

# Answers to exercises in How To Prove It

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This is to answer all the questions in the books ‘How to prove it’ by Velleman. Comments are appreciated!

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# 1 Introduction

Exercise 1.0.1.

- (a)  $a = 3, b = 5 \Rightarrow x = 2^5 - 1 = 31, y = 1 + 2^5 + 2^{10} = 1057$
- (b) Since 32,767 is not a prime,  $2^{32,767} - 1$  is not a prime either. Therefore, there exists a positive integer  $0 < x < 2^{32,767} - 1$  such that  $2^{32,767} - 1$  is divisible by  $x$ . Hence, by (a),  $x = 2^{31} - 1$  satisfies this.

Exercise 1.0.2.

$n$	$3^n - 1$	$3^n - 2^n$
2, prime	8, not prime	5, prime
3, prime	26, not prime	19, prime
4	80, not prime	65, not prime
5, prime	242, not prime	211, prime
6	728, not prime	665, not prime
7, prime	2,186, not prime	2,059, prime
8	6,560, not prime	6,305, not prime
9	19,682, not prime	19,171 = 19 · 1,009, not prime
10	59,048, not prime	58,025, not prime

*Conjecture 1.1.*  $3^n - 1$  is even for all  $n$ .

*Conjecture 1.2.* If  $n$  is prime,  $3^n - 2^n$  is prime.

*Conjecture 1.3.* If  $n$  is not prime,  $3^n - 2^n$  is not prime.

Exercise 1.0.3.

We have the following theorem.

**Theorem 1.1 (Theorem 3).** *There are infinitely many prime numbers.*

Its proof gives a method of finding a prime number  $n$  different from the ones in a given list. However, one needs to be careful of prime numbers *not* in the list and smaller than  $n$ , since the proof assumes that primes are finite. Example,  $n = 3 \cdot 5 + 1 = 15 + 1 = 16$  is not a prime number. Hence,

**Remark 1.1 (Method).** To find a prime different from the list, check if it is divisible by other primes missing from the list.

- (a) Let  $n = 2 \cdot 5 + 1 = 11$ .  $n$  is not divisible by 3, 7, a new prime number.
- (b) Let  $n = 2 \cdot 3 + 1 = 7$ .  $n$  is not divisible by 5, 11, a new prime number.

Exercise 1.0.4.

24, 25, 26, 27, 28

Exercise 1.0.5.

$$2^4 \cdot (2^5 - 1) = 496$$

$$2^6 \cdot (2^7 - 1) = 127.$$

Exercise 1.0.6.

*Conjecture 1.4.* 3, 5, 7 is the unique triplet prime.

I do not know now to prove it.

## 2 Chapter 1

### 2.1 Deductive reasoning and logical connectives

Exercise 2.1.1.

- (a)  $(R \vee H) \wedge \neg(H \wedge T)$
- (b)  $S = \text{“You go skiing”}$ ,  $N = \text{“There is snow”}$ .  
 $\neg S \vee (S \wedge \neg N)$
- (c)  $\neg((\sqrt{7} = 2) \vee (\sqrt{7} < 2))$

Exercise 2.1.2.

- (a)  $J = \text{John is telling the truth}$ ,  $B = \text{Bill is telling the truth}$   
 $(J \wedge B) \vee (\neg J \wedge \neg B)$
- (b)  $F = \text{“I have fish”}$ ,  $C = \text{“I have chicken”}$ ,  $M = \text{“I have mashed potatoes”}$   
 $(F \vee C) \wedge \neg(F \wedge M)$
- (c)  $(6:3) \wedge (9:3) \wedge (15:3)$

Exercise 2.1.3.

$A = \text{Alice is in the room}$ ,  $B = \text{Bob is in the room}$

- (a)  $\neg(A \wedge B)$
- (b)  $\neg A \wedge \neg B$