In [1]:

```
%load_ext autoreload
%autoreload 2
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
import time
import imagetools.assignment2 as im

%matplotlib notebook
```

1 Image Shift

Question 1

In [2]:

```
def shift(x, k, l, boundary):

    n1, n2 = x.shape[:2]
    if boundary is 'periodical_naive':
        xshifted = np.zeros(x.shape)
        # Main part
        for i in range(max(-k, 0), min(n1-k, n1)):
            for j in range(max(-l, 0), min(n2-l, n2)):
                xshifted[i, j] = x[i + k, j + l]
        # Corners
        for i in range(n1 - k, n1):
            for j in range(n2 - l, n2):
                xshifted[i, j] = x[i + k - n1, j + l - n2]
        for i in range(n1 - k, n1):
            for j in range(0, -l):
                xshifted[i, j] = x[i + k - n1, j + l + n2]
        return xshifted

    if boundary is 'periodical':
        xshifted = np.zeros(x.shape)
        irange = np.mod(np.arange(n1) + k, n1)
        jrange = np.mod(np.arange(n2) + l, n2)
        xshifted = x[irange, :][:, jrange]
        return xshifted
    k = -k
    l = -l
    if boundary is 'extension':
        xshifted = np.zeros(x.shape)
        irange = np.pad(np.arange(max(0,-k),min(n1-k,n1)),(max(k,0),max(0,-k)), 'constant', constant
_values=(0, n1-1))
        jrange = np.pad(np.arange(max(0,-l),min(n2-l,n2)),(max(l,0),max(0,-l)), 'constant', constant
_values=(0, n2-1))
        xshifted = x[irange, :][:, jrange]
        return xshifted

    if boundary is 'mirror':
        xshifted = np.zeros(x.shape)
        irange = np.pad(np.arange(max(0,-k),min(n1-k,n1)),(max(k,0),max(0,-k)), 'reflect')
        jrange = np.pad(np.arange(max(0,-l),min(n2-l,n2)),(max(l,0),max(0,-l)), 'reflect')
        xshifted = x[irange, :][:, jrange]
        return xshifted
```

In [3]:

```
x = plt.imread('assets/windmill.png')
x_shift_mirror = shift(x, 100, -50,'mirror')
x_shift_periodical = shift(x, 100, -50,'periodical')
x_shift_extension = shift(x, 100, -50,'extension')
```

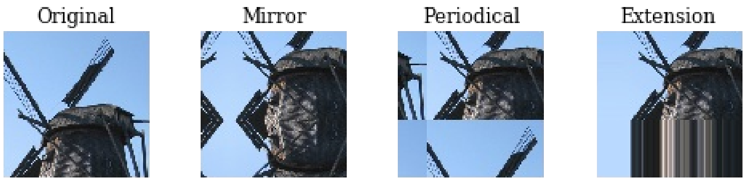
Question 2

Testing the functions

In [4]:

```
import imagetools as im
fig, axes = plt.subplots(ncols=4, figsize=(7, 2))
im.show(x, ax=axes[0])
axes[0].set_title('Original')
im.show(x_shift_mirror, ax=axes[1])
axes[1].set_title('Mirror')
im.show(x_shift_periodical, ax=axes[2])
axes[2].set_title('Periodical')
im.show(x_shift_extenstion, ax=axes[3])
axes[3].set_title('Extension')
fig.show()
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:418: UserWarning:
matplotlib is currently using a non-GUI backend, so cannot show the figure
  "matplotlib is currently using a non-GUI backend, "
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```



Question 3

Is the shifting operation linear?

In [5]:

```
y = plt.imread('assets/lake.png')
# shift(a * x + b * y, k, l) = a * shift(x, k, l) + b * shift(y, k, l)
lhs = shift(3*x+0.8*y, -50,100,'periodical')
rhs = 3*shift(x, -50,100,'periodical')+ 0.8*shift(y, -50,100,'periodical')
print('For a periodical shift linearity is ',np.array_equal(lhs, rhs))

lhs = shift(0.2*x+0.8*y, -50,100,'mirror')
rhs = 0.2*shift(x, -50,100,'mirror')+ 0.8*shift(y, -50,100,'mirror')
print('For a mirror shift linearity is ',np.array_equal(lhs, rhs))

lhs = shift(0.2*x+0.8*y, -50,100,'extension')
rhs = 0.2*shift(x, -50,100,'extension')+ 0.8*shift(y, -50,100,'extension')
print('For an extension shift linearity is ',np.array_equal(lhs, rhs))
```

```
For a periodical shift linearity is  True
For a mirror shift linearity is  True
For an extension shift linearity is  True
```

Question 4

One-to-one shift for the periodical shift, the nullspace of this operation is empty.

For the rest, the shit is not one to one and the nullspace is the elements in the shited region that are lost (move out of frame) in the shifting operation.

In [22]:

```
forward = shift(x, 100, -50,'periodical')
backward = shift(forward, -100, +50,'periodical')
np.allclose(x,backward)
```

Out[22]:

```
True
```

In [23]:

```
def kernel(name, tau, eps=1e-3):
    if name is 'gaussian':
        i =0
        while (np.exp(-(i**2)/(2*tau**2)) >eps):
            i+=1

        nx = i-1
        ny =i-1
        #points in the range
        i = np.arange(-nx, nx+1, 1)
        j = np.arange(-ny, ny+1, 1)

