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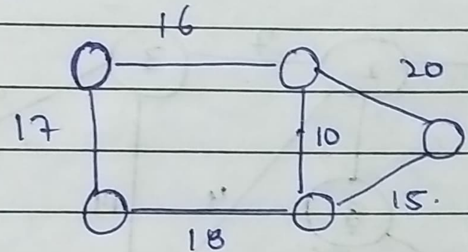
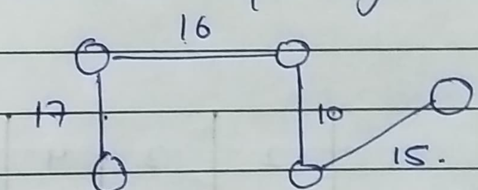
Q1 Minimum Spanning Tree

- A Spanning Tree of a undirected graph is a subgraph that is a tree and joined all vertices. One of those tree which has minimum total cost would be its minimum Spanning Tree.

for eg.



for the above connected, undirected graph. minimum cost spanning Tree would be.



Applications of MST.

Minimum spanning Trees have direct applications is the design of networks including computer networks, telecommunication networks, transportation networks etc.

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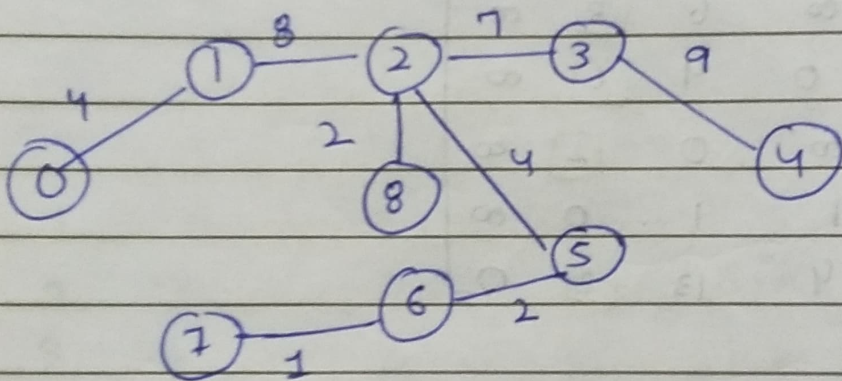
Q2

	Prim's Algorithm	Kruskal's Algorithm	Dijkstra's Algorithm	Bellman Ford Algo.
Time Complexity	$O(V^2)$	$O(E \log V)$	$O(V+E \log V)$	$O(VE)$
Space Complexity	$O(V+E)$	$O(\|E\|+\|V\|)$	$O(V^2)$	$O(V^2)$

Q3 Prim's Algo.

0	1	2	3	4	5	6	7	8
$\boxed{0}$	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~	~~∞~~
	$\boxed{4}$	$\boxed{8}$	$\boxed{7}$		$\boxed{4}$	~~6~~	~~8~~	$\boxed{2}$
				~~10~~		$\boxed{2}$	~~7~~	
				$\boxed{9}$			$\boxed{1}$	



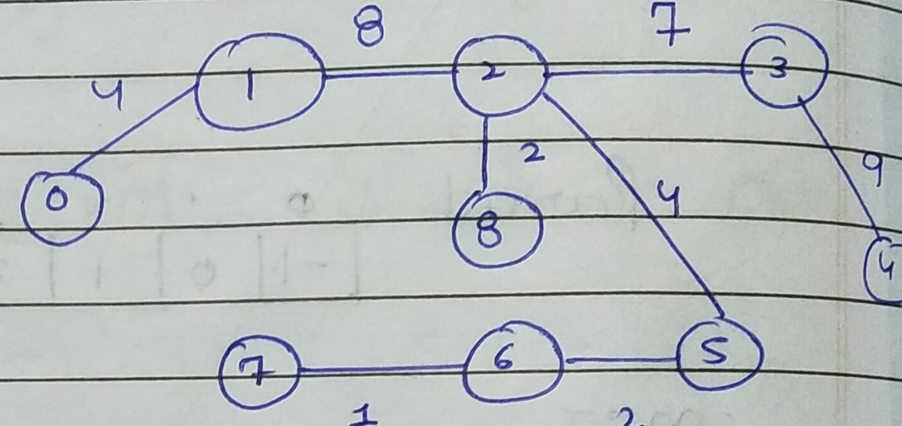
min weight = 37

Parent	0	1	2	3	~~4~~	5	6	~~7~~	8
	-1	~~-1~~	~~-1~~	~~-1~~	-1	~~-1~~	~~-1~~	~~-1~~	~~-1~~
		0	1	2		2		~~0~~	2
							~~8~~	~~8~~	
					~~7~~		5	6	
					3				

Parent	0	1	2	3	4	5	6	7	8
	-1	0	1	2	3	2	5	6	2

Q3 Krushkal's Algo.

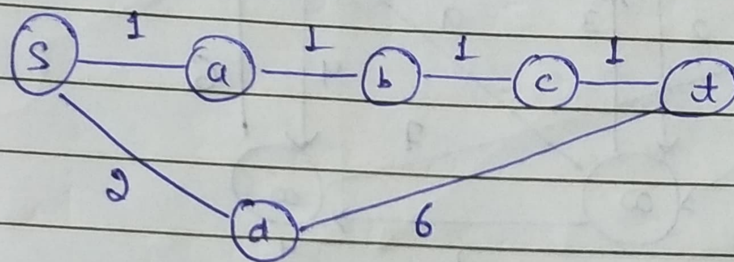
U	V	W	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	✗
7	8	7	✗
2	3	7	✓
1	2	8	✓
0	7	8	✗
3	4	9	✓
5	4	10	✗
1	7	11	✗
3	5	14	✗



Weight = 37

Q4 (i) If 10 units is added to each edge, the overall weight of the paths may ~~may~~ change.

