

Recap

- Ray Tracing:

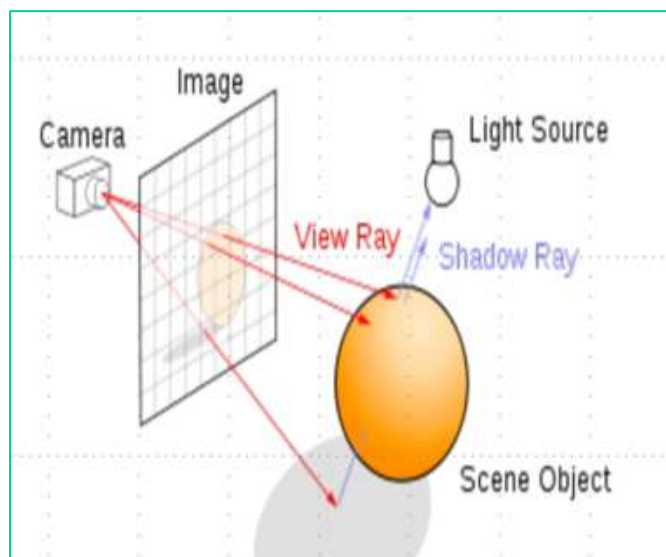
- 从视点出发对每像素发出光线, 交于场景物体找到最近交点, **计算交点颜色**作为像素颜色。

- Ray Casting:

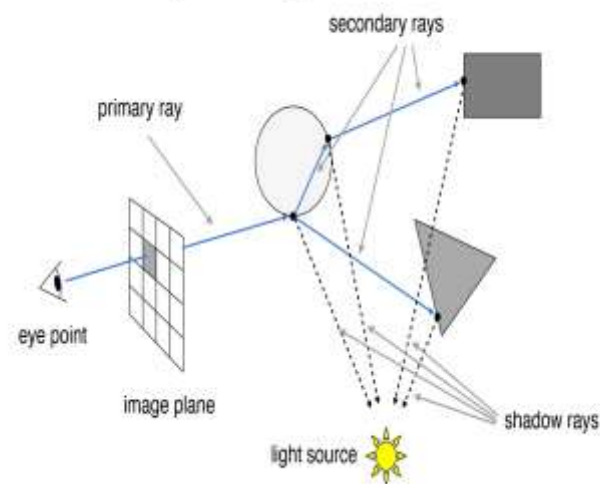
- 从视点出发对每像素发出光线, 交于场景物体找到最近交点, **计算交点颜色**: 交点弹射1次到光源, 判定是否阴影后进行简单光照着色

- Whitted Style Ray Tracing:

- 从视点出发对每像素发出光线, 交于场景物体找到最近交点, **计算交点颜色**: 包含以下两项:
 - 若交点是漫反射材质, 则弹射1次到光源, 判定是否被其它物体遮挡, 返回阴影后或简单光照着色。
 - 若交点是镜面或透明材质, 则沿着镜面反射或折射方向继续弹射发出光线跟踪下去(**递归**), 直到达到最大递归次数或光线衰减阈值停止跟踪。



Recursive Ray Tracing



递归的光线跟踪算法

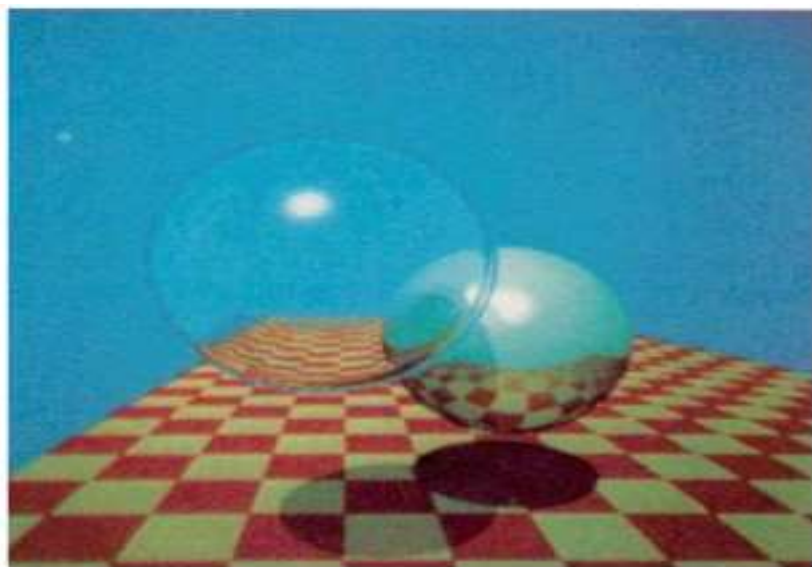
```
IntersectColor( vBeginPoint, vDirection)
{
    for each light
        Color = ambient color;
        Determine IntersectPoint;
        Color += local shading term;
    if(surface is reflective)
        color += reflect Coefficient *
            IntersectColor(IntersecPoint, Reflect Ray);
    else if ( surface is refractive)
        color += refract Coefficient *
            IntersectColor(IntersecPoint, Refract Ray);

    return color;
}
```

Recap(cont.)

◆ Whitted-Style RT 符合“物理原理”吗？



- Whitted-Style Ray Tracing is Wrong!
- PBR(Physical Based Rendering) is Correct!
 - Path Tracing is based on “Radiometry辐射度量学”



Spheres and Checkerboard, T. Whitted, 1979

Is Path Tracing Correct?

