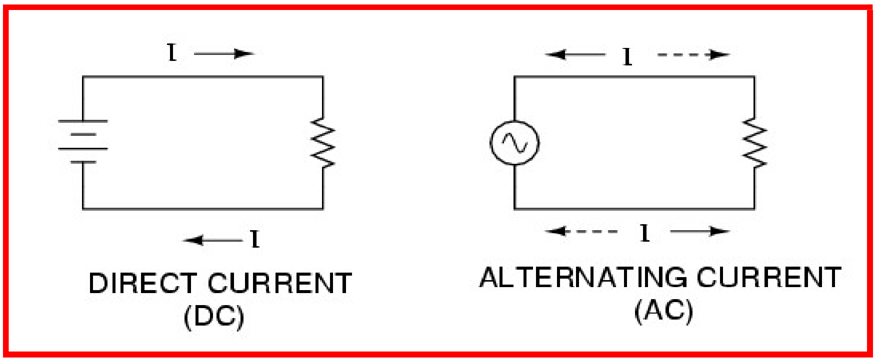
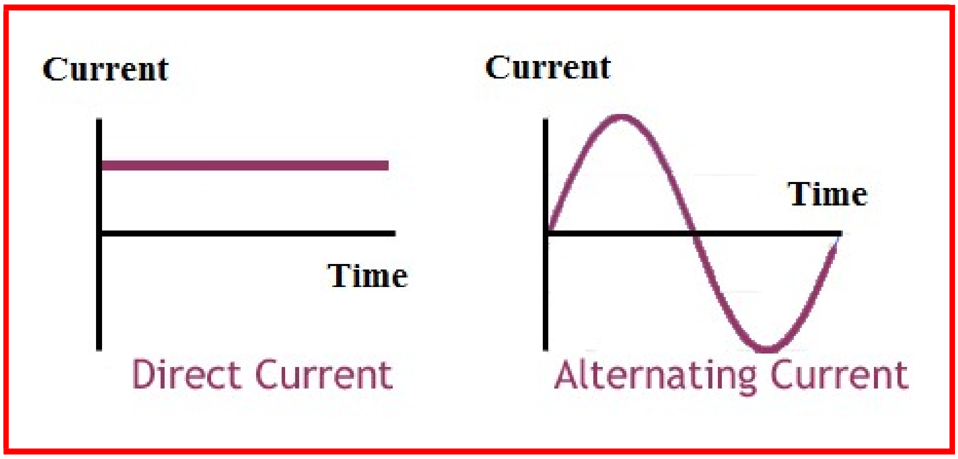
DC Circuits

Electrical Network: A combination of various electric elements (Resistor, Inductor, Capacitor, Voltage source, Current source) connected in any manner what so ever is called an electrical network.

A network or circuit that can be AC or DC is the combination of active elements (power supply sources) and passive elements (resistors, capacitors and inductors). The electricity is of two types, Alternating Current (AC) and Direct Current (DC). A circuit that deals with AC is referred to as AC circuit and a circuit with DC source is termed as DC circuit.

In direct current (DC), the electric charge (current) only flows in one direction with respect to time. Electric charge in alternating current (AC), on the other hand, changes direction periodically. The voltage in AC circuits also periodically reverses because the current changes direction.





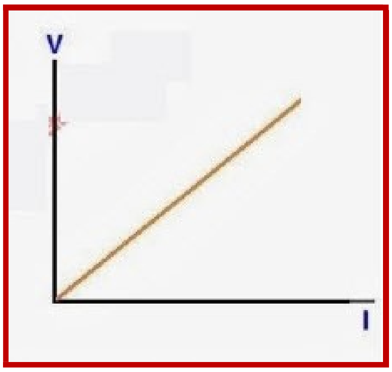
Types of Electric Circuit Elements

Linear elements:

Linearity is the property of an element describing a linear relationship between cause and effect.

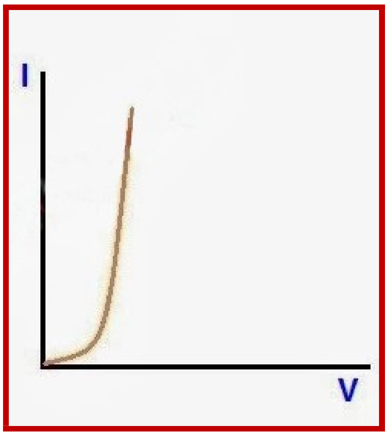
In an electric circuit, a linear element is an electrical element with a linear relationship between current and voltage. Resistors are the

most common example of a linear element; other examples include capacitors, inductors.



Non-linear elements:

A nonlinear element is one which does not have a linear input/output relation. In a diode, for example, the current is a non-linear function of the voltage.



Passive element

The element which consumes energy (or absorbs energy) rather than produce energy and then either converts it into heat or stored it in an electric or magnetic field called passive element. Example: Resistor, Inductor, Capacitor etc.

Passive network contains circuit elements without any energy sources.

Active element

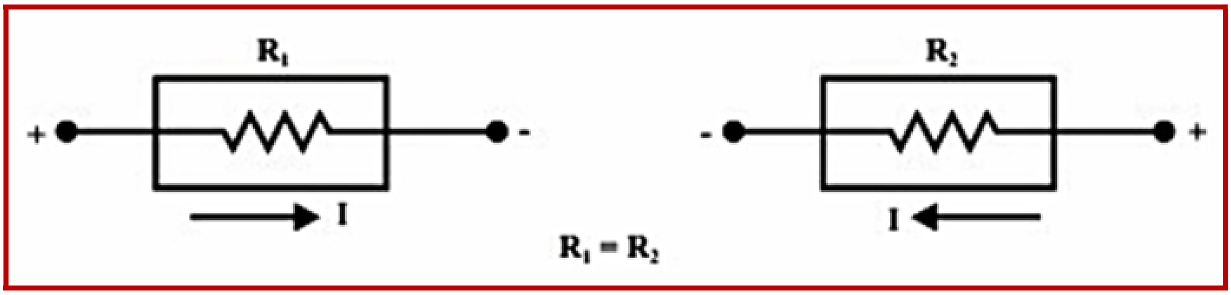
The elements which supply or generate energy is called Active element. Examples: Voltage and Current sources, Generators, Batteries etc.

Active network containing energy sources together with the other circuit elements.

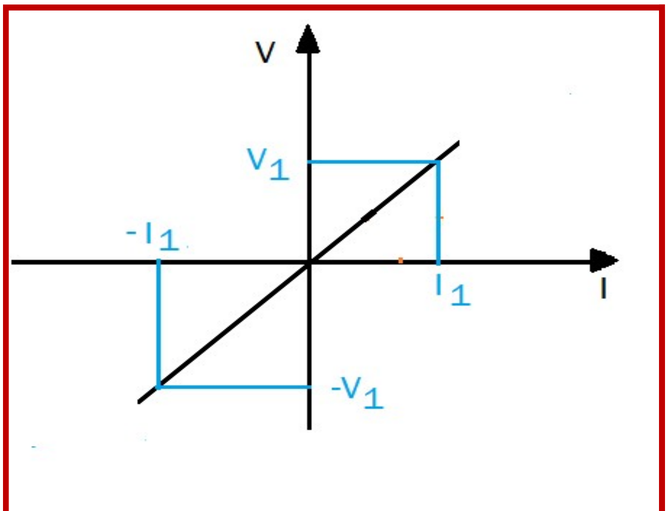
Bilateral element

Conduction of current in both directions in an element with same magnitude is termed as Bilateral Element. Example: Resistor, Inductor, Capacitor etc.

The element in which the voltage current relationship is same for current flowing in either direction is known as bilateral element. Example- Resistor, Inductor, Capacitor *etc.*



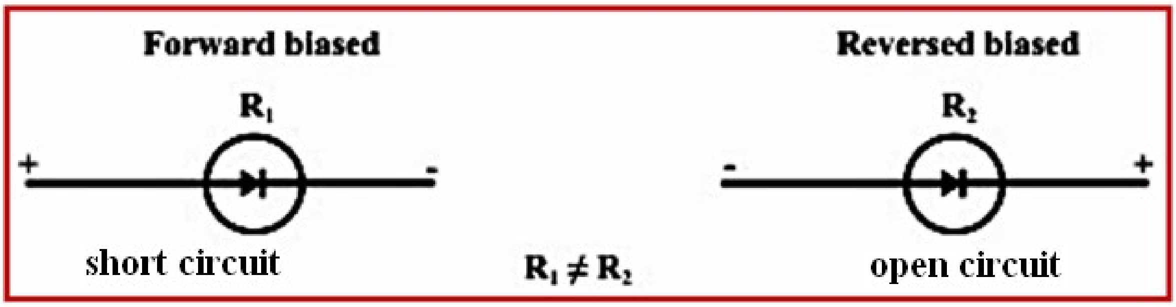
The above figure shows a bilateral element can conduct from both sides and offers same resistance for current from either side.

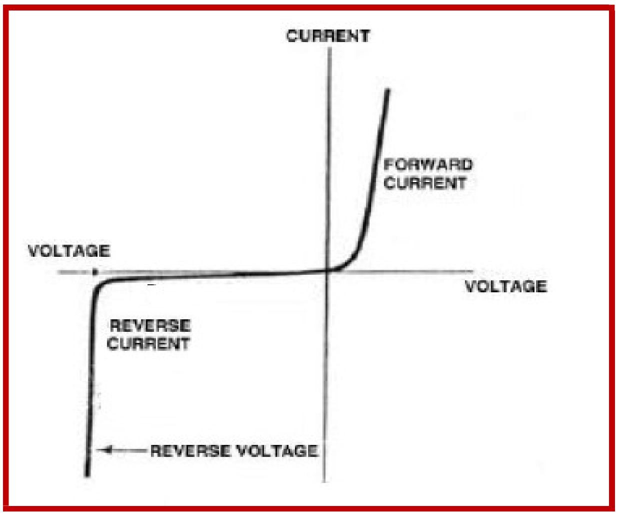


Bilateral element

Unilateral element

The element in which the voltage current relationship is not same for current flowing in either direction is known as unilateral element. Example- vacuum tubes, diodes, transistors etc.





Unilateral element

Linear and Nonlinear Circuits

Linear Circuit: Roughly speaking, a linear circuit is one whose parameters do not change its values with voltage or current.

The property is a combination of both the homogeneity (scaling) property and the additivity property.

The homogeneity property requires that if the input (also called the excitation) is multiplied by a constant, then the output (also called the response) is multiplied by the same constant.

If $v = iR$ this leads $\Rightarrow kv = kiR$

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

$v_1 = i_1R$, $v_2 = i_2R$ this leads to $\Rightarrow v = (i_1 + i_2)R = v_1 + v_2$

A resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties

Non-Linear Circuit: Roughly speaking, a non-linear circuit is that whose parameters change with voltage or current. More specifically, non-linear circuit does not obey the homogeneity and additive properties.

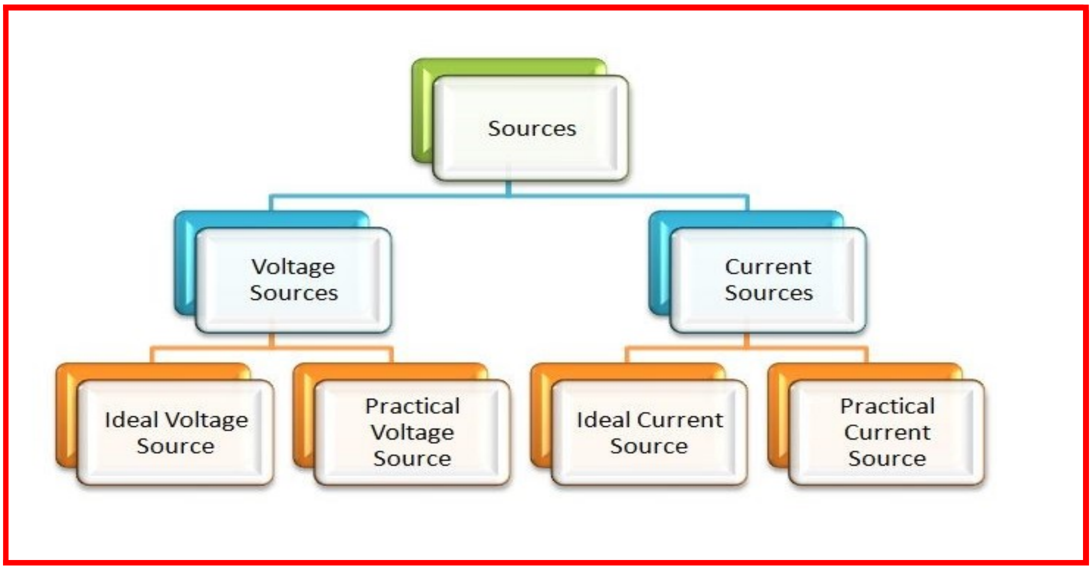
NOTE:

[**Linearity in words**

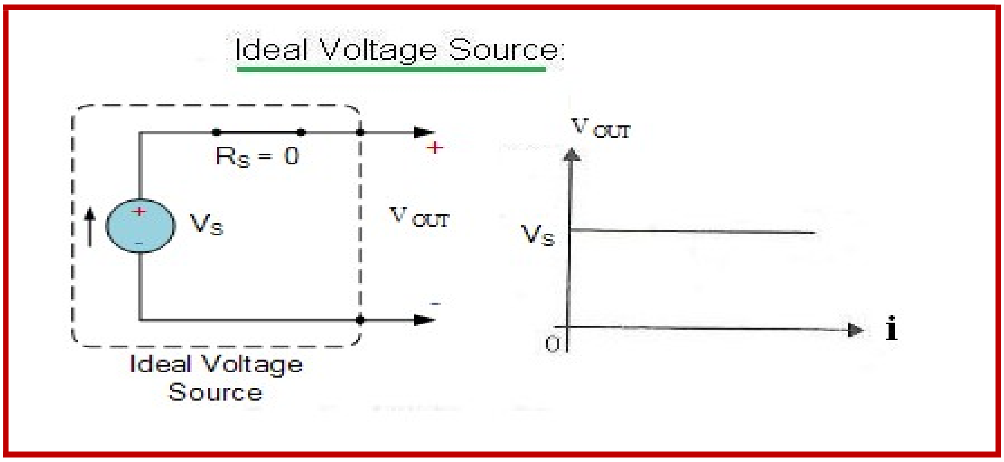
- **Scaling the input by a scale the output by a.**
- **Adding two inputs produces the same output as applying each input individually and adding the two separate outputs.**]

Sources of Electrical Energy

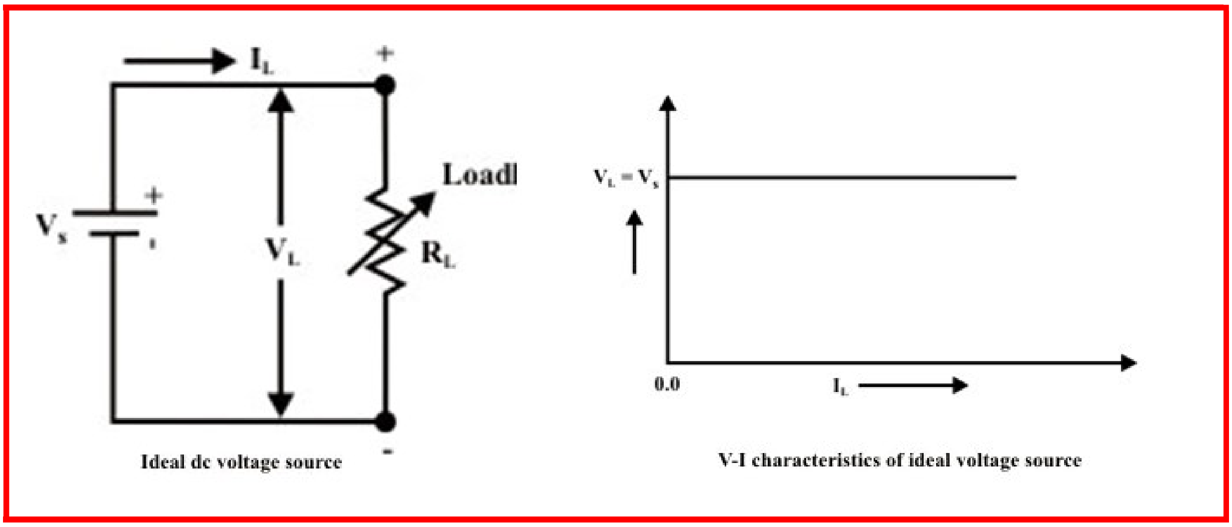
There are two types of sources of electrical energy: voltage source and current source.



An ideal voltage source is two-terminal element which maintains a constant terminal voltage regardless of the value of the current through its terminals. The terminal voltage of ideal voltage source is independent of the current flowing through it. It has zero resistance.



An ideal voltage source and its V-I characteristic



An ideal voltage source has following features: -

1) The output voltage remains absolutely constant whatever be the value of the output current.

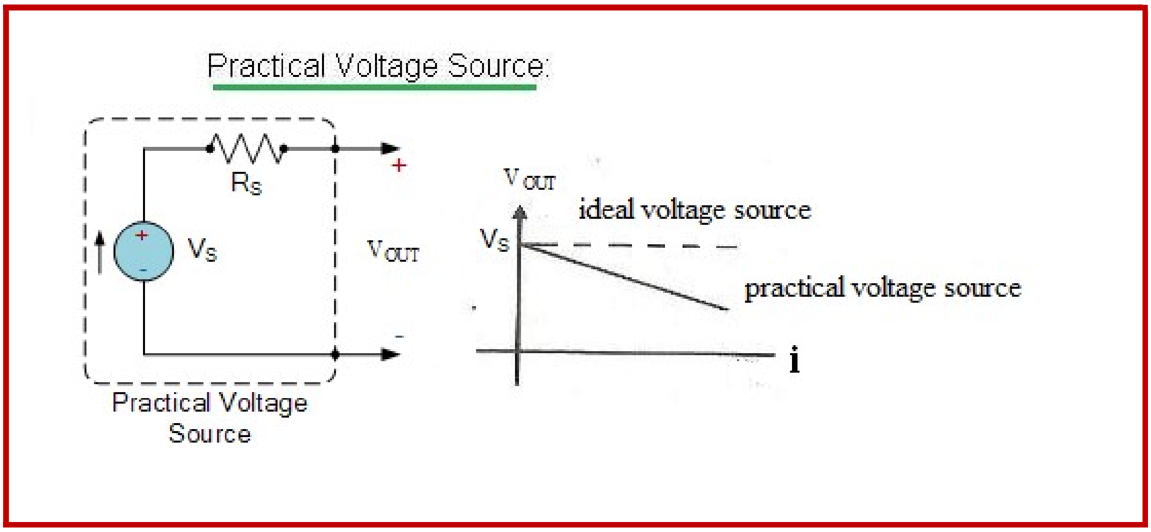
2) It has zero internal resistance so that voltage drop in the source is zero.

3) The power drawn by the source is zero.

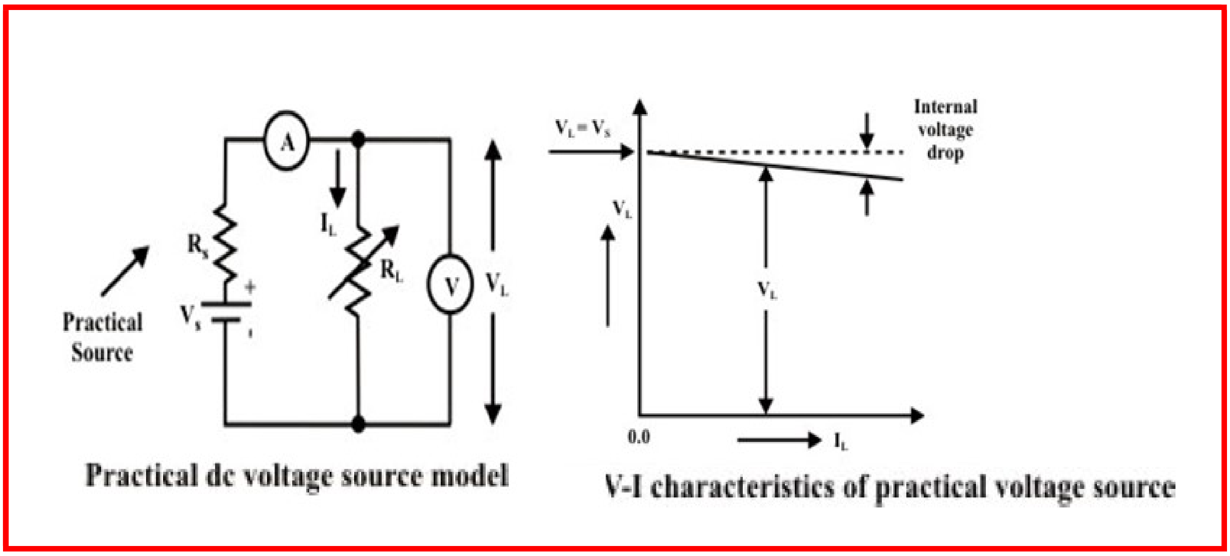
In a practical voltage source, voltage across the terminals of the source keeps falling as the current through it increases. This behavior can be explained by putting a resistance in series with an ideal voltage source. Then we have the terminal voltage V_{OUT} as

$V_{OUT} = V_S - iR_S$

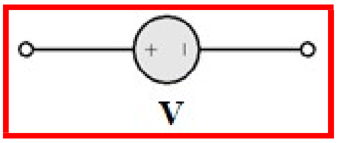
where i is the current flowing and R_S the internal resistance of the ideal voltage source of voltage V_S. The practical voltage source approaches the ideal voltage source in the limit R_S becoming zero.



A practical voltage source and V-I characteristic



Symbol of Ideal Voltage Source



Example Problem on Voltage Source

A certain voltage source has a terminal voltage of 50 V when I= 400 mA; when I rises current value 800 mA the output voltage is recorded as 40 V. Calculate (i) Internal resistance of the voltage source (R_s). (ii) No-load voltage (open circuit voltage V_s).

Solution

From equation $V_L = V_s - I_L R_s$ one can write the following expressions under different loading conditions:

$50 = V_S - 0.4R_S$ …. (1) and $40 = V_S - 0.8R_S$…..(2)

solving these equations, we get, $V_S = 60V$ & $R_S = 25$ ohm.

Example 2

A practical voltage source whose short-circuit current is 1.0A and open-circuit voltage is 24 Volts. What is the voltage across, and the value of power dissipated in the load resistance when this source is delivering current 0.25A?

Solution

From figure: $I_{sc} = V_{sc} / R_s = 1.0A$

$V_{oc} = V_s = 24$*volts*. Therefore, the value of internal source resistance is obtained as $R_s = V_{oc}\,(V_s) / I_{sc} = 24 / 1.0 = 24\Omega$.

The source is delivering current I_L= 0.25A when the load resistance is connected across the source terminals. Mathematically, we can write the following expression to obtain the load resistance R_L

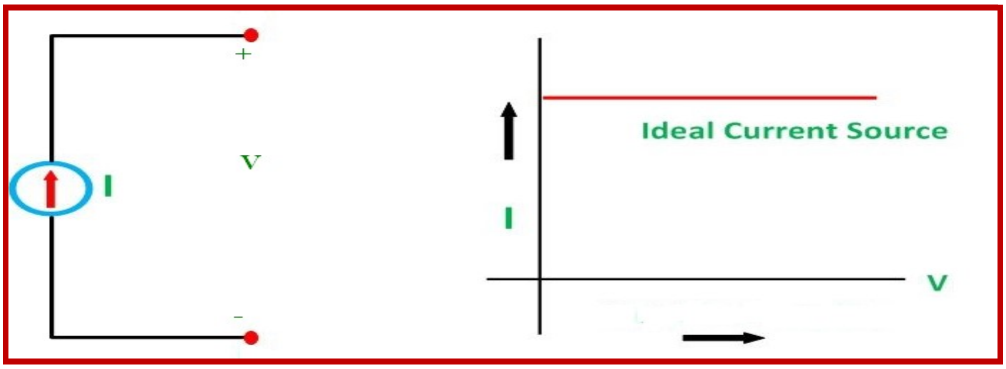
$I_L = V_s / R_s + R_L$

Where $I_L = 24 / 24+R_L= 0.25$, $R_L = 72\Omega$.

Now, the voltage across the load $V_L = I_L \text{ x } R_L = 0.25 \text{ x } 72 = 18$ volts.

The power consumed by the load, $P_L = I_L^2 \times R_L = 0.0625 \times 72 = 4.5$ watts.

