DP Intro

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These can't remember their past are condemned to repeat it.

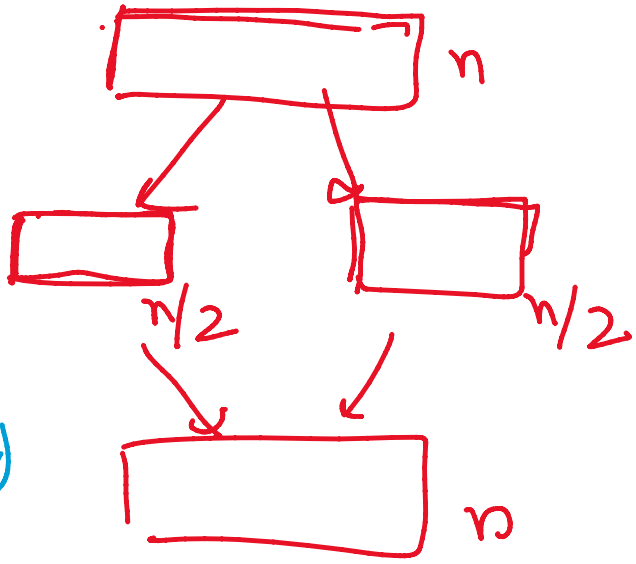
subproblems $1+1+1+1+1 = 5$ $O(N)$

problem $1+1+1+1+1+1 = 6$ $O(1)$

where!

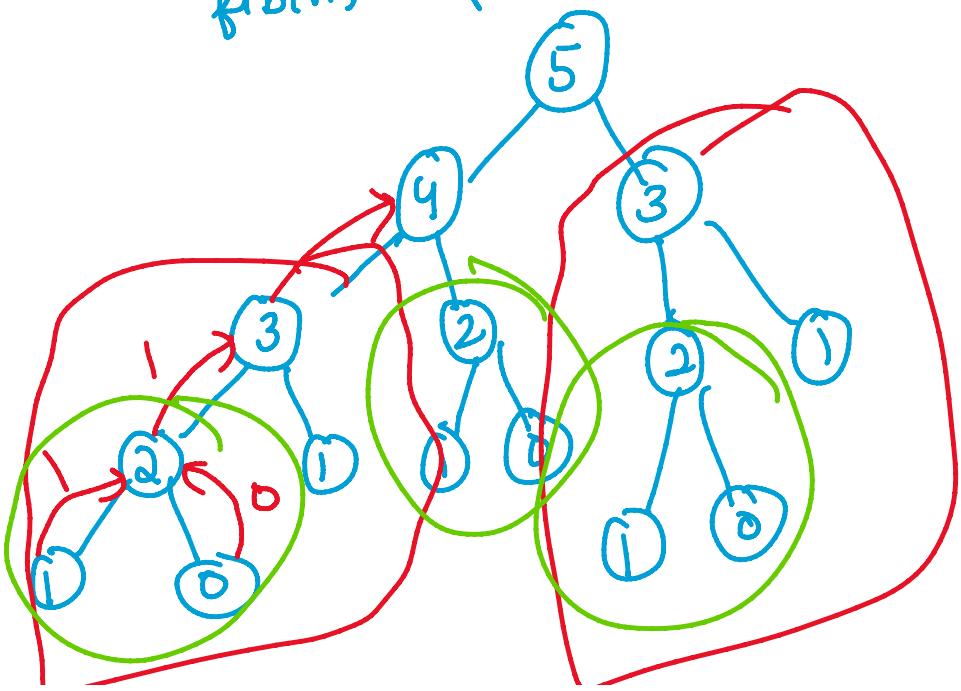
① optimal substructure.

$$\underbrace{sum(6)}_{bigger} = \underbrace{sum(5)}_{sum.} + 1$$



② Overlapping subproblems.

$fib(n) = fib(n-1) + fib(n-2)$



base case

$fib(0) = 0$

$fib(1) = 1$

→ Top down (recursion)

