ITS202: Algorithms and Data Structure

Heap and HeapSort

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Heap Data Structure

Definition

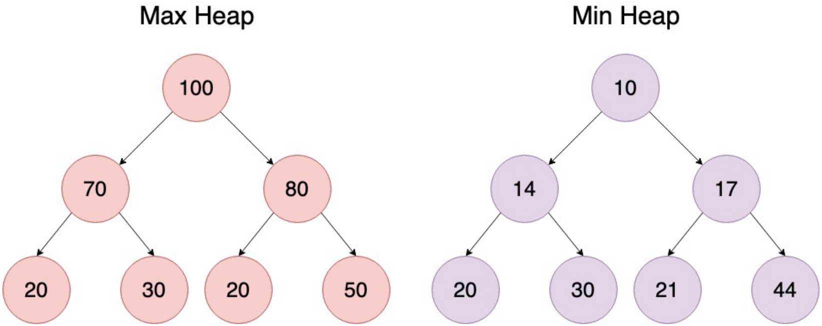
A Heap is a complete binary tree-based data structure.

Heap Data Structure

Heaps have specific ordering properties. The ordering can be one of two types:

- Max-Heap: The value of each node is less than or equal to the value of the parent. The greatest value is at the root. The same property must be true for all subtrees.
- Min-Heap: The value of each node is greater than or equal to the value of its parent. The smallest value is at the root. The same property must be true for all subtrees.

Heap Data Structure

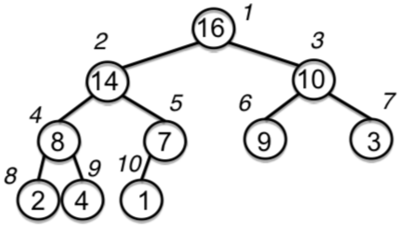


Heaps are not sorted

Heap as a Tree

- root of tree: first element in the array, corresponding to i = 1
- parent(i) =i/2: returns index of node's parent
- left(i)=2i: returns index of node's left child
- right(i)=2i+1: returns index of node's right child

Heap as a Tree



1	2	3	4	5	6	7	8	9	10
16	14	10	8	7	9	3	2	4	1

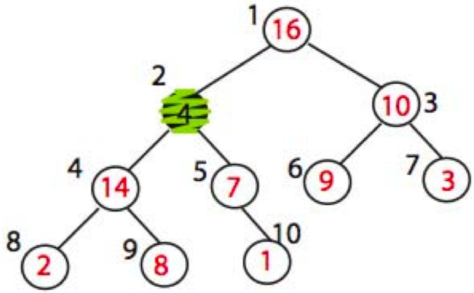
Heap Operation

- build_max_heap : produce a max-heap from an unordered array
- max_heapify : correct a single violation of the heap property in a subtree at its root

Max_Heapify

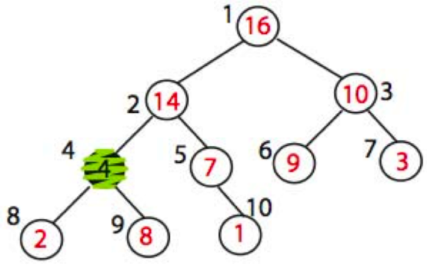
- Assume that the trees rooted at left(i) and right(i) are max-heaps
- If element A[i] violates the max-heap property, correct violation by “trickling” element A[i] down the tree, making the subtree rooted at index i a max-heapt

Max_Heapify Example



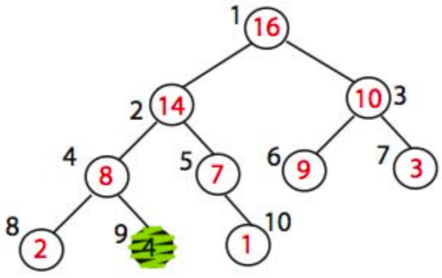
MAX_HEAPIFY (A,2)
heap_size[A] = 10

Max_Heapify Example



Exchange A[2] with A[4]
Call MAX_HEAPIFY(A,4)
because max_heap property
is violated

Max_Heapify Example



Exchange A[4] with A[9]
No more calls

Max_Heapify Running time???

