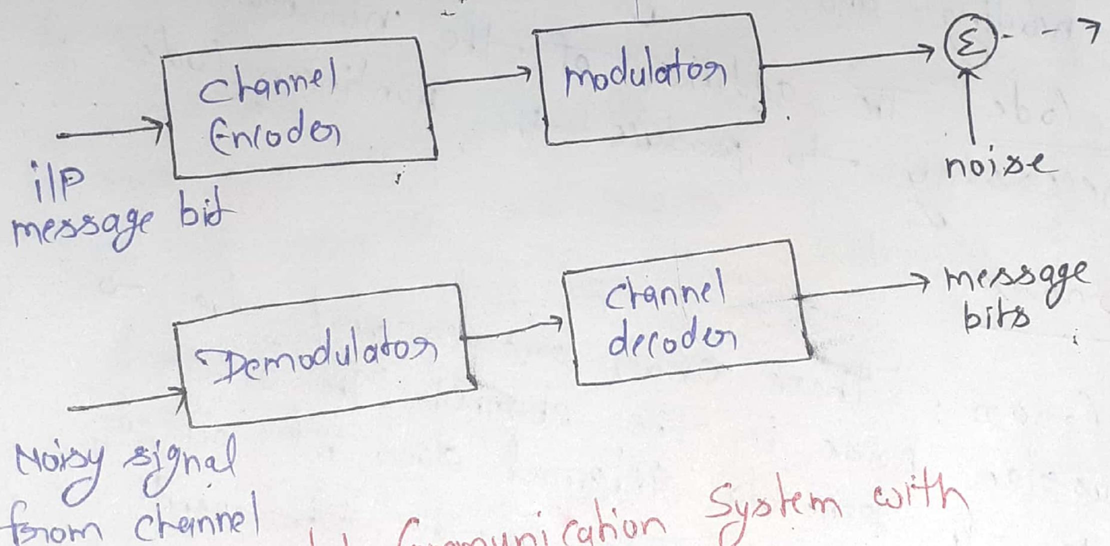
POORNIMA COLLEGE OF ENGINEERING

DETAILED LECTURE NOTES

Unit-3

Error Correcting Codes:

message that are transmitted from any communication channel can be damaged. There bits can be changed or masked. To achieve the error free communication error correcting codes are generated.



Digital Communication System with Channel Encoding

The channel Encoder add extra bit to the message bit. The encoded channel is transmitted over the noisy channel. The channel decoder identifies the redundant bits and use them to detects and correct the error.

Types of ~~Error~~ Code

1. **Block Code** :— These Codes consist n number of bits in one block or codeword. And this block consists of k message bit and (n-k) redundent bit, these are called (n,k) block codes.

ii. **Convolutional Code** The coding technig is discreate time convolution of Input sequenly with the impulse response of the encoder. The encoder accepts the message bit continuosly and generated the encoded sequence continuosly.

3. **Linear Code**: A linear code is error correcting code for which any linear combination of codeword is another codeword of the code. It allows more efficient encoding and decoding algorithm.

4. **Nonlinear Code** The addition of the nonlinear code is not necessary to produce the nonlinear code.

Type of Errors

i) Random Error : These error are created due to white Gaussian noise in the channel. The error generated in the particular interval does not affect the performance of the system in the subsequent interval.

ii) Burst Error: These errors are generated due to impulsive noise in the channel. It generated due to lightning and switching transients noise. It affects several successive symbols representation

Method of controlling Error

1. Forward acting Error Correction

In this method the errors are detected and corrected by proper coding techniques at the receiver side. The check bits and redundant bits has been used for checking the error. It is faster but the probablity of error is very high.

2. Error detection with retransmission

In this method the decoder check the input sequence, when it detects any error, it discard that part of sequence and request the transmitter for retransmission. It has lower probablity of error but it is slow.

~~Types of Errors~~: Terms used in Error Correcting Code

Two main type of error is generated in the transmission.