Yes, almost 100% correct, a.k.a. **PHOTO-REALISTIC**



Photo

Path traced:
global illumination

The Cornell box — <http://www.graphics.cornell.edu/online/box/compare.html>

GAMES101 AB Lingqi Yeri, UC Santa Barbara

Today' Theme

Ref: GAMES101 lecture 15, 清华胡事民课件

- PBR(Physical Based Rendering)
 - 光线亮度 I 的物理表示: radiance
 - 物体表面材质对光的反射系数 K 的表示: BRDF
 - 基于物理的反射光的计算模型: 反射方程, 以及渲染方程
 - 基于物理原理的照片级真实感渲染效果: 全局光照

Outline

- PBR(Physical Based Rendering)

- Radiometry(辐射度量学)
- BRDF(Bidirectional Reflectance Distribution Function双向反射分布函数)
- The Reflection Equation(反射方程)
- The Rendering Equation(渲染方程)
- Global Illumination(全局光照)

Radiometry(辐射度量学)

➤ Radiometry辐射度量学

- **Radiant energy**:辐射能量Q, (单位:焦耳Joule)
- **Radiant flux(power)**: 单位时间的光通量(能量), (单位:流明lm (= lumen) or 瓦特Watt)
 - Is the energy emitted , reflected, transmitted or received , per unit time
 - Photons flowing through a sensor in unit time在单位时间内流过传感器的光子数量

Radiant Energy and Flux (Power)

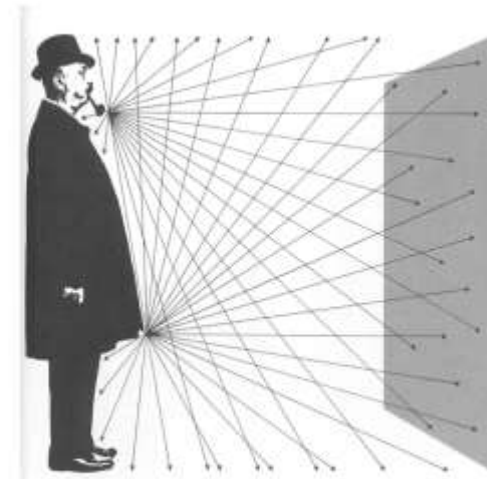
Definition: Radiant energy is the energy of electromagnetic radiation. It is measured in units of joules, and denoted by the symbol:

$$Q \text{ [J = Joule]}$$

Definition: Radiant flux (power) is the energy emitted, reflected, transmitted or received, per unit time.

$$\Phi \equiv \frac{dQ}{dt} \text{ [W = Watt] [lm = lumen]}^*$$

Flux – #photons flowing through a sensor in unit time

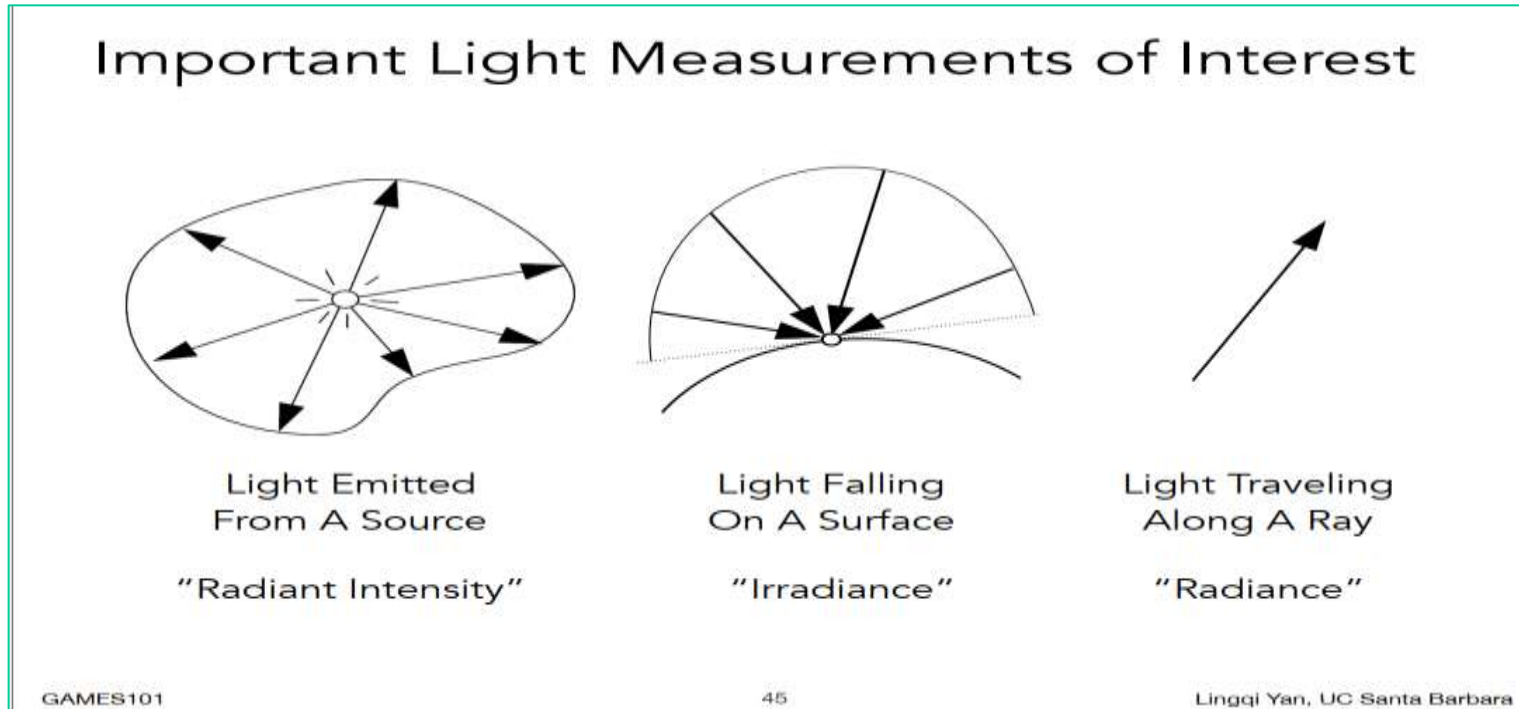


From London and Upton

Radiometry(cont.)

➤重点介绍三个对光照计算重要的物理量

1. Radiant Intensity: Light Emitted From A Source
2. Irradiance: Light Falling On A Surface
3. Radiance: Light Traveling Along A Ray *



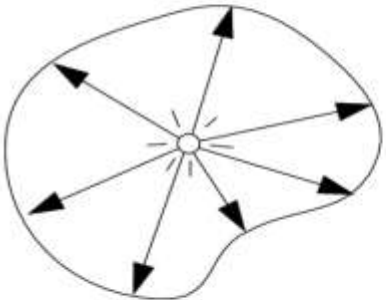
Radiometry(cont.)

