Monte Carlo Lab

Lab - 07

Somya Khandelwal (200123056)

Q1)

```
For M = 100-
95 percent confidence interval is - [ 1.9967792598921295 , 2.1653844213054625 ].
Actual value of I is 2.
Estimated value of I is 2.081081840598796.
```

```
For M = 1000-
95 percent confidence interval is - [ 1.9711977505067906 , 2.02533883806119 ].
Actual value of I is 2.
Estimated value of I is 1.9982682942839902.
```

```
For M = 10000-
95 percent confidence interval is - [ 1.9861740293078887 , 2.0034121991501066 ].
Actual value of I is 2.
Estimated value of I is 1.9947931142289976.
```

```
For M = 100000-
95 percent confidence interval is - [ 1.9994658132664074 , 2.0049265062445634 ].
Actual value of I is 2.
Estimated value of I is 2.0021961597554854.
```

For $X \sim U(0,1)$,

$$f_U(x) = 1, \; for\, x \in (0,1)$$

Now,

$$\begin{aligned}
E\left[e^{\sqrt{U}}\right] &= \int_0^1 e^{\sqrt{x}} f_U(x)dx \\
&= \int_0^1 e^{\sqrt{x}} dx \\
&= 2\int_0^1 te^t dt \qquad \left(using\ the\ substitution\ \sqrt{x} = t\right) \\
&= 2\left[te^t - e^t\right]_0^1 \\
&= 2
\end{aligned}$$

As we can see in all cases of M, the actual value of I and the estimated value of I is very similar and the values grow closer as M increases.

Also, the values of L and U in the confidence interval become closer to the mean value of I as M increases, i.e, we have the same confidence for a much smaller interval.

Q2)

f(X) = e^(X^(½))

U = random.random() generates a random number from the distribution U(0,1).

So, **seqY1** contains numbers of the form $e^{\sqrt{U}}$ and **seqY2** contains numbers of the form $e^{\sqrt{1-U}}$. And these are negatively correlated as $e^{\sqrt{1-U}}$ increases as $e^{\sqrt{U}}$ decreases and vice versa, hence variance will decrease if we take their average as our new estimator.

seqY = (**seqY1** + **seqY2**)/2 indicates that,

seqY[i] = (**seqY1[i]** + **seqY2[i]**)/2, for all i = 0 to M-1

So, **seqY** contains numbers of the form $\frac{e^{\sqrt{U}} + e^{\sqrt{1-U}}}{2}$.

Now,

```
For M = 100-
95 percent confidence interval is - [ 1.9933050492113589 , 2.006105215300541 ].
Actual value of I is 2.
Estimated value of I is 1.99970513225595.
The reduction in variance is 99.42364577757964 %
```

```
For M = 1000-
95 percent confidence interval is - [ 1.9986598364382338 , 2.002650737391663 ].
Actual value of I is 2.
Estimated value of I is 2.0006552869149483.
The reduction in variance is 99.45663964451305 %
```

```
For M = 10000-
95 percent confidence interval is - [ 1.99962381237389 , 2.0009067210818148 ].
Actual value of I is 2.
Estimated value of I is 2.0002652667278524.
The reduction in variance is 99.44612827482409 %
```

```
For M = 100000-
95 percent confidence interval is - [ 1.9998267750306216 , 2.000233511875644 ].
Actual value of I is 2.
Estimated value of I is 2.0000301434531327.
The reduction in variance is 99.4452063109237 %
```

The observations are similar to question 1, but we observe a variance reduction of ~99% in all the cases.

Q3)

f(X) is the same as question 2.

seqX is a sequence of numbers of the form $e^{\sqrt{U}}$ and seqY is a sequence of number of the form $\sqrt{U}$ (control variate).

cov is the $Cov(X, Y)$.

c_hat is $\frac{-Cov(X,Y)}{Var(Y)}$

Hence the new estimator is given by $X + \frac{-Cov(X,Y)}{Var(Y)}(Y - u_Y)$, where u_Y is the mean of **seqY** (stored in the variable **mu_y**)

So, **seqX_new** contains variables of the form $X + \frac{-Cov(X,Y)}{Var(Y)}(Y - u_Y)$.

```
For M = 100-
95 percent confidence interval is - [ 2.071563812571647 , 2.090599868625945 ].
Actual value of I is 2.
Estimated value of I is 2.081081840598796.
The reduction in variance is 98.72528757409258 %
```

```
For M = 1000-
95 percent confidence interval is - [ 1.995075746357498 , 2.0014608422104825 ].
Actual value of I is 2.
Estimated value of I is 1.9982682942839902.
The reduction in variance is 98.60914808824833 %
```

```
For M = 10000-
95 percent confidence interval is - [ 1.99377266694976 , 1.9958135615082362 ].
Actual value of I is 2.
Estimated value of I is 1.994793114228998.
The reduction in variance is 98.5982878920906 %
```

```
For M = 100000-
95 percent confidence interval is - [ 2.001873464077502 , 2.0025188554334696 ].
Actual value of I is 2.
Estimated value of I is 2.002196159755486.
The reduction in variance is 98.60314679092393 %
```

The observations are similar to question 1, and we observe that using the variance reduction is ~98.6 % using this method.