O(log n)

Max_Heapify Running time???

O(log n)

build_max_heap(A)

Converts A[1...n] to a max heap

```
Build_Max_Heap(A):
for i=n/2 downto 1
do Max_Heapify(A, i)
```

Why start at n/2?

Because elements A[n/2 + 1 ... n] are all leaves of the tree 2i > n, for i > n/2 + 1

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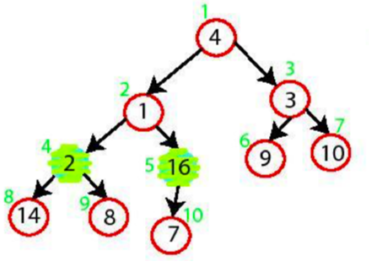
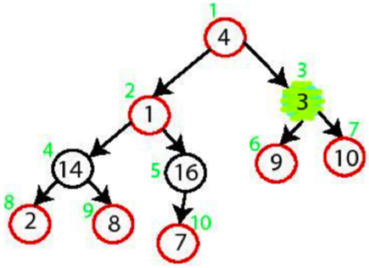
build_max_heap(A) Demo



A	4	1	3	2	16	9	10	14	8	7

MAX-HEAPIFY (A,5)
no change
MAX-HEAPIFY (A,4)
Swap A[4] and A[8]

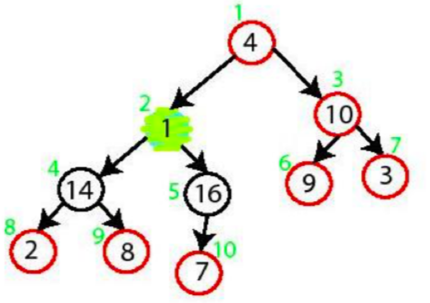
build_max_heap(A) Demo



MAX-HEAPIFY (A,3)
Swap A[3] and A[7]

15

build_max_heap(A) Demo



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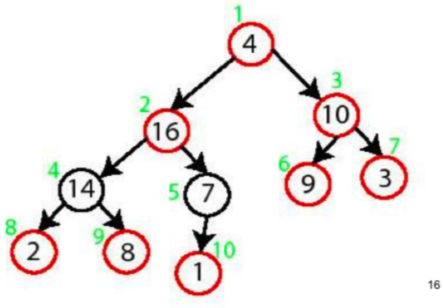
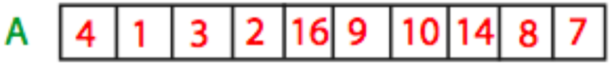
MAX-HEAPIFY (A,2)
Swap A[2] and A[5]
Swap A[5] and A[10]

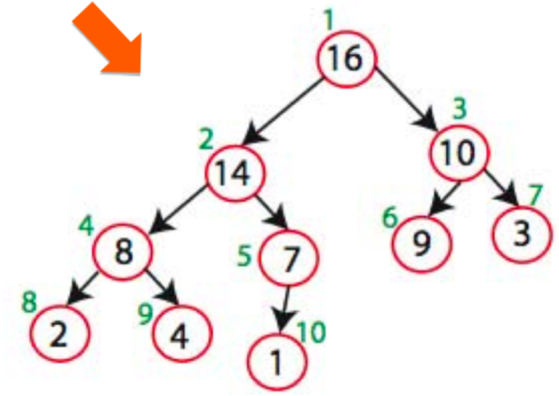
build_max_heap(A) Demo



MAX-HEAPIFY (A,1)
Swap A[1] with A[2]
Swap A[2] with A[4]
Swap A[4] with A[9]

build_max_heap(A) Demo





build_max_heap(A) Running time???

O(n log n)

build_max_heap(A) Running time???

O(n log n)

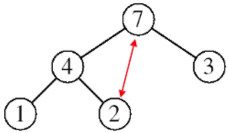
HeapSort

Sorting Strategy:

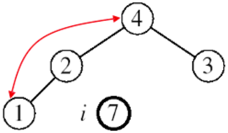
1. Build Max Heap from unordered array;
2. Find maximum element A[1];
3. Swap elements A[n] and A[1]: now max element is at the end of the array!
4. Discard node n from heap(by decrementing heap-size variable)
5. New root may violate max heap property, but its children are max heaps. Run max_heapify to fix this.
6. Go to Step 2 unless heap is empty.