An Ideal current source is a two-terminal circuit element which supplies the same current to any load resistance connected across its terminals regardless of the value of the terminal voltage. It is important to keep in mind that the current supplied by the current source is independent of the voltage of source terminals. It has infinite resistance.



An ideal current source and its V-I characteristic

It can be noted from model of the current source that the current flowing from the source to the load is always constant for any load

resistance i.e., whether R_L is small (V_L is small) or R_L is large (V_L is large).

An ideal current source has following features: -

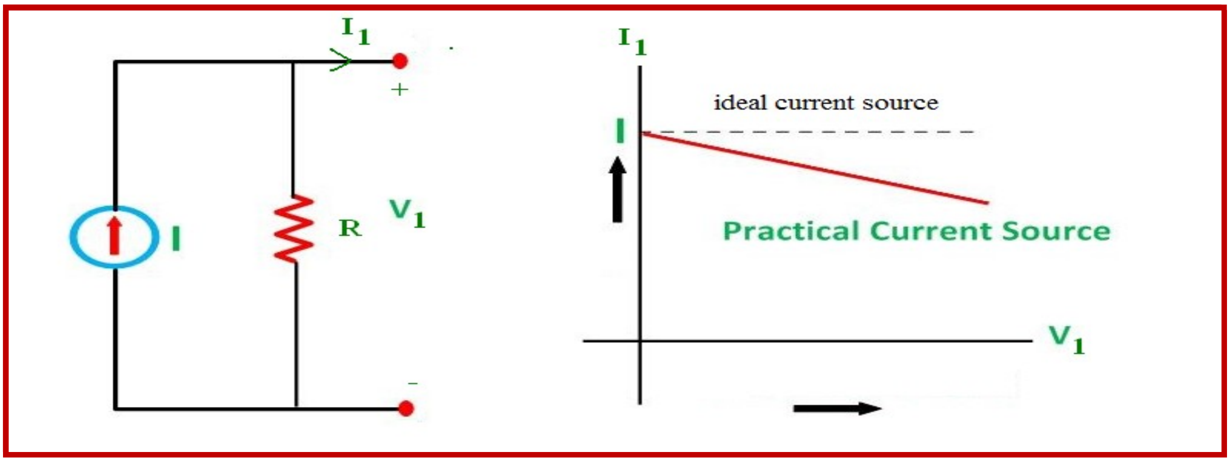
1) It produces a constant current irrespective of the value of the voltage across it.

2) It has infinite resistance.

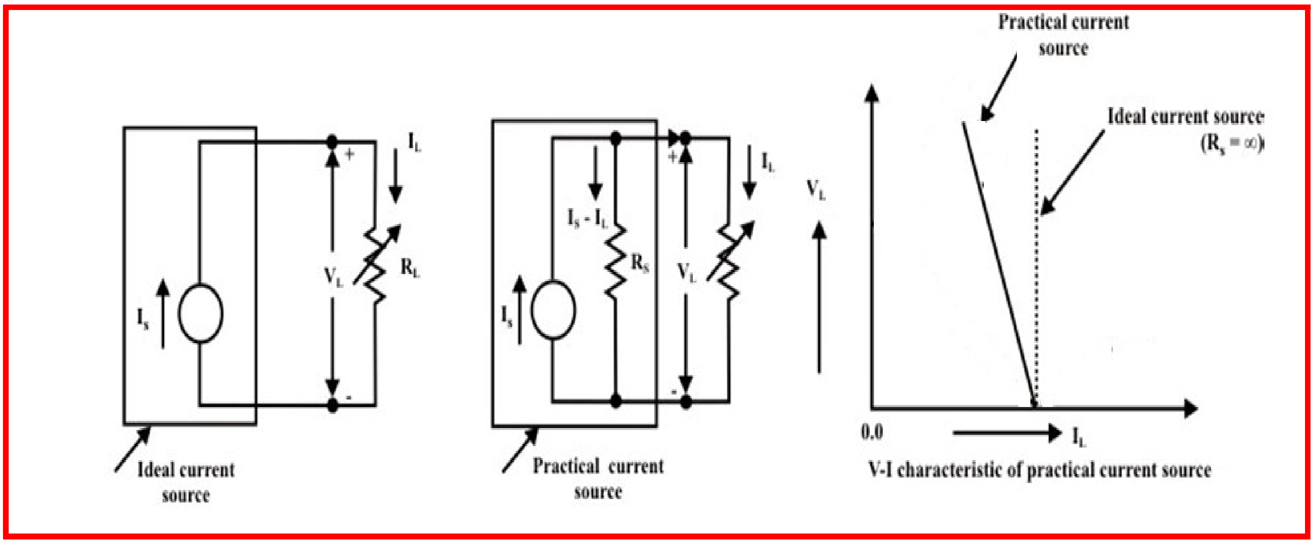
3) It is capable of supplying infinite power.

In a practical current source, the current through the source decreases as the voltage across it increases. This behavior can be explained by putting a resistance across the terminals of the source. Then the terminal current is given by

$$I_1 = I - (V_1/R)$$

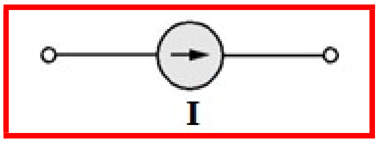


A practical current source and its V-I characteristic

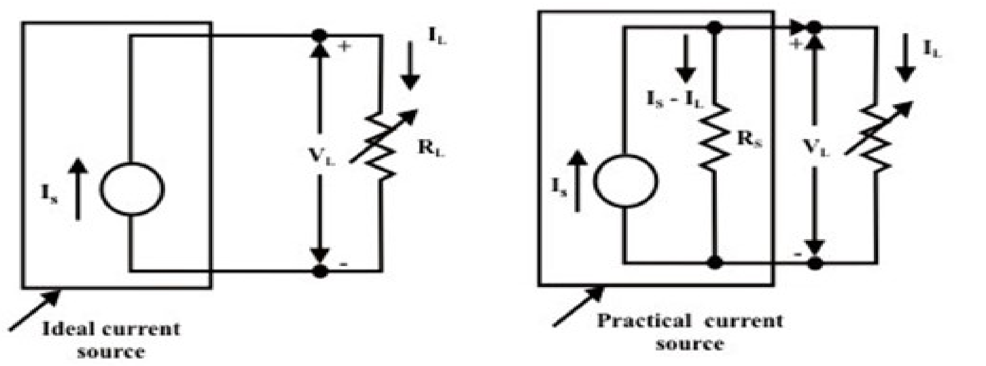


A **practical current source is represented as an ideal current source connected with the resistance in parallel. The symbolic representation is shown below:**

Symbol of Independent Current Source



When a load is connected across a practical current source, one can observe that the current flowing in load resistance R_L is reduced as the voltage across the current source's terminal is increased, by increasing the load resistance.

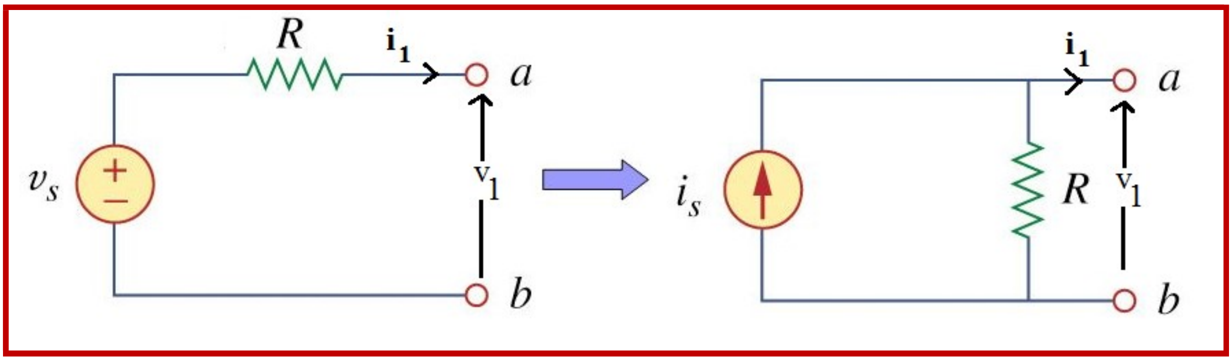


Ideal current source with variable load

Practical current source with variable load

Source Transformation

- Replace a voltage source v_s in series with a resistor R by a current source i_s in parallel with the **SAME** resistor R.



An ideal voltage source is one which gives a constant voltage v_s irrespective of the current drawn from it (source resistance is zero i.e., slope is zero).

An ideal current source is one which gives a constant current irrespective of the voltage across it (source resistance is infinite i.e., slope is infinity).

Referring to the first Fig., output voltage at the terminal a, b is

$$v_1 = v_s - i_1 R$$

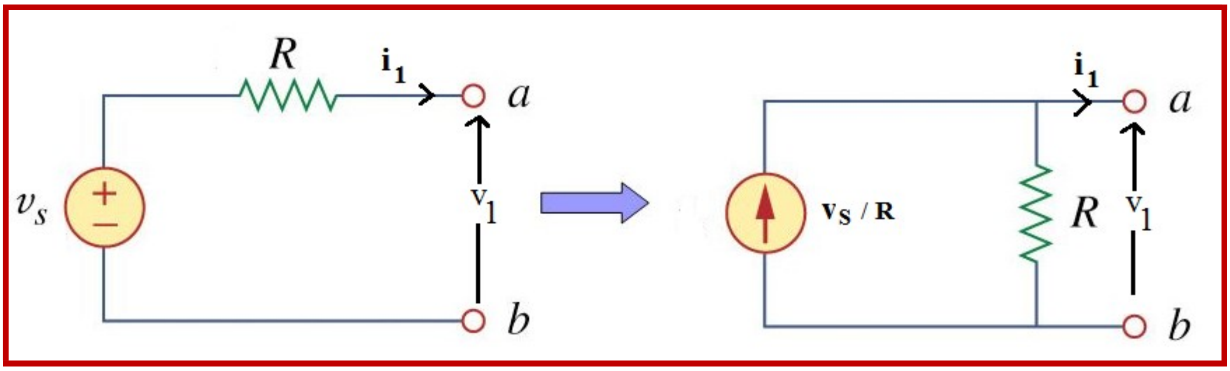
Referring to the second figure, the current flowing in the resistance R will be (i_s- i_1), so that voltage at the terminal a, b is

$v_1 = i_s R - i_1 R$

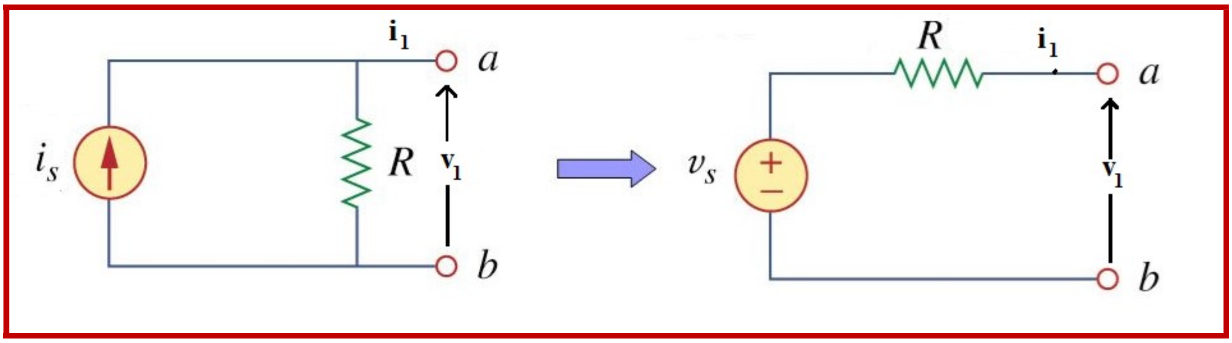
In order that the circuit in first and second figure are equivalent,

$v_s = i_s R$

Hence, if it is required to convert voltage source v_s in series with an internal resistance R into an equivalent current source, it is done by replacing the voltage source with a current source of value (v_s/R), placed in parallel with a resistance R.



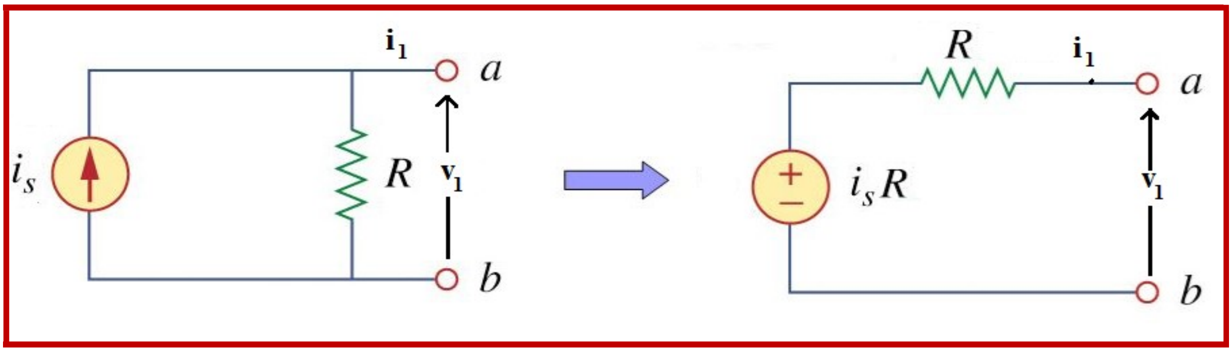
- Replace a current source i_s in parallel with a resistor R by a voltage source v_s in series with the **SAME** resistor R.



For practical current source as in the figure above, the current flowing through a resistor is (i_s- i_1). Therefore

$v_1 = (i_s - i_1)$ R or $v_1 = i_s$ R $-i_1$R

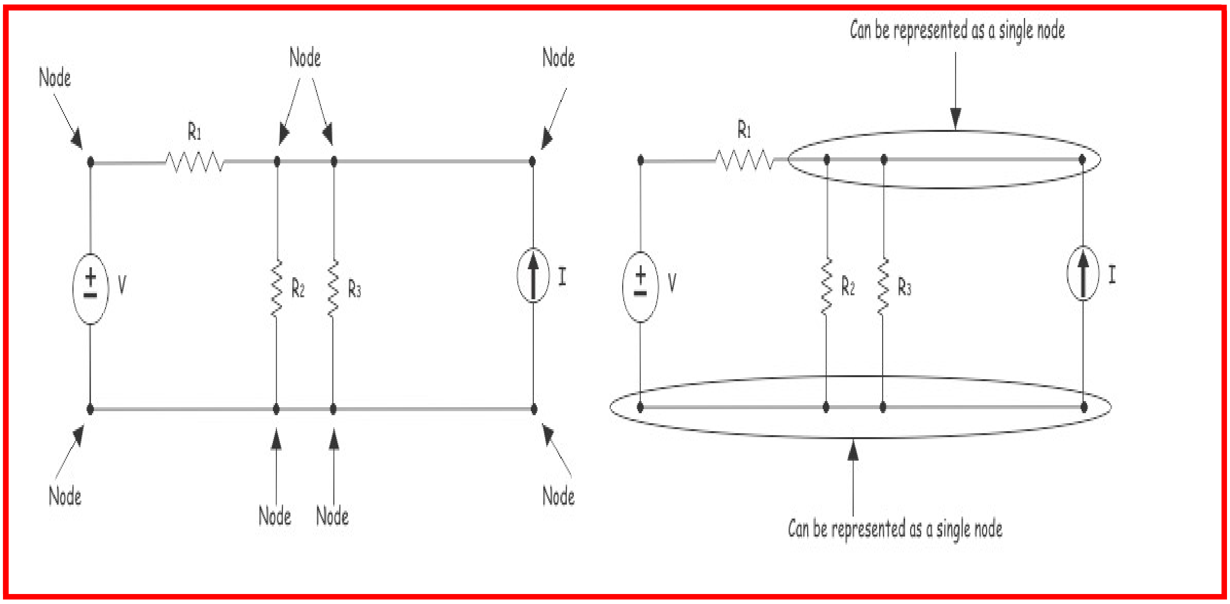
From second figure, $v_1 = v_s - i_1$R where $v_s = i_s$ R



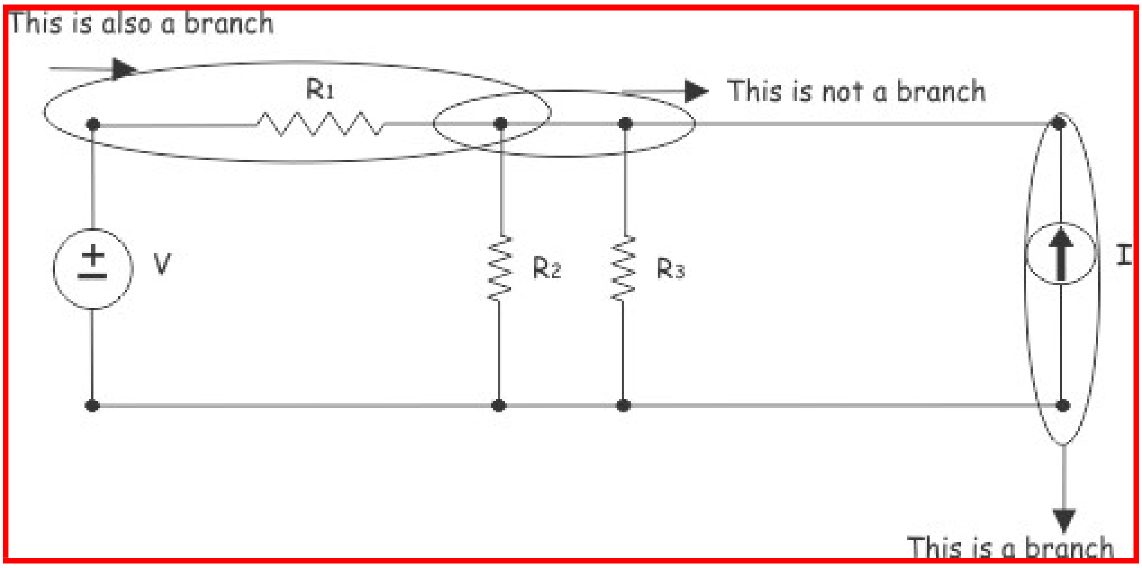
Some basic definitions of electric circuit

Elements that generally encounter in an electric circuit can be interconnected in various possible ways. Before discussing the basic analytical tools that determine the currents and voltages at different parts of the circuit, some basic definition of the following terms is considered.

Node is a point in an electric circuit where, terminal of two or more circuit elements are connected together. Node is a junction point in the circuit.

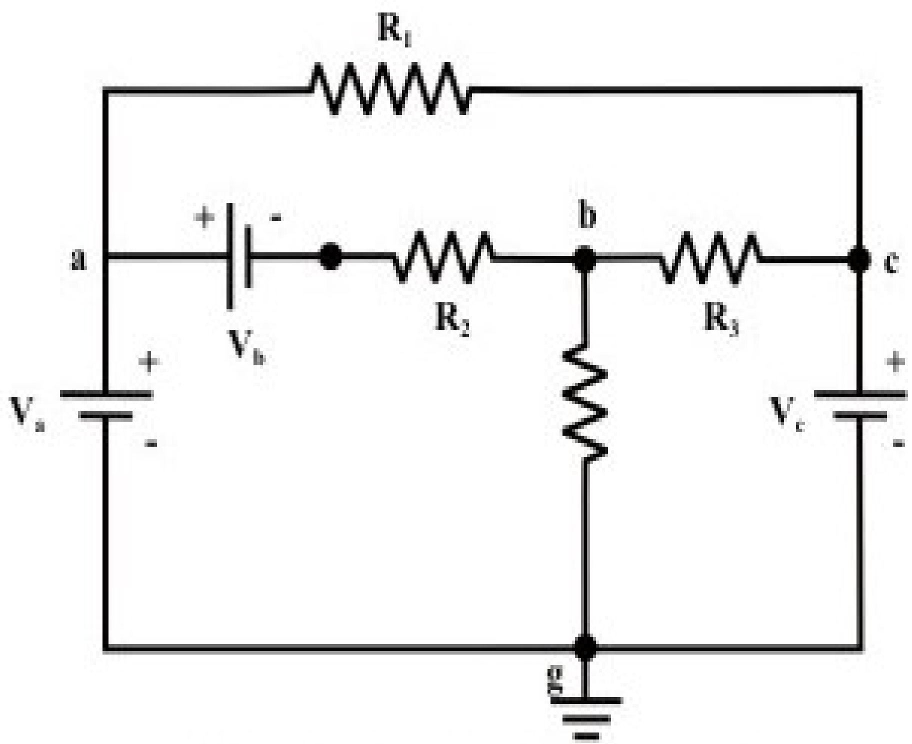


A branch is the conducting path between two nodes without crossing other nodes. The short circuit between two nodes is not referred as branch of electric circuit.



Loop- A loop is any closed path in an electric circuit.

Fig. below shows three loops or closed paths namely, a-b-g-a; b-c-g-b; and a-b-c-a. Further, it may be noted that the outside closed paths a-c-g-a and a-b-c-g-a are also form two loops.



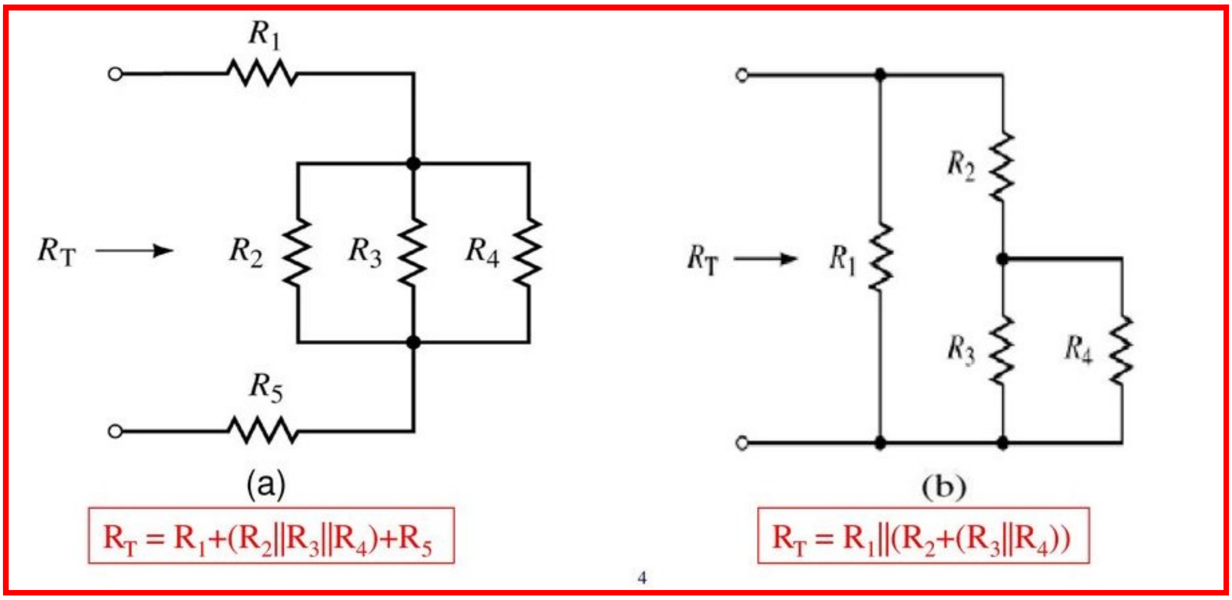
A simple resistive network

Mesh- a mesh is a **special case of loop** that does not have any other loops within it or in its interior.

Fig. above indicates that the first three loops (**a-b-g-a; b-c-g-b; and a-b-c-a**) just identified are also 'meshes' but other two loops (**a-c-g-a and a-b-c-g-a)** are not.

