[ESC] Bayes Week1 HW

SonJiwoo

2021 1 13

```
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
par(mfrow=c(1,2))

x<-0:10
plot(x,dpois(x,2.1), type="h",lwd=1, xlab=expression(italic(y)),ylab=
       #    expression(paste(italic("p(y"),"|",theta==2.1,")")) )
       expression(paste(italic("p"),"(",italic("y"),"|",theta==2.1,")")) )

x<-0:100
plot(x,dpois(x,21), type="h",lwd=1, xlab=expression(italic(y)),ylab=
       expression(paste(italic("p"),"(",italic("y"),"|",theta==21,")")) )
```

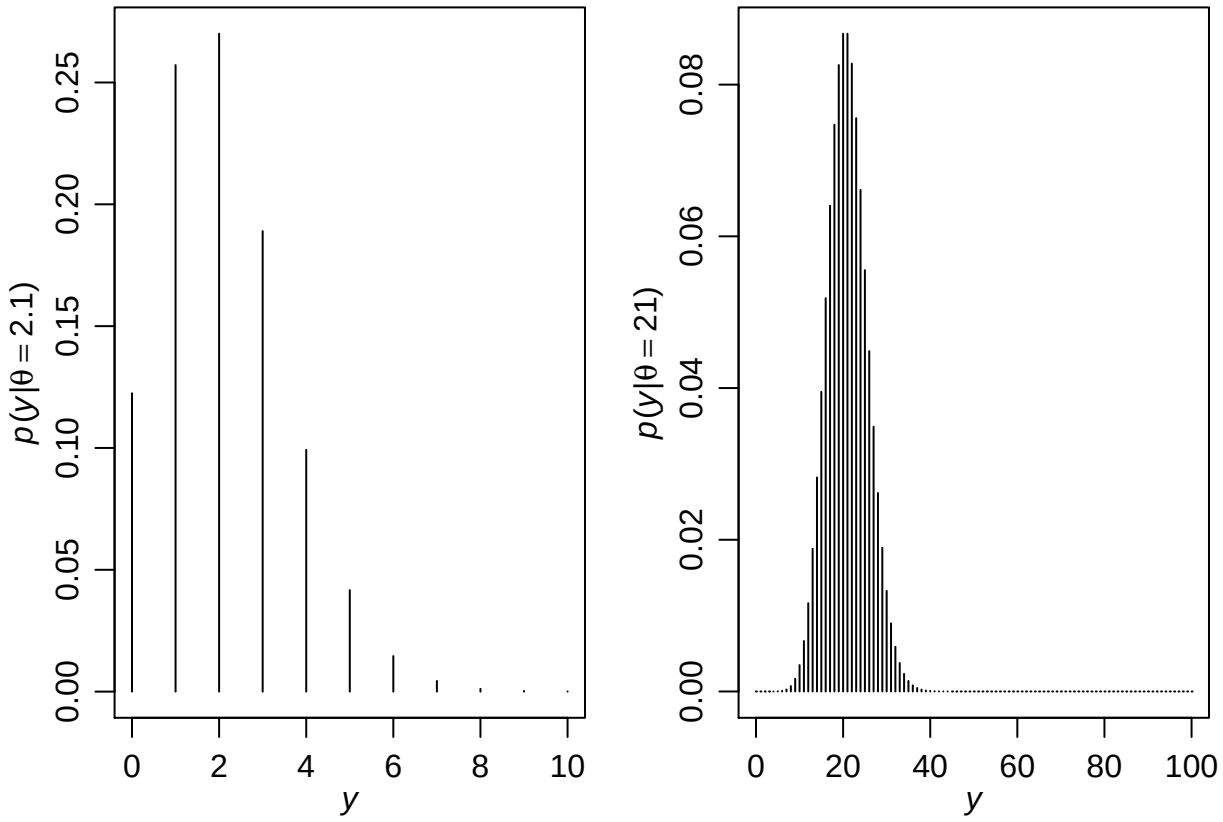

Fig 2.1

```
mu<-10.75
sig<- .8

par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
par(mfrow=c(1,2))

x<- seq(7.9,13.9,length=500)
plot(x,pnorm(x,mu,sig),type="l",ylab=expression(paste(italic("F"),"(",italic("y"),")")),xlab=
       expression(italic(y)),lwd=1)
abline(h=c(0,.5,1),col="gray")
plot(x,dnorm(x,mu,sig),type="l",ylab=expression(paste(italic("p"),"(",italic("y"),")")),
     xlab=
       expression(italic(y)),lwd=1)
abline(v=mu,col="gray")
```