→ Bottom up

Fibonacci series

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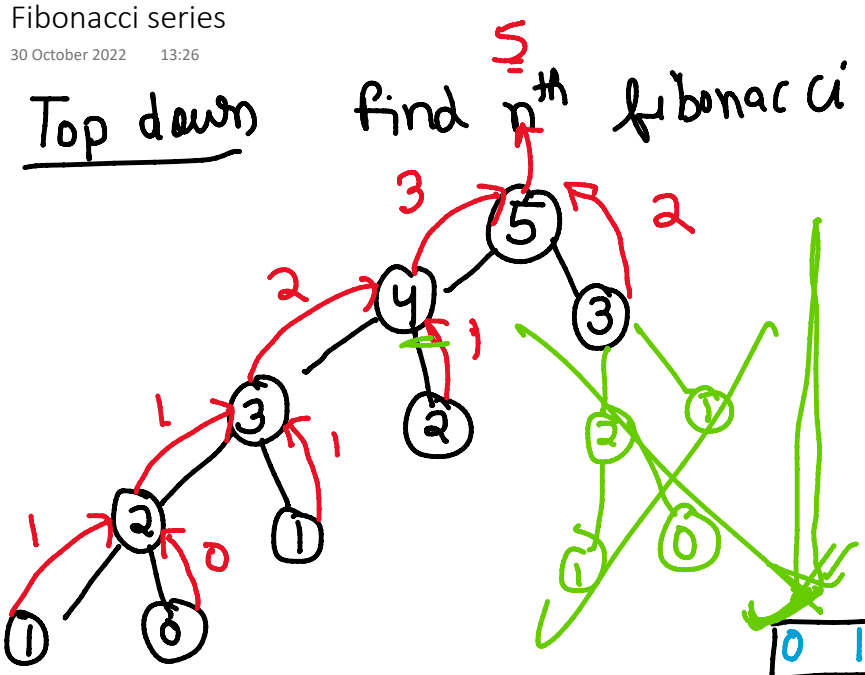
Top down

Find n^{th} fibonacci number. $n = 5$.

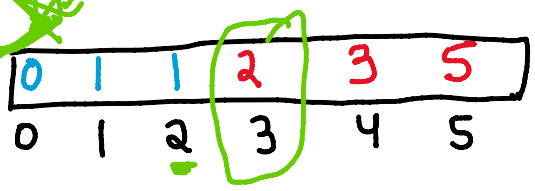


Base cases

$fib(0) = 0$

$fib(1) = 1$

$fib(n) = fib(n-1) + fib(n-2)$



dp array

Time :- O(N)

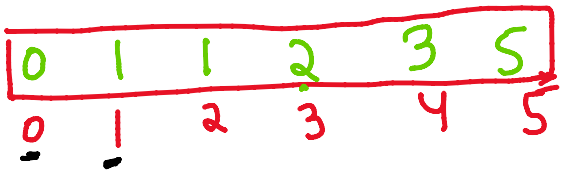
Space :- O(N)

(2^N)

Bottom Up

n^{th} fibonacci number. $n = 5$.

0 1 1 2 3 5 8 13 .....



$dp[i] = dp[i-1] + dp[i-2]$

$dp[4] = dp[3] + dp[2]$

$= 2 + 1$

$= 3$

O(N)

O(N)

3 variables

p	c	n	n	n	5
0	1	1	2	3	(5)
0^{th}	1	2	3	4	

$$\begin{cases} n = c + p \\ p = c \\ c = n \end{cases}$$

→ current

0 1 2 3 4 5

O(N)

O(1)

c → current

p → previous

r → result

Coin change

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coins = [1, 2, 5], amount = 11

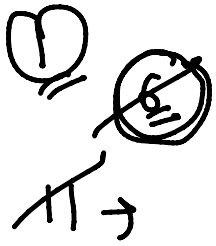
→ 1 coin 11 times = 11 coins

→ 1 coin 1 time and 2 coin 5 times = 6 coin

→ 1 coin 1 time and 5 coin 2 times = : 3 coins

5 + 5 + 1 = 11

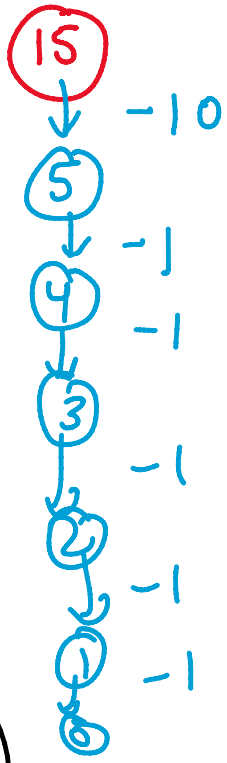
greedy!



