Solutions to Book Of Proof

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Preface

An attempt at solving all the exercises.

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Contents

Part I

Fundamentals

Chapter 1

Sets

1.1 Introduction to sets

1.1.1

$\{\ldots -16, -11, -6, -1, 4, 9, 14, \ldots\}$.

1.1.2

$\{\ldots -7, -4, -1, 2, 5, 8, 11, \ldots\}$.

1.1.3

$\{-2, -1, \ldots, 6\}$.

1.1.4

$\{1, 2, \ldots, 7\}$.

1.1.5

$\{\pm\sqrt{3}\}$.

1.1.6

$\{\pm 3\}$.

1.1.7

$\{-2, -3\}$.

1.1.8

$\{0, -2, -3\}$.

1.1.9

$\mathbb{Z}$.

1.1.10

$\{2\pi x : x \in \mathbb{Z}\}$.

1.1.11

$\{-4, -3, \ldots, 4\}$.

1.1.12

$\{-2, -1, \ldots, 2\}$.

1.1.13

$\{0\}$.

1.1.14

$\{-20, -15, -10, \ldots, 10, 15, 20\}$.

1.1.15

Let's call the set S. It's clear that every member of S is an integer. Conversely, note that $n = 5n + 2(-2n)$, $n \in \mathbb{Z}$. Therefore, $S = \mathbb{Z}$.

1.1.16

The reasoning is similar, but note that there exists no $a, b \in \mathbb{Z}$ such that either $n = 6n + 2b$ or $n = 6a + 2b$, $n \in \mathbb{Z}$. Also, note that $6a + 2b = 2(3a + b)$, in which $n = 3n - 2n$. Therefore, S is the set of even integers in $\mathbb{Z}$.

$$S = \{2n : n \in \mathbb{Z}\} \subset \mathbb{Z} \tag{1.1}$$

1.1.17

$\{2^n : n \in \mathbb{N}\}$.

1.1.18 Unsolved

Observation: Successive difference of each couple of numbers: $4, 12, 20, 28, 36, \ldots$ (a difference of 8 each).

1.1.19

$\{3n : n \in \mathbb{Z}\}$.

1.1.20

$\{5n + 2 : n \in \mathbb{Z}\}$.

1.1.21

$\{n^2 : n \in \mathbb{Z}\}$.

1.1.22 Unsolved

My first conjecture was $2^n + n$, but it is wrong for the fourth number.

1.1.23

$\{n \in \mathbb{N} : 3 \leq n \leq 8\}$.

1.1.24

$\{n \in \mathbb{Z} : -4 \leq n \leq 2\}$.

1.1.25

$\{2^n : n \in \mathbb{Z}\}$.

1.1.26

$\{3^n : n \in \mathbb{Z}\}$.

1.1.27

$\{\frac{n\pi}{2} : n \in \mathbb{Z}\}$.

1.1.28

$\{\frac{3}{4}n : n \in \mathbb{Z}\}$.

1.1.29

3. Namely, $\{1\}$, $\{2,\{3,4\}\}$, $\emptyset$.

1.1.30

5. Namely, $\{1, 4\}$, a, b, $\{\{3,4\}\}$, $\{\emptyset\}$.

1.1.31

1. Namely, the biggest set that includes all the others.

1.1.32

1. Same as above.

1.1.33

19. Namely, $-9, -8, \ldots, 8, 9$.

1.1.34

9. Namely, $1, \ldots, 9$.

1.1.35

7. Namely, $-3, \ldots, 3$.

1.1.36

3. Namely, $1, 2, 3$.

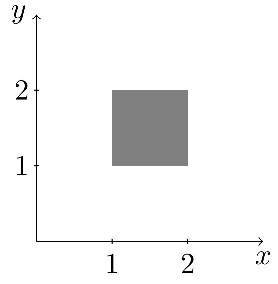
1.1.37

0. Namely, $\emptyset$.

1.1.38

4. Namely, $1, 2, 3, 4$.

1.1.39



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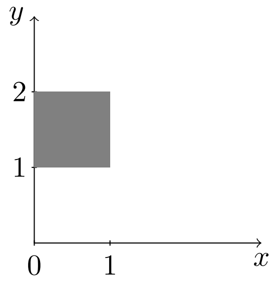

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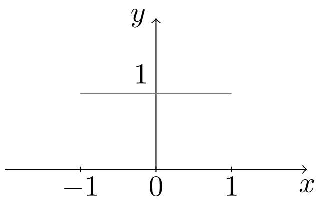

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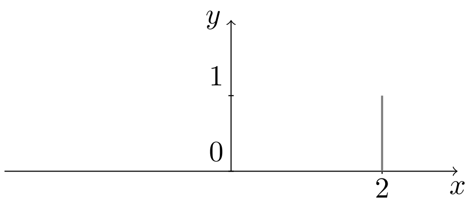

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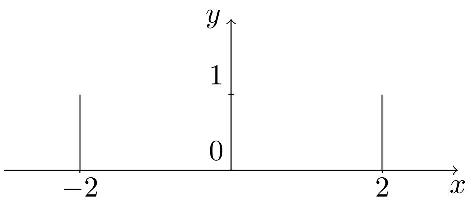

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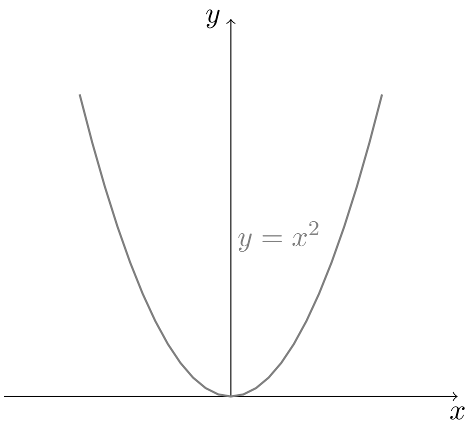