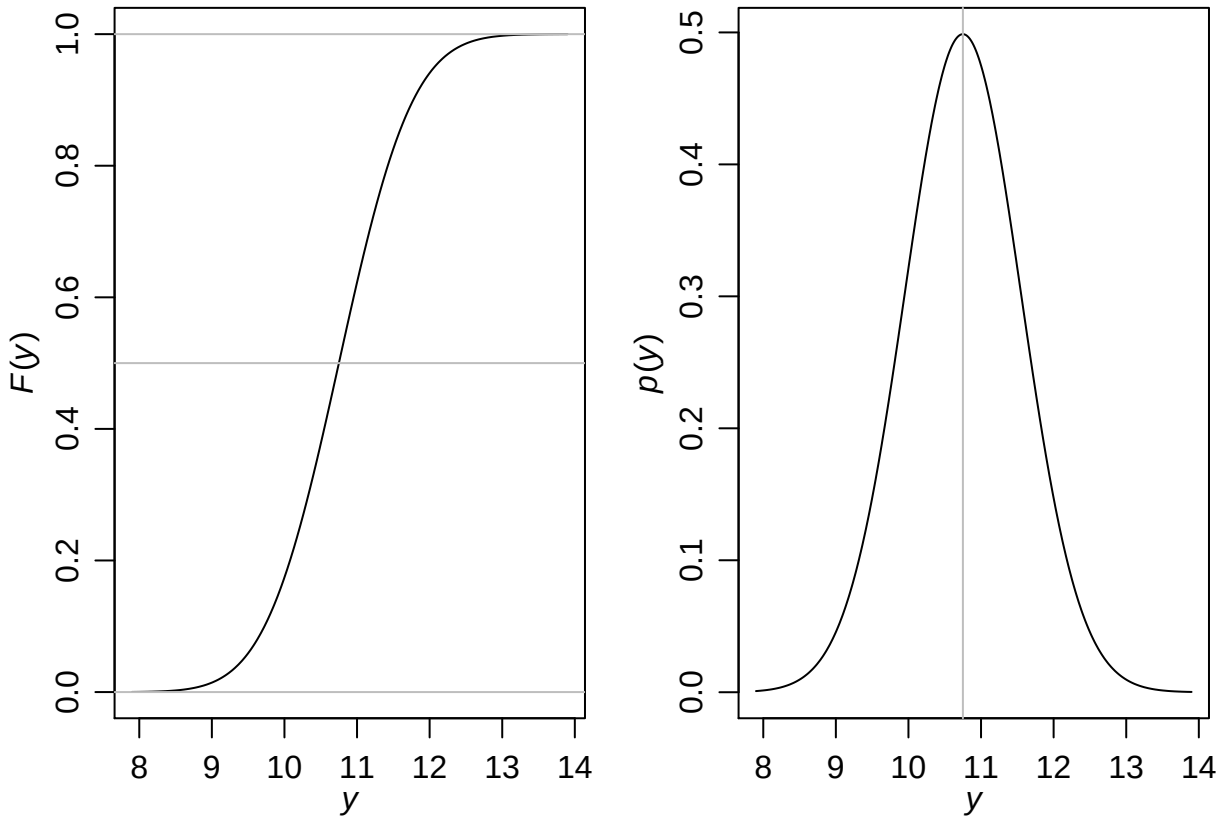

Fig 2.2

```
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
x<-seq(7.75,13.75,length=100)
mu<-10.75 ; sig<-.8

par(mfrow=c(1,2))
plot(x, dnorm(x,mu,sig),type="l",xlab=expression(italic(y)),
```

```
     ylab= expression(paste(italic("p"),"(",italic("y"),")")) )
abline(v=mu,lty=1,col=gray(0))
abline(v=mu,lty=2,col=gray(.33))
abline(v=mu,lty=4,col=gray(.66))

x<-seq( 0,300000,length=200)
mu<-10.75 ; sig<-.8
plot(x, dlnorm(x,mu,sig)*1e5,type="l", xlab=expression(italic(y)),
     ylab= expression( 10^5*paste(italic("p"),"(",italic("y"),")")) )
abline(v=24600,col=gray(0))
abline( v=qlnorm(.5,mu,sig),col=gray(.3),lty=2)
abline(v=exp(mu+.5*sig^2) , col=gray(.7),lty=4)
legend(150000,1.0,c("mode","median","mean"),
       lty=c(1,2,4),col=gray(c(0,.33,.66)),
       bty="n",cex=.85)
```

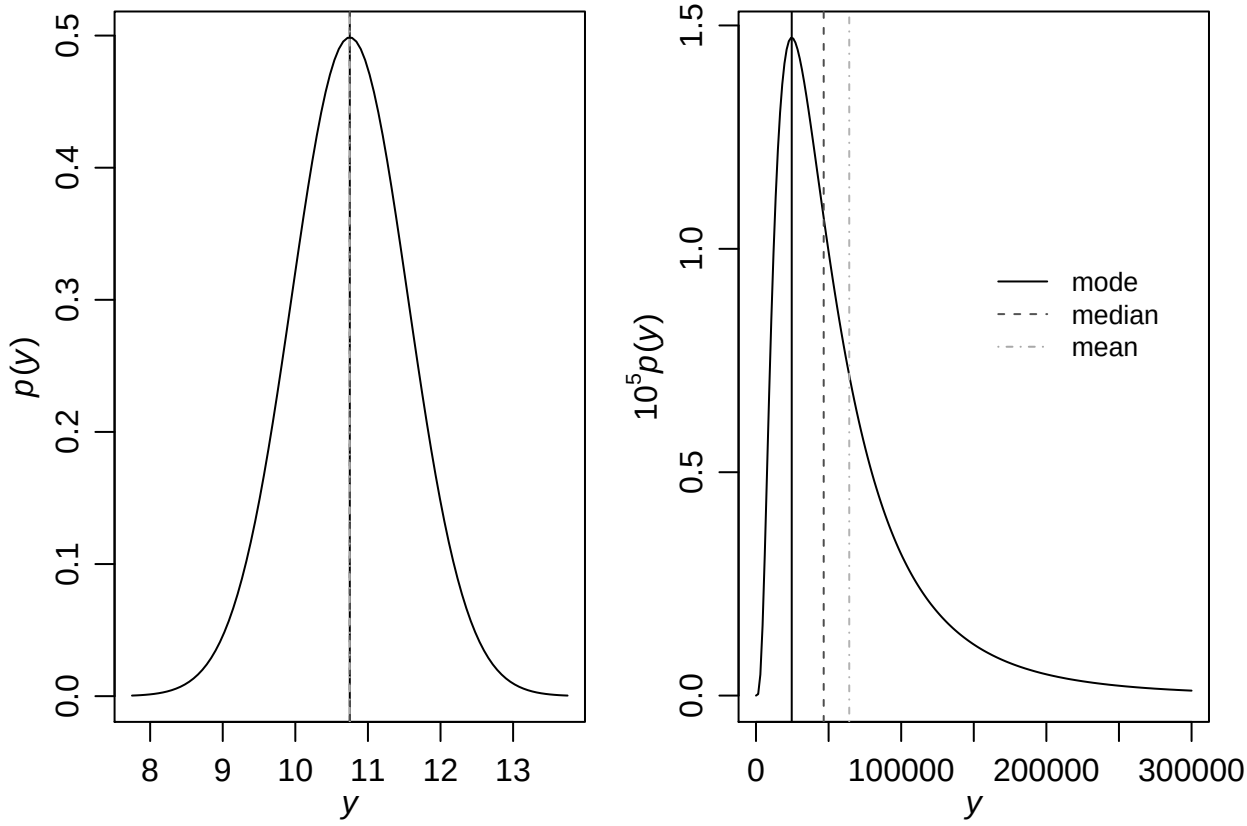

Fig 2.3

```
load("gss.RData")

y<-gss[gss$YEAR==1998 & gss$AGE>=65 & gss$FEMALE==1, ]$HAPUNHAP
y[y>4]<-NA
y[y<=2]<-1
y[y>2]<-0
```

```r
y<-y[!is.na(y) ]
sy<-sum(y)
n<-length(y)

par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
par(mfrow=c(2,1))

theta<-seq(0,1,length=200)
plot(theta,10^17*theta^sy*(1-theta)^(n-sy),type="l",ylab=
       expression(paste(10^27, paste(italic("p"),"(",italic(y[1]),",...,",
                                     italic(y[129]),sep=""), paste("|",theta,")",sep="")), sep=""),
     xlab=expression(theta))

plot(theta,dbeta(theta,sy+1,n-sy+1),type="l",ylab=
       expression(paste(italic("p"),"(",theta,"|",italic(y[1]),",...,",
                        italic(y[129]),")",sep="")),xlab=expression(theta))
abline(h=1,col="gray")
```

