9-1 The two-link RR Jacobian matrix is given by

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} \end{bmatrix}$$

The joint torque necessary to balance an end-effector force $F = (-1, -1)^T$ is given by

$$\begin{aligned} \tau &= J^T F \\ &= \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & a_1 c_1 + a_2 c_{12} \\ -a_2 s_{12} & a_2 c_{12} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} a_1(s_1 - c_1) + a_2(s_{12} - c_{12}) \\ a_2(s_{12} - c_{12}) \end{bmatrix} \end{aligned}$$

9-2 The solution proceeds as in Problem 9-1. The torque required to balance an end-effector force F is given by $\tau = J^T F$, where now J represents the Jacobian of the robot with remote drive. The $x - y$ coordinates of the end-effector in terms of the absolute angles θ_1 and θ_2 are easily seen to be

$$\begin{aligned} x &= a_1 c_1 + a_2 c_2 \\ y &= a_1 s_1 + a_2 s_2 \end{aligned}$$

The velocities therefore are

$$\begin{aligned} \dot{x} &= -a_1 s_1 \dot{\theta}_1 - a_2 s_2 \dot{\theta}_2 \\ \dot{y} &= a_1 c_1 \dot{\theta}_1 + a_2 c_2 \dot{\theta}_2 \end{aligned}$$

Thus, the Jacobian is given by

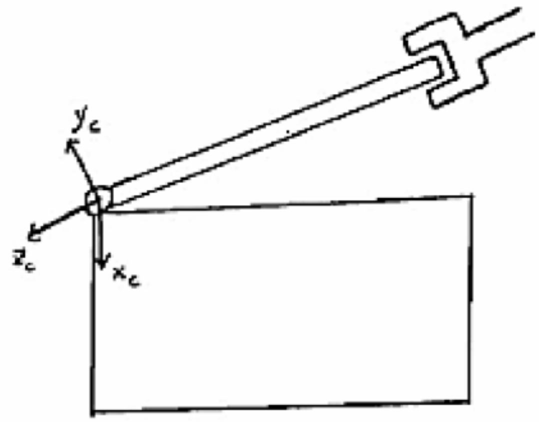
$$J = \begin{bmatrix} -a_1 s_1 & -a_2 s_2 \\ a_1 c_1 & a_2 c_2 \end{bmatrix}$$

9-3 Square peg in square hole

Natural constraints	Artificial constraints
$v_x = 0$	$f_x = 0$
$v_y = 0$	$f_y = 0$
$f_z = 0$	$v_z = v_d$
$w_x = 0$	$\tau_x = 0$
$w_y = 0$	$\tau_y = 0$
$w_z = 0$	$\tau_z = 0$

9-4 Opening a box with a hinged lid

Natural constraints	Artificial constraints
$v_x = 0$	$w_x = w_d$
$v_y = 0$	$\tau_x = I\dot{w}_d$
$v_z = 0$	$f_x = 0$
$w_y = 0$	$f_z = 0$
$w_z = 0$	$\tau_y = 0$
$f_y = 0$	$\tau_z = 0$



9-5 This is somewhat open ended and is a good question for classroom discussion. The notions of wedging and jamming are important to consider for this problem. The two manipulators should produce a straight-line motion and avoid rotating the drawer. Natural and artificial constraints are similar to the peg-in-hole problem.

9-6 Each task should be decomposed into single-DOF directions according to the compliance frame. In each direction the environment can be classified according to whether or not a significant inertia is to be moved, or significant compliance exits, and so on.

1. Turning a crank can be considered as inertial tangent to the circle defined by the crank rotation and capacitive along the crank direction.
2. Inserting a peg in a hole can be considered capacitive in directions with position constraints and inertial in other directions.
3. Polishing the hood of a car can be considered capacitive normal to the hood and inertial tangent to the hood.
4. Cutting cloth can be considered resistive in the cutting direction.
5. Shearing a sheep can be considered capacitive normal to the sheep and resistive in the shearing (tangent) direction.
6. Placing stamps on envelopes can be considered capacitive normal to the envelope.
7. Cutting meat can be considered resistive or capacitive in the cutting direction.