Random Error

1. Code Word: The encoded block of n bit is called a code word. It contains message bits and redundant bit.

2. Block length: The number of bit n after coding is called the block length of the code.

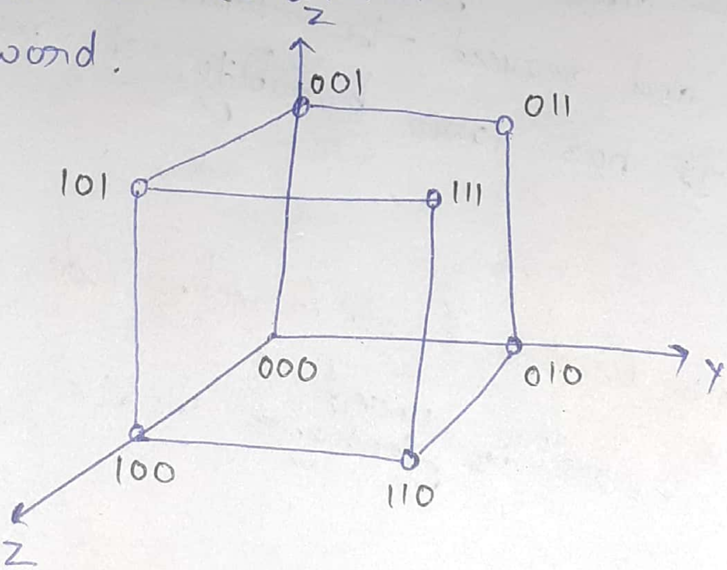
3. Code Rate: The ratio of message bits (k) and the encoder output bit (n) is called code rate. It is defined by r

$$\boxed{r = k/n}$$

4. Channel Data Rate: It is the bit rate at the o/p of the encoder. If the bit Rate at the input of encoder is Rs, than channel data Rate will be

$$\boxed{R_o = (n/k) R_s}$$

5. Code Vector: An n bit code word can be visualized in an n-dimensional space as a vector whose elements or co-ordinates bits are defined in the code word.



6. Hamming Distance: The hamming distence between Two code vectors is equal to no of element in which they differ for example x (101) y (110) the hamming distence $d(x, y) = 2$

7. Minimum Distance (d_{min}): It is the smallest hamming distance the valid code vectors.

S.No.	Name of Error	Distance Requirement
1.	Detect upto 's' error	$d_{min} \geq s+1$
2.	Correct upto 't' error	$d_{min} \geq 2t+1$
3.	Correct upto 't' error and detect $s > t$ error per word	$d_{min} \geq t+s+1$

8. Code Efficiency: The code efficiency is the ratio of message bits in a block to the transmitted bit for that block by the encoder

$$\boxed{\text{code efficiency} = \frac{\text{message bit in a block}}{\text{Transmitted bit for the block}}}$$

9. Weight of the Code:

The number of non zero element in the transmitted code vector is called vector weight. It is denoted by $w(x)$; for example if $x = 01110101$ then weight of this code will be

$w(x) = 5$.

ERROR Detection:

It is realized using checksum Algorithm. A hash function add a fixed length tag to a message., which enable receiver to verify the delivered message.

→ Repetition Code: Each block is transmitted by predetermined times. to send the 1011, it transmitted by 3 times. The received bit was

1011 1010 1011, where the 2nd block is unlike to other. It can be determined that an error has occurred.

Parity bits → A parity bit is a bit that is added to a group of source bit to ensure that the no of set bits in the outcome is even or odd.

It is simple scheme to detect the error.

Error Correction:

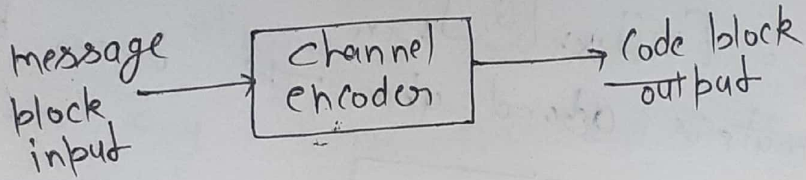
Automatic Repeat Request (ARQ) is an error control method for data transmission. that make use for error correction.