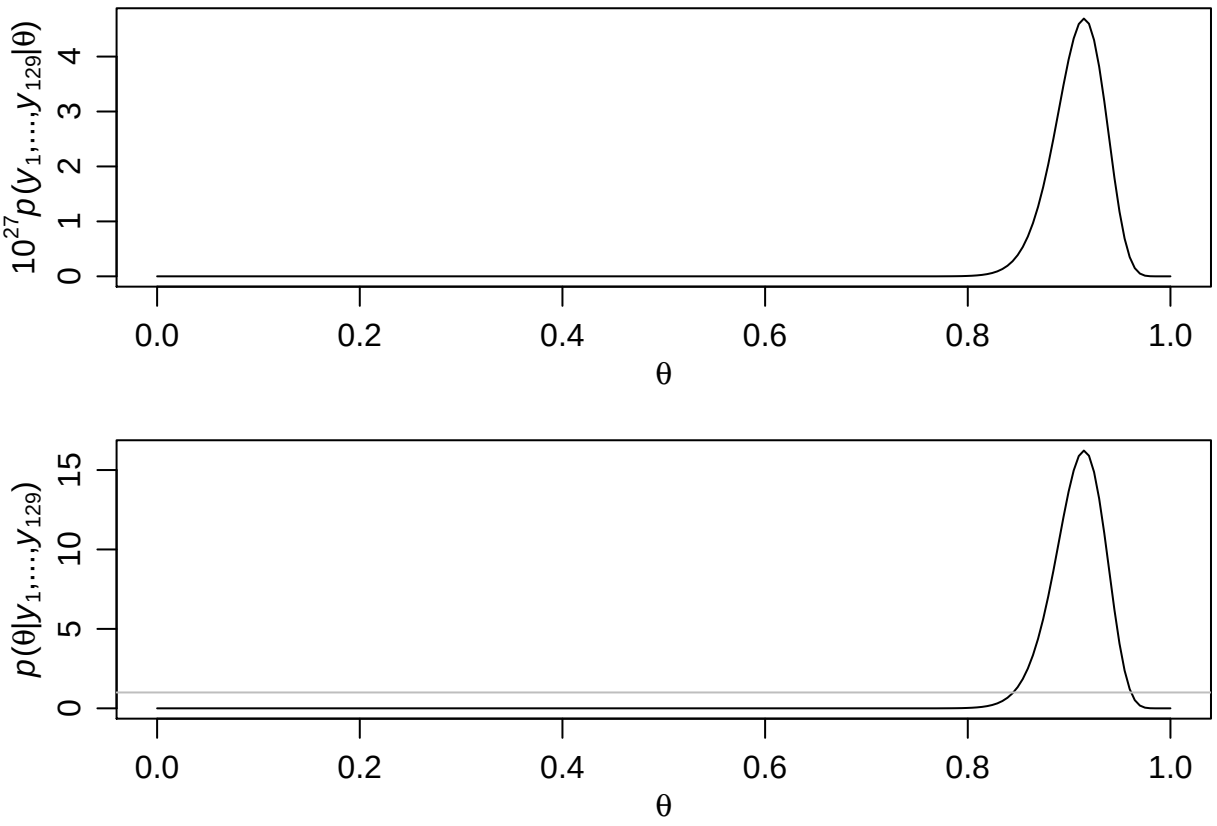

Fig 3.1

```r
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
par(mfrow=c(1,2))

n<-10
```

```r
theta<-.2
plot(0:n,dbinom(0:n,n,theta), type="h",lwd=2,xlab=expression(italic(y)),
     ylab=expression(paste("Pr(",italic("Y=y"),"|",theta==.2,
                           italic(", n="),"10)",sep="")))
n<-10
theta<-.8
plot(0:n,dbinom(0:n,n,theta), type="h",lwd=2,xlab=expression(italic(y)),
     ylab=expression(paste("Pr(",italic("Y=y"),"|",theta==.8,
                           italic(", n="),"10)",sep="")))
```

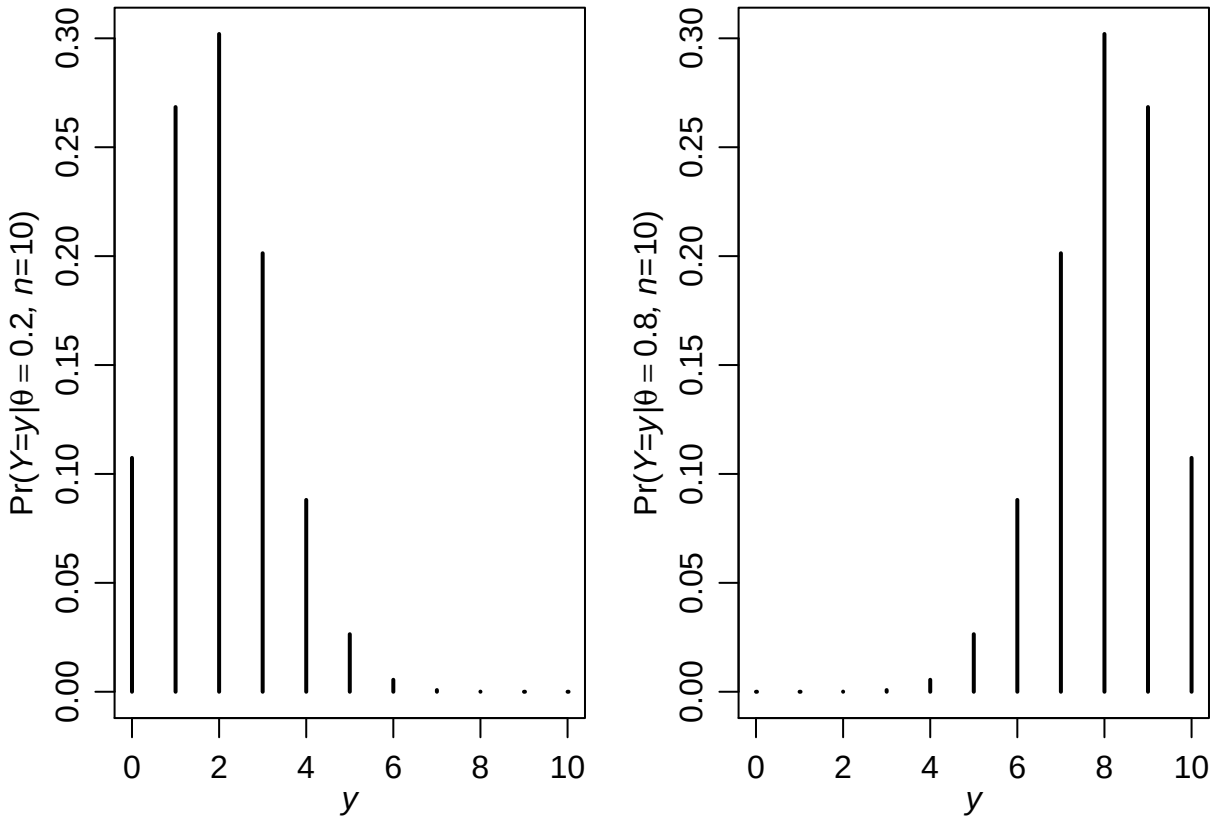

Fig 3.2

```r
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
par(mfrow=c(1,2))

n<-100
theta<-.2
plot(0:n,dbinom(0:n,n,theta), type="h",lwd=2,xlab=expression(italic(y)),
     ylab=expression(paste("Pr(",italic("Y=y"),"|",theta==.2,
                           italic(", n="),"100)",sep="")))

n<-100
theta<-.8
plot(0:n,dbinom(0:n,n,theta), type="h",lwd=2,xlab=expression(italic(y)),
```

```
    ylab=expression(paste("Pr(",italic("Y=y"),"|",theta==.8,
                          italic(", n="),"100)",sep="")))
```

