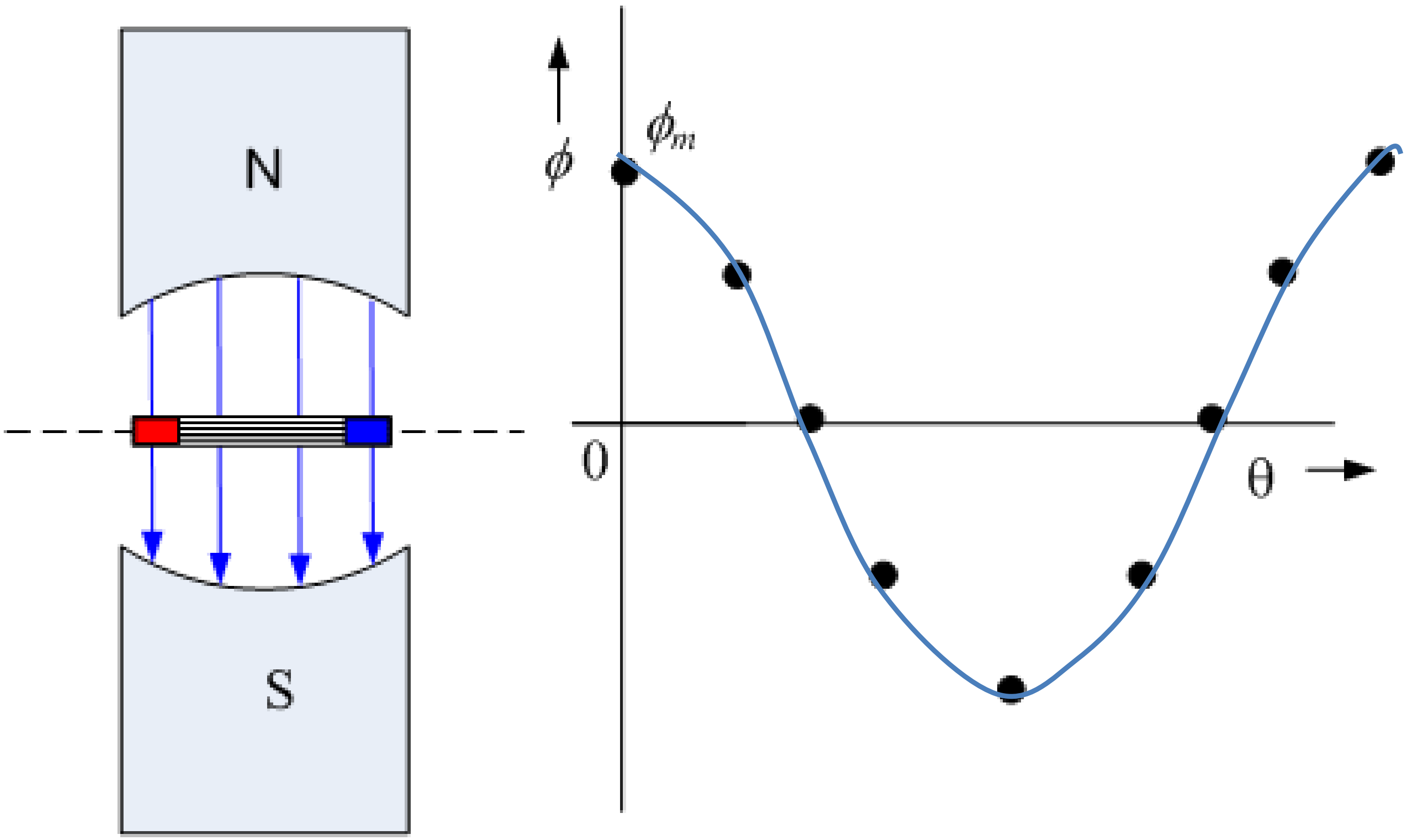
AC Voltage generation

The nature of flux variation with respect to the angle of rotation is :

