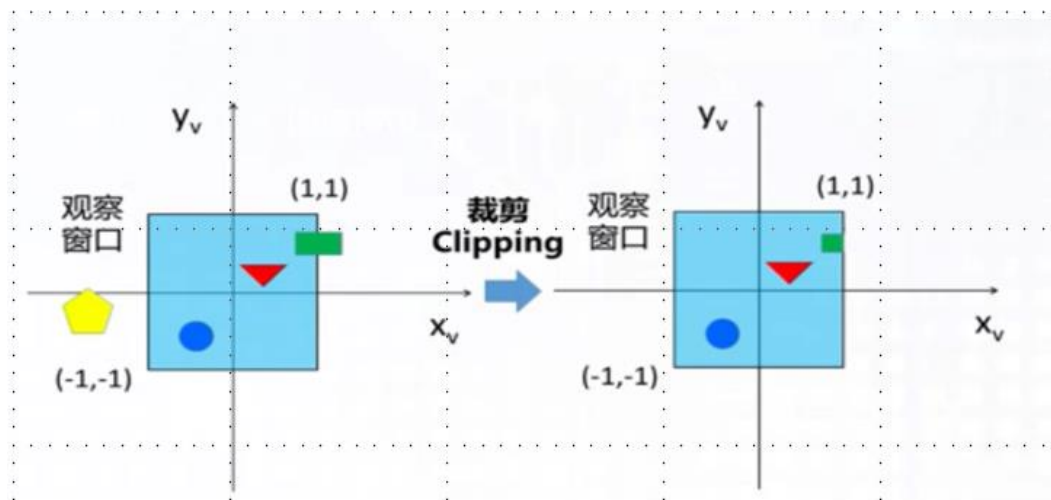
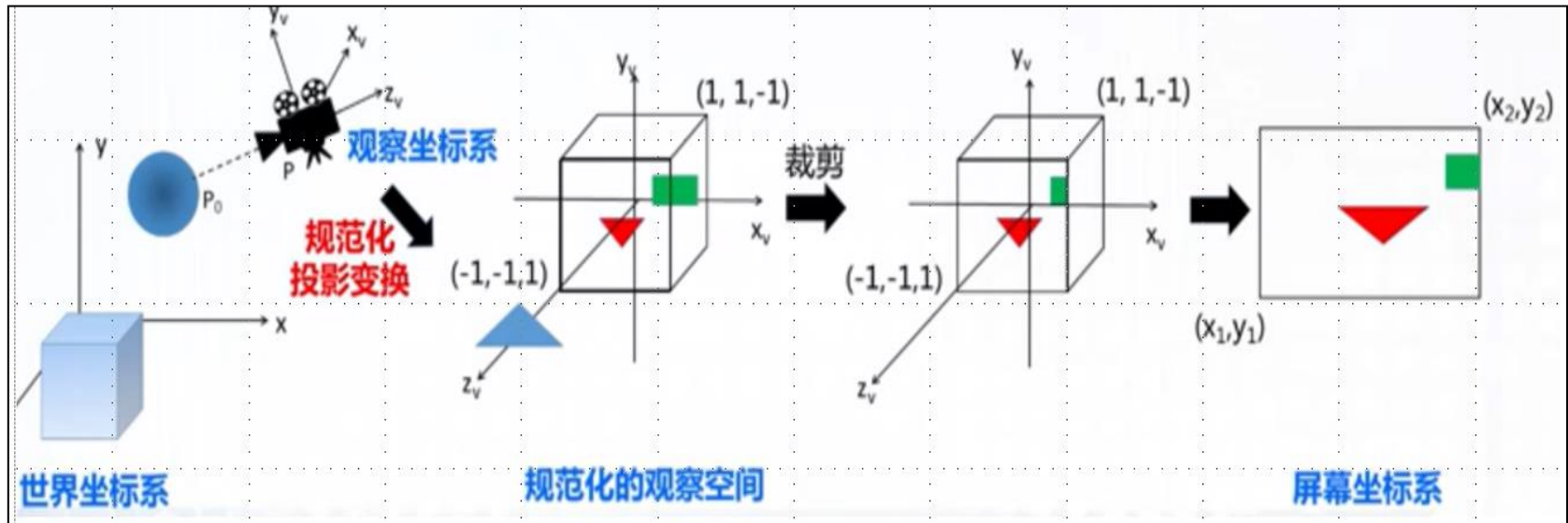




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# Recap

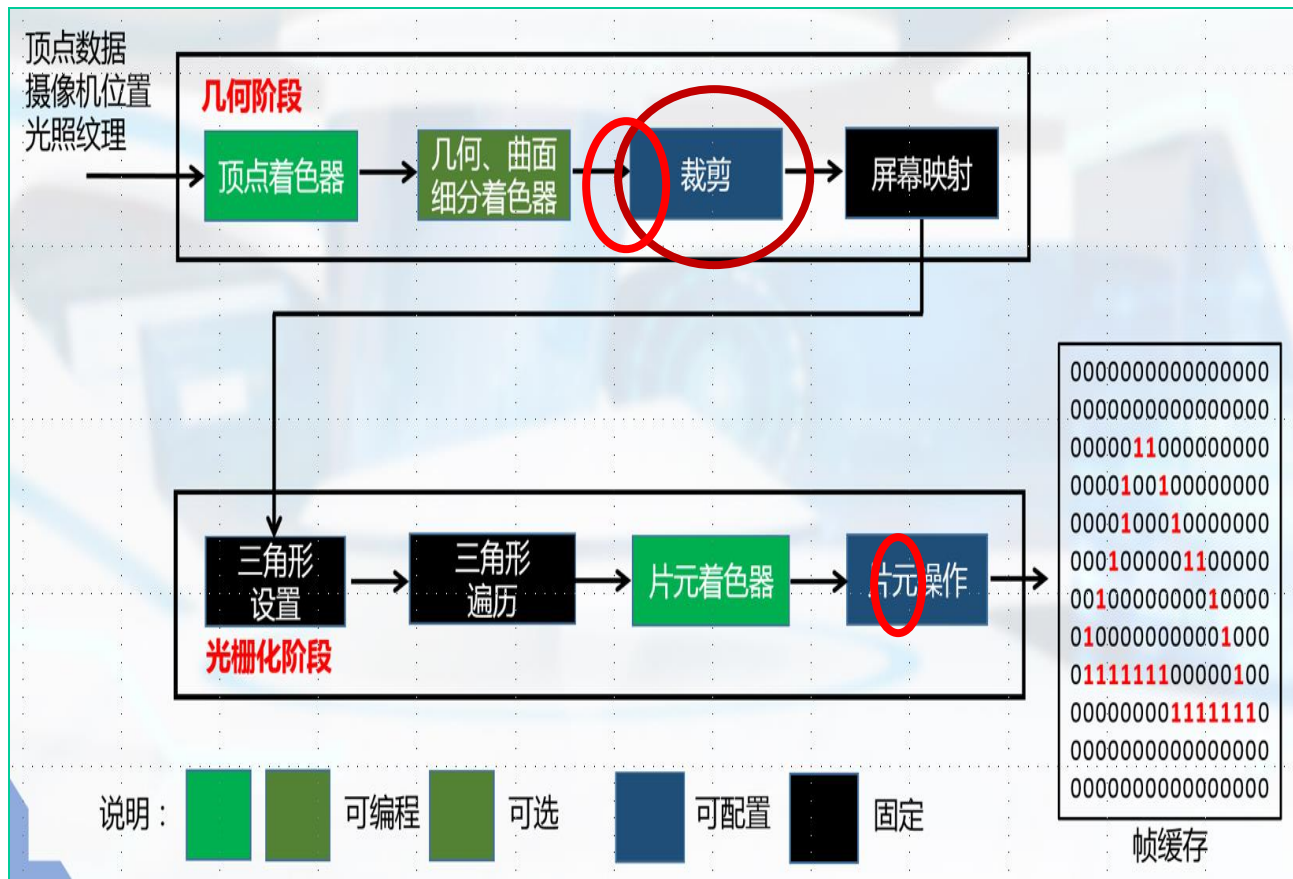




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# Recap(cont.)

## ➤ 目前学习位置: 裁剪和消隐



# Outline

- **Clipping Algorithm**

- Clipping Line Segments
  - Cohen-Sutherland Line Clipping 编码裁剪算法
  - Liang-Barsky Line Clipping 梁永栋裁剪算法
- Clipping Polygons
  - Sutherland-Hodgeman Polygon Clipping 逐边裁剪算法

- **Hidden Surface Removal Algorithm**

- Object Space Approach
  - Back-face culling 后向面剔除 (着色前就消隐掉)
- Image Space Approach
  - Depth-buffer 深度缓存算法 (光栅化渲染中的消隐算法)
  - Ray-casting 光线投射 (光线跟踪渲染中的消隐算法)

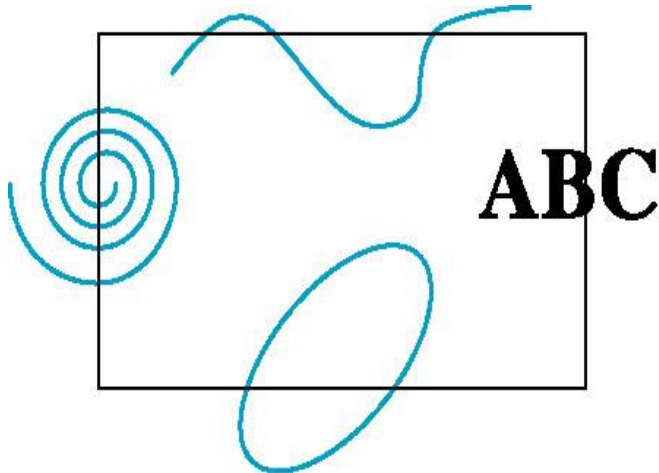
# Clipping Algorithm

## ➤ Clipper 裁剪器:

- 2D against **clipping window**, 3D against **clipping volume**
- 一般2D裁剪边选择主轴, 3D裁剪面选择主面

## ➤ Clipped Primitives 被裁剪图元:

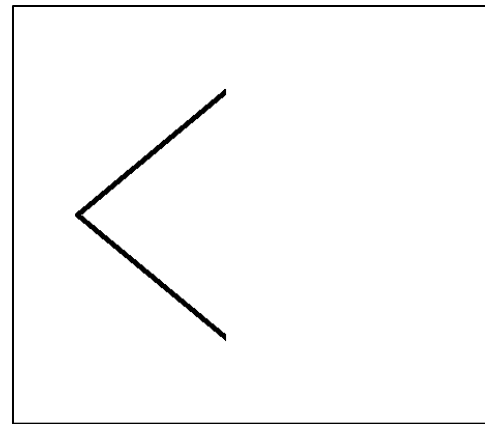
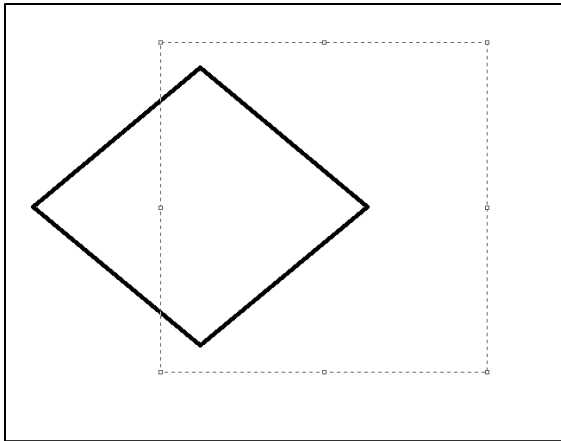
- Easy for **point**, **line segments**, **polygons**
- Hard for **curves** and **text**: Convert to lines and polygons first



# Clipping Algorithm (cont.)

## ➤ 裁剪算法使用广泛

- 裁剪算法在渲染管线中使用
- 裁剪算法可在交互软件中使用, 如“绘图”软件中



# Outline

- **Clipping Algorithm**

- **Clipping Line Segments**

- Cohen-Sutherland Line Clipping 编码裁剪算法
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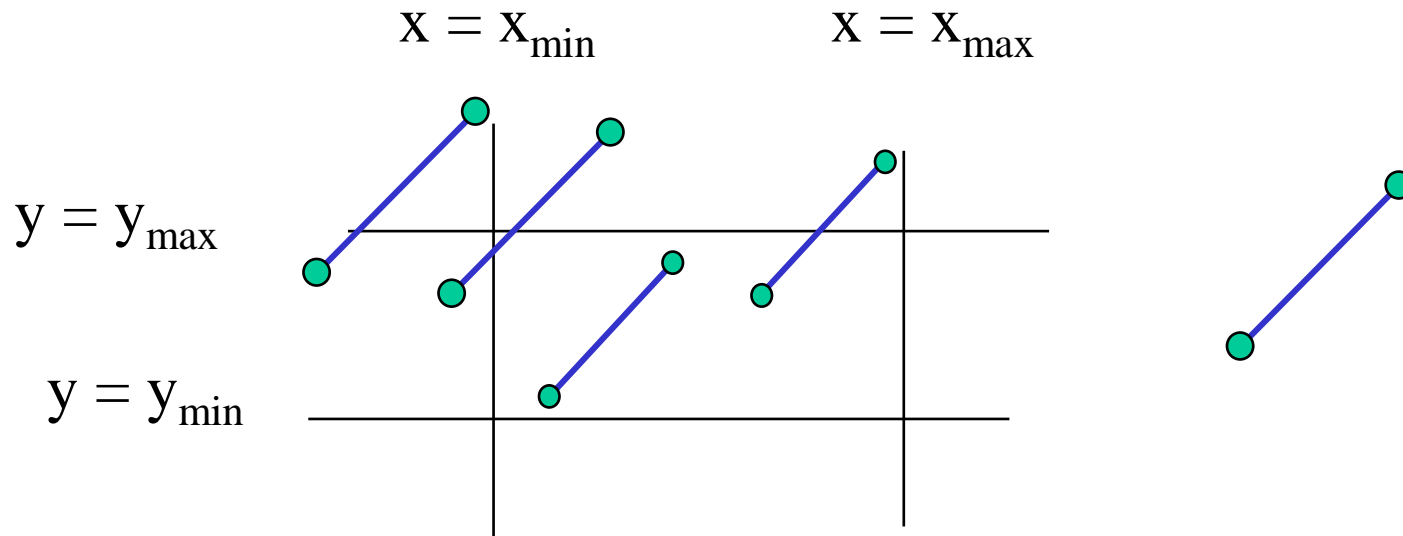


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# Clipping Line Segments

## ➤ Brute force approach

- Start with each segment, compute intersections with all sides of clipping window (对每条线段, 用裁剪窗口得四条裁剪边去求交, 每次求交就分割一次线段 (one division per intersection))
- 缺点: Inefficient (低效)