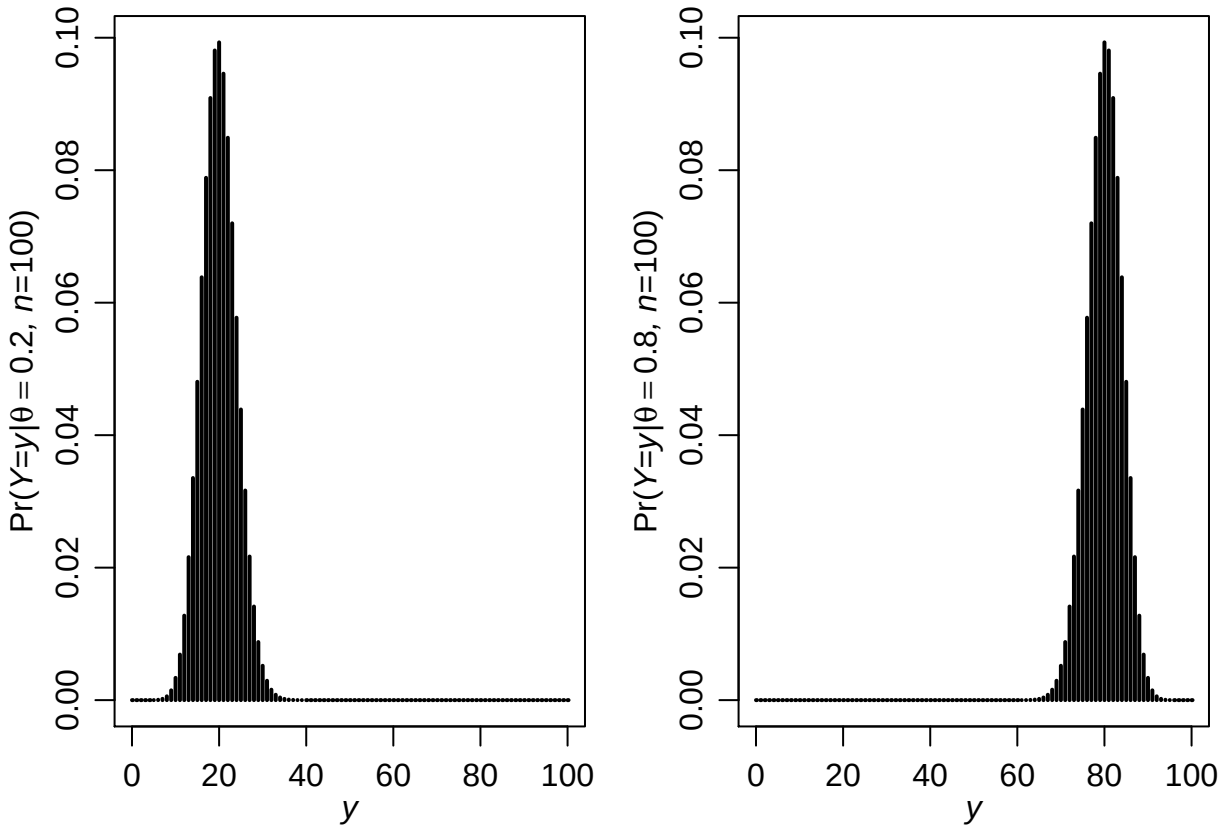

Fig 3.3

```
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0),oma=c(0,0,.5,0))
par(mfrow=c(2,2))
theta<-seq(0,1,length=100)
a<-1; b<-1
n<-5 ; y<-1
plot(theta,dbeta(theta,a+y,b+n-y),type="l",ylab=
       expression(paste(italic("p("),theta,"|y)",sep="")),
     xlab=expression(theta), lwd=2)
mtext(expression(paste("beta(1,1) prior,  ", italic("n"),"=5  ",
                       italic(sum(y[i])),"=1",sep="")), side=3,line=.1)
lines(theta,dbeta(theta,a,b),type="l",col="gray",lwd=2)
legend(.45,2.4,legend=c("prior","posterior"),lwd=c(2,2),
       col=c("gray","black"), bty="n")

a<-3; b<-2
n<-5 ; y<-1
plot(theta,dbeta(theta,a+y,b+n-y),type="l",ylab=
       expression(paste(italic("p("),theta,"|y)",sep="")), xlab=expression(theta),
     lwd=2)
```

```
mtext(expression(paste("beta(3,2) prior,  ", italic("n"),"=5  ",italic(sum(y[i])),"=1",sep="")), side=3
lines(theta,dbeta(theta,a,b),type="l",col="gray",lwd=2)

a<-1 ; b<-1
n<-100; y<-20
plot(theta,dbeta(theta,a+y,b+n-y),type="l",ylab=
       expression(paste(italic("p("),theta,"|y)",sep="")), xlab=expression(theta),
     lwd=2)
mtext(expression(paste("beta(1,1) prior,  ", italic("n"),"=100  ",
                       italic(sum(y[i])),"=20",sep="")), side=3,line=.1)
lines(theta,dbeta(theta,a,b),type="l",col="gray",lwd=2)

a<-3 ; b<-2
n<-100; y<-20
plot(theta,dbeta(theta,a+y,b+n-y),type="l",ylab=
       expression(paste(italic("p("),theta,"|y)",sep="")),
     xlab=expression(theta),lwd=2)
mtext(expression(paste("beta(3,2) prior,  ", italic("n"),"=100  ",
                       italic(sum(y[i])),"=20",sep="")), side=3,line=.1)
lines(theta,dbeta(theta,a,b),type="l",col="gray",lwd=2)
```