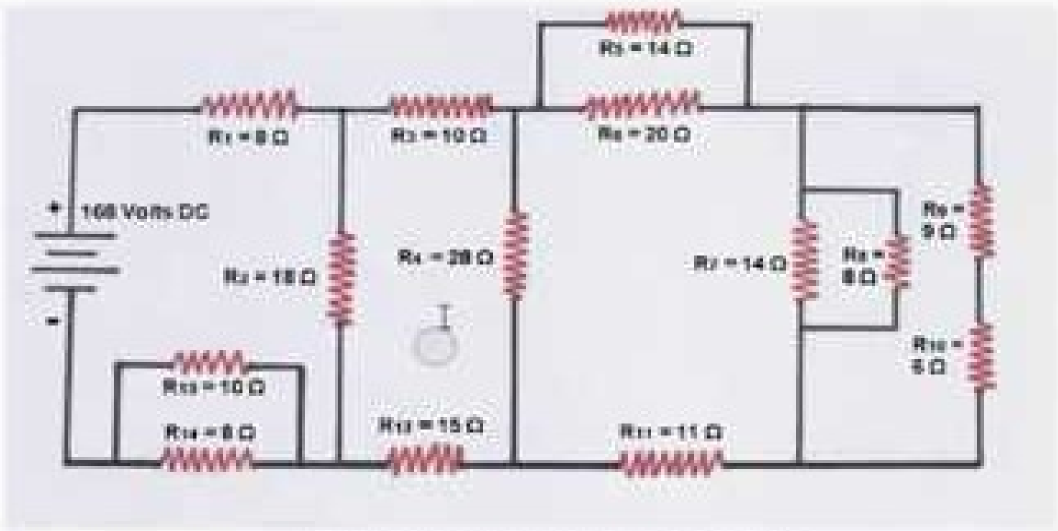
Solution of Electric Circuit Based on Mesh (Loop) Current Method

The Series-parallel reduction technique for analyzing DC circuit simplifies every step logically from the preceding step and leads on logically to the next step.



DC Series Parallel Circuit

Unfortunately, if the circuit is complicated, this method (the simplification and reconstruct) becomes mathematically laborious, time consuming and likely to produce mistake in calculations.



DC SERIES-PARALLEL CIRCUIT

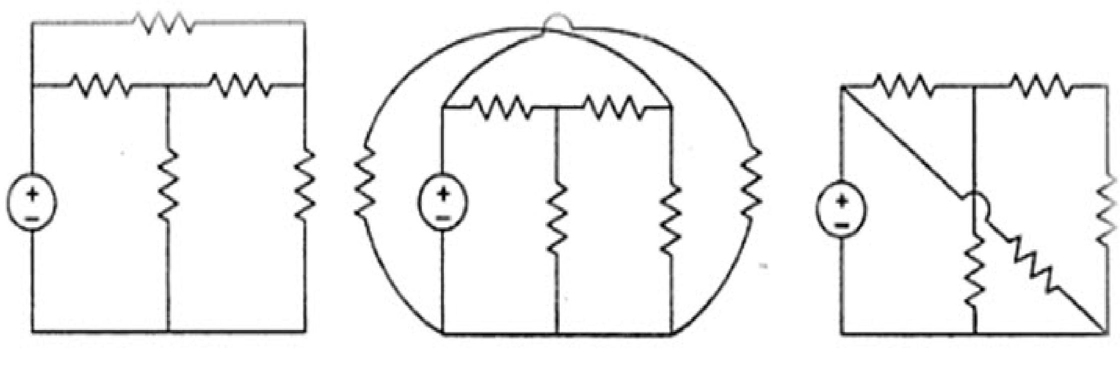
One most popular technique is known as 'mesh or loop' analysis method that based on the fundamental principles of circuits laws, namely, Ohm's law and Kirchhoff's voltage law.

MESH ANALYSIS (LOOP ANALYSIS)

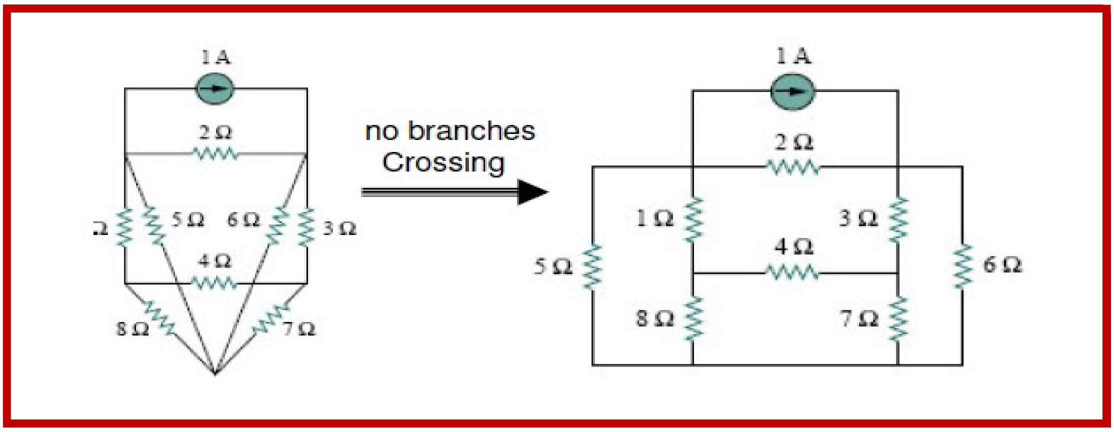
- **'Mesh or loop' analysis method is based on the fundamental principles of circuits laws, namely, Ohm's law and Kirchhoff's voltage law.**
- **Mesh analysis, is a method to calculate voltage drops and mesh currents that flow around loops in a circuit.**

- **Mesh (Loop) analysis results in a system of linear equations which must be solved for unknown currents.**
- **It reduces the number of required equations to the number of meshes.**
- **It is applicable to a circuit with no branches crossing each other or planar circuit.**

Figure (a) is a planar circuit. Figure (b) is a non-planar circuit and Fig. 5 (c) is a planar circuit which looks like a non-planar circuit.



(a) (b) (c)



Planar vs. Non-planar

- **Planar circuit**: it can be drawn on a plane surface where no element cross any other element
- **Non-planar circuit**: there is no way to redraw it **without** avoid the branches crossing

Steps in Mesh Analysis

1. Identify all of the meshes in the circuit.

2. Label the currents flowing in each mesh. Current direction needs to be same in all meshes either clockwise or anticlockwise.

3. Label the voltage across each component in the circuit.

4. Write the voltage loop equations using Kirchhoff's Voltage Law.

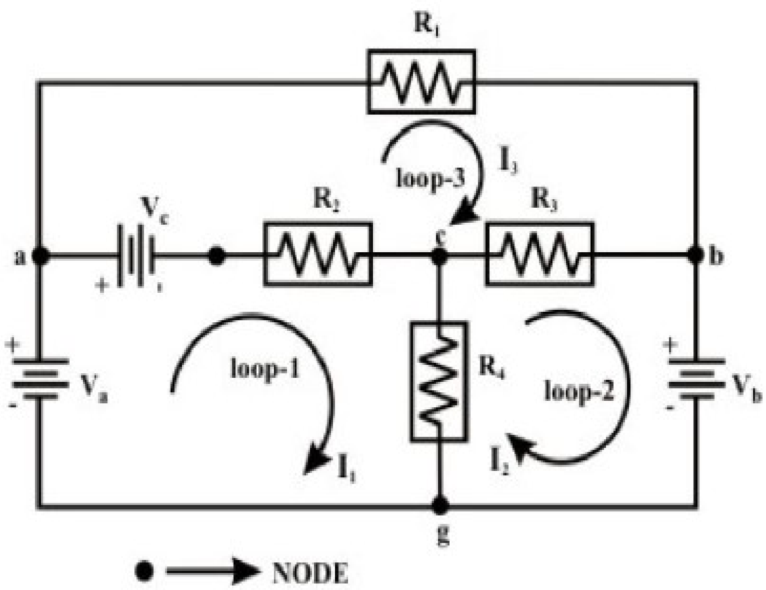
5. Use Ohm's Law to relate the voltage drops across each component to the sum of the currents flowing through them.

6. Solve for the mesh currents.

7. Once the mesh currents are known, calculate the voltage across all of the components.

Solution of Electric Circuit Based on Mesh (Loop) Current Method

Consider a simple dc network as shown in Figure to find the currents through different branches using Mesh (Loop) current method.



Mesh analysis is valid only for circuits that can be drawn in a two-dimensional plane in such a way that no element crosses over another

No. of independent mesh (loop) equations = (no. of branches (b) - no. of principle nodes (n) + 1).

In the above example, b=7, n=5 and no. of independent equations: 7-5+1=3.

Applying KVL in mesh (loop)-1 (in mesh-1, current I_1 is local current and I_2 and I_3 are foreign currents)

$$V_a - V_c - (I_1 - I_3)R_2 - (I_1 - I_2)R_4 = 0$$

or

$$V_a - V_c = (R_2 + R_4)I_1 - R_4 I_2 - R_2 I_3$$

Applying KVL in mesh (loop)-2 (in mesh-2, current I_2 is local current and I_1 and I_3 are foreign currents)

$$-V_b - (I_2 - I_3)R_3 - (I_2 - I_1)R_4 = 0$$

Or

$$-V_b = -R_4 I_1 + \left(R_3 + R_4\right) I_2 - R_3 I_3$$

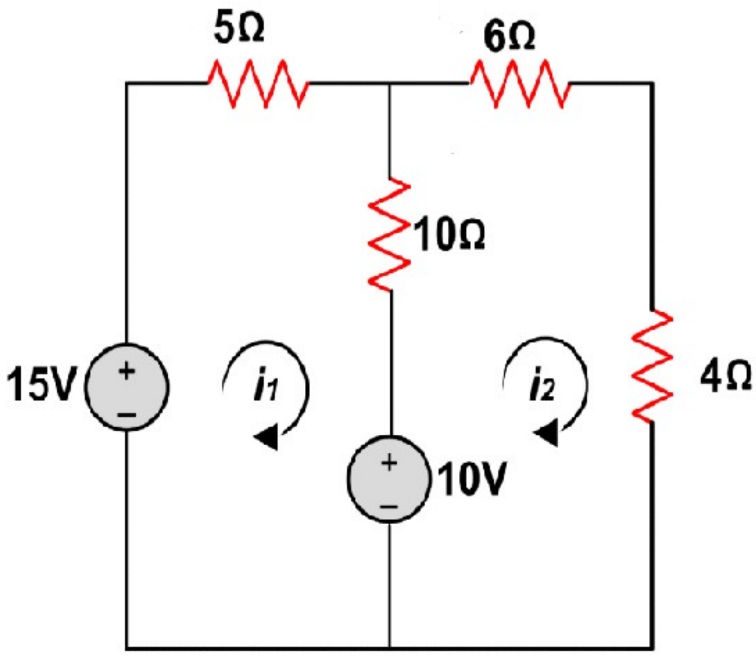
Applying KVL in mesh (loop)-3 (in mesh-3, current I_3 is local current and I_1 and I_2 are foreign currents)

$$V_c - I_3 R_1 - \left(I_3 - I_2\right) R_3 - \left(I_3 - I_1\right) R_2 = 0$$

Or

$$V_c = -R_2 I_1 - R_3 I_2 + \left(R_1 + R_2 + R_3\right) I_3$$

Example 1 Solve the circuit using mesh analysis.



Loop 1 Equation

$$-15+5i_1+10+10(i_1-i_2) = 0$$

$$-2i_2+3i_1=1 \quad \cdots \quad (1)$$

Loop 2 equation

$$-10+4i_2+6i_2+10(i_2-i_1) = 0$$

$$2i_2- i_1=1 \quad \cdots \quad (2)$$

By solving equation (1) and (2), we have

$$i_1 = 1A$$

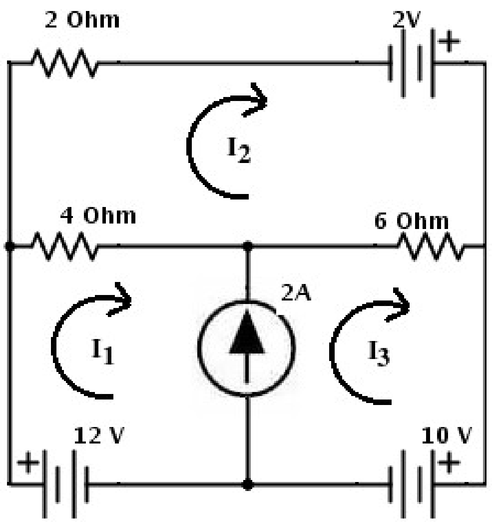
$$i_2 = 1A$$

Example: To apply Mesh Analysis Method in Super Mesh.

A super mesh occurs when a current source is contained between two essential meshes. The circuit is first treated as if the current source is not there. This leads to one equation that incorporates two mesh currents. Once this equation is formed, an equation is needed that relates the two mesh currents with the current source. This will be an equation where the current source is equal to one of the mesh currents minus the other.

The following is a simple example of dealing with a super mesh.

Solve the following circuit using Mesh Analysis or Loop Current Method:



Here,

Equations for Mesh with mesh current I_2:

$$2I_2 + 6(I_2 - I_3) + 4(I_2 - I_1) = 2$$

Or, $$-4I_1 + 12I_2 - 6I_3 = 2$$

Equation for the combined I_1, I_3 mesh (Because it is a super mesh) is:

$$4(I_1 - I_2) + 6(I_3 - I_2) = 12 - 10$$

Or, $$4I_1 - 10I_2 + 6I_3 = 2$$

And

$$I_3 - I_1 = 2$$

Thus, solving these equations:

$$I_1 = 1A, I_2 = 2A, I_3 = 3A$$

Superposition Theorem

The superposition theorem for electrical circuits states that

In a linear network with several sources, the overall response at any point in the network is equal to the algebraic sum of individual response of each source, acting independently, the other sources being made inoperative.

To ascertain the contribution of each individual source, all of the other sources first must be "turned off" (set to zero) by:

- **Replacing all other independent voltage sources with a short circuit (thereby eliminating difference of potential i.e., V=0; internal impedance of ideal voltage source is zero).**
- **Replacing all other independent current sources with an open circuit (thereby eliminating current i.e. I=0; internal impedance of ideal current source is infinite).**

Note:

1. **The principle of superposition is useful for linearity test of the system or network.**
2. **A linear network comprises independent sources, linear dependent sources and linear passive elements.**

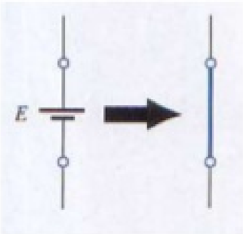
- ***In order to apply the superposition theorem to a network, certain conditions must be met :***

- All the components must be linear, for e.g.- the current is proportional to the applied voltage (for resistors), flux linkage is proportional to current (in inductors), etc.
- All the components must be bilateral, meaning that the current is the same amount for opposite polarities of the source voltage.
- Passive components may be used. These are components such as resistors, capacitors, and inductors, that do not amplify or rectify.
- Active components may not be used. Active components include transistors, semiconductor diodes, and electron tubes. Such components are never bilateral and seldom linear.

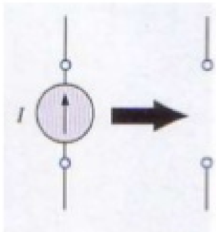
Procedure for applying Superposition Theorem

➢ **<u>Circuits Containing Only Independent Sources:-</u>**

- Consider only one source to be active at a time.
- Remove all other ideal voltage sources by short circuit & all other ideal current sources by open circuit.

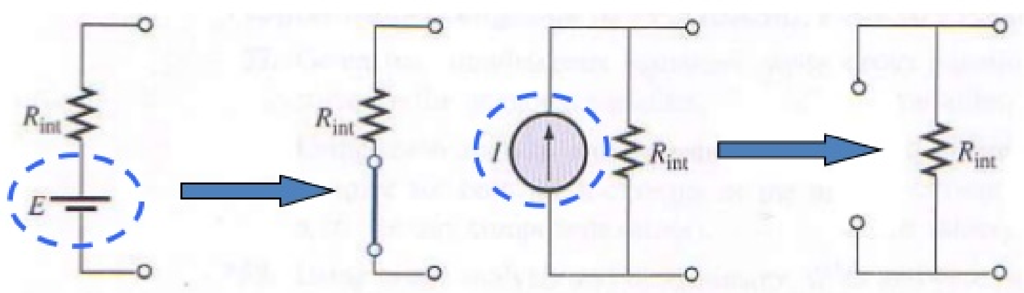


Voltage source is replaced by a Short Circuit

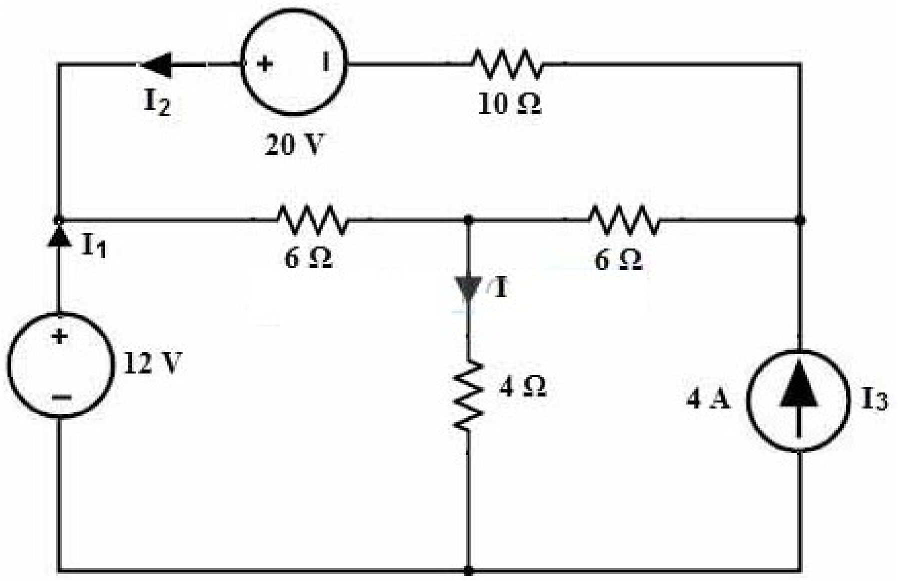


Current source is replaced by a Open Circuit

- If there are practical sources, replace them by the combination of ideal source and an internal resistances.
- After that, short circuit the ideal voltage source & open circuit the ideal current source..



Example: Determine the current I through the 4-ohm resistor using superposition theorem.

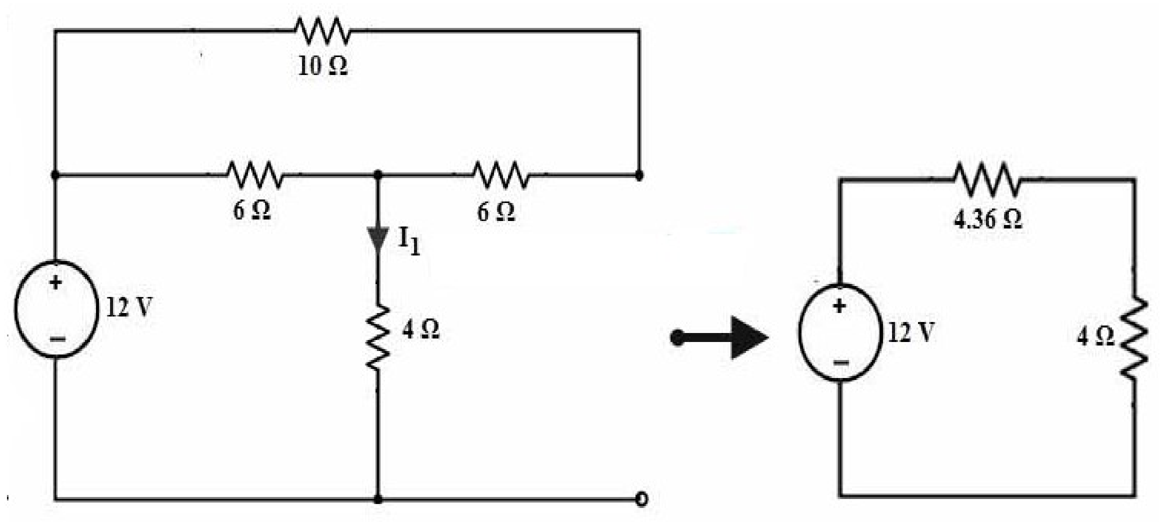


Consider I_1, I_2 and I_3 are the currents due to sources 12v, 20V and 4A sources respectively. Then, based on superposition theorem $I = I_1+I_2+I_3$. So, let's determine these currents with each individual sources acting alone.

Only with 12V Voltage source:

Consider the below circuit where only 12V source is retained in the circuit and other sources are replaced by their internal resistances.

By combining the resistance 6 ohm with 10 ohm we get 16-ohm resistance which is parallel with 6-ohm resistance. Then this combination produces, 16 × 6 / (16 + 6) = 4.36 ohms. Therefore, the equivalent circuit will be as shown in figure.



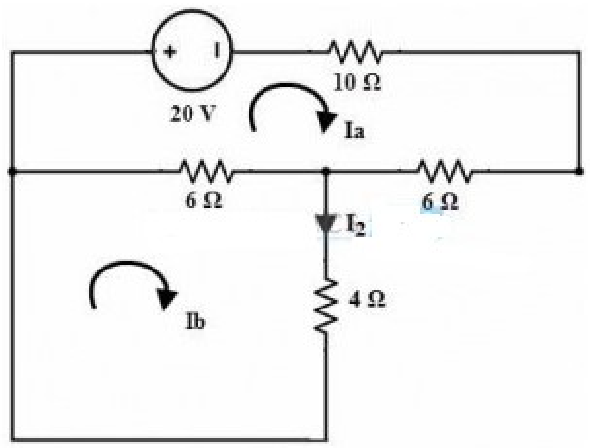
Then the current through 4 ohms resistance,

$$I_1 = 12 / 8.36$$

$$= 1.43 \text{ A}$$

Only with 20 V Voltage Source:

Retain only the 20V voltage source and replace other sources with their internal resistance, then the circuit becomes as shown below.



Apply the mesh analysis to the Loop a, we get

$$22I_a - 6I_b + 20 = -20$$

$$22I_a - 6I_b = -20 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

For Loop b, we get

$$10I_b - 6I_a = 0$$

$I_a = 10I_b / 6$

Substituting I_b in equation (1)

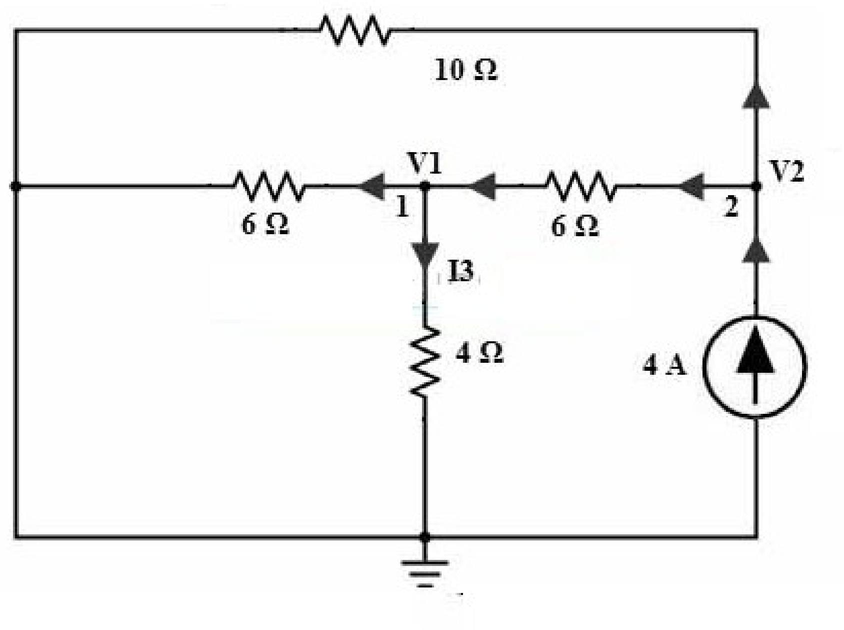
$22\ (10I_b/6) - 6I_b = -20$

$I_b = -0.65$

Therefore, $I_2 = I_b = -0.65A$

Only with 4A Current Source

Consider the below circuit where only current source is retained and other sources are replaced with their internal resistances.



At node 2 we get,

$4 = (V_2/10) + (V_2 - V_1)/6$ ………………..(2)

At node1,

$(V_1/6) + (V_1/4) = (V_2 - V_1)/ 6$

$V_2 = 3.496\ V_1$

Substituting V_2 in equation 2, we get

$V_1 = 0.766$ Volts.

Therefore, $I_3 = V_1/4$

$= 0.766/4$

$= 0.19$ Amps.

Thus, as per the superposition theorem, $I = I_1 + I_2 + I_3$

$= 1.43 – 0.65 + 0.19$

$= 0.97$ Amps.

Limitations of Superposition Theorem

Superposition theorem cannot be applied for non linear circuit (consists of Diode, Transistor).