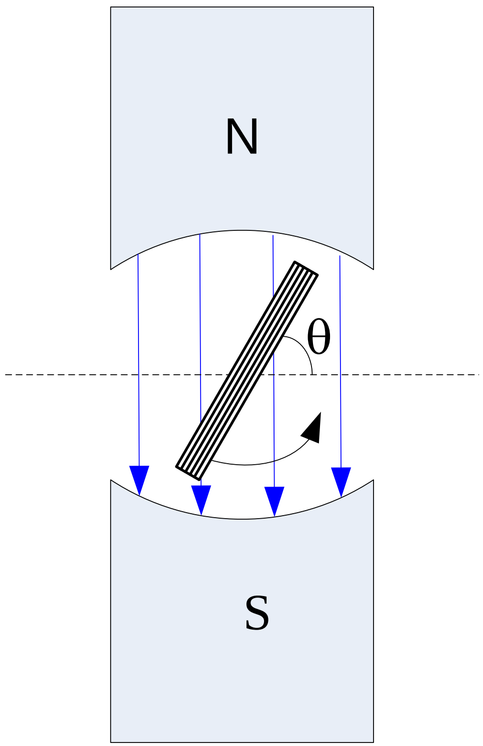


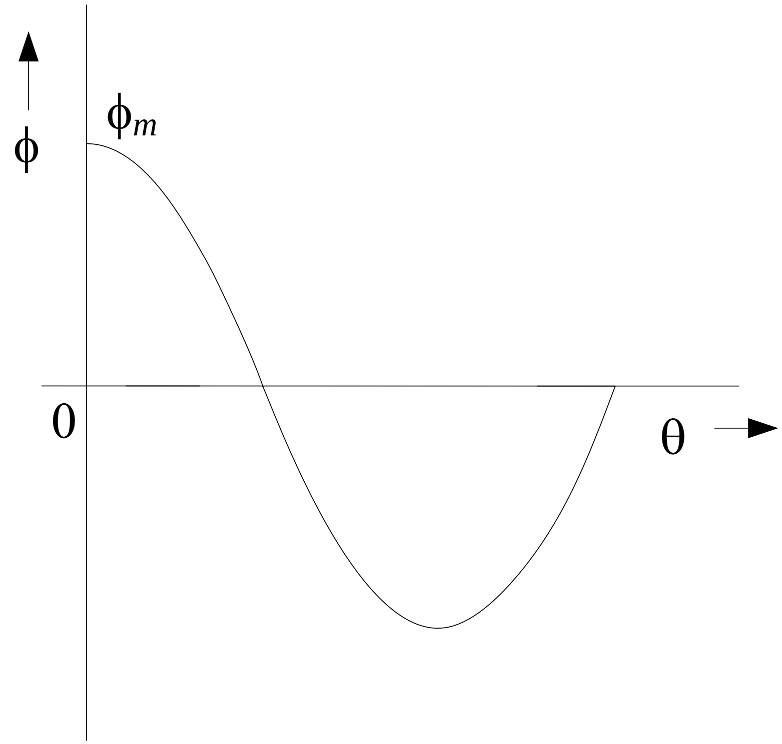
AC Voltage generation

The nature of flux variation with respect to the angle of rotation is :



AC Voltage generation





Expression for the flux wave is co-sinusoidal: $\phi = \phi_m \cos\theta$

Where, ϕ_m is the peak value of flux that links with the coil

AC Voltage generation

$$\phi = \phi_m \cos\theta$$

- If the coil rotates at an angular speed of ω rad/s,
- Then , $\theta = \omega t$
- Hence the flux variation with respect to time 't' can be written as:

$$\phi = \phi_m \cos\omega t$$

According to Faraday's law, EMF induced in the coil is thus:

$$e = -N\frac{d\phi}{dt}$$

$$e = -N\frac{d}{dt}\left(\phi_m \cos\omega t\right)$$

$$e = -N\phi_m \frac{d}{dt}\left(\cos\omega t\right)$$

$$e = \left(N\phi_m\omega\right)\sin\omega t$$

$$e = E_m \sin\omega t$$

Where,

$$E_m = N\phi_m\omega$$

is the peak value of the EMF wave which is a sinusoidal signal

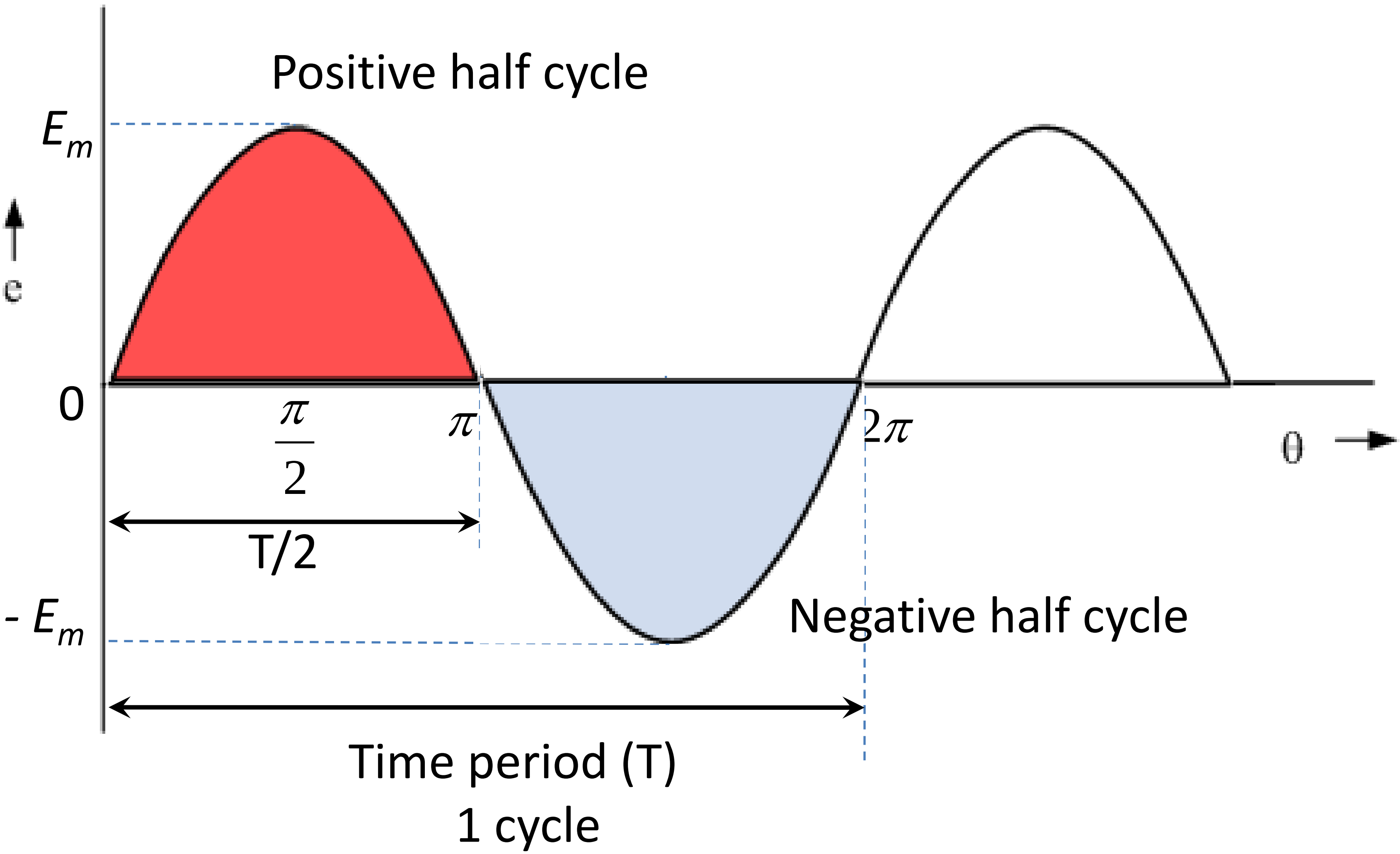
AC Voltage generation

$$e = E_m \sin\theta = E_m \sin\omega t$$

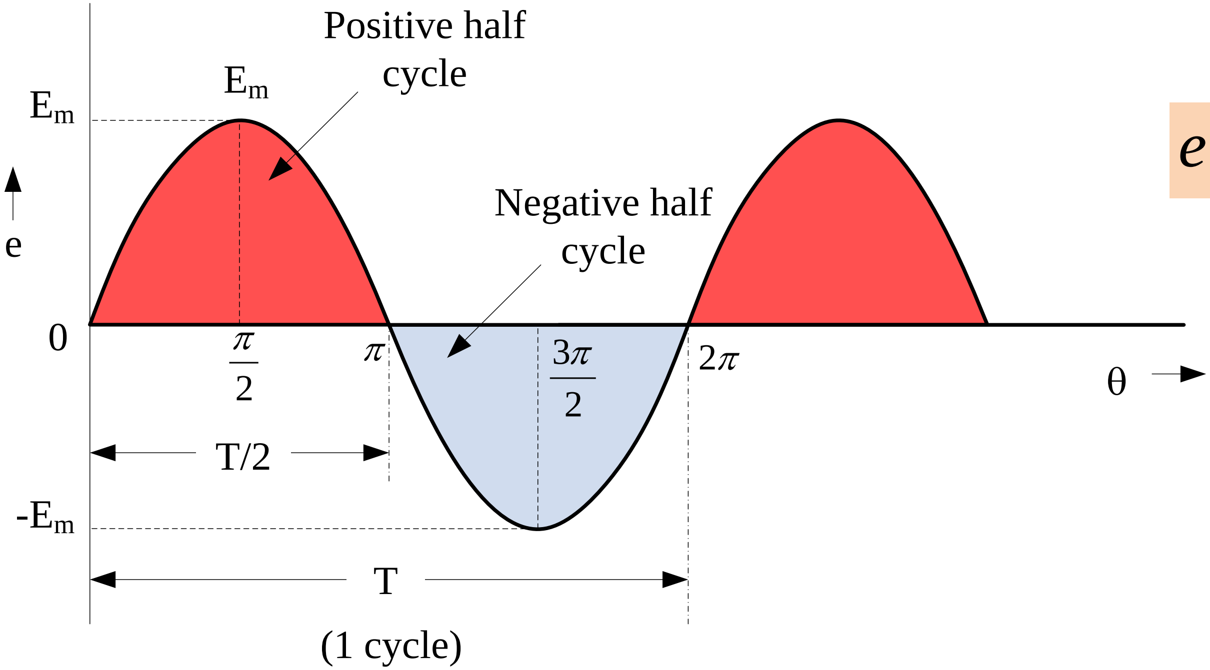
- The EMF generated in the coil is thus sinusoidal in nature
- This is the source of alternating current in AC circuits
- Continuous generation of AC signal is possible by rotating the coil continuously

