

Solutions to Book Of Proof

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Preface

An attempt at solving all the exercises.

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Part I

Fundamentals

Chapter 1

Sets

1.1 Introduction to sets

1.1.1

$\{\dots -16, -11, -6, -1, 4, 9, 14, \dots\}$.

1.1.2

$\{\dots -7, -4, -1, 2, 5, 8, 11, \dots\}$.

1.1.3

$\{-2, -1, \dots, 6\}$.

1.1.4

$\{1, 2, \dots, 7\}$.

1.1.5

$\{\pm\sqrt{3}\}$.

1.1.6

$\{\pm 3\}$.

1.1.7

$\{-2, -3\}$.

1.1.8

$\{0, -2, -3\}$.

1.1.9

\mathbb{Z} .

1.1.10

$\{2\pi x : x \in \mathbb{Z}\}$.

1.1.11

$\{-4, -3, \dots, 4\}$.

1.1.12

$\{-2, -1, \dots, 2\}$.

1.1.13

$\{0\}$.

1.1.14

$\{-20, -15, -10, \dots, 10, 15, 20\}$.

1.1.15

Let's call the set S . It's clear that every member of S is an integer. Conversely, note that $n = 5n + 2(-2n)$, $n \in \mathbb{Z}$. Therefore, $S = \mathbb{Z}$.

1.1.16

The reasoning is similar, but note that there exists no $a, b \in \mathbb{Z}$ such that either $n = 6n + 2b$ or $n = 6a + 2b$, $n \in \mathbb{Z}$. Also, note that $6a + 2b = 2(3a + b)$, in which $n = 3n - 2n$. Therefore, S is the set of even integers in \mathbb{Z} .

$$S = \{2n : n \in \mathbb{Z}\} \subset \mathbb{Z} \tag{1.1}$$

1.1.17

$\{2^n : n \in \mathbb{N}\}$.

1.1.18 Unsolved

Observation: Successive difference of each couple of numbers: 4, 12, 20, 28, 36, ... (a difference of 8 each).

1.1.19

$$\{3n : n \in \mathbb{Z}\}.$$

1.1.20

$$\{5n + 2 : n \in \mathbb{Z}\}.$$

1.1.21

$$\{n^2 : n \in \mathbb{Z}\}.$$

1.1.22 Unsolved

My first conjecture was $2^n + n$, but it is wrong for the fourth number.

1.1.23

$$\{n \in \mathbb{N} : 3 \leq n \leq 8\}.$$

1.1.24

$$\{n \in \mathbb{Z} : -4 \leq n \leq 2\}.$$

1.1.25

$$\{2^n : n \in \mathbb{Z}\}.$$

1.1.26

$$\{3^n : n \in \mathbb{Z}\}.$$

1.1.27

$$\left\{\frac{n\pi}{2} : n \in \mathbb{Z}\right\}.$$

1.1.28

$$\left\{\frac{3}{4}n : n \in \mathbb{Z}\right\}.$$

1.1.29

3. Namely, $\{1\}$, $\{2, \{3, 4\}\}$, \emptyset .

1.1.30

5. Namely, $\{1, 4\}$, a , b , $\{\{3, 4\}\}$, $\{\emptyset\}$.

1.1.31

1. Namely, the biggest set that includes all the others.

1.1.32

1. Same as above.

1.1.33

19. Namely, $-9, -8, \dots, 8, 9$.

1.1.34

9. Namely, $1, \dots, 9$.

1.1.35

7. Namely, $-3, \dots, 3$.

1.1.36

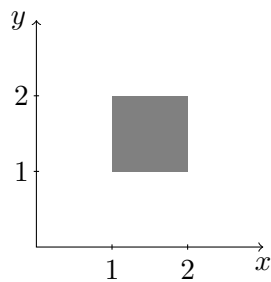
3. Namely, $1, 2, 3$.

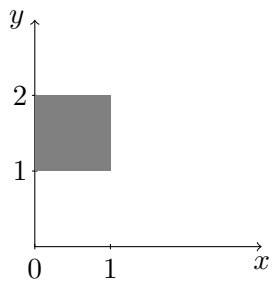
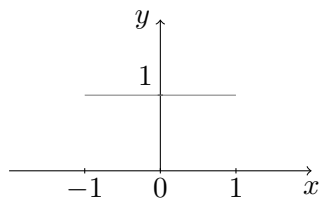
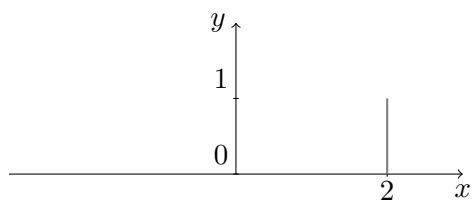
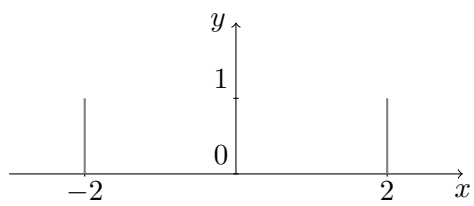
1.1.37