1. Radiant Intensity

- 单位立体角内的能量 Power per unit solid angle (单位: 坎德拉 $\text{cd} = \text{lm}/\text{sr}$)
 - 单位: W/sr : 功率瓦/球面度 或 lm/sr : 流明/球面度
 - //注: Steradian (sr): 球面度/立体弧度, 是立体角 solid angle 的单位

Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit **solid angle (?)** emitted by a point light source.
(立体角)


$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$
$$\left[\frac{\text{W}}{\text{sr}} \right] \left[\frac{\text{lm}}{\text{sr}} = \text{cd} = \text{candela} \right]$$

The candela is one of the seven SI base units.

GAMES101 47 Lingqi Yan, UC Santa Barbara

Radiometry(cont.)

1. Radiant Intensity(cont.)

➤单位立体角内的能量Power per unit solid angle (单位: 坎德拉cd=lm/sr)

➤立体角 Solid Angel: “球面子区域面积A” 和 “球半径r平方”的比值

//注: Steradian(sr):球面度/立体弧度, 是立体角solid angel的单位

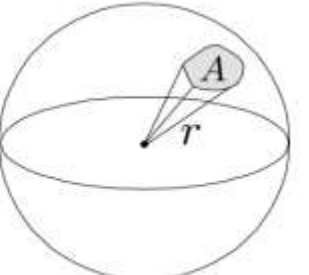
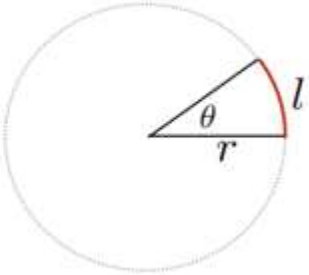
Angles and Solid Angles

Angle: ratio of subtended arc length on circle to radius

- $\theta = \frac{l}{r}$
- Circle has 2π radians

Solid angle: ratio of subtended area on sphere to radius squared

- $\Omega = \frac{A}{r^2}$
- Sphere has 4π steradians



GAMES101 48 Lingqi Yan, UC Santa Barbara

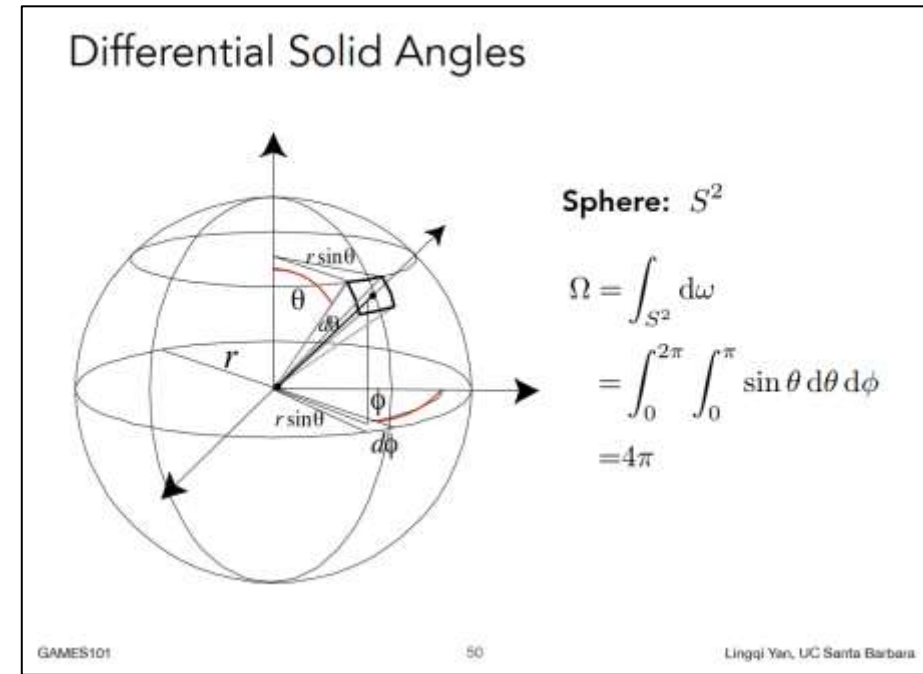
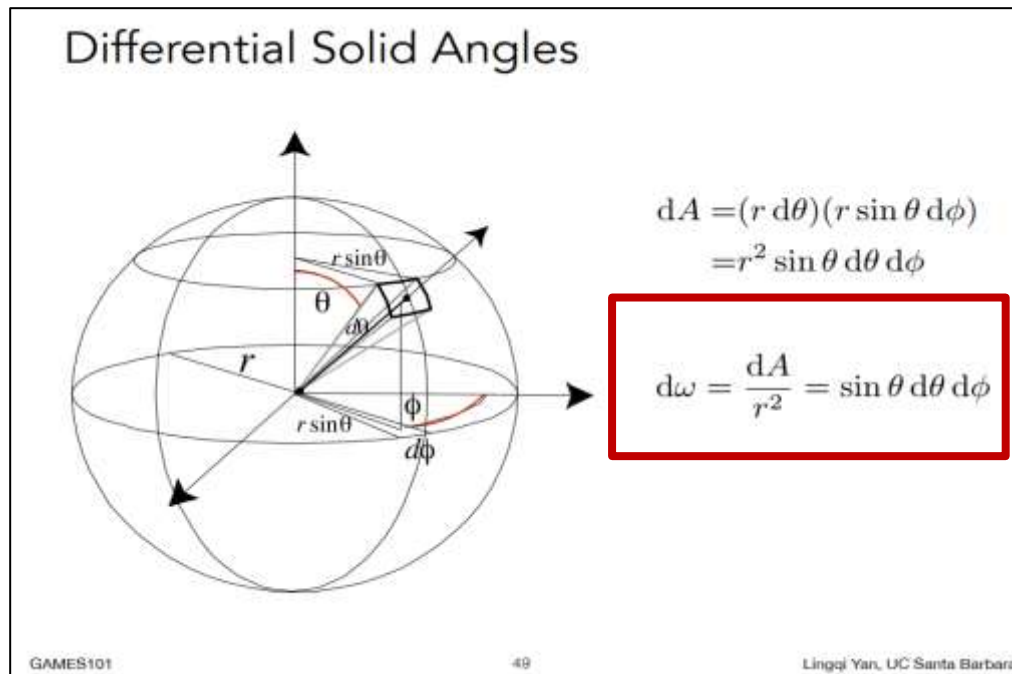
Radiometry(cont.)