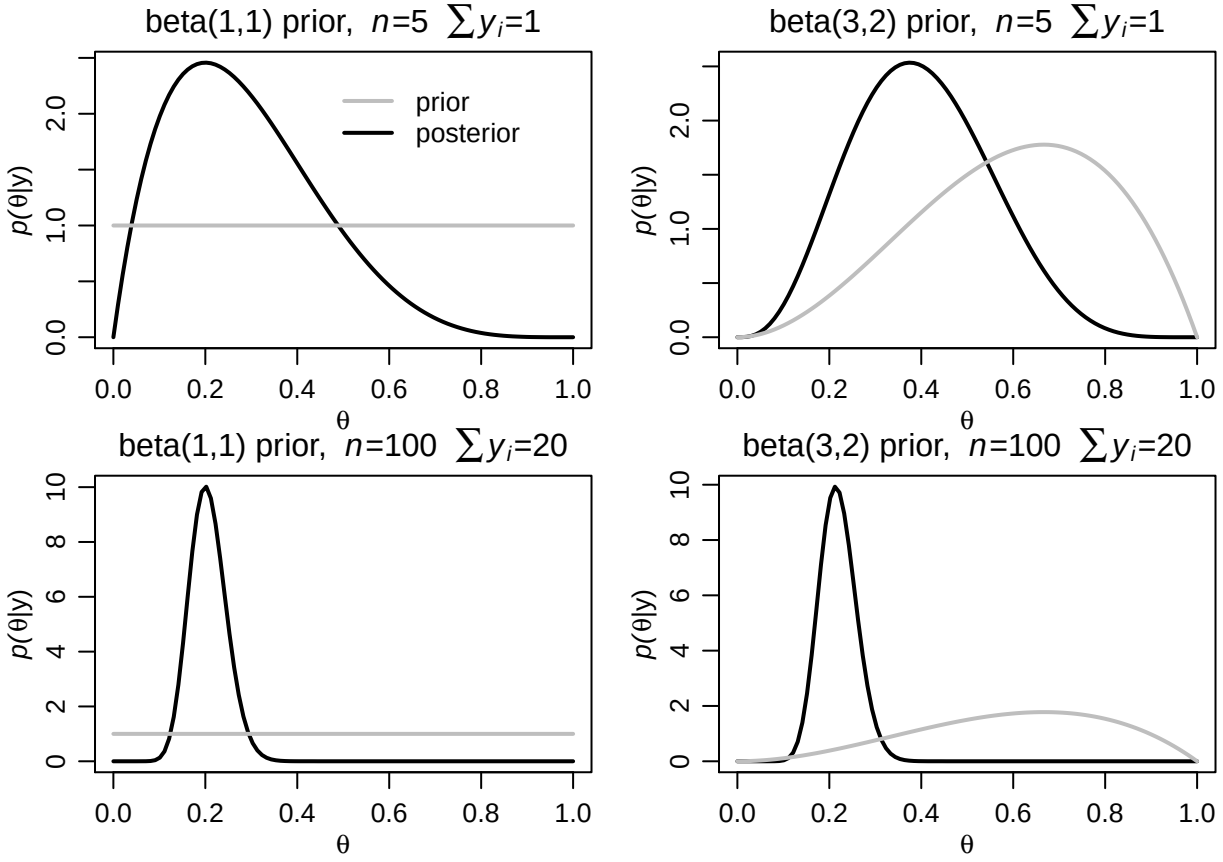

Fig 3.4

```
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
```

```r
a<-1  ; b<-1   #prior
n<-10 ; y<-2   #data
theta.support<-seq(0,1,length=100)
plot(theta.support, dbeta(theta.support, a+y, b+n-y), type="l",
     xlab=expression(theta),ylab=expression(paste(italic("p("),theta,"|y)")))
abline(v=qbeta( c(.025,.975), a+y,b+n-y))
```

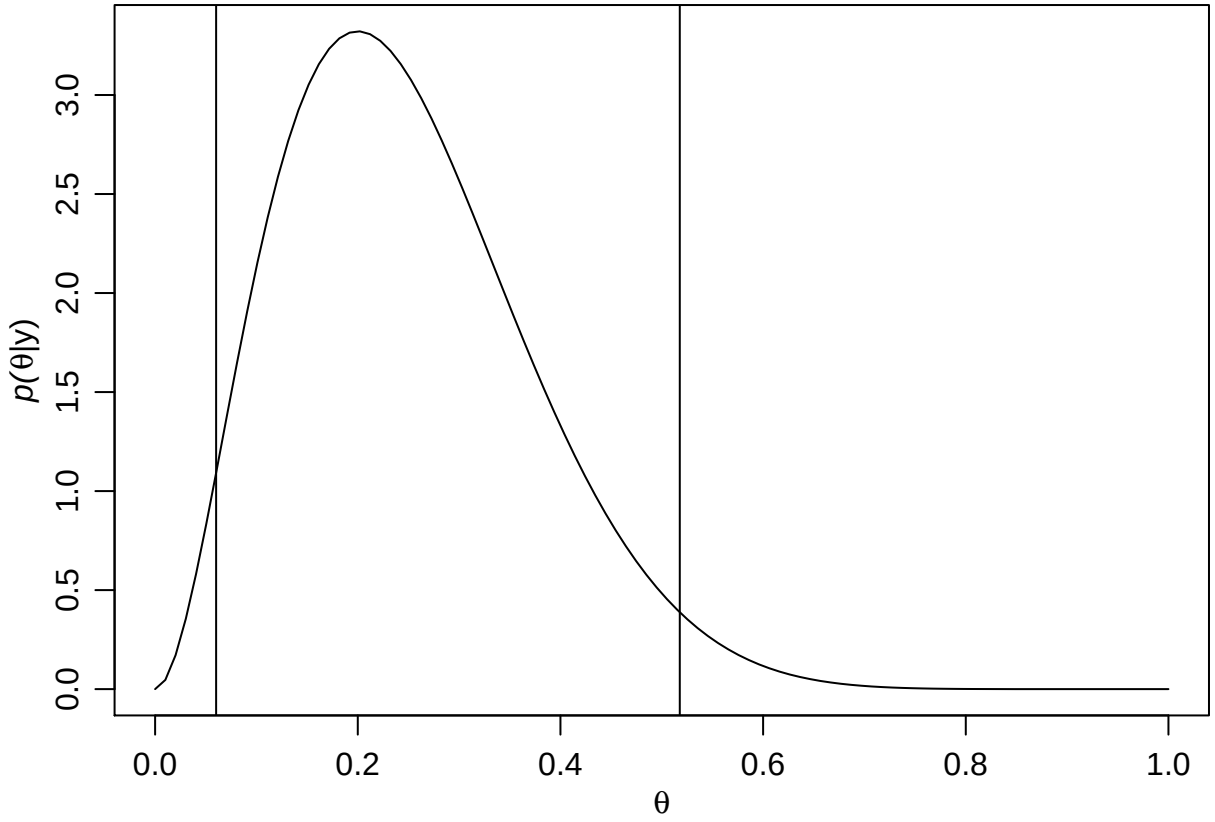

Fig 3.5

```r
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))

theta.support<-seq(0,1,length=5000)
plot(theta.support, dbeta(theta.support, a+y, b+n-y), type="l",
     xlab=expression(theta),ylab=expression(paste(italic("p("),theta,"|y)")))
pth<-dbeta(theta.support, a+y, b+n-y)
pth<-pth
ord<- order(-pth)
xpx<-cbind(theta.support[ord], pth[ord])
xpx<-cbind(xpx,cumsum(xpx[,2])/sum(xpx[,2]))

hpd<-function(x,dx,p){
  md<-x[dx==max(dx)]
  px<-dx/sum(dx)
  pxs<--sort(-px)
```

```
  ct<-min(pxs[cumsum(pxs)< p])
  list(hpdr=range(x[px>=ct]),mode=md) }

tmp<-hpd(xpx[,1],xpx[,2],.5)$hpdr
lines( x=c(tmp[1],tmp[1],tmp[2],tmp[2]),
       y=dbeta(c(0,tmp[1],tmp[2],0),a+y,b+n-y)  ,col=gray(.75),lwd=2   )
tmp<-hpd(xpx[,1],xpx[,2],.75)$hpdr
lines( x=c(tmp[1],tmp[1],tmp[2],tmp[2]),
       y=dbeta(c(0,tmp[1],tmp[2],0),a+y,b+n-y)  ,col=gray(.5),lwd=2   )
tmp<-hpd(xpx[,1],xpx[,2],.95)$hpdr
lines( x=c(tmp[1],tmp[1],tmp[2],tmp[2]),
       y=dbeta(c(0,tmp[1],tmp[2],0),a+y,b+n-y)  ,col=gray(0),lwd=2   )

tmp<-qbeta( c(.025,.975), a+y,b+n-y)
lines( x=c(tmp[1],tmp[1],tmp[2],tmp[2]),
       y=dbeta(c(0,tmp[1],tmp[2],0),a+y,b+n-y)  ,col=gray(0),lwd=2 ,lty=2  )


legend(.5, 2.75, c("50% HPD","75% HPD","95% HPD","95% quantile-based"),
       col=c(gray(.75),gray(.5),
             gray(0),gray(0)),lty=c(1,1,1,2),lwd=c(2,2,2,2),
       bty="n")
```