# Outline

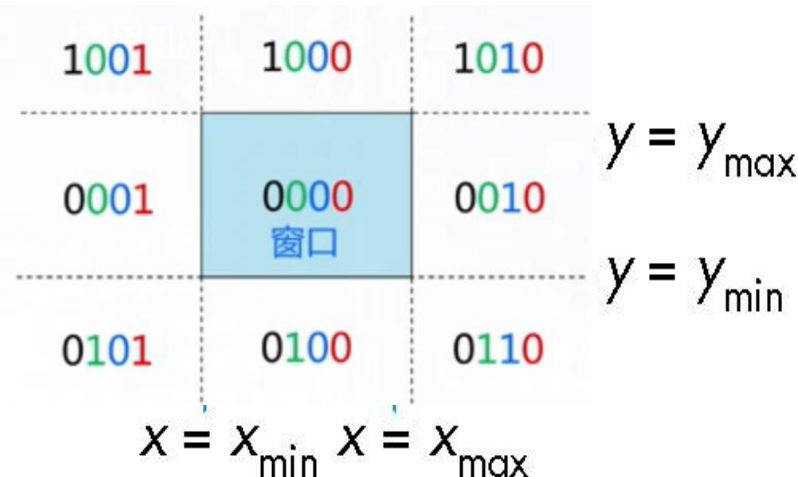
- **Clipping Algorithm**
  - Clipping Line Segments
    - **Cohen-Sutherland Line Clipping**编码裁剪算法
    - Liang-Barsky Line Clipping梁永栋裁剪算法
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# Cohen-Sutherland Line Clipping

- IDEA: eliminate as many cases as possible “without computing intersections” (消除尽可能多的“不用计算交点”的情况)
- ◆ Start with four lines that determine the sides of the clipping window:  $X_{min}$ ,  $X_{max}$ ,  $Y_{min}$ ,  $Y_{max}$ .
- ◆ Defining Outcodes  $b_0b_1b_2b_3$  : For each endpoint, define an outcode, Outcodes divide space into 9 regions

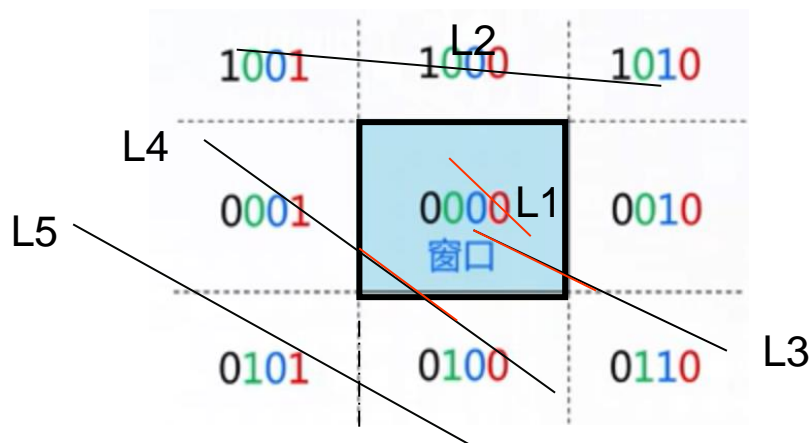


outcode:  $b_0b_1b_2b_3$

$b_0 = 1$  if  $y > y_{max}$ , 0 otherwise  
 $b_1 = 1$  if  $y < y_{min}$ , 0 otherwise  
 $b_2 = 1$  if  $x > x_{max}$ , 0 otherwise  
 $b_3 = 1$  if  $x < x_{min}$ , 0 otherwise

# Cohen-Sutherland Line Clipping(cont.)

- 根据线段两端点的outcodes, 可以判定线段和窗口的关系如下
  - Case 1: both endpoints of line segment inside all four lines
    - Draw (accept) line segment as is
  - Case 2: both endpoints outside all lines and on same side of a line
    - Discard (reject) the line segment
  - Case 3: One endpoint inside, one outside
    - Must do at least one intersection
  - Case 4: Both outside
    - May have part inside, Must do at least one intersection



L1(0000 0000) visible: **accept**  
 L2(1010 1001) invisible: **reject**  
 L3(0000 0110) part visible  
 L4(0001 0100) part visible  
 L5(0001 0100) invisible



# Cohen-Sutherland Line Clipping(cont.)

## ➤ 算法(演示)

## ➤ Algorithm Description

(1) 求出线段的两个端点 $P_1$ 、 $P_2$ 的区域码为code1,code2

(2) While(done){

1. 若 $code1|code2=0$ ,即两端点 $P_1$ 、 $P_2$ 的区域码位“或”操作结果为0000, 则完全可见(两端点在裁剪窗口内), 结束并输出线段done =true(简取)
2. 若 $code1\&code2\neq 0$ , 即两端点 $P_1$ 、 $P_2$ 的区域码位“与”操作结果不为0000, 则完全不可见(两端点同一外侧), 结束done =true(简弃)
3. 否则, 线段有可能是部分可见或完全不可见。这时用四条裁剪边逐个测是否与线段有交, 若有交则作如下操作:
  - a、先判断 $P_1$ 区域码code1, 如果 $P_1$ 在窗口内, 则交换 $P_1$ 、 $P_2$ , (保证 $P_1$ 在窗口外)。
  - b、再用线段 $P_1P_2$ 与该裁剪边的有效交点代替 $P_1$ , 并求出新 $P_1$ 的区域码code1后, 返回(2)进入下一轮循环。

}



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# Cohen-Sutherland Line Clipping(cont.)

## • Efficiency

- In many applications, the size of clipping window sides is small relative to the size of the entire data base (一般裁剪窗口边比被裁剪图元相对少得多)
  - Most line segments are outside one or more side of the window and can be eliminated based on their outcodes

## • Inefficiency

- when code has to be reexecuted for line segments that must be shortened in more than one step (有些线段不只一次被裁剪)

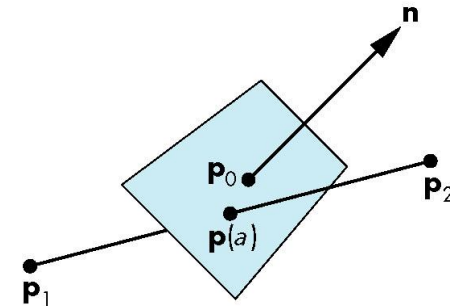
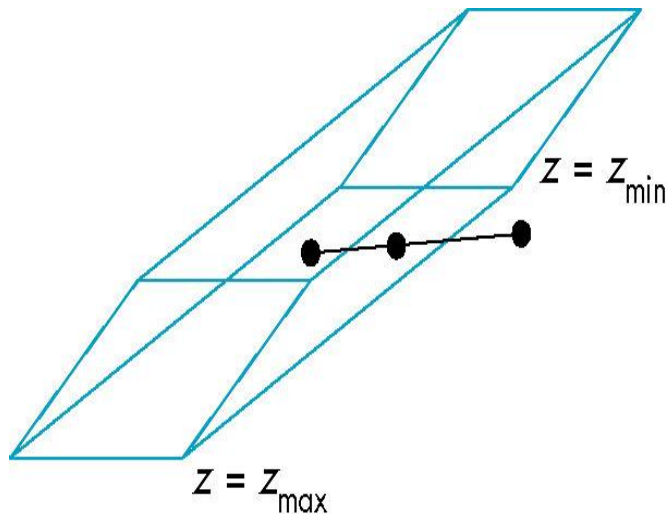


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# Cohen-Sutherland Line Clipping(cont.)

## ➤ Clipping 3D Line Segments

- General clipping in 3D requires intersection of line segments against arbitrary plane
- Example: oblique view(斜的观察)
  - Need Plane-Line Intersections(线面求交)



$$a = \frac{n \bullet (p_o - p_1)}{n \bullet (p_2 - p_1)}$$

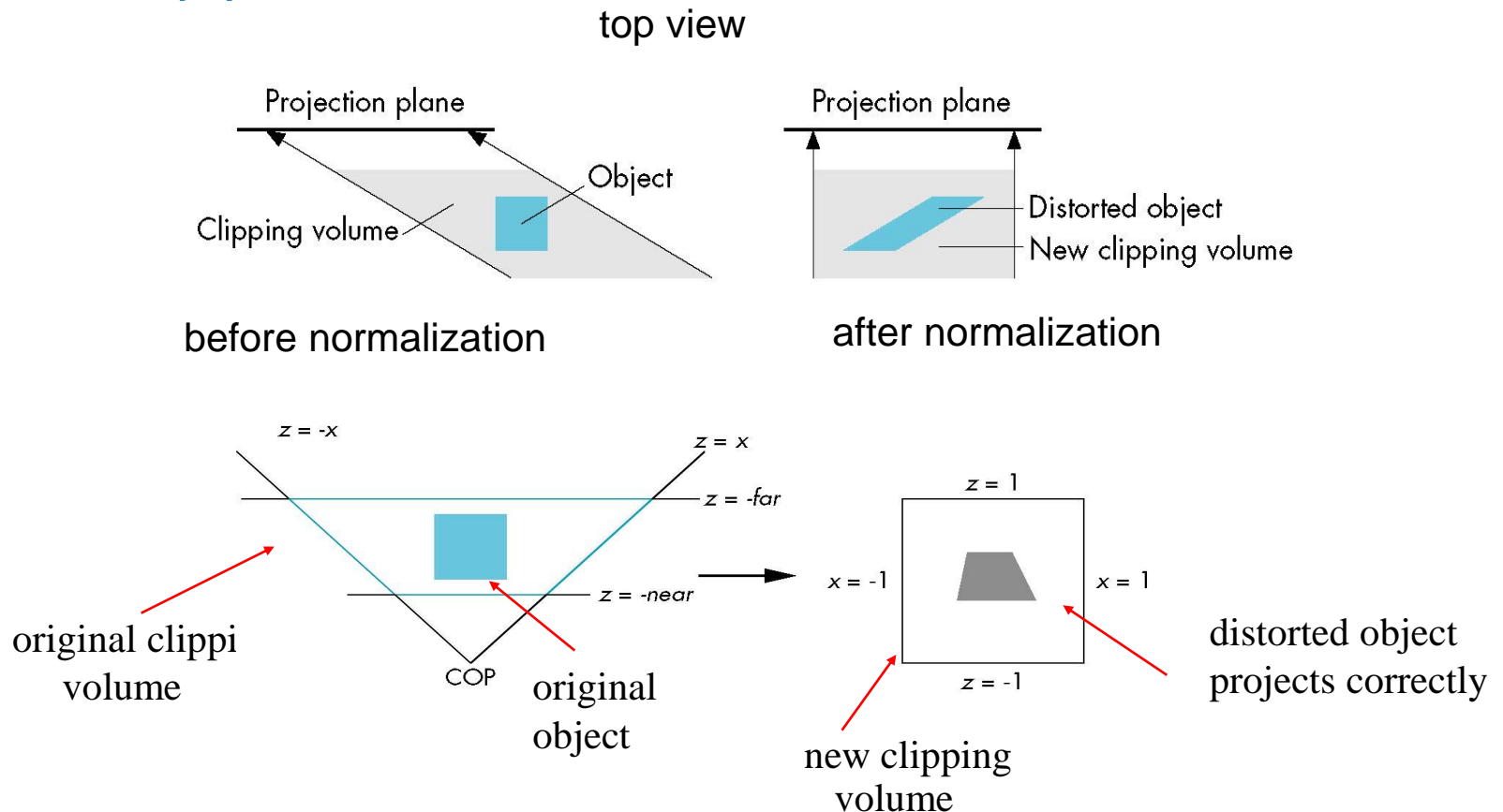


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# Cohen-Sutherland Line Clipping(cont.)

## ➤ Clipping 3D Line Segments (cont.)

- after normalization Projection, we clip against sides of right parallelepiped,

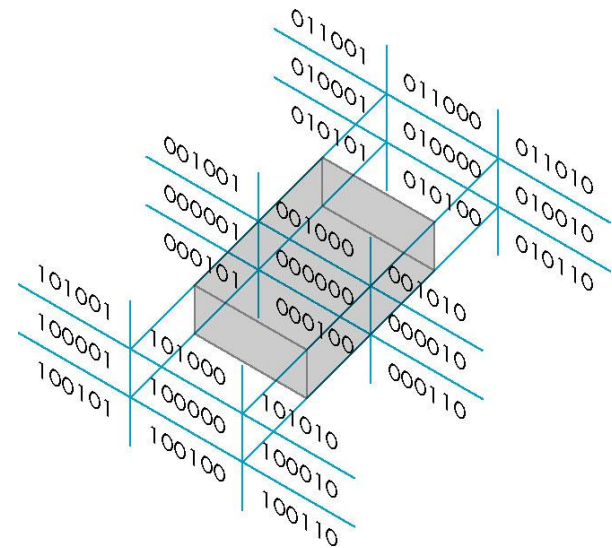
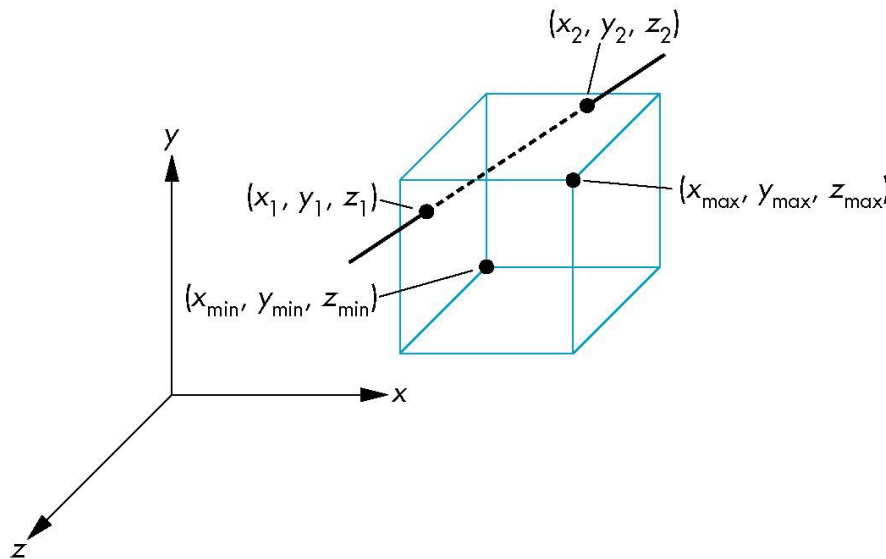




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# Clipping 3D Line Segments(cont.)