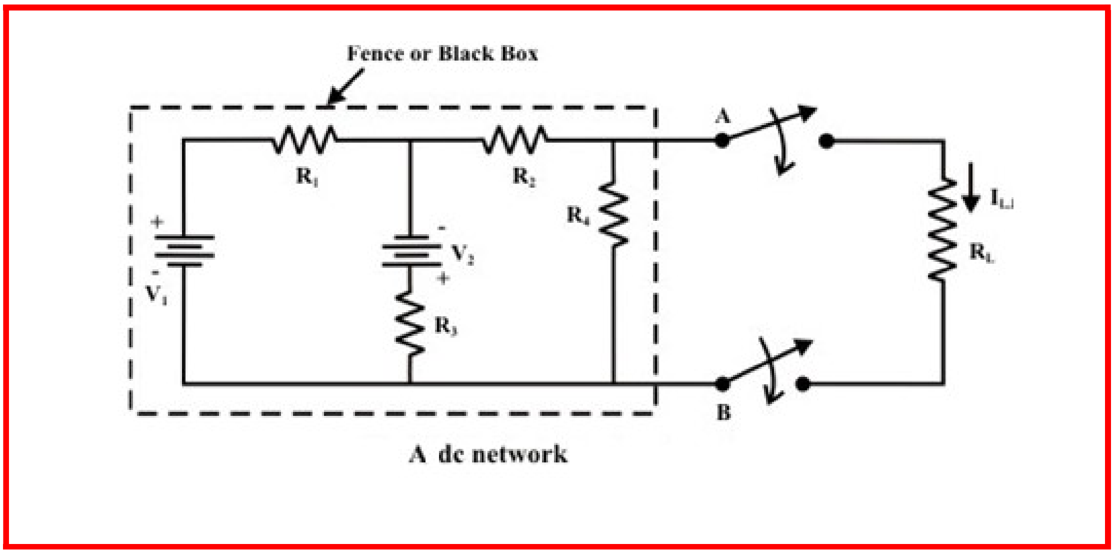
This method has weaknesses: -

In order to calculate load current, I or the load voltage V for the several choices of load resistance R of the resistive network, one needs to solve for every source voltage and current, perhaps several times. With the simple circuit, this is fairly easy but in a large circuit this method becomes a painful experience.

Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source.

We can't apply superposition theorem directly in order to find the amount of power delivered to any resistor that is present in a linear circuit, just by doing the addition of powers delivered to that resistor due to each independent source. Rather, we can calculate either total current flowing through or voltage across that resistor by using superposition theorem and from that, we can calculate the amount of power delivered to that resistor using I^2R or V^2/R.

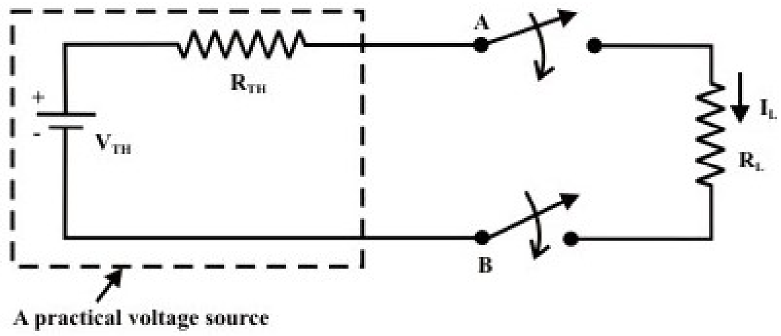
Thevenin's Theorem



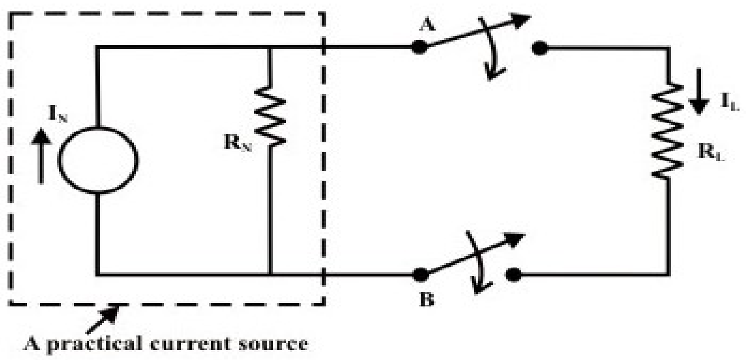
- Mesh current method needs 3 equations to be solved

- **Superposition method requires a complete solution through load resistance (*R_L*) by considering each independent source at a time and replacing other sources by their internal source resistances.**

If the value of *R_L* is changed then the three (mesh current method) equations need to be solved again to find the new current in *R_L*. Similarly, in case of superposition theorem each time the load resistance *R_L* is changed, the entire circuit has to be analyzed all over again. Much of the tedious mathematical work can be avoided if the fixed part of circuit or in other words, the circuit contained inside the imaginary fence or black box with two terminals *A&B*, is replaced by the simple equivalent voltage source (as shown in fig. below) or current source (as shown in fig. below).



a simple practical voltage source



by a simple practical current source

Thevenin's theorem tells that:

A linear two terminal network consists of active and passive components can be replaced with an equivalent circuit consisting of a voltage source E and in series with a resistance r. The value of E is the open circuit voltage between the terminals of the network and r

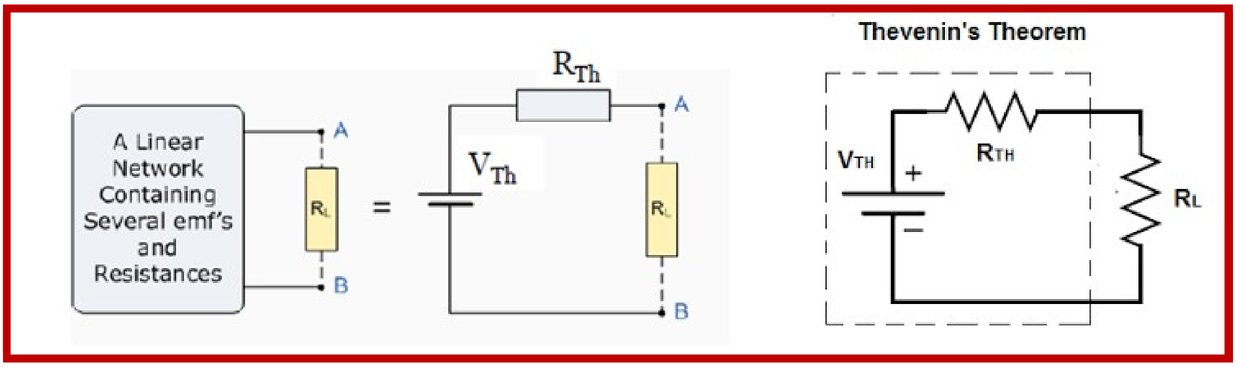
is the resistance of the network measured between the terminals of the network the energy sources replaced by their internal resistances.

OR

The current through a resistor R_L connected across any two points A and B of an active network [i.e., a network containing one or more sources of e.m.f.] is obtained by dividing the p.d. between A and B, with R_L disconnected, by (R_L + r), where r is the resistance of the network measured between points A and B with R_L disconnected and the sources replaced by their internal resistances.

Thevenin's theorem:

- ☞ **It provides replacing a given network by a single voltage source with a series resistance.**
- ☞ **It makes the solution of complicated networks quick and easy.**



In the Figure above $E = V_{TH}$ and $r = R_{TH}$.

The basic steps for Thevenin's Equivalent:

1. Remove the load resistor R_L or component concerned.

2. Find R_{Th} by shorting all voltage sources or by open circuiting all the current sources. The Thevenin equivalent resistance (R_{Th}) is the total resistance of the circuit appearing between two terminals in a given network with all voltage sources and current sources replaced by their internal resistance.

3. Find V_{Th} by the usual circuit analysis methods. The Thevenin equivalent voltage (V_{Th}) is the open circuit voltage (no-load) voltage between two terminals in a circuit. When a particular branch is

removed from a circuit, the open circuit voltage appears across the terminals of the circuit, is Thevenin equivalent voltage (V_{Th}).

4. Find the current flowing through the load resistor R_L.

5. The load current I_L through R_L is given by V_{TH} / (R_L + R_{TH})

Proof: Verify Thevenin's Theorem

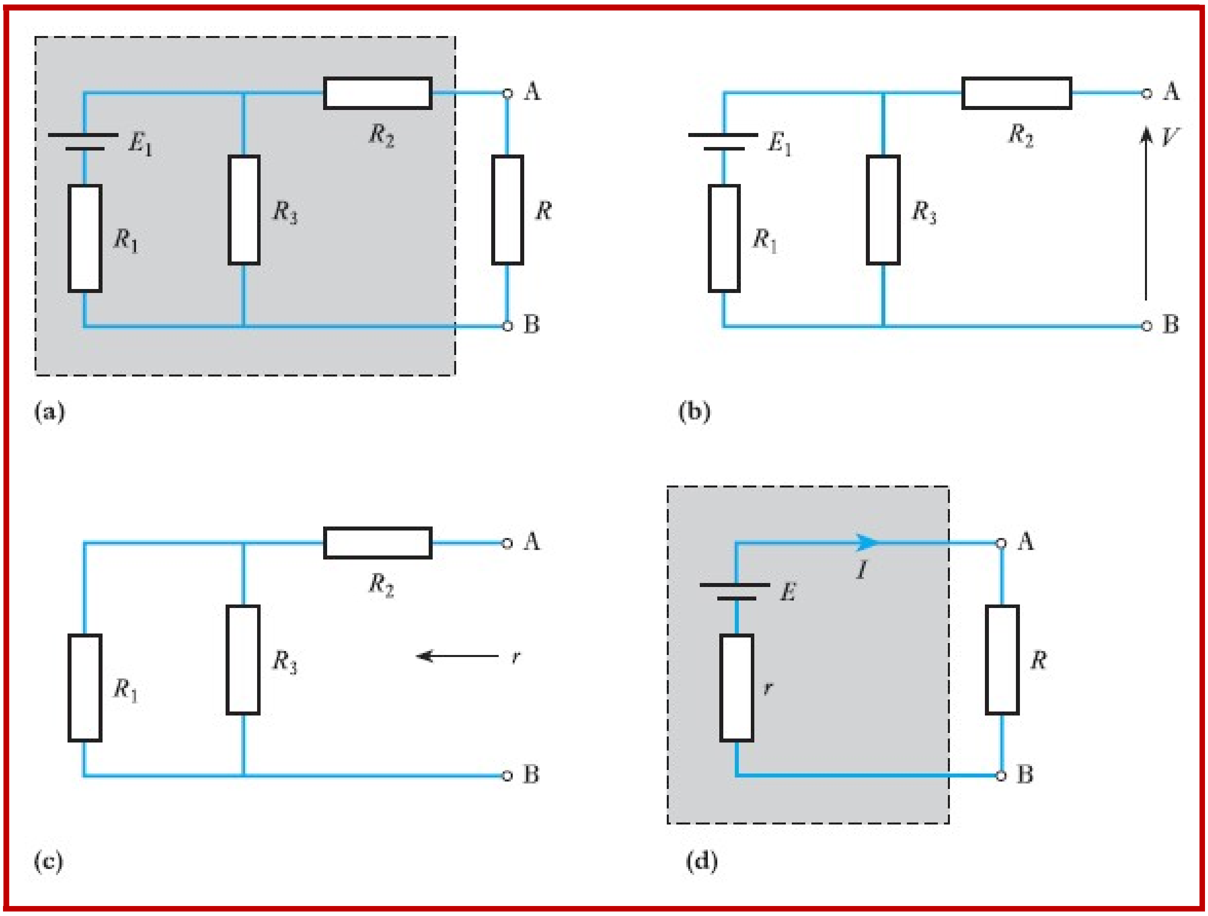


Fig. Network illustrates the Thevenin's Theorem

It is required to determine the current through a load of resistance *R* connected across AB.

Step1. Remove load resistor R

With the load disconnected as in Fig. (b)

Step2. Determine r or R_{Th}

Figure(c) shows the network with the load disconnected and the battery replaced by its internal resistance R_1. Resistance of network between A and B is

$$r = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Step3. Determine V_{Th} or E.

$$\text{Current through } R_3 = \frac{E_1}{R_1 + R_3}$$

and

$$\text{PD across } R_3 = \frac{E_1 R_3}{R_1 + R_3}$$

Since there is no current through R_2, p.d. across AB is

$$V = \frac{E_1 R_3}{R_1 + R_3}.$$

Here V = V_{Th}

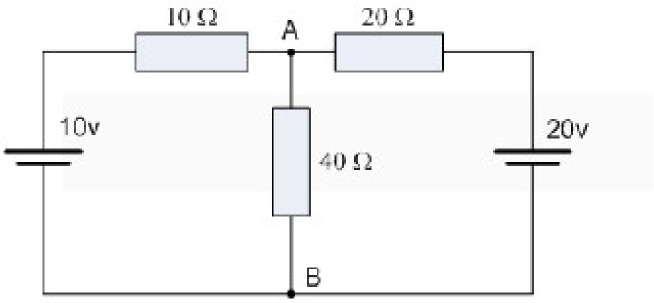
Step4. Determine the load current

Thevenin's theorem merely states that the active network enclosed by the dotted line in Fig.(a) can be replaced by the very simple circuit enclosed by the dotted line in Fig.(d) and consisting of a source having an e.m.f. *E* equal to the open-circuit potential difference *V* between A and B, and an internal resistance *r*, where *V* and *r* have the values determined above. Hence

$$\text{Current through } R = I = \frac{E}{r + R}.$$

Example:

Consider the circuit below:



Steps:

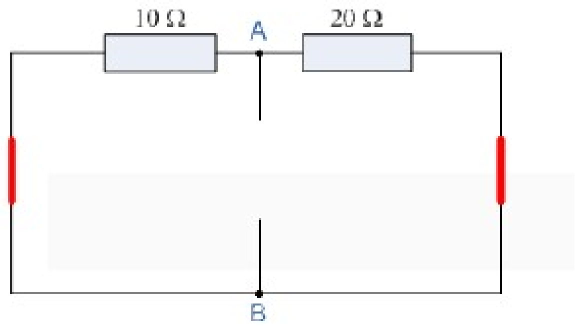
- **Remove the centre 40Ω resistor and short out all the emf′s connected to the circuit.**

- **The value of resistor R_{Th} is found by calculating the total resistance at the terminals A and B with all the emf′s removed.**

- **The value of the voltage required V_{Th} is the total voltage across terminals A and B with an open circuit and no load resistor R_{Th} connected.**

Find the Equivalent Resistance (R_{Th})

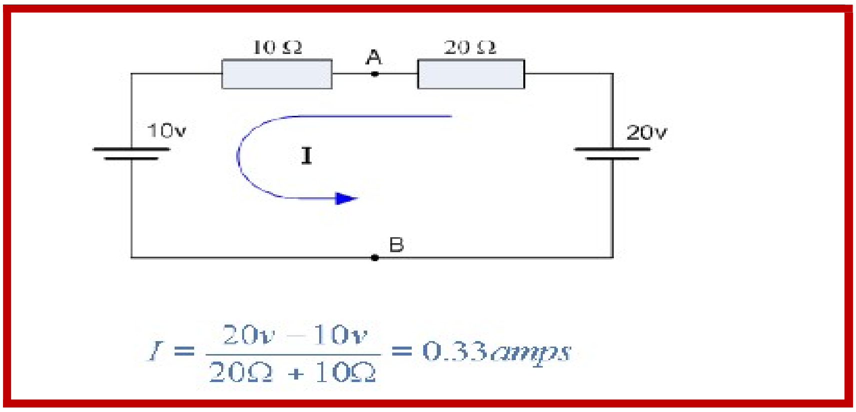
1. **Remove the load resistor R_L or component concerned**.



2. **Find R_{Th} by shorting all voltage sources or by open circuiting all the current sources.**

R_{Th} = 10x20 / (10+20) Ω = 6.67Ω.

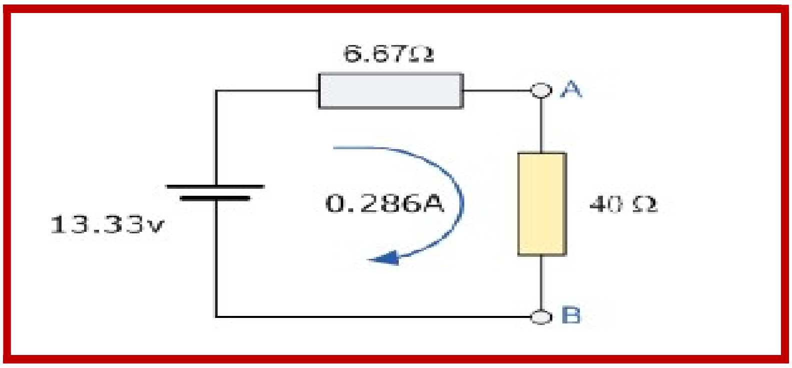
3. Find V_{Th} by the usual circuit analysis methods.



So the voltage drop across the 20Ω resistor can be calculated as:

$V_{AB} = V_{Th}$ = 20 - (20 x 0.33) = 13.33 volts.

Then the Thevenin's Equivalent circuit is shown below with the 40Ω resistor connected.



Thevenin Equivalent Circuit

The current flowing through the load resistor R_L

$$I = \frac{13.33v}{6.67\Omega + 40\Omega} = 0.286 amps$$

Limitations of Thevenin's Theorem

- **If the circuit consists of nonlinear elements, this theorem is not applicable.**
- **Also, to the unilateral networks it is not applicable.**
- **There should not be magnetic coupling between the load and the circuit to be replaced with the Thevenin's equivalent.**

Remarks:

(i) One great advantage of Thevenin's theorem over the normal circuit reduction technique or any other technique is this: once the Thevenin equivalent circuit has been formed, it can be reused in calculating load current (I_L), load voltage (V_L) and load power (P_L) for different loads.

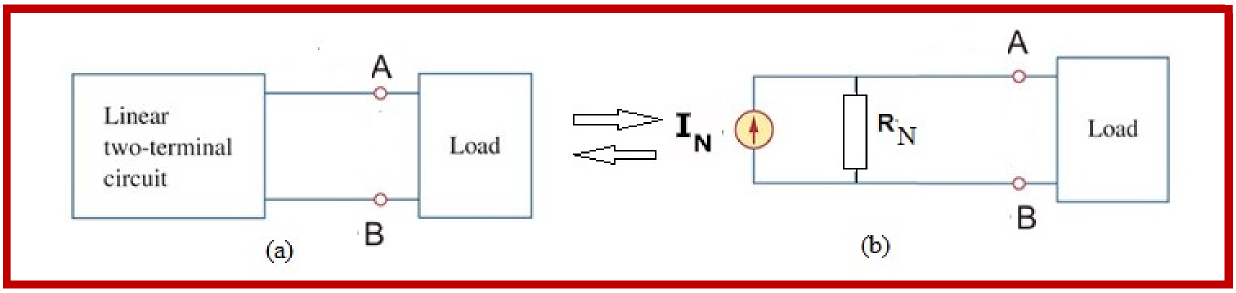
(ii) Fortunately, with help of this theorem one can find the choice of load resistance R_L that results in the maximum power transfer to the load. On the other hand, the effort necessary to solve this problem-using node or mesh analysis methods can be quite complex and tedious from computational point of view.

Norton's Theorem

A linear two terminal network consists of active and passive components can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistance R_N. The value of I_N is the short –circuit current between the terminals of the network and

R_N is the resistance measured between terminals with energy sources eliminated by their respective internal resistances.

Figure (a) circuit is a linear two terminal circuit. According to Norton's theorem we replace this circuit by a current source and an equivalent resistance that is shown in figure (b).



(a) Original circuit (b) Norton's Equivalent Circuit

Norton's equivalent circuit resembles a practical current source. Hence, it is having a current source in parallel with a resistor.

- The current source present in the Norton's equivalent circuit is called as Norton's equivalent current or simply Norton's current I_N.
- The resistor present in the Norton's equivalent circuit is called as Norton's equivalent resistor or simply Norton's resistor R_N.

Norton's Theorem Summary:

The basic procedure for solving a circuit using Norton's Theorem is as follows:

1. Remove the load.

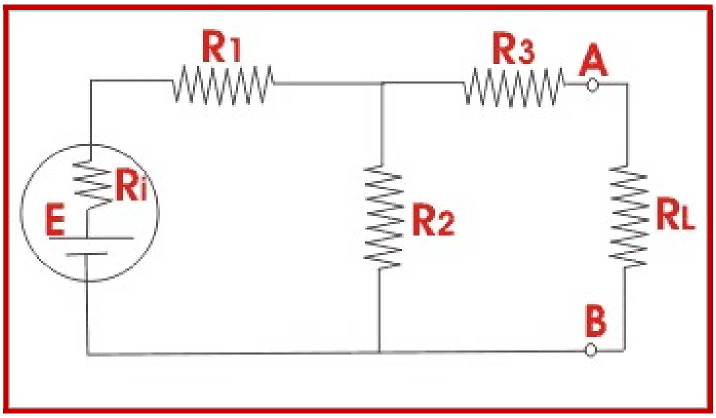
2. Find R_N by shorting all voltage sources or by open circuiting all the current sources.

3. Find I_N by placing a shorting link on the output terminals A and B.

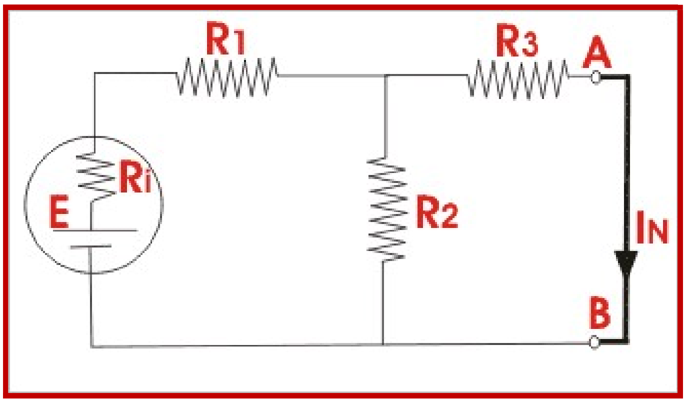
4. Find the current flowing through the load.

Proof of the Norton's Theorem

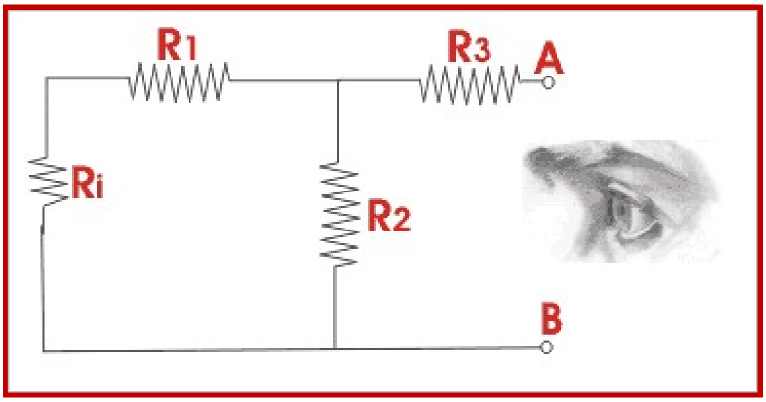
Find out the current through R_L by applying Norton theorem.



Step1: **Remove the resistor R_L from terminals A and B and make the terminals A and B short circuited by zero resistance.**



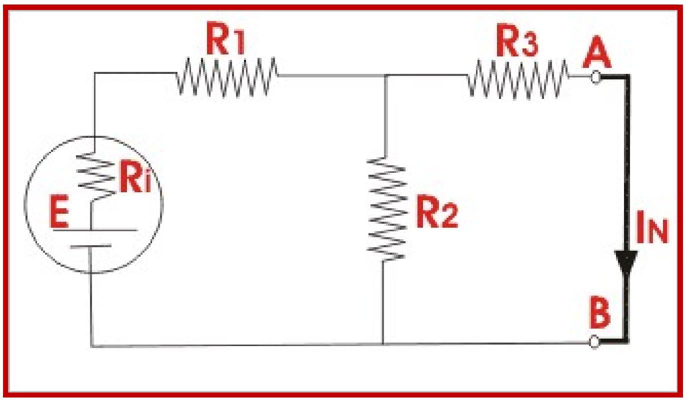
Step2: Determine internal resistance or Norton equivalent resistance R_N of the network under consideration, remove the branch between A and B and also replace the voltage source by its internal resistance. Now the equivalent resistance as viewed from open terminals A and B is R_N,



$$R_N = R_3 + R_2 || (R_1 + R_i) = R_3 + \frac{R_2(R_1 + R_i)}{R_1 + R_2 + R_i} = \frac{R_2.R_3 + (R_1 + R_i).(R_2 + R_3)}{R_1 + R_2 + R_i}$$

As per Norton theorem, when resistance R_L is reconnected across terminals A and B, the network behaves as a source of constant current I_N with shunt connected internal resistance R_N and this is Norton equivalent circuit.

