Statistics (MAL7060)

Assignment - 1

① (a) Mean, $\mu = \dfrac{\Sigma x_i}{n} = \dfrac{226.52}{50}$

$= 4.53$

Variance, $\sigma^2 = \dfrac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$

$= \dfrac{\sum_{i=1}^{50} (x_i - 4.53)^2}{50}$

$= \dfrac{11.204}{50} = 0.2241$

$\therefore$ Standard dev, $\sigma = \sqrt{0.2241}$

$= 0.4736$

(b) Median = 50th percentile = Q2.

$n = 50$, so median $= \dfrac{25^{th} + 26^{th}}{2}$

$= \dfrac{4.50 + 4.51}{2}$

$= 4.505$.

$Q1 = 25^{th}$ percentile

$= \dfrac{25}{100} \times (n+1)^{th}$

$= \dfrac{1}{4} \times 51$

$= 12.75^{th} = 12^{th} + 0.75\,(13^{th} - 12^{th})$

$= \cancel{12^{th} + \frac{75}{100} \times 13^{th}} = 4.28 + 0.75(4.30 - 4.28)$

$= \cancel{4.28 + \frac{3}{4} \times 4.30} = 4.28 + 0.75 \times 0.02$

$= \cancel{4.28 + 3.225} = 4.285$

$Q3 = 75^{th}$ percentile

$= \frac{75}{100} \times (n+1)$

$= \frac{3}{4} \times 51$

$= 37.75^{th}$

$= 37^{th} + 0.75(38^{th} - 37^{th})$

$= 4.70 + 0.75(4.70 - 4.70) = 4.70$

(c) 90^{th} percentile

$= 0.9 \times 51$

$= 45.9^{th}$

$= 45^{th} + 0.9(46^{th} - 45^{th})$

$= 4.80 + 0.9(5.07 - 4.80)$

$= 4.80 + 0.9 \times 0.27 = 5.043$

(d) $\bar{x} = 4.5074, \quad s = 0.3644.$

$\therefore \quad \bar{x} \pm s = (4.1430, 4.8718)$

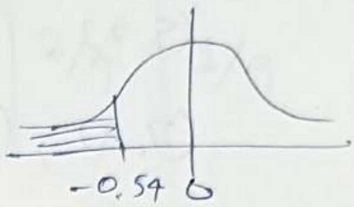
$\bar{x} \pm 2s = (3.7786, 5.2362)$

$\bar{x} \pm 3s = (3.4142, 5.6006)$

2.(a) Here, $\mu = 7.2$, $\sigma = 1.3$

$\therefore \; P(X < 6.5)$

$= P\left(Z < \frac{6.5 - 7.2}{1.3}\right)$



$= P(Z < -0.54)$

$= 0.5 - P(-0.54 < Z < 0)$

$= 0.5 - 0.2054$ ~~≈ 0.03 ≈ 30%~~

$= 0.2946 \simeq 29.46\%$

~~(b)~~ $\cancel{Z = \frac{X - \mu}{\sigma}}$

$\cancel{\Rightarrow X = \mu + Z \times \sigma}$

$\cancel{= 7.2 +}$

(b) 70th percentile = 0.7 approx on Z-table

≈ 0.52

$\therefore \; X = \mu + Z \times \sigma$

$= 7.2 + 0.52 \times 1.3$

$= 7.876$

$\therefore$ 70th percentile of distribution for hrs of sleep is 7.876 hrs approx.

③ (a) Set = {3, 5, 7}, sample size = 2.

∴ Possible samples :

(3, 3), (3, 5), (3, 7)

(5, 3), (5, 5), (5, 7)

(7, 3), (7, 5), (7, 7)

	$\bar{x}$	s^2
(3,3)	3	0
(3,5)	4	1
(3,7)	5	4
(5,3)	4	1
(5,5)	5	0
(5,7)	6	1
(7,3)	5	4
(7,5)	6	1
(7,7)	7	0

(b) For $\bar{x}$ distribution:

$\bar{x}$	3	4	5	6	7
$P(\bar{x})$	1/9	2/9	3/9	2/9	1/9

For s^2 distribution:

S^2	0	1	4
$P(s^2)$	3/9	4/9	2/9

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(4) Set = $\{0, 2, 4, 6\}$.

