12-1

$$L = \begin{bmatrix} -\frac{\lambda}{z_1} & 0 & \frac{u_1}{z_1} & \frac{u_1 v_1}{\lambda} & -\frac{\lambda^2 + u_1^2}{\lambda} & v_1 \\ 0 & -\frac{\lambda}{z_1} & \frac{v_1}{z_1} & \frac{\lambda^2 + v_1^2}{\lambda} & -\frac{u_1 v_1}{\lambda} & -u_1 \\ -\frac{\lambda}{z_2} & 0 & \frac{u_2}{z_2} & \frac{u_2 v_2}{\lambda} & -\frac{\lambda^2 + u_2^2}{\lambda} & v_2 \\ 0 & -\frac{\lambda}{z_2} & \frac{v_2}{z_2} & \frac{\lambda^2 + v_2^2}{\lambda} & -\frac{u_2 v_2}{\lambda} & -u_2 \end{bmatrix}$$

12-2 If the four rows of L are linearly independent, then L is rank 4, and its null space has rank 2. In general, the rank of the null space of L in problem 12-1 is given by $6 - \text{rank } L$.

12-3 For this problem, we shall let the coordinate frame of the left camera be the reference frame. Thus, the coordinates of the fixed point (x, y, z) are expressed relative to the (moving) left camera frame. It is common in stereo vision systems to choose a configuraion in which the position and orientation of the right camera w.r.t. the left camera is given by

$$H_r^l = \begin{bmatrix} 1 & 0 & 0 & B \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

in which B is called the *baseline distance* (see problem 11-3).

Suppose the left camera is moving with velocity $\xi_l = (v_l, \omega_l)$. The interaction matrix for u_l, v_l is just the usual interaction matrix for a point.

As we have seen in Chapter 4, if the two cameras are rigidly attached (i.e., their coordinate frames are rigidly attached), their angular velocities are the same, i.e., $\omega_l = \omega_r$. The velocity of the origin of the right camera frame is given by $v_r = \omega \times [B, 0, 0]^T + v_l$ which gives

$$\begin{bmatrix} v_{rx} \\ v_{ry} \\ v_{rz} \end{bmatrix} = \begin{bmatrix} 0 \\ B\omega_z \\ -B\omega_y \end{bmatrix} + \begin{bmatrix} v_{lx} \\ v_{ly} \\ v_{lz} \end{bmatrix}$$

The coordinates of the fixed point w.r.t. the coordinate frame of the right camera are given by $(x - B, y, z)$, since the left and right frames are related by a pure translation along the x-axis. The velocity of the fixed point relative to the moving right camera frame is therefore given by

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} &= -\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \times \begin{bmatrix} x - B \\ y \\ z \end{bmatrix} - \begin{bmatrix} v_{rx} \\ v_{ry} \\ v_{rz} \end{bmatrix} \\ &= \begin{bmatrix} y\omega_z - z\omega_y - v_{lx} \\ -(x - B)\omega_z + z\omega_x - v_{ly} - B\omega_z \\ (x - B)\omega_y - y\omega_x - v_{lz} + B\omega_y \end{bmatrix} \end{aligned}$$

which can be written as the system of three equations

$$\begin{aligned} \dot{x} &= y\omega_z - z\omega_y - v_{lx} \\ \dot{y} &= -x\omega_z + z\omega_x - v_{ly} \\ \dot{z} &= x\omega_y - y\omega_x - v_{lz} \end{aligned}$$

There is no published solution for Problem 12.4.

12-5 Suppose the end effector frame is moving with velocity $\xi = (v, \omega)$. Then, the origin of the frame has velocity v, since angular velocity of the end-effector frame does not induce motion of the frame's orign. Therefore, we have

$$\begin{aligned} \dot{x} &= v_x \\ \dot{y} &= v_y \\ \dot{z} &= v_z \end{aligned}$$

We use the quotient rule for differentiation with the equations of perspective projection to obtain

$$\dot{u} = \frac{d}{dt}\frac{\lambda x}{z} = \lambda \frac{z\dot{x} - x\dot{z}}{z^2}$$

and

$$\dot{v} = \frac{d}{dt}\frac{\lambda y}{z} = \lambda \frac{z\dot{y} - y\dot{z}}{z^2}$$

By solving the perspective projection equations for x and y we obtain

$$x = \frac{uz}{\lambda}, \qquad y = \frac{vz}{\lambda}$$

and substituting thees results into the above derivatives we obtain

$$\begin{aligned} \dot{u} &= \lambda \frac{z\dot{x} - x\dot{z}}{z^2} = \frac{\lambda}{z^2}(zv_x - \frac{uz}{\lambda}v_z) = \frac{\lambda}{z}(v_x - \frac{u}{\lambda}v_z) \\ \dot{v} &= \frac{\lambda}{z}(v_y - \frac{u}{\lambda}v_z) \end{aligned}$$

Writing this in matrix form gives

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \frac{\lambda}{z} & 0 & -\frac{u}{z} & 0 & 0 & 0 \\ 0 & \frac{\lambda}{z} & -\frac{v}{z} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

12-6 This problem is somewhat open ended. Derivations and discussion of this issue can be found in the following reference.

- F. Chaumette, "Image moments: a general and useful set of features for visual servoing," IEEE Trans. on Robotics, 20(4):713-723, August 2004.
- O. Tahri, F. Chaumette, "Point-based and region-based image moments for visual servoing of planar objects," IEEE Trans. on Robotics, 21(6), December 2005.

12-7 through 12-11 These problems are all simulation problems.

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