1.

Given that

$$X \sim N(\mu, 4)$$

$$\Rightarrow \quad Z = \frac{X-\mu}{2} \sim N(0,1)$$

Now

$$P(X<1) = 0.7$$

$$\Rightarrow \quad P\left(\frac{X-\mu}{2} < \frac{1-\mu}{2}\right) = 0.7$$

$$\Rightarrow \quad P\left(Z < \frac{1-\mu}{2}\right) = 0.7$$

$$\Rightarrow \quad \frac{1-\mu}{2} = 0.525 \quad (\text{approx.})$$

$$\Rightarrow \quad \mu = 1 - 1.01 = -0.01$$

(approx.).

2.

Given that

$$n = 49$$

$$\sigma = 49$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

let μ be the population mean.

$$P(\mu-2 < \bar{x} < \mu+2)$$

$$= P(-2 < \bar{x}-\mu < 2)$$

$$= P\left(-\frac{2}{\sigma/\sqrt{n}} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{2}{\sigma/\sqrt{n}}\right)$$

$$= P\left(-\frac{14}{49} < Z < \frac{14}{49}\right)$$

$$= 2P\left(Z < \frac{2}{7}\right)$$

$$= 2.P(Z < 0.2857)$$

$$= 2.\ 0.1122 = 0.2244$$

(approx.)

3. Given that

$$X_i \sim N(0,16)$$

$$Y_i \sim N(1,9) \quad \forall i=1,\dots,25$$

$$\Rightarrow \bar{X} = \frac{\sum X_i}{n} \sim N\left(0, \frac{16}{25}\right).$$

$$\bar{Y} = \frac{\sum Y_i}{n} \sim N\left(1, \frac{9}{25}\right).$$

$$\Rightarrow \bar{Y}-\bar{X} \sim N(1,1)$$

Consider.

$$P(\bar{Y} > \bar{X}) = P(\bar{Y}-\bar{X} > 0)$$

$$= P(\bar{Y}-\bar{X}-1 > -1).$$

$$= P(Z > -1) = 0.6587.$$

Q.4

We are given $\mu = 500$, $\sigma = 80$, $n = 100$

We have to find the interval, that covers the middle 95% of the distribution of sample mean, or we can say that,

$$P\left(\mu - 1.96 \cdot \frac{\sigma}{\sqrt{n}} < \bar{x} < \mu + 1.96 \cdot \frac{\sigma}{\sqrt{n}}\right) = 0.95$$

So, the interval will be,

$$\left(\mu - 1.96 \frac{\sigma}{\sqrt{n}}, \mu + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

$$= \left(500 - 1.96 \frac{80}{10}, 500 + 1.96 \frac{80}{10}\right)$$

$$= \left(500 - (1.96 \times 8), 500 + (1.96 \times 8)\right)$$

$$= (500 - 15.68, 500 + 15.68)$$

$$= (484.32, 515.68)$$

Q.5

We are given, $X \sim N(8, 9)$
$Y \sim N(12, 16)$

say, $\mu_x = 8$, $\mu_y = 12$
$\sigma_x^2 = 9$ $\sigma_y^2 = 16$

Since, X & Y are ~~ide~~ independent random variables that follow Normal distribution,

so, X+Y and X−Y also follows Normal distⁿ.

$\mu_{x+y} = E(X+Y) = E(X) + E(Y) = 8+12 = 20$

$\mu_{x-y} = E(X-Y) = E(X) - E(Y) = 8-12 = -4$

$\sigma_{x+y}^2 = Var(X+Y) = \sigma_x^2 + \sigma_y^2 = 9+16 = 25$

$\sigma_{x-y}^2 = Var(X-Y) = \sigma_x^2 + \sigma_y^2 = 9+16 = 25$

so, $(X+Y) \sim N(20, 25)$
$(X-Y) \sim N(-4, 25)$

we are given $P(X+Y \geq 2a) = P(X-Y \leq a)$

$$\Rightarrow P\left(\frac{(X+Y)-\mu_{X+Y}}{\sqrt{\sigma^2_{X+Y}}} \geq \frac{2a-20}{\sqrt{25}}\right) = P\left(\frac{(X-Y)-\mu_{X-Y}}{\sqrt{\sigma^2_{X-Y}}} \leq \frac{a+4}{\sqrt{25}}\right)$$

$$\Rightarrow P\left(Z \geq \frac{2a-20}{5}\right) = P\left(Z \leq \frac{a+4}{5}\right)$$

$$\Rightarrow P\left(Z \geq \frac{2a-20}{5}\right) = P\left(Z \geq -\frac{a+4}{5}\right)$$

$$\frac{2a-20}{5} = -\frac{a+4}{5}$$

$$\Rightarrow 2a-20 = -a-4$$

$$\Rightarrow 3a = 16$$

$$\Rightarrow a = \frac{16}{3}$$

Q.6

$X_1, \ldots, X_{100}$ are iid. with mean $\frac{1}{5}$, Var $-\frac{1}{9}$

so $\mu_{X_i} = \frac{1}{5}$, $Var(X_i) = \frac{1}{9}$

According to CLT, for a large number of iid random variables the sum can be approximated by a Normal distn.

Since, $n = 100$ (>30)

so, $\sum_{i=1}^{100} X_i$ can be approximated by normal distn. with

mean $(\mu) = 100 \times \frac{1}{5} = 20$

Var $(\sigma^2) = 100 \times \frac{1}{9} = \frac{100}{9}$

Now, $P(\Sigma X_i < 30)$

$$= P\left(\frac{\Sigma X_i - \mu}{\sqrt{\sigma^2}} < \frac{30-20}{\sqrt{\frac{100}{9}}}\right)$$

$$= P\left(Z < \frac{10}{10/3}\right) = P(Z < 3) = \underline{0.99}$$

Q.7 Given that, $\sigma = 4$

The Confidence Level is 95%.

So $1-\alpha = 0.95$

$\alpha = 1-0.95 = 0.05$

$\alpha/2 = 0.025$

Now $Z_{\alpha/2} = 1.96$

Maximum error $(d) = 0.8$

Thus, ~~we~~ we have,

$$Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = d$$

$$\Rightarrow \sqrt{n} = Z_{\alpha/2} \frac{\sigma}{d}$$

$$\Rightarrow \sqrt{n} = 1.96 \times \frac{\cancel{4}\,5}{0\cancel{8}} = 9.80$$

$$\Rightarrow n = (9.8)^2 = 96.04$$

$$n = 96.04$$