Identifying Preferences with Unobserved Individual Budget using Vertical Differentiation in Brands

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July 16, 2025

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Motivation

- Faint notion of **affordability** in discrete choice literature
- In many applied works, budget is only applied through price sensitivity
  - e.g., in BLP, utility of product j for individual i with budget y_i:

$$u_{ij} = \underbrace{x_j'}_{\substack{\text{observed}\\\text{attributes}\\\text{of } j}} \underbrace{\beta_i}_{\substack{\text{individual}\\\text{preference}\\\text{parameter}}} - \underbrace{\alpha}_{\substack{\text{marginal}\\\text{utility of}\\\text{numeraire}\\\text{(price sensitivity)}}} \cdot \frac{1}{y_i} \cdot \underbrace{p_j}_{\substack{\text{price}\\\text{of } j}} + \underbrace{\epsilon_{ij}}_{\substack{\sim\text{T1EV}\\\text{idiosyncratic}\\\text{preference}\\\text{shock for } j}}$$

    - ★ $\frac{\alpha}{y_i}$ is frequently written α_i to denote i's price sensitivity
  - i solves $\max_{j \in \mathcal{J}} u_{ij}$, where $\mathcal{J} = \{0, 1, \ldots, j, \ldots, J\}$ is total choice set

Motivation

$$\text{BLP: } \max_{j \in \mathcal{J}} u_{ij} = \max_{j \in \mathcal{J}} x_j' \beta_i - \underbrace{\frac{\alpha}{y_i}}_{=:\alpha_i} p_j + \epsilon_{ij}$$

- The model says we could have bought Ferrari, Gucci, and ________
- Thing that bothered were our α_i being too large
- More probable is i's consideration set $\mathcal{J}_i \subset \mathcal{J}$ didn't contain them
  - Major source of this will be the occasion: $y_i < p_{\text{Ferrari,Gucci,etc.}}$

$\Rightarrow$ Let's **model** $\mathcal{J}_i = \{j \in \mathcal{J} | p_j \leq y_i\}$ and **identify** preferences

Motivation

$$\text{BLP: } \max_{j\in\mathcal{J}} V_{ij} = \max_{j\in\mathcal{J}} x_j'\beta_i - \underbrace{\frac{\alpha}{y_i}} p_j + \epsilon_{ij}$$

$$\text{Our Model: } \max_{c\geq c^{\min}>0, j\in\mathcal{J}} U_{ij} = \max_{c\geq c^{\min}>0, j\in\mathcal{J}} c^{\alpha}\exp\left(x_j'\beta + \varepsilon_{in(j)}\right)$$

$$\text{s.t. } c + \sum_{j\in\mathcal{J}} p_j \mathbf{1}\{j \text{ purchased}\} \leq y_i \;\&\; \sum_{j\in\mathcal{J}} \mathbf{1}\{j \text{ purchased}\} = 1,$$

where $c^{\min}$: the minimum level of consumption of c,

$\varepsilon_{in(j)}$: the idiosyncratic preference shock for brand $n(j)$,

$n(j)$: the brand whose constituent is product j.

- BLP can also be thought of as coming from the following:

$$\text{Original BLP: } \max_{c\geq 0, j\in\mathcal{J}} U_{ij} = \max_{c\geq 0, j\in\mathcal{J}} c^{\alpha}\exp\left(x_j'\beta + \varepsilon_{ij}\right)$$

$$\text{s.t. } c + \sum_{j\in\mathcal{J}} p_j \mathbf{1}\{j \text{ purchased}\} \leq y_i \;\&\; \sum_{j\in\mathcal{J}} \mathbf{1}\{j \text{ purchased}\} = 1,$$

