# MI1124E CALCULUS II

Version: 2024.2.0

## 1. GENERAL INFORMATION

**Course title:** Calculus II

**Unit in charge** Faculty of Mathematics and Informatics

**Course ID:** MI1124E

**Course Units:** 3(2-2-0-6)

- Lecture: 30 hours
- Seminar: 30 hours

**Previous module:** MI1114E Calculus I

**Prerequisites:** MI1114E Calculus I

**Companion module:** MI1134E Calculus III

## 2. DESCRIPTION

This course provides some applications of differential calculus in geometry, the basic ideas and techniques of parameter-dependent integrals, double integrals and triple integrals, line integrals of scalar fields and vector fields, surface integrals of scalar fields and vector fields, and vector fields.

## 3. OBJECTIVES AND EXPECTED OUTCOMES

Students who complete this module have the abilities to:

| Objectives | Objectives description/Expected Outcomes | Outcome standard allocated for modules/ Levels (I/T/U) |
|---|---|---|
| [1] | [2] | [3] |
| **M1** | **Master the basic knowledge of Caculus II and apply in practice to solve related exercises** | |
| M1.1 | Master the basic concepts such as: double integrals, triple integrals, line integrals, surface integrals, vector fields as well as applications of differential calculus | I/T |
| M1.2 | Be able to apply the knowledge to solve exercises | T/U |
| **M2** | **Achieve serious attitude and necessary skills for highly effective work** | |
| M2.1 | Be skilled at analyzing and solving problems with strong logical thinking; working independently and staying focused | T/U |
| M2.2 | Identify some practical problems that can be solved by using tools of calculus. | I/T/U |
| M2.3 | Gain serious working attitude, proactive creativity, adaptation to highly competitive working environment | I/T |

## 4. COURSE MATERIALS

### Textbooks


[1] James Stewart (2016). *Calculus: Concepts and Contexts, eighth edition*. Thomson, Brooks/Cole Publishing Company

[2] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2015). *Toán học cao cấp tập 2.* NXB Giáo dục.


### References


[1] Trần Bình (2005). *Giải tích II*. NXB Khoa học và Kỹ thuật.

[2] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2017). *Bài tập Toán học cao cấp tập 2.* NXB Giáo dục.

[3] Trần Thị Kim Oanh, Phan Xuân Thành, Lê Chí Ngọc, Nguyễn Thị Thu Hương (2022), *Giải tích II: Hàm số nhiều biến số (bài giảng dành cho sinh viên các trường kĩ thuật)*, NXB Bách Khoa Hà Nội.

[4] Khoa Toán – Tin (2023), *Slides bài giảng Giải tích 2* (tài liệu lưu hành nội bộ).


## 5. ASSESSMENT

| Components | Evaluation method | Description | Assessed expected outcomes | Proportion |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| **A1. The process mark** | | | | **50%** |
| **A1.1. Attendance and performance*** | Attendance and performance in class | | M2.3 | **10%** |
| **A1.2. Continuous assessment** | Continuous assessment test | Online multiple choice tests | M1.1, M1.2 | **10%** |
| **A1.3. Midterm exam** | Midterm exam<br>**Content: From the 1st week to the 7th week** | Multiple choice and constructed response test | M1.1, M1.2, M2.1, M2.2, M2.3 | **30%** |
| **A2. Final exam** | **Final exam** | Essay | M1.1, M1.2, M2.1, M2.2, M2.3 | **50%** |

**Attendance and performance in class are evaluated according to the Rule of Faculty of Mathematics and Informatics accompanied with the Regulations of Higher Education of Hanoi University of Science and Technology.*

## 6. COURSE PLAN

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| 1 | **Chapter 1: Applications of differential calculus in geometry**<br>1.1. Applications in plane geometry<br>- Normal vector and equations for tangent lines and normal lines of a curve at a point.<br>- Curvature: mean curvature, curvature at a point, formula of curvature at a point (no proof) and examples.<br>- Envelope of a family of parametric curves: definition, formula, examples.<br>1.2. Applications in spatial geometry<br>- Vector functions, derivative of vector functions ($\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$) and properties. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | Lecturer:<br>- Self-introduce<br>- Introduce the course outline<br>- Explain teaching and learning methods; and forms of subject assessment<br>- Lecture, exchange questions and answers with students during the lecture<br>Student:<br>- Read in advance the next lesson<br>- Master the basic concepts and apply to solve exercises according to the content and progress of the subject | A1.1<br>A1.2<br>A.1.3<br>A2 |
| 2 | - Curves: equations of tangent lines and normal planes at a point of curves, curvature at a point of curves (formulas).<br>- Surfaces: equations of tangent planes and normal lines at a point of surfaces (formulas).<br>**Chapter 2. Multiple integrals**<br>2.1. Double integrals<br>- Definition, geometric meaning, properties.<br>- Calculations of double integrals in the Cartesian coordinate system. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | Lecturer<br>- Lecture, exchange questions and answers with students during the lecture<br>Student:<br>- Read in advance the next lesson | A1.1<br>A1.2<br>A.1.3<br>A2 |