$$\text{Radiant intensity } I(\omega) \equiv \frac{d\Phi}{d\omega}$$

- power per unit solid angle

1. Radiant Intensity(cont.)

- 单位立体角内的能量 Power per unit solid angle (单位: 坎德拉 cd = lm/sr)
- 立体角 Solid Angel: “球面子区域面积A” 和 “球半径r平方”的比值
- 微分立体角 Differential Solid Angle: $d\omega$



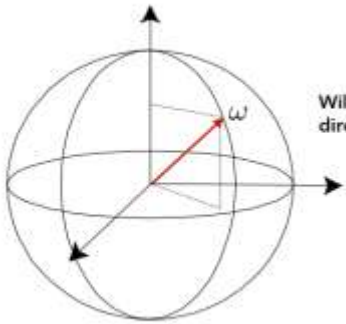
Radiometry(cont.)

Radiant intensity $I(\omega) \equiv \frac{d\Phi}{d\omega}$
• power per unit solid angle

1. Radiant Intensity(cont.)

- 单位立体角内的能量 Power per unit solid angle (单位: 坎德拉 $\text{cd} = \text{lm}/\text{sr}$)
 - 立体角 Solid Angle: “球面子区域面积A”和“球半径r平方”的比值
 - 微分立体角 Differential Solid Angle/单位立体角 unit solid angle
- “微分立体角”可表示任意空间中的“一个方向”;
- 若点光源各向同性, 则intensity表示光源沿着某个方向单位时间内所发射的能量

ω as a direction vector



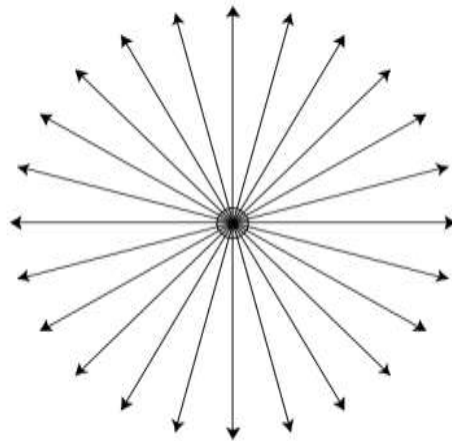
Will use ω to denote a direction vector (unit length)

GAMES101

51

Lingqi Yan, UC Santa Barbara

Isotropic Point Source



$$\Phi = \int_{S^2} I d\omega$$
$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

GAMES101

52

Lingqi Yan, UC Santa Barbara

Modern LED Light

Output: 815 lumens
(11W LED replacement
for 60W incandescent)

Radiant intensity?
Assume isotropic:
Intensity = 815 lumens / $4\pi \text{ sr}$
= 65 candelas



GAMES101

53

Lingqi Yan, UC Santa Barbara

Radiometry(cont.)

• 1. Radiant Intensity(cont.)

- 总结: Intensity是光源在单位时间内, 向某个方向(微立体角)发出的能量
- Intensity is power per unit solid angel

Radiant energy Q [J = Joule] (barely used in CG)

- the energy of electromagnetic radiation

Radiant flux (power) $\Phi \equiv \frac{dQ}{dt}$ [W = Watt] [lm = lumen]

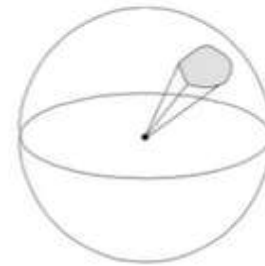
- Energy per unit time

Radiant intensity $I(\omega) \equiv \frac{d\Phi}{d\omega}$

- power per unit solid angle

Solid Angle $\Omega = \frac{A}{r^2}$

- ratio of subtended area on sphere to radius squared



Radiometry(cont.)

2. Irradiance

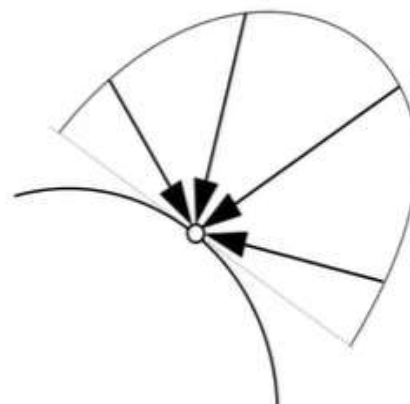
- “power(perpendicular/projected) per unit area” incident on a surface point
入射(垂直/投影)到表面点的单位面积的能量, (单位:勒克斯lux=lm / m²)

Irradiance

Definition: The irradiance is the power per (perpendicular/projected) unit area incident on a surface point.

$$E(\mathbf{x}) \equiv \frac{d\Phi(\mathbf{x})}{dA}$$

$$\left[\frac{\text{W}}{\text{m}^2} \right] \quad \left[\frac{\text{lm}}{\text{m}^2} = \text{lux} \right]$$



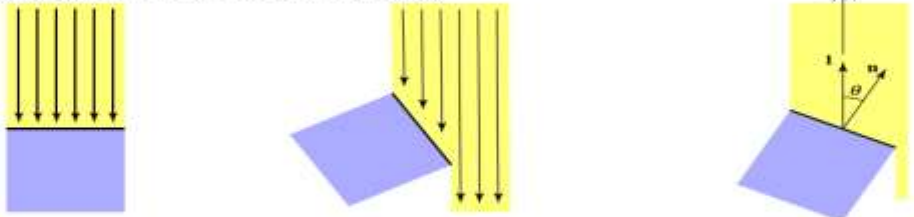
Radiometry(cont.)

2.Irradiance(cont.)