Motivation

- Best if data itself tells us $\Rightarrow$ treat budget distribution **unknown**
  - Extrapolate income data: income may not approximate well of budget
- Main strategy: exploit **vertical differentiation** within brands
  - Most brands typically form a line-up to ensnare all budget groups
  - e.g.: Smartphone line-up, Car line-up, Apple PC add-ons, etc.
  - The line-up within a brand will be well ordered w.r.t. your budget
- **Single-Crossing Property (SCP)** embeds the last point above
  - Your purchase conversely gives info of which budget group you are in
  - How much bought a product recovers budget distribution

Literature Review

- Berry et al. [1995] uses mixed logit which is known to mimic RUM
  - McFadden and Train [2000] but with *observed* choice set
  - Budget is unobserved: part of random consideration set models
  - More structure (vertical differentiation) will be used than BLP
- Does omitting affordability actually cause a problem?
  - Pesendorfer et al. [2023]: biases on price sensitivity and price elasticity
    OmitBudgetBias
- Interaction of price with latent budget which interacts with preference
  - Exogenous consideration set models applied: Goeree [2008] and a lot
  - Here, it is *Endogenous*: "price $>$ unobserved budget" determines
  - Dealt in Barseghyan et al. [2021] but with cognitive models
- Barseghyan et al. [2021] used SCP to identify type heterogeneity

LitRev

Literature Review

- Preference model aligns with Song [2015]'s Hybrid model
  - The idiosyncracy only applies to brand-level shock
  - Similar to random coefficient nested logit without product-level shock
  - Within brand type (budget) determines the best product
  - The model embodies both BLP and Pure Characteristic Model (PCM)
  - We **drop** *random coefficients* but **add** *consideration set* mechanism
  - Also, logit is not necessary for our identification
- Facilitates an approach to Hicksian demand in discrete choice
  - Price effect will now have additional income effects through affordability

Contributions

- First to solve omitted budget bias in choice set level
  - Identify homogenous preference parameters (price sensitivity)
  - Nonparametrically identify budget distribution (price elasticity)
- First to solve endogenous and non-cognitive consideration mechanism
- It generates a new data about consumer's wealth to analyze
  - It depends on utility function of choice
- Analyze capacity effects both on consideration set and preferences
  - Ways to discern preference distribution v.s. capacity distribution
  - Reduces tyranny of pref. hetero. (focus on homogeneity)

Contributions

- Provides the direct utility function that may allow the identification

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Model Setup

- A market consists of product set $\mathcal{J}$
- Brands $\mathcal{N} = \{n_1, \ldots, n_N\}$ are partition of $\mathcal{J}$, with an element n
- Let the example product j and $n(j)$ denote brand j is in
- Indirect utility: $U_{ij} = \underbrace{-\frac{\alpha}{y_i} p_j + \beta x_j + \xi_j}_{=:V(y_i, p_j, x_j, \xi_j)} + \epsilon_{in(j)}$ with $U_{i0} = 0$
- Consumer i's problem: $\max\limits_{j \in \mathcal{J}(y_i)} U_{ij}$ with $\mathcal{J}(y_i) = \{j \in \mathcal{J} : p_j \leq y_i\}$

Single-Crossing Property (SCP)

- It is not necessary to stick with the current utility if below holds

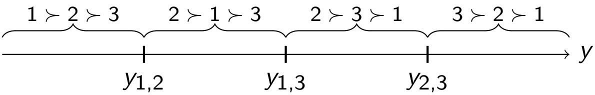
Assumption (SCP)

For all $j < k$, there exists $y_{j,k}$ such that

$$V(y, p_j, x_j, \xi_j) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} V(y, p_k, x_k, \xi_k) \text{ as } y \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} y_{j,k}, \text{respectively.}$$

- $y_{j,k}$ is cutoff budget where preference order between j and k revert

- Your type maps to certain preference ordering within each brand
  - Consider when there are only three goods (1, 2, and 3) for example;