3 coins

5, 5, 1 — 3 coins

greedy



coins [] = {1, 7, 10} amount = 15

6 coins greedy

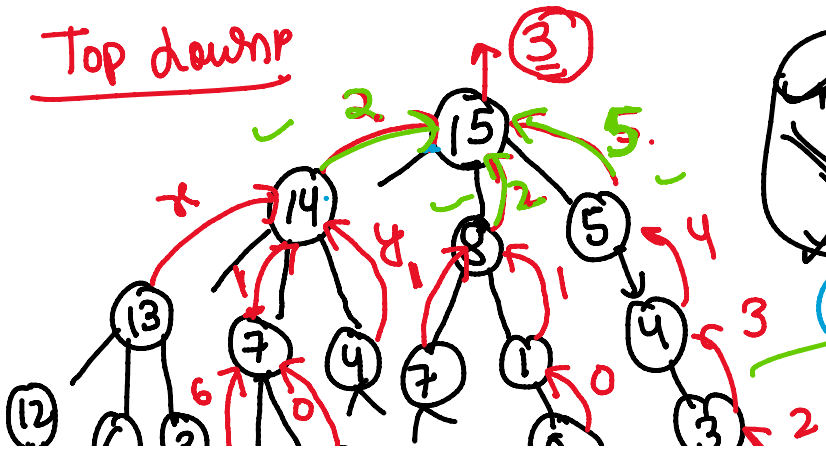
3 coins dP

7, 7, 1

amount = 15

coins = [1, 7, 10]

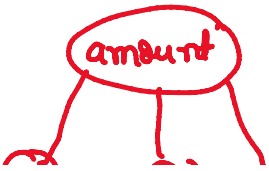
Top down

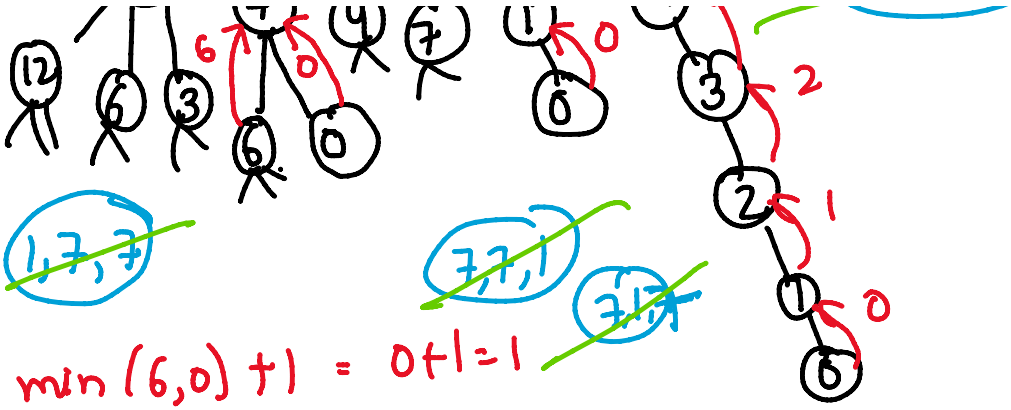


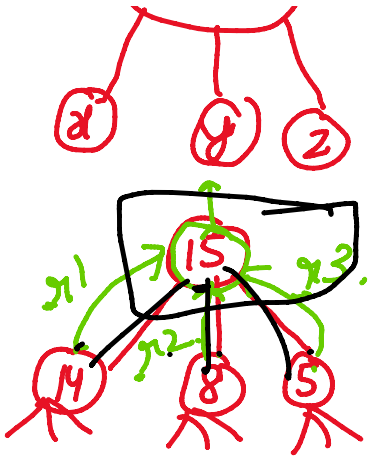
min(2, 2, 5)

min(14, 8, 5)

10, 1, 1, 1, 1, 1, 1.







