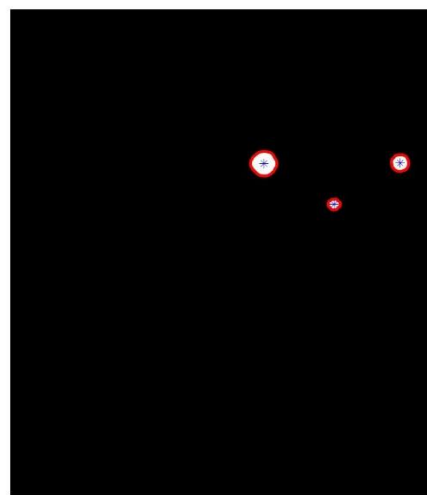
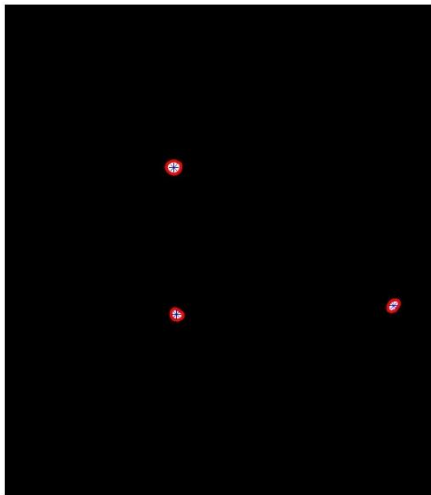


LabReport8_JiaweiGe

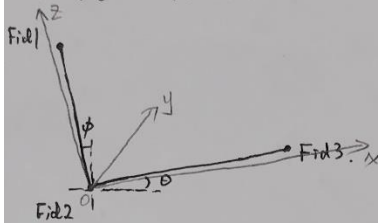
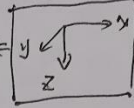
The dimensions of each red slice equal **215.6 mm x 247.92 mm**. The length **215.6 mm** is yield by **154 slices x 1.4 mm**. The width **247.92 mm** is yield by **240 pixels x 1.033 mm**. The distance between fiducials' plane and targets' plane equals **54.899 mm**, which is yield by adding up the depth information in the upper-right box in the screenshots (**33.629 mm + 21.270 mm**).

The fiducials' centers are **[619.2565, 596.6149]**, **[629.9568, 1.1356e+03]**, **[1.4237e+03, 1.1028e+03]** in the Matlab results. Because the bw size is **1799 x 1569**. Using the proportion of **240/1799**, we can get the y value of fiducials' centers, which are **79.59, 151.50, 147.12 = 80th, 152th, 147th pixel**, respectively. Using the proportion of **154/1569**, we can get the slice number of fiducials' centers, which are **60.78, 61.83, 139.74 = 61, 62, 140**, respectively. The true length and width have a **215.6/1569** and **247.92/1799** proportion, respectively. The true positions of the fiducials' centers in red slices, therefore, are **(85.09, 82.22)**, **(86.56, 156.50)**, **(195.63, 151.98)** mm.

The targets' centers are **[934.5005, 567.5116]**, **[1.1933e+03, 717.6362]**, **[1.4366e+03, 565.3621]** in the Matlab results. Because the bw size is **1791 x 1567**. Using the proportion of **240/1791**, we can get the y values of targets' centers are **78.05, 96.17, 75.76 = 78th, 96th, 76th pixel**, respectively. Using the proportion of **154/1567**, we can get the slice number of fiducials' centers, which are **91.84, 117.23, 141.18 = 92, 117, 141**, respectively. The true length and width have a **215.6/1567** and **247.92/1791** proportion, respectively. The true positions of the targets' centers in red slices, therefore, are **(128.58, 78.56)**, **(164.18, 99.34)**, **(197.66, 78.26)** mm.



Fid1 (85.09, 82.22), Fid2 (86.56, 156.50), Fid3 (195.63, 151.98).
 Tar1 (128.58, 78.56), Tar2 (164.18, 99.34), Tar3 (197.66, 78.26).



$$\tan \theta = \frac{156.50 - 151.98}{195.63 - 86.56} \approx 0.04$$

$$\tan \phi = \frac{86.56 - 85.09}{156.50 - 82.22} \approx 0.02$$

Theoretically, $\tan \theta$ should equals $\tan \phi$.

Assume Fid2 to be the origin of Fid frame, and set tilting angle ψ satisfy $\tan \psi = \frac{0.04 + 0.02}{2} = 0.03$

Then the rotation matrix is $\begin{bmatrix} c\psi & s\psi \\ -s\psi & c\psi \end{bmatrix} \approx \begin{bmatrix} 1 & 0.03 \\ -0.03 & 1 \end{bmatrix}$