  - We can see the single-crossing along the path of budget (type) growth

Single-Crossing Property (SCP)

$$\overbrace{\qquad\qquad}^{1 \succ 2 \succ 3}\underset{y_{1,2}}{|}\overbrace{\qquad\qquad}^{2 \succ 1 \succ 3}\underset{y_{1,3}}{|}\overbrace{\qquad\qquad}^{2 \succ 3 \succ 1}\underset{y_{2,3}}{|}\overbrace{\qquad\qquad}^{3 \succ 2 \succ 1}\rightarrow y$$

- Back to the utility function: $U_{ij} = -\frac{\alpha}{y_i} p_j + \beta x_j + \xi_j + \epsilon_{in(j)}$

- Cutoff formula: $y_{j,k} = \frac{\alpha(p_j - p_k)}{\beta(x_j - x_k) + \xi_j - \xi_k}$

- The fact that cutoffs give first-best region bears integration region
  - If there is only one brand in a market, even no outside option:

    $$P_1 = F_y(y_{1,2}) - F_y(\underline{y}) \quad P_2 = F_y(y_{2,3}) - F_y(y_{1,2}) \quad P_3 = F_y(\overline{y}) - F_y(y_{2,3})$$

  - If $y_{j,k}$ is a known function, $F_y(y)$ will be identified

Choice Probability and Integration Regions

- Let $V_n^*(y) := \max_{j \in n} V(y, p_j, x_j, \xi_j)$, the first-best utility in n given y
- Assume T1EV brand-level shock for explanary purpose
  - Conditional choice probability (given y):

$$P_j(y) = \begin{cases} \frac{e^{V_n^*(y)}}{\sum_{n'} e^{V_{n'}^*(y)}}, & \text{if } j = \arg\max_{k \in n} V(y, p_k, x_k, \xi_k) \text{ and } p_j \leq y \\ 0, & \text{otherwise} \end{cases}$$

  - Unconditional: $P_j = \int_{S_j} \frac{e^{V_n^*(y)}}{\sum_{n'} e^{V_{n'}^*(y)}} f(y) dy$
- $S_j = \{y : j = \arg\max_{k \in n} V(y, p_k, x_k, \xi_k) \, \& \, p_j \leq y\}$
  - $S_2 = [y_{1,2}, y_{2,3}]$ in the example above
  - But **1. affordability issue** and **2. other possible cases** are *ignored*

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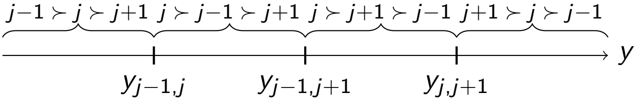
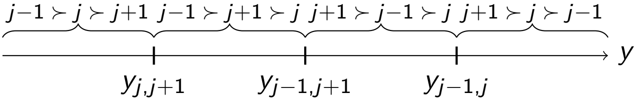
Main Challenge: Identifying S_j

- Consider products $j{-}1$, j, and $j{+}1$ in a category. Possible orderings:



Ordering 1



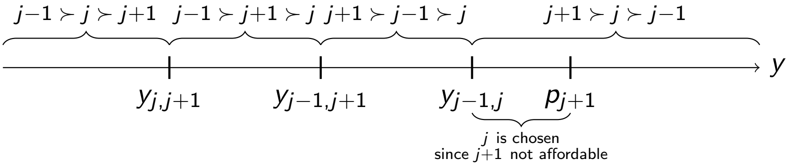
Ordering 2

- Ordering 2 does not allow j being chosen ignoring affordability
  - If $P_j > 0$, we can rule out Ordering 2
  - $S_j = [y_{j-1,j}, y_{j,j+1}]$ in this case, ignoring affordability issue

- Complication arises when **budget constraint** takes place

Main Challenge: Identifying S_j

- $p_{j+1} > y_{j-1,j}$ allows j to have positive market share in **Ordering 2**



BestUnaffordable Asymmetry

- Even worse: the number of cutoff ordering grows super-exponentially
  - Known as "half-period of allowable sequences" $= \dfrac{\binom{|n|}{2}!}{\prod_{k=1}^{|n|-1}(2k-1)^{|n|-k}}$
  - $|n| = 3$ then 2, $|n| = 4$ then 16, $|n| = 5$ then 768