AC Voltage generation

$$e = E_m \sin\theta = E_m \sin\omega t$$



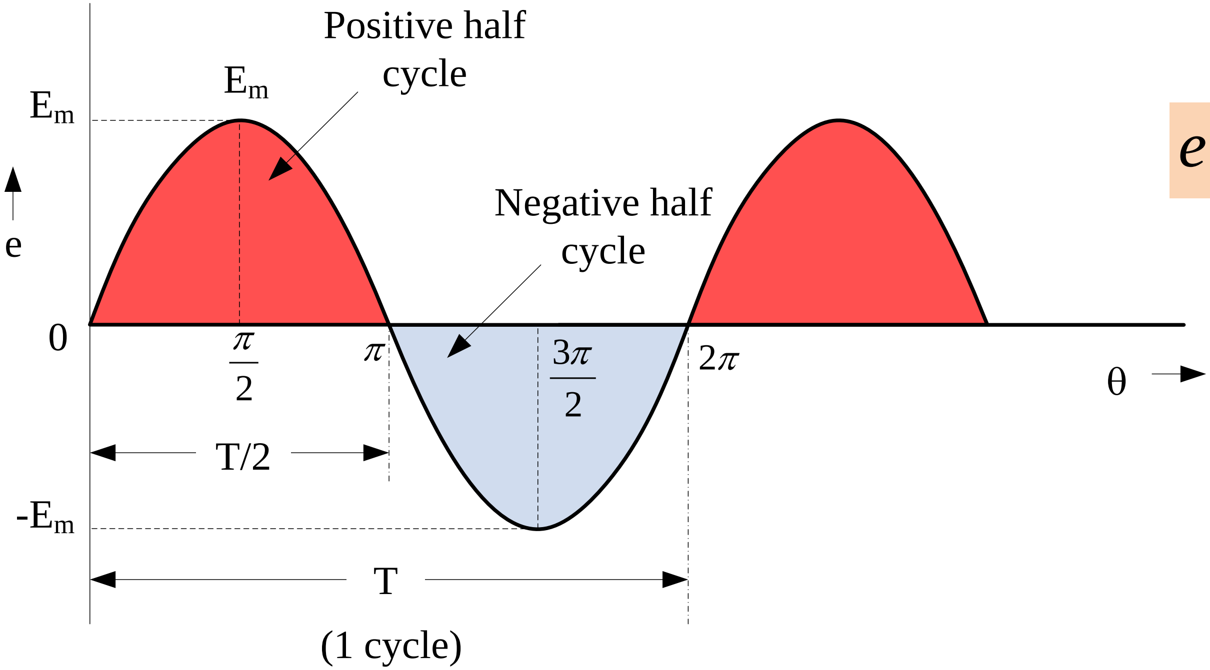
AC Voltage generation



$$e = E_m \sin\theta = E_m \sin\omega t$$

- The time taken to complete one full cycle is T, the time period.
- A term also commonly used to denote the rate of alteration of an AC signal is its frequency 'f'
- Frequency indicates how many such complete cycles the signal can make in 1 second time
- Frequency is thus the number of cycles per second or Hertz (Hz)

AC Voltage generation



$$e = E_m \sin\theta = E_m \sin\omega t$$

- The relationship between T and f is thus:

$$T = \frac{1}{f}$$

- Also we have the relation:

$$\omega T = 2\pi$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$T = \frac{2\pi}{\omega}$$

Alternate expressions

$$e = E_m \sin\theta = E_m \sin\omega t$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

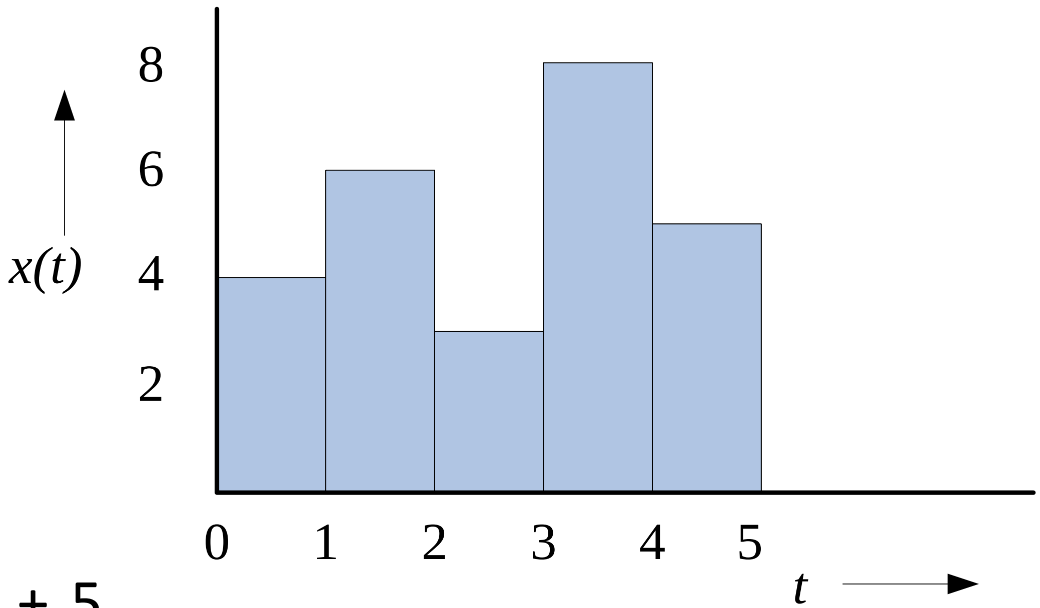
Alternate expressions:

$$\boxed{e = E_m \sin 2\pi f t}$$

$$\boxed{e = E_m \sin \frac{2\pi}{T} t}$$

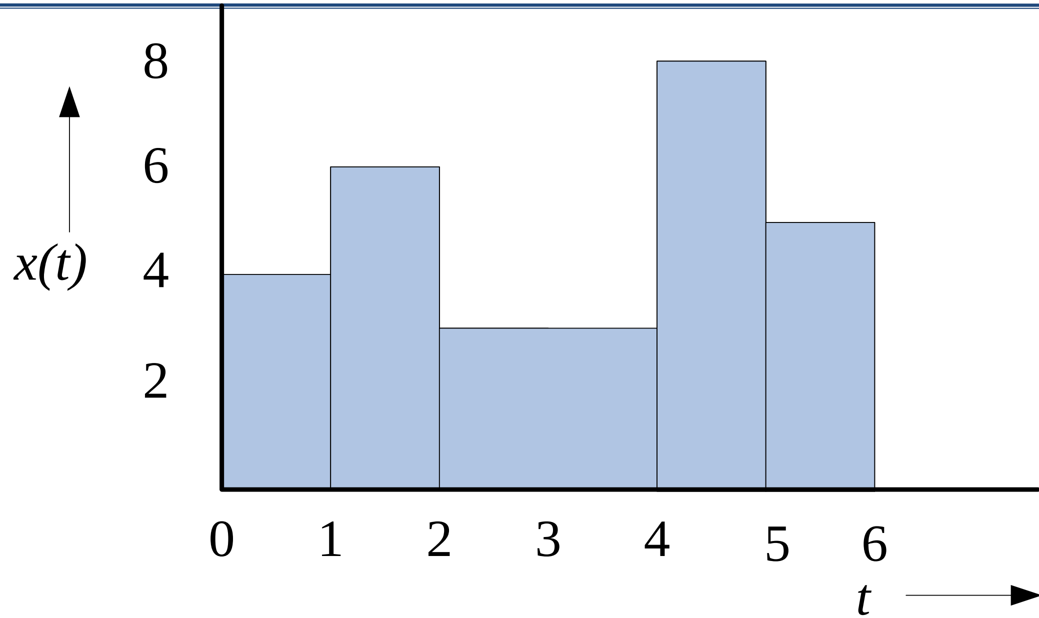
Average value of a signal

Average value of a signal during a certain time is the total strength of the signal divided by the total time



$$X_{av} = \frac{4 + 6 + 3 + 8 + 5}{5} = 5.2$$

Average value of a signal

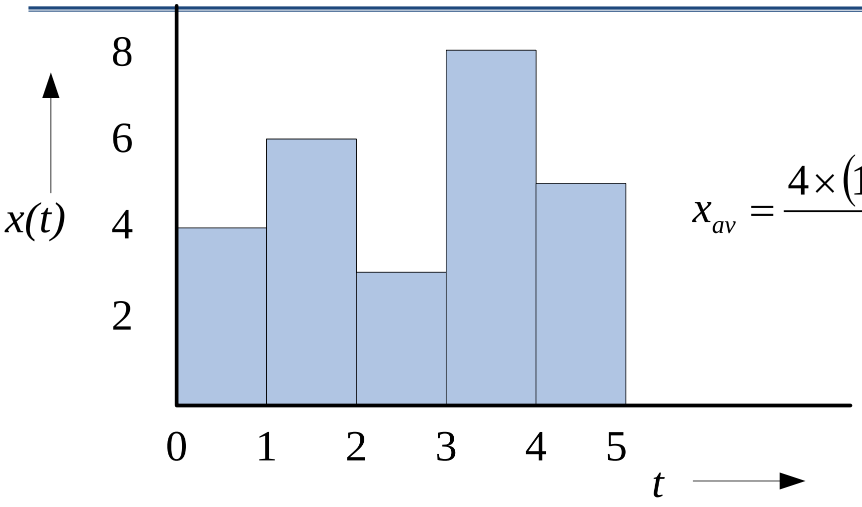


$$x_{\text{av}} = \frac{4 + 6 + 3 + 3 + 8 + 5}{6} = 4.83$$

$$x_{av} = \frac{4\times(1-0)+6\times(2-1)+3\times(4-2)+8\times(5-4)+5\times(6-5)}{6} = 4.83$$

Note that average value of the signal is nothing but summation of all the rectangular areas divided by the total time

Average value of a signal



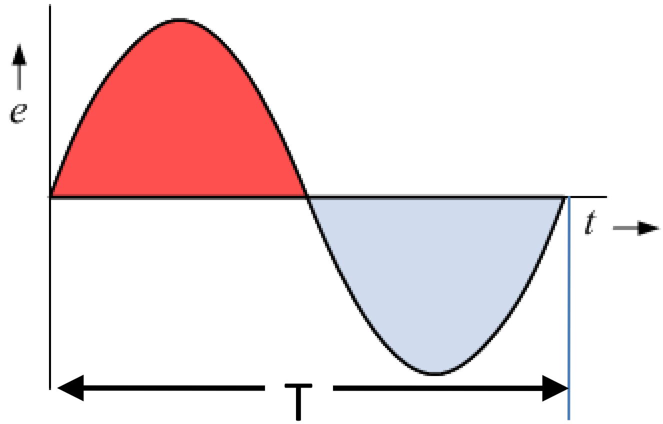
$$x_{av} = \frac{4\times(1-0)+6\times(2-1)+3\times(3-2)+8\times(4-3)+5\times(5-4)}{5} = 5.2$$

