Finding an Optimal Design

Sonali Manjunath
11/18/2024
SE 310

TABLE OF CONTENTS:

I. Introduction & Problem Description

This report will walk through my process of designing a truss structure while minimizing the objective quality function. The quality objective function is given by C = $(1 + n)\sum_{1}^{m} x_i^2 l_i$, where n is the number of joints, x_i^2 is the cross-sectional area of the specific truss, and l_i is the respective length. The given design constraints was the given outline of the truss structure (with a few dimension requirements), a max stress of 150 MPa, a Young's Modulus of 200E9, a slenderness ratio less than 500, and all trusses in compression must pass the buckling test.There is also a load in the y-direction on joint G of 98634 N.

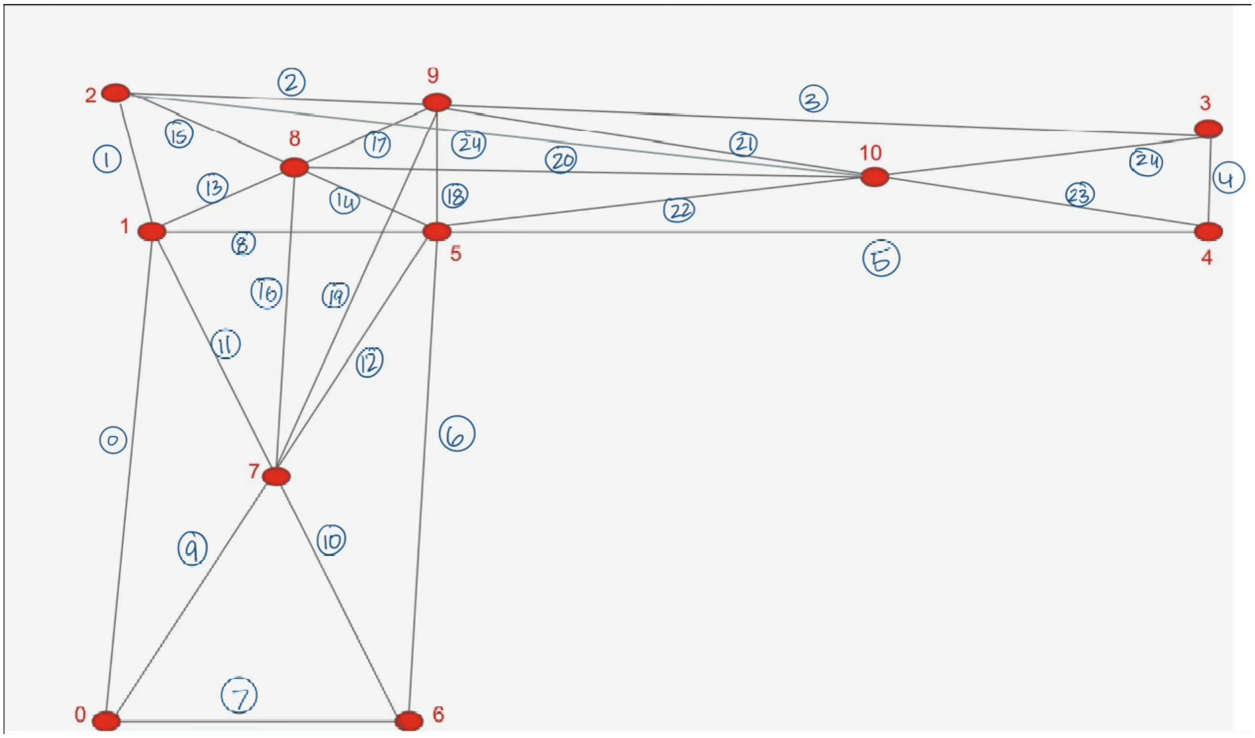
Minimizing the objective function will yield the optimal truss structure as it will have a lower cost and amount of material, while still being stable. All my calculations for this project were done on a Google Sheet, which I have linked below.

II. Design 1

A. Design and Ideas

For my initial design, I really prioritized stability in my design. I thought about the real-life implementation while deciding my lengths and joints. For example, I kept Lef to be very small because in real life each truss would have a weight and I didn't want a top heavy design. This is because it would cause a larger moment, making my structure more likely to collapse.