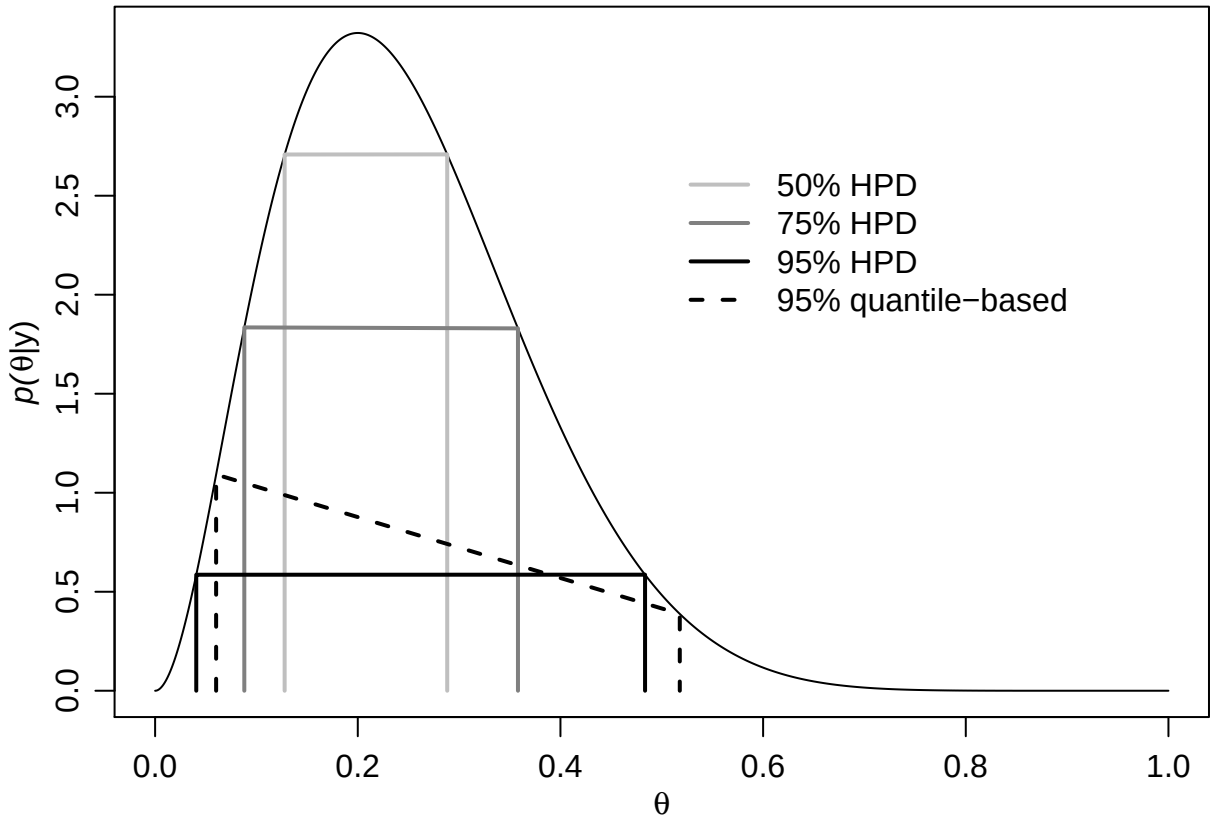

Fig 3.6

```
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
par(mfrow=c(1,2))

CHILDS<-gss$CHILDS[gss$FEMALE==1 & gss$YEAR>=1990 & gss$AGE==40 & !is.na(gss$DEG)]
CHILDS<-CHILDS[!is.na(CHILDS)]
ecdf<-as.vector((table(c(CHILDS,0:8))-1 )/sum(table(CHILDS)))
plot(0:8+.1,ecdf,type="h",lwd=5,xlab="number of children",
     ylab=expression(paste("Pr(",italic(Y[i])==y[i],")",sep="")),col="gray")
points(0:8-.1, dpois(0:8,mean(CHILDS,na.rm=T)),lwd=5,col="black",type="h")

legend(2.25,.31,
       legend=c("Poisson model","empirical distribution"),lwd=c(2,2),col=
         c("black","gray"),bty="n",cex=.75)

plot(0:52,dpois(0:52,10*mean(CHILDS)),lwd=3,col="black",type="h",
     xlab="number of children",
     ylab=expression(paste("Pr(",italic(sum(Y[i])==y),"|",theta==1.83,")",
                           sep="")))
```