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| 3 | - Change of variables in double integrals: general change of variables formula, change of variables in polar coordinate system. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | - Master the basic concepts and apply to solve exercises according to the content and progress of the subject | A1.1<br>A1.2<br>A.1.3<br>A2 |
| 4 | - Applications of double integrals: Calculate the volume of an object, the area of a plane domain, the area of a surface (formulas and examples).<br>2.2. Triple integrals<br>- Definition, geometric meaning, properties. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A.1.3<br>A2 |
| 5 | - Calculations of triple integrals in the Cartesian coordinate system.<br>- Change of variables in triple integrals: general change of variables formula, change of variables in cylindrical coordinate system, change of variables in spherical coordinate system. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A.1.3<br>A2 |
| 6 | - Applications: Calculate the volume of an object.<br>**Chapter 3. Parameter Dependent Integrals**<br>3.1. Definite Integrals depending on parameters<br>- Definition<br>- Theorems on continuity. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A.1.3<br>A2 |
| 7 | - Theorems on differentiation under integral sign, integration under integral sign.<br>3.2. Improper Integrals depending on parameters<br>- Definition<br>- Uniform convergence, Weierstrass theorem. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A.1.3<br>A2 |
| 8 | - Properties: continuity, differentiation under integral sign, integration under integral sign.<br>3.3. Euler's integrals<br>- Introduce Gamma function ( $\Gamma(p)$) and properties: definiteness, continuity, infinite differentiability. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A2 |

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| | $\Gamma(p+1) = p\Gamma(p) \quad \forall p > 0,$<br>$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin(p\pi)} \quad (0 < p < 1)$<br>(no proof).<br>- Beta function: Introduce Beta function ( $\mathrm{B}(p,q)$) with its two types and properties (no proof): symmetry.<br>$\mathrm{B}(p,q) = \frac{p}{p+q-1}\mathrm{B}(p,q-1),$<br>$\mathrm{B}(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$ | | | |
| 9 | **Chapter 4. Line Integrals**<br>4.1. Line integrals of scalar fields<br>- Definition<br>- Calculation<br>4.2. Line integrals of vector fields<br>- Definition, physical meaning.<br>- Properties | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | - Lecture, exchange questions and answers with students during the lecture<br>Student:<br>- Read in advance the next lesson<br>- Master the basic concepts and apply to solve exercises according to the content and progress of the subject | A1.1<br>A1.2<br>A2 |
| 10 | - Relation of line integrals of scalar fields and line integrals of vector fields.<br>- Calculation<br>- Green's Theorem (proof for the case of a simple region). | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A2 |
| 11 | - Path independence of line integrals (no proof); find a function $u(x,y)$such that $du = Pdx + Qdy$.<br>**Chapter 5. Surface integrals**<br>5.1 Surface integrals of scalar fields<br>- Definition<br>- Calculation | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A2 |
| 12 | 5.2 Surface integrals of vector fields<br>- Definition, properties.<br>- Relation of surface integrals of scalar fields and surface integrals of vector fields.<br>- Calculation | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A2 |
| 13 | - Ostrogradsky's Theorem, Stoke's Theorem (no proof).<br>**Chapter 6.** Field Theory<br>6.1 Scalar Fields | M1.1<br>M1.2<br>M2.1<br>M2.2 | | A1.1<br>A1.2<br>A2 |

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| | - Notions of scalar fields and level surfaces.<br>- Directional derivative: Definition, Theorem on relation between directional derivative and partial derivative. | M2.3 | | |
| 14 | - Gradient: Definition of vector $\text{grad}u$ and theorem $\frac{\partial u}{\partial \vec{\ell}} = ch_{\vec{\ell}}\text{grad}u$ (no proof) and properties.<br>6.2 Vector Fields<br>- Notions of vector fields and flow lines, system of differential equations of flow lines.<br>- The flux, div, incompressible fields: the flux of a vector field across oriented surface *S*, div (divergence), properties, incompressible fields, source (point), sink (point). | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A2 |
| 15 | - Circulation and curl vector: the circulation of a vector field around an oriented closed curve, curl vector, curly point.<br>- Conservative vector fields: notions of conservative vector fields $\vec{F}$, the potential function for $\vec{F}$, conditions for a vector field to be conservative, conditions for an expression to be the total differential, path independence of spatial line integrals. | M1.1<br>M1.2<br>M2.1<br>M2.2<br>M2.3 | | A1.1<br>A1.2<br>A2 |
| 16 | **Summary and revision** | | | |

## 7. RULES OF THE MODULE

(Regulations of the course if any)

## 8. DATE OF APPROVAL: …….

**Faculty of Mathematics and Informatics**