$$\begin{aligned} \overrightarrow{\text{Fid2} \cdot \text{Fid1}} &= \begin{pmatrix} -1.4 \\ 0 \\ 74.28 \end{pmatrix} \\ \overrightarrow{\text{Fid2} \cdot \text{Fid3}} &= \begin{pmatrix} 109.07 \\ 0 \\ 45.2 \end{pmatrix} \end{aligned} \Rightarrow \text{multiply with } R \Rightarrow \begin{aligned} \text{Fid1}^* &= \begin{pmatrix} 0.76 \\ 0 \\ 74.31 \end{pmatrix} \\ \text{Fid3}^* &= \begin{pmatrix} 109.21 \\ 0 \\ 1.25 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \overrightarrow{\text{Fid2} \cdot \text{Tar1}} &= \begin{pmatrix} 42.02 \\ 54.90 \\ 77.94 \end{pmatrix} \\ \overrightarrow{\text{Fid2} \cdot \text{Tar2}} &= \begin{pmatrix} 77.62 \\ 54.90 \\ 57.16 \end{pmatrix} \\ \overrightarrow{\text{Fid2} \cdot \text{Tar3}} &= \begin{pmatrix} 111.10 \\ 54.90 \\ 78.24 \end{pmatrix} \end{aligned} \xrightarrow{\text{multiply by } R} \begin{aligned} \text{Tar1}^* &= \begin{pmatrix} 44.36 \\ 54.90 \\ 76.68 \end{pmatrix} \\ \text{Tar2}^* &= \begin{pmatrix} 79.33 \\ 54.90 \\ 54.83 \end{pmatrix} \\ \text{Tar3}^* &= \begin{pmatrix} 113.45 \\ 54.90 \\ 74.91 \end{pmatrix} \end{aligned}$$

Look up to the reference frame, both x and y values are accurate.
 all z values are 3 mm less than the reference.

The center position of Fid1 equals **[3.9548e+02, 3.8029e+02]**, the depth is **-20.271mm**, and the bw size is **1154 x 997**. The center position of Fid2 equals **[4.0654e+02, 7.2571e+02]**, the depth is **-23.371mm**, and the bw size is **1153 x 1001**. The center position of Fid3 equals **[9.1061e+02, 7.0301e+02]**, the depth is **-18.204mm**, and the bw size is **1151 x 997**. Using the same methods as mentioned above (**215.6 / bw(2)**) * **Fid1(1) = x_fid; depth = y_fid; -247.92 / bw(1)** * **Fid1(2) = z_fid**, we can easily yield that the fiducials' centers locate at **(85.52, -20.27, 81.70), (87.56, -23.37, 156.04), (196.92, -18.20, 151.43) mm**.

Fid1 (85.52, -20.27, 81.70), Fid2 (87.56, -23.37, 156.04), Fid3 (196.92, -18.20, 151.43)
 Tar1 (128.58, 33.63, 78.56), Tar2 (164.18, 33.63, 99.34), Tar3 (197.66, 33.63, 78.26)

$$\tan \phi_1 = \frac{156.04 - 151.43}{196.92 - 87.56} \approx 0.042; \tan \phi_2 = \frac{87.56 - 85.52}{156.04 - 81.70} \approx 0.027;$$

$$\tan \theta_1 = \frac{23.37 - 18.20}{196.92 - 87.56} \approx 0.047; \tan \theta_2 = \frac{87.56 - 85.52}{23.37 - 20.27} \approx 0.658;$$

$$\tan \psi_1 = \frac{156.04 - 151.43}{23.37 - 18.20} \approx 0.892; \tan \psi_2 = \frac{23.37 - 20.27}{156.04 - 81.70} \approx 0.042.$$

$$\Rightarrow \tan \phi = \frac{0.042 + 0.027}{2} \approx 0.035; \tan \theta = 0.047; \tan \psi = 0.042.$$

$$\Rightarrow R_{y,\phi} \approx \begin{bmatrix} 1 & 0.035 \\ 0.035 & 1 \end{bmatrix}, R_{z,\theta} \approx \begin{bmatrix} 0.999 & -0.047 \\ 0.047 & 0.999 \end{bmatrix}, R_{x,\psi} = \begin{bmatrix} 1 & 0.042 \\ 0.042 & 0.999 \end{bmatrix}.$$

$$\Rightarrow R = R_{x,\psi} \cdot R_{z,\theta} \cdot R_{y,\phi} \approx \begin{bmatrix} 0.999 & 0.048 & 0.033 \\ -0.047 & 0.998 & -0.092 \\ -0.035 & 0.040 & 0.999 \end{bmatrix}$$

Assume Fid2 is the origin of the fiducial frame.

$$\begin{aligned} \overrightarrow{\text{Fid2} \cdot \text{Fid1}} &= \begin{pmatrix} -2.04 \\ 3.10 \\ 74.34 \end{pmatrix} \\ \overrightarrow{\text{Fid2} \cdot \text{Fid3}} &= \begin{pmatrix} 109.36 \\ 5.17 \\ 4.61 \end{pmatrix} \end{aligned} \xrightarrow{\text{multiply by } R} \begin{cases} \text{Fid1}^* = \begin{pmatrix} 0.565 \\ 0.071 \\ 74.467 \end{pmatrix} \\ \text{Fid3}^* = \begin{pmatrix} 109.653 \\ -0.174 \\ 0.991 \end{pmatrix} \end{cases}$$

$$\begin{aligned} \overrightarrow{\text{Fid2} \cdot \text{Tar1}} &= \begin{pmatrix} 41.02 \\ 57 \\ 77.48 \end{pmatrix} \\ \overrightarrow{\text{Fid2} \cdot \text{Tar2}} &= \begin{pmatrix} 76.62 \\ 57 \\ 56.7 \end{pmatrix} \\ \overrightarrow{\text{Fid2} \cdot \text{Tar3}} &= \begin{pmatrix} 110.1 \\ 57 \\ 77.78 \end{pmatrix} \end{aligned} \xrightarrow{\text{multiply by } R} \begin{cases} \text{Tar1}^* = \begin{pmatrix} 46.295 \\ 51.707 \\ 78.274 \end{pmatrix} \\ \text{Tar2}^* = \begin{pmatrix} 81.174 \\ 50.906 \\ 56.269 \end{pmatrix} \\ \text{Tar3}^* = \begin{pmatrix} 115.316 \\ 48.448 \\ 76.158 \end{pmatrix} \end{cases}$$