Solutions to Book Of Proof

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Preface

An attempt at solving all the exercises.

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Contents

\mathbf{P}	refac	e																				i
C	onte	nts																				ii
Ι	Fur	ndamen	ıta	ıls	3																	1
1	Set	\mathbf{s}																				3
	1.1	Introdu	ıct	io	n	to	se	ets	3													3
		1.1.1																				3
		1.1.2																				3
		1.1.3																				3
		1.1.4																				3
		1.1.5																				3
		1.1.6																				3
		1.1.7																				3
		1.1.8																				3
		1.1.9																				4
		1.1.10																				4
		1.1.11																				4
		1.1.12																				4
		1.1.13																				4
		1.1.14																				4
		1.1.15																				4
		1.1.16																				4
		1.1.17																				4
		1.1.18																				4
		1.1.19																				4

Part I Fundamentals

Chapter 1

Sets

1.1 Introduction to sets

1.1.1

 $\{\ldots -16, -11, -6, -1, 4, 9, 14, \ldots\}.$

1.1.2

 $\{\ldots -7, -4, -1, 2, 5, 8, 11, \ldots\}.$

1.1.3

 $\{-2, -1, \dots, 6\}.$

1.1.4

 $\{1, 2, \dots, 7\}.$

1.1.5

 $\{\pm\sqrt{3}\}.$

1.1.6

 $\{\pm 3\}.$

1.1.7

 $\{-2, -3\}.$

1.1.8

 $\{0, -2, -3\}.$

1.1.9

 $\mathbb{Z}.$

1.1.10

 $\{2\pi x:x\in\mathbb{Z}\}.$

1.1.11

$$\{-4, -3, \dots, 4\}.$$

1.1.12

$$\{-2, -1, \dots, 2\}.$$

1.1.13

 $\{0\}.$

1.1.14

$$\{-20, -15, -10, \dots, 10, 15, 20\}.$$

1.1.15

Let's call the set S. It's clear that every member of S is an integer. Conversely, note that $n = 5n + 2(-2n), n \in \mathbb{Z}$. Therefore, $S = \mathbb{Z}$.

1.1.16

The reasoning is similar, but note that there exists no $a, b \in \mathbb{Z}$ such that either n = 6n + 2b or n = 6a + 2b, $n \in \mathbb{Z}$. Also, note that 6a + 2b = 2(3a + b), in which n = 3n - 2n. Therefore, S is the set of even integers in \mathbb{Z} .

$$S = \{2n : n \in \mathbb{Z}\} \subset \mathbb{Z} \tag{1.1}$$

1.1.17

$$\{2^n:n\in\mathbb{N}\}.$$

1.1.18

1.1.19