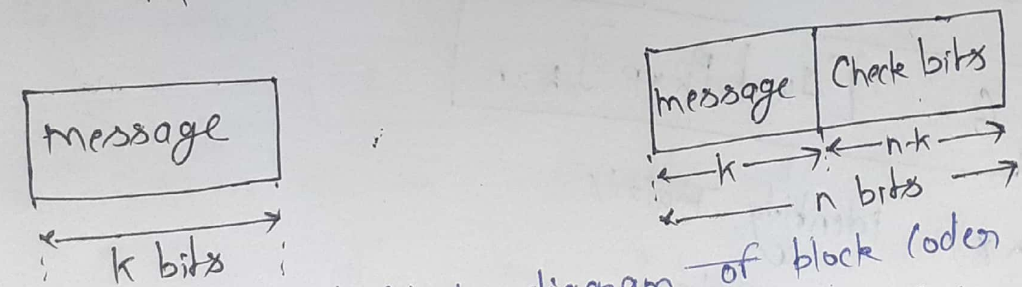
ARQ is appropriate if the communication channel has varying and unknown capacity.

Forward Error Correction (FEC) It the error control technique for data transmission. sender adds systematically generated redundant data to the message. The carefully designed redundancy allows the receiver to detect and correct the limited number of errors.

Linear Block Code :

Principal of Block Coding for the block of k message bits (n-k) parity bits or check bits are added. Hence the total bit at the o/p of the channel encoder are 'n'. these codes are called (n,k) block codes.





functional block diagram of block coder

In systematic codeword the message bit appear at the begning of the code word.

Linear Code is known as linear beacuse the sum of any two code vector produce another code vector. The code word consist with $m_1, m_2, m_3 \ldots m_k$ message bit and $c_1, c_2 \ldots c_k$ check bit.

Then this code vector can be written as

$$X = (m_1, m_2 \cdots m_k \; c_1, c_2 \cdots c_q)$$

$$\boxed{q = n-k}$$

q are the number of redundant bit add by the encoder. The above code word can be written as

fi

$$\boxed{X = (m \mid c)}$$

m = k bit message Vector
c = q. bit check Vector

Matrix Description of Linear Block Code.

$$X = mG$$

X = Code Vector of $1 \times n$ size or n bits

m = message Vector of $1 \times k$ size or k bits

G = Generator matrix of $k \times n$ size.

$$[X]_{1 \times n} = [m]_{1 \times k} [G]_{k \times n}$$

(The generator matrix depend upon the linear block used

$$\boxed{G = [I_k \mid P_{k \times p}]_{k \times n}}$$

I_k = $k \times k$ identity matrix

P = $k \times q$ Submatrix

$$\boxed{C = mP}$$

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Linear Block Code:

— A block code is said to be linear code if its codeword satisfy this condition that the sum of two codeword given another codeword

$$C_p = C_i + C_k$$

Property: (i) The all zero word [0 0 ... 0] is always a codeword

(ii) Given any three codeword C_i, C_j and C_k such that $\boxed{C_p = C_i + C_k}$ then $d(c_i, c_j) = w(c_p)$

(iii) minimum distance of the code

$$\boxed{d_{min} = \omega_{min}}$$

Example

(7,4) hamming code

$C_1 = 0001011$
$C_{10} = 1010011$

$C_{11} = C_1 + C_{10}$

C_{11} 1011000

$d(c_1, c_{10}) = 3$

$\omega(c_{11}) = 3$

So thus condition satisfy

$$\boxed{d(c_1, c_{10}) = \omega(c_{11})}$$

While for transmission of

$C_{15} = [1\,1\,1\,1\,1\,1\,1]$ $\omega = 7$

other than C_{15} codes are having weight 3.

$$\boxed{d_{min} = \omega_{min}}$$

It analyze the three property of the code word

- → Code word length
- → Total number of valid codeword
- → The minimum distance between two code words, using mainly the hamming distance and covered the entire codeword.

