Systematic Study of Shell and Sub-Shell Closures in the isotopic chain of "No" Element

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Introduction:

We extend our keen interest to reveal the shell/sub-shell closure of nuclei from the isotopic shift. Since by adding nucleons one by one, the distribution of charge inside the nucleus changes accordingly due to quantum shell effects like pairing energy, large shell gap, and change in shape i.e rms radii, as a result of which kinks or peaks appear in the continuous trend. We have determined the ground state properties of even-even isotopes of Nobelium (Z=102) using the NL3* force parameter [1] within relativistic mean field theory (RMF) [2]. Also using binding energy (B.E.), $\Delta_{1n}^{(3)}B.E.(N)$,dS_{2n}, ΔE are calculated.

Theoretical Formulation:

Relativistic Mean Field Theory(RMF)

The Lagrangian density is

$$\mathscr{L} = \bar{\psi}_i(i\gamma_\mu \partial_\mu - M)\psi_i + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\partial^2 - \frac{1}{3}g_2\partial^3 - \frac{1}{4}g_3\partial^4 - g_5\bar{\psi}_i\psi_i\sigma - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2V^\mu V_\mu + \frac{1}{4}C_3(V_\mu V^\mu)^2 - g_\omega\bar{\psi}_i\gamma^\mu\psi_iV_\mu - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}^\mu.\vec{R}_\mu - g_\mu\bar{\psi}_i\gamma^\mu\vec{\tau}\psi_i\vec{R}^\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}_i\gamma^\mu\frac{(1-\tau_{3i})}{2}\psi_iA_\mu \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The symbols have their usual meaning. The static solutions of the field equations give us the ground state properties such as the binding energies (B.E), nuclear radii, etc. Using the B.E values we obtain the following quantities [2, 3, 4].

$$S_{2n}(N,Z) = B.E(N,Z) - B.E(N-2,Z) \ldots\ldots\ldots\ldots(2)$$

$$dS_{2n} = S_{2n}(N,Z) - S_{2n}(N+2,Z) \ldots\ldots\ldots\ldots(3)$$

$$\Delta E = 2[\Delta_{1n}^{(3)}B.E(N,Z) - \Delta_{1n}^{(3)}B.E(N+1,Z)] \ldots\ldots(4)$$

$$\Delta_{1n}^{(3)}B.E.(N) = \frac{1}{2}(-1)^N[B.E.(N-1) - 2B.E.(N) + B.E.(N+1)] \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Results and Discussions:

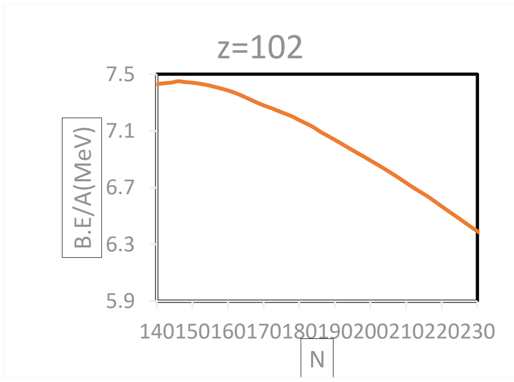

Fig.1: B.E./A as a function of neutron number N.

Always B.E./A takes a major role in deciding the stability, From the figure it is clear that for N=146 it's value is 7.45 MeV & is maximum. So it again Proves that $N = 146$ is the most stable one in this isotopic chain. The nuclei change their shape means rms radii above and below the relatively stable nucleus due to the quantum shell effect & redistribution of charge. So rms radii play a key role in the search for stability.

