MI1134E DIFFERENTIAL EQUATIONS AND SERIES

Version: 2024.2.0

Objective: To provide the knowledge and calculation skills on infinite series and basic differential equations, one-sided Laplace transform, to formulate Mathematical foundations for students majored in technology, to provide mathematical tools for students.

Contents: Infinite number series, series of functions, Fourier series, first-order differential equations, second-order linear differential equations, systems of first-order differential equations, Laplace transforms, some models and modeling of technical problems.

1. GENERAL INFORMATION

Course title: Differential Equations and Series

Unit in charge: Faculty of Mathematics and Informatics

Course ID: MI1134E

Course Units: 3(2-2-0-6)
- Lecture: 30 hours
- Seminar: 30 hours

Previous module:

Prerequisites:
- MI1111E Calculus I

Companion module:
- MI1121E Calculus II

2. DESCRIPTION

This course provides basic knowledge on infinite series, ordinary differential equations, and the Laplace operator method.

3. OBJECTIVES AND EXPECTED OUTCOMES

Students who complete this module have the abilities to:

Objectives	Objectives description/Expected Outcomes	Outcome standard allocated for modules/ Levels (I/T/U)
M1	**Master the basic knowledge on infinite series, ordinary differential equations and Laplace operator method**	
M1.1	Master the basic knowledge	I/T
M1.2	Have the ablility to apply knowledge in solving problems related to the content of the course	T/U
M2	**Achieve serious work attitude and necessary skills to work effectively**	
M2.1	Be skillful in analyzing and solving problems by logical thinking; in working independently and staying focused	T/U
M2.2	Identify some practical problems that can be solved by using the tools of series, differential equations and the Laplace operator method	I/T/U
M2.3	Gain serious working attitude, proactive creativity, adaptation to highly competitive working environment	I/T

4. COURSE MATERIALS

Textbooks

[1] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2015). *Toán học cao cấp tập 3: Chuỗi và phương trình vi phân.* NXB Giáo dục VN.

[2] Nguyễn Đình Trí, Trần Việt Dũng, Trần Xuân Hiển, Nguyễn Xuân Thảo (2017). *Bài tập Toán học cao cấp tập 3: Chuỗi và phương trình vi phân.* NXB Giáo dục VN.

[3] Nguyễn Đình Trí, Tạ Văn Đĩnh, Nguyễn Hồ Quỳnh (2000). *Bài tập Toán học cao cấp tập II.* NXB Giáo dục.

[4] Nguyễn Đình Trí, Tạ Văn Đĩnh, Nguyễn Hồ Quỳnh (1999). *Bài tập Toán học cao cấp tập III.* NXB Giáo dục.

References

[1] Nguyễn Thiệu Huy, Bùi Xuân Diệu, Đào Tuấn Anh: *Giải tích III, chuỗi vô hạn và phương trình vi phân.* NXB Bách Khoa Hà Nội, 2022.

[2] Nguyễn Xuân Thảo (2010). *Bài giảng Phương pháp Toán tử Laplace* (tài liệu lưu hành nội bộ).

[3] Nguyen Thieu Huy, Vu Thi Ngoc Ha: *Infinite series and differential equations*, Hanoi University of Science and Technology, Elite Technology program, 2022.

[4] Trần Bình (2005). *Giải tích II và III*, NXB KH và KT.

[5] J. Stewart, D. Clegg, S. Watson, *Multivariable Calculus,* 9th Edition, Cengage Learning, 2020.

[6] W. E. Boyce, R.C. DiPrima, D.B. Meade, *Elementary Differential Equations and Boundary Value Problems,* 11th Edition, Wiley, 2017.

[7] R. Bronson, G. B. Costa, *Differential Equations,* 4th Edition, The McGraw-Hill, 2014.