It is known as error correcting code for any linear combination of the pair circuit.

It is the example of (7,4) matrix terminology.

Parity Check matrix:

H is a matrix associated with each (n,k) block code.

$H = [P^T : I_{n-k}]_{(n-k \times n)}$ $G = [I : P]$

$H = [I : P^T] \Rightarrow G [P : I]$

If c is the codeword in (n,k) block code generated by $G = [I_k : P]$ if and only if

$$\boxed{CH^T = 0}$$

H^T is transpose of parity check matrix H

Considering a (n,k) block code with generator matrix G and a parity matrix H. Let c is the code that transmitted over a noisy channel and R is the noise corrupted vector.

$$R = C + E$$

$$S = RH^T$$

$$S = (C+E)H^T$$

$$S = CH^T + EH^T$$

$CH^T = 0$

$$\boxed{S = EH^T}$$

Syndrome of Received vector is zero if R is the valid code word. If error occurs than S of Received H^T word is non zero.

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

find out the error vector?

Ans – step 1 $\quad H = [P^T / I_{n-k}]_{n-k \times n}$

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$\underbrace{\qquad\qquad}_{P^T} \quad \underbrace{\qquad}_{I_3}$

step 2 $\quad k = 4$

$n = 7$

$2^k = 2^4$ code words for 2^4 message (0000 — 1111)

step 3 from any value of D for Ex: 1011

$C = DG$

$$= [1011] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$= 1011001$

step 4 Calculate Syndrome.

$$S = CH^T = 1011001 \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}^T$$

$$= [1011001] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Step 5 If $R = 1001001$ is given find

$S = RH^T$

$$(1001001)\begin{bmatrix}1&1&1\\1&1&0\\1&0&1\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$= 101$

Step 6

This value compare with H^T. now 101 is equal to third Row of H^T, so third bit is in error so the transmitted word.

$C = 1011001$

Error Vector $E =$ ~~$R+C$~~

$$\boxed{E = 0010000}$$

Q2 A parity check code has parity check matrix as show

$$H = \begin{bmatrix}1&0&1&1&1&0&0\\1&1&0&1&0&1&0\\0&1&1&1&0&0&1\end{bmatrix}$$

(a) Determine Generator matrix

$H = [P^T \mid I_{n-k}]$

$\therefore P^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$G = [I_k \,\vdots\, P]$

$k = 3$

$G = [I_3 \,\vdots\, P]$

$= \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 1 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 0 & 1 \end{bmatrix}$

(b) find error when Received word is 110110
Decode this Received word

$S = RH^T$

$R = 110110$

H is given

$H^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$S = \therefore RH^T$

$S = [1\,1\,0\,1\,1\,0] \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= [0\,1\,1]$

s is in second row of H^T error in second bit

$C = 100110$

Error Detection and Correction Capabilities of Linear Block Code.

Theoram 1 The minimum distance of linear block code is equal to minimum weight of any non zero word in code.

Proof
$$d(c_i, c_j) = w(c_i \oplus c_j)$$

$$d_{min} = \min d(c_i, c_j) = \min w(c_i \oplus c_j)$$

$c_i \neq c_j$
$c_j \neq c_i$

$$\boxed{d_{min} = \min w(c)}$$

$c \neq 0$

Theoram 2

A linear block code with a minimum distance d_{min} can correct upto $(d_{min}-1)/2$ errors and detect upto $(d_{min}-1)$ errors.

Proof Let R be the Received codeword and c is the transmitted codeword. c' is any other codeword. The H/w distance between two codeword can be defined by $d(c, c')$

Satisfies

$$d(c,R) + d(c',R) \geq d(c,c') \quad \cdots \text{(i)}$$

so $d(u,v)$, weight of $u \oplus v$, this addition is on bit by bit basis in modulo 2 arithmetic.

