Answers to exercises in How To Prove It

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This is to answer all the questions in the books ‘‘How to prove it’’ by Velleman. Comments are appreciated!

Contents

1 Introduction

Exercise 1.1.

(a) $a = 3$, $b = 5 \Rightarrow x = 2^5 - 1 = 31$, $y = 1 + 2^5 + 2^{10} = 1057$

(b) Since $32,767$ is not a prime, $2^{32,767} - 1$ is not a prime either. Therefore, there exists a positive integer $0 < x < 2^{32,767} - 1$ such that $2^{32,767} - 1$ is divisible by x. Hence, by (a), $x = 2^{31} - 1$ satisfies this.

Exercise 1.2.

n	$3^n - 1$	$3^n - 2^n$
2, prime	8, not prime	5, prime
3, prime	26, not prime	19, prime
4	80, not prime	65, not prime
5, prime	242, not prime	211, prime
6	728, not prime	665, not prime
7, prime	$2,186$, not prime	$2,059$, prime
8	$6,560$, not prime	$6,305$, not prime
9	$19,682$, not prime	$19,171 = 19 \cdot 1,009$, not prime
10	$59,048$, not prime	$58,025$, not prime

Conjecture 1.1. $3^n - 1$ is even for all n.

Conjecture 1.2. If n is prime, $3^n - 2^n$ is prime.

Conjecture 1.3. If n is not prime, $3^n - 2^n$ is not prime.

Exercise 1.3.
We have the following theorem.

Theorem 1.1 (Theorem 3). *There are infinitely many prime numbers.*

Its proof gives a method of finding a prime number n different from the ones in a given list. However, one needs to be careful of prime numbers *not* in the list and smaller than n, since the proof assumes that primes are finite. Example, $n = 3 \cdot 5 + 1 = 15 + 1 = 16$ is not a prime number. Hence,

Remark 1.1 (Method). To find a prime different from the list, check if it is divisible by other primes missing from the list.

(a) Let $n = 2 \cdot 5 + 1 = 11$. n is not divisible by $3, 7$, a new prime number.

(b) Let $n = 2 \cdot 3 + 1 = 7$. n is not divisible by $5, 11$, a new prime number.

Exercise 1.4.
$24, 25, 26, 27, 28$

Exercise 1.5.
$2^4 \cdot (2^5 - 1) = 496$
$2^6 \cdot (2^7 - 1) = 127$.

Exercise 1.6.

Conjecture 1.4. $3, 5, 7$ is the unique triplet prime.

I do not know now to prove it.