5. ASSESSMENT

Components	Evaluation method	Description	Assessed expected outcomes	Proportion
[1]	[2]	[3]	[4]	[5]
A1. The process mark				**50%**
A1.1. Attendance and performance*	Attendance and performance in class		M2.3	**10%**
A1.2. Continuous assessment	Continuous assessment test	Online multiple choice tests	M1.1, M1.2	**10%**
A1.3. Midterm exam	Midterm exam **Content: From chapter 1 to the section 2.2 of**	Multiple choice and constructed	M1.1, M1.2, M2.1,	**30%**

	chapter 2	response test	M2.2, M2.3	
A2. Final exam	**Final exam**	Essay	M1.1, M1.2, M2.1, M2.2, M2.3	**50%**

**Attendance and performance in class are evaluated according to the Rule of Faculty of Mathematics and Informatics accompanied with the Regulations of Higher Education of Hanoi University of Science and Technology.*

6. COURSE PLAN

Week	Topics	Objective	Activities	Exercises
[1]	[2]	[3]	[4]	[5]
1	**Chapter 1**. Series **(11+ 11)** **1.1** Infinite number series - Definitions: Number series, general term, partial sums, remainder, convergence, divergence, sum of a series. Note: including geometric series $\sum_{n=0}^{+\infty} aq^n$. - Necessary condition for convergence (with proof). Note: including the harmonic series $\sum_{n=1}^{+\infty} \frac{1}{n}$. - Fundamental properties of convergent series (proofs for self-study) **1.2** Series of non-negative terms - Definition - Comparison tests (including proof of the first comparison test, proof of the second one is for self-study) - Tests for convergence (D'Alembert's test, Cauchy's test, integral test) (including the proof of D'Alembert's test, the proofs of the other are for self-study). Note: including $\sum_{n=1}^{+\infty} \frac{1}{n^\alpha}$	M1, M2	Lecturer: - Self-introduce - Introduce the course outline - Explain teaching and learning methods; and forms of subject assessment - Lecture, exchange questions and answers with students during the lecture Student: - Read in advance the next lesson - Master the basic concepts and apply to solve exercises according to the content	A1.1 A1.2 A1.3 A2

Week	Topics	Objective	Activities	Exercises
[1]	[2]	[3]	[4]	[5]
			and progress of the subject	
2	**1.3** Series of sign-changing terms - Series of sign-changing terms: definitions of absolute convergence, conditional convergence. Theorems on absolutely convergent series (proofs for self-study) - Alternating series: definition, Leibniz's test (with proof) - Properties of absolutely convergent series. Properties of rearrangement of terms and the product of two series (proofs for self-study)	M1, M2	Lecturer: - Lecture, exchange questions and answers with students during the lecture Student: - Read in advance the next lesson - Master the basic concepts and apply to solve exercises as well as some practical models connected with the subject	A1.1 A1.2 A1.3 A2
3	**1.4** Series of functions - Definitions: series of functions, domain of convergence (pointwise convergence), sum of a function series - Uniform convergence: definition, Cauchy's test, Weierstrass' test (without proof) - Properties of uniformly convergent function series: continuity, differentiation, integration (proofs of the last two properties are for self-study)	M1, M2		A1.1 A1.2 A1.3 A2
4	**1.5** Power series - Definition, Abel's theorem (with proof), radius, interval and domain of convergence - Properties: uniform convergence, continuity of the sum, termwise differentiation and integration (proofs for self-study). Applications in finding sum of a series (one example, self-study) - Representation of functions by power series (Taylor's series, Maclaurin's series). Theorems on expandability of a function in a power series (without proof)	M1, M2		A1.1 A1.2 A1.3 A2
5	- Expansion of some elementary functions. Applications in approximating the value of functions and definite integrals (for self-study) **1.6** Fourier series - Trigonometric series, Fourier series	M1, M2		A1.1 A1.2 A1.3 A2