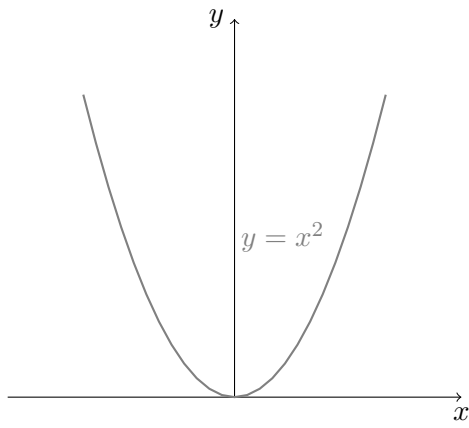
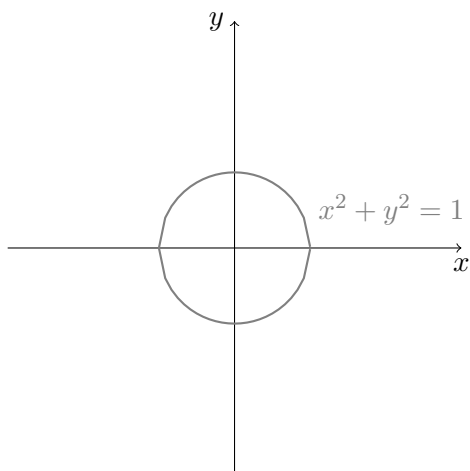
0. Namely, \emptyset .

1.1.38

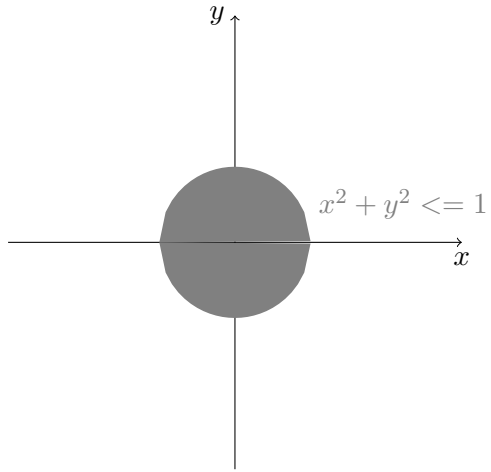
4. Namely, $1, 2, 3, 4$.

