

A Note of Calculus-Michael Spivak

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Preface

This is the note for the book Calculus written by Michael Spivak, citing what I think the most interesting and important subjects mentioned in the book.

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Part I

Prologue

Chapter 1

Basic properties of number

(P1) If a , b , and c are any numbers, then

$$a + (b + c) = (a + b) + c$$

See *problem 24* for the generalization of $a_1 + a_2 + a_3 + \cdots + a_n$ for (P1).

The number 0 has important properties.

(P2) If a is any number, then

$$a + 0 = 0 + a = a$$

(P3) For every number a , there is also a number $-a$ such that

$$a + (-a) = (-a) + a = 0$$

We now prove Lemma 1.

Lemma 1. *If $a + x = a$, then $x = 0$*

Proof.

| | | |
|------------|---------------------------------|-----------------|
| If | $a + x = a$ | |
| then | $(-a) + (a + x) = (-a) + a = 0$ | (by (P3)) |
| hence | $((-a) + a) + x = 0$ | (by (P1)) |
| hence | $0 + x = 0$ | (by (P3) again) |
| therefore, | $x = 0$ | (by (P2)) |

□

Also, remember that the order of addition does not matter.

(P4) If a and b are any numbers, then

$$a + b = b + a$$

However, with only (P1)-(P4), we are powerless to figure out what conditions needed to have $a - b = b - a$. Therefore, we need to set new properties, and, oddly, they involve multiplication.

(P5) If a , b and c are any numbers, then

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

(P6) If a is any number, then

$$a \cdot 1 = 1 \cdot a = a$$

Moreover, $1 \neq 0$ (This cannot be proved by other properties listed!)

(P7) For every number $a \neq 0$, there is a number a^{-1} such that

$$a \cdot a^{-1} = a^{-1} \cdot a = 1 (\Leftrightarrow 0 \cdot b = 0 \ \forall b)$$

This is why $1/0$ is meaningless!

(P8) If a and b are any numbers, then

$$a \cdot b = b \cdot a$$

From (P5), (P6) and (P7), we have two lemmas:

Lemma 2. *If $a \cdot b = a \cdot c$ then $a = 0 \vee b = c$*

Proof. If $a = 0$ then the lemma is trivial. Suppose now $a \neq 0$,

Multiply a^{-1} to both sides, $(a^{-1}) \cdot (a \cdot b) = (a^{-1}) \cdot (a \cdot c)$

By (P5), $(a^{-1} \cdot a) \cdot b = (a^{-1} \cdot a) \cdot c$

By (P7), $1 \cdot b = 1 \cdot c$

By (P6), $b = c$

□

Lemma 3. *If $a \cdot b = 0$ then $a = 0 \vee b = 0$*

Proof. If $a = 0$, there is nothing to prove. Suppose now $a \neq 0$, follow the proof of Lemma 2 by consecutively applying (P5), (P7) and (P6) in that order to finish the proof. □