# MI1131E          CALCULUS III

***Objective:*** To provide the knowledge and calculation skills on infinite series and basic differential equations, one-sided Laplace transform, to formulate Mathematical foundations for students majored in technology, to provide mathematical tools for students.

***Contents:*** Infinite number series, series of functions, Fourier series, first-order differential equations, second-order linear differential equations, systems of first-order differential equations, Laplace transforms, some models and modeling of technical problems.

## 1. GENERAL INFORMATION

| | |
|---|---|
| **Course title:** | Calculus III |
| **Unit in charge:** | Faculty of Mathematics and Informatics |
| **Course ID:** | MI1131E |
| **Course Units:** | 3(2-2-0-6) |
| | - Lecture: 30 hours |
| | - Seminar: 30 hours |
| **Previous module:** | |
| **Prerequisites:** | - MI1111E Calculus I |
| **Companion module:** | - MI1121E Calculus II |

## 2. DESCRIPTION

This course provides students with the basic knowledge on infinite series, differential equations, and the Laplace operator method.

## 3. OBJECTIVES AND EXPECTED OUTCOMES

Students who complete this module have the abilities to:

| Objectives | Objectives description/Expected Outcomes | Outcome standard allocated for modules/ Levels (I/T/U) |
|---|---|---|
| **M1** | **Master the basic knowledge about series, ordinary differential equations** | |
| M1.1 | Master the basic concepts | I/T |
| M1.2 | Be able to apply the knowledge to solve exercises | T/U |
| **M2** | **Achieve serious attitude and necessary skills for highly effective work** | |
| M2.1 | Be skilled at analyzing and solving problems with strong logical thinking; working independently and staying focused | T/U |
| M2.2 | Identify some practical problems that can be solved by using tools of series, differential equations and Laplace operator method | I/T/U |
| M2.3 | Gain serious working attitude, proactive creativity, adaptation to highly competitive working environment | I/T |

## 4. COURSE MATERIALS

**Textbooks**

[1] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2015). *Toán học cao cấp tập 3: Chuỗi và phương trình vi phân.* NXB Giáo dục VN.

[2] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2017). *Bài tập Toán học cao cấp tập 3: Chuỗi và phương trình vi phân.* NXB Giáo dục VN.

[3] Nguyễn Đình Trí, Tạ Văn Đĩnh, Nguyễn Hồ Quỳnh (2000). *Bài tập Toán học cao cấp tập II.* NXB Giáo dục.

[4] Nguyễn Đình Trí, Tạ Văn Đĩnh, Nguyễn Hồ Quỳnh (1999). *Bài tập Toán học cao cấp tập III.* NXB Giáo dục.

**References**

[1] Nguyễn Thiệu Huy, Bùi Xuân Diệu, Đào Tuấn Anh: *Giải tích III, chuỗi vô hạn và phương trình vi phân.* NXB Bách Khoa Hà Nội, 2022.

[2] Nguyễn Xuân Thảo (2010). *Bài giảng Phương pháp Toán tử Laplace* (tài liệu lưu hành nội bộ).

[3] Nguyen Thieu Huy, Vu Thi Ngoc Ha: *Infinite series and differential equations*, Hanoi University of Science and Technology, Elite Technology program, 2022.

[4] Trần Bình (2005). *Giải tích II và III*, NXB KH và KT.

[5] J. Stewart, D. Clegg, S. Watson, *Multivariable Calculus,* 9th Edition, Cengage Learning, 2020.

[6] W. E. Boyce, R.C. DiPrima, D.B. Meade, *Elementary Differential Equations and Boundary Value Problems,* 11th Edition, Wiley, 2017.

[7] R. Bronson, G. B. Costa, *Differential Equations,* 4th Edition, The McGraw-Hill, 2014.

## 5. ASSESSMENT

| Components | Evaluation method | Description | Assessed expected outcomes | Proportion |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
| **A1. The process mark** | | | | **50%** |
| **A1.1. Attendance and performance*** | Attendance and performance in class | | M2.3 | **10%** |
| **A1.2. Continuous assessment** | Continuous assessment test | Online multiple choice tests | M1.1, M1.2 | **10%** |
| **A1.3. Midterm exam** | Midterm exam **Content: From the 1st** | Multiple choice and constructed | M1.1, M1.2, M2.1, | **30%** |

| | week to the 7th week | response test | M2.2, M2.3 | |
|---|---|---|---|---|
| **A2. Final exam** | **Final exam** | Essay | M1.1, M1.2, M2.1, M2.2, M2.3 | **50%** |