HeapSort Example

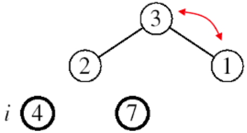
Example: A=[7, 4, 3, 1, 2]



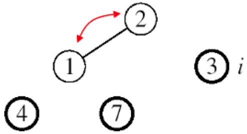
MAX-HEAPIFY(A, 1, 4)



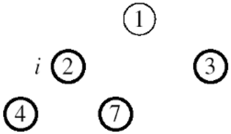
MAX-HEAPIFY(A, 1, 3)



MAX-HEAPIFY(A, 1, 2)



MAX-HEAPIFY(A, 1, 1)



A

1	2	3	4	7

HeapSort Example

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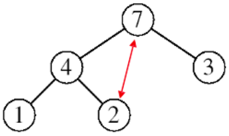

MAX-HEAPIFY(A, 1, 4)

HeapSort Example

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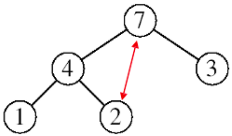
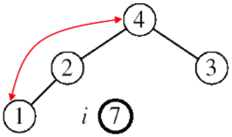


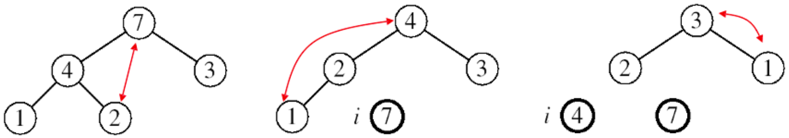
MAX-HEAPIFY(A, 1, 4)



MAX-HEAPIFY(A, 1, 3)

HeapSort Example

Example: A=[7, 4, 3, 1, 2]



MAX-HEAPIFY(A, 1, 4) MAX-HEAPIFY(A, 1, 3) MAX-HEAPIFY(A, 1, 2)

HeapSort Example

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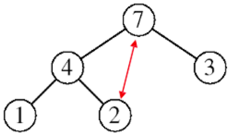
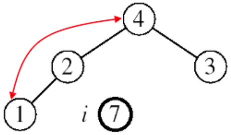
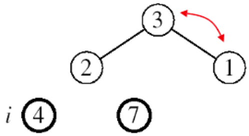


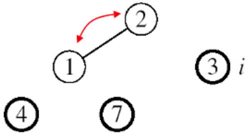
MAX-HEAPIFY(A, 1, 4)



MAX-HEAPIFY(A, 1, 3)



MAX-HEAPIFY(A, 1, 2)



MAX-HEAPIFY(A, 1, 1)

HeapSort Example

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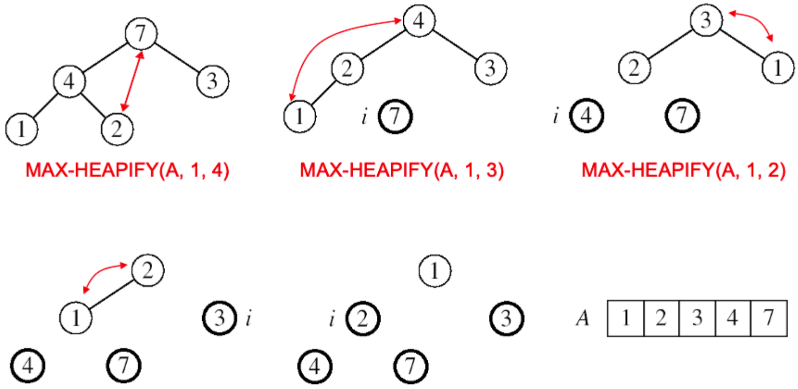

MAX-HEAPIFY(A, 1, 4)

MAX-HEAPIFY(A, 1, 3)

MAX-HEAPIFY(A, 1, 2)

MAX-HEAPIFY(A, 1, 1)

THANK YOU FOR YOUR TIME. BEST OF LUCK IN YOUR EXAM