Tutorial - 4.

Masters theorem :-

$T(n) = aT(n/b) + f(n)$

where $a \geq 1$ and $b > 1$

$f(n)$ is asymptotically positive function.

$c = \log_b a$

There can be 3 possible case.

1. If $f(n) = \Theta(n^c)$, $T(n) = \Theta(n^c \log n)$

2. If $f(n) < n^c$, $T(n) = \Theta(n^c)$

3. If $f(n) > n^c$, $T(n) = \Theta(f(n))$.

Q1 $T(n) = 3T(n/2) + n^2$

$a = 3$, $b = 2$.

$c = \log_2 3 = 1.58$

$f(n) = n^2$.

$n^c = n^{1.58}$.

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(f(n)) = \Theta(n^2)$

Q2 $T(n) = 4T(n/2) + n^2$.

$a = 4$, $b = 2$, $f(n) = n^2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\Rightarrow f(n) = n^c = n^2$

$\therefore T(n) = \Theta(n^2 \log n)$.

3. $T(n) = T(n/2) + 2^n$

$a = 1,\ b = 2,\ f(n) = 2^n$

$c = \log_b a = \log_2 3 = 1.5$

$n^c = n^{1.5}$

$n^c < f(n)$

$\therefore T(n) = O(f(n)) = O(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n,\ b = 2,\ f(n) = n^n$

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

$\Rightarrow n^c = f(n)$.

$\therefore T(n) = O(n^c \cdot \log n)$

$= O(n^n \log n)$

5. $T(n) = 2T(n/2) + n\log n$.

$a = 2,\ b = 2,\ f(n) = n\log n$.

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$, $\cancel{f(n)}$

$\Rightarrow f(n) > n^c$

$\therefore T(n) = O(f(n)) = O(n\log n)$

6. $T(n) = 16T(n/4) + n$.

$a = 16,\ b = 4,\ f(n) = n$.

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

$n^2 > n$

$\Rightarrow n^c > f(n)$

$\therefore T(n) = O(n^c) = O(n^2)$

7. $T(n) = 2T(n/2) + n/\log n.$

$a=2,\ b=2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1$

$f(n) < n^c$

$\therefore T(n) = O(n^c) = O(n)$

8. $T(n) = 2T(n/4) + n^{0.51}$

$a=2,\ b=4,\ f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^c > f(n)$

$\therefore T(n) = O(n^c) = O(n^{0.5})$

9. $T(n) = 0.5T(n/2) + 1/n.$

$a=0.5,\ b=2,\ f(n) = 1/n.$

$\log_b a = \log_2 0.5 = -1.0$

$n^c = n^{-1.0}$

$n^c = f(n)$

$\therefore T(n) = O(n^c \log n)$

$= O(\log n/n)$

10. $T(n) = 16T(n/4) + n!$

$a=16,\ b=4,\ f(n) = n!$

$\log_b a = \log_4 16. = 2$

$n^c = n^2$

$f(n) > n^c$

$\therefore T(n) = O(n!)$

11. $T(n) = 4T(n/2) + \log n$

$a = 4, \; b = 2, \; f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n^c > f(n)$

$\therefore T(n) = O(n^c) = O(n^2)$

12. $T(n) = 8qrt(n) \, T(n/2) + \log n$

$a = sqrt(n) = n^{1/2}, \; b = 2, \; f(n) = \log n$

$c = \log_b a = \log_2 n^{1/2} = \frac{1}{2} \log_2 n$

$n^c = n^{1/2 \log_2 n}$

$n^c > f(n)$

$\therefore T(n) = \cancel{of} \; O(n^c) = O(n^{1/2 \log_2 n})$

13. $T(n) = 3T(n/2) + n$

$a = 3, \; b = 2, \; f(n) = n$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58}$

$n^c > f(n)$

$\therefore T(n) = O(n^c) = O(n^{1.58})$

14. $T(n) = 3T(n/3) + sqrt(n)$

$a = 3, \; b = 3, \; f(n) = sqrt\, n = n^{1/2}$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$n^c > n^{1/2}$

$\therefore T(n) = O(n^c) = O(n)$

15. $T(n) = 4T(n/2) + cn$

$a=4$, $b=2$, $f(n) = cn = n$.

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

16. $T(n) = 3T(n/4) + n\log n$.

$a=3$, $b=4$, $f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.79$.

$n^c = n^{0.79}$

$f(n) > n^c$

$\therefore T(n) = O(f(n))$

$= O(n\log n)$.

17. $T(n) = 3T(n/3) + n/2$.

$a=3$, $b=3$, $f(n) = n/2$.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1$.

$n^c > f(n)$

$\therefore T(n) = O(n^c)$

$= O(n)$

16. $T(n) = 3T(n/4) + n\log n$.

$a=3$, $b=4$, $f(n) = n\log n$.

$c = \log_b a = \log_4 3 = 0.7$

$n^c = n^{0.7}$

$f(n) > n^c$

$\therefore T(n) = O(n^c) = O(n^{0.7})$

17. $T(n) = 3T(n/3) + n/2.$

$a = 3, b = 3, f(n) = n/2.$

$c = \log_3 3 = 1$

$n^c = n^1 = n.$

$n^c > f(n)$

$\therefore T(n) = O(n^c) = O(n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, b = 3, f(n) = n^2 \log n.$

$c = \log_3 6 = 1.6$

$n^c = n^{1.6}$

$f(n) > n^c$

$\therefore T(n) = O(f(n))$

$= O(n^2 \log n)$

19. $T(n) = 4T(n/2) + n/\log n.$

$a = 4, b = 2, f(n) = n/\log n$

$c = \log_2 4 = 2$

$n^c = n^2.$

$n^c > f(n)$

$\therefore T(n) = O(n^c) = O(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n.$

$a = 64, b = 8, f(n) = -n^2 \log n.$

$c = \log_8 64 = 2$

$n^c = n^2$

21. $T(n) = 7(T(n/3)) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_3 7 = 1.77$

$n^c = n^{\log_3 7} = n^{1.77}$

$f(n) > n^c$

$\therefore T(n) = O(f(n))$

$= O(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2, f(n) = (2 - \cos n)n.$

$c = \log_2 1 = 0$

$n^c = n^0 = 1.$

$f(n) > n^c$

$\therefore T(n) = O(f(n)) = O((2 - \cos n)n).$