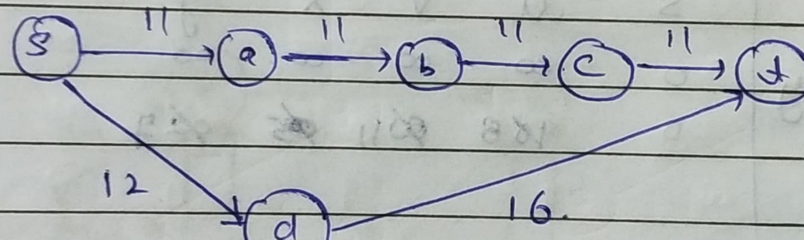
for eg.



Shortest path is ~~a~~ $S \rightarrow a \rightarrow b \rightarrow c \rightarrow t$

weight $1+1+1+1 = 4$.

now if 10 unit weight is added to each edge.

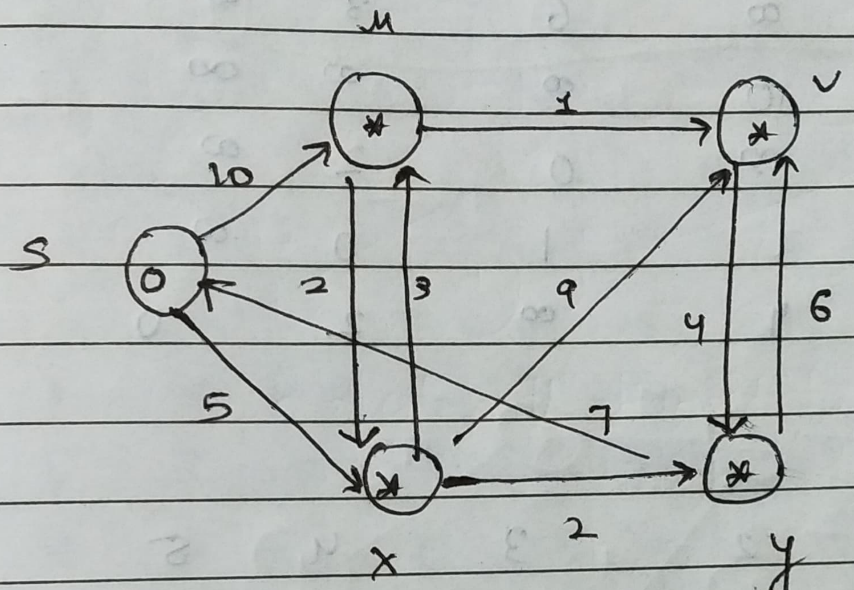


Shortest path changed to $S \rightarrow d \rightarrow t$

weight 28

(ii) Multiplying the weight of each edge by 10 will have no impact on the shortest path.

Q5. dijkstra's Algo.



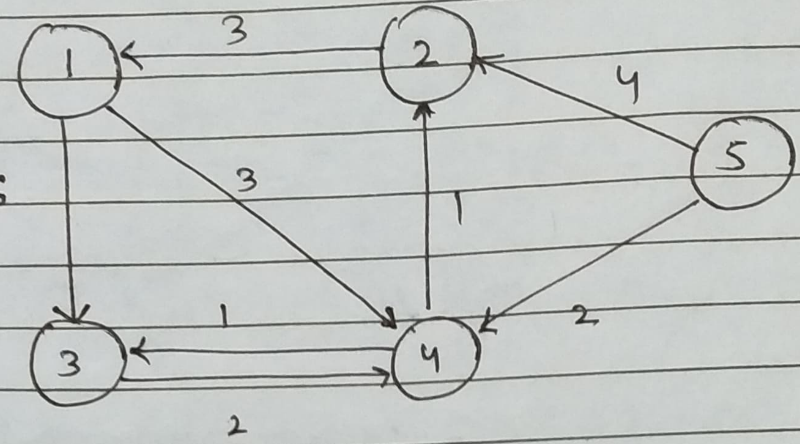
Queue : ~~s~~ ~~u~~ ~~x~~ ~~v~~ ~~y~~

visited : s u v x y.

✓. ✓ ✓ ✓ ✓

S	u	v	x	y
0	~~∞~~	∞	~~∞~~	∞.
0	10	~~∞~~	5	∞
0	10	11	5	~~∞~~
0	10	11	5	7

Q6 All pair shortest path algorithm. - Floyd Warshall.



$A^0 =$

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	∞	∞	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$A^1 =$

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$A^0[2,3] = \infty$

$A^0[2,1] + A^0[1,3] = 3 + 6 = 9$

$9 < \infty$

Similarly, $A^0[2,4] = \infty$

$A^0[2,1] + A^0(1,4) = 3 + 3 = 6$

$6 < \infty$

$A^0[2,5] = \infty$

$A^0(2,1) + A^0[1,5] = 3 + \infty$

$$A^2 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & \boxed{6} & 3 & \infty \\ 3 & 0 & \boxed{9} & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & \boxed{1} & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$A^1[1,3] = 6$

$A^1[1,2] + A^1[2+3] = \infty + 9.$

$6 < \infty + 9.$

$$A^3 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & \boxed{3} & \infty \\ 3 & 0 & 9 & \boxed{6} & \infty \\ \infty & \infty & 0 & \boxed{2} & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & \boxed{2} & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1\ \emptyset \\ 2\ \cancel{1} \\ 3\ 2 \\ 4\ 3 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 8 & \boxed{\infty} \\ 3 & 0 & 7 & 6 & \infty. \\ \infty & 3 & 0 & 2 & \infty. \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^5 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$