- “power(perpendicular/projected) per unit area” incident on a surface point
入射到物体表面的单位面积接收的能量（单位：勒克斯lux=lm/m²）
 - “Irradiance”正比于“光线方向和物体表面法向量之间的夹角余弦”或可看作物体表面接收的是光线垂直于物体表面的能量分量（如：阳光垂直照射区域光能接收多，如赤道附近多，而两级少）

Lambert's Cosine Law

Irradiance at surface is proportional to cosine of angle between light direction and surface normal.
(Note: always use a unit area, the cosine applies on Φ)



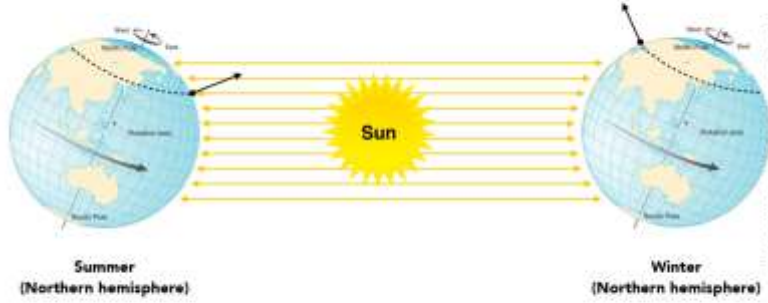
Top face of cube receives a certain amount of power
$$E = \frac{\Phi}{A}$$

Top face of 60° rotated cube receives half power
$$E = \frac{1}{2} \frac{\Phi}{A}$$

In general, power per unit area is proportional to $\cos \theta = l \cdot n$
$$E = \frac{\Phi}{A} \cos \theta$$

GAMES101 10 Lingqi Yan, UC Santa Barbara

Why Do We Have Seasons?



Summer (Northern hemisphere) Winter (Northern hemisphere)

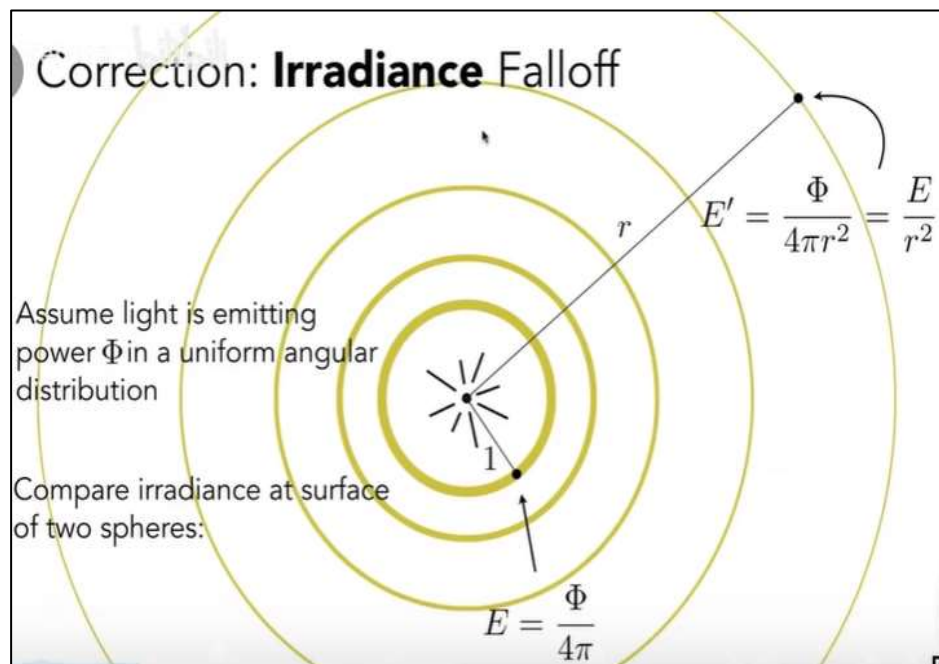
Earth's axis of rotation: ~23.5° off axis

GAMES101 11 Lingqi Yan, UC Santa Barbara

Radiometry(cont.)

2. Irradiance

- “power(perpendicular/projected) per unit area” incident on a surface point
- 入射到物体表面的单位面积接收的能量（单位：勒克斯lux=lm/m²）
 - “Irradiance”正比于“光线方向和物体表面法向量之间的夹角余弦”或可看作物体表面接收的是光线垂直于物体表面的能量分量（如：阳光垂直照射区域光能接收多，如赤道附近多，而两级少）
 - 光能量传播会有衰减：随着接收光的物体表面离光源越远，球面面积会越来越大，而单位面积接收的能量会越来越小。如图：半径为1的球面的能量E和半径为r的球面能量E'之间的关系是： $E' = E/r^2$



Radiometry(cont.)


3. Radiance*

- The fundamental field quantity that describes the distribution of light in an environment(描述光在环境中的分布的基本场量)
 - Radiance is the quantity associated with a ray(指与一条光线相关联的量)
 - Rendering is all about computing radiance(渲染就是关于计算radiance的)
- Definition: Power emitted, reflected, transmitted or received by a surface , Per unit solid angle , per projected unit area. 单位: 尼特 $nit = cd / m^2 = lm/sr m^2$

Radiance

Radiance is the fundamental field quantity that describes the distribution of light in an environment


- Radiance is the quantity associated with a ray
- Rendering is all about computing radiance



Light Traveling Along A Ray

Radiance

Definition: The radiance (luminance) is the power emitted, reflected, transmitted or received by a surface, per unit solid angle, per projected unit area.


$$L(p, \omega) \equiv \frac{d^2 \Phi(p, \omega)}{d\omega dA \cos \theta}$$

$\cos \theta$ accounts for projected surface area

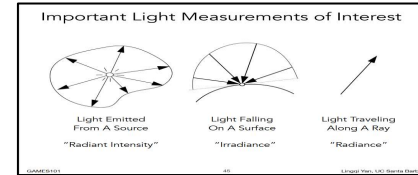
$$\left[\frac{W}{sr m^2} \right] \left[\frac{cd}{m^2} = \frac{lm}{sr m^2} = nit \right]$$

Radiometry(cont.)

3. Radiance(cont.)

➤ Radiance可以表示入射光线, 也可表示出射光线~

- Incident Radiance(入射辐射/入射光线) $L(p, \omega)$: 从一个微立体角进来, 被一个微面积所接收到的能量
- Exiting Radiance(出射辐射/出射光线) $L(p, \omega)$: 从一个微面积发出, 向一个微立体角发射出去的能量



$$L(p, \omega) \equiv \frac{d^2\Phi(p, \omega)}{d\omega dA \cos \theta}$$

$\cos \theta$ accounts for projected surface area

$$\left[\frac{W}{sr \cdot m^2} \right] \left[\frac{cd}{m^2} = \frac{lm}{sr \cdot m^2} = nit \right]$$

Irradiance

Definition: The irradiance is the power per (perpendicular/projected) unit area incident on a surface point.

$$E(x) \equiv \frac{d\Phi(x)}{dA}$$

$$\left[\frac{W}{m^2} \right] \left[\frac{lm}{m^2} = lux \right]$$

Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle (?) emitted by a point light source.