*\*Attendance and performance in class are evaluated according to the Rule of Faculty of Mathematics and Informatics accompanied with the Regulations of Higher Education of Hanoi University of Science and Technology.*

## 6. COURSE PLAN

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| **[1]** | **[2]** | **[3]** | **[4]** | **[5]** |
| 1 | **Chapter 1**. Series (11+ 11) <br><br> **1.1** Infinite number series <br><br> - Definitions: Number series, general term, partial sums, remainder, convergence, divergence, sum of a series. Note: including geometric series $\sum_{n=0}^{+\infty} aq^n$. <br><br> - Necessary condition for convergence (with proof). Note: including the harmonic series $\sum_{n=1}^{+\infty} \frac{1}{n}$. <br><br> - Fundamental properties of convergent series (proofs for self-study) <br><br> **1.2** Series of non-negative terms <br><br> - Definition <br><br> - Comparison tests (including proof of the first comparison test, proof of the second one is for self-study) <br><br> - Tests for convergence (D'Alambert's test, Cauchy's test, integral test) (including the proof of D'Alambert's test, the proofs of the other are for self-study). Note: including $\sum_{n=1}^{+\infty} \frac{1}{n^\alpha}$ | M1, M2 | Lecturer: <br><br> - Self-introduce <br><br> - Introduce the course outline <br><br> - Explain teaching and learning methods; and forms of subject assessment <br><br> - Lecture, exchange questions and answers with students during the lecture <br><br> Student: <br><br> - Read in advance the next lesson <br><br> - Master the basic concepts and apply to solve exercises according to | A1.1 A1.2 A1.3 A2 |

| Week | Topics | Objective | Activities | Exercises |
|------|--------|-----------|------------|-----------|
| [1] | [2] | [3] | [4] | [5] |
| | | | the content and progress of the subject | |
| 2 | **1.3** Series of sign-changing terms<br><br>- Series of sign-changing terms: definitions of absolute convergence, conditional convergence. Theorems on absolutely convergent series (proofs for self-study)<br><br>- Alternating series: definition, Leibniz's test (with proof)<br><br>- Properties of absolutely convergent series. Properties of rearrangement of terms and the product of two series (proofs for self-study) | M1, M2 | Lecturer:<br>- Lecture, exchange questions and answers with students during the lecture<br><br>Student:<br>- Read in advance the next lesson<br>- Master the basic concepts and apply to solve exercises as well as some practical models connected with the subject | A1.1<br>A1.2<br>A1.3<br>A2 |
| 3 | **1.4** Series of functions<br><br>- Definitions: series of functions, domain of convergence (pointwise convergence), sum of a function series<br><br>- Uniform convergence: definition, Cauchy's test, Weierstrass' test (without proof)<br><br>- Properties of uniformly convergent function series: continuity, differentiation, integration (proofs of the last two properties are for self-study) | M1, M2 | | A1.1<br>A1.2<br>A1.3<br>A2 |
| 4 | **1.5** Power series<br><br>- Definition, Abel's theorem (with proof), radius, interval and domain of convergence<br><br>- Properties: uniform convergence, continuity of the sum, termwise differentiation and integration (proofs for self-study). Applications in finding sum of a series (one example, self-study)<br><br>- Representation of functions by power series (Taylor's series, Maclaurin's series). Theorems on expandability of a function in a power series (without proof) | M1, M2 | | A1.1<br>A1.2<br>A1.3<br>A2 |
| 5 | - Expansion of some elementary functions. Applications in approximating the value of functions and definite integrals (for self-study)<br><br>**1.6** Fourier series | M1, M2 | | A1.1<br>A1.2<br>A1.3<br>A2 |