- With affordability issue present, **infeasible** to identify the range

Using Luxury Goods for Identification

- Identify preference parameters $\Rightarrow$ pins down the cutoff ordering

- Let's investigate in which condition this is possible

Definition (Luxury Goods)

Given a preference parameter vector, product k is called a luxury good if $p_k = \inf\{y : V_n^*(y) = V(y - p_k, x_k, \xi_k)\}$.

- Luxury: $\exists$people who think it is the first best but cannot afford it

- Price of k gives upper bound for S_j for j **suboptimal** to k
  - $P_j = \int_{\underline{S_j}}^{p_k} \frac{e^{V_n^*(y)}}{\sum_{n'} e^{V_{n'}^*(y)}} f(y)dy \cdots$ Imagine derivative wrt p_k

- But how do we detect a luxury good?

Main Idea of Identification of Preference Parameters

Asymmetry | BestUnaffordable

- We are going to exploit price-quality asymmetry at budget constraint

Proposition (Price-Quality Asymmetry)

For a product $j \in n$, let product $k = \min\{k' \in n | k' > j, P_{k'} > 0\}$. Then, if $\frac{\partial P_j}{\partial x_k} = 0$, k is a luxury good, i.e., $\overline{S_j} = p_k$.

- We have to go through possible orderings to show this is true

Proposition (No Luxury Goods pins down regions)

If there is no luxury goods among products with positive market share, then the region S_j is uniquely determined, with $\underline{S_j} = y_{l,j}$ and $\overline{S_j} = y_{j,k}$ for l and k s.t. $l < j < k$ closest to j, for each j.

- Without luxury good, the theorem validates approach in Song [2015]

Identification from Cross-Brand Comparison

- Now, assume K brand with same-priced luxury goods

Assumption

There exist products $j_1, \ldots, j_K$ in distinct categories (brands) $n_1, \ldots, n_K \subset \mathcal{N}$ such that they are luxury and $p_{j_1} = p_{j_2} = \cdots = p_{j_K}$.

- Let $j_k^\circ = \max\{j' \in n_k : j' < j_k, P_{j'} > 0\}$: the suboptimal of j_k in n_k

- $$P_{j_k^\circ} = \int_{\underline{S_{j_k^\circ}}}^{p_k} \frac{e^{V_n^*(y)}}{\sum_{n'} e^{V_{n'}^*(y)}} f(y)dy \Rightarrow \partial_{p_{j_k}} P_{j_k^\circ} = \frac{e^{V_{j_k^\circ}(p_{j_k})}}{\sum_{n'} e^{V_{n'}^*(p_{j_k})}} f(p_{j_k})$$

- We can focus on preference distribution:

$$\ln \frac{\partial_{p_{j_k}} P_{j_k^\circ}}{\partial_{p_{j_l}} P_{j_l^\circ}} = V_{j_k^\circ}(p_{j_k}) - V_{j_l^\circ}(p_{j_k}) = \frac{\alpha}{p_{j_k}}(p_{j_k^\circ} - p_{j_l^\circ}) + (x_{j_k^\circ} - x_{j_l^\circ})'\beta + \xi_{j_k^\circ} - \xi_{j_l^\circ}$$

Main Theorem

- Below is imperfect and still working on it
- Concern: parametric identification argument typically uses likelihood
- We use log of partial derivative ratios

Theorem (Parametric Local Identification of Preference Parameters)

Let real-valued parameter vector θ be in the parameter space $\Theta \subseteq \mathbb{R}^m$, which is open. ϵ_{in} follows iid a distribution that has density, $f_\Theta(\cdot)$, s.t. $f_\Theta(\cdot)$ and $\ln f_\Theta(\cdot)$ continuously differentiable wrt θ. Also, assume that the information matrix $I(\theta) = \mathbb{E}\left[\frac{\partial \ln f_\Theta}{\partial \theta}\frac{\partial \ln f_\Theta}{\theta'}\right]$ exists and continuous in $\theta \in \Theta$. Then, by the above assumption, preference parameters α, β can be identified.