Average value of a signal can thus be defined as the average area enclosed by the signal between X and Y axes over the given time

Mathematical expression for the average value of a periodic signal $x(t)$ over one complete cycle, i.e. over one time period T is thus given as:

$$\boxed{x_{av} = \frac{1}{T}\int_0^T x(t)\,dt}$$

Average value of the sine wave



$$e = E_m \sin\frac{2\pi}{T}t$$

$$e_{av} = \frac{1}{T}\int_0^T e\,dt$$

$$e_{av} = \frac{1}{T}\int_0^T E_m \sin\frac{2\pi}{T}t\,dt$$

$$e_{av} = -\frac{E_m}{T}\frac{T}{2\pi}\cos\frac{2\pi}{T}t\Big|_0^T$$

$$e_{av} = -\frac{E_m}{2\pi}\left(\cos\frac{2\pi}{T}T - \cos\frac{2\pi}{T}0\right)$$

$$e_{av} = -\frac{E_m}{2\pi}(\cos 2\pi - \cos 0)$$

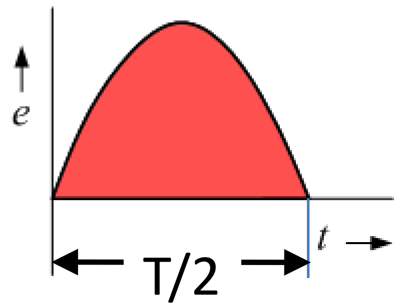
$$e_{av} = -\frac{E_m}{2\pi}(1-1)$$

$$\boxed{e_{av} = 0}$$

Thus, average value of a sinusoidal signal over its complete time period is ZERO

Average value of the sine wave

It is thus more sensible to express the average value of a sinusoidal signal computed over half its time period, rather than one complete time period:



$$e = E_m \sin\frac{2\pi}{T}t$$

$$e_{av} = \frac{1}{T/2}\int_0^{T/2} e\,dt$$

$$e_{av} = \frac{2}{T}\int_0^{T/2} E_m \sin\frac{2\pi}{T}t\,dt$$

$$e_{av} = -\frac{2E_m}{T}\frac{T}{2\pi}\cos\frac{2\pi}{T}t\Big|_0^{T/2}$$

$$e_{av} = -\frac{E_m}{\pi}\left(\cos\frac{2\pi}{T}\frac{T}{2} - \cos\frac{2\pi}{T}0\right)$$

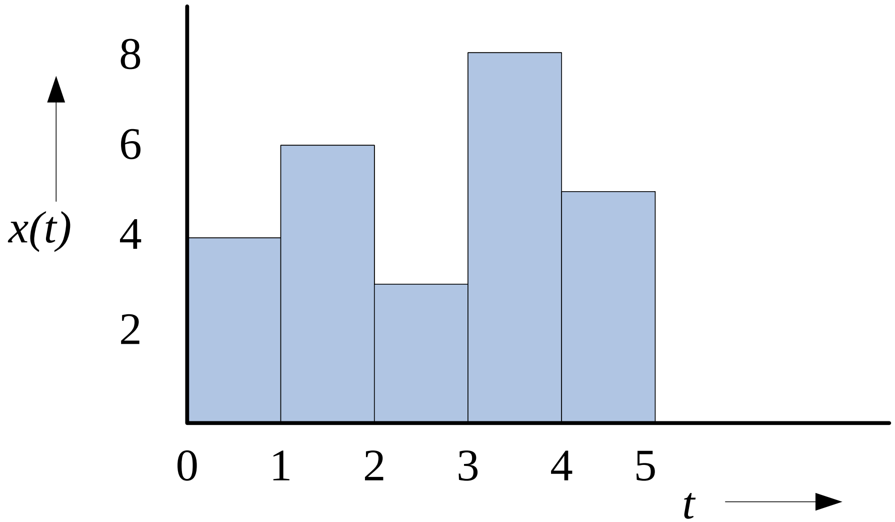
$$e_{av} = -\frac{E_m}{\pi}\left(\cos\pi - \cos 0\right)$$

$$e_{av} = -\frac{E_m}{\pi}\left(-1-1\right)$$

$$\boxed{e_{av} = \frac{2E_m}{\pi} = 0.637E_m}$$

Root Mean Square (RMS) value of a signal

RMS means square root of the average (mean) of the squared values:



