

Solutions to Book Of Proof

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Preface

An attempt at solving all the exercises.

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Part I

Fundamentals

Chapter 1

Sets

1.1 Introduction to sets

1.1.1

$\{\dots -16, -11, -6, -1, 4, 9, 14, \dots\}$.

1.1.2

$\{\dots -7, -4, -1, 2, 5, 8, 11, \dots\}$.

1.1.3

$\{-2, -1, \dots, 6\}$.

1.1.4

$\{1, 2, \dots, 7\}$.

1.1.5

$\{\pm\sqrt{3}\}$.

1.1.6

$\{\pm 3\}$.

1.1.7

$\{-2, -3\}$.

1.1.8

$\{0, -2, -3\}$.

1.1.9

\mathbb{Z} .

1.1.10

$\{2\pi x : x \in \mathbb{Z}\}$.

1.1.11

$\{-4, -3, \dots, 4\}$.

1.1.12

$\{-2, -1, \dots, 2\}$.

1.1.13

$\{0\}$.

1.1.14

$\{-20, -15, -10, \dots, 10, 15, 20\}$.

1.1.15

Let's call the set S . It's clear that every member of S is an integer. Conversely, note that $n = 5n + 2(-2n)$, $n \in \mathbb{Z}$. Therefore, $S = \mathbb{Z}$.

1.1.16

The reasoning is similar, but note that there exists no $a, b \in \mathbb{Z}$ such that either $n = 6n + 2b$ or $n = 6a + 2b$, $n \in \mathbb{Z}$. Also, note that $6a + 2b = 2(3a + b)$, in which $n = 3n - 2n$. Therefore, S is the set of even integers in \mathbb{Z} .

$$S = \{2n : n \in \mathbb{Z}\} \subset \mathbb{Z} \tag{1.1}$$

1.1.17

$\{2^n : n \in \mathbb{N}\}$.

1.1.18 Unsolved

Observation: Successive difference of each couple of numbers: 4, 12, 20, 28, 36, ... (a difference of 8 each).

1.1.19

$$\{3n : n \in \mathbb{Z}\}.$$

1.1.20

$$\{5n + 2 : n \in \mathbb{Z}\}.$$

1.1.21

$$\{n^2 : n \in \mathbb{Z}\}.$$

1.1.22 Unsolved

My first conjecture was $2^n + n$, but it is wrong for the fourth number.

1.1.23

$$\{n \in \mathbb{N} : 3 \leq n \leq 8\}.$$

1.1.24

$$\{n \in \mathbb{Z} : -4 \leq n \leq 2\}.$$

1.1.25

$$\{2^n : n \in \mathbb{Z}\}.$$

1.1.26

$$\{3^n : n \in \mathbb{Z}\}.$$

1.1.27

$$\left\{\frac{n\pi}{2} : n \in \mathbb{Z}\right\}.$$

1.1.28

$$\left\{\frac{3}{4}n : n \in \mathbb{Z}\right\}.$$

1.1.29

3. Namely, $\{1\}$, $\{2, \{3, 4\}\}$, \emptyset .

1.1.30

5. Namely, $\{1, 4\}$, a , b , $\{\{3, 4\}\}$, $\{\emptyset\}$.

1.1.31

1. Namely, the biggest set that includes all the others.

1.1.32

1. Same as above.

1.1.33

19. Namely, $-9, -8, \dots, 8, 9$.

1.1.34

9. Namely, $1, \dots, 9$.

1.1.35

7. Namely, $-3, \dots, 3$.

1.1.36

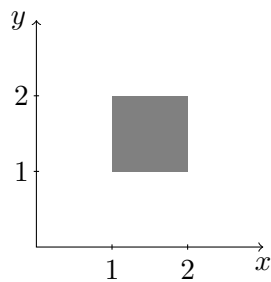
3. Namely, $1, 2, 3$.

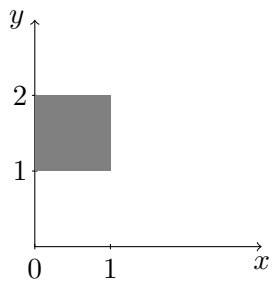
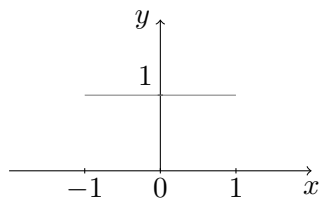
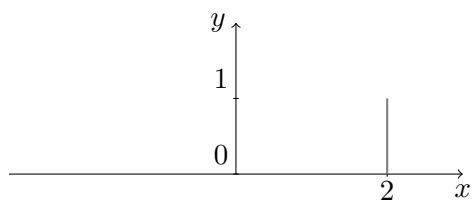
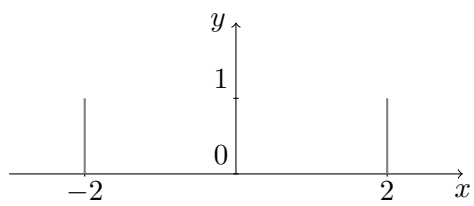
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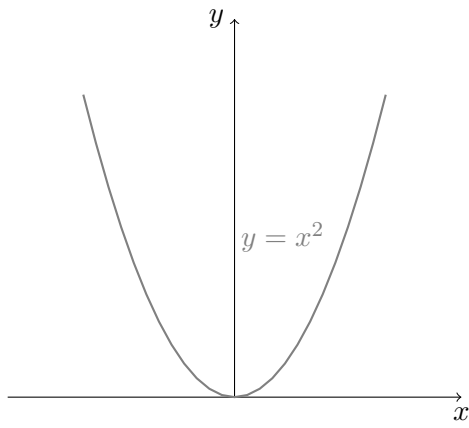
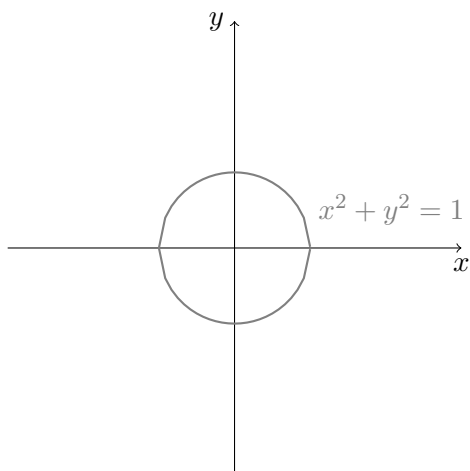
0. Namely, \emptyset .

