1.2-1.6 Practice Problems

Ex: Compute the following limits.

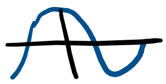
1. $\lim\limits_{x\to 0} \cos(x+\sin(x))$

Sol: $$\lim_{x\to 0} \cos(x+\sin x) \underset{u=x+\sin x}{=} \lim_{u\to 0} \cos(u)$$

$$\lim_{x\to 0}(x+\sin x) \underset{\text{ADD}}{=} \lim_{x\to 0} x + \lim_{x\to 0} \sin x$$

$$\underset{\text{BASIC}}{=} 0 + \sin 0 = 0$$


$$\underset{\text{BASIC}}{=} \cos 0 = \boxed{1}$$



2. $\lim\limits_{x\to -3} \dfrac{x^2-9}{x^2+2x-3}$

Sol: First, note that

$$\lim_{x\to -3} x^2+2x-3 \underset{\text{ADD}}{=} \underset{\text{BASIC}}{\lim_{x\to -3} x^2} + \underset{\text{MULT}}{\lim_{x\to -3} 2x} + \underset{\text{BASIC}}{\lim_{x\to -3} -3}$$

$$= (-3)^2 + 2\underset{\text{BASIC}}{\lim_{x\to -3} x} + (-3)$$

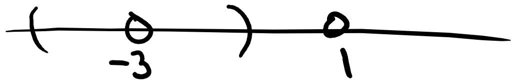
$$= 9 + 2\cdot(-3) - 3$$

$$= 9 - 6 - 3 = 0.$$

Since the limit of the bottom is zero, then we cannot use the Division Rule. In this case, we must first use the Simplifcation Rule. To do this, we note that

$$\frac{x^2-9}{x^2+2x-3} = \frac{(x-3)(x+3)}{(x-1)(x+3)} = \frac{x-3}{x-1} \qquad \text{for } x \neq 1, -3$$

In particular, this identity is true near x=-3 (but not at -3).



This means we can use the Simplification Rule to compute

$$\ell_{x\to -3} \frac{x^2-9}{x^2+2x-3} = \ell_{x\to-3} \frac{x-3}{x-1}$$

$$\underset{\text{DIV}}{=} \frac{\ell_{x\to-3}\, x-3}{\ell_{x\to-3}\, x-1} = \boxed{\frac{-3-3}{-4}}$$

$$\ell_{x\to-3}\, x-1 = -3-1 = -4 \neq 0$$

3. $\lim\limits_{x\to 0} \dfrac{(x-1)^3+1}{x}$

Sol: Again, we cannot use the Division Rule. We compute

$$* \quad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(x-1)^3 = x^3 + 3x^2(-1) + 3x(-1)^2 + (-1)^3$$

$$= x^3 - 3x^2 + 3x - 1$$

Using the Simplification Rule,

$$\lim_{x\to 0} \frac{(x-1)^3+1}{x} = \lim_{x\to 0} \frac{x^3-3x^2+3x-1+1}{x}$$

$$= \lim_{x\to 0} \frac{x^3-3x^2+3x}{x}$$

$$= \lim_{x\to 0} x^2-3x+3$$

$$\underset{\substack{\text{ADD}\\ \text{MULT}\\ \text{BASIC}}}{=} 0^2-3\cdot 0+3 = \boxed{3}$$

4. $\lim_{x\to 4^+} \frac{4-x}{|4-x|}$

Sol: Let's first use the u-Substitution Rule,

$$\lim_{x\to 4^+} \frac{4-x}{|4-x|} \underset{u=4-x}{=} \lim_{u\to 0^-} \frac{u}{|u|}$$

$$\lim_{x\to 4^+} 4-x = 0^-$$

$$x>4 \Rightarrow 4-x<0$$

Recall the definition of the absolute value,

$$|u| = \begin{cases} u & \text{if } u\geq 0 \\ -u & \text{if } u<0 \end{cases}$$

This means that for u<0,

$$\frac{u}{|u|} = \frac{u}{-u} = -1$$

We compute using the Simplification Rule that

$$\lim_{u \to 0^-} \frac{u}{|u|} = \lim_{u \to 0^-} -1 = -1$$

$$\Rightarrow \quad \lim_{x \to 4^+} \frac{4-x}{|4-x|} = \boxed{-1}$$