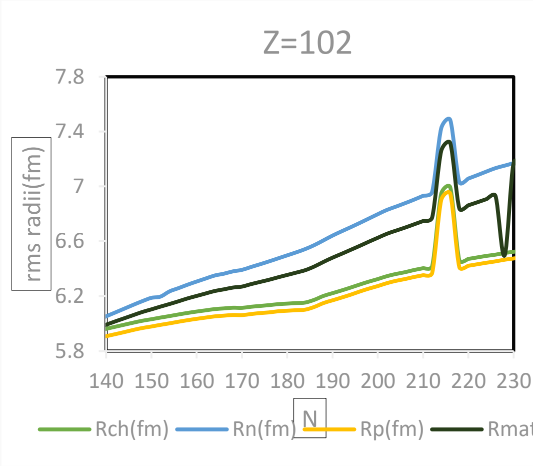

Fig.2: Rms radii as a function of neutron number N

From Fig.2 it is clear that at N=184 , prominent kinks occur for charge and proton radius. After which it increases & sharp peak occurs at N=216 for all rms radii . So N=184, 216 are the stable isotopes.

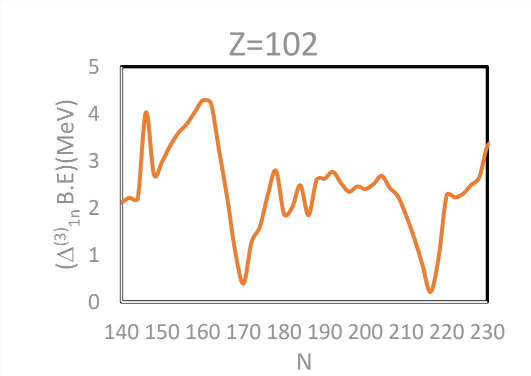

Fig.3: $\Delta^{(3)}_{1n}$B. E.(N) as a function of neutron number N.

A sharp peak at N=154 in Fig.3 indicates shell closure at N=154. Again from Fig.4, three sharp peaks are observed from N=146, N=184, and N=216, which shows the shell or sub-shell closures. This S_{2n}differential is most important to test the shell closures in an isotopic chain. So N=146, 184. and,216 are the possible shell or sub-shell closures. From Fig.5, it produces a dip at N=146, N=184, 188, & N=216, which indicates a shell/sub-shell closure.

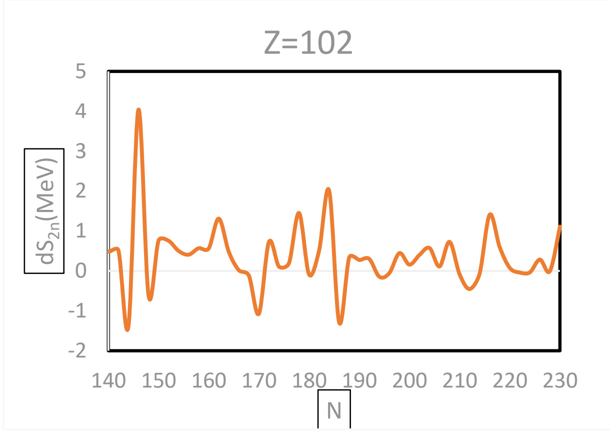

Fig.4: dS_{2n} as a function of neutron number N.

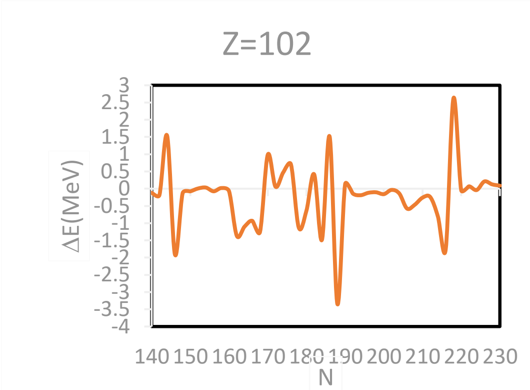

Fig.5: ΔE as a function of neutron number N.

Conclusion:

So, to get a clearer picture of the possible neutron shell closures, we calculate B.E/A, radii r_{rms}, two neutron separation energy (S_{2n}), differential variation of S_{2n} (dS_{2n}), the energy gap (ΔE), and also three-point differences of B.E $\Delta^{(3)}_{1n}$(B.E). We got the signatures of shell closures at N=146, 184, 188, and 216 with Z=102.

References:

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