3.7,4.2-4.5 Practice Problems

Ex: Compute the most general antiderivative of the given f over the largest possible open subset I of R, and give I.

1. $f(x)=\cos^2 x$

Sol: We compute

$$\int \cos^2 x\,dx = \int \tfrac{1}{2}(1+\cos 2x)\,dx$$

$$= \tfrac{1}{2}\int 1+\cos 2x\,dx$$

$$\underset{\substack{u=2x \\ du=2dx \\ \Rightarrow \frac{1}{2}du=dx}}{=} \tfrac{1}{2}\int (1+\cos u)\tfrac{1}{2}\,du \Big|_{u=2x}$$

$$= \tfrac{1}{4}\int 1+\cos u\,du \Big|_{u=2x}$$

$$= \tfrac{1}{4}(u+\sin u)+c \Big|_{u=2x}$$

$$= \boxed{\begin{array}{c}\tfrac{1}{4}(2x+\sin 2x)+c \\ \text{over } I=(-\infty,\infty)\end{array}}$$

We similarly compute $\int \sin^2 x\,dx$

2. f(x)=sin(x) cos(x)

Sol: We compute

$$\int \underbrace{\sin x \cos x}_{\substack{\| \\ \frac{\sin 2x}{2}}} dx = \int u\,du \Big|_{u=\sin x}$$

$u = \sin x$

$du = \cos x\, dx$

& $u = \cos x$

$$= \frac{u^2}{2} + c \Big|_{u=\sin x}$$

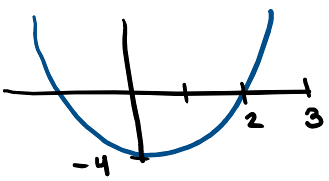
$$= \boxed{\frac{\sin^2 x}{2} + c \quad \text{over } I=(-\infty,\infty)}$$

Ex: Compute $\int_0^3 |x^2-4|\,dx$

Sol: The absolute value is defined to be

$$|u| = \begin{cases} u & \text{if } u \geq 0 \\ -u & \text{if } u < 0 \end{cases}$$

Note that



$$\Rightarrow \begin{cases} x^2-4 > 0 & \text{if } x \in [2,3] \\ x^2-4 < 0 & \text{if } x \in [0,2] \end{cases}$$

$$\Rightarrow |x^2-4| = \begin{cases} x^2-4 & \text{if } x \in [2,3] \\ -(x^2-4) & \text{if } x \in [0,2] \end{cases}$$

Using the Base Rule,

$$\int_0^3 |x^2-4|\,dx = \int_0^2 |x^2-4|\,dx + \int_2^3 |x^2-4|\,dx$$

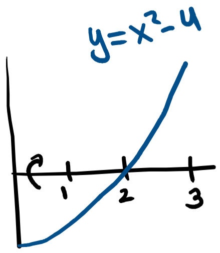
$$= \int_0^2 -(x^2-4)\,dx + \int_2^3 (x^2-4)\,dx$$

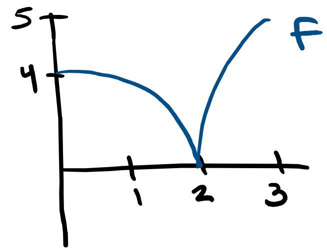
$$= -\int_0^2 x^2-4\,dx + \int_2^3 x^2-4\,dx$$

$$= -\left(\frac{x^3}{3}-4x\right)\Bigg|_{x=0}^{2} + \left(\frac{x^3}{3}-4x\right)\Bigg|_{x=2}^{3}$$

$$= \boxed{-\left(\left(\frac{8}{3}-8\right)-(0-0)\right) + \left(\left(\frac{27}{3}-12\right)-\left(\frac{8}{3}-8\right)\right)}$$

check this number is >0





Ex: Compute $\frac{d}{dx}\int_0^{x^2} e^{t^2}\,dt$

Sol: We cannot use the Area Function Thm directly. To use the Area Function Thm, we need

$$\frac{d}{dx}\int_{c}^{x} f(t)\,dt$$

x ← x by itself

c ↑ constant ✓

$f(t)$ ↑ continuous ✓

We must use the Chain Rule.

$$\frac{d}{dx}\int_{0}^{x^2} e^{t^2}\,dt = \frac{d}{dx}\left(\int_{0}^{u} e^{t^2}\,dt\,\Big|_{u=x^2}\right)$$

$$= \left(\frac{d}{du}\int_{0}^{u} e^{t^2}\,dt\right)\Big|_{u=x^2}\frac{d}{dx}x^2$$

$$\underset{AFT}{=} e^{u^2}\Big|_{u=x^2}(2x)$$

$$= 2x\,e^{(x^2)^2} = \boxed{2xe^{x^4}}$$