- The theorem comes from Rothenberg [1971]

Identification of Budget Distribution

- Identification of budget heterogeneity follows Barseghyan et al. [2021]
- The difference between our approach and Barseghyan et al. [2021]'s
  - Identification of the cutoff budgets needed here (main contribution)
  - Cognitive (attentional) consideration set mechanism used there
- Cutoff budgets play a role of special regressors
  - Many cutoff budgets here $\Rightarrow$ relax full support assumption

Theorem (Nonparametric Identification of Budget Distribution)

Let the support of x, ξ, p and y be $\mathcal{X}, \Xi, \mathcal{P}$ and $\mathcal{Y}$. Under the main assumption, F_y is nonparametrically identified if $F_y(y)$ is cont. in $y \in \mathcal{Y}$ and $\mathcal{Y} \subseteq \cup_{n \in N} \cup_{j \in n} \{\underline{S_j}(x, \xi, p), \overline{S_j}(x, \xi, p) : x \in \mathcal{X}, \xi \in \Xi, p \in \mathcal{P}\}$.

Identification of Budget Distribution

- The intuition comes from the following:

$$\frac{\partial}{\partial x_{j+1}} P_j = \frac{\partial}{\partial x_{j+1}} \int_{\underline{S_j}}^{\overline{S_j}} \frac{e^{V_j(y)}}{\sum_{n'} e^{V^*_{n'}(y)}} f(y) dy = \underbrace{\frac{e^{V_j(\overline{S_j})}}{\sum_{n'} e^{V^*_{n'}(\overline{S_j})}} \frac{\partial \overline{S}_j}{\partial x_{j+1}}}_{\text{identified function}} f(\overline{S_j})$$

Similarly,

$$\frac{\partial}{\partial x_{j-1}} P_j = \underbrace{\frac{e^{V_j(\underline{S_j})}}{\sum_{n'} e^{V^*_{n'}(\underline{S_j})}} \frac{\partial \underline{S}_j}{\partial x_{j-1}}}_{\text{identified function}} f(\underline{S_j})$$

- The more the brands/products, the easier to estimate the distribution

Criticism from Mike Abito

- Note that our specification started from "indirect utility"
- Consumers account for budget on "direct utility" maximization
- Mike "Budget haunting consideration set is delusional"
  - It should be bluntly reflected in the indirect utility, not consideration set
  - Same criticism possible for BLP used in applied works
- Consider the following maximization of direct utility:

$$\max_{j \in \mathcal{J}} c^{\alpha} \exp(x_j \beta + \varepsilon_j)$$

$$\text{s.t. } c + \sum_j p_j \mathbf{1}\{j \text{ purchased}\} \leq y, \& \sum_j \mathbf{1}\{j \text{ purchased}\} = 1$$

Criticism from Mike Abito

- Consider the following maximization of direct utility:

$$\max_{j \in \mathcal{J}} c^{\alpha} \exp(x_j \beta + \varepsilon_j)$$

$$\text{s.t. } c + \sum_j p_j \mathbf{1}\{j \text{ purchased}\} \leq y, \& \sum_j \mathbf{1}\{j \text{ purchased}\} = 1$$

- The indirect utility given purchasing j is given as follows:

$$V_j(x, p, y) = (y - p)^{\alpha} \exp(x_j \beta + \varepsilon_j) \underbrace{\Longrightarrow}_{\text{take } log} \alpha \ln(y - p) + x_j \beta + \varepsilon_j$$

