Statistical Simulation Assignment 1

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Q2. (b) Given : MGF of X is g(t)
To find: MGF of Y=kX+d, k,d $\in$ R;
$M_Y(t) = \int_{-\infty}^{\infty} e^{yt} f_Y(y)\, dy$=

$$\int_{-\infty}^{\infty} e^{(ax+b)t} \frac{f_X(x)}{a}\, adx$$

= $e^{bt} \int_{-\infty}^{\infty} e^{axt} f_x(x)\, dx$= $e^{bt} M_x(at)$

(d) Let X have a normal distribution with mean μ_x, variance σ_x^2, and standard deviation σ_x.
Let Y have a normal distribution with mean μ_y, variance σ_y^2, and standard deviation σ_y.
If X and Y are independent, then $X - Y$ will follow a normal distribution with mean $\mu_x - \mu_y$, variance $\sigma_x^2 + \sigma_y^2$, and standard deviation $\sqrt{\sigma_x^2 + \sigma_y^2}$

(e) Part (i) : $M(t) = \frac{1}{6}e^{3t} + \frac{1}{2}e^{5t} + \frac{1}{3}e^{7t}$

$M_x(t) = E[e^{tx}] = \sum_{x \in S} P(X = x)e^{xt} - eq(1)$

x	p_x
3	$\frac{1}{6}$
5	$\frac{1}{2}$
7	$\frac{1}{3}$

Comparing to eq. 1, we see that S = {3, 5, 7}, and for each for those x values, $P(X = x) = p_x$ from the table above.

Part (ii): $M_x(t) = \frac{5}{5-t}$

Let X be a continuous random variable with an exponential distribution with parameter β for some $\beta \in R > 0$
Then the moment generating function M_X of X is given by: $M_x(t) = \frac{1}{1-\beta t}$

Our given eqaution can be written as $\frac{1}{1-5t}$, thus $\beta = \frac{1}{5}$
From the definition of the Exponential distribution, X has probability density function: $f_x(x) = \frac{1}{\beta}e^{\frac{-x}{\beta}}$
Thus $pdf\, f_x(x) = 5e^{-5x}$ or $X \sim Exponential(1/5)$

Q4. By looking at the support S of the DF of a RV that we can determine whether it is continuous or discrete. If the support is an interval then the DF is continuous, if the support is a set of discrete values then the DF is discrete.

