2.1-2.5,2.8,3.1 Practice Problems

Ex: Compute the following derivatives.

1. $\frac{\partial}{\partial x} 4\sin^2 x$

Sol: There are two ways of doing this problem. First,

$$\frac{\partial}{\partial x} 4\sin^2 x = 4 \frac{\partial}{\partial x} \sin^2 x = 4 \frac{\partial}{\partial x} (\sin x)^2$$

$$= 4 \frac{\partial}{\partial x} \left(u^2 \Big|_{u=\sin x} \right)$$

$$= 4 \left(\frac{\partial}{\partial u} u^2 \Big|_{u=\sin x} \right) \frac{\partial}{\partial x} \sin x$$

$$= 4 \left(2u \Big|_{u=\sin x} \right) \cos x$$

$$= \boxed{4 \cdot 2 \sin x \cos x}$$

We can also use the trigonometric identity

$$\frac{\partial}{\partial x} 4\sin^2 x = \frac{\partial}{\partial x} 4 \cdot \tfrac{1}{2}(1-\cos 2x)$$

$$= \frac{\partial}{\partial x} 2(1-\cos 2x)$$

$$= 2 \frac{\partial}{\partial x} (1-\cos 2x)$$

$$= 2 \cdot \left(-\frac{\partial}{\partial x} \cos 2x \right)$$

$$= -2 \frac{\partial}{\partial x}\left(\cos u\,\big|_{u=2x}\right)$$

$$= -2\left(\frac{\partial}{\partial u}\cos u\right)\Big|_{u=2x} \cdot \frac{\partial}{\partial x} 2x$$

$$= -2(-\sin u)\big|_{u=2x} \cdot 2$$

$$= \boxed{+4\sin 2x}$$

$2\sin x\cos x = \sin 2x$

2. $\frac{\partial}{\partial x} \sec(1+x^2)$

Sol: By definition,

or use the Quotient Rule

$$\frac{\partial}{\partial x}\sec(1+x^2) = \frac{\partial}{\partial x}\frac{1}{\cos(1+x^2)}$$

$$= \frac{\partial}{\partial x}\left(\cos(1+x^2)\right)^{-1}$$

$$= \frac{\partial}{\partial x}\left(u^{-1}\,\big|_{u=\cos(1+x^2)}\right)$$

$$= \left(\frac{\partial}{\partial u}u^{-1}\right)\Big|_{u=\cos(1+x^2)} \frac{\partial}{\partial x}\cos(1+x^2)$$

$$= (-1)\, u^{-2} \Big|_{u=\cos(1+x^2)} \frac{\partial}{\partial x} \cos(1+x^2)$$

$$= \frac{-\frac{\partial}{\partial x}\cos(1+x^2)}{\left(\cos(1+x^2)\right)^2}$$

$$= \frac{-\frac{\partial}{\partial x}\left(\cos u \,\big|_{u=1+x^2}\right)}{\cos^2(1+x^2)}$$

$$= \frac{-\left(\frac{\partial}{\partial u}\cos u\right)\Big|_{u=1+x^2} \frac{\partial}{\partial x}(1+x^2)}{\cos^2(1+x^2)}$$

$$= \frac{-(-\sin u)\big|_{u=1+x^2}\,(0+2x)}{\cos^2(1+x^2)}$$

$$= \boxed{\frac{2x\sin(1+x^2)}{\cos^2(1+x^2)}}$$

You can check this by instead using the Quotient Rule.

Ex: Estimate the value of $(8.06)^{\frac{2}{3}}$.

Sol: One answer is to say that

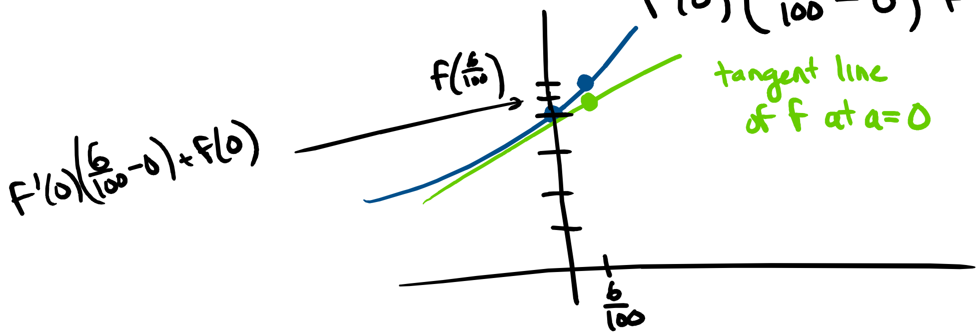
$$(8.06)^{\frac{2}{3}} \approx 8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4$$

Let's give an even better estimate, using linear approximation. Consider the function

$$f(x) = (x+8)^{\frac{2}{3}}$$

We want to estimate the value of f(6/100). This means that

$$f\left(\tfrac{6}{100}\right) \approx \underbrace{\text{the tangent line of } f \text{ at } a=0 \text{ evaluated at } x=\tfrac{6}{100}}_{= \; f'(0)\left(\frac{6}{100}-0\right)+f(0)}$$



We compute

$$f(0) = (0+8)^{\frac{2}{3}} = 4$$

$$f'(0) = \frac{d}{dx}(x+8)^{\frac{2}{3}} \Big|_{x=0}$$

$$= \left(\frac{2}{3}\right)(x+8)^{\frac{2}{3}-1} \cdot \frac{\partial}{\partial x}(x+8) \Big|_{x=0}$$

$$= \left(\frac{2}{3}\right)(x+8)^{-\frac{1}{3}} \Big|_{x=0}$$

$$= \left(\frac{2}{3}\right)\left(8^{\frac{-1}{3}}\right) = \frac{2}{3} \cdot (2^3)^{\frac{-1}{3}}$$

$$= \frac{2}{3} \cdot 2^{-1} = \frac{1}{3}$$

We conclude that

$$(8.06)^{\frac{2}{3}} = f\left(\frac{6}{100}\right) \approx \frac{1}{3}\left(\frac{6}{100}-0\right)+4 = \boxed{\frac{201}{50}}$$

Using a calculator to check,

$$(8.06)^{\frac{2}{3}} = 4.0199\ldots$$

$$\frac{201}{50} = 4.02 !$$

Ex: Find the absolute extremum points and values of

$$f(x)=x^3-3x^2+1$$

over [1,4].

Sol: We must compare the values of f(1),f(4), and f’(c) for c in (1,4) with f’(c)=0.

We compute

$$f(1) = 1^3 - 3\cdot 1^2 + 1 = 1 - 3 + 1 = -1$$

$$f(4) = 4^3 - 3\cdot 4^2 + 1 = 4^2(4-3) + 1 = 4^2 + 1 = 17$$

$$0 = f'(c) = 3c^2 - 6c \Rightarrow 3c^2 - 6c = 0$$

$$\Rightarrow 3c(c-2) = 0$$

$$\Rightarrow c = 0, 2$$

$$\Rightarrow c = 2 \in (1,4)$$

$$\Rightarrow f(2) = 2^3 - 3\cdot 2^2 + 1$$

$$= 8 - 12 + 1 = -3$$

x=2 is the absolute minimum point of f over [1,4] with absolute minimum value f(2)=-3.
x=4 is the absolute maximum point of f over [1,4] with absolute maximum value f(4)=17.