Step 3: Calculate the short circuit current or Norton equivalent current I_N through the points A and B.



The equivalent resistance of the network,

$$R = (R_1 + R_i) + R_2 \,||\, R_3 = (R_1 + R_i) + \frac{R_2 \cdot R_3}{R_2 + R_3}$$

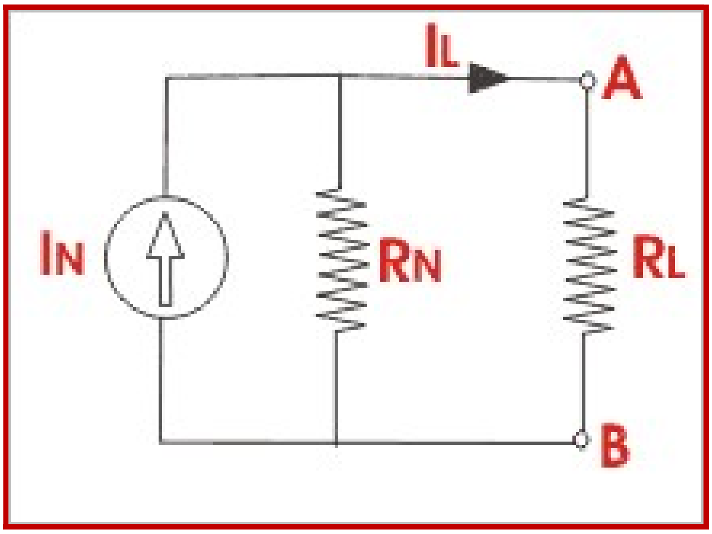
$$Current\ supplied\ by\ the\ Battery\ I_1 = \frac{E}{R}$$

$$Norton\ equivalent\ current\ I_N = Current\ flowing\ through\ R_3$$

$$I_N = I_1 \times \frac{R_2}{R_2 + R_3}$$

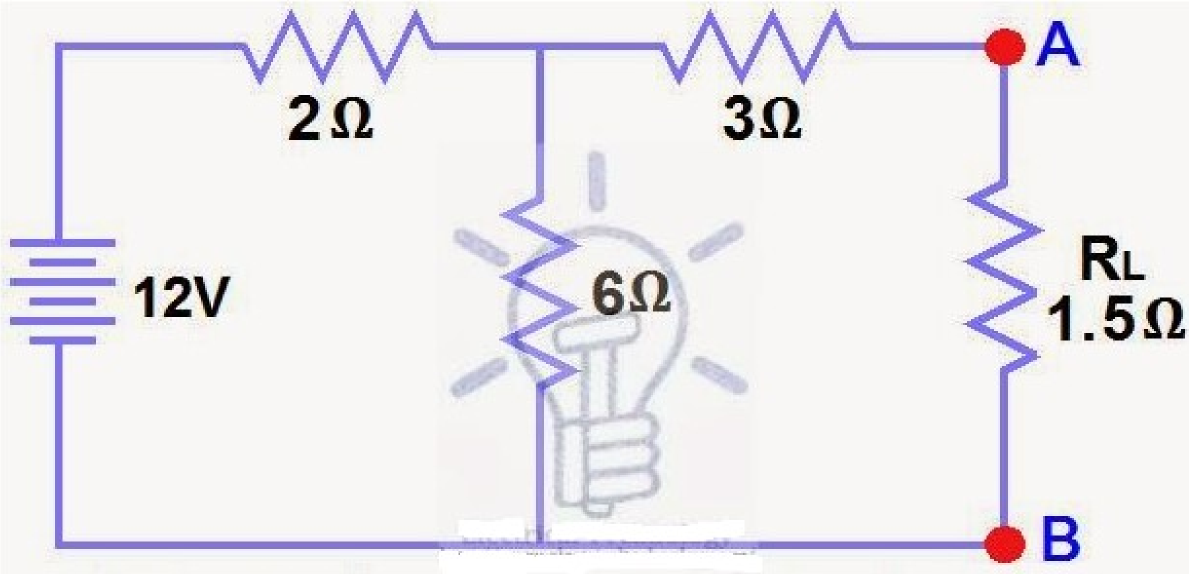
$$= \frac{E}{R_1 + R_i + R_2 \,||\, R_3} \times \frac{R_2}{R_2 + R_3} = \frac{E \cdot R_2}{(R_1 + R_i) \cdot (R_2 + R_3) + R_2.R_3}$$

Norton Equivalent Circuit



$$\text{The current through } R_L \text{ is given as, } I_L = \frac{I_N . R_N}{R_N + R_L}$$

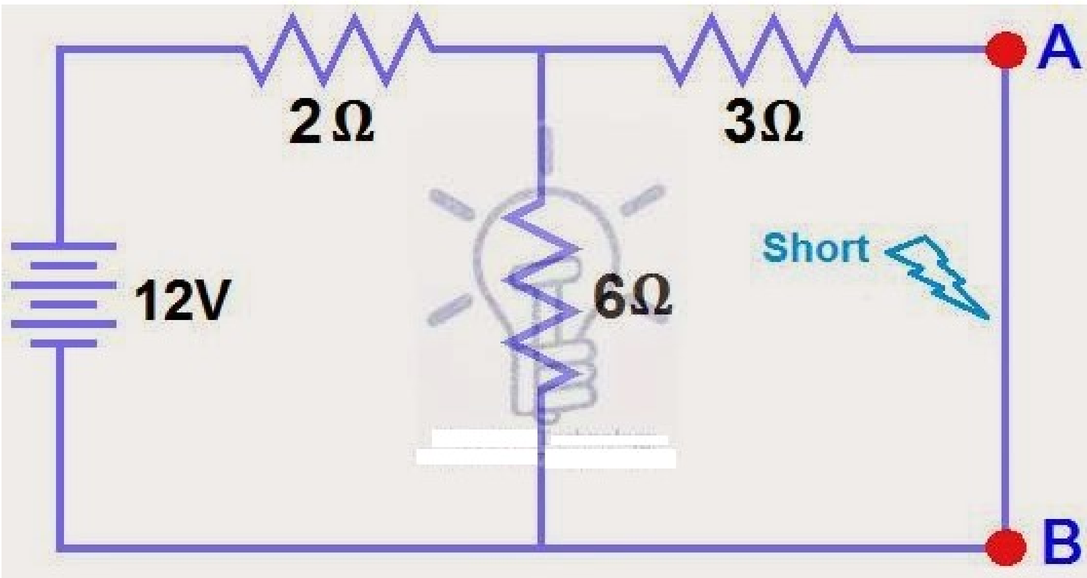
Example: **Find the current flowing through and Load Voltage across the load resistor R_L in figure below by using Norton's Theorem.**



Solution: -

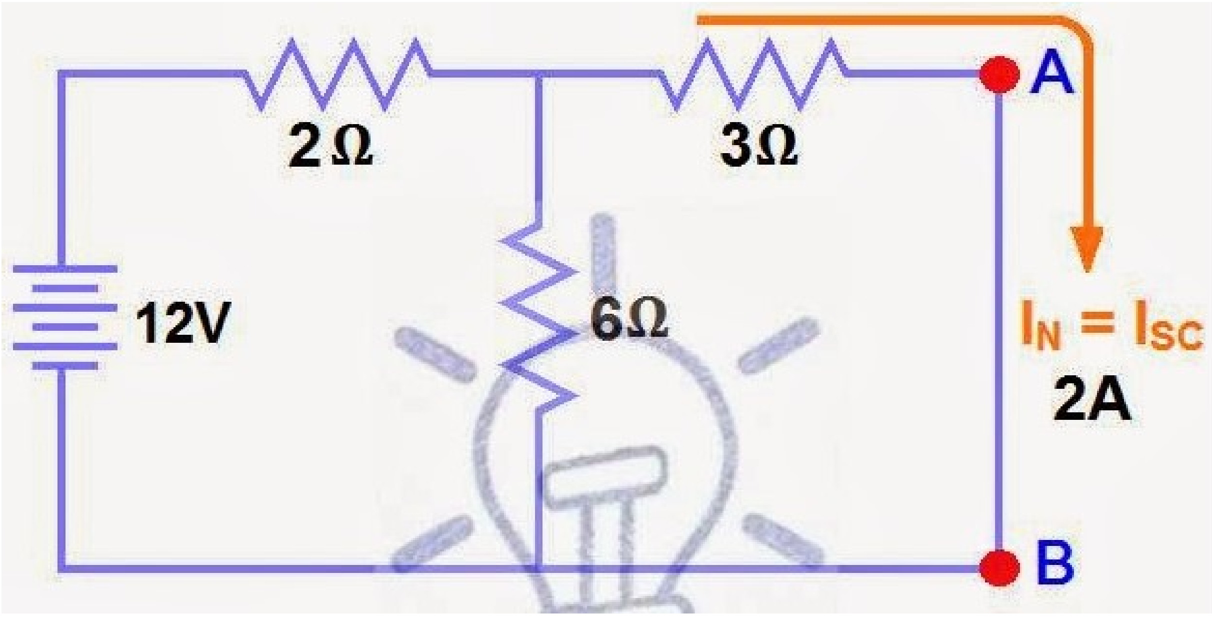
Step1.

Short the 1.5Ω load resistor as shown in Figure.



Step2.

Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).



$$R_T = 2\Omega + \frac{3\Omega \times 6\Omega}{3\Omega + 6\Omega} \rightarrow 2\Omega + 2\Omega = 4\Omega$$

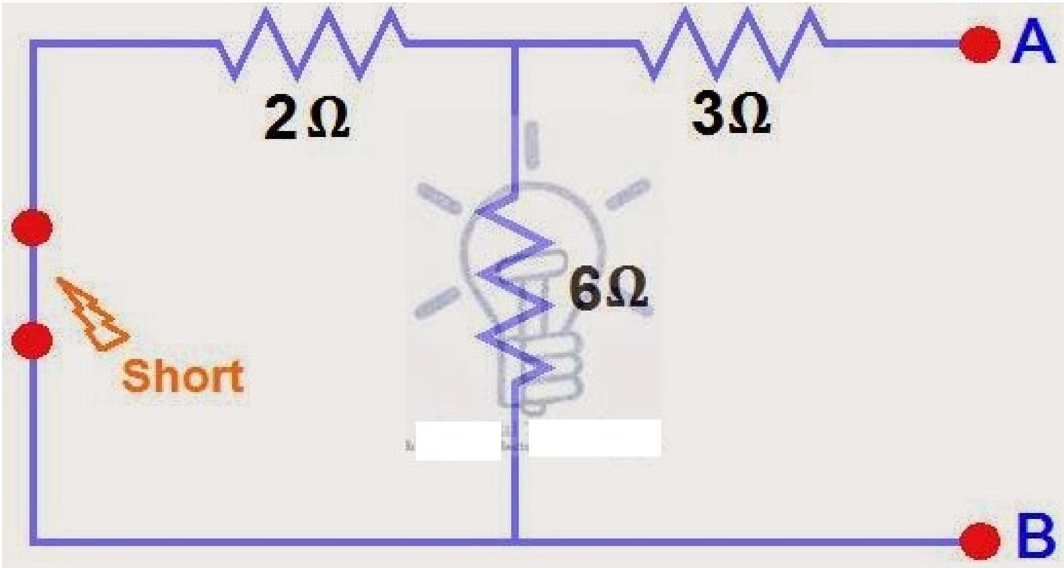
$$R_T = 4\Omega$$

$$I_T = V / R_T = 12/4 = 3A$$

$$I_N = I_{SC} = 3A \times \frac{6\Omega}{3\Omega + 6\Omega} = 2A$$

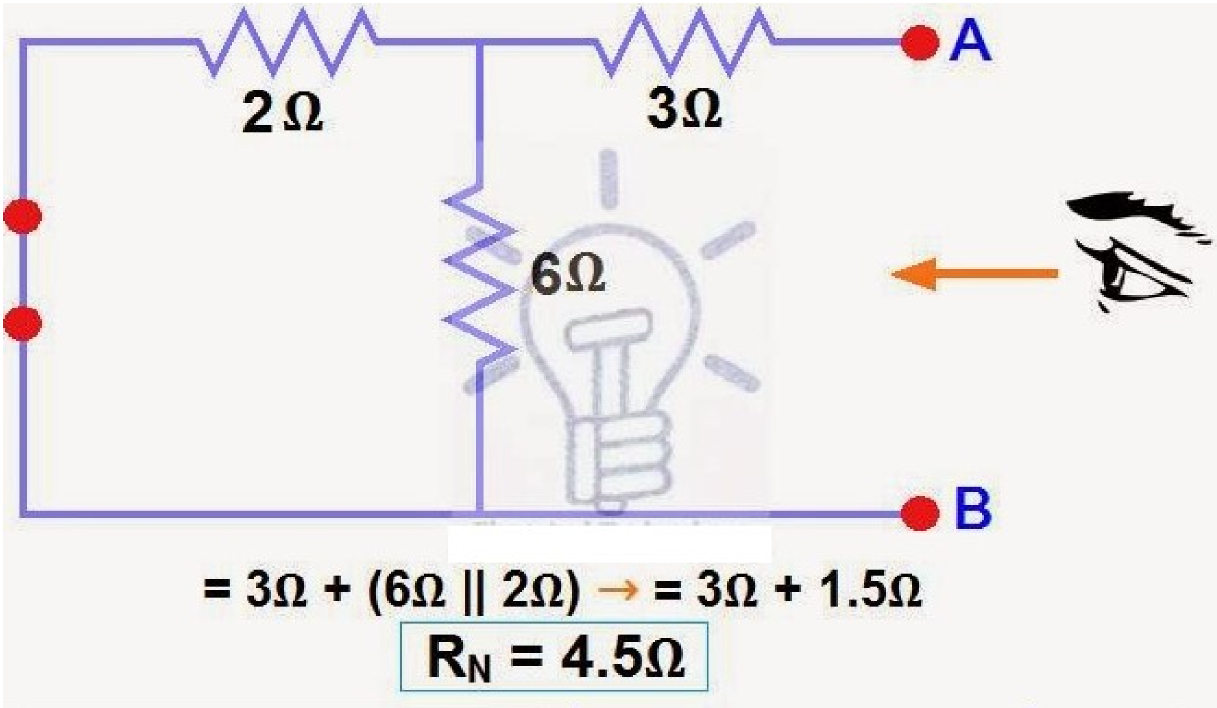
Step3.

Short Voltage Sources and Open Load Resistor. Figure below



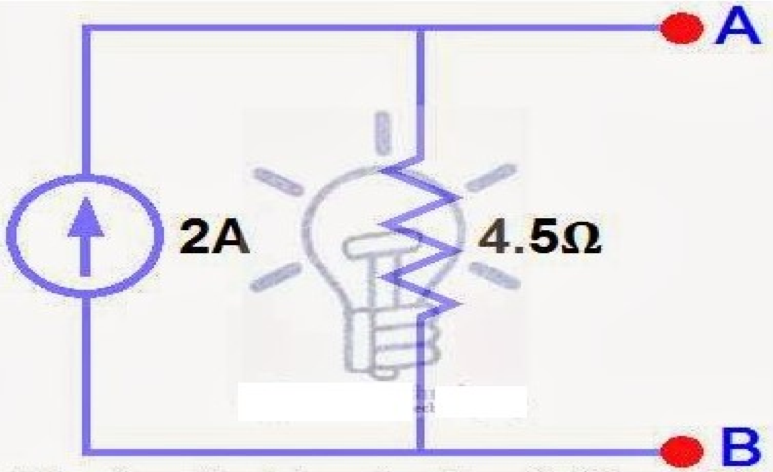
Step4.

Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N).



Step5.

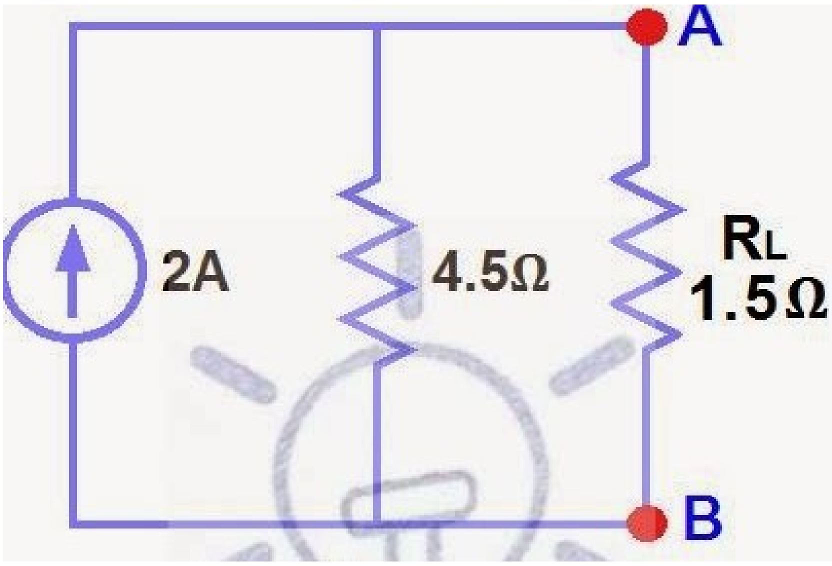
Connect the R_N in Parallel with Current Source I_N and re-connect the load resistor. This is shown in fig (6) i.e. Norton Equivalent circuit with load resistor.



Norton's Equivalent Circuit

Step6.

Now apply the last step i.e., calculate the load current through and Load voltage across load resistor by Ohm's law as shown in figure below.



$$I_L = I_N \times \frac{R_N}{(R_N+R_L)}$$

$$= 2A \times \frac{4.5\Omega}{(4.5\Omega + 1.5\Omega)} = \boxed{I_L = 1.5A}$$

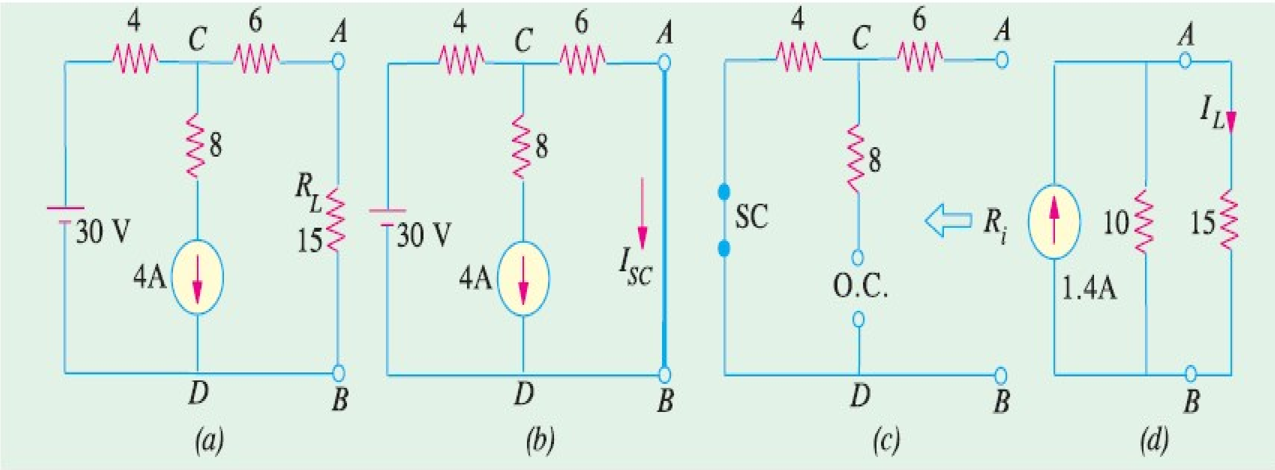
$$V_L = I_L \times R_L$$

$$V_L = 1.5A \times 1.5\Omega$$

$$\boxed{V_L = 2.25V}$$

Example:2

Using Norton's theorem, calculate the current flowing through the 15 Ω load resistor in the circuit of Fig. (a). All resistance values are in ohm.



Solution:

Find out Short-Circuit Current I_{SC}

Step1: **As shown in Fig. (*b*), terminals *A* and *B* have been shorted after removing 15 Ω resistor.**

Step 2: **Superposition theorem is used to find I_{SC}.**

(*i*) When Only Current Source is Present

In this case, 30-V battery is replaced by a short-circuit. The 4 A current divide at point *D* between parallel combination of 4 Ω and 6 Ω. Current through 6 Ω resistor is

$I_{SC'}$ = 4 × 4/(4 + 6) = 1.6 A flowing from *B* to *A*

(*ii*) When Only Battery is Present

In this case, current source is replaced by an open-circuit so that no current flows in the branch *CD*. The current supplied by the battery constitutes the short-circuit current

$I_{SC''} = 30/(4 + 6) = 3$ **A flowing from** A **to** B

$I_{SC} = I_{SC''} - I_{SC'} = 3 - 1.6 = 1.4$ **A – flowing from** A **to** B

Find out Norton's Resistance

Step:3 As seen from Fig. (c) $R_i = 4 + 6 = 10\ \Omega$**. The 8 Ω resistance does not come into the picture because of an open in the branch *CD*. Fig. (*d*) shows the Norton's equivalent circuit along with the load resistor.**

Step:4 The current flowing through the 15Ω resistor is

$I_L = 1.4 \times 10\ (10 + 15) = 0.56$ **A**

Remarks:

(i) Similar to the Thevenin's theorem, Norton's theorem has also a similar advantage over the normal circuit reduction technique or any other technique when it is used to calculate load current (I_L**), load voltage (**V_L**) and load power (**P_L**) for different loads.**

(ii)Fortunately, with help of either Norton's theorem or Thevenin's theorem one can find the choice of load resistance R_L that result in the maximum power transfer to the load.

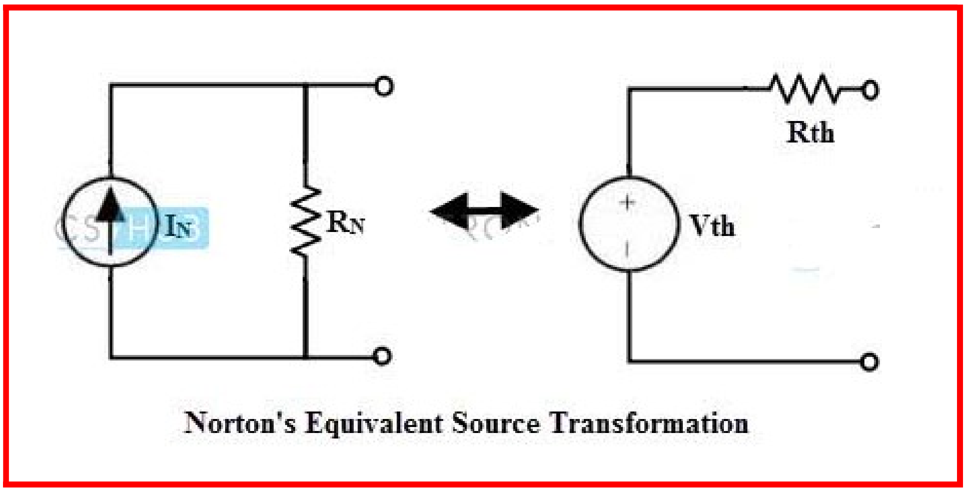
(iii) Norton's current source may be replaced by an equivalent Thevenin's voltage source. The magnitude of voltage source (V_{Th}) and its internal resistances (R_{Th}) are expressed by the following relations

$$V_{Th} = I_N \times R_N; \; R_{Th} = R_N \text{ (with proper polarities at the terminals)}$$

In other words, a source transformation converts a Thevenin equivalent circuit into a Norton equivalent circuit or vice-verse.

Relation between Norton's and Thevenin's Theorems

Norton's equivalent circuit of a linear network constitutes a Norton current source I_N in parallel with a resistance R_N. Therefore, it is possible to perform a source transformation of Norton's equivalent circuit to get the Thevenin's equivalent circuit or vice-versa.