- Just as Mike said, the budget constraint is inside the indirect utility
  - More worse? Assume p is much smaller than y

$$\ln V_j = \alpha(\ln(y) + \ln(1 - \frac{p_j}{y})) + x_j \beta + \varepsilon_j \approx -\alpha \frac{p_j}{y} + x_j \beta + \varepsilon_j$$

Identification Possible Direct Utility Function

- Assuming smaller ticket and then say it makes affordability issue?
  - $\alpha \ln(y - p) + x_j\beta + \varepsilon_j$ was where I was stuck in the first place
  - This doesn't allow luxury good since affordability is endogenized

- There is small modification that defies Mike's criticism

$$\max_{j \in \mathcal{J}} (c + \eta)^\alpha \exp(x_j\beta + \varepsilon_j)$$

$$\text{s.t. } c + \sum_j p_j \mathbf{1}\{j \text{ purchased}\} \leq y, \& \sum_j \mathbf{1}\{j \text{ purchased}\} = 1$$

$$\ln V_j = \ln(y - p_j + \eta) + x_j\beta + \varepsilon_j$$

- Assume $\eta > 0$ is a transferred consumption (not possible to re-sell)
  - $p_j > y$ can still happen when $\ln(y - p_j + \eta) \nrightarrow -\infty$
  - Every identification strategy works if we know what η is

Identification Possible Direct Utility Function

- Observe that if p is much smaller than $y+\eta$, then the approximation:

$$\ln V_j = -\alpha \frac{p_j}{y+\eta} + x_j \beta + \varepsilon_{n(j)}$$

- There may be ways to identify the fixed η through panel data
  - Possible but with strong assumption: assume fixed unobserved utility

$$\underbrace{\ln \frac{\partial_{p_{j_k}} P_{j_k^\circ,t}}{\partial_{p_{j_l}} P_{j_l^\circ,t}}}_{=:y_{kl,t}} = \frac{\alpha}{p_{j_k,t}+\eta} \underbrace{(p_{j_k^\circ,t} - p_{j_l^\circ,t})}_{=:\Delta_{kl} p_t} + \underbrace{(x_{j_k^\circ,t} - x_{j_l^\circ,t})'}_{=:\Delta_{kl} x_t} \beta + \xi_{j_k^\circ} - \xi_{j_l^\circ}$$

$$\Delta_{t+1,t} y_{kl} = \alpha \left(\frac{1}{p_{j_k,t+1}+\eta} - \frac{1}{p_{j_k,t}+\eta} \right) \Delta_{kl} p_t$$

for $\Delta_{kl} p_t = \Delta_{kl} p_{t+1} \& \Delta_{kl} x_t = \Delta_{kl} x_{t+1}$

  - If $\exists q, r \in \mathcal{N}$ that also allows the above $\Rightarrow$1 equation, 1 unknown

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Further Challenges

- Obvious extension of the model: Random Coefficients
  - Identification having random coefficients not straightforward
  - Change of market share from luxury goods' quality is mixed:
    - Silence of unaffordable groups + Renewal of first best groups
    - Smaller than non-luxury counterparts
    - But how to discern:
      renewal groups being small vs. existence of an unaffordable group
- Application to empirical data:
  - Candidate: automobile market used in Pesendorfer et al. [2023]
  - Afraid of the identification strategy not working due to above

Some Notes for Random Coefficients

- If we stick with the utility function in Song [2015], we get

$$\frac{\partial y_{j,j+1}}{\partial p_{j+1}} = \frac{\alpha}{\beta \Delta_{j+1,j} x} \qquad \frac{\partial y_{j,j+1}}{\partial x_{j+1}} = -\frac{\alpha \Delta_{j+1,j} p}{\beta (\Delta_{j+1,j} x)^2}$$

$$\Rightarrow \frac{\partial y_{j,j+1}}{\partial p_{j+1}} \Bigg/ \frac{\partial y_{j,j+1}}{\partial x_{j+1}} = -\frac{\Delta_{j+1,j} x}{\Delta_{j+1,j} p}$$

