In [1]:

```
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
import imagetools as im

%matplotlib notebook
```

Part 1 - Bob's Watermarking Technique

Question 1

In [28]:

```
#load the images and the mask
xalice = plt.imread('assets/img_alice.png')
ybob = plt.imread('assets/img_bob.png')

mask = np.load('assets/mask.npy')
```

In [31]:

```
shape_x = xalice.shape
shape_y = ybob.shape
shape_m = mask.shape
print("shape of xalice, ybob, mask respectively",shape_x,shape_y, shape_m)

max_x, min_x = np.max(xalice),np.min(xalice)
max_y,min_y = np.max(ybob),np.min(ybob)
max_m, min_m = np.max(mask),np.min(mask)
print("range of xalice, ybob and mask respectively (%d, %d), (%f, %d), (%d, %d)" % (min_x, max_x,
min_y, max_y,min_m, max_m))
```

```
shape of xalice, ybob, mask respectively (540, 720, 3) (540, 720, 3) (540, 720, 3)
range of xalice, ybob and mask respectively (0, 1), (0.007843, 1), (0, 1)
```

Shape of xalice, ybob, mask respectively (540, 720, 3) (540, 720, 3) (540, 720, 3)

Range of xalice, ybob and mask respectively (0, 1), (0.007843, 1), (0, 1)

Question 2

In [34]:

```
plt.figure()
plt.imshow(xalice)
```

Out[34]:

```
<matplotlib.image.AxesImage at 0x7f4649e7cd68>
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```

There are no buttons after my picture

Question 3

In [32]:

```
plt.figure()
im.show(ybob)
```

Out[32]:

```
<matplotlib.image.AxesImage at 0x7f4649b09860>
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```

The difference is that the in-house im.show does not show the axes

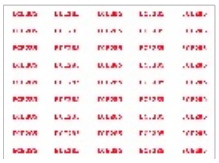
Question 4

In [36]:

```
fig, axes = plt.subplots(ncols=3, figsize=(7, 2))
im.show(xalice, ax=axes[0])
im.show(ybob, ax=axes[1])
im.show(mask, ax=axes[2])
fig.show()
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:418: UserWarning:
matplotlib is currently using a non-GUI backend, so cannot show the figure
  "matplotlib is currently using a non-GUI backend, "
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```





The first line defines the figure and its axis with 3 different subpolots each with a defined size, the next three lines define what array contents are displayed where (axis locations). The final command displays the result

Question 5 and 6

In [41]:

```
def watermark(x, mask, p):
    lambd = 1/((1-p)*mask + p)
    y = lambd*x
    return y

y_test = watermark(xalice,mask, 0.1)
plt.figure()
im.show(y_test)
```

Out[41]:

```
<matplotlib.image.AxesImage at 0x7f46497defd0>
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```

Question 7

Changing p leads to variations in the colour and intensity of the overlaid watermark.

In [43]:

```
fig, axes = plt.subplots(nrows=2, ncols=3, figsize=(7, 3),
                         sharex='all', sharey='all')
for k in range(6):
    im.show(watermark(xalice, mask, p=(k+1)/3), ax=axes[k % 2][int(k / 2)])
fig.show()
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:418: UserWarning:
matplotlib is currently using a non-GUI backend, so cannot show the figure
  "matplotlib is currently using a non-GUI backend, "
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```

Part 2 - Alice's Watermarking Removal

Question 8

In [63]:

```
def iwatermark(y, m, p):
    x = ((1-p)*m +p)*y
    return x
```

Question 9

In [77]:

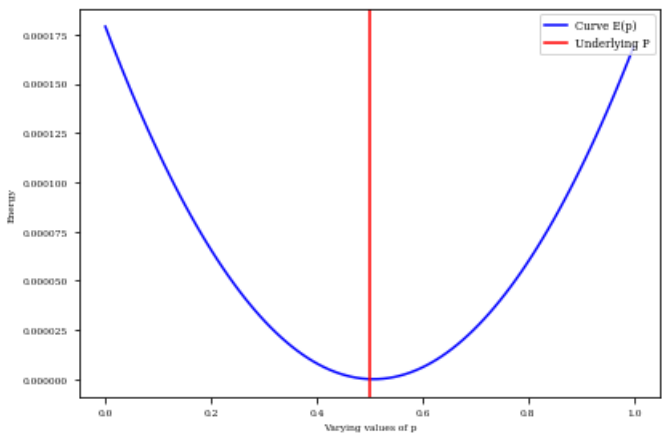
```
def energy(y, m, p):
    e = np.zeros_like(p)
    for i in range(len(p)):
        x = iwatermark(y,m,p[i])
        m_bar = np.mean(m)
        mu_bar = np.mean(x)
        #print(((m-m_bar)*(x-mu_bar)).shape)
        e_root = np.mean((m-m_bar)*(x-mu_bar))
        e[i] = e_root*e_root
    return e
```

Question 10

In [97]:

