

In [17]:

```
%load_ext autoreload
%autoreload 2
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
import time
import imageio.assignment4 as im

%matplotlib notebook
```

The autoreload extension is already loaded. To reload it, use:
%reload_ext autoreload

In [2]:

```
y = plt.imread('assets/leopard.png')
```

1 Bilateral Filter

Question 1

In [3]:

```
x = plt.imread('assets/castle.png')
sig = 10 / 255
y = x + sig * np.random.randn(*x.shape)
```

In [4]:

```
np.maximum([1,-4,3],0)
```

Out[4]:

```
array([1, 0, 3])
```

Question 2 and 3

In [5]:

```
def kernel_func(alpha,C, h, sig):
    num = -np.maximum((alpha-2*h*sig**2),0)*(C**0.5)
    den = (2**1.5)*h*sig**2
    expon = np.exp(num/den)
    return expon
```

In [6]:

```
def bilateral_naive(y, sig, s1=2, s2=2, h=1):
    n1, n2 = y.shape[:2]
    c = y.shape[2] if y.ndim == 3 else 1
    x = np.zeros(y.shape)
    Z = np.zeros((n1, n2, *[1] * (y.ndim - 2)))
    for i in range(s1, n1-s1):
        for j in range(s2, n2-s2):
            for k in range(-s1, s1 + 1):
                for l in range(-s2, s2 + 1):
                    dist2 = ((y[i + k, j + l] - y[i, j])**2).mean()
                    # complete
                    alpha =dist2

                    kern_alpha = ((kernel_func(alpha,c,h,sig)))
```

```

        r0 = (kern_alph*y[i + k, j + l])

        x[i,j,...] += (r0)
        Z[i,j,...] += (kern_alph)

Z[Z == 0] = 1
x = x / Z
return x

```

In [9]:

```

t1 = time.time()
naive_filt = bilateral_naive(y,sig,s1 =2, s2=2, h=1)
t2 = time.time()
tn =t2-t1
print(" Bilateral naive takes {:.2f} s ".format(tn))

```

Bilateral naive takes 145.87 s

Question 4

In [7]:

```

def bilateral(y, sig, s1=10, s2=10, h=1, boundary='mirror'):
    n1, n2 = y.shape[:2]
    c = y.shape[2] if y.ndim == 3 else 1
    x = np.zeros(y.shape)
    Z = np.zeros((n1, n2, *[1] * (y.ndim - 2)))

    for k in range(-s1, s1 + 1):
        for l in range(-s2, s2 + 1):
            y_shift = im.shift(y,-k,-l,boundary)
            dist2 = np.mean(((y_shift - y)**2),axis =-1)
            # complete
            alpha =dist2

            kern_alph = ((kernel_func(alpha,c,h,sig)))
            r0 = (y_shift.transpose()*kern_alph.transpose()).transpose()
            #r0 = (kern_alph*y_shift)
            kern_alph = kern_alph.reshape(Z.shape)
            x += (r0)
            Z += (kern_alph)

    Z[Z == 0] = 1
    x = x / Z
    return x

```

Question 5

In [165]:

```

x = plt.imread('assets/castle.png')
sig = 10 / 255
y = x + sig * np.random.randn(*x.shape)

t1 = time.time()
bil_filt = bilateral(y,sig,s1 =7, s2=7, h=1,boundary = 'mirror')
t2 = time.time()
tn =t2-t1
print(" Bilateral takes {:.2f} s ".format(tn))

```

Bilateral takes 4.13 s

In [166]:

```

fig, axes = plt.subplots(ncols=4, figsize=(10, 10))
im0.show(x, ax=axes[0])
axes[0].set_title('Original')
im0.show(y, ax=axes[1])

```

```

axes[1].set_title('Noisy')
im0.show(naive_filt, ax=axes[2])
axes[2].set_title('Naive')
im0.show(bil_filt, ax=axes[3])
axes[3].set_title('Bilateral')
fig.show()

```

Original



Noisy



Naive



Bilateral



Question 6

The function is slower with $s1=s2=7$ but the filtered output appears superior. If the noise is 10 times greater then the bilateral filter appears to blur the image and does not do much to reduce the noise

In [167]:

```

x = plt.imread('assets/castle.png')
sig = 100 / 255
y = x + sig * np.random.randn(*x.shape)

t1 = time.time()
bil_filt = bilateral(y,sig,s1=2, s2=2, h=1,boundary = 'mirror')
t2 = time.time()
tn =t2-t1
print(" Bilateral takes {:.2f} s ".format(tn))

```

```
im0.show(bil_filt)
```

Bilateral takes 0.44 s



Out[167]:

<matplotlib.image.AxesImage at 0x7f54779dd278>

2 NL-means Filter

Question 7

In [14]:

```
def kernel_func2(alpha,C, h, sig,P):
    num = -np.maximum((alpha-2*h*sig**2),0)*((C*P)**0.5)
    den = (2**1.5)*h*sig**2
    expon = np.exp(num/den)
    return expon

def nlmeans_naive(y, sig, s1=2, s2=2, p1=1, p2=1, h=1):
    n1, n2 = y.shape[:2]
    c = y.shape[2] if y.ndim == 3 else 1
    x = np.zeros(y.shape)
    Z = np.zeros((n1, n2, *[1] * (y.ndim - 2)))
    P = (2*p1+1)*(2*p2+1)
    for i in range(s1, n1-s1-p1):
        for j in range(s2, n2-s2-p2):
            for k in range(-s1, s1 + 1):
                for l in range(-s2, s2 + 1):
                    dist2 = 0
                    for u in range(-p1, p1 + 1):
                        for v in range(-p2, p2 + 1):
                            # complete
                            dist2 += ((y[i + k+u, j + l+v] - y[i, j])**2).mean()
                    # complete
                    alpha =dist2/P

                    kern_alph = ((kernel_func2(alpha,c,h,sig,P)))

                    r0 = (kern_alph*y[i + k, j + l])

                    x[i,j,...] += (r0)
                    Z[i,j,...] += (kern_alph)

    Z[Z == 0] = 1
    x = x / Z
    return x
```