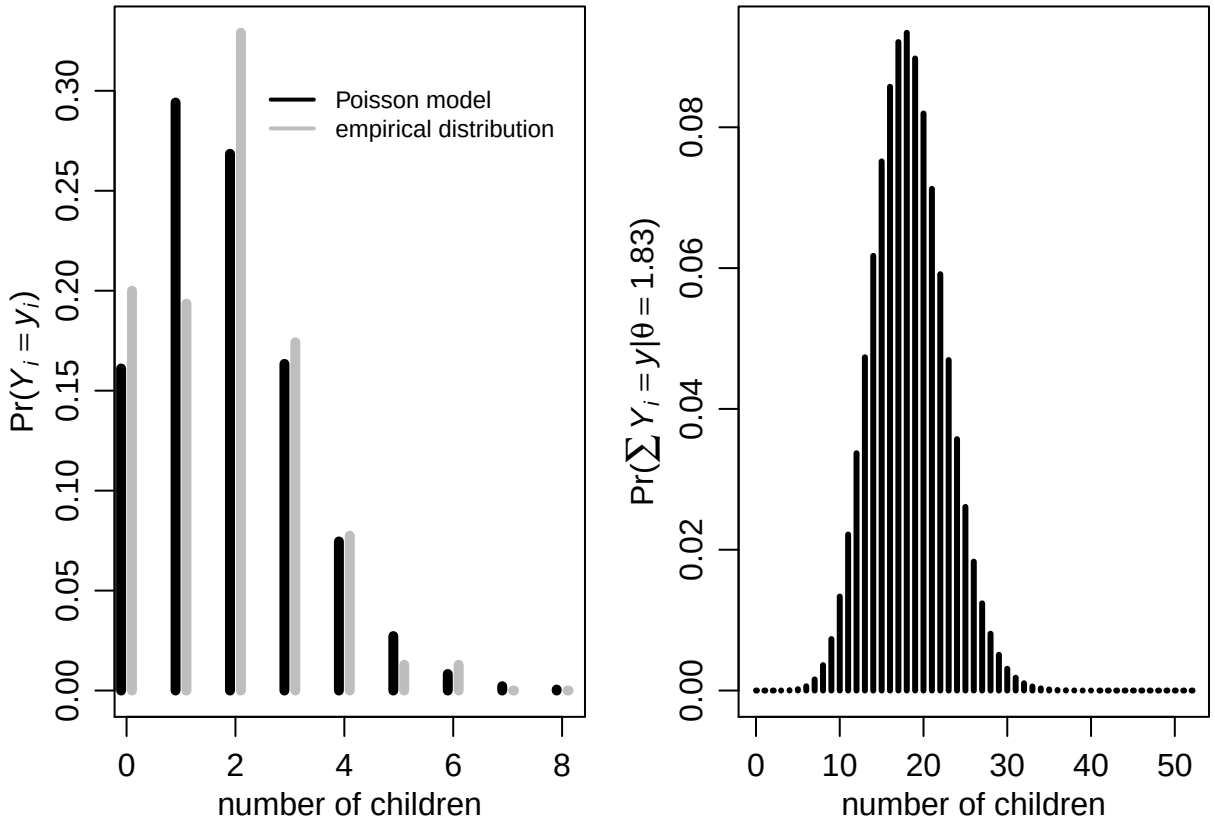

Fig 3.7

```
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
par(mfrow=c(2,3))
```

```
a<-1 ; b<-1
x<-seq(.001,10,length=100)
plot(x, dgamma(x,a,b),type="l",
     xlab=expression(theta), ylab=expression(italic(paste("p(",theta,")",sep=""))))
mtext(expression(italic(paste("a=",1," b=",1,sep=""))),side=3,line=.12,cex=.8)

a<-2 ; b<-2
x<-seq(.001,10,length=100)
plot(x, dgamma(x,a,b),type="l",
     xlab=expression(theta), ylab=expression(italic(paste("p(",theta,")",sep=""))))
mtext(expression(italic(paste("a=",2," b=",2,sep=""))),side=3,line=.12,cex=.8)

a<-4 ; b<-4
x<-seq(.001,10,length=100)
plot(x, dgamma(x,a,b),type="l",
     xlab=expression(theta), ylab=expression(italic(paste("p(",theta,")",sep=""))))
mtext(expression(italic(paste("a=",4," b=",4,sep=""))),side=3,line=.12,cex=.8)

a<-2 ; b<-1
x<-seq(.001,10,length=100)
plot(x, dgamma(x,a,b),type="l",
     xlab=expression(theta), ylab=expression(italic(paste("p(",theta,")",sep=""))))
mtext(expression(italic(paste("a=",2," b=",1,sep=""))),side=3,line=.12,cex=.8)

a<-8 ; b<-4
x<-seq(.001,10,length=100)
plot(x, dgamma(x,a,b),type="l",
     xlab=expression(theta), ylab=expression(italic(paste("p(",theta,")",sep=""))))
mtext(expression(italic(paste("a=",8," b=",4,sep=""))),side=3,line=.12,cex=.8)

a<-32 ; b<-16
x<-seq(.001,10,length=100)
plot(x, dgamma(x,a,b),type="l",
     xlab=expression(theta), ylab=expression(italic(paste("p(",theta,")",sep=""))))
mtext(expression(italic(paste("a=",32," b=",16,sep=""))),side=3,line=.12,cex=.8)
```

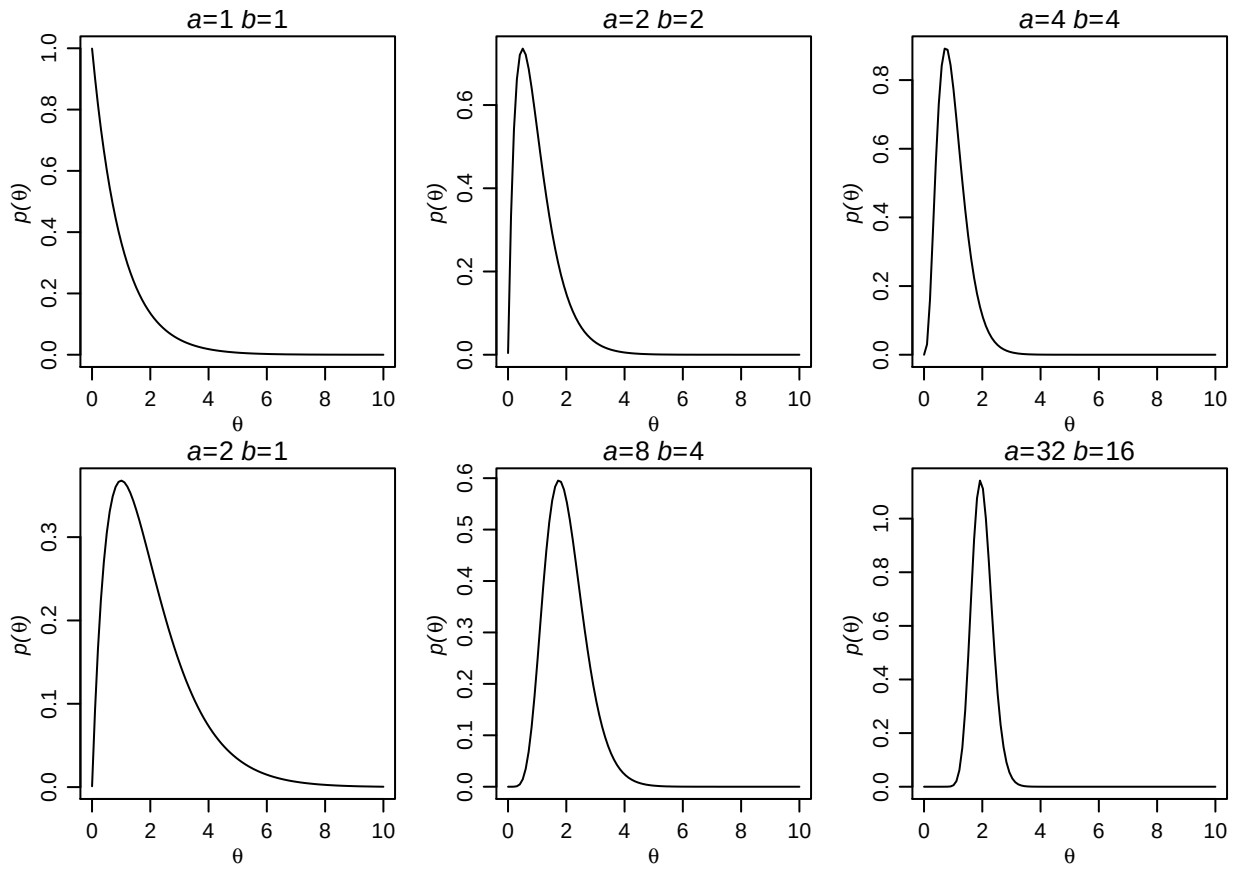

Fig 3.8

```
y2<-gss$CHILDS[gss$FEMALE==1 &  gss$YEAR>=1990  & gss$AGE==40 & gss$DEG>=3 ]
y1<-gss$CHILDS[gss$FEMALE==1 &  gss$YEAR>=1990  & gss$AGE==40 & gss$DEG<3 ]

y2<-y2[!is.na(y2)]
y1<-y1[!is.na(y1)]

par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
par(mfrow=c(1,2))

set.seed(1)
n1<-length(y1) ; n2<-length(y2)
s1<-sum(y1)
s2<-sum(y2)

par(mfrow=c(1,2),mar=c(3,3,1,1),mgp=c(1.75,.75,0))
plot(table(y1), type="h",xlab=expression(italic(y)),ylab=expression(italic(n[1](y))),col=gray(.5) ,lwd=3
mtext("Less than bachelor's",side=3)
plot(table(y2), type="h",xlab=expression(italic(y)),ylab=expression(italic(n[2](y))),col=gray(0),lwd=3)
mtext("Bachelor's or higher",side=3,lwd=3)
```