Like I described above, I really prioritized stability and strike resistance for this design. I wanted to create a truss so that if even 1 of my trusses or joints fail, my structure would not collapse. To do this, I had to make sure my determinacy was greater than 1 so that it could stay strong even if a truss failed. I also had to make sure my determinacy was at least 1 less than the maximum trusses connected to a joint. For example, Joint 5 or Joint C had 7 trusses connected to it, so I had to make my determinacy at least 6. In order to satisfy these requirements, I designed the following truss:



Please note that there is a downwards load on joint 4 of 98634 N. As you can see, this structure has 26 trusses and 11 joints. Given the initial problem, there are 3 reaction forces (2 from the hinge joint on A and 1 on the roller joint on B). This makes my determinacy equal to d = b + r -2j = 26 + 3 - 2(11) = 7.

From the code, I could also determine the structure's stability by looking at the joint deflections:

Joints	Deflection[x] (m)	Deflection[y] (m)
0	0	0
1	0.00511	0.001308
2	0.011117	0.002861
3	0.005388	-0.055591
4	0.004466	-0.05572
5	0.004956	-0.000475
6	-0.000085	0
7	0.000336	0.000601
8	0.007146	0.001657
9	0.010948	-0.000921

10	0.006028	-0.02474

B. Determining Areas (Slenderness Ratio and Buckling)

Next, I had to decide on how big to make my areas. I know that I wanted to minimize area (x_i^2) in order to minimize c, but I had to use the slenderness ratio and buckling forces to constrain how small I could make my area.

I used the "L_elem" function to get the length of each of my trusses. I then made an educated guess for each area depending on the length of the truss. For example, longer trusses needed bigger areas and vice versa. I ran my code again with my guesses, increasing the values if I found there to be an absurdly large displacement. Then I calculated the buckling force for each truss that was in compression. If a truss was failing the buckling test, I decreased the area slightly until it passed. I kept doing this until all the internal forces in the trusses that were in compression were less than the calculated buckling force for that truss.

In the table below, you can observe the length, final area, slenderness ratio, internal force, and buckling force for each truss.

Truss	Length(m)	Area(m^2)	Slenderness Ratio	Internal Force(N)	Buckling Load(N)
0	30	0.04329784244	499.4347422	377685.36470769	
1	5	0.003471043703	293.9886653	215611.06571751	
2	5.04535678	0.00437816347	264.1414266	58773.9648673	
3	24.95464386	0.03149769433	487.0826018	447928.49649394	
4	1	0.002314029135	72.012222	59682.7692412	
5	24.73213749	0.08584633009	292.4094529	-340127.50257119	494957.8516
6	30	0.2082626222	227.7226409	-659986.97341025	1979831.406
7	5	0.004338804628	262.951456	-14693.40869621	30934.86572
8	5	0.01157014568	161.0242236	-71244.86613644	219981.2674
9	15.13274595	0.01313160563	457.4561266	111175.81047268	
10	15.29705854	0.0112971827	498.5554584	74921.97766315	
11	15.13274595	0.01313160563	457.4561266	12134.84785376	
12	15.29705854	0.01327418968	459.9329693	-26019.75118145	30934.86572
13	2.6925824	0.002336517796	192.9634606	187514.34523075	
14	4.71699057	0.02183050722	110.5920015	-673342.58075165	879925.0695
15	2.6925824	0.009346071183	96.48173031	-247605.22871307	494957.8516
16	17.52854814	0.01483959918	498.4544567	112713.36924873	
17	4.39666066	0.003815250324	246.5767088	414477.37478899	

18	4.325	0.01501226401	122.2794954	-309429.73310418	494957.8516
19	19.55647271	0.06788137144	260.019456	85637.49271683	494957.8516
20	20.05617112	0.01933773513	499.6154337	165625.96817055	
21	16.34183665	0.012891643	498.5828697	4628.59704446	
22	16.03121954	0.05564506363	235.4202752	-307813.06488182	494957.8516
23	8.78921073	0.007626933639	348.6306105	342350.57526305	
24	8.73213749	0.03030963084	173.7484203	-443934.57291252	494957.8516
25	21.37755833	0.02214998186	497.5789917	34318.79411071	

C. Checking Max Stress

Based on the minimum safety factor of 4, and max allowable stress as 600 MPa, all stress values must be less than 150 MPa. Below are the recorded stress values for each truss.