| Week | Topics | Objective | Activities | Exercises |
|------|--------|-----------|------------|-----------|
| [1] | [2] | [3] | [4] | [5] |
| | - Trigonometric series, Fourier series<br><br>- Conditions for expanding a function to Fourier series. Dirichlet's theorem (without proof) | | | |
| 6 | - Fourier expansion of odd and even $2\pi$ periodic functions<br><br>- Fourier expansion of $2\pi$ periodic functions, $2l$ period functions. Fourier expansion of functions defined on an interval $[a,b]$<br><br>**Chapter 2. Ordinary differential equations (11+ 12)**<br><br>**2.1** Introduction<br><br>- Definition: ordinary differential equations (ODEs), order of an ODE, solutions to an ODE<br><br>**2.2** First order ODEs<br><br>- Outlines about first order ODEs: general forms, existence and uniqueness theorem (without proof), Cauchy problem, general solutions, particular solutions. Introductory practical examples of first order ODEs | M1, M2 | | A1.1<br>A1.2<br>A1.3<br>A2 |
| 7 | - Equations without $x$ or $y$<br><br>- Separable equations<br><br>- Homogeneous equations<br><br>- Linear equations<br><br>- Bernoulli equations<br><br>- Exact equations | M1, M2 | | A1.1<br>A1.2<br>A1.3<br>A2 |
| 8 | **2.3** Second order differential equations<br><br>- Outlines about first order ODEs: general forms, existence and uniqueness theorem (without proof), Cauchy problem, general solutions, particular solutions. Introductory practical examples of second order ODEs<br><br>- Equations without $y$ and $y$'; Equations without $y$; Equations without $x$<br><br>- Linear equations $y'' + p(x)y' + q(x)y = f(x)$<br><br>Homogeneous linear equations: structure of general solutions (proofs of the theorem | M1, M2 | | A1.1<br>A1.2<br>A2 |

| Week | Topics | Objective | Activities | Exercises |
|------|--------|-----------|------------|-----------|
| **[1]** | **[2]** | **[3]** | **[4]** | **[5]** |
| | yielding the formula $y = C_1 y_1(x) + C_2 y_2(x)$) | | | |
| 9 | Nonhomogeneous linear equations: structure of general solutions (proof for self-study)<br><br>Lagrange method of variation of parameters Superposition principle<br><br>- Second order linear ODEs with constant coefficients<br>Homogeneous linear equations | M1, M2 | Lecturer:<br>- Lecture, exchange questions and answers with students during the lecture<br><br>Student:<br>- Read in advance the next lesson<br>- Master the basic concepts and apply to solve exercises as well as some practical models connected with the subject | A1.1<br>A1.2<br>A2 |
| 10 | Nonhomogeneous linear equations with right-hand side of the forms<br>$f(x) = e^{\alpha x} P_n(x)$<br>$f(x) = e^{\alpha x}[P_n(x)\cos \beta x + Q_m(x)\sin \beta x]$ | M1, M2 | Lecturer:<br>- Lecture, exchange questions and answers with students during the lecture<br><br>Student:<br>- Read in advance the next lesson<br>- Master the basic concepts and apply to solve | A1.1<br>A1.2<br>A2 |
| 11 | - Euler equations (introduction by examples)<br><br>**2.4** Systems of first order ODEs<br><br>- Definition, general form, solutions, convert higher order ODEs into systems of first order equations and vice versa. Existence and uniqueness theorem.<br><br>- Solving by substitution: illustrated by a simple example (this part is for self-study) | M1, M2 | | A1.1<br>A1.2<br>A2 |
| 12 | **Chapter 3**. **Laplace transform and applications (8+7)** | M1, M2 | | A1.1<br>A1.2 |

| Week | Topics | Objective | Activities | Exercises |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] | [5] |
|  | **3.1** Laplace transform and inverse Laplace transform<br><br>- Laplace transform, linearity property, tables of Laplace transform, piecewise continuous functions, existence of Laplace transform. Examples<br><br>- Inverse Laplace transform, uniqueness of inverse Laplace transform. Examples |  | exercises as well as some practical models connected with the subject | A2 |
| 13 | **3.2** Transform of initial value problems<br><br>- Transform of the derivative of a function, solutions of initial value problems, examples of solving second order linear ODEs with constant coefficients<br><br>- Systems of second order linear ODEs, introduction to mathematical modeling<br>- Transform of the integral of a function | M1, M2 |  | A1.1<br>A1.2<br>A2 |
| 14 | **3.3** Shifting properties and partial fractions<br><br>- Linear partial fractions, irreducible quadratic partial fractions, $s$- shifting<br><br>- Solving higher order (greater than or equal to 3) ODEs with constant coefficients | M1, M2 |  | A1.1<br>A1.2<br>A2 |
| 15 | **3.4** Derivatives, integrals and product of Laplace transforms<br><br>- Convolution of two functions, Laplace transform of convolution<br><br>- Derivative of Laplace transform<br><br>- Integral of Laplace transform<br><br>- Solving homogeneous linear second order ODEs with variable coefficients<br><br>- Solving linear second order ODEs with constant coefficients and piecewise continuous righthand side | M1, M2 |  | A1.1<br>A1.2<br>A2 |
| 16 | Revision – Summary | M1, M2 |  | A2 |

## 7. COURSE REGULATIONS

(Regulations of the course if any)

## 8. DATE OF APPROVAL: …………………..

**Faculty of Mathematics and Informatics**