The magnitude of the voltage source (V_{th}) and a series resistance (R_{th}) from Norton's equivalent circuit using source transformation are determined as

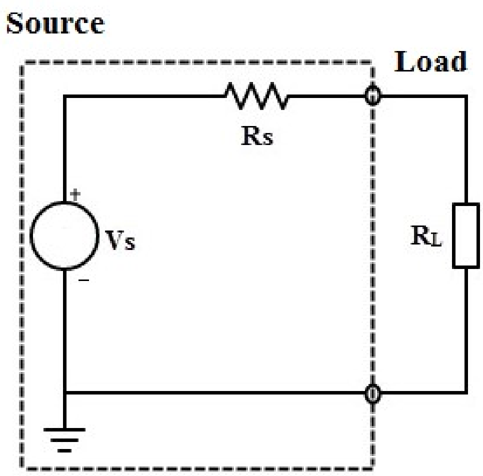
$V_{th} = R_N \times I_N$ and $R_{th} = R_N$

Maximum Power Transfer Theorem

The maximum power transfer theorem states that in a linear, bilateral DC network, maximum power is delivered to the load when the load resistance is equal to the internal resistance of a source.

If it is an independent voltage source, then its series resistance (internal resistance R_s) or if it is independent current source, then

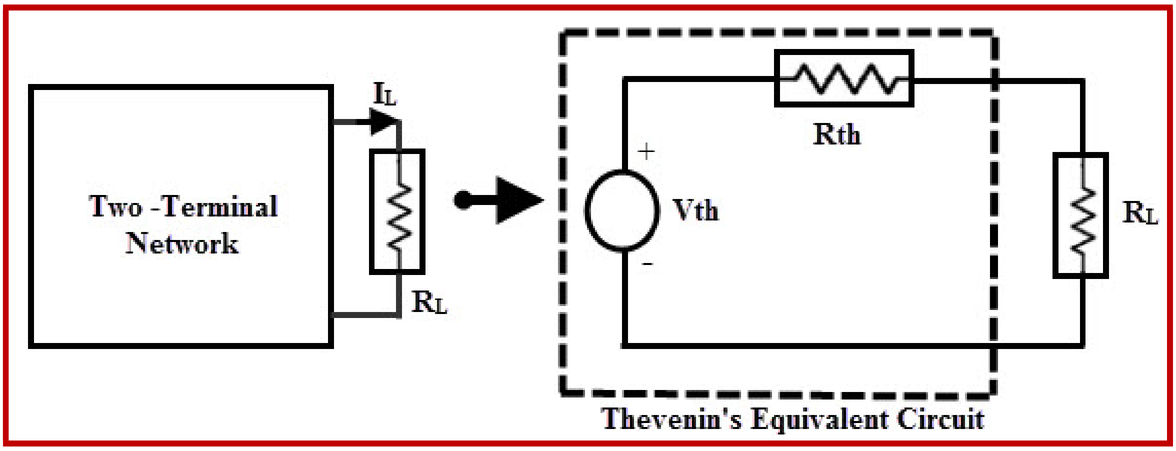
its parallel resistance (internal resistance R_s) must equal to the load resistance R_L to deliver maximum power to the load.



Proof of Maximum Power Transfer Theorem

The maximum power transfer theorem ensures the value of the load resistance, at which the maximum power is transferred to the load. Consider the below DC two terminal network (left side circuit), to which the condition for maximum power is determined, by obtaining the expression of power absorbed by load with use of mesh or nodal current methods and then deriving the resulting expression with respect to load resistance R_L. But this is quite a

complex procedure. But in previous articles we have seen that the complex part of the network can be replaced with a Thevenin's equivalent as shown below.



The original two terminal circuit is replaced with a Thevenin's equivalent circuit across the variable load resistance. The current through the load for any value of load resistance is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

The power absorbed by the load is

$$P_L = I_L^{\ 2} \times R_L$$

$$= \left[\frac{V_{Th}}{R_{Th} + R_L} \right]^2 \times R_L$$

Form the above expression the power delivered depends on the values of R_{TH} and R_L. However, the Thevenin's equivalent is constant, the power delivered from this equivalent source to the load entirely depends on the load resistance R_L. To find the exact value of R_L, we apply differentiation to P_L with respect to R_L and equating it to zero as

$$\frac{dP_L}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th}+R_L)^2 - 2R_L \times (R_{Th}+R_L)}{(R_{Th}+R_L)^4} \right] = 0$$

$$\Rightarrow (R_{Th}+R_L) - 2R_L = 0$$

$$\Rightarrow R_L = R_{Th}$$

Therefore, this is the condition of matching the load where the maximum power transfer occurs when the load resistance is equal to the Thevenin's resistance of the circuit. By substituting the $R_{Th} = R_L$ in equation 1 we get

The maximum power delivered to the load is,

$$P_{max} = \left[\frac{V_{Th}}{R_{Th}+R_L} \right]^2 \times R_L \Bigg|_{R_L = R_{Th}}$$

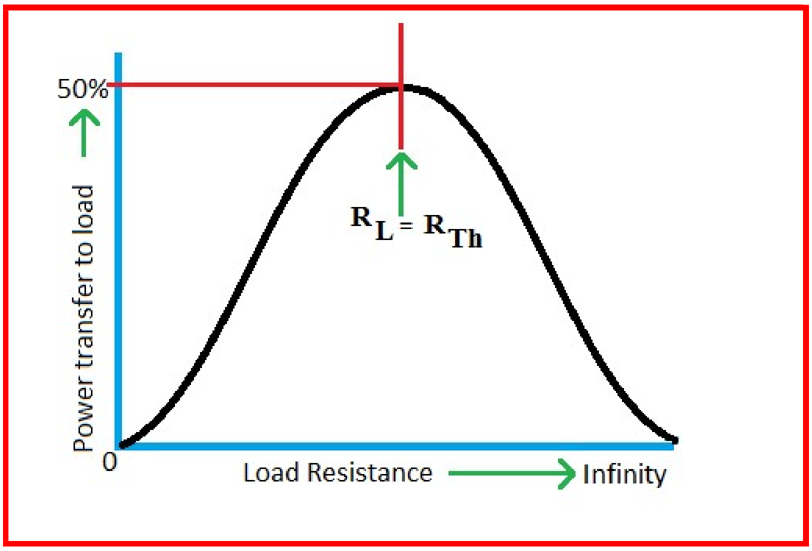
$$= \frac{V_{Th}^2}{4R_{Th}}$$

Input power = $V_{Th} \times I_L = V_{Th} \times (V_{Th} / 2R_{Th}) = V_{Th}^2 / 2R_{Th}$

Efficiency = η = output power / input power

$$= (V_{Th}^2 / 4R_{Th}) / (V_{Th}^2 / 2R_{Th}) = 50\%$$

The graph shows Power transfer to Load resistance R_L



Condition for maximum current, maximum voltage and maximum power in load

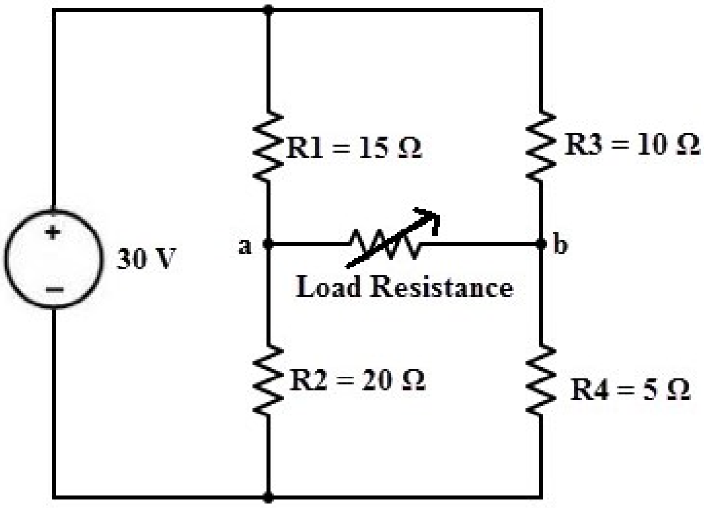
- Maximum current I_L occurs when $R_L = 0$ (shorted terminals)
- The maximum voltage V_L occurs when $R_L = \infty$ (open circuited terminals)
- Yet load power $P_L = 0$ for both cases
- P_L is maximum when R_L equals the Thevenin equivalent resistance of the source, I.e. when $R_L = R_{TH}$

Power Transfer Efficiency

This theorem results maximum power transfer but not a maximum efficiency.

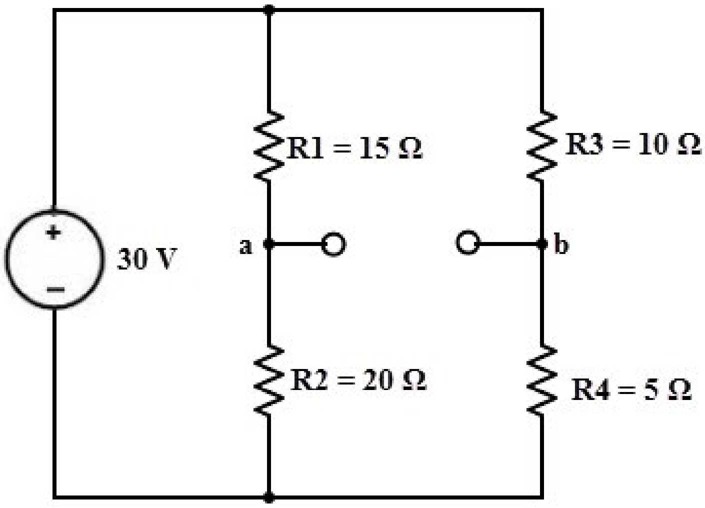
- **If the load resistance is smaller than source resistance, power dissipated at the load is reduced than the power generated at the source then the efficiency becomes lower.**
- **when the load resistance is equal to the internal resistance of the circuit or source (or Thevenin's equivalent resistance) i.e. at the condition of maximum power transfer, half percentage of generated power is delivered (or dissipated) to the load then the efficiency is 50%.**
- **when the load resistance is much larger value than internal source resistance, the power delivered will be less though the efficiency is high.**

Example: **Determine the value of the load resistance that receives the maximum power from the supply and the maximum power under the maximum power transfer condition.**



Step1.

Disconnect the load resistance connected across the load terminals a and b. To represent the given circuit as Thevenin's equivalent, determine the Thevenin's voltage V_{TH} and Thevenin's equivalent resistance R_{TH}.



The Thevenin's voltage or voltage across the terminal's ab is V_{ab} = $V_a - V_b$

$V_a = V \times R2 / (R1 + R2)$

$= 30 \times 20 /(20 + 15)$

$= 17.14$ V

$V_b = V \times R4/ (R3 + R4)$

$= 30 \times 5 /(10 + 5)$

$= 10$ V

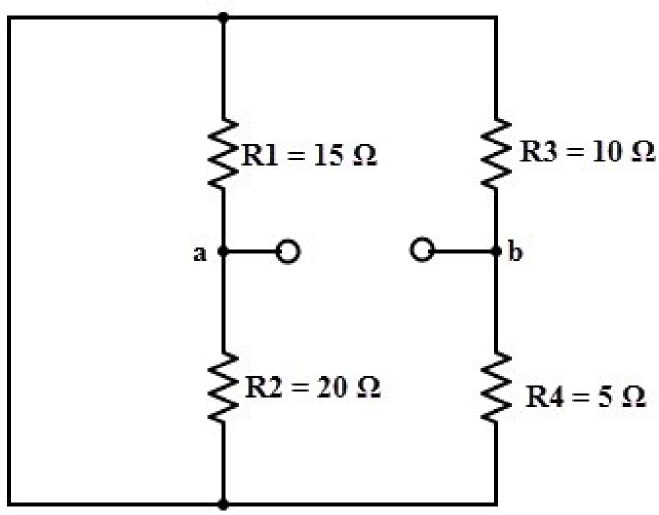
$V_{ab} = 17.14 - 10$

$= 7.14$ V

$V_{TH} = V_{ab} = 7.14$ **Volts**

Step2.

Calculate the Thevenin's equivalent resistance R_{TH} by replacing sources with their internal resistances (here voltage source has zero internal resistance so it becomes a short circuited).



Thevenin's equivalent resistance or resistance across the terminals ab is

$R_{TH} = R_{ab} = [R1R2 / (R1 + R2)] + [R3R4 /(R3 + R4)]$

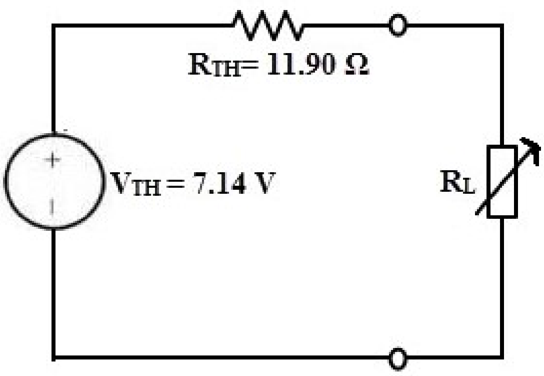
$= [(15 \times 20) / (15 + 20)] + [(10 \times 5) / (10+ 5)]$

$= 8.57 + 3.33$

R_{TH} = 11.90 Ohms

Step3.

The Thevenin's equivalent circuit with above calculated values by reconnecting the load resistance is shown below.



From the maximum power transfer theorem, R_L value must equal to the R_{TH} to deliver the maximum power to the load.

Therefore, $R_L = R_{TH}$= 11.90 Ohms

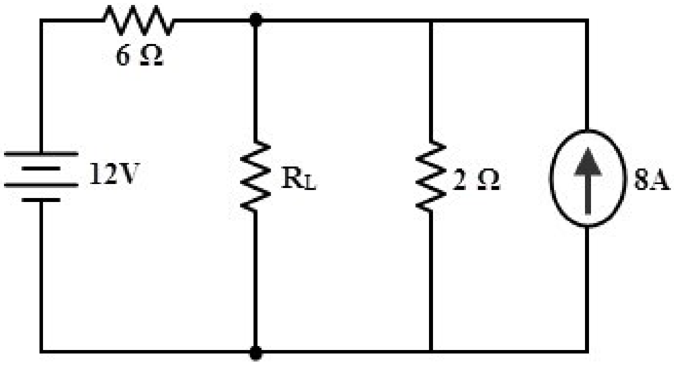
And the maximum power transferred under this condition is,

$P_{max} = V^2_{TH} / 4\ R_{TH}$

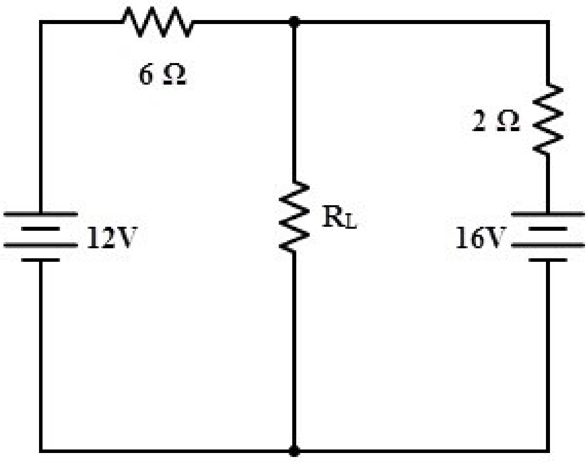
$= (7.14)^2 / (4 \times 11.90)$

= 50.97 / 47.6 = 1.07 Watts.

Example: Determine the value of load resistance, R_L for which maximum power will transfer from source to load.

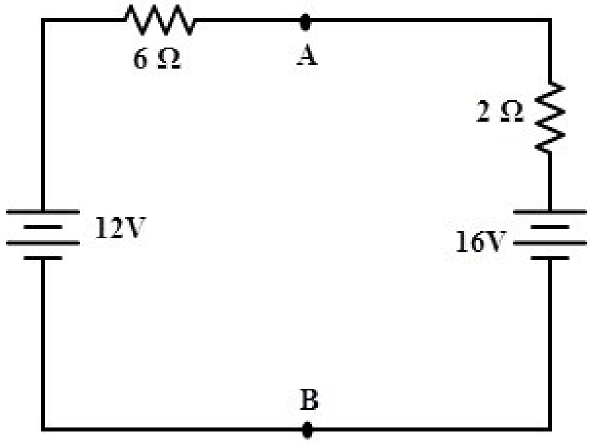


Solution: The given circuit can be further simplified by converting the current source into equivalent voltage source.



Find the Thevenin's equivalent voltage V_{th} and Thevenin's equivalent resistance R_{th} across the load terminals in order to get the condition for maximum power transfer.

By disconnecting the load resistance, the open-circuit voltage across the load terminals can be calculated as;



By applying, Kirchhoff's voltage law we get

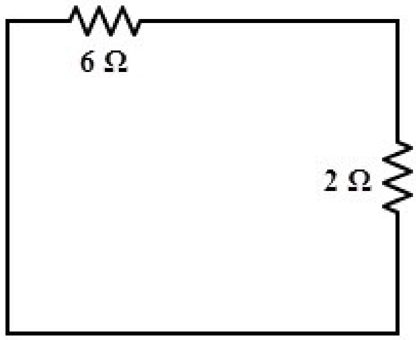
$$6I + 2I = -16+12$$

$$8I = -4$$

$$I = -0.5\ A$$

The open-circuit voltage across the terminals A and B, $V_{AB} = 16 - 2 \times 0.5 = 15$ V

Thevenin's equivalent resistance across the terminals A and B is obtained by short circuiting the voltage sources



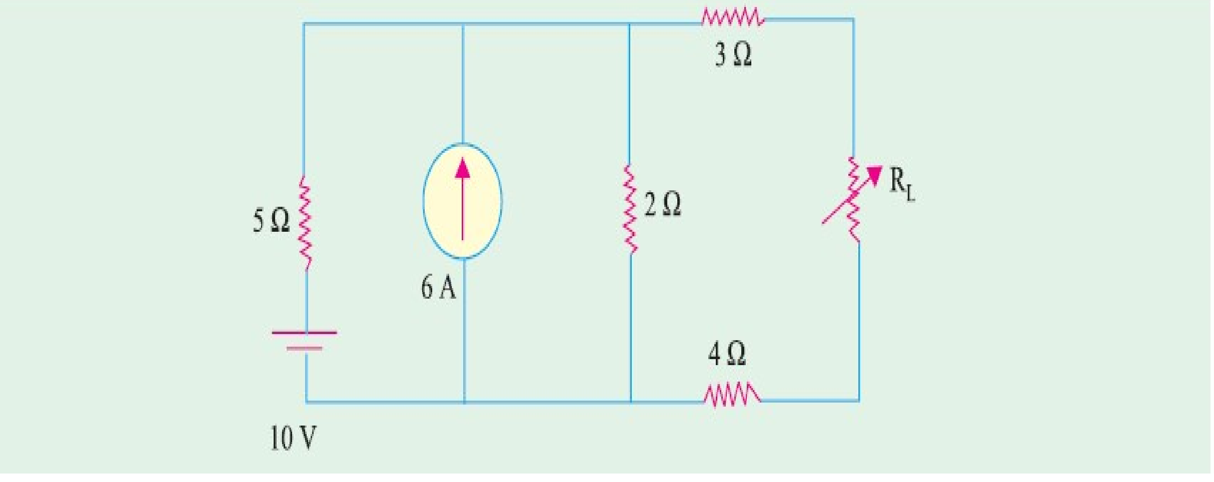
$R_{th} = (6 \times 2) / (6 + 2) = 1.5\ \Omega$

So, the maximum power will be transferred to the load when R_L = 1.5 ohm.

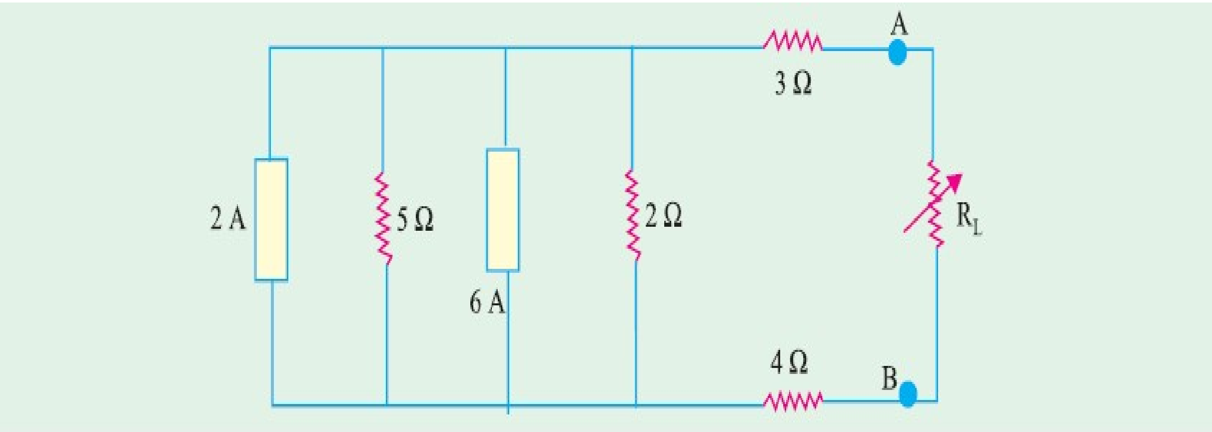
Current through the circuit, $I = 15 / (1.5 + 1.5) = 5$ A

Therefore, the maximum power = $5^2 \times 1.5 = 37.5$ W

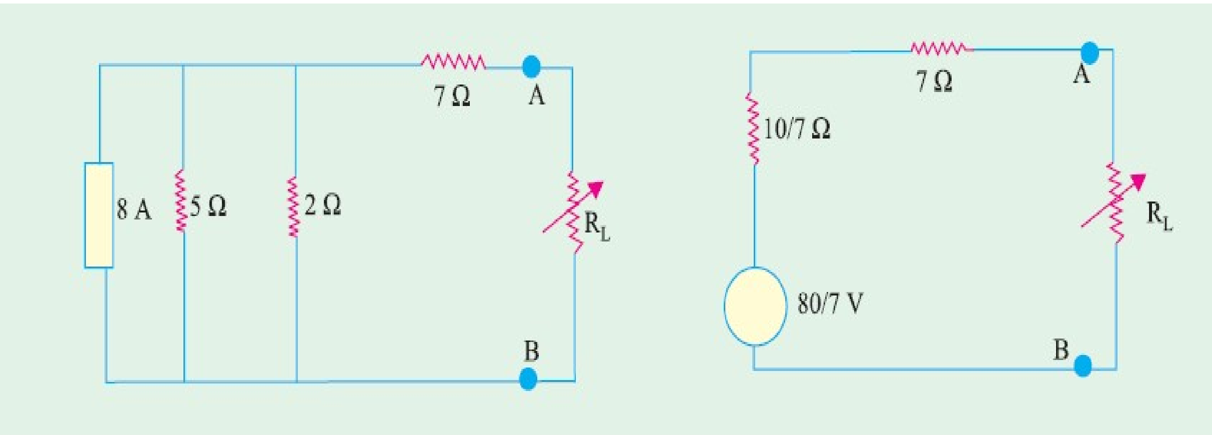
Example: Determine the value of load resistance, R_L for which maximum power will transfer from source to load.



Solution: **Simplify by source transformations, as done in Figs (*b*), (*c*), (*d*)**



(a)



(b) **(c)**

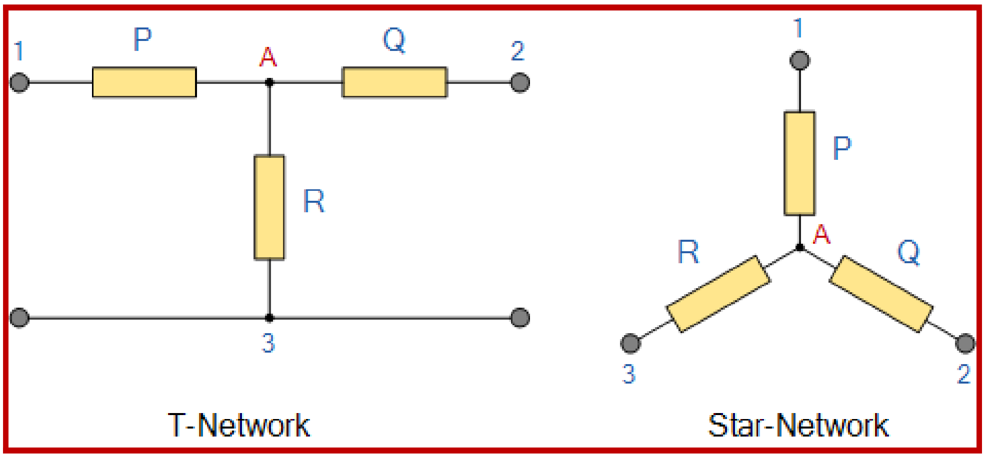
For maximum power, R_L = 7 + (10/7) = 8.43 Ω

Maximum power = [(80/7)/16.68]2 × 8.43 = 3.87 watts.

