POORNIMA
COLLEGE OF ENGINEERING

DETAILED LECTURE NOTES

PAGE NO. ....

Unit - I

Introduction to information theory

Information Theory: a theory that was initiated by one man - The American Electrical Engineer Claude E. Shannon, whose ideas appeared in the article "The Mathematical Theory of Communication" in the Bell system Technical Journal (1948).

The chief concern of information theory is to discover mathematical laws governing the systems, designed to communicate or manipulate information. It sets up quantitative measures of information and the capacity of various systems to transmit, store and process information.

The performance of the communication system is measured in terms of its error probability. An errorless transmission is possible when probability of error at the receiver approaches zero.

The performance of the system depends upon available signal power, channel noise and bandwidth. Based on these parameters it is possible to establish the ~~theorems~~ condition for errorless transmission.

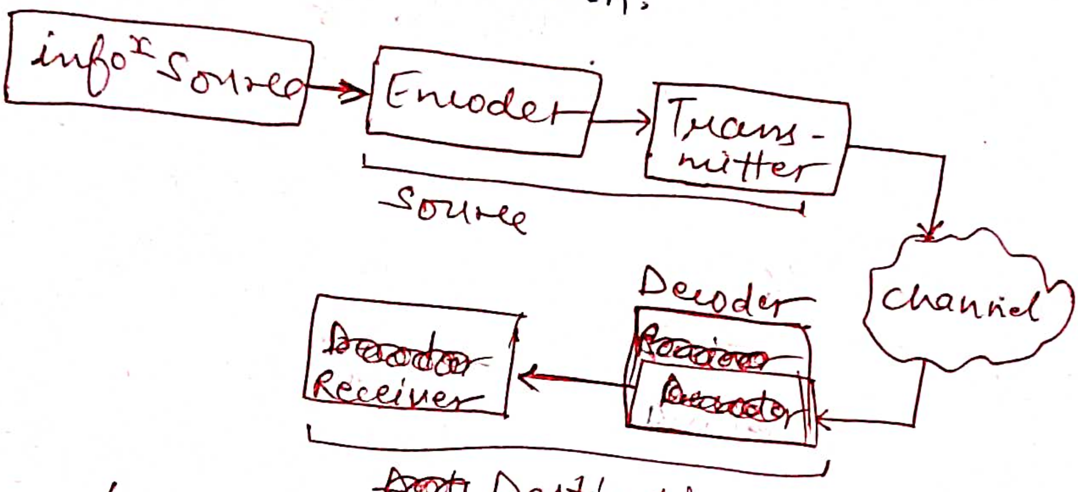
These conditions are ~~&~~ referred as Shannon's theorems.

Information: It is the intelligence / ideas or message in information theory.

message:
- Electrical (Vol, curr.)
- Speech / Voice
- Picture / image
- Video / audio

Source of messages (information source)

In communication System, info^n is transmitted from Source to destination.



(Digital communication System)

Uncertainty:

Consider the source which emits the discrete symbols randomly from the set of fixed alphabet i.e.

$$X = \{x_0, x_1, x_2 \ldots x_{K-1}\}$$

The various symbols in 'x' have probabilities of ~~Po~~ P_0, P_1, P_2 --- etc, ~~ato~~ which can be written as,

$$P(X = x_K) = P_K \qquad K = 0, 1, 2, \ldots K-1$$

This set of probabilities satisfy the following condition,

$$\sum_{K=0}^{K-1} P_K = 1$$

Such information source is called discrete information

DETAILED LECTURE NOTES

source. The concept/idea of information is related to 'Uncertainty' or 'surprise'.

The probability P_K is low, there is more surprise or uncertainty. Befor the event $x = x_K$ is emitted, there is an amount of uncertainty. When the symbol $x = x_K$ Occurs, there is an amount of surprise. After the occurrence of the symbol $x = x_K$, there is the gain in amount of information.

Measure of Information: Let us consider the communication system which transmits the messages m_1, m_2, m_3 -- with probabilities of occurrence P_1, P_2, P_3 .... The amount of information transmitted through the message m_K with probability P_K is given as,

Amount of information: $I_K = \log_2 \left(\frac{1}{P_K}\right)$

$$\log_2 x = \frac{\log_{10} x}{\log_{10} 2}$$

unit of information = bit

Properties of Information:

1. If there is more uncertainty about the message, information carried is also more.
2. If ~~...~~ receiver knows the message being transmitted, the amount of information carried is zero.
3. If I_1 is the information carried by message m_1 and I_2 is the information carried by m_2, then amount of information carried combindly due to m_1 and m_2 is $I_1 + I_2$.
4. If there are $M = 2^N$ equally likely messages, then amount of information carried by each messages will be N bits.

Proof: 1 Let we take two symbol 0 occur with probability $\frac{1}{4}$ and 1 occur with probability 3/4. assume $0 \to m_1$

$1 \to m_2$

$m_1 \to 1/4$, $m_2 \to 3/4$

$(U_{m_1} > U_{m_2})$, $(I_{m_1} > I_{m_2})$

$I_{m_1} = \log_2 \left(\frac{1}{P_K}\right)$

$I_{m_1} = \log_2 \left(\frac{1}{1/4}\right)$

$= \log_2 (4)$

$= \log_2 (2)^2$

$= 2 \log_2 2$

$= 2 \quad [\log_2 2 = 1]$

$= 2$ bits

$I_{m_2} = \log_2 (4/3)$

$= \dfrac{\log_{10} (4/3)}{\log_{10} 2}$

$= 0.415$ bits

Hence $I_{m_1} > I_{m_2}$ Proved.

POORNIMA
COLLEGE OF ENGINEERING

DETAILED LECTURE NOTES

Proof: 2 Receiver 'knows' the message. This means only one message is transmitted. Hence probability of occurrence of this message will be $P_K = 1$.

The amount of infoⁿ carried by this type of message is,

$$I_K = \log_2\left(\frac{1}{P_K}\right)$$

$$I_K = \log_2(1)$$

$$I_K = \frac{\log_{10}(1)}{\log_{10}(2)}$$

$$= 0 \text{ bits}$$

Hence proved, the statement that if receiver knows message, the amount of information carried is zero.

Proof: 3 I_1 is infoⁿ of m_1, I_2 is infoⁿ of m_2 $\Big] I_1 + I_2$

P_1 is probability of message m_1 and P_2 is probability of message m_2.

The individual amount carried by messages m_1 and m_2 are

$$I_1 = \log_2\left(\frac{1}{P_1}\right) \text{ and } I_2 = \log_2\left(\frac{1}{P_2}\right)$$

Since messages m_1 and m_2 are independent
So composite prob. $= p_1 p_2$

$$I_{1,2} = \log_2\left[\frac{1}{p_1 p_2}\right]$$

$$= \log_2\left[\left(\frac{1}{p_1}\right)\left(\frac{1}{p_2}\right)\right]$$

$$= \underbrace{\log_2\left(\frac{1}{p_1}\right)}_{I_1} + \underbrace{\log_2\left(\frac{1}{p_2}\right)}_{I_2} \quad [\log(xy) = \log$$

~~Prod~~ $I_{1,2} = I_1 + I_2$

Proof: 4 $\quad M = 2^N, \quad I = N$ bits

Since we 'M' equal likely messages.
'p' of each mess. $= \frac{1}{M}$

$$I_k = \log_2\left(\frac{1}{p_k}\right) = \log_2(M)$$

$$M = 2^N$$

$$I_k = \log_2(2^N)$$

$$= N \cancel{\log_2(2)} \rightarrow 1$$

$I_k = N$ bits $\qquad$ Hence Proved.