Truss	Stress (Pa)	Stress (MPa)
0	8,722,960	8.72296
1	62,117,070	62.11707
2	13,424,340	13.42434
3	14,220,990	14.22099
4	25,791,710	25.79171
5	-3,962,051	-3.962051
6	-3,169,013	-3.169013
7	-3,386,511	-3.386511
8	-6,157,646	-6.157646
9	8,466,277	8.466277
10	6,631,917	6.631917
11	924,095	0.9240948
12	-1,960,176	-1.960176
13	80,253,760	80.25376
14	-30,844,110	-30.84411
15	-26,492,970	-26.49297
16	7,595,446	7.595446
17	108,637,000	108.637
18	-20,611,800	-20.6118
19	1,261,576	1.261576
20	8,564,910	8.56491

21	359,039	0.3590386
22	-5,531,723	-5.531723
23	44,887,050	44.88705
24	-14,646,650	-14.64665
25	1,549,382	1.549382

D. Checking for Crash Resistant

Besides the determinacy equation I used in section I.A, I manually deleted each truss in my code to check that it was crash resistant. I recorded the maximum displacement of a joint each time I did this, which you can see below. I have also attached a pdf of the plot for each deletion in the zip folder, to prove that it is crash resistant.

Truss	Max Displacement Without (m)
0	-0.0810
1	-0.0629
2	-0.0562
3	-0.0971
4	-0.0971
5	-0.0733
6	-0.2295
7	-0.0579
8	-0.0590
9	-5.79E-02
10	-0.0579
11	-0.0557
12	-0.0558
13	-0.0598
14	-0.1358
15	-0.0629
16	-0.0571
17	-0.0921
18	-0.0624
19	-0.0562
20	-0.0580
21	-0.0557

22	-0.0687
23	-0.0733
24	-0.0971
25	-0.0559

E. Quality Factor (C value) Analysis

My calculated objective function value turned out to be around 185.38 for my first design. The calculations are shown below.

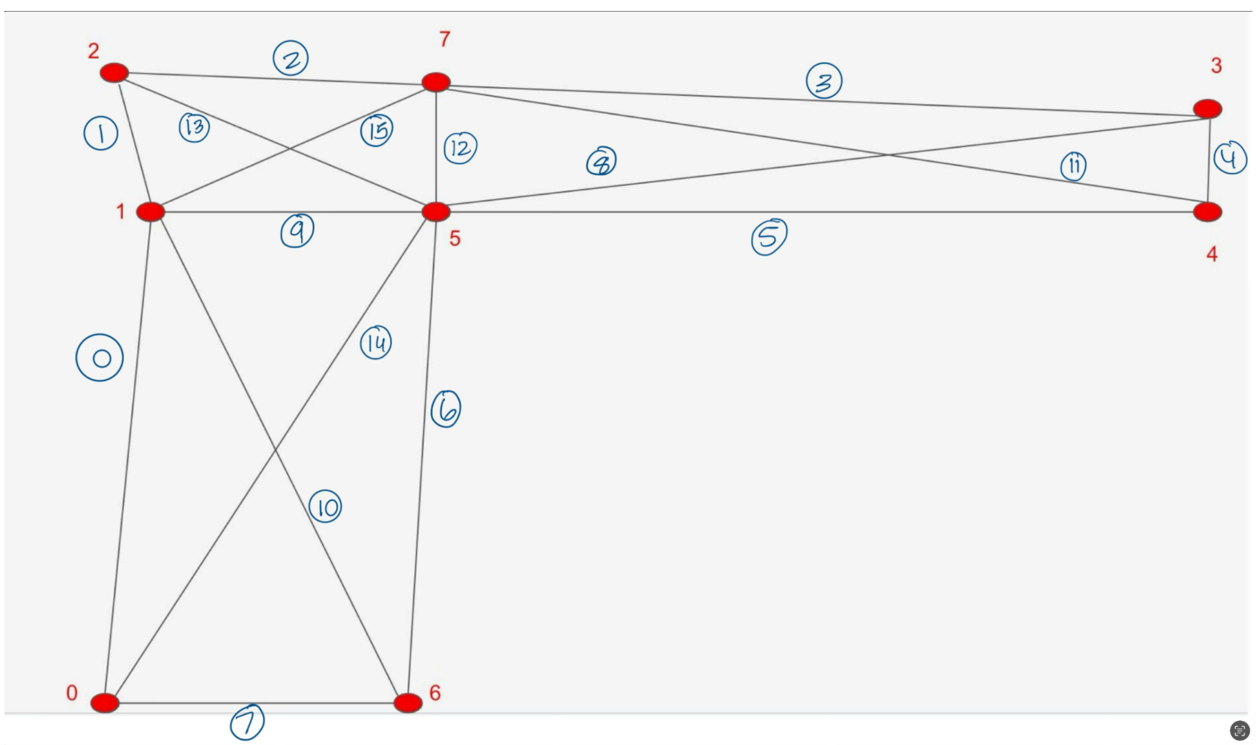
Truss	Length(m)	Area(mˆ2)	C Calculations
0	30	0.04329784244	1.298935273
1	5	0.003471043703	0.01735521851
2	5.04535678	0.00437816347	0.02208939675
3	24.95464386	0.03149769433	0.7860137445
4	1	0.002314029135	0.002314029135
5	24.73213749	0.08584633009	2.123163239
6	30	0.2082626222	6.247878665
7	5	0.004338804628	0.02169402314
8	5	0.01157014568	0.05785072838
9	15.13274595	0.01313160563	0.198717252
10	15.29705854	0.01117826499	0.170994574
11	15.13274595	0.01313160563	0.198717252
12	15.29705854	0.01327418968	0.2030560566
13	2.6925824	0.002336517796	0.006291266694
14	4.71699057	0.02183050722	0.1029742967
15	2.6925824	0.009346071183	0.02516506678
16	17.52854814	0.01483959918	0.2601166286
17	4.39666066	0.003815250324	0.01677436101
18	4.325	0.01501226401	0.06492804186
19	19.55647271	0.06788137144	1.327520188
20	20.05617112	0.01933773513	0.3878409248
21	16.34183665	0.012891643	0.210673124
22	16.03121954	0.05564506363	0.8920582313
23	8.78921073	0.007626933639	0.06703472697

