MI1121E

CALCULUS II

Version: 2024.2.0

1. GENERAL INFORMATION

Course title: Calculus II

Unit in charge Faculty of Mathematics and Informatics

Course ID: MI1121E
Course Units: 3(2-2-0-6)

Lecture: 30 hoursSeminar: 30 hours

Previous module: MI1111E Calculus I
Prerequisites: MI1111E Calculus I
Companion module: MI1131E Calculus III

2. DESCRIPTION

This course provides some applications of differential calculus in geometry, the basic ideas and techniques of parameter-dependent integrals, double integrals and triple integrals, line integrals of scalar fields and vector fields, surface integrals of scalar fields and vector fields, and vector fields.

3. OBJECTIVES AND EXPECTED OUTCOMES

Students who complete this module have the abilities to:

| Objectives | ojectives Objectives description/Expected Outcomes | |
|------------|--|-------|
| [1] | [2] | [3] |
| M1 | Master the basic knowledge of Caculus II and apply in practice to solve related exercises | |
| M1.1 | Master the basic concepts such as: double integrals, triple integrals, line integrals, surface integrals, vector fields as well as applications of differential calculus | I/T |
| M1.2 | Be able to apply the knowledge to solve exercises | T/U |
| M2 | Achieve serious attitude and necessary skills for highly effective work | |
| M2.1 | Be skilled at analyzing and solving problems with strong logical thinking; working independently and staying focused | T/U |
| M2.2 | Identify some practical problems that can be solved by using tools of calculus. | I/T/U |
| M2.3 | Gain serious working attitude, proactive creativity, adaptation to highly competitive working environment | I/T |

4. COURSE MATERIALS

Textbooks

- [1] James Stewart (2016). Calculus: Concepts and Contexts, eighth edition. Thomson, Brooks/Cole Publishing Company
- [2] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2015). *Toán học cao cấp tập 2*. NXB Giáo dục.

References

- [1] Trần Bình (2005). Giải tích II. NXB Khoa học và Kỹ thuật.
- [2] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2017). *Bài tập Toán học cao cấp tập 2*. NXB Giáo dục.
- [3] Trần Thị Kim Oanh, Phan Xuân Thành, Lê Chí Ngọc, Nguyễn Thị Thu Hương, Giải tích II: Hàm số nhiều biến số (bài giảng dành cho sinh viên các trường kĩ thuật), NXB Bách Khoa Hà Nôi, 2022.
- [4] Khoa Toán Tin (2023), Slides bài giảng Giải tích 2 (tài liệu lưu hành nội bộ).

5. ASSESSMENT

| Components | Evaluation method | Description | Assessed expected outcomes | Proportion |
|-----------------------------------|--|--|--|------------|
| [1] | [2] | [3] | [4] | [5] |
| A1. The process mark | | | | 50% |
| A1.1. Attendance and performance* | Attendance and performance in class | | M2.3 | 10% |
| A1.2. Continuous assessment | Continuous assessment test | Online multiple choice tests | M1.1, M1.2 | 10% |
| A1.3. Midterm exam | Midterm exam Content: From the 1 st week to the 7 th week | Multiple choice and constructed response test | M1.1, M1.2, M2.1, M2.2, M2.3 | 30% |
| A2. Final exam | Final exam | Essay | M1.1, M1.2, M2.1, M2.2, M2.3 | 50% |

^{*}Attendance and performance in class are evaluated according to the Rule of Faculty of Mathematics and Informatics accompanied with the Regulations of Higher Education of Hanoi University of Science and Technology.