- Clipping 3D Line Segments(cont.)
- Use 6-bit outcodes 区域码, 27 个区域
- “clip line segment 裁剪边” against “clip planes 裁剪面”



# Outline

- **Clipping Algorithm**

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    - **Liang-Barsky Line Clipping 梁永栋裁剪算法**

- **Clipping Polygons**

- Sutherland-Hodgeman Polygon Clipping 逐边裁剪算法

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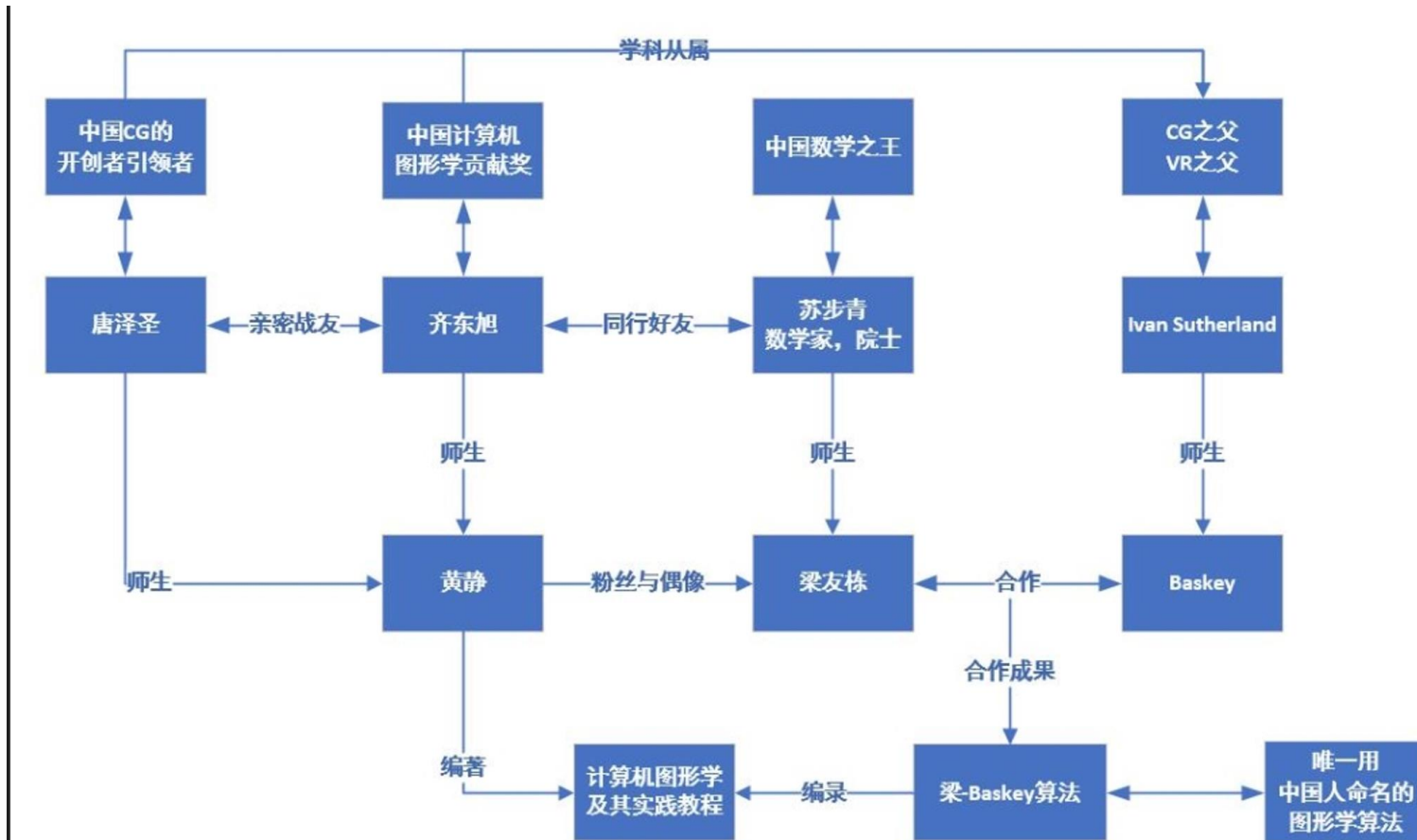




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# Liang-Barsky Line Clipping

## • 梁永栋-Barsky线段裁剪算法





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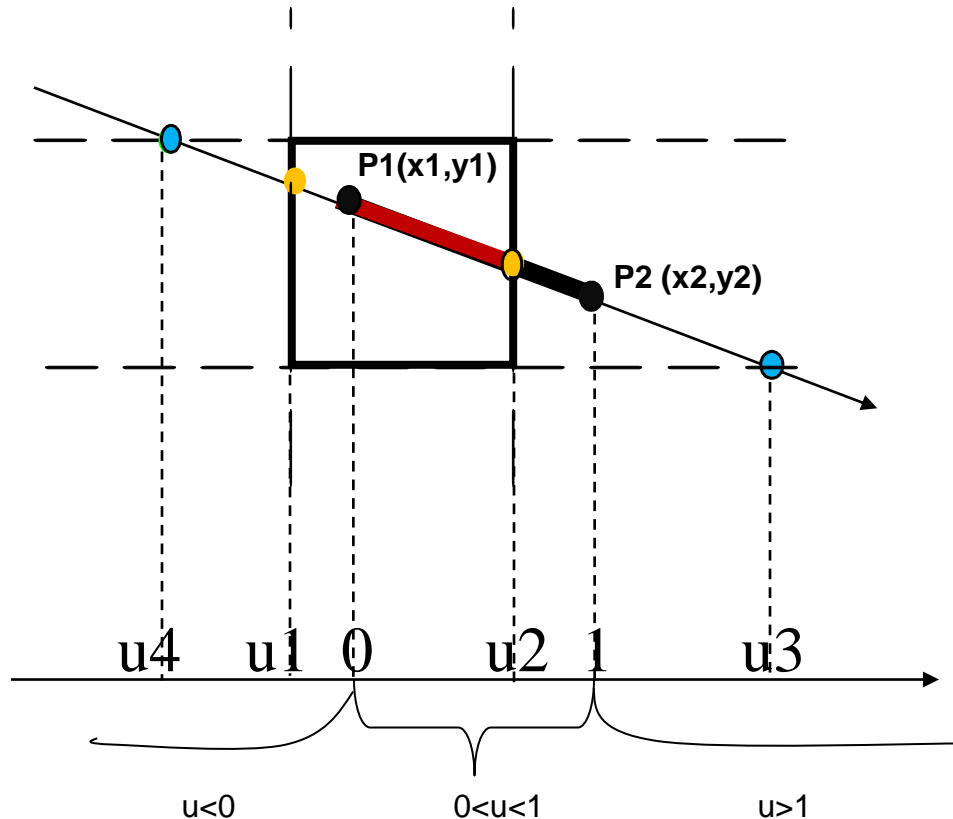
# Liang-Barsky Line Clipping(cont.)

## ➤ Idea:

- 采用直线的参数方程表示线段  $P(u)=P1+u(P2-P1)$   $0 \leq u \leq 1$ ;
- 把找被裁剪后的线段端点问题转换为找端点对应的参数  $Umin, Umax$

如下图中:  $Umin = \text{MAX}(0, u1, u4) = 0$ ,  $Umax = \text{MIN}(1, u2, u3) = u2$ ,

则裁剪后线段为  $P1'P2'$ :  $P1' = P1 + Umin * (P2 - P1) = P1$ ;  $P2' = P1 + Umax * (P2 - P1)$





# Liang-Barsky Line Clipping(cont.)

➤ 既在裁剪窗口中又在线段上的点的参数, 需要满足三个不等式:

1) on the line segment  $p_1p_2$  :  $P(u)=P_1+u(P_2-P_1)$

■  $0 \leq u \leq 1$

2) in the clipping windows :

- 根据:  $w_{min} \leq x \leq X_{wmax}$ ,  $Y_{wmin} \leq y \leq Y_{wmax}$
- 以及:  $X = x_1 + u(x_2 - x_1) = x_1 + u\Delta x$  和  $y = y_1 + u(y_2 - y_1) = y_1 + u\Delta y$
- 得到两个不等式:

■  $X_{wmin} \leq x_1 + u\Delta x \leq X_{wmax}$

■  $Y_{wmin} \leq y_1 + u\Delta y \leq Y_{wmax}$



# Liang-Barsky Line Clipping(cont.)

## ➤ 线段有裁剪结果的情况判定：

➤ If  $U_{min} < U_{max}$  , have clipped line segment in the window

- Bring  $U_{min}$  ,  $U_{max}$  in the following formula, get the clipped line segment endpoints:

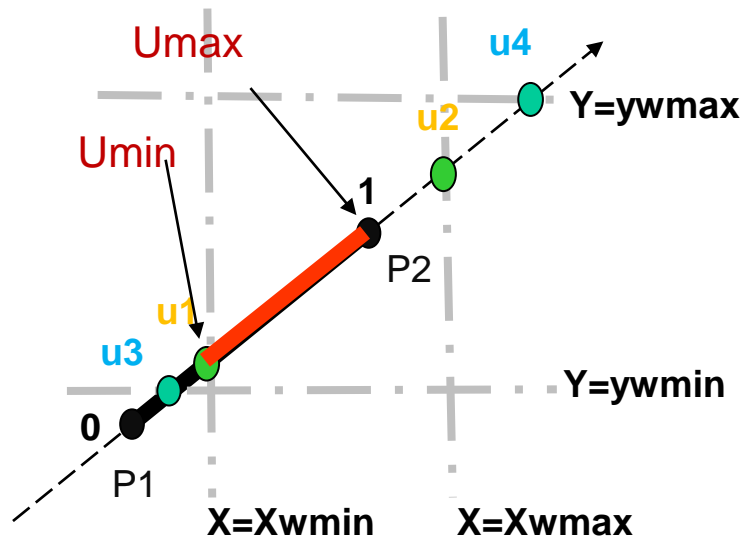
- 例如下图中情况满足三个不等式

$$0 \leq u \leq 1, \quad u_1 \leq u \leq u_2, \quad u_3 \leq u \leq u_4$$

则：  $U_{min} = \max(0, u_1, u_3) = u_1$  ,  $U_{max} = \min(1, u_2, u_4) = 1$  ,

因：  $U_{min} < u < U_{max}$  , 有裁剪结果为  $P(U_{min})P(U_{max})$  , 端点坐标计算如下：

- $x = x_1 + U_{min}(x_2 - x_1) = x_1 + U_{min} \Delta x$  ,  $y = y_1 + U_{min}(y_2 - y_1) = y_1 + U_{min} \Delta y$
- $x = x_1 + U_{max}(x_2 - x_1) = x_1 + U_{max} \Delta x$  ,  $y = y_1 + U_{max}(y_2 - y_1) = y_1 + U_{max} \Delta y$





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# Liang-Barsky Line Clipping(cont.)

## ➤ 线段完全在窗口外的判定:

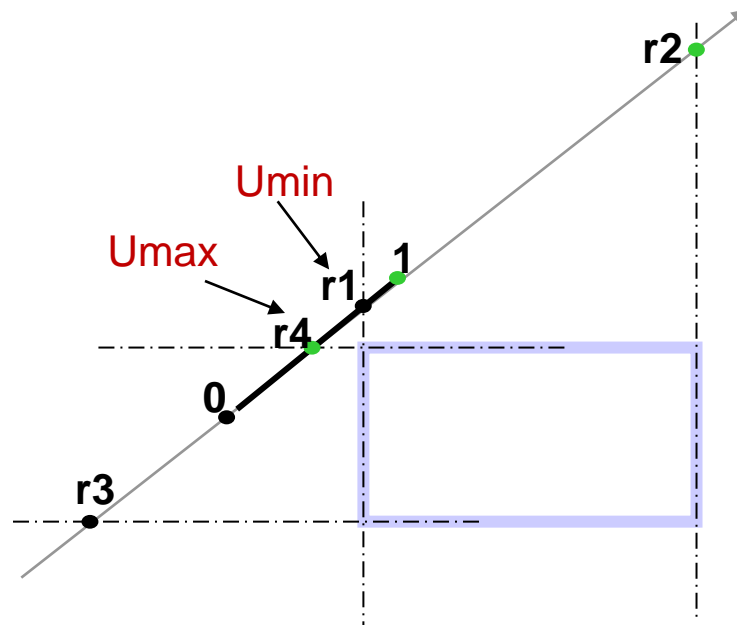
➤ If  $U_{min} > U_{max}$  this line segment is outside the window

例如下图情况:

-  $U_{min} = \text{MAX}(0, r1, r3)$  // here,  $r1$

-  $U_{max} = \text{MIN}(1, r2, r4)$ ; // here,  $r4$

因为  $U_{min} > U_{max}$ , 所以没有满足三个条件不等式的参数  $u$  存在, 即没有点





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# Liang-Barsky Line Clipping(cont.)

## ➤ 算法大致思路

根据:  $0 \leq u \leq 1$ ,  $xw_{\min} - x_1 \leq u\Delta x \leq xw_{\max} - x_1$ ,  $yw_{\min} - y_1 \leq u\Delta y \leq yw_{\max} - y_1$

后两个不等式转换为4个单边不等式

$u(-\Delta x) \leq x_1 - xw_{\min}$ ,  $u(\Delta x) \leq xw_{\max} - x_1$ ,  $u(-\Delta y) \leq y_1 - yw_{\min}$ ,  $u(\Delta y) \leq yw_{\max} - y_1$

为便于讨论, 四个不等式统一形式为:  $u \cdot p_k \leq q_k$  ( $k=1,2,3,4$ ),

- If  $p_k=0$ , *there is no corresponding  $r_k$*
- If  $p_k \neq 0$ , compute  $r_k = q_k / p_k$  ( $k=1,2,3,4$ )

$$p_1 = -\Delta x \quad q_1 = x_1 - xw_{\min} \quad r_1 = q_1 / p_1$$

$$p_2 = \Delta x \quad q_2 = xw_{\max} - x_1 \quad r_2 = q_2 / p_2$$

$$p_3 = -\Delta y \quad q_3 = y_1 - yw_{\min} \quad r_3 = q_3 / p_3$$

$$p_4 = \Delta y \quad q_4 = yw_{\max} - y_1 \quad r_4 = q_4 / p_4$$

- Compute  $U_{\min}, U_{\max}$ , Group by the sign of  $p_k$

初始:  $U_{\min}=0, U_{\max}=1$

$p_k < 0$ :  $U_{\min} = \max(U_{\min}, r_k)$ ,  $p_k > 0$ :  $U_{\max} = \min(U_{\max}, r_k)$