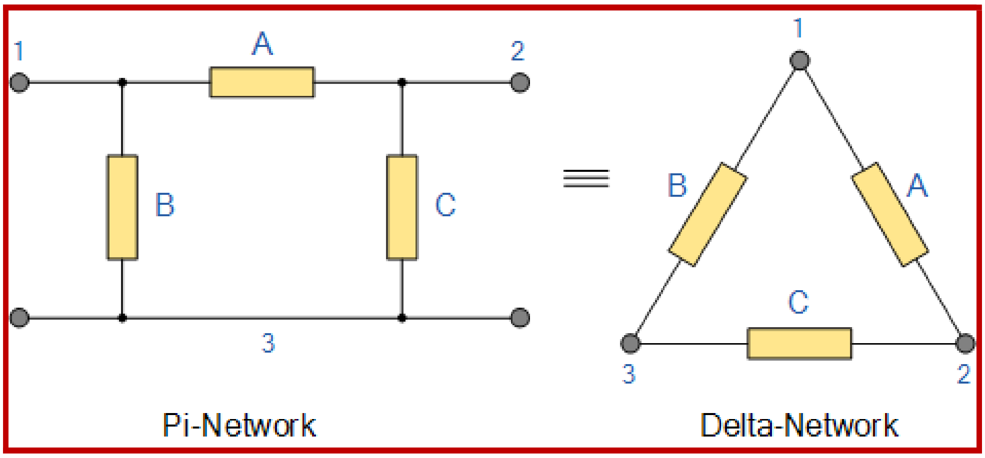
Star Delta Transformation

There are certain circuit configurations that cannot be simplified by series-parallel combination alone. A simple transformation based on mathematical technique is readily simplifies the electrical circuit configuration.

T-connected and Equivalent Star Network



Pi-connected and Equivalent Delta Network



Delta – Wye conversion (Δ) - (Y)

Let us consider the network shown in fig.1(e) and assumed the resistances (R_{AB}, R_{BC}, R_{CA}) in 1(*c*)→ network is known. Our problem is to find the values of R_A, R_B, R_C in Wye (*Y*) network (see fig.1(e)) that will produce the same resistance when measured between similar pairs of terminals. We can write the equivalence resistance between any two terminals in the following form.

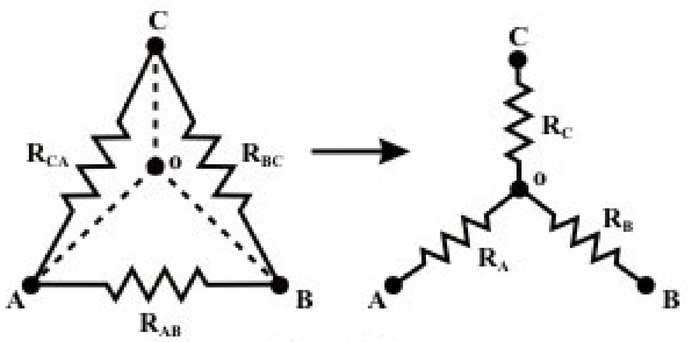


Fig. 1 (e)

Between A & C terminals:

$$R_A + R_C = \frac{R_{CA}\left(R_{AB} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad \textbf{........... (1)}$$

Between C & B terminals:

$$R_C + R_B = \frac{R_{BC}\left(R_{AB} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad \textbf{.............(2)}$$

Between B & A terminals:

$$R_B + R_A = \frac{R_{AB}\left(R_{CA} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad \textbf{.............(3)}$$

By combining above three equations, one can write an expression as given below.

$$R_A + R_B + R_C = \frac{R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \qquad \textbf{............(4)}$$

Subtracting equations (2), (1), and (3) from (4) equations, we can write the express for unknown resistances of Wye (*Y*) network

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad \text{..........(5)}$$

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \quad \text{...............(6)}$$

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad \text{.............(7)}$$

These relationships may be expressed thus: *the equivalent star resistance connected to a given terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta resistances.*

Conversion from Star or Wye to Delta

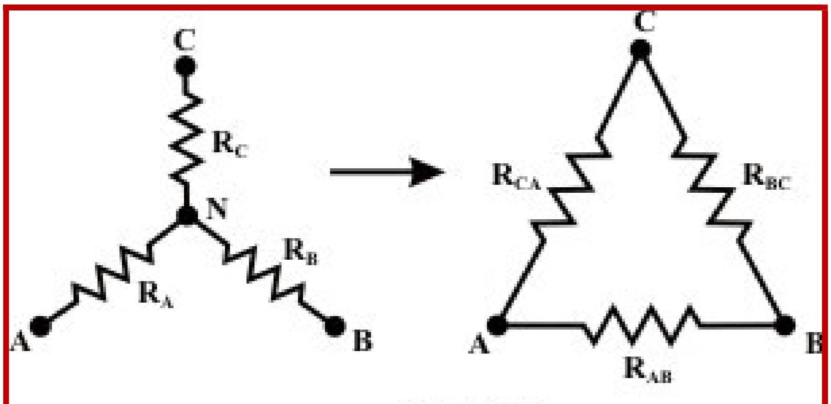


Fig. 6.1 (f)

To convert a Wye (*Y*) to a Delta (Δ), the relationships R_{AB}, R_{BC} and R_{CA} must be obtained in terms of the Wye (R_A, R_B, R_C) resistances (referring to fig.6.1 (f)).

Let us next consider how to replace the star-connected network of Fig. 6.1(f) by the equivalent delta-connected network.

Dividing equation [5] by equation [6], we have

$$\frac{R_A}{R_B} = \frac{R_{CA}}{R_{BC}}$$

$$\therefore \qquad R_{CA} = \frac{R_A}{R_B} R_{BC}$$

Similarly, dividing equation [5] by equation [7], we have

$$\frac{R_A}{R_c} = \frac{R_{AB}}{R_{BC}}$$

$$\therefore \qquad R_{AB} = \frac{R_A}{R_c} R_{BC}$$

Substituting for R_{AB} and R_{CA} in equation [5], we have

$$R_{BC} = R_B + R_c + \frac{R_c \; R_B}{R_A}$$

Similarly, we get

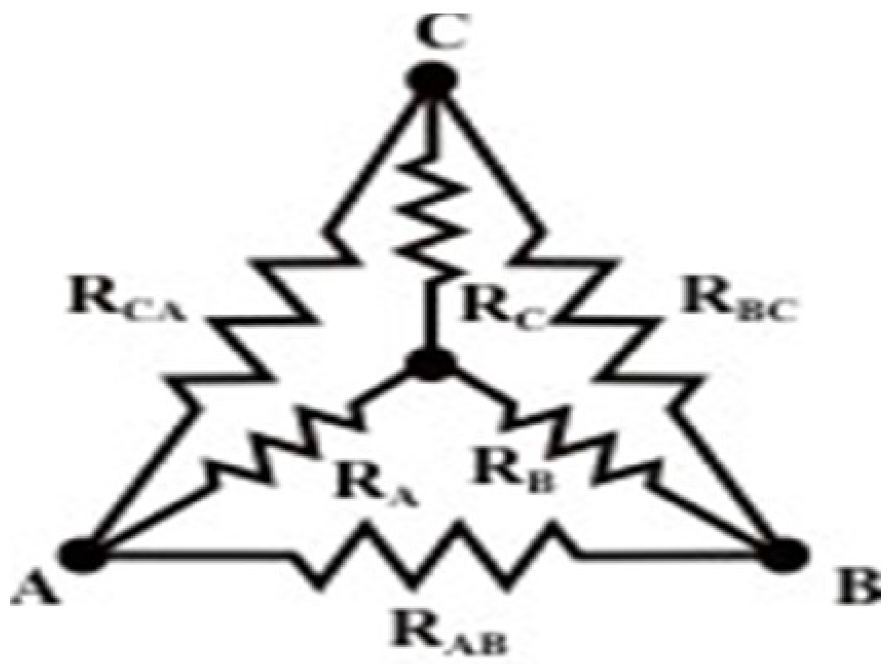
$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B}$$

These relationships may be expressed thus: ***the equivalent delta resistance between two terminals is the sum of the two star resistances connected to those terminals plus the product of the same two star resistances divided by the third star resistance.***

Observations

In order to note the symmetry of the transformation equations, the Wye (*Y*) and Delta (Δ) networks have been superimposed on each other as shown in fig.



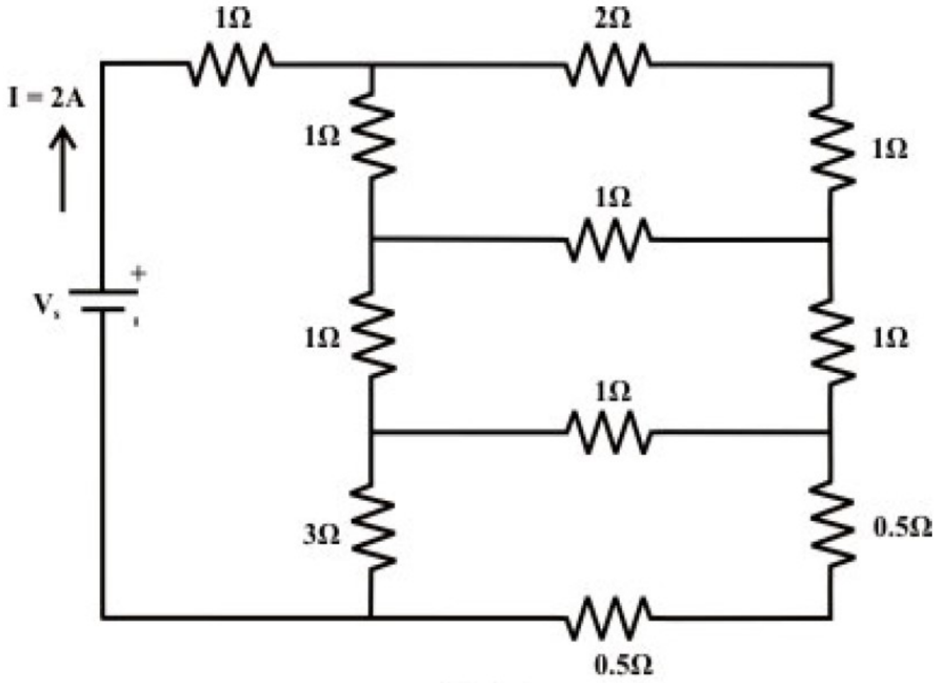
- **The equivalent star (Wye) resistance connected to a given terminal is equal to the product of the two Delta (Δ) resistances connected to the same terminal divided by the sum of the Delta (Δ) resistances (see fig.).**
- **The equivalent Delta (Δ) resistance between two-terminals is the sum of the two-star (Wye) resistances connected to those terminals plus the product of the same two-star (Wye)**

resistances divided by the third star (Wye (*Y*)) resistance (see fig.).

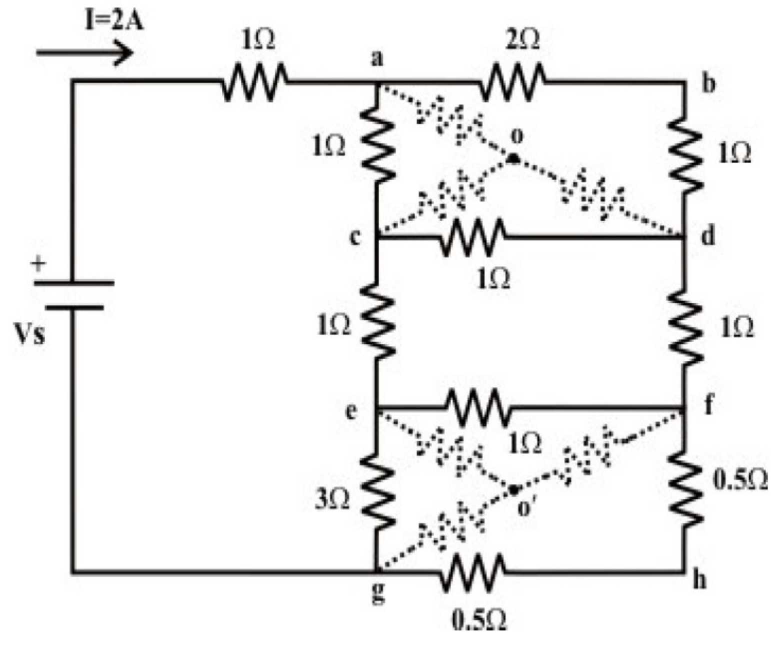
- **One final point about converting a star resistive network to an equivalent delta network. If all the resistors in the star network are all equal in value then the resultant resistors in the equivalent delta network will be three times the value of the star resistors and equal, giving: $R_{DELTA} = 3R_{STAR}$.**
- **If the three resistors in the delta network are all equal in value then the resultant resistors in the equivalent star network will be equal to one third the value of the delta resistors, giving each branch in the star network as: $R_{STAR} = (1/3)\ R_{DELTA}$**

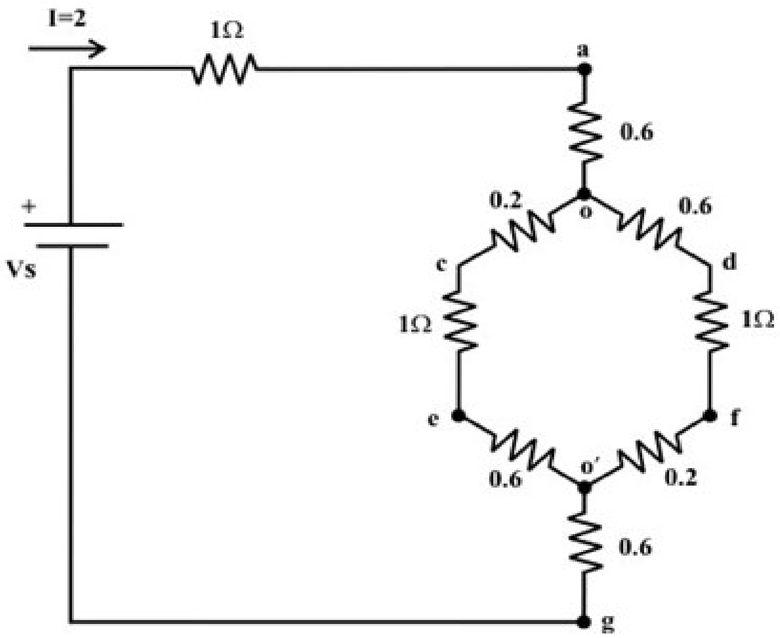
Example: 1

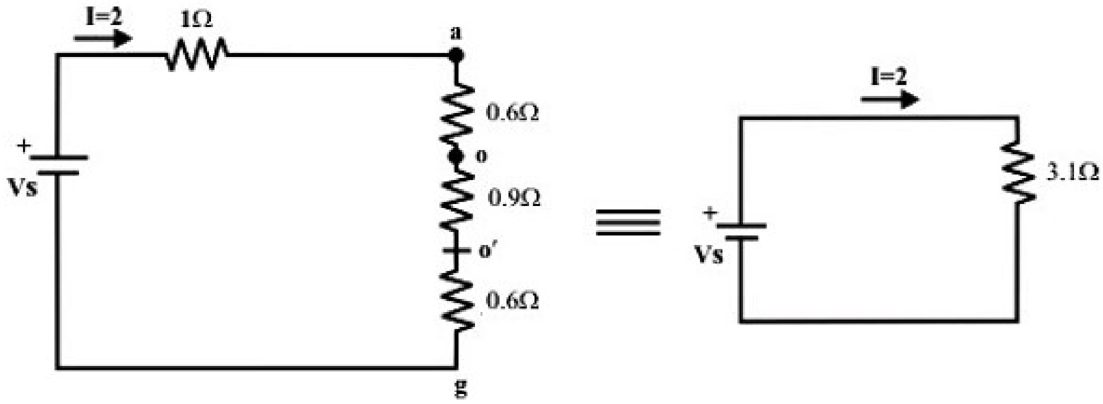
Find the value of the voltage source (V_s) that delivers 2 Amps current through the circuit as shown in Figure below.


1Ω
2Ω
I = 2A
1Ω
1Ω
1Ω
+
V_s
1Ω
1Ω
1Ω
3Ω
0.5Ω
0.5Ω

Solution:


I=2A
1Ω
a
2Ω
b
1Ω
o
1Ω
c
d
1Ω
+
Vs
1Ω
1Ω
e
f
1Ω
0.5Ω
3Ω
o′
g
0.5Ω
h

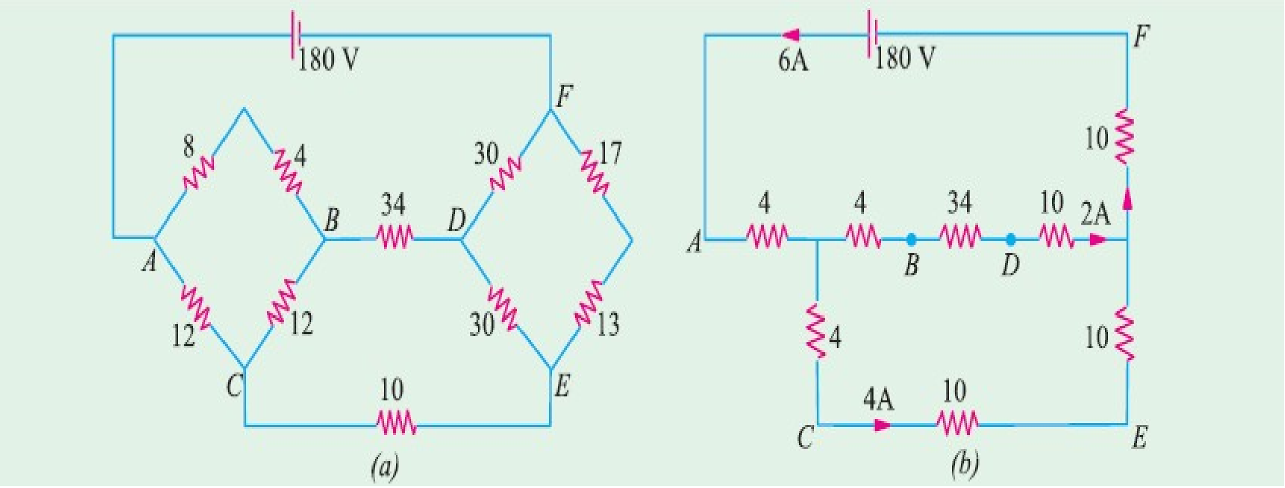




The source V_s that delivers 2*A* current through the circuit can be obtained as, Vs = 2x3.1 = 6.2V.

Example: 2

Calculate the current flowing through the 10 Ω resistor of Figure below.



Solution:

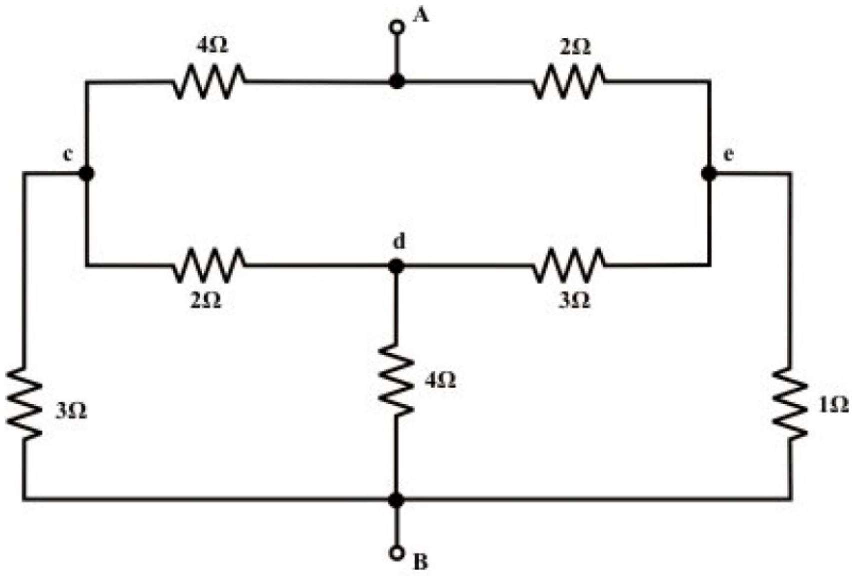
Here are two deltas in the circuit *i.e.*, *ABC* and *DEF*. They have been converted into their equivalent stars as shown in Figure (*b*). Each arm of the delta *ABC* has a resistance of 12 Ω and each arm of the equivalent star has a resistance of 4 Ω. Similarly, each arm of the delta *DEF* has a resistance of 30 Ω and the equivalent star has a resistance of 10 Ω per arm.

The total circuit resistance between *A* and *F* = 4 + 48 || 24 + 10 = 30 Ω. Hence, *I* = 180/30 = 6 A.

Current through 10 Ω resistor as given by current-divider rule = 6 × 48/(48 + 24) = 4 A.

Example: 3

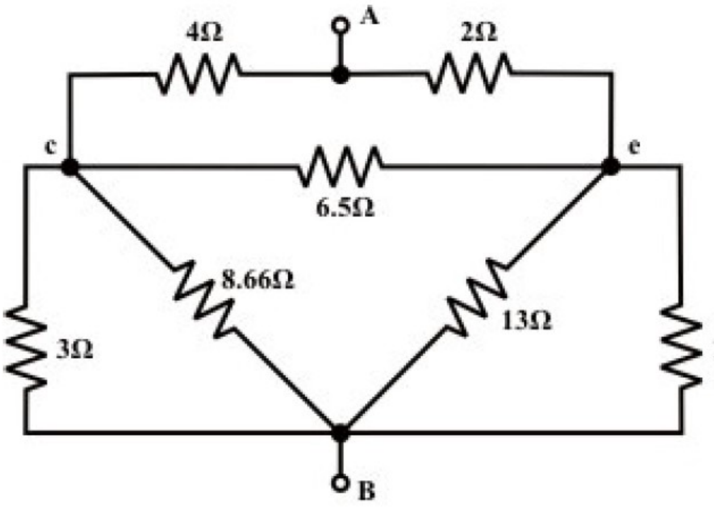
Determine the equivalent resistance between the terminals *A and B* of network shown in Figure below.



Solution:

A 'Δ' is substituted for the 'Y' between points c, d, and e as shown in fig. then unknown resistances value for Y *to* Δ transformation are computed below.

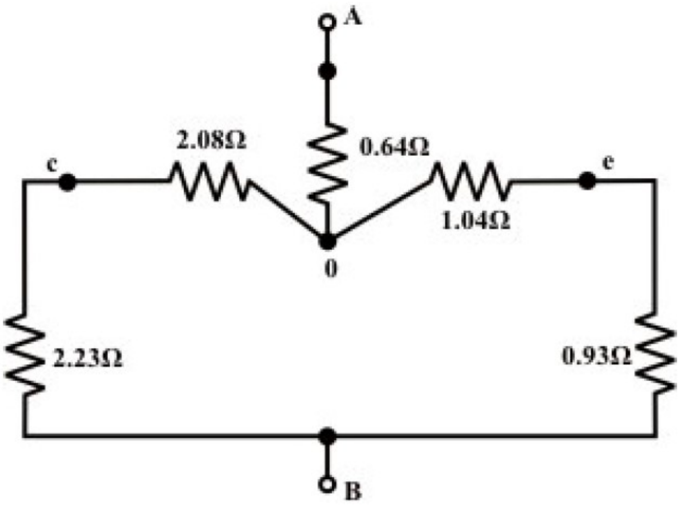
$$R_{cB} = 2+4+\frac{2\times 4}{3} = 8.66\Omega;\ R_{eB} = 3+4+\frac{4\times 3}{2} = 13\Omega;\ R_{ce} = 2+3+\frac{2\times 3}{4} = 6.5\Omega$$



Next we transform 'Δ'connected 3-terminal resistor to an equivalent 'Y' connected network between points 'A'; 'c' and 'e' and the corresponding Y connected resistances value are obtained using the following expression.Simplified circuit after conversion is shown in fig.

$$R_{Ao}=\frac{4\times 2}{4+2+6.5}=0.64\Omega;\ R_{co}=\frac{4\times 6.5}{4+2+6.5}=2.08\Omega;\ R_{eo}=\frac{6.5\times 2}{4+2+6.5}=1.04\Omega;$$

The circuit shown in fig can further be reduced by considering two pairs of parallel branches $3\|8.66$ *and* $13\|1$ and the corresponding simplified circuit is shown in fig.