If an error pattern t' occur, than hamming distance c and R is $d(c,R) = t'$ . . . (ii)

Now distance of two codeword is either d_{min} or greater than d_{min}

$$\therefore \quad d(c,c') \geq d_{min} \quad \cdots \text{(iii)}$$

by equation (1), (2) and (3)

$$t' + d(c',R) \geq d_{min}$$

$$d(c',R) \geq d_{min} - t' \quad \cdots \text{(iv)}$$

The decoder will identify c as transmitted vector $d(c,R) < d(c',R)$

$$t' < d_{min} - t'$$

$$t' < \frac{d_{min}}{2}$$

L.B.C can detect $d_{min} - 1$ error.

$\therefore$ Number of error corrected.

$$\boxed{\frac{d_{min} - 1}{2}}$$

Example

for a (6,3) code The generator matrix G is

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

find (i) All corresponding code vector
(ii) Minimum hamming distance (d_{min})
(iii) Error detection and correction capablity
(iv) Parity check matrix
(v) find error if received code is (100011)

Answer

$$G = [I_k : P]$$

$$I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$[C] = [m : P_c]$$

$$[P_c] = [i_m][P]$$

$$[P_c] =$$

$$[P_0 \quad P_1 \quad P_2] = [i_0 \quad i_1 \quad i_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$P_0 = i_0 \oplus i_2 \oplus$

$P_1 = i_1 \oplus i_2$

$P_2 = i_0 \oplus i_1$

C	i_0	i_1	i_2	P_0	P_1	P_2	W
C_1	0	0	0	0	0	0	—
C_2	0	0	1	1	1	0	3
C_3	0	1	0	0	1	1	3
C_4	0	1	1	1	0	1	4
C_5	1	0	0	1	0	1	3
C_6	1	0	1	0	1	1	4
C_7	1	1	0	1	1	0	4
C_8	1	1	1	0	0	0	3

⇒ minimum hamming distance $\boxed{d_{min} = 3}$

(iii) $d_{min} = 3$

Error detection

$d_{min} \geq s + 1$

$3 \geq s + 1$

$\Rightarrow s \leq 2$

— it can detect 2 bit error

Error Correction

$d_{min} \geq 2t + 1$

$3 \geq 2t + 1$

$t \leq 1$

— it can correct 1 bit error

(iv) Parity check matrix

$$H = [P^T : I_{n-k}]$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad I_{6-3} = I_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \boxed{H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}}$$

(v) Error Syndrome

$$S = Y(H^T)$$

$$S = (100011)(H^T)$$

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$S = y H^T$

$$= [1\,0\,0\,0\,1\,1] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [1\,1\,0]$$

position of this syndrome we find in the H^T matrix

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ \boxed{1 \quad 1 \quad 0} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ — it occurs in third line

So the transmitted 3rd bit is here error

$e = [0\,0\,1\,0\,0\,0]$

$y = [1\,0\,0\,0\,1\,1]$

To extract origined information

$x = e + y$

$$\boxed{x = [1\,0\,1\,0\,1\,1]}$$

Q The parity check matrix of a particular (7,4) linear block code is given by

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(i) find all generator matrix (G)
(ii) list all code vector
(iii) what is minimum distance between code vector
(iv) How many error can be detected? How many error can be corrected.

Ans $n = 7 \quad k = 4$

no of check bit $n-k = 7-4 \Rightarrow q = 3$

$$n = 2^q - 1 = 2^3 - 1 = 7$$

To find the P submatrix

$$[H]_{3\times 7} = \begin{bmatrix} P_{11} & P_{21} & P_{31} & P_{41} & \cdots & 1 & 0 & 0 \\ P_{12} & P_{22} & P_{32} & P_{42} & \cdots & 0 & 1 & 0 \\ P_{13} & P_{23} & P_{33} & P_{43} & \cdots & 0 & 0 & 1 \end{bmatrix}$$

$$= [P^T : I_3]$$

P submatrix can be obtained as

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{4\times 3}$$

(i) To obtain the generator matrix G

$$G = [I_k : P_{k\times q}]_{k\times n}$$

with $k=4$, $q=3$ and $n=7$

$$G = [I_4 : P_{4\times 3}]_{4\times 7}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & \vdots & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & \vdots & 0 & 1 & 1 \end{bmatrix}_{4\times 7}$$