- Now, with random coefficients, the choice probabilities will be

$$P_{j_k^\circ} = \int_{\beta_i} \int_{\underline{S_{j_k^\circ}(\beta_i)}}^{\overline{S_{j_k^\circ}(\beta_i)}} \frac{e^{V_{j_k^\circ}(y_i,\beta_i)}}{\sum_{n'} e^{V^*_{n'}(y_i,\beta_i)}} f(y_i|\beta_i) dy_i f(\beta_i) d\beta_i$$

- If every product is available at its first best region (no luxury for $\forall \beta_i$)

$$P_{j_k^\circ} = \int_{\beta_i} \int_{y_{j_k^\circ-1,j_k^\circ}(\beta_i)}^{y_{j_k^\circ,j_k^\circ+1}(\beta_i)} \frac{e^{V_{j_k^\circ}(y_i,\beta_i)}}{\sum_{n'} e^{V^*_{n'}(y_i,\beta_i)}} f(y_i|\beta_i) dy_i f(\beta_i) d\beta_i$$

Some Notes for Random Coefficients

- If every product is available at its first best region (no luxury for $\forall \beta_i$)

$$P_{j_k^\circ} = \int_{\beta_i} \int_{y_{j_k^\circ - 1, j_k^\circ}(\beta_i)}^{y_{j_k^\circ, j_k^\circ + 1}(\beta_i)} \frac{e^{V_{j_k^\circ}(y_i, \beta_i)}}{\sum_{n'} e^{V_{n'}^*(y_i, \beta_i)}} f(y_i | \beta_i) dy_i f(\beta_i) d\beta_i$$

- Consider $\frac{\partial P_{j_k^\circ}}{\partial p_{j_k^\circ + 1}} \Big/ \frac{\partial P_{j_k^\circ}}{\partial x_{j_k^\circ + 1}}$ with $\frac{\partial y_{j,j+1}}{\partial p_{j+1}} = \frac{\alpha}{\beta \Delta_{j+1,j} x}$ and $\frac{\partial y_{j,j+1}}{\partial x_{j+1}} = -\frac{\alpha \Delta_{j+1,j} p}{\beta (\Delta_{j+1,j} x)^2}$

$$\frac{\partial P_{j_k^\circ}}{\partial p_{j_k^\circ + 1}} = \int_{\beta_i} \frac{e^{V_{j_k^\circ}(y_{j_k^\circ, j_k^\circ + 1}(\beta_i), \beta_i)}}{\sum_{n'} e^{V_{n'}^*(y_{j_k^\circ, j_k^\circ + 1}(\beta_i), \beta_i)}} f(y_{j_k^\circ, j_k^\circ + 1}(\beta_i) | \beta_i) \frac{\partial y_{j_k^\circ, j_k^\circ + 1}(\beta_i)}{\partial p_{j_k^\circ + 1}} f(\beta_i) d\beta_i$$

$$\Rightarrow \frac{\partial P_{j_k^\circ}}{\partial p_{j_k^\circ + 1}} \Big/ \frac{\partial P_{j_k^\circ}}{\partial x_{j_k^\circ + 1}} = -\frac{\Delta_{j_k^\circ + 1, j_k^\circ} x}{\Delta_{j_k^\circ + 1, j_k^\circ} p}$$