24	8.73213749	0.03030963084	0.2646678638
25	21.37755833	0.02214998186	0.4735125291
		SUM:	15.4483367
		Calculated C Value:	185.3800404

III. Design 2

A. Design and Ideas

My priority for this design was to minimize the quality function by using all my observations of the first design. For example, I knew that I wanted to reduce my number of joints and keep it to the minimum number of joints allowed, which was 8. Because c is dependent on length as well, I got rid of unnecessarily long trusses, as well as keeping my number of trusses to a minimum. Because I still wanted my determinacy to be at least 1, I had to have at least 14 trusses. I found that I needed to have at least 16, because if I didn't, there would be zero force members present, which would cause the structure to not be crash-resistant. For example, I was hesitant on keeping the truss connecting joints 1 and 6 because it had a large length and would not minimize my quality factor. Without that truss, the member connecting joints 0 and 6 would become a zero force member. I know this because using joint analysis, there would be no force in the x - direction to cancel out the force produced by the member connecting joint 0 and 6. Zero force members might as well be erased, which would make my determinacy value misleading even though it was a positive number. Taking all of this into account, I designed the following structure.



Please note that there is a downwards load on joint 4 of 98634 N. There are 16 trusses and 8 joints. So checking the determinacy, we get d = b + r -2j = 16 + 3 - 2(8) = 3.
From the code, I could also determine the structure's stability by looking at the joint deflections:

Joints	Deflection[x] (m)	Deflection[y] (m)
0	0	0
1	0.006714	0.00141
2	0.013261	0.004924
3	0.008812	-0.064166
4	0.00619	-0.064397
5	0.006693	-0.000831
6	-0.000048	0
7	0.016206	-0.000839

B. Determining Areas (Slenderness Ratio and Buckling)

Following the same procedure as my first design, I used the slenderness ratio and buckling load to determine my area for each truss.

Truss	Length (m)	Area (m^2)	Slenderness Ratio	Internal Force (N)	Buckling Force(N)
0	30	0.04329784244	499.4347422	413,047.89	
1	5	0.003471043703	293.9886653	257,397.29	
2	5.04535678	0.003891700862	280.1642909	300,268.48	
3	24.95464386	0.03464746377	464.4150387	309,057.20	
4	1	0.001157014568	101.840661	53,564.07	
5	24.73213749	0.06358987414	339.7490824	-257,728.46	271,581.81
6	30	0.1203830186	299.5223698	-661,357.97	661,509.37
7	5	0.006942087405	207.8813788	-12,473.01	79,193.26
8	24.75234585	0.08591647419	292.5288908	-306,551.79	494,957.85
9	5	0.01577747138	137.8929069	-278,933.86	409,056.08
10	30.41381265	0.04454332189	499.1949558	75,870.34	
11	25.10745407	0.03041852368	498.6826214	261,639.56	
12	4.325	0.03002452803	86.46466043	-276,566.05	1,979,831.41
13	7.07106781	0.04908797078	110.5574684	-420,826.29	1,979,831.41
14	30.4138126	0.0444495464	499.721255	75,870.34	

