# Problems In Mathematics for Computer Science

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# Preface

This is a research project in which I try to read the notes and solve all the problems from [1]

# Contents

# Part I

# Notes

# Chapter 1

# What is a Proof?

## 1.1 Propositions

**Definition 1.1.** A *proposition* is a statement (communication) that is either true or false.

**Claim 1.1.1.** $\forall\ n \in \mathbb{N},\ p ::= n^2 + n + 41$ *is prime*

**Question:** Is this claim true or false?

**Claim 1.1.2.** *No polynomial with integer coefficients can map all nonnegative numbers into primes, unless it's a constant.*

**Question:** Is this true or false?

**Claim 1.1.3** (Euler's Conjecture)**.** $\forall a, b, c, d \in \mathbb{Z}^+.\ a^4 + b^4 + c^4 \neq d^4$

**Claim 1.1.4.** $313(x^3 + y^3) = z^3$ *has no solution when* $x, y, z \in \mathbb{Z}^+$

**Claim 1.1.5** (Four Color Theorem)**.** *Every map can be colored with* 4 *colors so that adjacent regions have different colors.*

**Claim 1.1.6** (Fermat's Last Theorem)**.** $\forall a, b, c \in \mathbb{Z}^+\ \forall n > 2, n \in \mathbb{Z}.\ a^n + b^n \neq c^n$

**Claim 1.1.7** (Goldbach)**.** *Every even integer greater than* 2 *is the sum of two primes.*

## 1.2 Predicates

**Definition 1.2.** A *predicate* is a proposition whose truth depends on the value of one or more variables.

If $P$ is a predicate, then $P(n)$ is either *true* or *false*, depending on the value of $n$.

## 1.3 The Axiomatic Method

**Definition 1.3.** A *proof* is a sequence of logical deductions from a set of axioms and previous proved propositions that concludes with the proposition in question.

- *Theorems*
- *Lemma*
- *Corollary*

$\Rightarrow$ Axiomatic Method

## 1.4 Our axioms

### 1.4.1 Logical deductions

Keywords: *Logical deductions(inference rules), antecedents, conclusion, modus ponens*

### 1.4.2 Patterns of Proof

Many proofs follow specific templates... Many special techniques later on.

## 1.5 Proving an Implication

**Definition 1.4.** *Implications* means $P \Rightarrow Q$

### 1.5.1 Method #1: $P \Rightarrow Q$

### 1.5.2 Method #2: Contrapositive: $\neg Q \Rightarrow \neg P$

## 1.6 Proving an "if and only if"

### 1.6.1 Method #1: Prove each statement implies the other

### 1.6.2 Method #2: Construct a chain of iffs

## 1.7 Proof by Cases

Amusing theorem

**Theorem 1.7.1.** *Every collection of 6 people includes a club of 3 people or a group of 3 strangers.*

*Proof.* The proof is by case analysis. Let $x$ be one of those 6 people. Among 5 other people, there are two scenarios:

1. At least 3 people have met $x$
2. At least 3 people have not met $x$

We argue that these two cases are exhaustive since we are dividing the 5 people into two groups: those who have met $x$ and those who have not.

**Case 1:** Suppose that at least 3 people have met $x$
This is divided further more into two subcases:

**Case 1.1:** No pairs among those people have met each other. In this case, they form a group of at least 3 strangers. Thus, the theorem holds in this subcase.

**Case 1.2:** At least one pair in those people have met. Adding $x$ to such pair forms a club of at least 3 people. The theorem is proved in this subcase.

This implies that the theorem holds for Case 1.

**Case 2:** Suppose that at least 3 people have not met $x$
This again splits the case into two subcases:

**Case 2.1:** All pairs among those people have met each other. In this case, they form a club of at least 3 people. Thus the theorem holds in this subcase.

**Case 2.2:** At least one pair in those people have not met. Adding $x$ to such pair forms a group of at least 3 strangers. The theorem holds in this subcase.

This implies that the theorem holds for Case 2.

We have proved the theorem. □

# Part II

# Problems and Exercises

Chapter 1

# What is a Proof?

# Bibliography

[1] Eric Lehman, Tom Leighton, and Albert Meyer. *Mathematics for Computer Science.* MIT OCW, 2018.