1.1.39

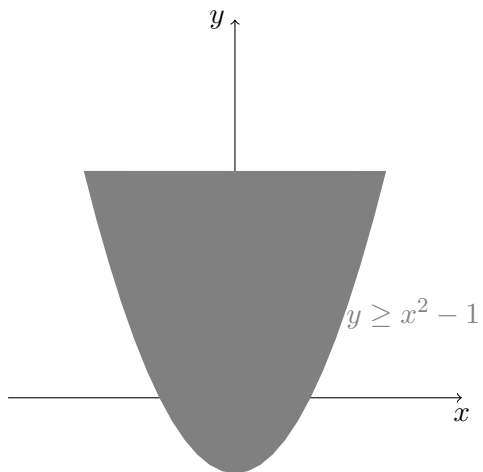
1.1.40**1.1.41****1.1.42****1.1.43**

1.1.44**1.1.45**

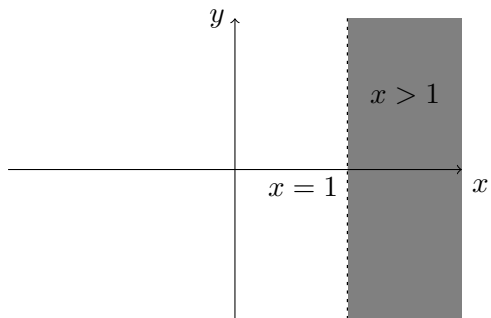
1.1.46

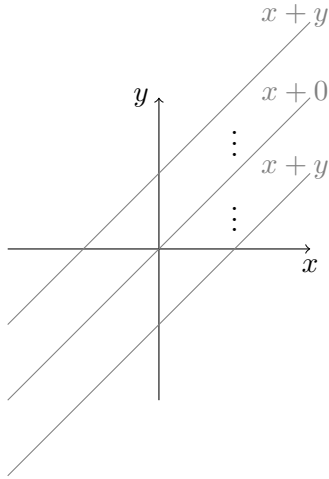
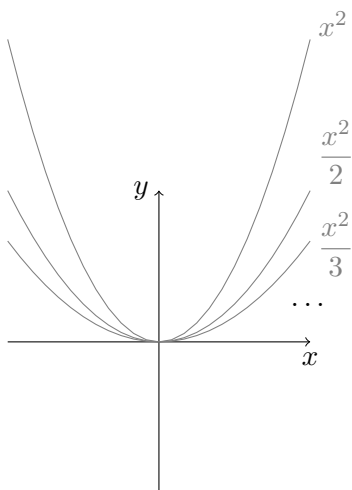
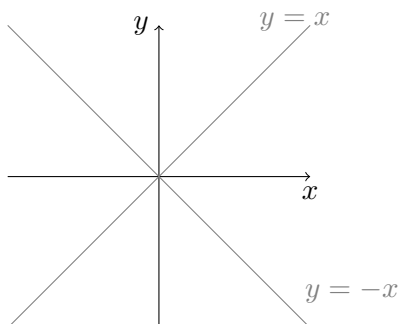


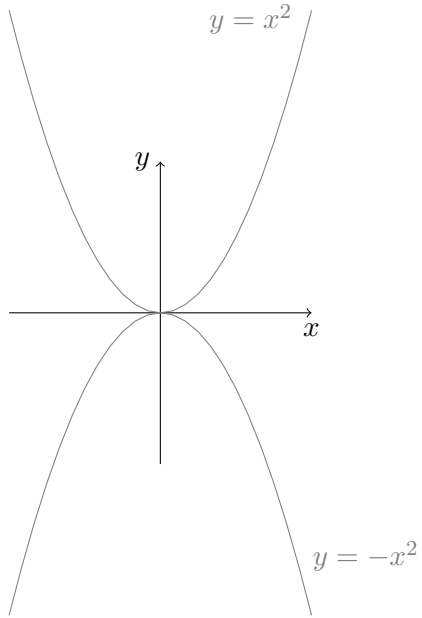
1.1.47



1.1.48



1.1.49**1.1.50****1.1.51**

1.1.52

1.2 The Cartesian Product

1.2.1

- (a) $\{(1, a), (1, c), (2, a), (2, c), \dots, (4, a), (4, c)\}$.
- (b) $\{(a, 1), (a, 2), \dots, (c, 4)\}$.
- (c) $\{(1, 1), (1, 2), \dots, (4, 4)\}$.
- (d) $\{(a, a), (a, c), (c, a), (c, c)\}$.
- (e) \emptyset .
- (f) $\{((1, a), a), ((1, c), a), ((2, a), a), ((2, c), a), \dots, ((4, c), c)\}$.
- (g) $\{(1, (a, a)), (1, (a, c)), (1, (c, a)), (1, (c, c)), \dots, (4, (c, c))\}$.
- (h) $\{(a, a, a), (a, a, c), (a, c, a), (a, c, c), (c, a, a), (c, a, c), (c, c, a), (c, c, c)\}$.

1.2.2

- (a) $\{(\pi, 0), (\pi, 1), (e, 0), (e, 1), (0, 0), (0, 1)\}$.
- (b) $\{(0, \pi), (1, \pi), (0, e), (1, e), (0, 0), (1, 0)\}$.
- (c) Easy
- (d) Easy
- (e) \emptyset
- (f) Easy
- (g) Easy
- (h) Easy

1.2.3

$$\{(\sqrt{2}, a), (\sqrt{2}, c), (\sqrt{2}, e), (-\sqrt{2}, a), (-\sqrt{2}, c), (-\sqrt{2}, e)\}.$$

1.2.4

$$\{(3, -5), (3, 5), (4, -5), (4, 5)\}.$$

1.2.5

$$\{(\sqrt{2}, -2), (\sqrt{2}, 2), (-\sqrt{2}, -2), (-\sqrt{2}, 2)\}.$$

1.2.6

$\{(0, 1), (1, 1)\}$.

1.2.7

$\{(\emptyset, 0, 0), (\emptyset, 0, 1), (\emptyset, \emptyset, 0), (\emptyset, \emptyset, 1)\}$.

1.2.8

$\{(0, 0, 0, 0), (0, 0, 0, 1), (0, 0, 1, 0), (0, 1, 0, 0), (1, 0, 0, 0), ()\}$.