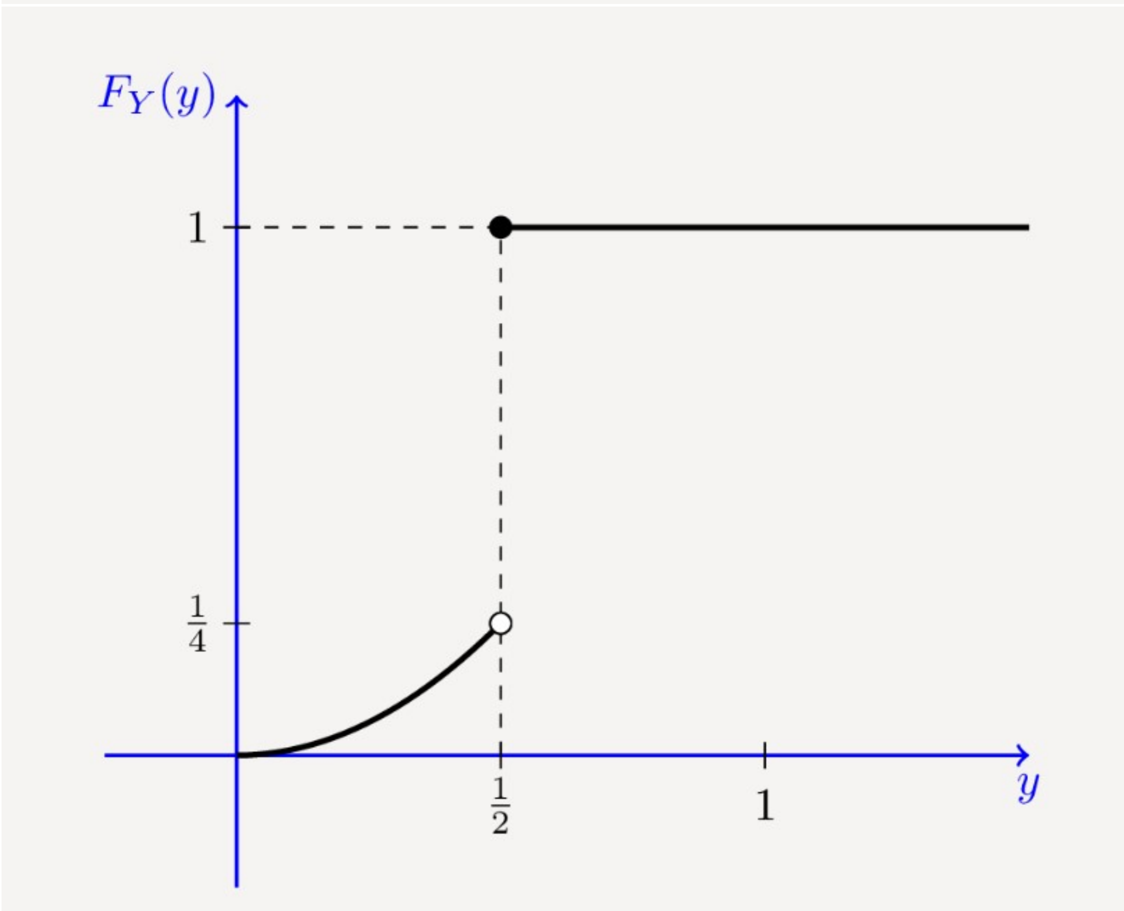
Yes, a RV can be neither discrete or continuous. Such RVs are called mixed random variables.
Example:

Let X be a continuous random variable with the following PDF:

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let also

$$Y = g(X) = \begin{cases} X & 0 \leq X \leq \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$



The CDF of Y has a continuous part and a discrete part.

Q5. DF,mean,variance of following distributions:

(a) $f(x) = \lambda e^{-\lambda x}; \forall x \geq 0$ or $X \sim Exponential(\frac{1}{\lambda})$ - Valid pdf

DF $= F(x) = P(X \leq x)$
$= \int_0^\infty f(w)\, dw = \int_0 \lambda e^{-\lambda w}\, dw = 1 - e^{-\lambda x} \;\; x > 0$

Mean $= E[X] = \int_0^\infty x\lambda e^{-\lambda x}\, dx$

$$= \lambda \left[\left| \frac{-xe^{-\lambda x}}{\lambda} \right|_0^\infty + \frac{1}{\lambda} \int_0^\infty e^{-\lambda x} dx \right]$$
$$= \lambda \left[0 + \frac{1}{\lambda} \frac{-e^{-\lambda x}}{\lambda} \right]_0^\infty$$
$$= \lambda \frac{1}{\lambda^2}$$
$$= \frac{1}{\lambda}$$

Hence, the mean of the exponential distribution is $\frac{1}{\lambda}$.

For variance: $E[X^2] = \int_0^\infty x^2 \lambda e^{-\lambda x}\, dx = \frac{2}{\lambda^2}$

Variance $= E[X^2] - (E[X])^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$

(b) $f(x) = \lambda^x \frac{e^{-\lambda}}{x!}; \forall x \in N_0$ or $X \sim Poisson(\lambda)$ - Valid pdf
For a poisson distribution, Mean and Variance both are equal to λ.

(d) $f(x) = \frac{1}{\pi} \frac{1}{1+x^2}; \forall x \in (-\infty, \infty)$ or $X \sim Cauchy(0, 1)$ - Valid pdf

$$\int_{-\infty}^\infty f_X(x)\, dx = \frac{1}{\pi} \int_{-\infty}^\infty \frac{1}{1+y^2}\, dy = \frac{2}{\pi} \int_0^\infty \frac{1}{1+y^2}\, dy = \frac{2}{\pi} \tan^{-1}(y) \,|_0^\infty = 1.$$

Mean and variance do not exist for a cauchy distribution.