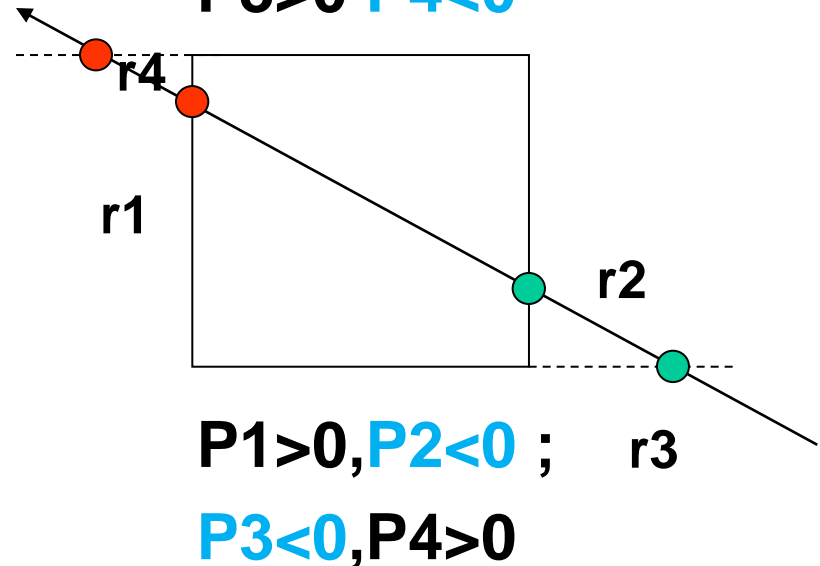
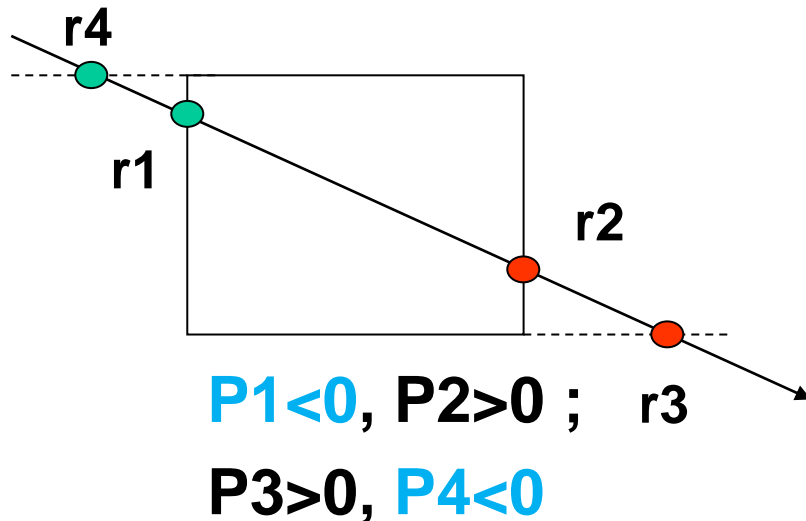
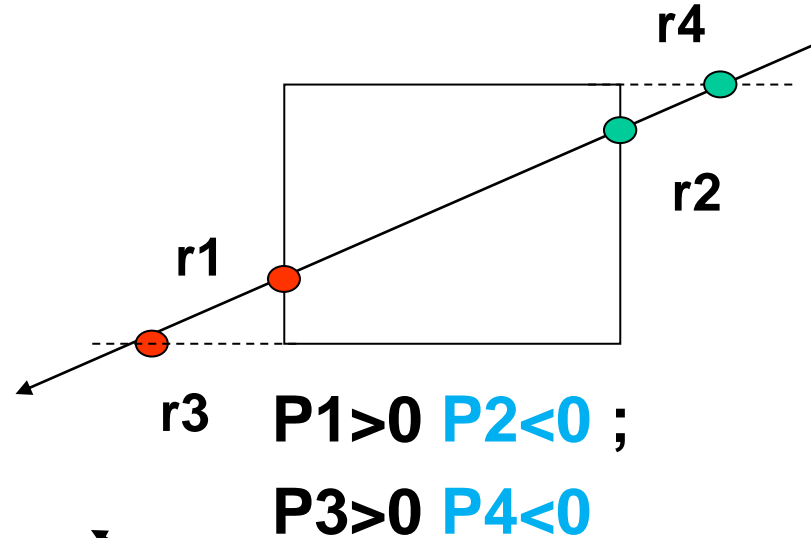
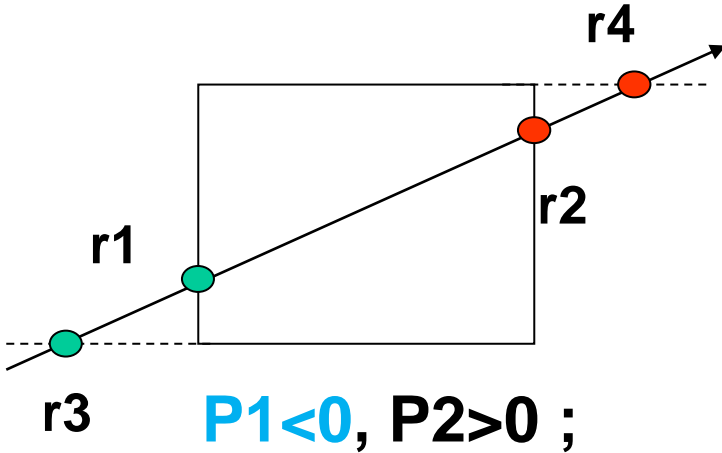
- Compute result line segment endpoints using  $U_{\min}$  and  $U_{\max}$ 
  - If  $U_{\min} \geq U_{\max}$ : this line segment is outside the clipping window.
  - If  $U_{\min} < U_{\max}$ : compute  $(X(u_{\min}), Y(u_{\min}))$ ,  $(X(u_{\max}), Y(u_{\max}))$



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# Liang-Barsky Line Clipping(cont.)

➤ If  $P_k \neq 0$ , 有  $r_k = q_k / p_k$ ,

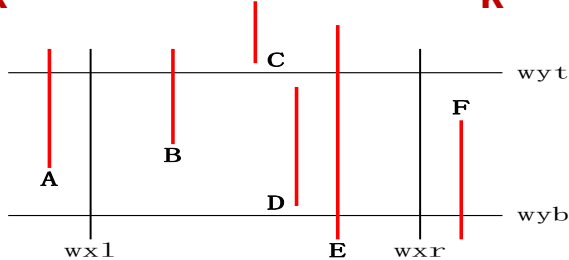




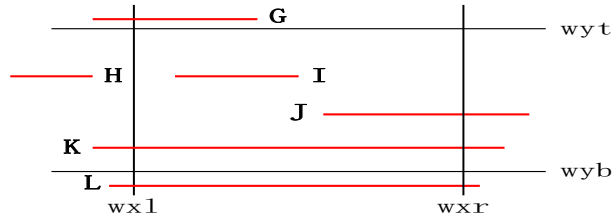
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# Liang-Barsky Line Clipping(cont.)

➤ If  $P_k = 0$ , 没有对应的  $r_k$



(a) 直线段与窗口边界  $w_{x1}$  和  $w_{xr}$  平行的情况



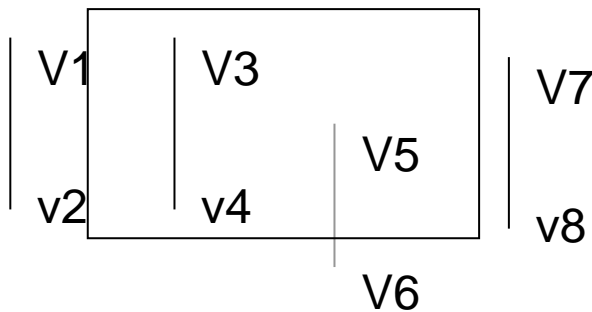
(b) 直线段与窗口边界  $w_{yb}$  和  $w_{yt}$  平行的情况

□ 当  $p_1 = -\Delta x = 0$ , 必有  $p_2 = \Delta x = 0$ , 线与Y平行;

□ 当  $p_3 = -\Delta y = 0$ , 必有  $p_4 = \Delta y = 0$ , 线与X平行

$p_k=0$ 时, 如果对应的某个  $q_k < 0$ , 则该线段完全不可见, 退出裁剪

- 对于线段  $V_1V_2$ ,  $p_1=p_2=0$ , ( $k=1,2$ ), 线与Y平行  
此时  $q_1 = x_1 - xw_{\min} < 0$ ,  $q_2 = xw_{\max} - x_1 > 0$ , 因此  $V_1V_2$  在窗口外。
- 对于线段  $V_7V_8$ ,  $p_1=p_2=0$ , ( $k=1,2$ ), 线与Y平行  
此时  $q_1 = x_1 - xw_{\min} > 0$ ,  $q_2 = xw_{\max} - x_1 < 0$ , 因此  $V_7V_8$  在窗口外。



$p_k=0$ 时, 如果其对应的两个  $q_k > 0$ , 则必有窗口内线段, 继续裁剪。

- 对于直线  $V_3V_4$ ,  $p_1=p_2=0$ , ( $k=1,2$ ), 线与Y平行  
此时,  $q_1 > 0$ ,  $q_2 > 0$ , 则  $V_3V_4$  在窗口X方向的两裁剪边内。
- 对于直线  $V_5V_6$ ,  $p_1=p_2=0$ , ( $k=1,2$ ), 线与Y平行  
此时,  $q_1 > 0$ ,  $q_2 > 0$ , 则  $V_5V_6$  在窗口X方向的两裁剪边内。





# Liang-Barsky Line Clipping(cont.)

## ➤ 总结: 完整算法描述

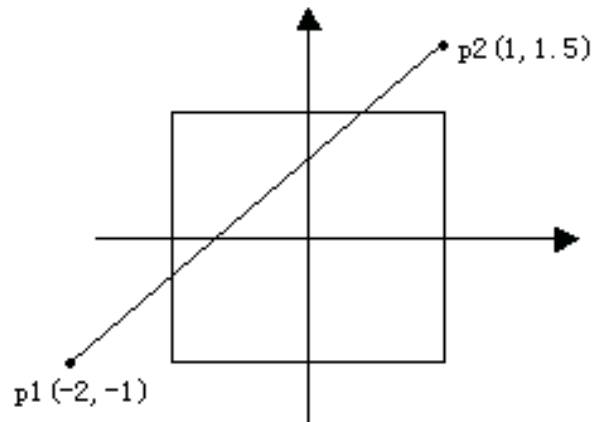
- (1) 输入直线段的两端点坐标 $(x_1, y_1)$ 和 $(x_2, y_2)$ , 以及窗口的四条边界坐标:  $wyt$ 、 $wyb$ 、 $wxl$ 和 $wxr$ 。 令 $Umin=0$ ;  $Umax=1$ ;  $\Delta x=x_2-x_1$ ;  $\Delta y=y_2-y_1$ ;
- (2) 若 $\Delta x=0$ , 则 $p_1=p_2=0$ 。若 $q_1<0$ 或 $q_2<0$ , 则该直线段不在窗口内, 算法转(7)。否则 $q_1>0$ 且 $q_2>0$ 必有裁剪结果, 算法转(4)。
- (3) 若 $\Delta y=0$ , 则 $p_3=p_4=0$ 。若 $q_3<0$ 或 $q_4<0$ , 则该直线段不在窗口内, 算法转(7)。否则 $q_3>0$ 且 $q_4>0$ 必有裁剪结果, 算法转(4)。
- (4) 若 $\Delta x=0$ , 则有 $p_k \neq 0 (k=3,4)$ , 计算 $r_3, r_4$ ,  
若 $\Delta y=0$ ,  $p_k \neq 0 (k=1,2)$ , 计算 $r_1, r_2$ 。  
若 $p_k \neq 0 (k=1,2,3,4)$ , 计算 $r_1, r_2, r_3, r_4$ 。  
将 $p_k < 0$ 的 $r_k$ 与 $Umin$ 比较后取大值赋予 $Umin$ , 将 $p_k > 0$ 的 $r_k$ 与 $Umax$ 比较后取小值赋予 $Umax$ 。
- (5) 若 $Umin > Umax$ , 则直线段在窗口外, 算法转(7)。
- (6) 若 $Umin < Umax$ , 利用直线的参数方程求得直线段在窗口内的两端点坐标, 调用画线程序绘制裁剪后线段, 结束。
- (7) 线段在窗口外, 无裁剪结果, 结束。

# Liang-Barsky Line Clipping(cont.)

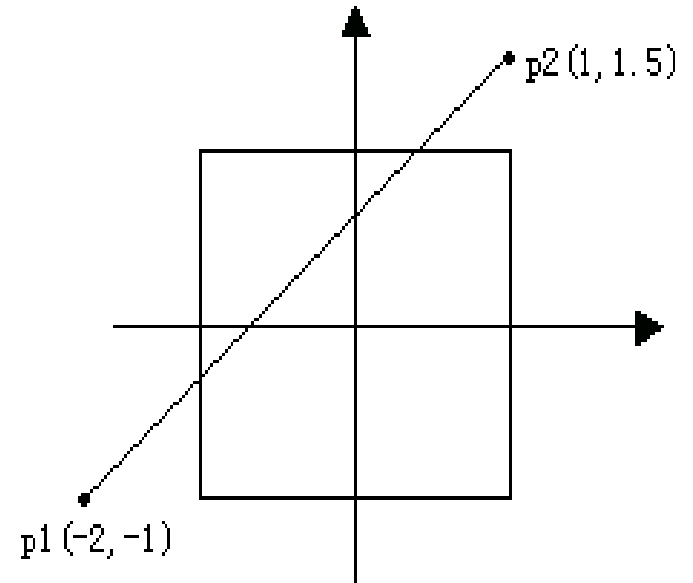
- **Practice:**

Clipping window:  $(-1, -1), (1, 1)$

Line segment:  $p_1 p_2 (-2, -1)(2, 1.5)$



$$\begin{aligned}
 p_1 &= -\Delta x & q_1 &= x_1 - xw_{\min} & r_1 &= q_1/p_1 \\
 p_2 &= \Delta x & q_2 &= xw_{\max} - x_1 & r_2 &= q_2/p_2 \\
 p_3 &= -\Delta y & q_3 &= y_1 - yw_{\min} & r_3 &= q_3/p_3 \\
 p_4 &= \Delta y & q_4 &= yw_{\max} - y_1 & r_4 &= q_4/p_4
 \end{aligned}$$



- $\Delta x=3, \Delta y=2.5$
- $p_1=-3 \quad q_1=-1;$
- $p_2=3 \quad q_2=3$
- $p_3=-2.5 \quad q_3=0;$
- $p_4=2.5 \quad q_4=2$
- 对于  $p<0$ , 计算小端  $u$ 
  - $r_1=1/3, r_3=0, (X_1, Y_1)$  对应的  $u=0$ ,
  - 取三个数值中的最大值  $u_1=\max(1/3, 0, 0)=1/3$ 。
- 对于  $p>0$ , 计算大端  $u$ 
  - $r_2=1, r_4=4/5, (X_2, Y_2)$  对应的  $u=1$ ,
  - 取三个数值中的最小值  $u_2=\min(1, 4/5, 1)=4/5$
- 因为  $u_1 < u_2$ , 则可见线段的端点坐标可计算方法出:
  - $x=x_1+u_1*\Delta x=-1, y=y_1+u_1*2.5=-1/6$  即  $(-1, -1/6)$
  - $x=x_1+u_2*\Delta x=2/5, y=y_1+u_2*2.5=1$  即  $(2/5, 1)$



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# Liang-Barsky Line Clipping(cont.)