(ii) To find all the code word

$$C = mP$$

$$[C_1\ C_2\ C_3]_{1\times 3} = [m_1\ m_2\ m_3\ m_4]_{1\times 4}\ [P]_{4\times 3}$$

$$C_1 = m_1 \oplus m_2 \oplus m_3$$
$$C_2 = m_1 \oplus m_2 \oplus m_4$$
$$C_3 = m_1 \oplus m_3 \oplus m_4$$

for determining the code vector $m_1 m_2 m_3 m_4 = 1011$

$$C_1 = 1 \oplus 0 \oplus 1 = 0$$
$$C_2 = 1 \oplus 0 \oplus 1 = 0$$
$$C_3 = 1 \oplus 1 \oplus 1 = 1$$

So $C_1 C_2 C_3 = 001$

(iii) minimum distance between code vector

The smallest weight of any non zero code vector 3.

The minimum distance of any linear block coder is equal to the minimum weight of any non zero vecto

$$d_{min} = [w(x)]_{min} \; ; \; x \neq (0, 0 \ldots 0)$$

Q1 For a (6,3) systematic LBC the three parity check bits C_4, C_5, C_6 are formed from the following

$$C_4 = d_1 \oplus d_3$$
$$C_5 = d_1 \oplus d_2 \oplus d_3$$
$$C_6 = d_1 \oplus d_2$$

Ans
(a) Write down the generator ~~matrix~~

$$C_4 = P_{11} d_1 \oplus P_{21} d_2 \oplus P_{31} d_3$$
$$C_5 = P_{12} d_1 \oplus P_{22} d_2 \oplus P_{32} d_3$$
$$C_6 = P_{13} d_1 \oplus P_{23} d_2 \oplus P_{33} d_3$$

$P_{11} = 1$ $\quad P_{21} = 0$ $\quad P_{31} = 1$

$P_{12} = 1$ $\quad P_{22} = 1$ $\quad P_{32} = 1$

$P_{13} = 1$ $\quad P_{23} = 1$ $\quad P_{33} = 0$

Now $G = [I_k | P]$

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

$$G = [I_3 | P]$$

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

2 Construct all possible code word

Construct c using G

000	000
001	110
010	011
011	101
100	111
101	001
110	100
111	010

3 Suppose R = 010111 find the location of error

$RH^T =$ $R = [010111]$

$H = [P^T | I_{n-k}]$

$$H = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$RH^T = (010111) \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= 100$

which is equalent to 4th ~~bit~~ Row of H^T

So the fourth bit is in error

010011

Ans.

Conversion of non systematic form of matrices into systematic form.

Systematic Generator matrix

A generator matrix $[G] = [I_k : P]$ is said to be in a systematic form If it generate the systematic codeword.

I_k = k x k matrix

P = k x (n-k) matrix

G = k x n matrix

In these matrix information bits are placed together

$$[C] = [i][G]$$

$$= [i_1, i_2, i_3, i_4] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

So when we get codeword

$$C = (i_1, i_2, i_3, i_4, P_1, P_2, P_3)$$

Identity bits keeps information together and parity matrix generate parity bits.

