Lab Assignment - 07

Instructor: Dr. Arabin Kumar Dey

1 Due date:

- 30/9/2022

2 Notes:

- Make a proper documentation preferably in latex or using some other software and submit the printout of the report in .pdf form.

- Each student needs to write his/ her own solutions, even though discussions of the assignments between students are encouraged.

3 Assignments:

1. Use the following Monte Carlo estimator to approximate the expected value

$$I = E(\exp(\sqrt{U}))$$

where $U \sim \mathcal{U}[0,1]$: $I_M = \frac{1}{M}\sum_{i=1}^{M} Y_i$, where $Y_i = \exp(\sqrt{U_i})$ with $U_i \sim \mathcal{U}[0,1]$.

Take all values of M to be $10^2, 10^3, 10^4$ and 10^5. Determine the 95% confidence interval for I_M for all the four values of M that you have taken.

2. Repeat the above exercise using antithetic variates via the following estimator and calculate the percentage of variance reduction:

$$\hat{I}_M = \frac{1}{M} \sum_{i=1}^{M} \hat{Y}_i$$

where

$$\hat{Y}_i = \frac{\exp(\sqrt{U_i}) + \exp(\sqrt{1 - U_i})}{2}$$

with $U_i \sim \mathcal{U}[0, 1]$

3. Use $\sqrt{U}$ to construct control variate estimate and repeat the above exercise. Calculate the percentage of variance reduction.