6. COURSE PLAN

| Week | Topics | Objective | Activities | Exercises |
|------|--|--------------------------------------|---|-----------------------------|
| [1] | [2] | [3] | [4] | [5] |
| 1 | Chapter 1: Applications of differential calculus in geometry 1.1. Applications in plane geometry - Normal vector and equations for tangent lines and normal lines of a curve at a point. - Curvature: mean curvature, curvature at a point, formula of curvature at a point (no proof) and examples. - Envelope of a family of parametric curves: definition, formula, examples. 1.2. Applications in spatial geometry - Vector functions, derivative of vector functions $(\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k})$ and properties. | M1.1 M1.2 M2.1 M2.2 M2.3 | Lecturer: - Self- introduce - Introduce the course outline - Explain teaching and learning methods; and forms of subject assessment - Lecture, exchange questions and answers with students during the lecture Student: - Read in advance the next lesson - Master the basic concepts and apply to solve exercises according to the content and | 5 A1.1 |
| 2 | Curves: equations of tengent lines and | M1.1 | progress of the subject | A 1 1 |
| 2 | Curves: equations of tangent lines and normal planes at a point of curves, curvature at a point of curves (formulas). Surfaces: equations of tangent planes and normal lines at a point of surfaces (formulas). Chapter 2. Multiple integrals Double integrals Definition, geometric meaning, properties. | M1.1 M1.2 M2.1 M2.2 M2.3 | Lecturer - Lecture, exchange questions and answers with students during the lecture Student: - Read in advance the | A1.1 A1.2 A.1.3 A2 |
| | - Calculations of double integrals in the Cartesian coordinate system. | | advance the next lesson | |

| Week | Topics | Objective | Activities | Exercises |
|------|---|-----------|------------------------------|-----------|
| [1] | [2] | [3] | [4] | [5] |
| 3 | - Change of variables in double | M1.1 | - Master the | A1.1 |
| | integrals: general change of variables | M1.2 | basic concepts | A1.2 |
| | formula, change of variables in polar | M2.1 | and apply to solve exercises | A.1.3 |
| | coordinate system. | M2.2 | according to | A2 |
| | | M2.3 | the content and | |
| 4 | - Applications of double integrals: | M1.1 | progress of the | A1.1 |
| | Calculate the volume of an object, the | M1.2 | subject | A1.2 |
| | area of a plane domain, the area of a | M2.1 | | A.1.3 |
| | surface (formulas and examples). | M2.2 | | A2 |
| | 2.2. Triple integrals | M2.3 | | |
| | - Definition, geometric meaning, properties. | | | |
| 5 | - Calculations of triple integrals in the | M1.1 | | A1.1 |
| | Cartesian coordinate system. | M1.2 | | A1.2 |
| | - Change of variables in triple integrals: | M2.1 | | A.1.3 |
| | general change of variables formula, | M2.2 | | A2 |
| | change of variables in cylindrical coordinate system, change of variables | M2.3 | | |
| | in spherical coordinate system. | | | |
| 6 | - Applications: Calculate the volume of | M1.1 | | A1.1 |
| | an object. | M1.2 | | A1.2 |
| | Chapter 3. Parameter Dependent | M2.1 | | A.1.3 |
| | Integrals | M2.2 | | A2 |
| | 3.1. Definite Integrals depending on | M2.3 | | |
| | parameters | | | |
| | - Definition | | | |
| | - Theorems on continuity. | | | |
| 7 | - Theorems on differentiation under | M1.1 | | A1.1 |
| | integral sign, integration under integral | M1.2 | | A1.2 |
| | sign. 3.2. Improper Integrals depending on | M2.1 | | A.1.3 |
| | parameters | M2.2 | | A2 |
| | - Definition | M2.3 | | |
| | - Uniform convergence, Weierstrass | | | |
| | theorem. | | | |
| 8 | - Properties: continuity, differentiation | M1.1 | | A1.1 |
| | under integral sign, integration under | M1.2 | | A1.2 |
| | integral sign. | M2.1 | | A2 |
| | 3.3. Euler's integrals | M2.2 | | |
| | - Introduce Gamma function ($\Gamma(p)$) and | M2.3 | | |
| | properties: definiteness, continuity, | | | |
| | infinite differentiability. | | | |