## ➤ Liang-Barsky Line Segment Clipping in 3D

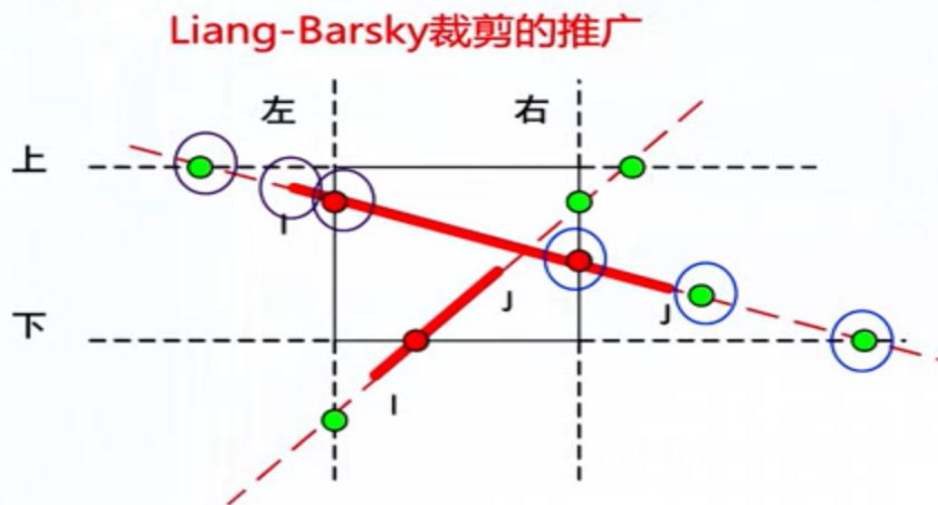
1) on the line segment:  $0 \leq u \leq 1$

2) in the clipping view volume

$$X_{wmin} \leq X_1 + u \Delta X \leq X_{wmax}, \quad \Delta x = x_2 - x_1$$

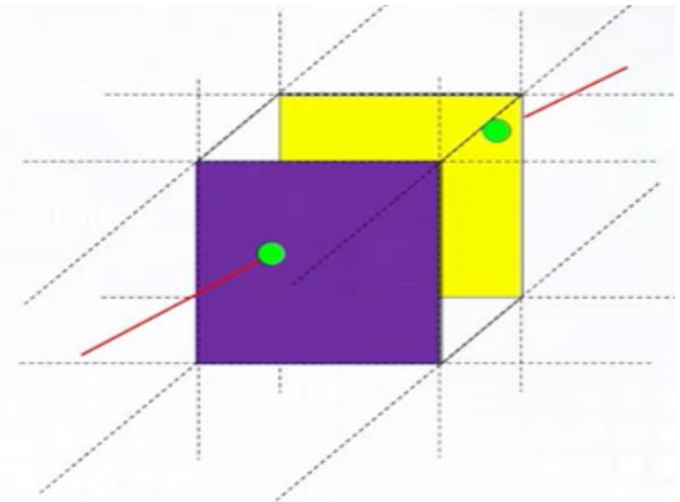
$$Y_{wmin} \leq Y_1 + u \Delta Y \leq Y_{wmax}, \quad \Delta y = y_2 - y_1$$

$$Z_{wmin} \leq Z_1 + u \Delta Z \leq Z_{wmax}, \quad \Delta z = z_2 - z_1$$



$$U_{one} = \max(0, u_{k|pk < 0}, u_{k|pk < 0})$$

$$U_{two} = \min(1, u_{k|pk > 0}, u_{k|pk > 0})$$



$$U_{one} = \max(0, u_{k|pk < 0}, u_{k|pk < 0}, u_{k|pk < 0})$$

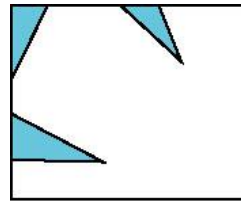
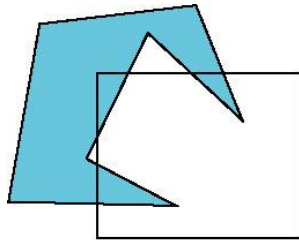
$$U_{two} = \min(1, u_{k|pk > 0}, u_{k|pk > 0}, u_{k|pk > 0})$$

# Outline

- **Clipping Algorithm**
  - **Clipping Line Segments**
    - Cohen-Sutherland Line Clipping 编码裁剪算法
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    - Sutherland-Hodgeman Polygon Clipping 逐边裁剪算法
- **Hidden Surface Removal Algorithm**
  - Object Space Approach
    - Back-face culling 后向面剔除 (着色前就消隐掉)
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    - Depth-buffer 深度缓存算法 (光栅化渲染中的消隐算法)
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# Polygon Clipping

- Not as simple as line segment clipping
  - Clipping a line segment yields at most one line segment (裁剪多边形可能产生“多”条线段)
  - Clipping a polygon can yield multiple polygons (裁剪凹多边形可能产生“多”个多边形)

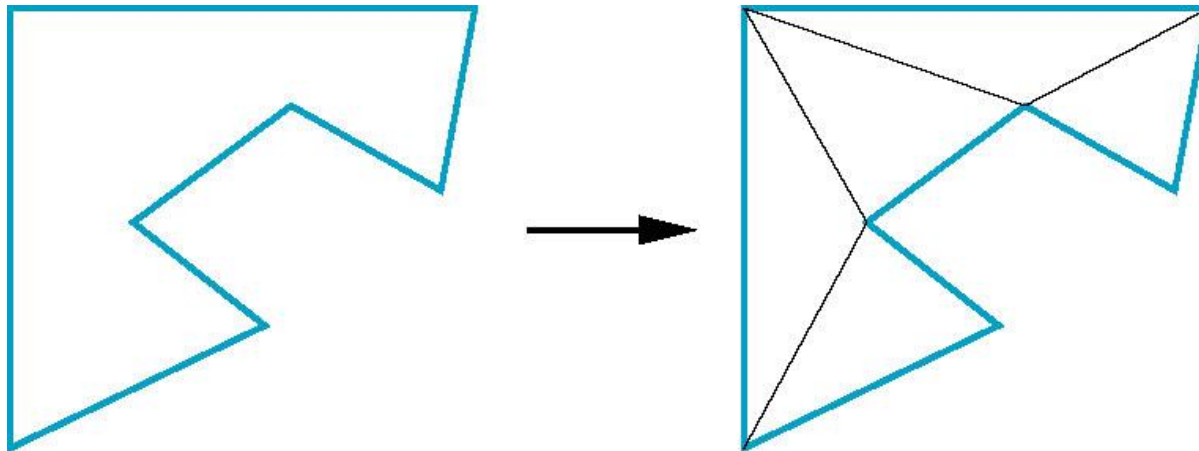


- However, clipping a convex(凸) polygon can yield at most one other polygon (但是裁剪凸多边形只产生“一”个多边形)

# Polygon Clipping (cont.)

## • Tessellation and Convexity (细分和凸性)

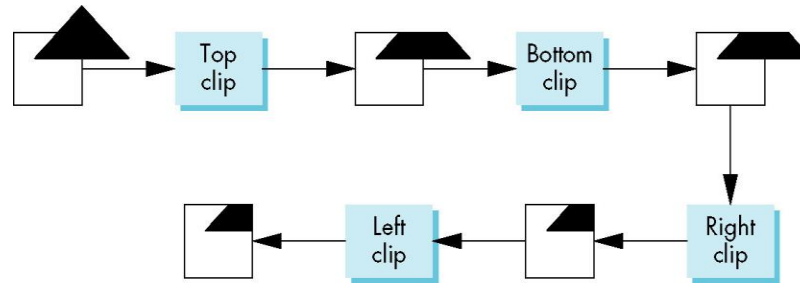
- One strategy is to replace nonconvex (concave) polygons with a set of triangular polygons (a tessellation), (将非凸多边形替换为一组三角多边形)
- Also makes fill easier (使得后续填充变得容易)
- Tessellation through “tessellation shaders” (网格细分可通过“细分着色器”完成)



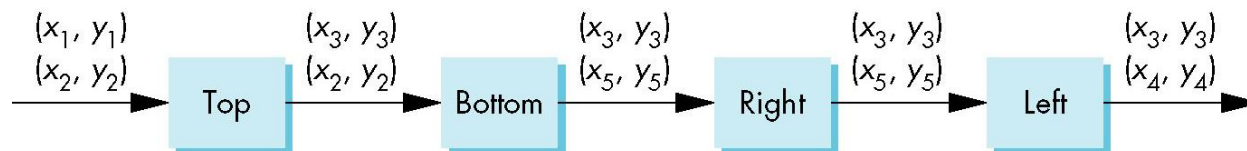
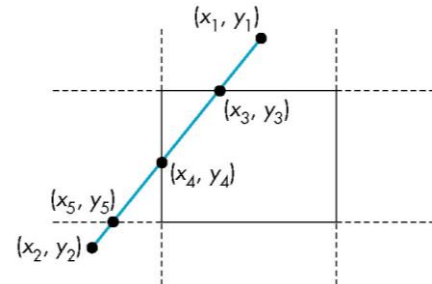
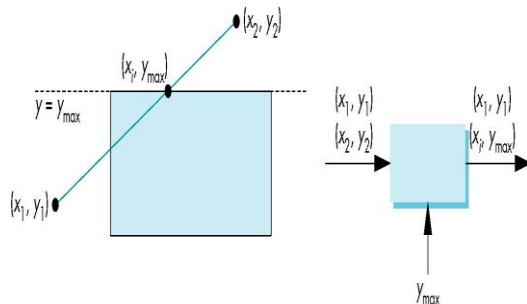


# Polygon Clipping(cont.)

- Sutherland-Hodgeman Polygon Clipping
  - Clipping against each side of window is independent of other sides



- Can use four independent clippers in a pipeline







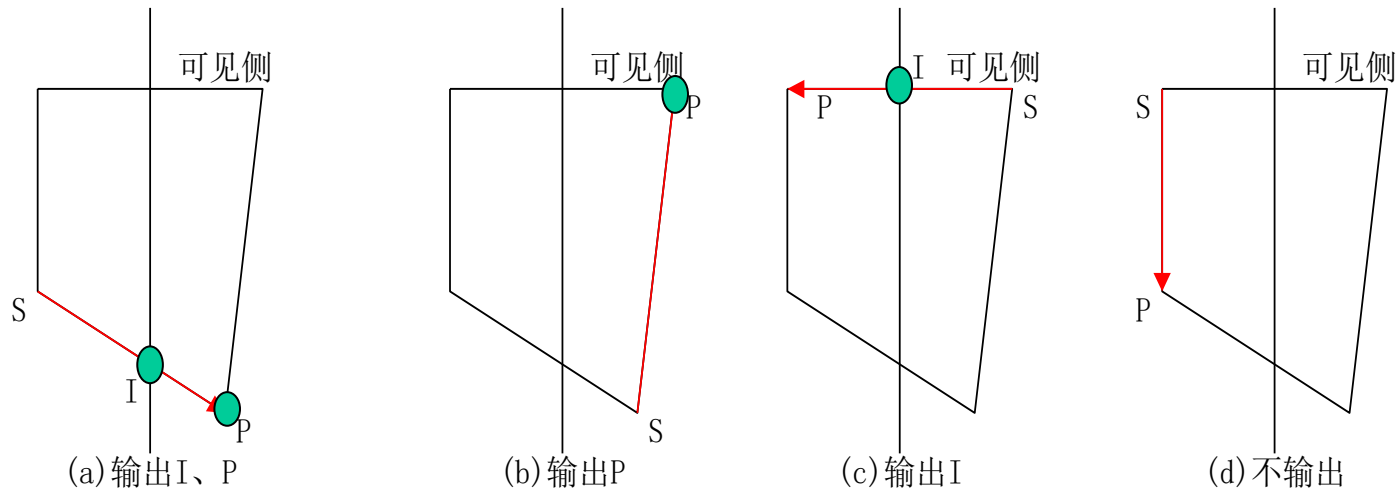
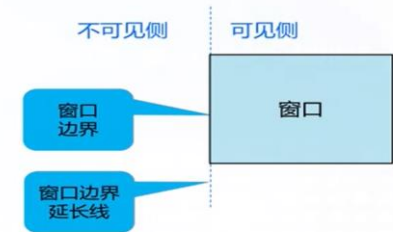
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# Polygon Clipping(cont.)

## Sutherland-Hodgeman Polygon Clipping(cont.)

**How to output intersection and vertex?**

**Notice: 不可重复输出线段的可见端点!**



此图中，输出“线段与裁剪边的交点” 和“可见边的结束顶点”

# Polygon Clipping(cont.)

## • Sutherland-Hodgeman Polygon Clipping(cont.)

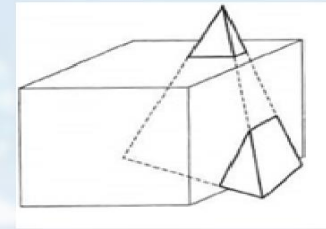
- Three Dimensions: add front and back clippers
  - Strategy used in SGI Geometry Engine

Sutherland-Hodgeman算法的推广



逐边裁剪

逐边裁剪多边形:  
逐边裁剪多边形的每条边  
输出: 顶点序列构成多边形



逐面裁剪

逐面逐个裁剪多个多边形:  
逐面裁剪多边形的每条边  
输出: 顶点序列, 构成多面体

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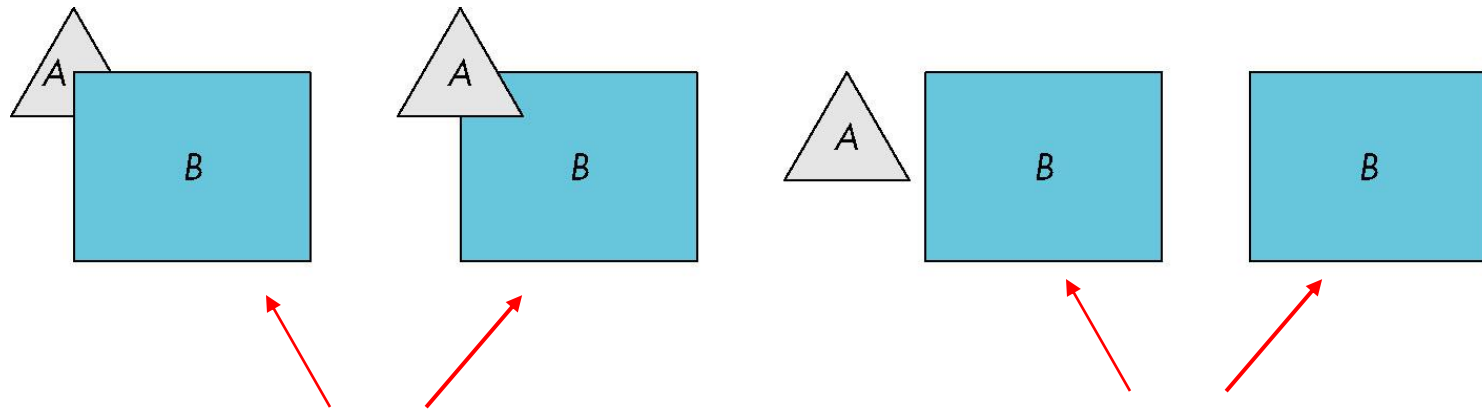
# Hidden Surface Removal