Squared

Mean (average)

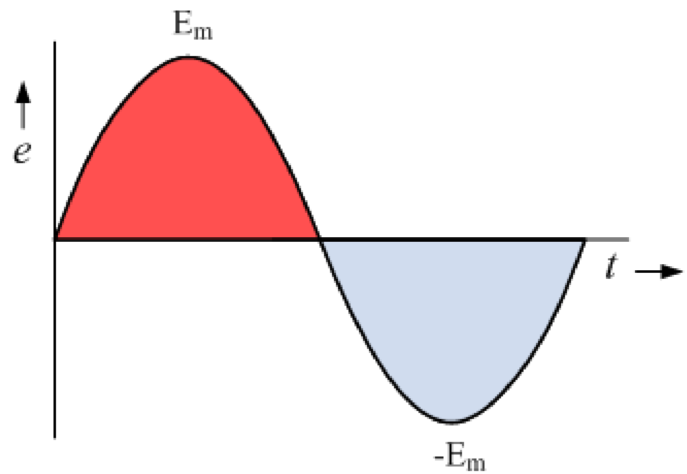
Root

$$x_{\mathrm{RMS}} = \sqrt{\frac{4^2 \mathrm{x}(1\text{-}0) + 6^2 \mathrm{x}(2\text{-}1) + 3^2 \mathrm{x}(3\text{-}2) + 8^2 \mathrm{x}\,(4\text{-}3) + 5^2 \mathrm{x}\,(5\text{-}4)}{5}} = 5.48$$

Mathematical expression for the RMS value of a periodic signal *x(t)* over one complete cycle, i.e. over one time period *T* is thus given as:

$$\boxed{x_{RMS} = \sqrt{\frac{1}{T}\int_0^T x(t)^2 dt}}$$

RMS value of the sine wave



$$e_{RMS} = \sqrt{\frac{1}{T}\int_0^T e^2\,dt}$$

$$e_{RMS} = \sqrt{\frac{1}{T}\int_0^T \left(E_m \sin\frac{2\pi}{T}t\right)^2 dt}$$

$$e_{RMS} = \sqrt{\frac{E_m^{\ 2}}{T}\int_0^T \frac{1}{2}\left(1-\cos\frac{4\pi}{T}t\right)dt}$$

$$e_{RMS} = \sqrt{\frac{E_m^{\ 2}}{2T}\left(t-\frac{T}{4\pi}\sin\frac{4\pi}{T}t\right)\Bigg|_0^T}$$

$$e_{RMS} = \sqrt{\frac{E_m^{\ 2}}{2T}\left[\left(T-\frac{T}{4\pi}\sin\frac{4\pi}{T}T\right)-\left(0-\frac{T}{4\pi}\sin\frac{4\pi}{T}0\right)\right]}$$

$$e_{RMS} = \sqrt{\frac{E_m^{\ 2}}{2T}\left[\left(T-\frac{T}{4\pi}\sin 4\pi\right)-\left(0-\frac{T}{4\pi}\sin 0\right)\right]}$$

$$e_{RMS} = \sqrt{\frac{E_m^{\ 2}}{2T}\left[(T-0)-(0-0)\right]}$$

$$e_{RMS} = \sqrt{\frac{E_m^{\ 2}}{2T}T} = \sqrt{\frac{E_m^{\ 2}}{2}}$$

$$\boxed{e_{RMS} = \frac{E_m}{\sqrt{2}} = 0.707E_m}$$

Peak divided by √2

Peak factor and Form factor of a signal

The peak factor of an alternating waveform is defined as:

$$\boxed{K_p = \frac{\text{Maximum value of the signal}}{\text{RMS value of the signal}}}$$

Thus, for a sinusoidal waveform, the peak factor is:

$$K_p = \frac{E_m}{E_m / \sqrt{2}} = \sqrt{2} = 1.414$$

Peak factor and Form factor of a signal

The form factor of an alternating wave is defined as:

$$\boxed{K_f = \frac{\text{RMS value of the signal}}{\text{Average value of the signal}}}$$

Thus, for a sinusoidal waveform, the form factor is:

$$K_f = \frac{E_m / \sqrt{2}}{2E_m / \pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$