15	6.61102299	0.004589429943	338.0492043	352,315.76	

C. Checking Max Stress

Below are the recorded stress values for each truss. All magnitudes of stresses are less than the maximum allowed (150 MPA).

Truss	Stress (Pa)	Stress (MPA)
0	9,540,000	9.54
1	74,200,000	74.2
2	77,200,000	77.2
3	8,920,000	8.92
4	46,300,000	46.3
5	-4,050,000	-4.05
6	-5,490,000	-5.49
7	-1,800,000	-1.8
8	-3,570,000	-3.57
9	-17,700,000	-17.7
10	1,700,000	1.7
11	8,600,000	8.6
12	-9,210,000	-9.21
13	-8,570,000	-8.57
14	1,710,000	1.71
15	76,800,000	76.8

D. Checking for Crash Resistant

Like in Section II, I manually deleted each truss in my code to check that it was crash resistant. I recorded the maximum displacement of a joint each time I did this, which you can see below. I have also attached a pdf of the plot for each deletion in the zip folder, to prove that it is crash resistant.

Truss	max displacement without
0	-0.064555
1	-0.069435
2	-0.069435
3	-0.054893
4	-0.054893

5	-0.051715
6	-0.096679
7	-0.044685
8	-0.054893
9	-0.065109
10	-0.044685
11	-0.051715
12	-0.065889
13	-0.069435
14	-4.47E-02
15	-0.064397

E. Quality Factor (C value) Analysis

My calculated objective function value turned out to be around 122.43 for my second design. I improved by 62.95 (around 33%). The calculations are shown below.

Truss	Length (m)	Area (mˆ2)		C Calculations (L*A)
0	30	0.04329784244		1.298935273
1	5	0.003471043703		0.01735521851
2	5.04535678	0.003891700862		0.01963501933
3	24.95464386	0.03464746377		0.864615119
4	1	0.001157014568		0.001157014568
5	24.73213749	0.06358987414		1.57271351
6	30	0.1203830186		3.611490558
7	5	0.006942087405		0.03471043703
8	24.75234585	0.08591647419		2.126634283
9	5	0.01577747138		0.07888735688
10	30.41381265	0.04454332189		1.354732247
11	25.10745407	0.03041852368		0.7637316861
12	4.325	0.03002452803		0.1298560837
13	7.07106781	0.04908797078		0.3471043701
14	30.4138126	0.0444495464		1.351880174
15	6.61102299	0.004589429943		0.03034082687
			TOTAL:	13.60377918

			calculated c:	122.4340126

IV. Conclusion and Recommendations

While on the mission to minimize the quality function, I found that the areas are very important. This was less intuitive to me because at first I thought that the number of joints and lengths of the trusses would have more of an impact. I found that if I had a lot of trusses or joints, and long trusses I could still obtain a small quality factor if I had very small areas. This is why I realized we have the most constraints on the area.

This project really challenged my creativity and determination. There were many designs that I thought of, and I had to use my engineering analysis skills to understand why it wouldn't be reasonable. An example of this is the zero-force member issue I had while creating my optimal design. I used a guess and check approach for my area, which really tested my determination as an engineer, but I found this approach allowed me to learn more about how the internal forces and buckling loads change in respect to the area.

I do not believe I reached the most minimized quality factor but I do believe that I am pretty close. My load was pretty big, which meant that I had a harsher constraint on my areas and could not make them as small as I could have if I had a smaller load. Overall, I am proud that I was able to decrease my objective function value by 33% through designing my optimal solution.

My future recommendations would be to design a truss with more joints and fewer trusses. This would cause lengths of trusses to decrease, and would allow for the areas to be smaller. I think this would improve even my optimal design.

V. Google Sheets Calculations and Code Outputs

Code Outputs
Design 1
Design 2
Design 1 Crash/Strike Resistant Plots
Design 2 Crash/Strike Resistant Plots