Question 8

Question 8

In [123]:

```
t1 = time.time()
naive_n1 = nlmeans_naive(y,sig,s1 =2, s2=2, p1 =1, p2 =1, h=1)
t2 = time.time()
tn =t2-t1
print(" N1 means naive takes {:.2f} s ".format(tn))
```

N1 means naive takes 592.04 s

Question 9

The patch size is the size of the box kernel used in convolution given by $(2p_1 + 1) * (2p_2 + 1)$

In [98]:

```
def nlmeans(y, sig, s1=7, s2=7, p1=None, p2=None, h=1, boundary='mirror'):
    p1 = (1 if y.ndim == 3 else 2) if p1 is None else p1
    p2 = (1 if y.ndim == 3 else 2) if p2 is None else p2
    n1, n2 = y.shape[:2]
    c = y.shape[2] if y.ndim == 3 else 1
    x = np.zeros(y.shape)
    Z = np.zeros((n1, n2, *[1] * (y.ndim - 2)))
    P = (2*p1+1)*(2*p2+1)

    for k in range(-s1,s1+1):
        for l in range(-s2,s2+1):

            y_shift = im.shift(y,-k,-l,boundary)
            y_diff = y_shift -y

            y_norm = np.mean((((y_diff)**2),axis=-1)

            box = im.kernel('box', p1)
            #convolve with an appropriate box function
            y_conv = im.convolve(y_norm, box, boundary, separable=None)
            # complete
            alpha =y_conv

            kern_alph = ((kernel_func2(alpha,c,h,sig,P)))
            r0 = (y_shift.transpose()*kern_alph.transpose()).transpose()
            #r0 = (kern_alph*y_shift)
            kern_alph = kern_alph.reshape(Z.shape)
            x += (r0)
            Z += (kern_alph)

    Z[Z == 0] = 1
    x = x / Z
    return x
```

Question 10 and 11

The implementation is 10 times faster and the complexity of the operation is $O((2s_1)^2)$ while that of the naive convolution is $O((n^2)(2s_1)^2(2p_1)^2)$

In [168]:

```
x = plt.imread('assets/leopard.png')
sig = 20 / 255
y = x + sig * np.random.randn(*x.shape)

t1 = time.time()
n1_means = nlmeans(y,sig,s1 =2, s2=2, p1 =1, p2 =1, h=1)
t2 = time.time()
tnn1 =t2-t1
print(" N1 means takes {:.2f} s ".format(tnn1))
```

Nl means takes 0.82 s

In [169]:

```
x = plt.imread('assets/leopard.png')
sig = 20 / 255
y = x + sig * np.random.randn(*x.shape)

t1 = time.time()
nl_means = nlmeans(y,sig,s1 =7, s2=7, p1 =1, p2 =1, h=1)
t2 = time.time()
tnnl =t2-t1
print(" Nl means takes {:.2f} s ".format(tnnl))

t1 = time.time()
bil_filt = bilateral(y,sig,s1 =7, s2=7, h=1,boundary = 'mirror')
t2 = time.time()
tb =t2-t1

print(" Bilateral takes {:.2f} s ".format(tb))
```

Nl means takes 7.30 s
Bilateral takes 4.60 s

Question 12

In [146]:

```
def psnr(x,x0):
    diff =np.linalg.norm(x-x0)
    R = np.ptp(x)
    n = np.prod(x.shape)
    psn = 10*np.log10(R**2*n/(diff)**2)
    return psn
```

Question 13

In [147]:

```
psnr_nl = psnr(x,nl_means)
psnr_bl = psnr(x,bil_filt)

import imageio as im0
fig, axes = plt.subplots(ncols=4, figsize=(10, 5))
im0.show(x, ax=axes[0])
axes[0].set_title('Original')
im0.show(naive_nl, ax=axes[1])
axes[1].set_title('Bilateral {:.2f} s and PSNR of {:.2f}dB'.format(tb,psnr_bl))
im0.show(nl_means, ax=axes[2])
axes[2].set_title('Nl means {:.2f} s and PSNR of {:.2f}dB'.format(tnnl,psnr_nl))
im0.show(y, ax=axes[3])
axes[3].set_title('Noisy')
fig.show()
```

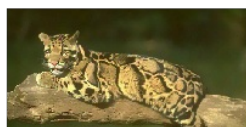
Original



Bilateral 4.12 s and PSNR of 29.82dB



Nl means 7.02 s and PSNR of 30.45dB



Noisy





Question 14

The psnr drops upon increasing or decreasing h , $h=1$ seems to be a local maximum. PSNR drops as the noise is increased.

In [186]:

```
x = plt.imread('assets/leopard.png')
sig = 160 / 255
y = x + sig * np.random.randn(*x.shape)

t1 = time.time()
nl_means = nlmeans(y, sig, s1=7, s2=7, p1=1, p2=1, h=2)
t2 = time.time()
tnnl = t2 - t1
print(" Nl means takes {:.2f} s ".format(tnnl))
```

Nl means takes 6.99 s

In [187]:

```
psnr_nl = psnr(x, nl_means)
print(psnr_nl)
```

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