1.1.38

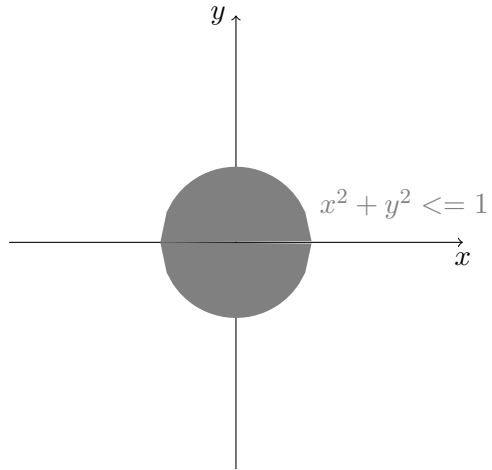
4. Namely, $1, 2, 3, 4$.

1.1.39

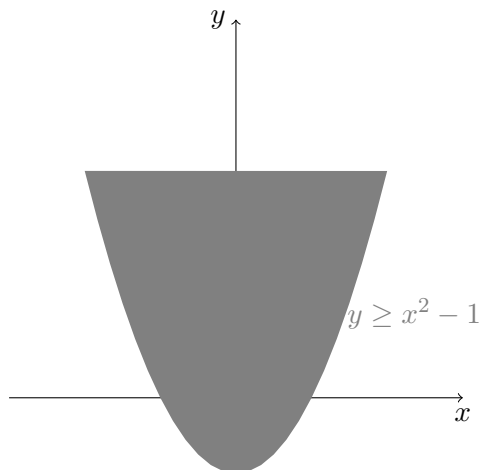
1.1.40**1.1.41****1.1.42****1.1.43**

1.1.44**1.1.45**

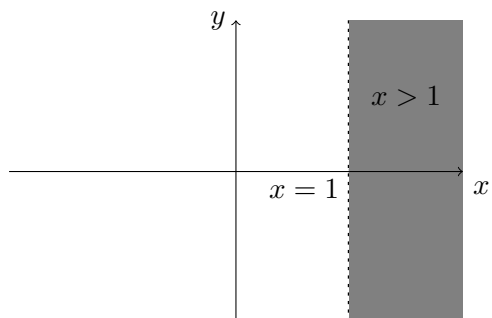
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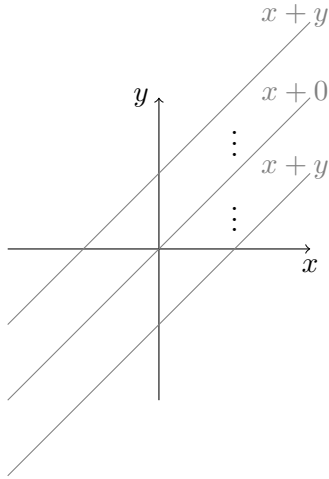
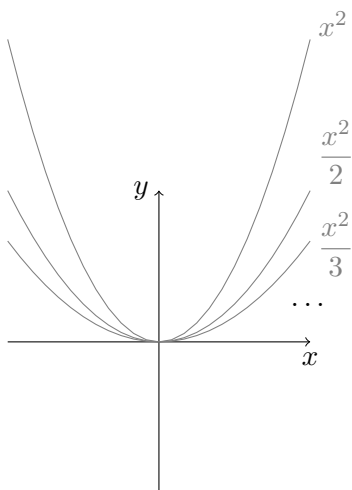
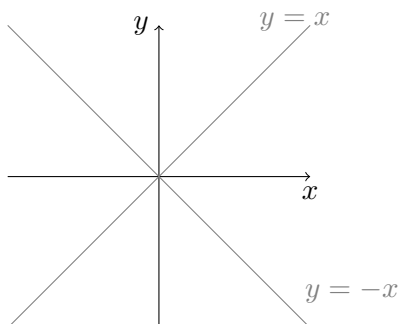


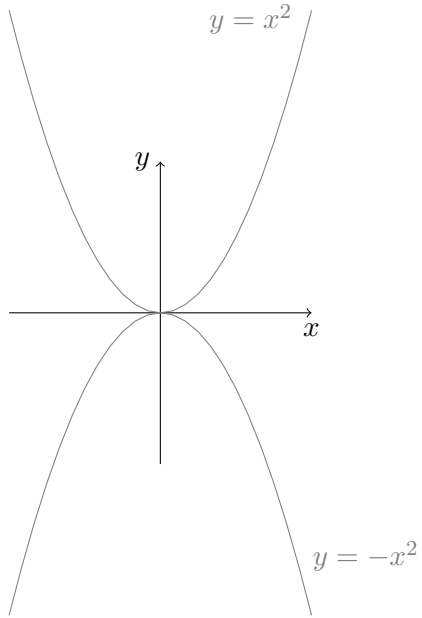
1.1.47



1.1.48



1.1.49**1.1.50****1.1.51**

1.1.52

1.2 The Cartesian Product