        #create a grid and evaluate the kernel atevery i,j
        xx, yy = np.meshgrid(i, j, indexing='ij')
        f_ij = np.exp(-(xx**2+yy**2 )/(2*tau**2))


    if name is 'exponential':
        i =0
        while (np.exp(-np.sqrt((i**2))/(tau)) >eps):
            i+=1

        nx = i-1
        ny =i-1
        #points in the range
        i = np.arange(-nx, nx+1, 1)
        j = np.arange(-ny, ny+1, 1)
        #create a grid and evaluate the kernel atevery i,j
        xx, yy = np.meshgrid(i, j, indexing='ij')
        f_ij = np.exp(np.sqrt(xx**2 + yy**2)/-tau)


    if name is 'box':
        f_ij =np.ones((tau,tau))


    Z =np.sum(f_ij)
    nu = (1/Z)*f_ij
    return nu
```

In [46]:

```
nu = kernel('exponential', 1, 1e-3)
nu.shape
```

Out[46]:

```
(13, 13)
```

Question 6

Performing the naive convolution

In [12]:

```
def convolve_naive(x, nu):
    n1, n2 = x.shape[:2]
    s1 = int((nu.shape[0] - 1) / 2)
    s2 = int((nu.shape[1] - 1) / 2)
    xconv = np.zeros(x.shape)
    for i in range(s1, n1-s1):
        for j in range(s2, n2-s2):

            for k in range(-s1, s1+1):
                for l in range(-s2, s2+1):
                    # complete
#                     print(i-k, j-l)
                    xconv[i,j] += (nu[k,l]*x[i-k,j-l])
    return xconv
```

In [29]:

```
start = time.time()
xconv =convolve_naive(x,nu)
end= time.time()
print("Time taken by convolve naive is ",(end-start))
xconv.shape
```

```
Time taken by convolve naive is  19.90034294128418
```

Out[29]:

```
(256, 256, 3)
```

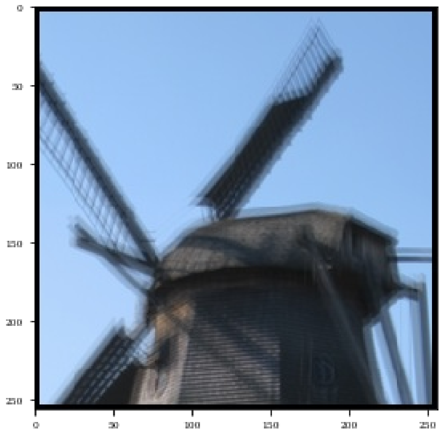
In [14]:

```
plt.imshow(xconv)
```

Out[14]:

```
<matplotlib.image.AxesImage at 0x7fe931953e80>
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```



Question 7

In [15]:

```
def convolve(x, nu, boundary):
    n1, n2 = x.shape[:2]
    s1 = int((nu.shape[0] - 1) / 2)
    s2 = int((nu.shape[1] - 1) / 2)
    xconv = np.zeros(x.shape)
    for k in range(-s1, s1+1):
        for l in range(-s2, s2+1):
            # complete

            xshift = shift(x, -k,-l,boundary)
            xconv += (nu[k,l]*xshift)
    return xconv
```

In [27]:

```
start0 = time.time()
xconv0 =convolve(x,nu,'mirror')
```

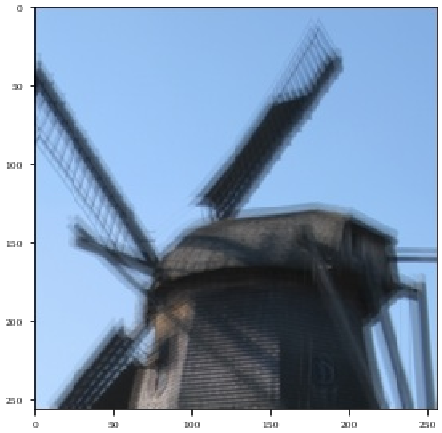
```
end0= time.time()
print("Time taken by convolve is ",(end0-start0))
xconv0.shape

plt.imshow(xconv0)
```

```
Time taken by convolve is  0.09414267539978027
```

Out[27]:

```
<matplotlib.image.AxesImage at 0x7fe931484da0>
```

```
/home/nasha/opt/anaconda3/lib/python3.6/site-packages/matplotlib/figure.py:2022: UserWarning: This
figure includes Axes that are not compatible with tight_layout, so results might be incorrect.
  warnings.warn("This figure includes Axes that are not compatible "
```



Question 8

Time the two operations, naive convolution takes 19s, and the one using the spatial shift takes around 0.09s

Question 9

Is the operation linear? The operation is linear for all boundary conditions implemented and for all the kernels.

In [52]:

```
lhs = convolve(21*x+8*y,nu,'periodical')
rhs = 21*convolve(x,nu,'periodical')+ 8*convolve(y,nu,'periodical')
print('For a periodical shift linearity is ',np.allclose(lhs, rhs))
```

```
For a periodical shift linearity is  True
```

In [48]:

```
lhs = convolve(0.2*x+0.8*y,nu,'extension')
rhs = 0.2*convolve(x,nu,'extension')+ 0.8*convolve(y,nu,'extension')
print('For a extension shift linearity is ',np.allclose(lhs, rhs))
```

```
For a extension shift linearity is  True
```

In [49]:

```
lhs = convolve(0.2*x+0.8*y,nu,'mirror')
rhs = 0.2*convolve(x,nu,'mirror')+ 0.8*convolve(y,nu,'mirror')
print('For a mirror shift linearity is ',np.allclose(lhs, rhs))
```

```
For a mirror shift linearity is  True
```