

# Advanced Data Structures and Algorithms

## Quiz 2

For each of the below statements answer whether it is True or False. For each statement, you will get **2 mark** for the right answer and **-1** for the wrong answer. Write the answers on this sheet and return it back. You do not need to submit the rough sheets. Make sure to read through all the questions, as they are **NOT** arranged in ascending order of difficulty.

**Name:**

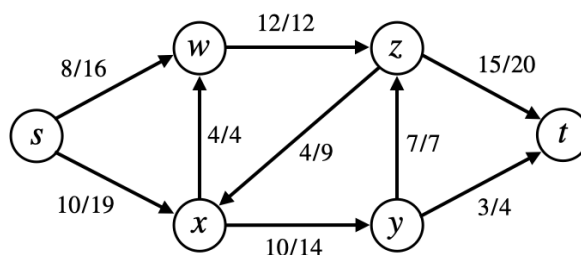
**Duration: 30 Minutes**

**Roll No:**

**Marks:**

1. Let  $G$  be a weighted directed graph with integer weights and  $s$  be a vertex in  $G$ . If for every vertex  $v$  in  $G$  there is a shortest path from  $s$  that does not contain a negative weight edge, then Dijkstra's algorithm will correctly find the weight of the shortest paths from  $s$  to every other vertex. **True.** Suppose for the sake of contradiction there exists a vertex, say  $u$ , in  $G$  whose distance is incorrectly computed by Dijkstra algorithm. Consider a shortest path from  $s$  to  $u$  that does not contain any negative weight edge. Let  $v$  be the first vertex on this path starting from  $s$  for which the distance is incorrectly computed. Then, the predecessor of  $v$ , say  $w$ , on this path must have the correct distance computed for it. But when  $w$  was dequeued, it must have set the  $\pi[v]$  to  $\delta(s, v)$ , which is a contradiction.

2. The number of edges in the residual network of the below flow network is 15.

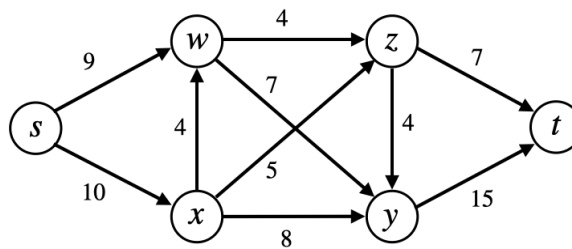


**True.**

3. For any two binary strings of length  $n$ , the top-down dynamic programming version of LCS that we have seen in the class will run in  $O(n)$  time.

**False.** Take two binary strings  $000\dots 0$  and  $111\dots 1$  of  $n$  length. Then, top-down DP on both the strings will make  $O(n^2)$  recursive calls and hence run for  $O(n^2)$  time.

4. The value of the max flow in the below flow network is 18.



**False.** A flow of 19 value is possible.

5. In any flow network if we decrease the capacity of any edge, then the value of the max flow will necessarily decrease.

**False.** If the minimum capacity does not get changed, then max flow will also not change.

6. In the activity selection problem, for any given set of activities an activity of least duration may not be a part of any largest-size set of mutually compatible activities.

**True.** Suppose we have three activities with the following start and finish time  $\{(1,10), (9,13), (12,20)\}$ . Then, the only largest-size set of mutually compatible activities is  $\{(1,10), (12,20)\}$ , which does not contain the least duration activity  $(9,13)$ .

7. The below algorithm to find the weight of the longest paths from  $s$  to every other vertex in any weighted directed graph  $G$  where every edge has a positive weight is incorrect.

**Longest-Path( $G, s$ ):**

1. Multiply weights of all the edges in  $G$  with  $-1$ .
2. Run Bellman-Ford on  $G$  with  $s$  as the source.
3. Return weights of the computed shortest paths from  $s$  to every other vertex of  $G$  in step 2 after multiplying them with  $-1$ .

**True.** Multiplying weights of all the edges with  $-1$  can create a cycle of negative weight, hence, Bellman-Ford algorithm will not be able to find the shortest paths correctly.

8. During the run of Ford-Fulkerson on any flow network  $G = (V, E)$ , if an edge in the residual network disappears, then it can reappear in any future residual network.

**True.** We discussed this during Edmond-Karp algorithm.

9. Longest common subsequence of a sequence  $X$  and its reverse is always the longest palindromic subsequence of  $X$ .