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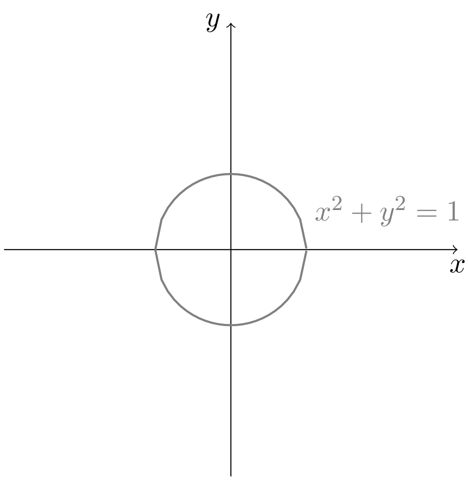

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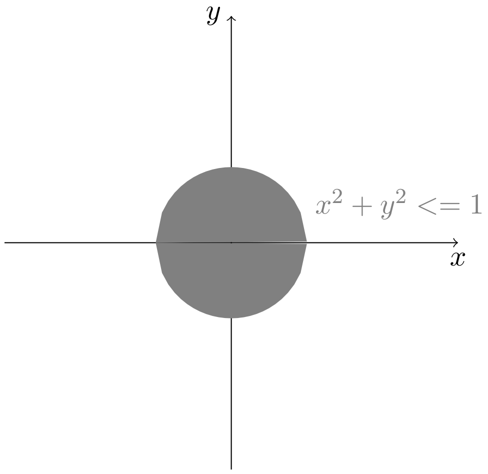

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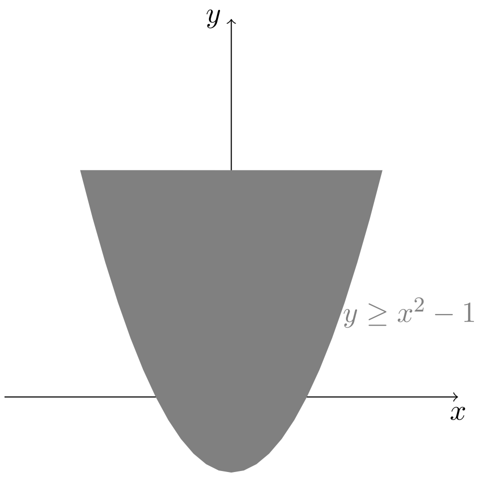

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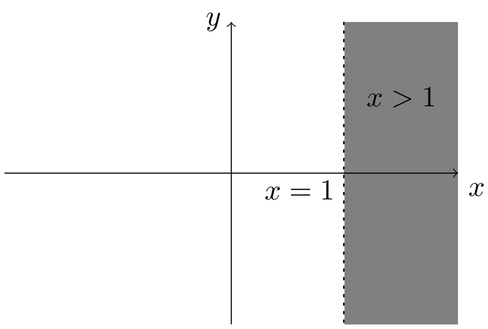

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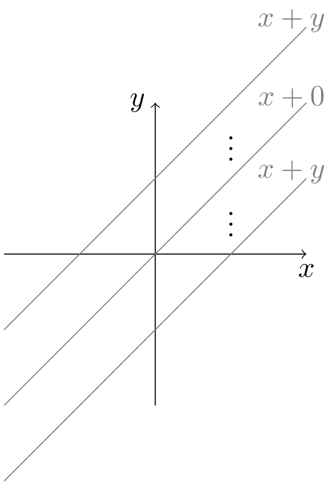

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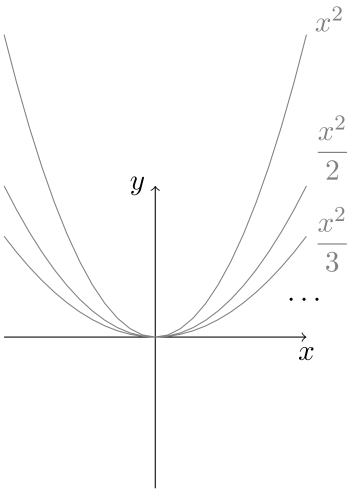

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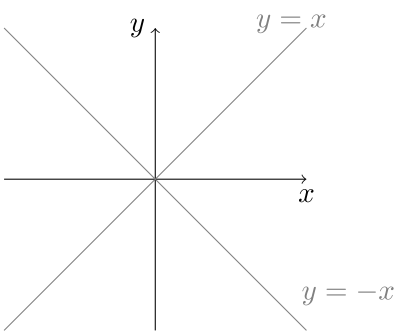

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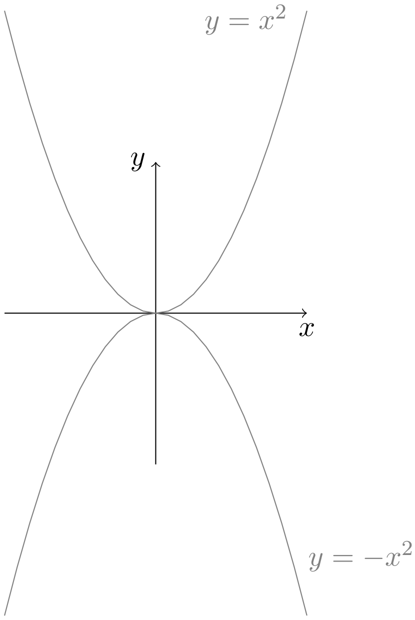

1.2 The Cartesian Product