(立体角)

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$

$$\left[\frac{W}{sr} \right] \left[\frac{lm}{sr} = cd = candelas \right]$$

Incident Radiance

Incident radiance is the irradiance per unit solid angle arriving at the surface.

$$L(p, \omega) = \frac{dE(p)}{d\omega \cos \theta}$$

i.e. it is the light arriving at the surface along a given ray (point on surface and incident direction).

Exiting Radiance

Exiting surface radiance is the intensity per unit projected area leaving the surface.

$$L(p, \omega) = \frac{dI(p, \omega)}{dA \cos \theta}$$

e.g. for an area light it is the light emitted along a given ray (point on surface and exit direction).

Radiometry(cont.)

3. Radiance(cont.)

➤ Irradiance vs. Radiance

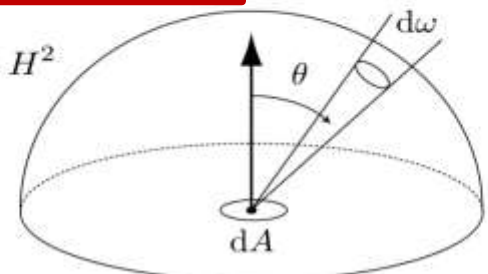
- Irradiance: total power received by area dA
 - 单位面积/微表面收到半球内入射的所有光线能量 $E(p)$ = 从半球立体角 2π (各个可见方向) 来的光能量和
 - Radiance: power received by area dA from "direction" $d\omega$
 - 单位面积/微表面入射/出射的某方向光线能量 $L(p, \omega)$ = 单位立体角(某方向)单位平面的入射/出射的光能
- 两者之间的关系(从入射光角度看): $dE(p, \omega) = L_i(p, \omega) * \cos\theta * d\omega$

Irradiance vs. Radiance

Irradiance: total power received by area dA
Radiance: power received by area dA from "direction" $d\omega$

$$dE(p, \omega) = L_i(p, \omega) \cos \theta d\omega$$
$$E(p) = \int_{H^2} L_i(p, \omega) \cos \theta d\omega$$

Unit Hemisphere: H^2



GAMES101 19 Lingqi Yan, UC Santa Barbara

Outline

- PBR(Physical Based Rendering)

- Radiometry(辐射度量学)
- BRDF(Bidirectional Reflectance Distribution Function双向反射分布函数)
- The Reflection Equation(反射方程)
- The Rendering Equation(渲染方程)
- Global Illumination(全局光照)

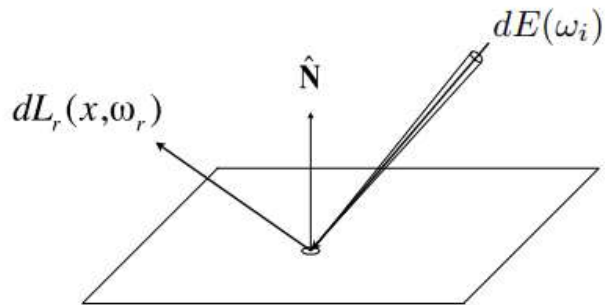
BRDF

➤ BRDF(Bidirectional Reflectance Distribution Function)双向反射分布函数

- 定义: $f(\omega_i \rightarrow \omega_r) = dL_r(\omega_r) / dE_i(\omega_i)$ // 出射方向光线能量/入射方向光线能量
- represents how much light is reflected into outgoing direction from each incoming direction
- 描述光线如何和物体表面作用, 表示光从每个入射方向朝每个出射方向反射了多少光(即描述物体表面材质属性的物理量, 即“反射率”)

Reflection at a Point

Radiance from direction ω_i turns into the power E that dA receives
Then power E will become the radiance to any other direction ω_o .

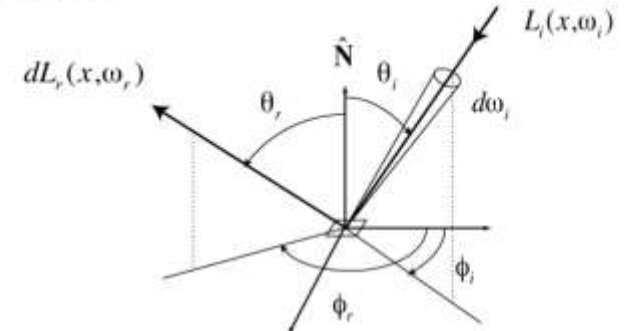


Differential irradiance incoming: $dE(\omega_i) = L(\omega_i) \cos \theta_i d\omega_i$

Differential radiance exiting (due to $dE(\omega_i)$): $dL_r(\omega_r)$

BRDF

The Bidirectional Reflectance Distribution Function (BRDF) represents how much light is reflected into each outgoing direction ω_r from each incoming direction



$$f_r(\omega_i \rightarrow \omega_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)} = \frac{dL_r(\omega_r)}{L_i(\omega_i) \cos \theta_i d\omega_i} \left[\frac{1}{\text{sr}} \right]$$

BRDF(cont.)

➤BRDF(Bidirectional Reflectance Distribution Function)双向反射分布函数

➤不同材质的BRDF大不相同：

- 如果是光滑的镜面物体表面，那么在理想镜面反射方向上的值会比较大，而其它出射方向上小。
- 如果是粗糙的漫反射物体表面，各个出射方向的比值都相同，即分布均匀。

BRDF 的定义

- BRDF 是关于入射光方向和反射光方向的**四维实值**函数，它等于反射方向的光亮度和沿入射方向的入射光的辉度之比：

$$f(\omega_i \rightarrow \omega_r) = \frac{dL_r(\omega_r)}{dE_i}$$

– 可以写成关于入射光的光亮度的形式：

$$f(\omega_i \rightarrow \omega_r) = \frac{dL_r(\omega_r)}{L_i(\omega_i) \cos \omega_i d\omega_i}$$

大千世界，物质材料多种多样，如何形式化描述这些物体的表面属性？如何真实地绘制渲染？



今天的主题：一个关于光线如何被表面反射的物理描述，也就是**BRDF**

BRDF(cont.)