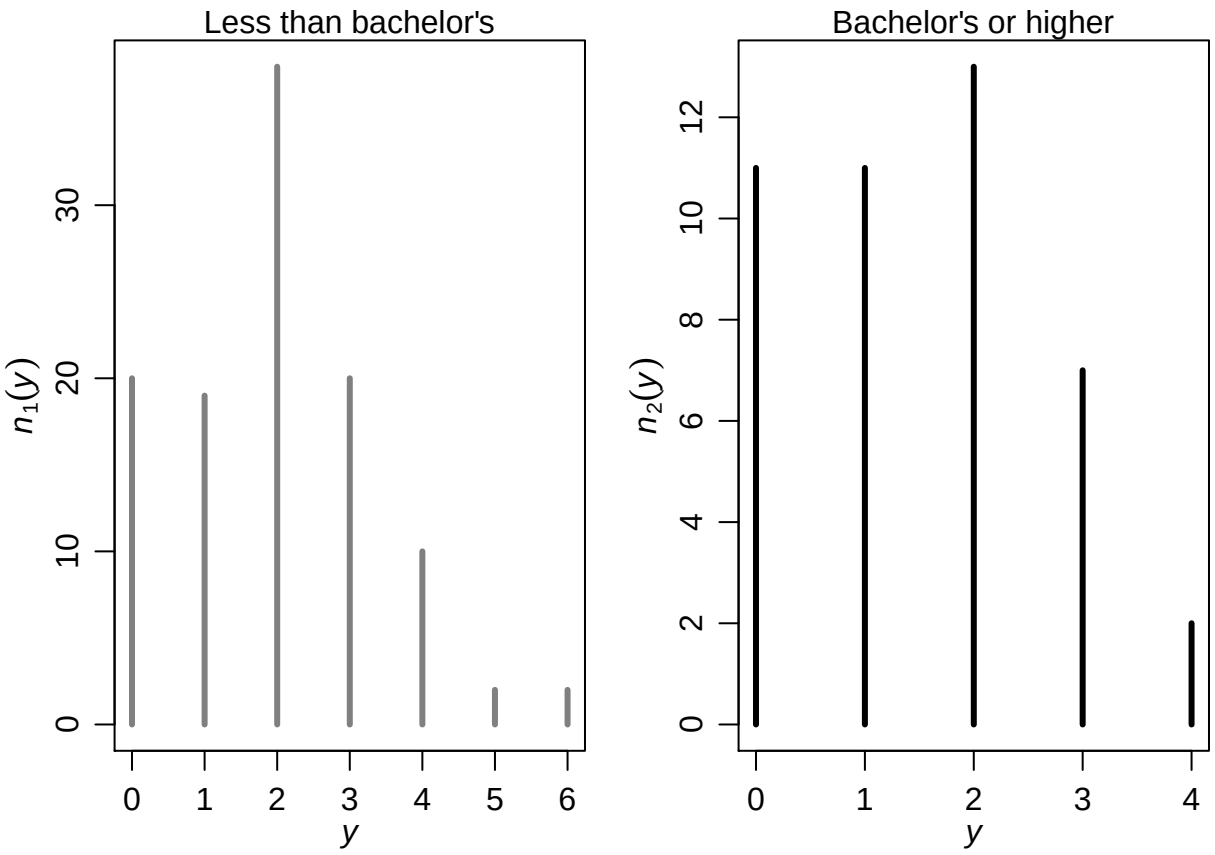

Fig 3.9

```
par(mar=c(3,3,1,1),mgp=c(1.75,.75,0))
par(mfrow=c(1,2))
a<-2
b<-1
xtheta<-seq(0,5,length=1000)
plot(xtheta,dgamma(xtheta,a+s1,b+n1),type="l",col=gray(.5),xlab=expression(theta),
     ylab=expression(paste(italic("p("),theta,"|",y[1],"...",y[n],")",sep="")))
lines(xtheta,dgamma(xtheta,a+s2,b+n2),col=gray(0),lwd=2)
lines(xtheta,dgamma(xtheta,a,b),type="l",lty=2,lwd=2)
abline(h=0,col="black")

y<-(0:12)
plot(y-.1, dnbinom(y, size=(a+s1), mu=(a+s1)/(b+n1)) , col=gray(.5) ,type="h",
     ylab=expression(paste(italic("p("),y[n+1],"|",y[1],"...",y[n],")",sep="")),
     xlab=expression(italic(y[n+1])),ylim=c(0,.35),lwd=3)
points(y+.1, dnbinom(y, size=(a+s2), mu=(a+s2)/(b+n2)) , col=gray(0) ,type="h",lwd=3)
legend(1,.375,legend=c("Less than bachelor's","Bachelor's or higher"),bty="n",
       lwd=c(3,3),col=c(gray(.5),gray(0)))
```

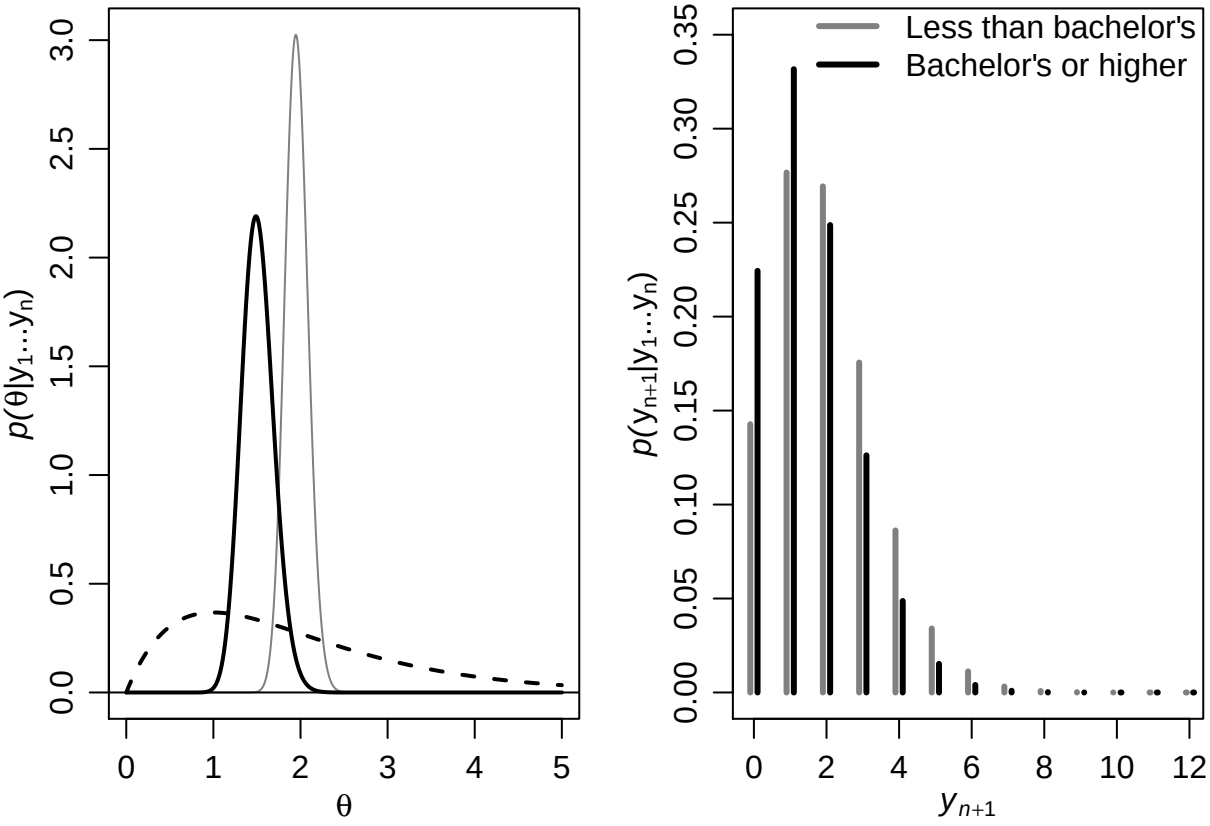
$p(\theta|y_1...y_n)$
θ
$p(y_{n+1}|y_1...y_n)$
y_{n+1}
Less than bachelor's
Bachelor's or higher

Fig 3.10