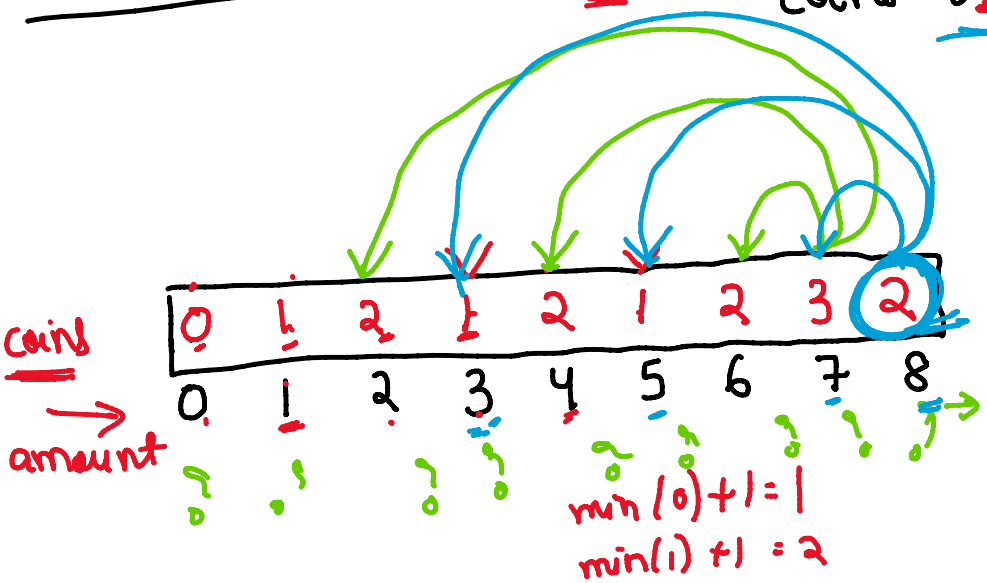
N T = min(x1, x2, x3) + 1

Bottom up

amount = 8

coins = [1, 3, 5]

2 coins



Time: O(N*T)
Space: O(N)

min(0,2)+1 = 0+1=1
min(1,1)+1 = 2
min(2,2,0)+1 = 1
min(1,1,3)+1 = 2

size = 9
initi q :-

[1, 5, 20, 2, 100, 10, 500, 1000]
[1, 2, 5, 10, 20, 100, 500, 1000]
10

Minimum path sum

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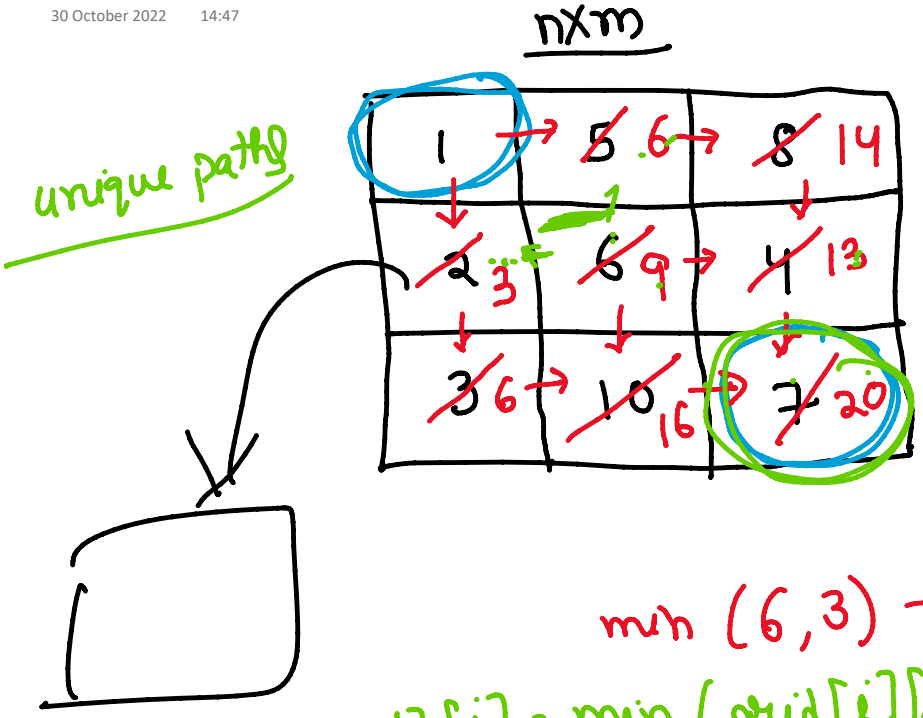

1+2+6+4+7=20

down
right

O(mxn)

~~O(mxn)~~ O(1)

min (6,3) + 6 = 9

grid[i][j] = min(grid[i][j-1], grid[i-1][j]) + grid[i][j]

Longest increasing subsequence (LIS)

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5, 1, 4, 3, 10, 2, 7, 8, 0, 11

[1, 3, 7, 8, 11] ⇒ 5

[1, 2, 7, 8, 11] ⇒ 5

[1, 4, 7, 8, 11] ⇒ 5

[2, 7, 8, 11]
x longest

[1, 4, 10, 11] ⇒ 4 !