Week	Topics	Objective	Activities	Exercises
[1]	[2]	[3]	[4]	[5]
	- Conditions for expanding a function to Fourier series. Dirichlet's theorem (without proof)			
6	- Fourier expansion of odd and even 2π periodic functions - Fourier expansion of 2π periodic functions, $2l$ period functions. Fourier expansion of functions defined on an interval $[a,b]$ **Chapter 2. Ordinary differential equations (11+ 12)** **2.1** Introduction - Definition: ordinary differential equations (ODEs), order of an ODE, solutions to an ODE **2.2** First order ODEs - Outlines about first order ODEs: general forms, existence and uniqueness theorem (without proof), Cauchy problem, general solutions, particular solutions. Introductory practical examples of first order ODEs	M1, M2		A1.1 A1.2 A1.3 A2
7	- Equations without x or y - Separable equations - Homogeneous equations - Linear equations - Bernoulli equations - Exact equations	M1, M2		A1.1 A1.2 A1.3 A2
8	**2.3** Second order differential equations - Outlines about first order ODEs: general forms, existence and uniqueness theorem (without proof), Cauchy problem, general solutions, particular solutions. Introductory practical examples of second order ODEs - Equations without y and y'; Equations without y; Equations without x - Linear equations $y''+p(x)y'+q(x)=f(x)$ Homogeneous linear equations: structure of general solutions (proofs of the theorem yielding the formula $y=C_1y_1(x)+C_2y_2(x)$)	M1, M2		A1.1 A1.2 A2

Week	Topics	Objective	Activities	Exercises
[1]	[2]	[3]	[4]	[5]
9	Nonhomogeneous linear equations: structure of general solutions (proof for self-study) Lagrange method of variation of parameters Superposition principle - Second order linear ODEs with constant coefficients Homogeneous linear equations	M1, M2	Lecturer: - Lecture, exchange questions and answers with students during the lecture Student: - Read in advance the next lesson - Master the basic concepts and apply to solve exercises as well as some practical models connected with the subject	A1.1 A1.2 A2
10	Nonhomogeneous linear equations with right-hand side of the forms $f(x) = e^{\alpha x} P_n(x)$ $f(x) = e^{\alpha x}[P_n(x)\cos\beta x + Q_m(x)\sin\beta x]$	M1, M2	Lecturer: - Lecture, exchange questions and answers with students during the lecture Student: - Read in advance the next lesson - Master the basic concepts and apply to solve exercises as well as	A1.1 A1.2 A2
11	- Euler equations (introduction by examples) **2.4** Systems of first order ODEs - Definition, general form, solutions, convert higher order ODEs into systems of first order equations and vice versa. Existence and uniqueness theorem. - Solving by substitution: illustrated by a simple example (this part is for self-study)	M1, M2		A1.1 A1.2 A2
12	**Chapter 3. Laplace transform and applications (8+7)** **3.1** Laplace transform and inverse Laplace	M1, M2		A1.1 A1.2 A2

Week	Topics	Objective	Activities	Exercises
[1]	[2]	[3]	[4]	[5]
	transform - Laplace transform, linearity property, tables of Laplace transform, piecewise continuous functions, existence of Laplace transform. Examples - Inverse Laplace transform, uniqueness of inverse Laplace transform. Examples		some practical models connected with the subject	
13	**3.2** Transform of initial value problems - Transform of the derivative of a function, solutions of initial value problems, examples of solving second order linear ODEs with constant coefficients - Systems of second order linear ODEs, introduction to mathematical modeling - Transform of the integral of a function	M1, M2		A1.1 A1.2 A2
14	**3.3** Shifting properties and partial fractions - Linear partial fractions, irreducible quadratic partial fractions, *s*- shifting - Solving higher order (greater than or equal to 3) ODEs with constant coefficients	M1, M2		A1.1 A1.2 A2
15	**3.4** Derivatives, integrals and product of Laplace transforms - Convolution of two functions, Laplace transform of convolution - Derivative of Laplace transform - Integral of Laplace transform - Solving homogeneous linear second order ODEs with variable coefficients - Solving linear second order ODEs with constant coefficients and piecewise continuous righthand side	M1, M2		A1.1 A1.2 A2
16	Revision – Summary	M1, M2		A2

7. COURSE REGULATIONS

(Possible course regulations)

8. DATE OF APPROVAL: ……..

Faculty of Mathematics and Informatics