- Assuming no luxury for $\forall \beta_i$ has a **testable** implication

Some Notes for Random Coefficients

- The counterpart for situations where $\beta_i > \underline{\beta}_{j_k^\circ}$ suffer from affordability

$$\frac{\partial P_{j_k^\circ}}{\partial p_{j_k^\circ+1}} \Bigg/ \frac{\partial P_{j_k^\circ}}{\partial x_{j_k^\circ+1}} = -\frac{\Delta_{j_k^\circ+1,j_k^\circ} x}{\Delta_{j_k^\circ+1,j_k^\circ} p} + \underbrace{\frac{\int_{\underline{\beta}_{j_k^\circ}}^{\overline{\beta_i}} P_{j_k^\circ}(p_{j_k^\circ+1}, \beta_i) f(p_{j_k^\circ+1}|\beta_i) f(\beta_i)\, d\beta_i}{-\alpha \frac{\Delta_{j+1,j} p}{(\Delta_{j+1,j} x)^2} \int_{\underline{\beta_i}}^{\underline{\beta}_{j_k^\circ}} P_{j_k^\circ}(y_{j_k^\circ, j_k^\circ+1}, \beta_i) f(y_{j_k^\circ, j_k^\circ+1}) \frac{f(\beta_i)}{\beta_i}\, d\beta_i}}_{\text{Identified}}$$

- Looking forward to full-identification with discrete random coefficients

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Bias Comming from Budget Omission

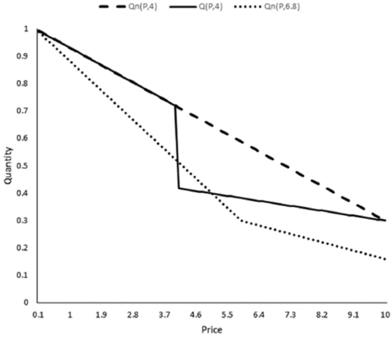

Fig. 1. Aggregate demand functions: income in {4, 10} with equal probability.

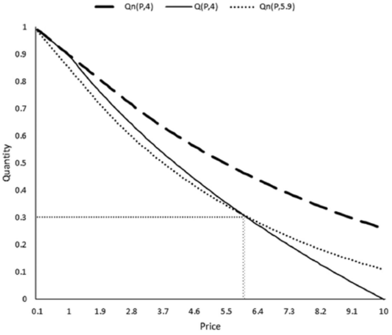

Fig. 2. Aggregate demand functions: income uniformly distributed on [1,10].

Figure: Pesendorfer et al. (2023) Aggregate Demand Curves

- *Solid* line shows the **true** demand curve
- *Dotted* line shows when it was **estimated** with full consideration
- *Dashed* line is when we **match** the price sensitivity α

OmitBudgetBias

Bias Comming from Budget Omission

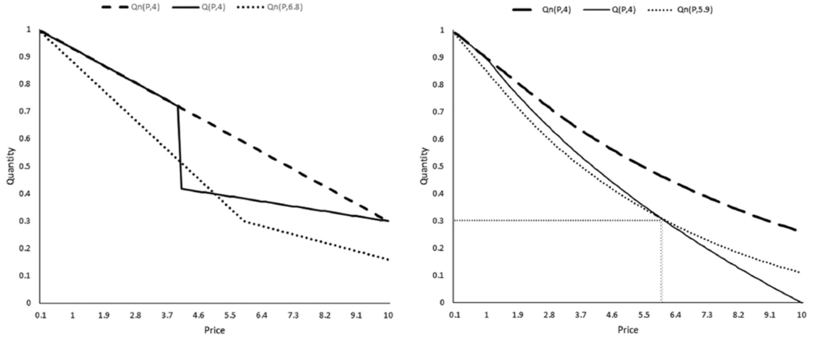

Fig. 1. Aggregate demand functions; income in {4, 10} with equal probability.

Fig. 2. Aggregate demand functions; income uniformly distributed on [1,10].

Figure: Pesendorfer et al. (2023) Aggregate Demand Curves

- Budget affordability omission means **assuming** full consideration
- Full consideration set cannot approximate the limited consideration
- **Shape** of budget distribution matters for the *direction* of bias
  - Bipolar (Fig.1.) v.s. Uniform (Fig.2.)

Back to Literature