- hidden-surface removal (隐藏面消除) has much in common with Clipping (裁剪), In both cases, we are trying to remove objects that are not visible to the camera (消隐和裁剪都是去除相机不可见的对象)
- Often we can use “visibility可见性” or “occlusion testing阻塞测试” early in the process to eliminate as many polygons as possible before going through the entire pipeline (通常, 越早使用使用“可见性”测试, 可在通过整个管道之, 尽可能消除不可见的多边形)

# Hidden Surface Removal (cont.)

## ➤ Object-space approach 对象空间算法

- use pairwise testing between polygons (objects)
- Worst case complexity  $O(n^2)$  for  $n$  polygons



partially obscuring 掩盖

can draw independently



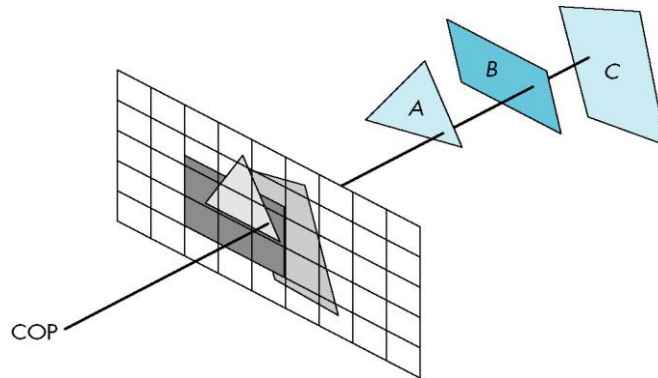
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# Image Space Approach

## ➤ Image Space Approach:

Idea: Look at each projector ( $nm$  for an  $n \times m$  frame buffer) and find closest of  $k$  polygons, Complexity  $O(nmk)$

从投影中心点(COP即视点)出发, 与 $m*n$ 分辨率的每个像素发出投影线(projector), 与场景中的 $k$ 个多边形求交, 再判定每个像素的颜色。复杂度 $O(nmk)$



✓在像素级别上确定“可见性”

✓Algorithms: **Depth Buffer** , **Ray-casting**

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- **Hidden Surface Removal Algorithm**

- **Object Space Approach**

- **Back-face culling 后向面剔除 (裁剪前就消掉)**

- **Image Space Approach**

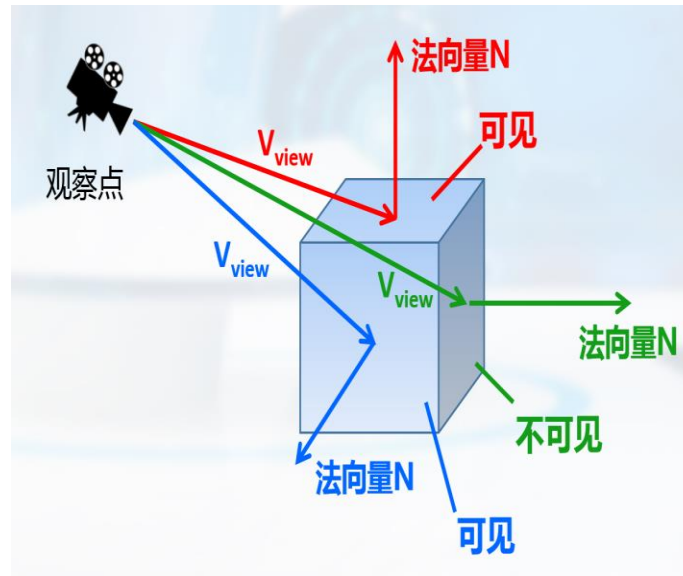
- **Depth-buffer 深度缓存算法 (光栅化渲染中的消隐算法)**
    - **Ray-casting 光线投射 (光线跟踪渲染中的消隐算法)**



# Back-Face Culling

## ➤ Back-Face Removal (Culling)后向面消除(剔除)

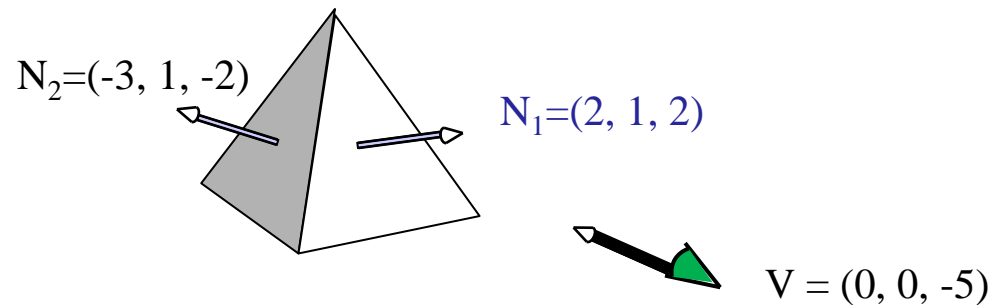
- 对每个凸多边形面，其外法向量 $\mathbf{n}$ 和观察方向 $\mathbf{v}$ 的夹角若小于90度，则是后向面back-Face，可判定为“不可见”，直接剔除culling
- Back face判定的计算方法：
  - face is invisible iff  $90 \geq \theta \geq -90$ ,
  - face is invisible  $\cos \theta \geq 0$  or  $\mathbf{v} \cdot \mathbf{n} \geq 0$  equivalently





# Back-Face Culling (cont.)

## ➤ Example



## ✓ solution :

$N_1 \bullet V = (2, 1, 2) \bullet (0, 0, -5) = -10$ , so  $N_1 \bullet V < 0$ ,  
→  $N_1$  is not back-face polygon, it is visible

$N_2 \bullet V = (-3, 1, -2) \bullet (0, 0, -5) = 10$ , so  $N_2 \bullet V > 0$ ,  
→  $N_2$  is Back-face polygon, it is invisible, need culling!

# Back-Face Culling (cont.)

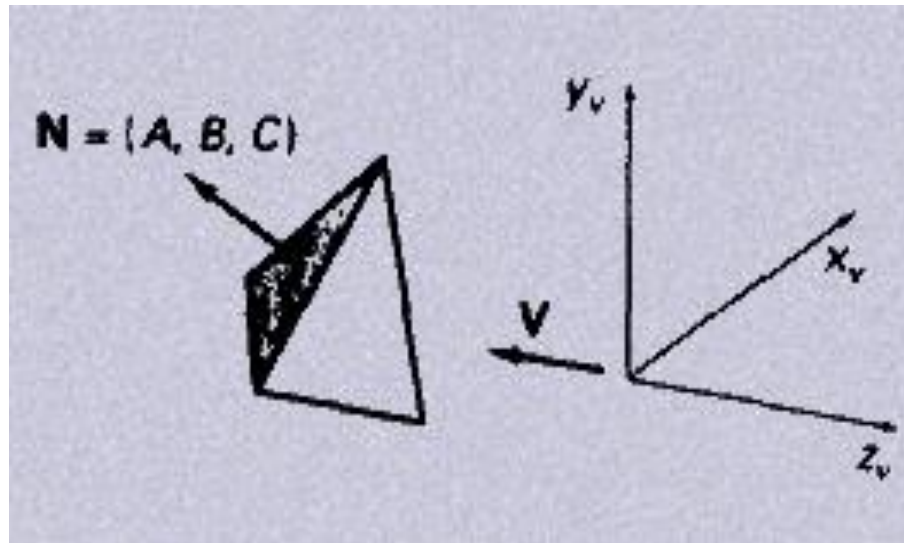
- **Special case :**

若:  $V = (0, 0, V_z)$  ,  $N = (N_x, N_y, N_z) = (A, B, C)$

则:  $v \cdot n = V_x N_x + V_y N_y + V_z N_z = 0 \cdot N_x + 0 \cdot N_y + V_z \cdot N_z = V_z \cdot C$

**If  $V_z \cdot C > 0$ , then this face is Back-face.**

➤ **So , if  $V_z < 0$ , need  $C < 0$  , this face is back face.**



# Back-Face Culling (cont.)

- In OpenGL we can simply enable culling
- **Open/Close culling function 开启/关闭剔除功能**
  - `gl.enable (gl.CULL_FACE);` //开启后向面剔除
  - `gl.disable(gl.CULL_FACE);` //关闭后向面剔除 , Default
- **Choose which face to be culled 选择哪个面被剔除**
  - `gl.cullFace( face);`//face: gl.BACK or gl.FRONT;  
// default is gl.BACK

# Back-Face Culling (cont.)

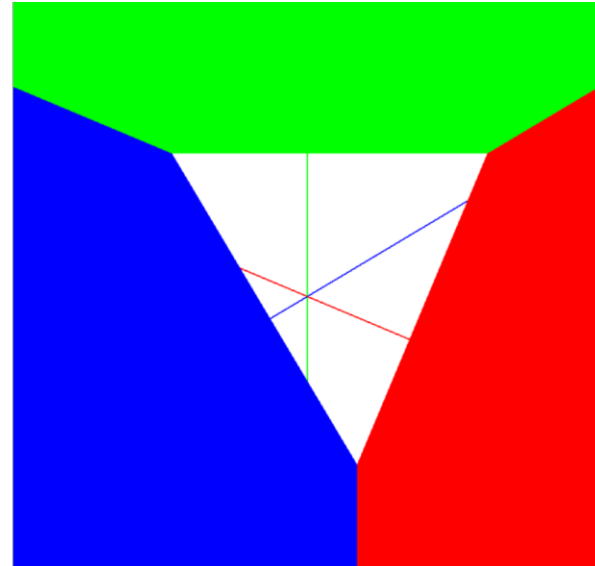
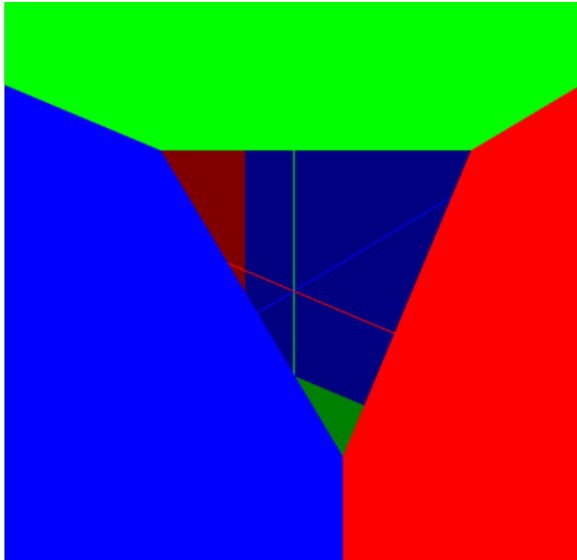
## ➤ 后向面剔除实例：

➤ 左边图：没有开启后向面剔除，默认情况

➤ `gl.disable(gl.CULL_FACE); // Default`

➤ 右边图：开启了后向面剔除，可加快渲染速度。

➤ `gl.enable (gl.CULL_FACE);`



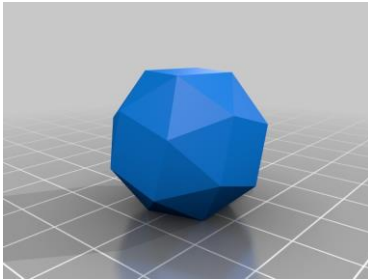
# Back-Face Culling(cont.)

## ➤ 优点:

- 在光栅化之前, 可剔除掉物体的不会显示的后向面

## ➤ 局限:

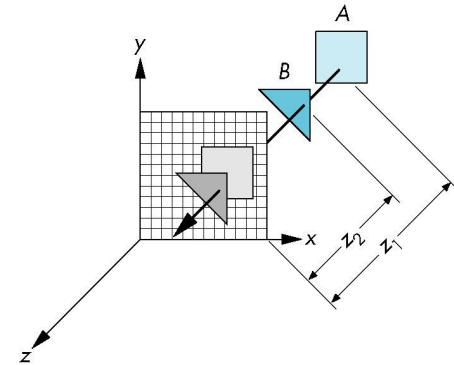
- 凹多面体的可见面或不同物体上的可见面的遮挡, 不能进行判定消隐。



凸多面体



凹多面体



不能判定“可见面”之间的遮挡

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# Z-Buffer Algorithm

- Ref: GAME101 Lecture7 (注: 下例左手坐标系 $Z > 0$ )
  - Use a buffer called the “z-buffer” or “depth buffer” to store the depth of the closest object at each pixel found so far
  - As we render each polygon, compare the depth of each pixel to depth in z buffer, If less, place shade of pixel in color buffer and update z buffer

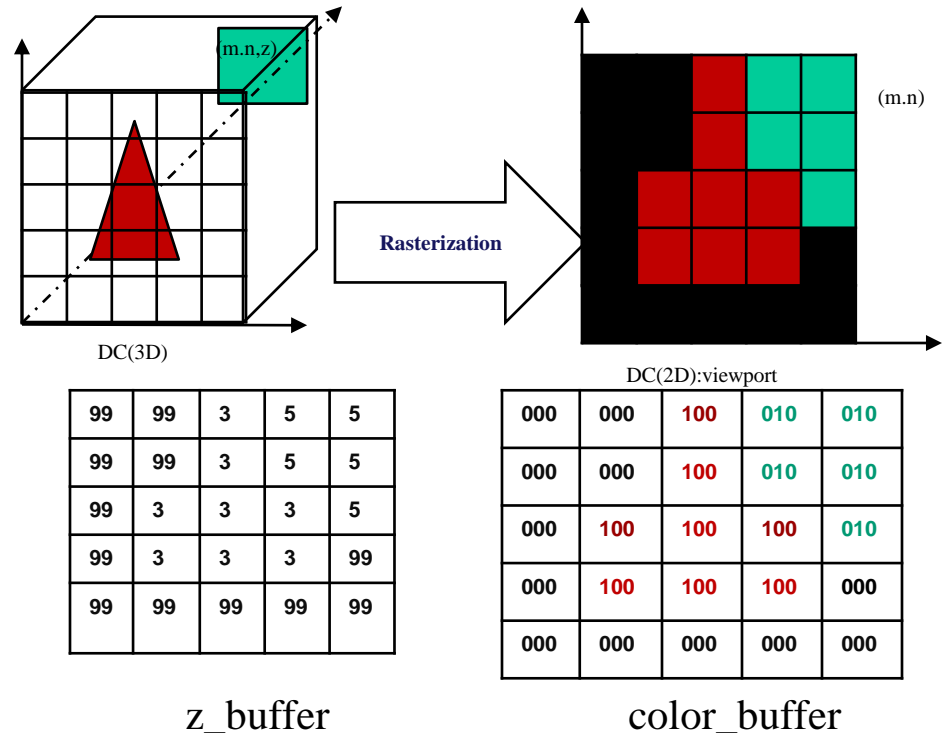
## Z-Buffer Algorithm

Initialize depth buffer to  $\infty$

During rasterization:

```

for (each triangle T)
  for (each sample (x,y,z) in T)
    if (z < zbuffer[x,y])           // closest sample so far
      framebuffer[x,y] = rgb;       // update color
      zbuffer[x,y] = z;             // update depth
    else
      ;                             // do nothing, this sample is occluded
    
```





# Z-Buffer Algorithm

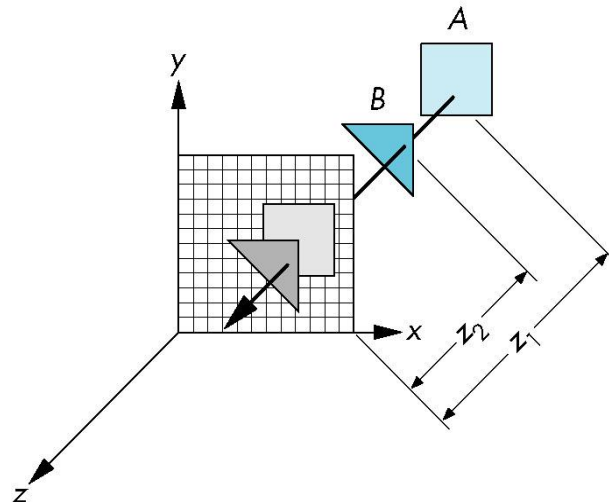
例如下图:对场景中三角形A,B进行绘制填充颜色(即写帧缓存)  
(注意:本例中采用右手坐标系: $Z < 0$ )( $Z_1, Z_2$ 表示为深度)

1) 若先光栅化填充A时,

对像素 $\text{pixel}(x,y)$ : 有 $z(x,y)=z_1 < \text{Dbuffer}(x,y) = \text{初始化为最大深度}$   
则:  $\text{Dbuffer}(x,y)=z_1$  ,  $\text{ColorBuffer}(x,y)=\text{“浅蓝”}$

2) 再光栅化填充B时,

对像素 $\text{pixel}(x,y)$ : 有 $z(x,y)=z_2 < \text{Zbuffer}(x,y) = z_1$   
则:  $\text{Dbuffer}(x,y)=z_2$  ,  $\text{ColorBuffer}(x,y)=\text{“深蓝”}$







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# Z-Buffer Algorithm (cont.)