➤ BRDF(Bidirectional Reflectance Distribution Function)双向反射分布函数

➤ 可逆性: 交换入射光和出射光角色, 不会改变BRDF值

➤ 能量守恒性质: 入射光能量=吸收能量+出射光能量, 即: 出射光线能量<=入射光线能量

BRDF 的可逆性



• 可逆性 (Reciprocity)

- BRDF 的可逆性源自于 Helmholtz 光路可逆性 (Helmholtz Reciprocity Rule)
- BRDF 的可逆性是说: 交换入射光与反射光的角色, 并不会改变 BRDF 的值:

$$f(\omega_i \rightarrow \omega_r) = f(\omega_r \rightarrow \omega_i)$$

能量守恒性质



- BRDF 需要遵循的另一个物理定律是能量守恒定律, 能量守恒定律指出: 入射光的能量与出射光的总能量应该相等
- 能量守恒方程如下:

$$Q_{incoming} = Q_{reflected} + Q_{absorb} + Q_{transmitted}$$

- 由此我们知道:

$$Q_{reflected} \leq Q_{incoming}$$

Outline

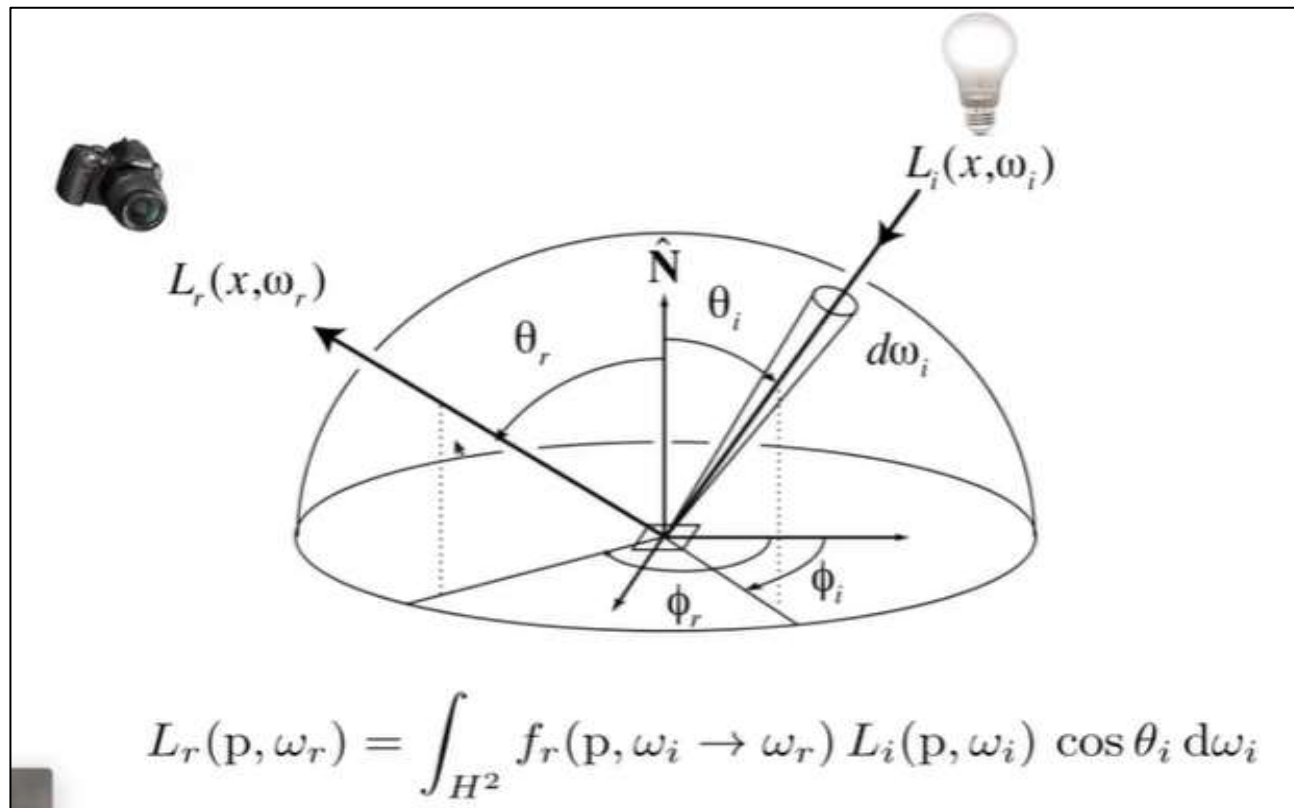
- PBR(Physical Based Rendering)

- Radiometry(辐射度量学)
- BRDF(Bidirectional Reflectance Distribution Function双向反射分布函数)
- The Reflection Equation(反射方程)
- The Rendering Equation(渲染方程)
- Global Illumination(全局光照)

Reflection Equation

• The Reflection Equation(反射方程)

- 对某表面某点p, 对半球内所有入射方向光在反射方向上所反射的能量求和, 得到反射光能量
 - 即: 出射光 L_r = 半球积分{反射率 $f_r(w_i \rightarrow w_r)$ *入射光 L_i 的垂直分量} // $f(w_i \rightarrow w_r)$ 采用BRDF
- (注:这里出射光里只考虑该点的反射光, 不考虑折射光)



Reflection Equation(cont.)