$\therefore$ Possible samples and their rang

	R
(0,0)	0
(0,2)	2
(0,4)	4
(0,6)	6
(2,0)	2
(2,2)	0
(2,4)	2
(2,6)	4
(4,0)	4
(4,2)	2
(4,4)	0
(4,6)	2
(6,0)	6
(6,2)	4
(6,4)	2
(6,6)	0

$\therefore$ Distribution of R:

R	0	2	4	6
P(R)	4/16	6/16	4/16	2/16

(5) Load on airplane, $X \sim N(1000, 14400)$.

Critical load, $Y \sim N(1260, 2500)$.

$\therefore \quad Z = Y - X$ (critical load − encountered load)

$\Rightarrow Z \sim N(\mu_z, \sigma_z^2)$

$\therefore \quad \mu_z = \mu_y - \mu_x$

$= 1260 - 1000 = 260.$

$\sigma_z^2 = \sigma_y^2 + \sigma_x^2 = 2500 + 14400$

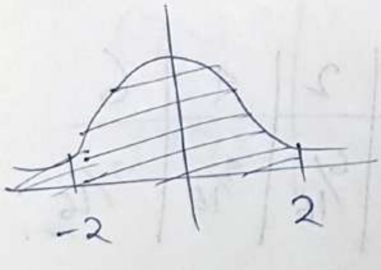
$= 16900.$

$\therefore \quad Z \sim N(260, 16900).$

$\therefore$ For $P(X < Y)$, $P(Z > 0)$.

$\therefore \quad P(Z>0) = P\left(\frac{Z-\mu_z}{\sigma_z} > \frac{0-260}{\sqrt{16900}}\right)$

$= P\left(\frac{Z-\mu_z}{\sigma_z} > \frac{-260}{130}\right)$



$= P\left(\frac{Z-\mu_z}{\sigma_z} > -2\right)$

$= P(Z' > -2) = P(Z' < 2).$

where $Z' = \frac{Z-\mu_z}{\sigma_z}$ & $Z \sim N(0,1)$.

$\therefore \quad P(Z' < 2) \approx 0.9772$

$\Rightarrow P(X < Y) \approx 97.72\%.$

⑥ $P(X > 2160) = 92.5\% = 0.925$

$\cancel{P(X > 17040) = 3.92\% = 0.039}$

$$\Rightarrow P\left(\frac{X-\mu}{\sigma} > \frac{2160-\mu}{\sigma}\right)$$

$$\Rightarrow P\left(Z > \frac{2160-\mu}{\sigma}\right).$$

$P(X > 17040) = 3.92\% = 0.0392$

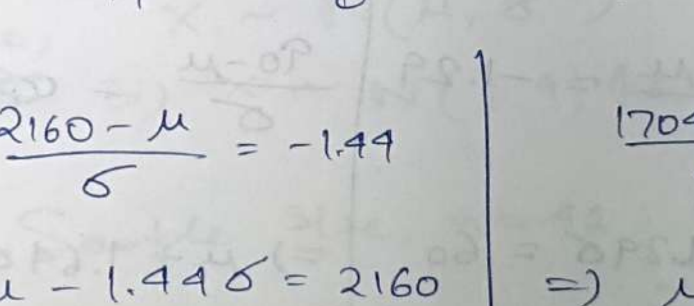
$$\Rightarrow P\left(\frac{X-\mu}{\sigma} > \frac{17040-\mu}{\sigma}\right)$$

$$\Rightarrow P\left(Z > \frac{17040-\mu}{\sigma}\right).$$

$$\therefore P\left(Z < \underbrace{\frac{2160-\mu}{\sigma}}_{}\right)^{x_1} = 1 - 0.925 = 0.075 \quad (-1.44)$$

$$P\left(Z < \frac{17040-\mu}{\sigma}\right) = 1 - 0.0392 = 0.9608. \quad (1.76)$$

x_2



$$\therefore \frac{2160-\mu}{\sigma} = -1.44 \qquad \Big| \qquad \frac{17040-\mu}{\sigma} = 1.76$$

$\Rightarrow \mu - 1.44\sigma = 2160$ — (I)

$\Rightarrow \mu + 1.76\sigma = 17040.$ — (II)

(11) − (1): $3.2\sigma = 17040 - 2160$

$\Rightarrow 3.2\sigma = \cancel{18080}\ 14880.$

$\Rightarrow \sigma = \dfrac{14880}{3.2} = 4650$

$\therefore \mu = 2160 + 1.44 \times 4650$

$= 8856.$

$\therefore$ Mean = 8856 hrs,

Std dev = 4650 hrs.

