# Solutions to Book Of Proof

Son To <son.trung.to@gmail.com>

Fazer Oy, Arcada Ammattikorkeakoulu January 12, 2020

# Preface

An attempt at solving all the exercises.

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# Part I Fundamentals

# Chapter 1

# Sets

# 1.1 Introduction to sets

# 1.1.1

 $\{\ldots -16, -11, -6, -1, 4, 9, 14, \ldots\}.$ 

# 1.1.2

 $\{\ldots -7, -4, -1, 2, 5, 8, 11, \ldots\}.$ 

#### 1.1.3

 $\{-2, -1, \dots, 6\}.$ 

# 1.1.4

 $\{1, 2, \dots, 7\}.$ 

# 1.1.5

 $\{\pm\sqrt{3}\}.$ 

# 1.1.6

 $\{\pm 3\}.$ 

# 1.1.7

 $\{-2, -3\}.$ 

# 1.1.8

 $\{0, -2, -3\}.$ 

#### 1.1.9

 $\mathbb{Z}$ .

#### 1.1.10

 $\{2\pi x:x\in\mathbb{Z}\}.$ 

#### 1.1.11

 $\{-4, -3, \dots, 4\}.$ 

#### 1.1.12

 $\{-2, -1, \dots, 2\}.$ 

#### 1.1.13

 $\{0\}.$ 

#### 1.1.14

$$\{-20, -15, -10, \dots, 10, 15, 20\}.$$

#### 1.1.15

Let's call the set S. It's clear that every member of S is an integer. Conversely, note that n = 5n + 2(-2n),  $n \in \mathbb{Z}$ . Therefore,  $S = \mathbb{Z}$ .

#### 1.1.16

The reasoning is similar, but note that there exists no  $a, b \in \mathbb{Z}$  such that either n = 6n + 2b or n = 6a + 2b,  $n \in \mathbb{Z}$ . Also, note that 6a + 2b = 2(3a + b), in which n = 3n - 2n. Therefore, S is the set of even integers in  $\mathbb{Z}$ .

$$S = \{2n : n \in \mathbb{Z}\} \subset \mathbb{Z} \tag{1.1}$$

#### 1.1.17

 $\{2^n:n\in\mathbb{N}\}.$ 

#### 1.1.18 Unsolved

Observation: Successive difference of each couple of numbers:  $4, 12, 20, 28, 36, \ldots$  (a difference of 8 each).

#### 1.1. INTRODUCTION TO SETS

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# 1.1.19

 ${3n:n\in\mathbb{Z}}.$ 

# 1.1.20

 $\{5n+2:n\in\mathbb{Z}\}.$ 

# 1.1.21

 ${n^2:n\in\mathbb{Z}}.$ 

# 1.1.22 Unsolved

My first conjecture was  $2^n + n$ , but it is wrong for the fourth number.

# 1.1.23

 $\{n \in \mathbb{N} : 3 \le n \le 8\}.$ 

# 1.1.24

 $\{n \in \mathbb{Z} : -4 \le n \le 2\}.$ 

# 1.1.25

 $\{2^n:n\in\mathbb{Z}\}.$ 

# 1.1.26

 ${3^n:n\in\mathbb{Z}}.$ 

# 1.1.27

 $\{\frac{n\pi}{2}:n\in\mathbb{Z}\}.$ 

# 1.1.28

 $\{\frac{3}{4}n:n\in\mathbb{Z}\}.$