## ➤ WEBGL中实现深度检测算法:

- 设置Z-BUFFER初值(无穷大(左手坐标), 无穷小(右手坐标))

**`gl.clear(gl.DEPTH_BUFFER_BIT);`**

*//也常和颜色缓存一起进行初始化:设置背景颜色和深度值*

*//`gl.clear( gl.COLOR_BUFFER_BIT | gl.DEPTH_BUFFER_BIT );`*

- 开启Z-BUFFER算法

**`gl.enable(gl.DEPTH_TEST);`**

- 关闭Z-BUFFER算法

**`gl.disable(gl.DEPTH_TEST);`**

# Z-Buffer Algorithm (cont.)

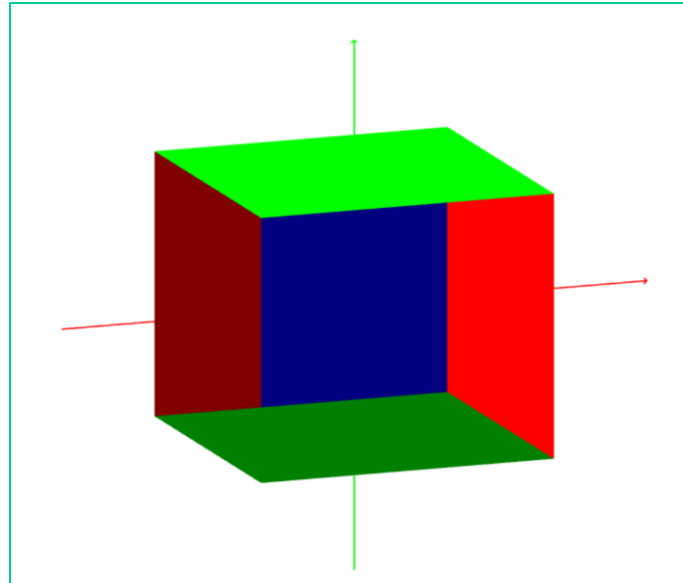
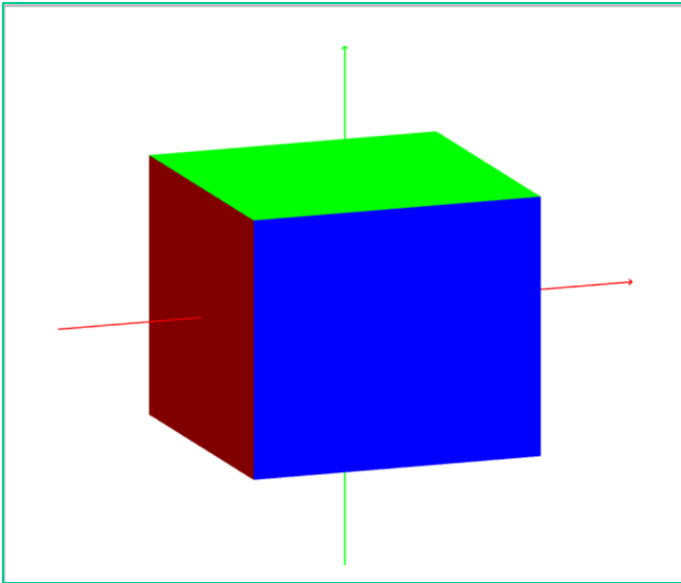
## ➤ 深度缓存的实例

➤ 左边: 开启了深度缓存消隐算法

➤ 渲染结果与图元绘制的先后顺序“无关”, 正确显示

➤ 右边: 没有开启深度缓存算法(默认)

➤ 渲染结果与图元绘制的先后顺序“相关”, 错误显示



```
function generateCube()  
{  
    quad( 1, 0, 3, 2 ); //Z正-前  
    quad( 4, 5, 6, 7 ); //Z负-后  
  
    quad( 2, 3, 7, 6 ); //X正-右  
    quad( 5, 4, 0, 1 ); //X负-左  
  
    quad( 6, 5, 1, 2 ); //Y正-上  
    quad( 3, 0, 4, 7 ); //Y负-下  
}
```

# Z-Buffer Algorithm (cont.)

## 优点:

- 不需要考虑整个场景的多边形面之间关系
- 简单稳定，利于硬件实现

## 缺点:

- 需要一个分辨率大小的额外的Z缓存空间。
- 在每个多边形占据的每个像素处都要计算深度值，计算量大，但可以优化。

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# Ray-casting Algorithm

## 光线投射:

模拟人的视觉效果，沿视线的路径跟踪场景的可见面，计算颜色并写入帧缓存  
(光线投射可以直接实现可见性判定而实现了消隐)

## 算法步骤:

1. 通过视点相机和投影平面上的象素点作一入射线。“发射投影线或发射光线”
2. 将任一投影线与场景中的所有多边形求交。“求交” (关键步骤)
3. 若有交点，则将所有交点按 $z$ 值的大小进行排序，取最近交点（离视点/相机最近）所属多边形的颜色作为像素颜色；若没有交点，则取背景的颜色。  
“输出最近交点颜色”

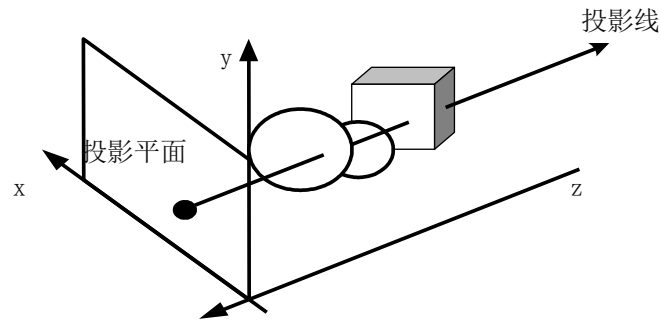


图 光线投射算法

# Ray-casting Algorithm(cont.)

## ➤ “求交” 举例：“光射线和平面求交”

1、假设射线起点为  $(x_0, y_0, z_0)$  ,方向向量为  $(d_1, d_2, d_3)$  ,则射线为

$$x = d_1 * t + x_0$$

$$y = d_2 * t + y_0$$

$$z = d_3 * t + z_0$$

2、假设物体表面是“平面”，设平面方程为  
 $ax + by + cz + d = 0$ .

3、将光线“射线”代入物体表面“平面方程”得  
 $a(d_1 * t + x_0) + b(d_2 * t + y_0) + c(d_3 * t + z_0) + d = 0$

即： $(a * d_1 + b * d_2 + c * d_3) * t + (a * x_0 + b * y_0 + c * z_0 + d) = 0$  //关于t的一次方程

➤ 若 $a * d_1 + b * d_2 + c * d_3 = 0$ 时，射线与平面平行，无交点。

➤ 否则解出 $t = -(a * x_0 + b * y_0 + c * z_0 + d) / (a * d_1 + b * d_2 + c * d_3)$

— 当 $t < 0$ 时，交点在射线的反向延长线上，视线看不到不求交。

— 当 $t \geq 0$ 时，交点在射线的正向上，视线看得见，算出交点。

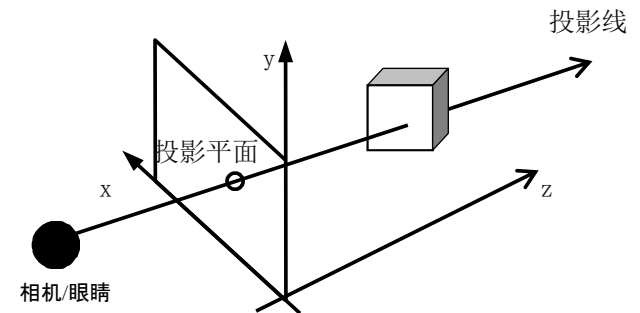


图 光线投射算法

# Summary

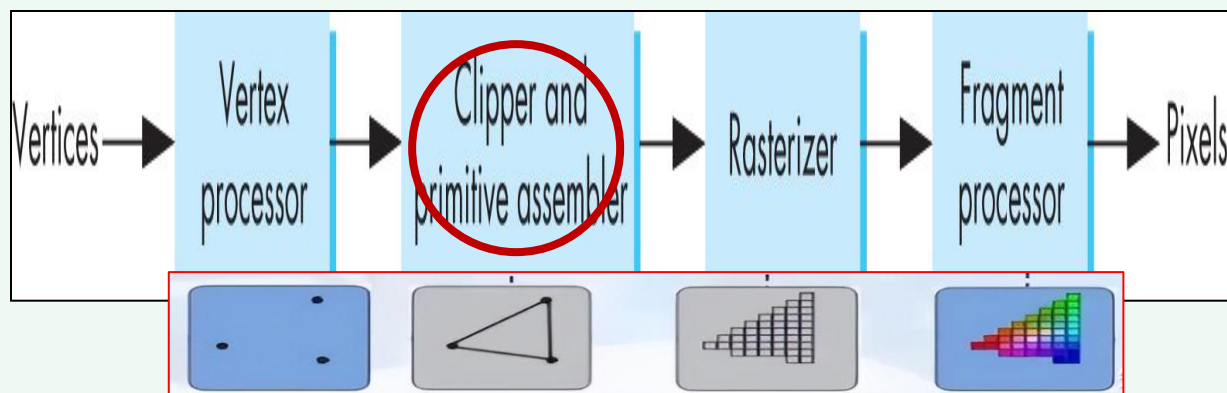
## ➤ 裁剪和图元组装：视见体外对象被剔除

### - 线段的裁剪算法

- Cohen-Sutherland Line Clipping 编码裁剪算法
- Liang-Barsky Line Clipping 梁永栋裁剪算法

### - 简单多边形的裁剪算法

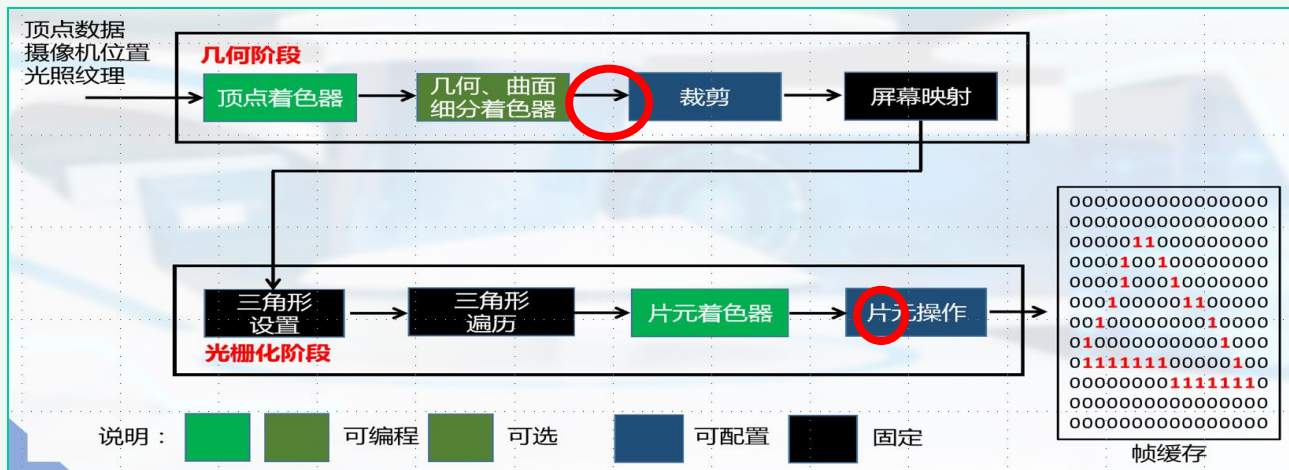
- Sutherland-Hodgeman Polygon Clipping 逐边裁剪算法