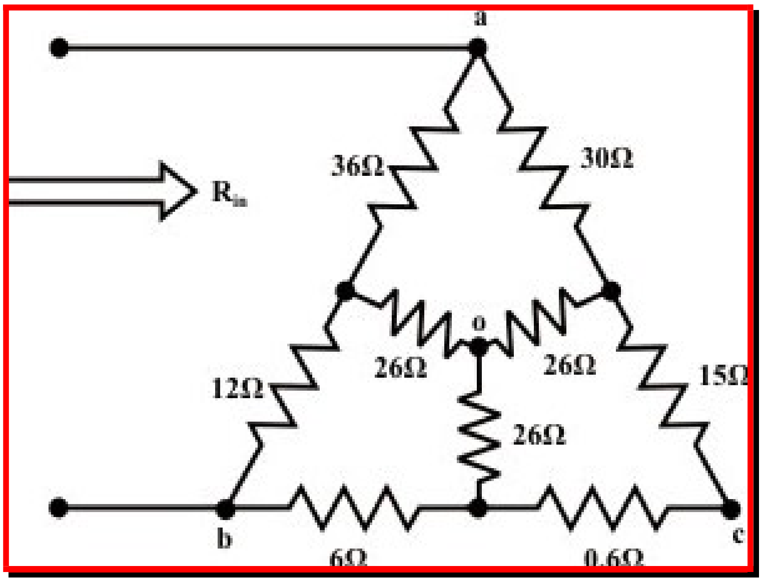


Now one can find the equivalent resistance between the terminals 'A' and 'B' as

$$R_{AB}=(2.23+2.08)\|(1.04+0.93)+0.64=2.21\Omega.$$

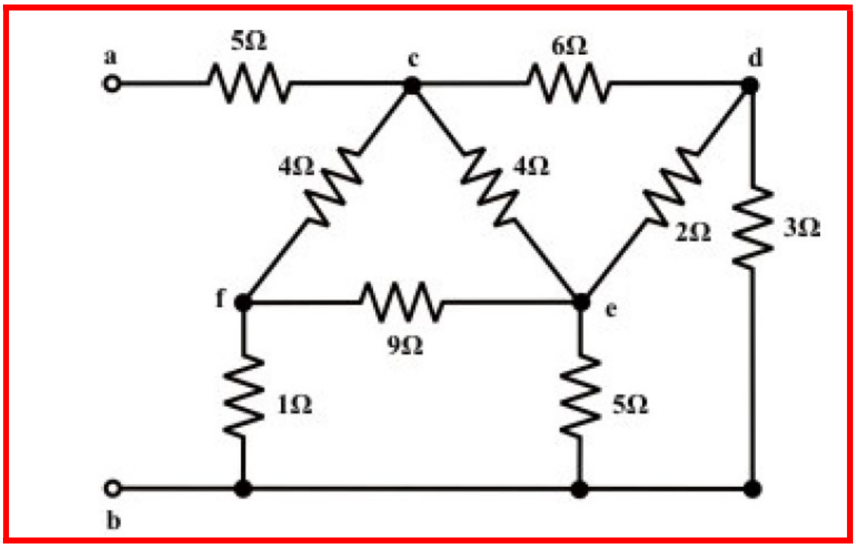
Problem 1

Find R_{in}



Problem 2

Find the equivalent resistance R_{eq} of the network at the terminals 'a' & 'b' using star-delta transformations.



Nodal Analysis

In electric circuit analysis, nodal analysis, node-voltage analysis is a method of determining the voltage (potential difference) between "nodes" (connection of two or more branches) in an electric circuit in terms of the branch currents.

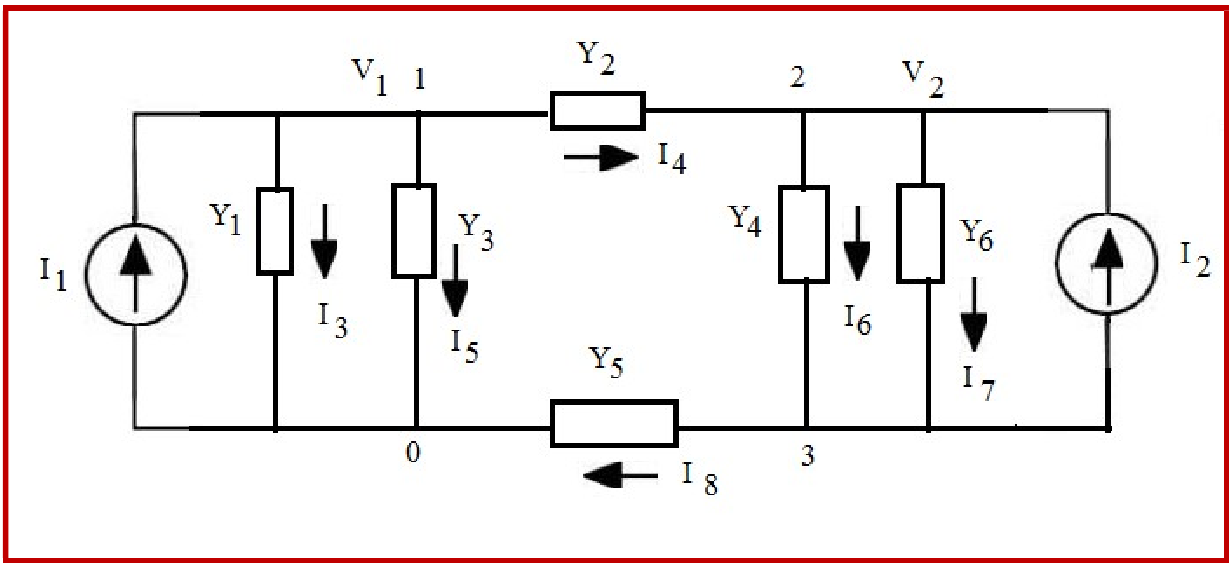
KCL is used to develop the method referred to as nodal analysis.

Steps to determine node voltage and node currents

1. **Convert all voltage sources (if any) to current sources through source transformation.**

2. **Identify the principal nodes and choose one of them as reference node and indicate it by '0'(preferably a common point or ground connection is chosen as reference). Number the remaining (n-1) nodes in sequence from 1 to (n-**1).
3. **Label the node voltages with respect to Ground for all the principal nodes except the reference node. Assign remaining (n-1) nodes with their branch voltages (w.r.t. reference node) for an "n" node network.**
4. **Assign branch currents to each branch.**
5. **Write current equation at each of (n-1) nodes (except reference node) using KCL. The sign of current is taken as positive if it is flowing towards the node.**
6. **Solve the simultaneous equations for the unknown node voltages.**

Example



According to KCL

$\sum I = 0$ at a node.

At node 1

$I_1 - I_3 - I_5 - I_4 = 0$

Or

$I_1 - V_1Y_1 - V_1Y_3 - (V_1 - V_2)\,Y_2 = 0$

Or

$(Y_1 + Y_2 + Y_3)V_1 + (-Y_2)V_2 + (0)V_3 = I_1$ ………………..(1)

Where Y_1, Y_2………Y_6 are admittances which are inverse of resistances i.e. $Y_1 = 1 / R_1$.

At node 2

$I_2 + I_4 - I_6 - I_7 = 0$

Or

$I_2 + (V_1 - V_2)Y_2 - (V_2 - V_3)Y_4 - (V_2 - V_3)Y_6 = 0$

Or

$(-Y_2)V_1 + (Y_2+Y_4+Y_6)V_2 + (-Y_4-Y_6)V_3 = I_2$ ………………….(2)

At node 3

$-I_2 + I_6 + I_7 - I_8 = 0$

Or

$-I_2 + (V_2-V_3)Y_4 + (V_2-V_3)Y_6 - V_3Y_5 = 0$

Or

$(0)V_1 + (-Y_4-Y_6)V_2 + (Y_4+Y_5+Y_6)V_3 = -I_2$ ………………..(3)

From relations (1), (2) and (3)

$$\begin{bmatrix} Y_1+Y_2+Y_3 & -Y_2 & 0 \\ -Y_2 & Y_2+Y_4+Y_6 & -Y_4-Y_6 \\ 0 & -Y_4-Y_6 & Y_4+Y_5+Y_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ -I_2 \end{bmatrix}$$

Or

[Y][V] = [I]

where [Y] is a square matrix, the admittance matrix.

Alternatively, $[V] = [Y]^{-1}[I]$

By using Cramer's rule,

$V_1 = D_1 / D$, $V_2 = D_2 / D$ **and** $V_3 = D_3 / D$

where

$$\mathbf{D} = \begin{vmatrix} Y_1 + Y_2 + Y_3 & -Y_2 & 0 \\ -Y_2 & Y_2 + Y_4 + Y_6 & -Y_4 - Y_6 \\ 0 & -Y_4 - Y_6 & Y_4 + Y_5 + Y_6 \end{vmatrix}$$

$$\mathbf{D}_1 = \begin{vmatrix} I_1 & -Y_2 & 0 \\ I_2 & Y_2 + Y_4 + Y_6 & -Y_4 - Y_6 \\ -I_2 & -Y_4 - Y_6 & Y_4 + Y_5 + Y_6 \end{vmatrix}$$

$$\mathbf{D}_2 = \begin{vmatrix} Y_1 + Y_2 + Y_3 & I_1 & 0 \\ -Y_2 & I_2 & -Y_4 - Y_6 \\ 0 & -I_2 & Y_4 + Y_5 + Y_6 \end{vmatrix}$$

$$\mathbf{D}_3 = \begin{vmatrix} Y_1 + Y_2 + Y_3 & -Y_2 & I_1 \\ -Y_2 & Y_2 + Y_4 + Y_6 & I_2 \\ 0 & -Y_4 - Y_6 & -I_2 \end{vmatrix}$$

The generalized representation with (n+1) nodes

[Y][V] = [I]

Where the square matrix Y is called the admittance matrix, having y_{ij} as elements i= 1, 2, ….. m and j=1,2,……m, and I is the column matrix of the input currents sources I = 1,2,…..m. The elements y_{ij} of the admittance matrix Y are

(i) Y_{ii}, the self admittance of the ith node

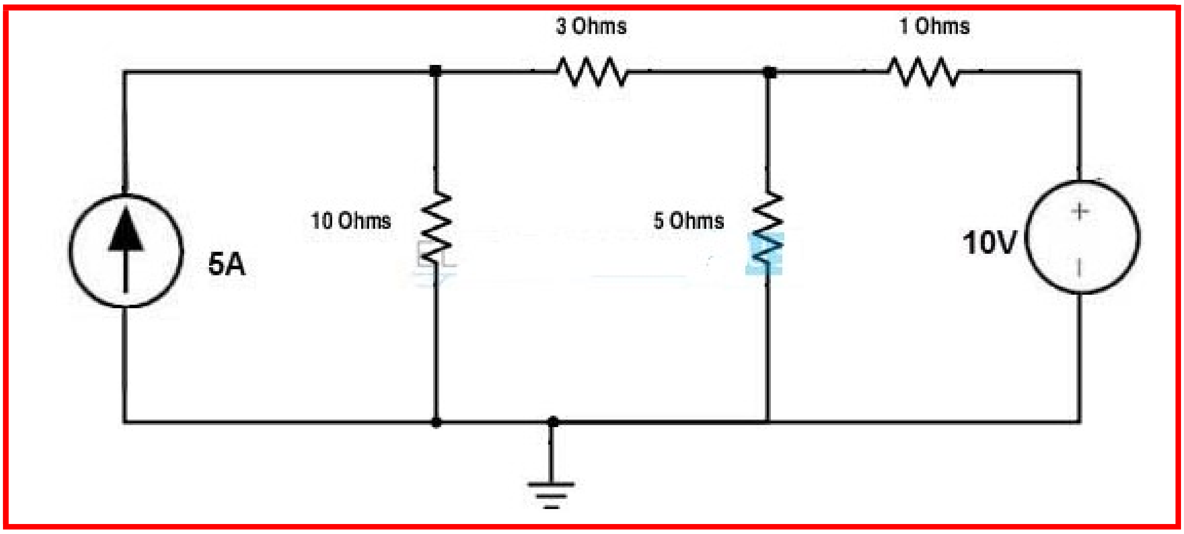
(ii) Y_{ij}, the mutual admittance between the ith and jth nodes.

V is the node voltage vector of order (mx1).

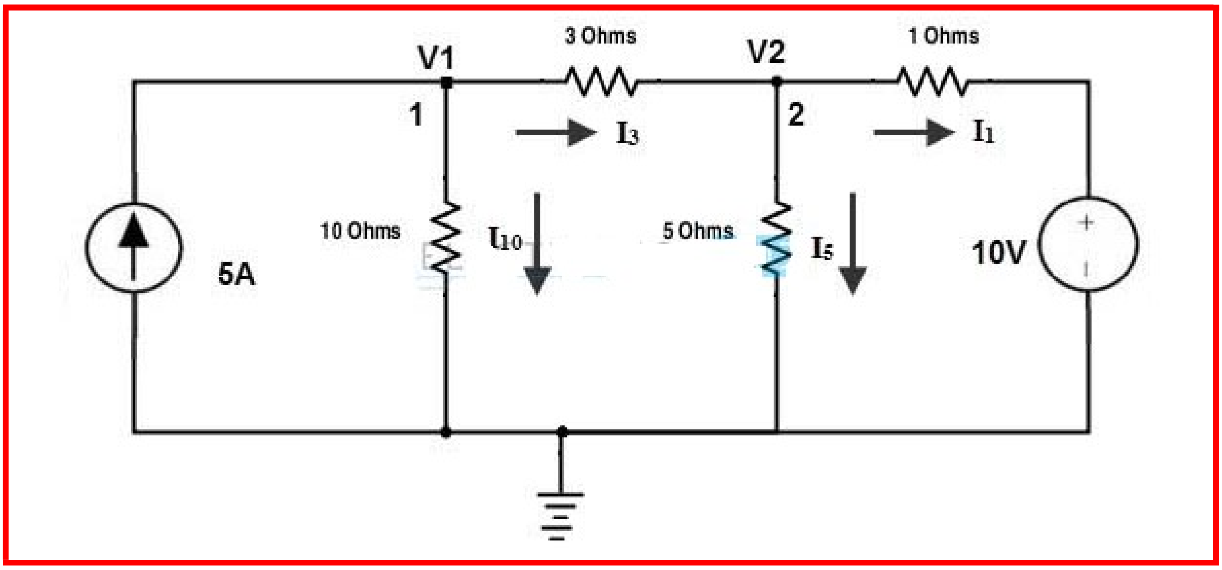
For networks having only passive elements and without any dependent source, the admittance matrix becomes symmetric, i.e. $y_{ij} = y_{ji}$.

Example

Determine the node voltages and currents in each branch using nodal analysis method in the given circuit.



Label the nodes present in the given circuit. By choosing the bottom node as reference node, there are two other nodes in the given circuit. So, these nodes are labeled as V1 and V2 as shown in below figure. And also, current directions in each branch are represented.



By applying KCL at node 1, we get

$$5 = I_3 + I_{10}$$

$$5 = (V1/10) + (V1 - V2)/3$$

$$13V_1 - 10V_2 = 150 \text{ ....(1)}$$

By applying KCL at node 2, we get

$$I_3 = I_5 + I_1$$

$$(V1 - V2)/3 = (V2/5) + (V2 - 10)/1$$

$$5V1 - 23V2 = -150 \text{ ....(2)}$$

By solving above two equations, we get

$$V1 = 19.85 \text{ Volts and } V2 = 10.9 \text{ Volts}$$

The currents in each branch is given as

$I10 = V1/10 = 19.85/10 = 1.985A$

$I3 = V1 – V2/3 = 19.85 – 10.9/3 = 2.98 \text{ A}$

$I5 = V2/5= 10.9/5$

$$= 2.18 \text{ A}$$

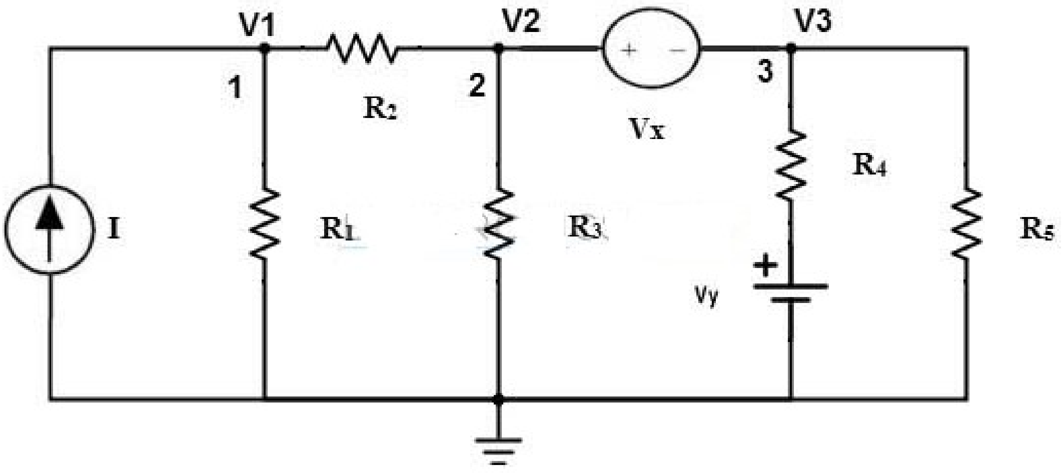
$$I1 = V2 – 10$$

$$= 10.9 – 10$$

$$= 0.9 \text{ A}$$

Concept of Super node

Sometimes it becomes difficult to apply nodal analysis when any voltage source is present in between two nodes in a circuit. One way to overcome this problem is by applying a super node technique. In super node technique, voltage source is connected between two adjacent nodes is shorted to reduce the two nodes to form a single super node.



In the above example a voltage source is connected between the 2 and 3 nodes. The calculations become more difficult if we analyze the circuit with voltage source. The analysis of this circuit becomes easier if we create a super node by shorting 2 and 3 nodes.

By applying Kirchhoff's current law at the node 1 we get,

$$I = (V1/R_1) + ((V1-V2)/R_2) \;\ldots\ldots(1)$$

The super node technique can be applied to the given circuit by shorting the 2 and 3 nodes and by applying KCL we get

$$((V1-V2)/R_2) + (V2/R_3) + ((V3-Vy)/R_4) + (V3/R_5) = 0$$

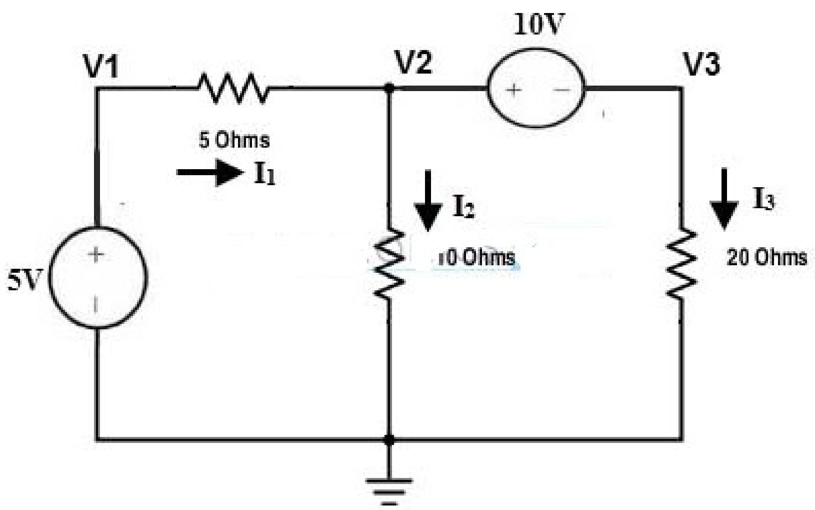
And also, voltage in the voltage source is given as

$$V_x = V2 - V3$$

From the above three equations, it can easily find out the three unknown voltages in the circuit.

Example of Super node

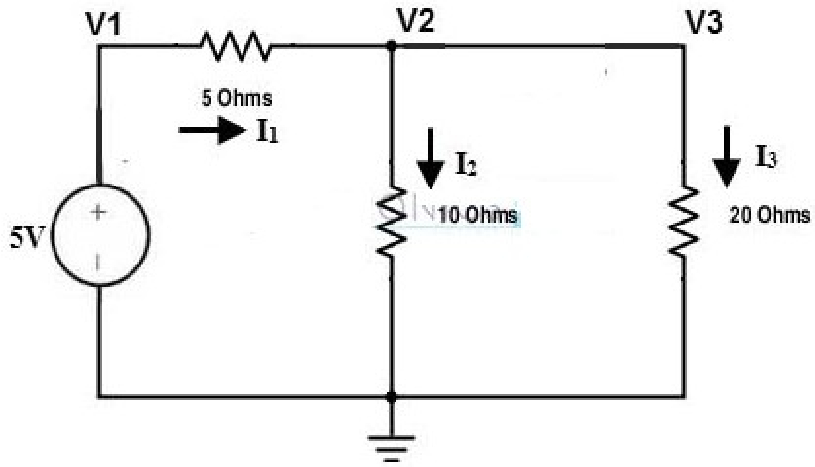
Consider the circuit below and find the three unknown node voltages V1, V2 and V3 by using super node technique.



At node 1, source is connected to the reference node and hence V1 becomes 5v

$$V1 = 5\ V$$

A super node is formed by enclosing the nodes 2 and 3. By applying KCL at this super node we get



$$I_1 = I_2 + I_3$$

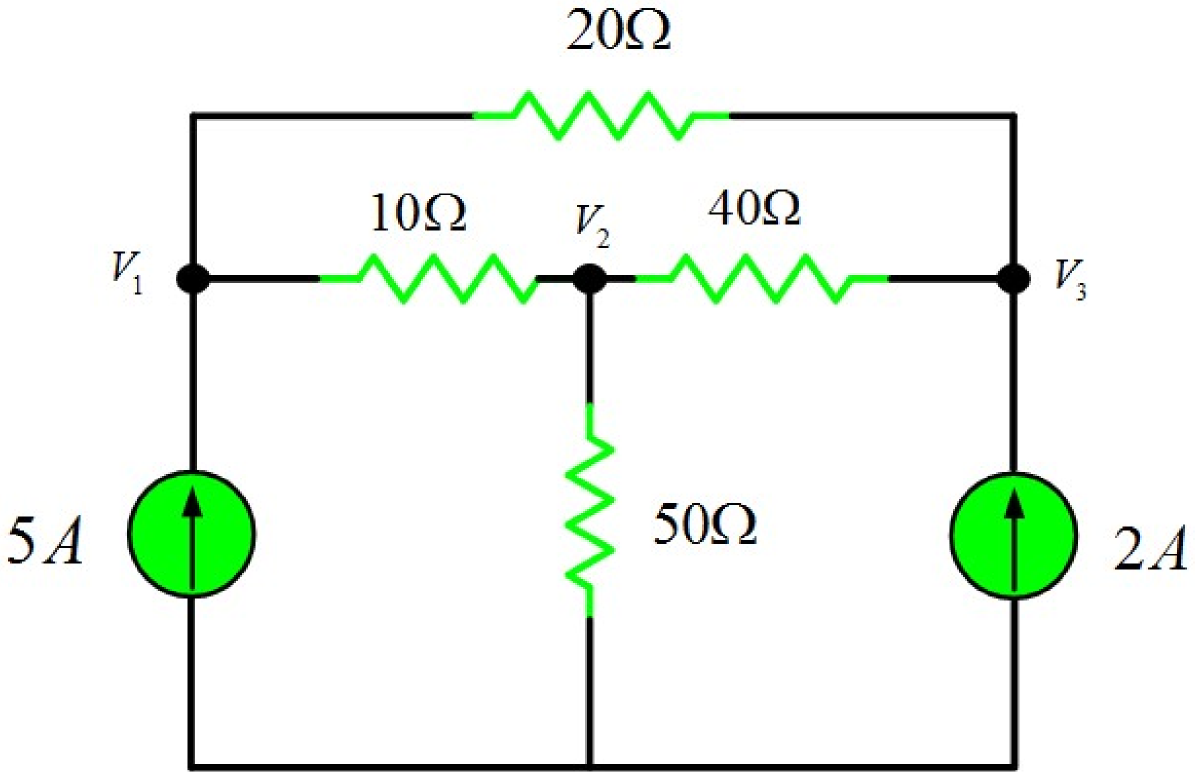
$$(V1 - V2)/5 = (V2 / 10) + (V3/20) \text{ .... } (1)$$

And also, KVL at super node gives,

$$V2 - V3 = 10\text{..............} \qquad (2)$$

By solving above equations, we get V2 = 4.29V and V3 = -5.71 V

Find node voltages for a very simple resistive circuit using Nodal Analysis.


20Ω
10Ω
V_2
40Ω
V_1
V_3
5A
50Ω
2A