$P_1 = i_1 + i_2 + i_3$

$P_2 = i_2 + i_3 + i_4$

$P_3 = i_1 + i_2 + i_4$

for Ex The (5, 3) linear code has the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

→ Determine Systematic form of G

→ Generate Codeword for information

→ systematic to nonsystematic codeword

Ans for systematic matrix the identity matrix must be follow by the next matrix

In the above example the identity matrix not easily funded So it is not the systematic matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

The 1st third row is false

Add R_2 to R_3 for R_3 →

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Add R_3 to R_1 for R_1 →

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

I P

$G = [I_k | P]$

(ii) for Non systematic G

$$C = [i][G]$$

$$= [0\ 1\ 1]\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0\ 0\ 1\ 1\ 1]$$

for systematic G

$$C = [i][G]$$

$$= [0\ 1\ 1]\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$= [\underbrace{0\ 1\ 1}_{\text{information}}\ \underbrace{1\ 0}_{\text{Parity}}]$$

Thus the result has the information first and after that parity bit is present.

Q The generator ~~matrix~~ for a (6,3) block code is given below. find all code ~~vectors of this~~ lode.

$$G = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Ans The codevector can be obtained through following steps

→ (i) Determine the p submatrix from generator matrix
(ii) Obtain equation for check bit using $C = mP$
(iii) Determine check bit for every message vector.

i) To obtain p submatrix

$$G = [I_k : P_{k \times p}]$$

$$I_k = I_{3\times3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{k \times q} = P_{3\times3} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$G = [I : P]$
$H = [P^T : I]$

$G = [P : I]$
$H = [I : P^T]$

(ii) To obtain the equation for check bits

Here $k = 3$, $q = 3$ and $n = 6$

Here $k=3$, $q=3$ and $n=6$

S.No	Bit of message m_1	m_2	m_3
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
6	1	0	1
7	1	1	0
8	1	1	1

P submatrix is given by $P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$[C_1 \; C_2 \; C_3] = (m_1 m_2 m_3)\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$C_1 = 0 \times m_1 \oplus m_2 \oplus m_3$$
$$C_2 = m_1 \oplus 0 \times m_2 \oplus m_3$$
$$C_3 = m_1 \oplus m_2 \oplus 0 \times m_3$$

$$C_1 = m_2 \oplus m_3$$
$$C_2 = m_1 \oplus m_3$$
$$C_3 = m_1 \oplus m_2$$

(iii) To determine check bit and code vector for every message vector

$(m_1 m_2 m_3) = 000$ we have

$$C_1 = 0 \oplus 0 = 0$$
$$C_2 = 0 \oplus 0 = 0$$
$$C_3 = 0 \oplus 0 = 0$$

$(m_1, m_2 m_3) = 001$

$$C_1 = 0 \oplus 1 = 1$$
$$C_2 = 0 \oplus 1 = 1$$
$$C_3 = 0 \oplus 0 = 0$$

(iv) Error Detection and Correction Capabilities

Since $d_{min} = 3$

$d_{min} \geq s+1$

$3 \geq s+1$

$s \leq 2$

Thus two errors will be detected

$d_{min} \geq 2t+1$

$3 \geq 2t+1$

$\boxed{t \leq 1}$

The one error will be corrected.

Syndrome detecting

It is the method to correct error in the linear block code. Let the transmitted vector code vector represented by be 'x' and received code vector represented by 'y' then

$X = Y$ if there no transmission error

$X \neq Y$ if there are error created in transmission detector detect that error by using the stored bit in the decoder.

$$H = [P^T : I_q]_{q \times n}$$

$$H^T = \begin{bmatrix} P \\ \vdots \\ I_q \end{bmatrix}_{n \times q}$$

Important Property used in syndrome detecting

$$XH^T = (0\,0\,0 \cdots 0)$$

$$[H]_{1 \times n} [H^T]_{n \times q} = (0\,0\,0 \cdots 0)_{1 \times q}$$

for syndrome checking check the

$$XH^T$$

So If $YH^T = \{0,0,0\}$ no error and Y is valid code vector

$YH^T =$ non zero than some error occurs in the detector.

Defination of Syndrome

When YH^T is non zero, some error are present in Y. Thus the nonzero output of the product YH^T is called syndrome and is used to dected errors in the Y

$$S = YH^T$$

$$\boxed{[S]_{1 \times q} = [Y]_{1 \times n} [H^T]_{n \times q}}$$