**False.** If  $X$  is  $abcabc$ , then one LCS of  $X$  and its reverse is  $bac$  which is not the longest palindromic subsequence. However, it is true that length of a longest common subsequence of a sequence  $X$  and its reverse is always the length of a longest palindromic subsequence of  $X$ . I wanted

to ask about length but made a mistake. Hence, everyone will get marks for this question irrespective of their answer.

**10.** Let  $G$  be any bipartite graph with partitions  $L$  and  $R$ , such that both  $L$  and  $R$  contain at least one vertex. Let  $G'$  be the corresponding flow network of  $G$ . During the execution of Ford-Fulkerson on  $G'$ , the length of an augmenting path can always be at most 3.

**False.** An augmenting path can go from  $L$  to  $R$  and  $R$  to  $L$  multiple times making its length more than 3.

# Advanced Data Structures and Algorithms

## Quiz 2

For each of the below statements answer whether it is True or False. For each statement, you will get **2 mark** for the right answer and **-1** for the wrong answer. Write the answers on this sheet and return it back. You do not need to submit the rough sheets. Make sure to read through all the questions, as they are **NOT** arranged in ascending order of difficulty.

**Name:**

**Duration: 30 Minutes**

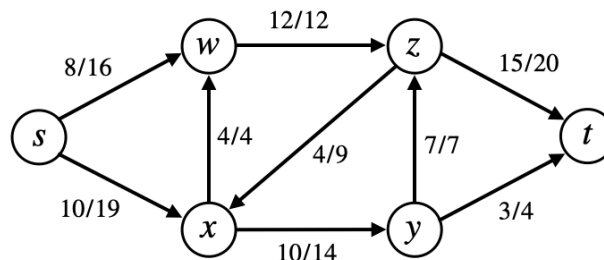
**Roll No:**

**Marks:**

1. Let  $G$  be any bipartite graph with partitions  $L$  and  $R$ , such that both  $L$  and  $R$  contain at least one vertex. Let  $G'$  be the corresponding flow network of  $G$ . During the execution of Ford-Fulkerson on  $G'$ , the length of an augmenting path can always be at most 3.

**False.**

2. The number of edges in the residual network of the below flow network is 14.

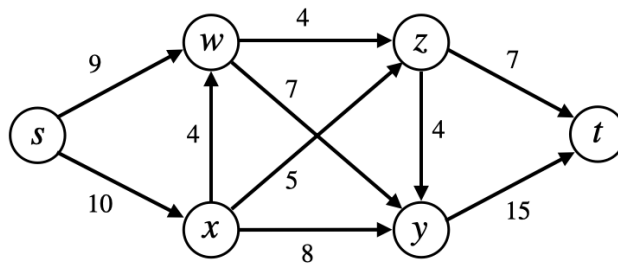


**False.**

3. During the run of Ford-Fulkerson on any flow network  $G = (V, E)$ , if an edge in the residual network disappears, then it can never reappear in any future residual network.

**False.**

4. The value of the max flow in the below flow network is 17.



**False.**

5. In the activity selection problem, for any given set of activities an activity of least duration may not be a part of any largest-size set of mutually compatible activities.

**True.**

6. Longest common subsequence of a sequence  $X$  and its reverse is always the longest palindromic subsequence of  $X$ .

**False.**

7. The below algorithms correctly finds the weight of the longest paths from  $s$  to every other vertex in any weighted directed graph  $G$  where every edge has a positive weight.

**Longest-Path( $G, s$ ):**

1. Multiply weights of all the edges in  $G$  with  $-1$ .
2. Run Bellman-Ford on  $G$  with  $s$  as the source.
3. Return weights of the computed shortest paths from  $s$  to every other vertex of  $G$  in step 2 after multiplying them with  $-1$ .

**False.**

8. For any two binary strings of length  $n$ , the top-down dynamic programming version of LCS that we have seen in the class will not run in  $O(n)$  time.

**False.**

9. In any flow network if we decrease the capacity of any edge, then the value of the max flow will necessarily decrease.

**False.**

10. Let  $G$  be a weighted directed graph with integer weights and  $s$  be a vertex in  $G$ . If for every vertex  $v$  in  $G$  there is a shortest path from  $s$  that does not contain a negative weight edge, then Dijkstra's algorithm will correctly find the weight of the shortest paths from  $s$  to every other vertex.

**True.**