```
yalice = watermark(xalice, mask, p =0.5)
p =0.5
p_test =np.linspace(0, 2 * p, 100)
E_p = energy(yalice, mask,p_test)
p_e = plt.plot(p_test, E_p,'b-',label='Curve E(p)')
p_p = plt.axvline(x=p, label='Underlying P', c='r')
plt.xlabel('Varying values of p')
plt.ylabel('Energy')
plt.legend(loc='upper right')
plt.show()
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```



Question 11

$\Sigma_k(m_k - \bar{m})(\hat{x_k} - \bar{\mu})$

Since,

$\bar{\mu} = \frac{1}{n}\Sigma_k[m_k - \hat{p}(1 - m_k)]y_k$

We can write:

$\Sigma_{k'} m_{k'}\bar{\mu} = \Sigma_{k'} \frac{m_{k'}}{n} \Sigma_k[m_k - \hat{p}(1 - m_k)]y_k$

And we note that

$\Sigma_{k'} \frac{m_{k'}}{n} = \bar{m}$

$\Sigma_{k'} \bar{m}\bar{\mu} = \Sigma_{k'} \frac{m}{n} \Sigma_k[m_k - \hat{p}(1 - m_k)]y_k$

Similarly the first term equals $\bar{m}$

And we can write that: $\Sigma_k(m_k - \bar{m})(\hat{x_k} - \bar{\mu}) = \Sigma_k(m_k - \bar{m})\hat{x_k}$

Then we proved the $E(p)$ part:

$E(p) = \frac{1}{n}\Sigma_k m_k(m_k - \bar{m})y_k + \hat{p}(1 - m_k)(m_k - \bar{m})y_k$

taking the derivative wrt p and set to zero:

$\frac{\delta E(p)}{\delta p} = 0 = [2^{\frac{1}{n}}\Sigma_k m_k(m_k - \bar{m})y_k + \hat{p}(1 - m_k)(m_k - \bar{m})y_k]. \frac{1}{n}\Sigma_k(1 - m_k)(m_k - \bar{m})$

This implies that the minimum is found when:

$p = -\frac{\Sigma_k m_k(m_k - \bar{m})y_k}{\Sigma_k(1 - m_k)(m_k - \bar{m})y_k}$

Question 12

In [100]:

```
def estimate_p(y, m):
    m_bar = np.mean(m)
    num = np.mean(m*(m-m_bar)*y)
    den = np.mean((1-m)*(m-m_bar)*y)
    p = -num/den
    return p
```

In [102]:

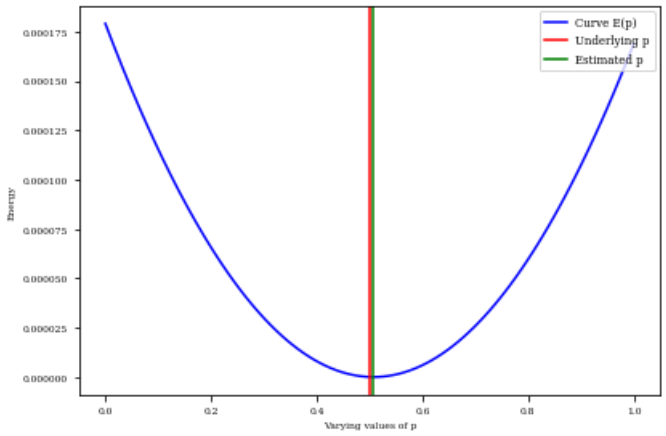
```
p_hat = estimate_p(yalice,mask)
```

In [106]:

```
yalice = watermark(xalice, mask, p =0.5)
p =0.5
p_test =np.linspace(0, 2 * p, 100)
E_p = energy(yalice, mask,p_test)
p_e = plt.plot(p_test, E_p,'b-',label='Curve E(p)')
p_p = plt.axvline(x=p, label='Underlying p', c='r')
p_phat = plt.axvline(x=p_hat, label='Estimated p', c='g')
```

```
plt.xlabel('Varying values of p')
plt.ylabel('Energy')
plt.legend(loc='upper right')
plt.show()
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```



Question 13

In [110]:

```
p_bob = estimate_p(ybob,mask)
print(p_bob)
xbob = iwatermark(ybob, mask, p_bob)
plt.figure()
im.show(xbob)
```

```
3.141587142445284
```

Out[110]:

```
<matplotlib.image.AxesImage at 0x7f4637de9160>
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```

Bob has been using a value of 3.1415

Question 14

In [112]:

```
fig, axes = plt.subplots(ncols=2, figsize=(10, 10))
im.show(xbob, ax=axes[0])
im.show(ybob, ax=axes[1])

fig.show()
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:418: UserWarning:
matplotlib is currently using a non-GUI backend, so cannot show the figure
  "matplotlib is currently using a non-GUI backend, "
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```

Question 15

Bob was right that Alice could not get the exact image back but Alice could get an estimate close enough for the two to be indistinguishable.