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(7) Here, $X \sim N(\mu, \sigma^2)$.

$P(X < 60) = 10\% = 0.1$

$\Rightarrow P\left(Z < \dfrac{60-\mu}{\sigma}\right) = 0.1 = \cancel{0.5398}$ (−1.29)

$P(X > 90) = 5\% = 0.05$

$\Rightarrow P(X < 90) = 0.95$

$\Rightarrow P\left(Z < \dfrac{90-\mu}{\sigma}\right) = 0.95 = \cancel{0.9495}$ (1.64)

$\therefore \dfrac{60-\mu}{\sigma} = -1.29$	$\dfrac{90-\mu}{\sigma} = 1.64$
$\Rightarrow \mu - 1.29\sigma = 60$ — (1)	$\Rightarrow \mu + 1.64\sigma = 90.$ — (11)

$\therefore$ (ii) − (i) : $2.93\sigma = 30$

$\Rightarrow \sigma = \dfrac{30}{2.93} = 10.23.$

$\therefore$ $\mu = 60 + 1.29 \times 10.23$

$= 73.19$

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(8) (a) $\mu = 8$ yrs, $\sigma = 2$ yrs.

$P(X < 10)$

$= P\left(Z < \dfrac{10-8}{2}\right)$

$= P(Z < 1) = 0.84134 \approx 0.84.$

$\therefore$ $P(Z > 1) = 0.16$

$\Rightarrow P(X > 10) = 16\%.$

(b) $P(X < 4)$

$= P\left(Z < \dfrac{4-8}{2}\right)$

$= P(Z < -2) = 0.2275 = 22.75\%.$

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(10) Here, $X \sim N(\mu, \sigma^2)$

$\Rightarrow X \sim N(1.9, (1.6)^2).$

(a) Sample size, $n = 41$. (large).

sample mean $= \overline{X}$.

$\therefore \quad P(1.7 < \bar{x} < 2.1)$

$= P\left(\frac{1.7-1.9}{1.6/\sqrt{41}} < \frac{\bar{x}-1.9}{1.6/\sqrt{41}} < \frac{2.1-1.9}{1.6/\sqrt{41}}\right)$

$= P\left(-\frac{\sqrt{41}}{8} < Z < \frac{\sqrt{41}}{8}\right)$

$= P(-0.8 < Z < 0.8)$

$= 0.78814 - 0.21186$

$= 0.57628 = 57.628\%$

(b) Here, Sample size, $n = 100$

$\therefore \quad P(1.7 < \bar{x} < 2.1)$

$= P\left(\frac{1.7-1.9}{1.6/10} < \frac{\bar{x}-1.9}{1.6/10} < \frac{1.7-1.9}{1.6/10}\right)$

$= P(-1.25 < Z < 1.25)$

$= 0.89435 - 0.10565$

$= 0.78870 = 78.87\%$

(9) $X_1 \sim N(0,1)$; $X_2 \sim N(1,1)$; $X_3 \sim N(2,1)$.

$\therefore \quad \mu_1 = 0, \quad \sigma_1^2 = 1.$

$\mu_2 = 1, \quad \sigma_2^2 = 1.$

$\mu_3 = 2, \quad \sigma_3^2 = 1.$

Let Y be the random variable such that

$Y = X_1 + X_2 + X_3$ and having

parameters μ and σ^2.

$\therefore \quad Y \sim N(\mu, \sigma^2)$

$\mu = \mu_1 + \mu_2 + \mu_3$

$= 0 + 1 + 2 = 3.$

$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$

$= 1 + 1 + 1 = 3.$

$\therefore \quad Y \sim N(3,3).$

$\therefore \quad P(X_1 + X_2 + X_3 > 1)$

$= P(Y > 1)$

$= P\left(\frac{Y-3}{\sqrt{3}} > \frac{1-3}{\sqrt{3}}\right)$

$= P(Z > \frac{-2}{\sqrt{3}})$

$= P(Z > -1.154)$

$= 1 - P(Z \leq -1.154)$

$= 1 - 0.125 = 0.875$

$= 87.5\%.$