5, 1, 4, 3, 10, 2, 7, 8, 0, 11 -1

dp[]: 1 1 2 2 3 2 3 4 1 (5) 1

[5, 10]

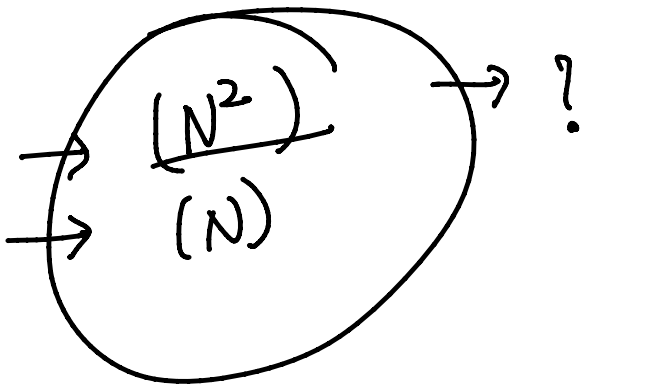
[1, 10]

[1, 4, 10]

[1, 3, 10]

smaller values on left

Binary search ?



[dp + binary search]

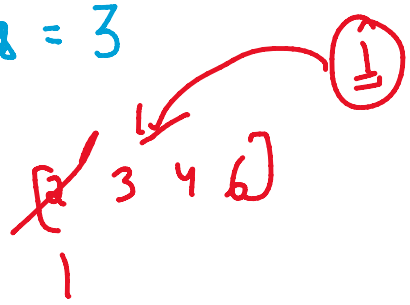
2 5 3 4 6 1

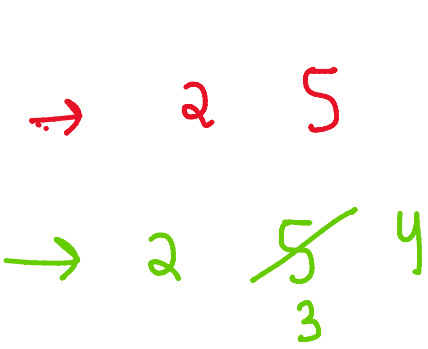
[2, 3, 4]

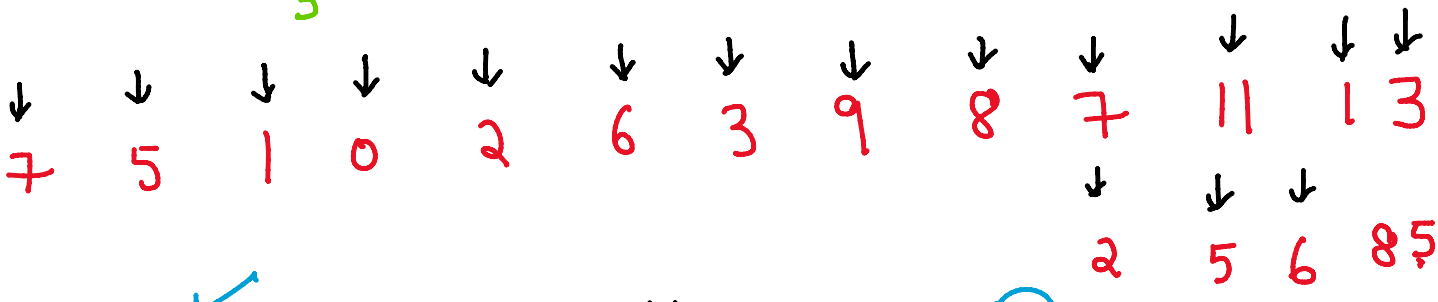
[2, 3, 4]

lis = 3

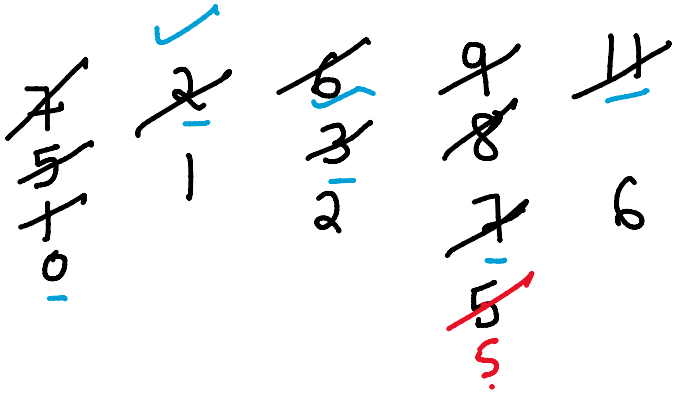
arr







dpt



(N log N)

(N)

1, 2, 3, 7, 11

0, 2, 3, 9, 11

0, 2, 6, 8, 11

lis = 5

7, 8, 9, 6, 5, 4, 3, 2, 1, 0