# Summary(cont.)

## ➤ 消隐算法：剔除后限免和消除隐藏的对象

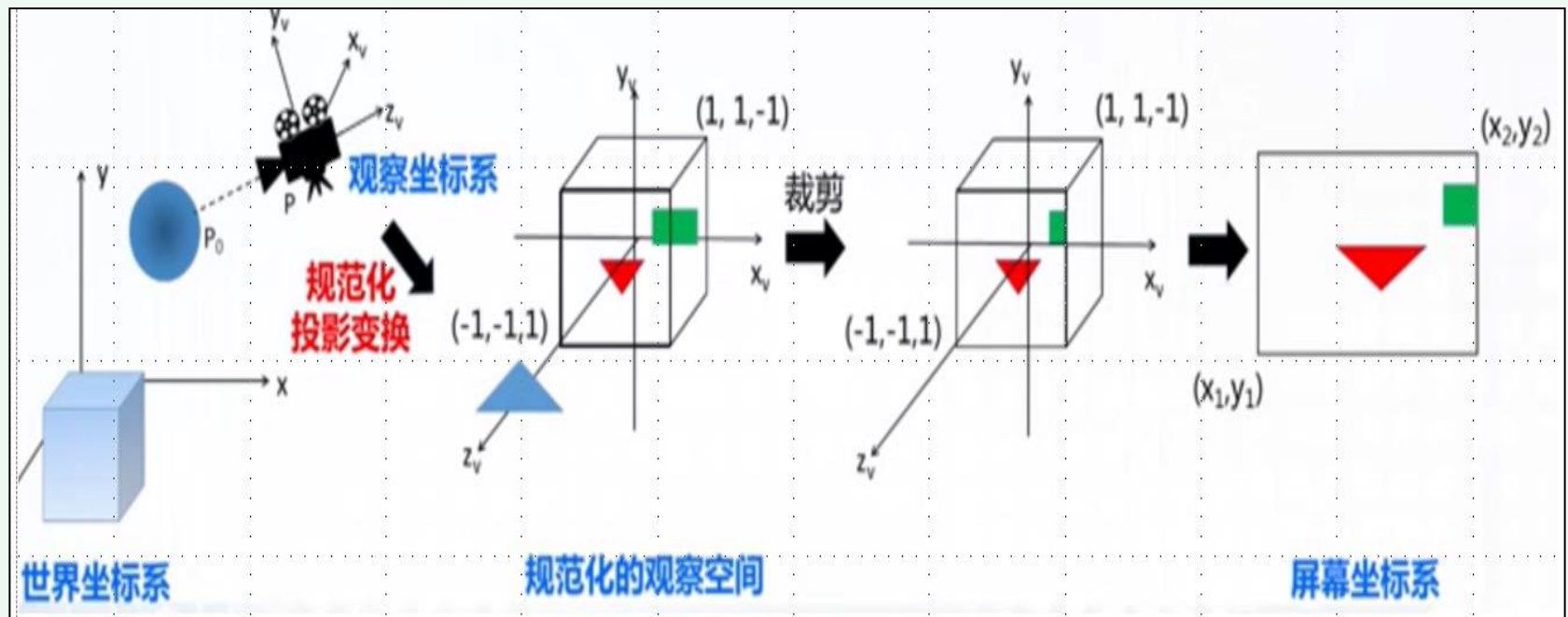
- 对象空间中的消隐算法
  - Back-face culling后向面剔除(着色前就消隐掉)
- 图像空间中的消隐算法
  - Depth-buffer 深度缓存算法(光栅化渲染中的消隐算法)
  - Ray-casting光线投射(光线跟踪渲染中的消隐算法)





# Summary (cont.)

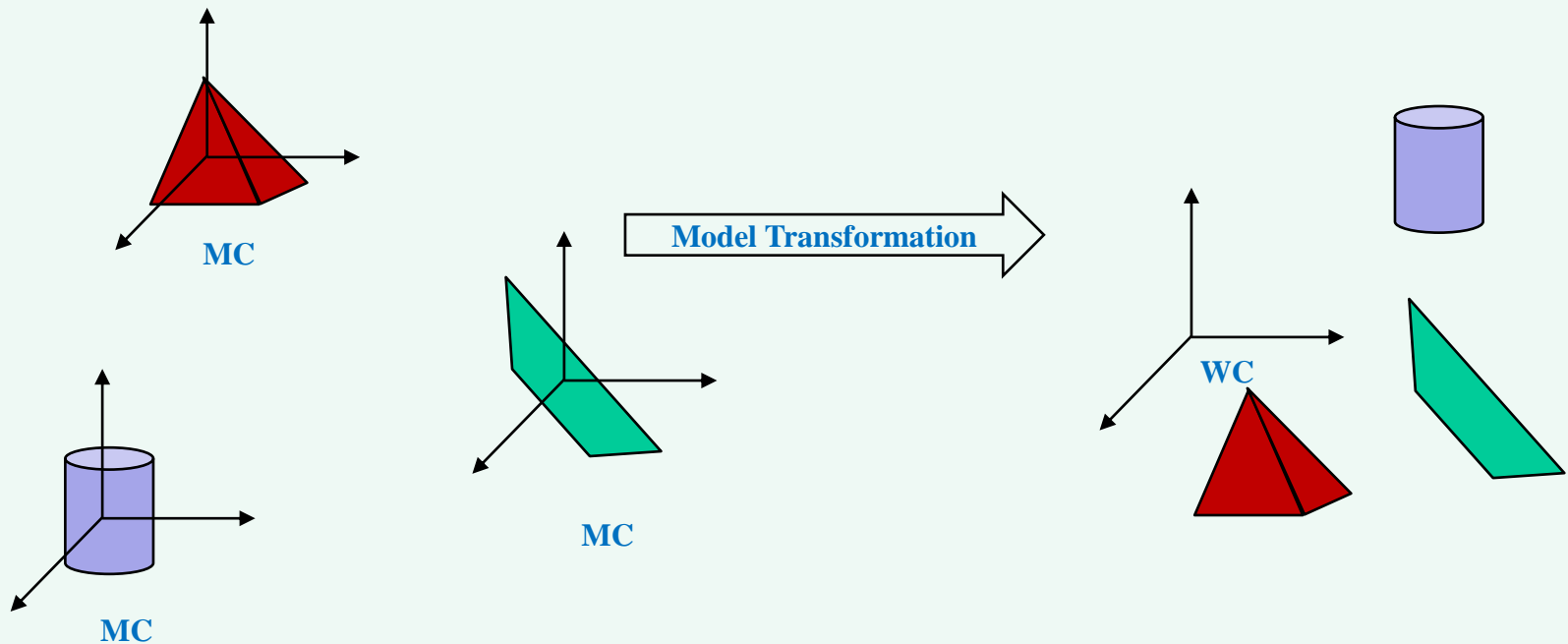
- View Process Transformations(观察流程中的变换)
- Clipping and Primitive Assembling (裁剪和图元组装)
- Hidden Removal(隐藏(对象)的消除)



# Summary(cont.)

## 1.from MC to WC

- Model Transformation (Instance Transformation)
  - Common : TRS

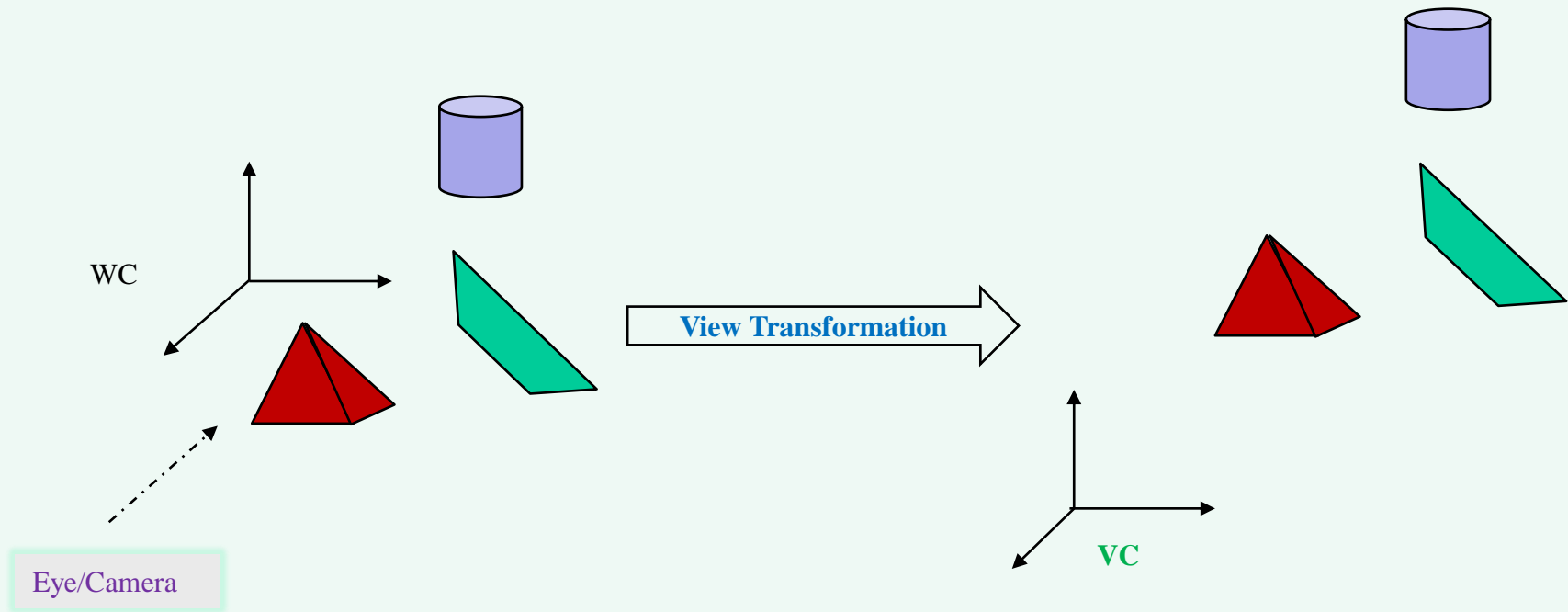


# Summary(cont.)

## 2. from WC to VC

### - View Transformation

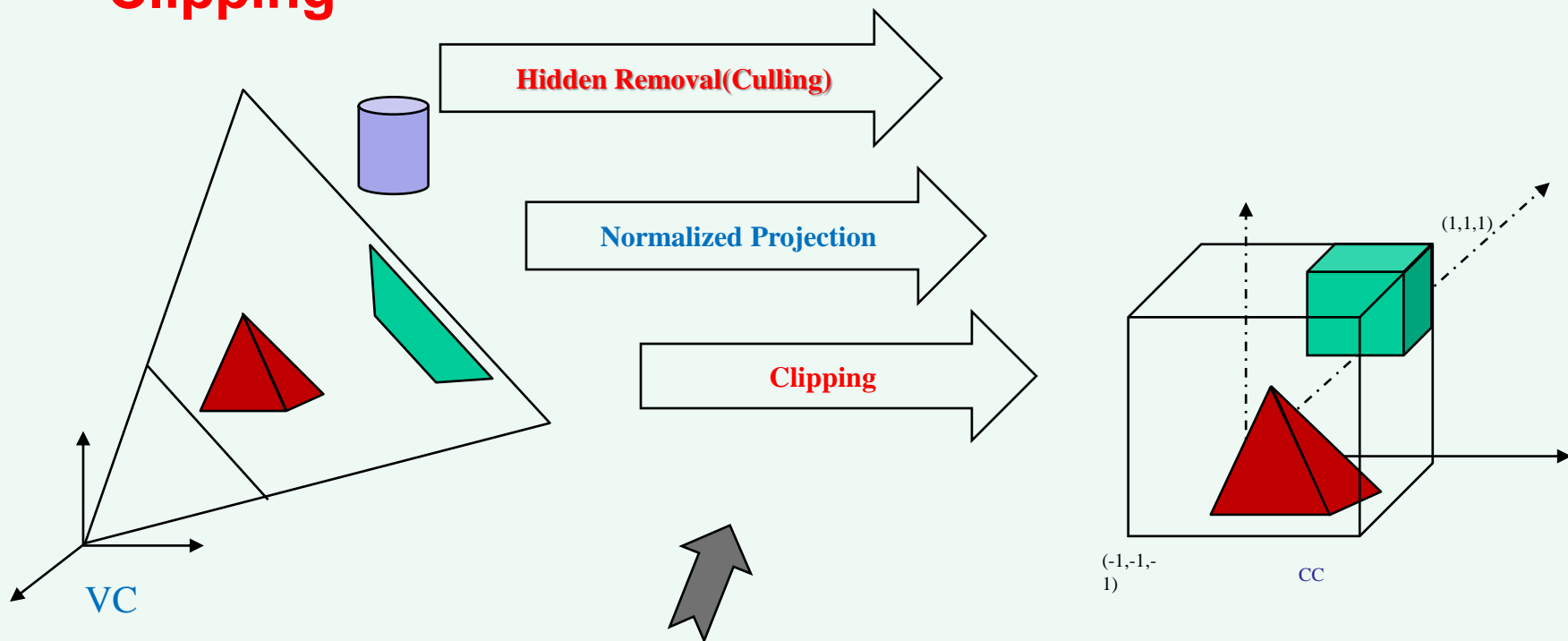
- *Common Lookat() =  $R(u,v,n)T(-eye)$*



# Summary(cont.)

## 3. from VC to CC

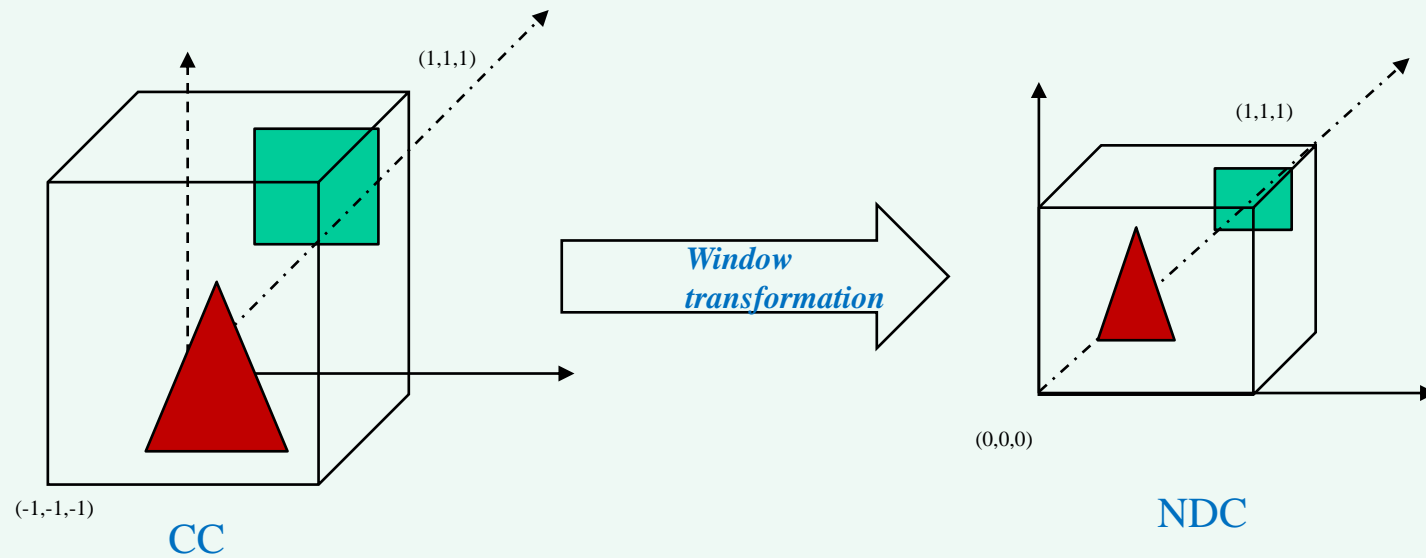
- **Hidden Removal(Culling剔除不可见面)**
- View Normalization Projection Transformation
- **Clipping**



# Summary(cont.)

## 4. from CC to NDC

### - *Window Viewport Transformation*



# Summary(cont.)

## 5.from NDC to DC

- *Window Viewport Transformation*
- *Perspective Division, Orthogonal Projection, Rasterization*
- *Hidden Removal Algorithm (Z-buffer algorithm)*