| Week | Topics | Objective | Activities | Exercises |
|------|---|-----------------|---------------------------------|--------------|
| [1] | [2] | [3] | [4] | [5] |
| | $\Gamma(p+1) = p\Gamma(p) \ \forall p > 0,$ | | | |
| | $\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin(p\pi)} (0$ | | | |
| | (no proof). | | | |
| | - Beta function: Introduce Beta function | | | |
| | (B(p,q)) with its two types and | | | |
| | properties (no proof): symmetry. | | | |
| | $B(p,q) = \frac{p}{p+q-1}B(p,q-1),$ | | | |
| | $B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$ | | | |
| | $\Gamma(p+q) = \frac{\Gamma(p+q)}{\Gamma(p+q)}$ | | | |
| 9 | Chapter 4. Line Integrals | M1.1 | - Lecture, | A1.1 |
| | 4.1. Line integrals of scalar fields | M1.2 | exchange | A1.2 |
| | - Definition | M2.1 | questions and | A2 |
| | - Calculation | M2.2 | answers with | |
| | 4.2. Line integrals of vector fields | M2.3 | students during | |
| | - Definition, physical meaning. | | the lecture | |
| | - Properties | | Student: | |
| 10 | - Relation of line integrals of scalar | M1.1 | - Read in | A1.1 |
| 10 | fields and line integrals of vector fields. | M1.1 | advance the next lesson | A1.1 A1.2 |
| | - Calculation | M2.1 | | A1.2 A2 |
| | - Green's Theorem (proof for the case of | M2.2 | - Master the basic concepts | A2 |
| | a simple region). | M2.3 | and apply to | |
| 1.1 | Dati to the control of | | solve exercises | A 1 1 |
| 11 | - Path independence of line integrals (no proof); find a function $u(x, y)$ such that | M1.1 | according to | A1.1 |
| | du = Pdx + Qdy. | M1.2 | the content and progress of the | A1.2 |
| | ~ 7 | M2.1 | subject | A2 |
| | Chapter 5. Surface integrals | M2.2 | | |
| | 5.1 Surface integrals of scalar fields- Definition | M2.3 | | |
| | - Calculation | | | |
| 10 | | 2.61.1 | | 4.1.1 |
| 12 | 5.2 Surface integrals of vector fields | M1.1 | | A1.1 |
| | - Definition, properties. | M1.2 | | A1.2 |
| | - Relation of surface integrals of scalar | M2.1 | | A2 |
| | fields and surface integrals of vector fields. | M2.2 | | |
| | - Calculation | M2.3 | | |
| 13 | - Ostrogradsky's Theorem, Stoke's | M1.1 | | A1.1 |
| | Theorem (no proof). | M1.2 | | A1.1 A1.2 |
| | Chapter 6. Field Theory | M2.1 | | A1.2 A2 |
| | 6.1 Scalar Fields | M2.1 M2.2 | | <i>A</i> 2 |
| | 15 | 1 V1 ∠.∠ | | |

| Week | Topics | Objective | Activities | Exercises |
|------|---|--------------------------------------|------------|--------------------|
| [1] | [2] | [3] | [4] | [5] |
| | Notions of scalar fields and level surfaces. Directional derivative: Definition, Theorem on relation between directional derivative and partial derivative. | M2.3 | | |
| 14 | - Gradient: Definition of vector gradu | M1.1 | | A1.1 |
| | and theorem $\frac{\partial u}{\partial \vec{l}} = ch_{\vec{l}} \operatorname{grad} u$ (no proof) | M1.2 | | A1.2 |
| | and properties. | M2.1 | | A2 |
| | 6.2 Vector Fields | M2.2 | | |
| | Notions of vector fields and flow lines, system of differential equations of flow lines. The flux, div, incompressible fields: the flux of a vector field across oriented surface <i>S</i>, div (divergence), properties, incompressible fields, source (point), sink (point). | M2.3 | | |
| 15 | Circulation and curl vector: the circulation of a vector field around an oriented closed curve, curl vector, curly point. Conservative vector fields: notions of conservative vector fields \$\vec{F}\$, the potential function for \$\vec{F}\$, conditions for a vector field to be conservative, conditions for an expression to be the total differential, path independence of spatial line integrals. | M1.1 M1.2 M2.1 M2.2 M2.3 | | A1.1 A1.2 A2 |
| 16 | Summary | | | A1.1 A3.1 |

7. RULES OF THE MODULE

(Regulations of the course if any)

8. DATE OF APPROVAL:

Faculty of Mathematics and Informatics