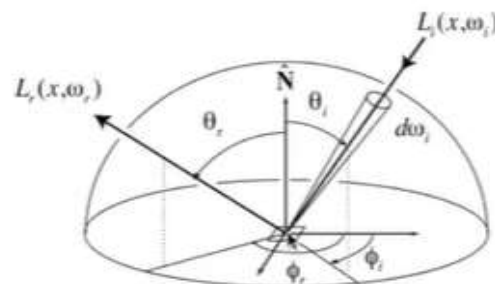
➤ Recursive Reflection Equation(带递归的反射方程)

- 入射光 L_i 可能是从“光源”来的一次弹射光(直接光照:光源光)
- 入射光 L_i 可能是从其它“物体”来的多次弹射后光(间接光照:镜面反射光, 漫反射光)➔递归

Challenge: Recursive Equation

Reflected radiance depends on incoming radiance

$$\boxed{L_r(p, \omega_r)} = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) \boxed{L_i(p, \omega_i)} \cos \theta_i d\omega_i$$



But incoming radiance depends on reflected radiance (at another point in the scene)

Outline

- PBR(Physical Based Rendering)

- Radiometry(辐射度量学)
- BRDF(Bidirectional Reflectance Distribution Function双向反射分布函数)
- The Reflection Equation(反射方程)
- The Rendering Equation(渲染方程)
- Global Illumination(全局光照)

Rendering Equation(渲染方程)

➤ Kaijiya: 1986 “The Rendering Equation”

➤ 反射方程和渲染方程都是由吉姆·卡吉雅提出的。

渲染方程之父：Jim Kajiya



- Jim Kajiya在渲染和计算机图形硬件设计领域有着诸多贡献，获得Steven Anson Coons奖。
- 他最为著名的工作就是1986年在SIGGRAPH以独立作者身份发表的论文：The Rendering Equation。

THE RENDERING EQUATION

James T. Kajiya
California Institute of Technology
Pasadena, Ca. 91125



Rendering Equation (Kajiya 86)



Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Rendering Equation(cont.)

• The Rendering Equation(渲染方程)(cont.)

- 反射方程是渲染方程的一个特例，渲染方程是反射方程的扩展和一般化形式

Rendering Equation

Surfaces (interreflection)

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

GAMES101 30 Lingqi Yan, UC Santa Barbara

The Rendering Equation

Re-write the reflection equation:

$$L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i d\omega_i$$

by adding an Emission term to make it general!

The Rendering Equation

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega^+} L_i(p, \omega_i) f_r(p, \omega_i, \omega_o) (n \cdot \omega_i) d\omega_i$$

Note: now, we assume that all directions are pointing **outwards**!

Outline

- PBR(Physical Based Rendering)

- Radiometry(辐射度量学)
- BRDF(Bidirectional Reflectance Distribution Function双向反射分布函数)
- The Reflection Equation(反射方程)
- The Rendering Equation(渲染方程)
- Global Illumination(全局光照)

Global Illumination(全局光照)

➤ Global illumination = Emission + Direct illumination + Indirect Illumination

Rendering Equation as Integral Equation

$$L_i(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_i(x, \omega_i) f(x, \omega_i, \omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v) K(u, v) dv$$

Kernel of equation

Linear Operator Equation

$$l(u) = e(u) + \int l(v) K(u, v) dv$$

Kernel of equation
Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation [or system of simultaneous linear equations] (L, E are vectors, K is the light transport matrix)

Ray Tracing and extensions

- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1}E$$

Binomial Theorem

$$L = (I + K + K^2 + K^3 + \dots)E$$

$$L = E + KE + K^2E + K^3E + \dots$$

Ray Tracing

$$L = E + KE + K^2E + K^3E + \dots$$

↓

Emission directly
From light sources

↓

Direct Illumination
on surfaces

↓

Indirect Illumination
(One bounce indirect)
[Mirrors, Refraction]

↓

(Two bounce indirect illum.)

Shading in Rasterization



取自《Physically Based Rendering, Second Edition》From Theory to Implementation 的封面

Global Illumination(全局光照)(cont.)

➤ Global Illumination 全局光照

$$L = E + KE + K^2E + K^3E + \dots$$

➤ 弹射1次得到直接光照效果, 弹射2次以上就包含了间接光照, 才是全局光照。

➤ 思考: 无限次弹射后, 会收敛到某个亮度不再变化, 还是会一直变亮, 直到过曝全白?



Global Illumination(全局光照)(cont.)

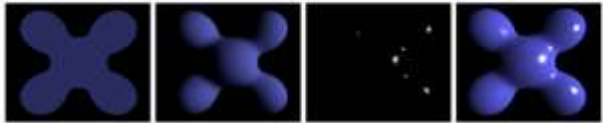
各种光照模型比较:

Blinn-Phong: local illumination

Whitted-Style Model: fake-global illumination

Rendering Equation: real-global illumination

Blinn-Phong Reflection Model



Ambient + Diffuse + Specular = Blinn-Phong Reflection

$$L = L_a + L_d + L_s$$
$$= k_a I_a + k_d (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{l}) + k_s (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{h})^p$$

GAMES101 10 Jingyi Yan, USC Santa Barbara

Whitted光透射模型 Phong模型 + 透射光强 + 反射光强

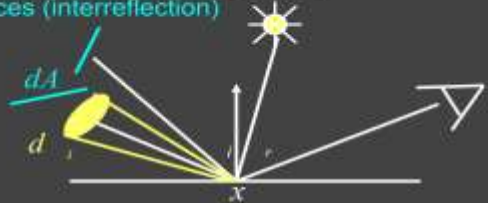
$$I = I_a K_a + I_p K_d (L \cdot N) + I_p K_s (R \cdot V)^n + I_t K_t + I_r K_r$$

Blin-Phong模型 + 透射光强 + 反射光强

$$I = I_a K_a + I_p K_d (L \cdot N) + I_p K_s (H \cdot N)^n + I_t K_t + I_r K_r$$

Rendering Equation

Surfaces (interreflection)


$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_r) f(x, \omega_r, \omega_r') \cos \theta_r d\omega_r'$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

GAMES101 30 Jingyi Yan, USC Santa Barbara

Summary

- PBR(Physical Based Rendering)

- Radiometry: intensity, irradiance, radiance
- BRDF(Bidirectional Reflectance Distribution Function)
- The Reflection Equation(反射方程)
- The Rendering Equation(渲染方程)
- Global Illumination(全局光照)
-(next)

Summary(cont.)

➤ **Next:** How to solve an integral numerically ?

But the rendering equation is correct

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega^+} L_i(p, \omega_i) f_r(p, \omega_i, \omega_o) (n \cdot \omega_i) d\omega_i$$

But it involves

- Solving an integral over the hemisphere, and
- Recursive execution

How do you solve an integral numerically?