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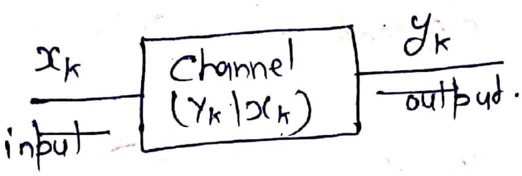
DETAILED LECTURE NOTES

Discreate memoryless channel:



the i/p and o/p are x_k and y_k discrate number. means these are not vary with the continuos range

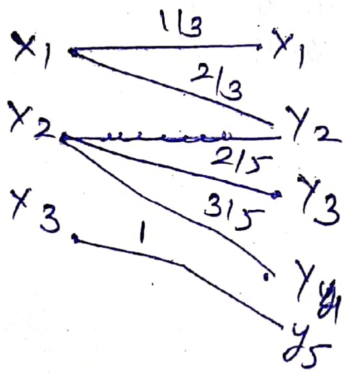
These values are not uniform.

Memoryless: it depends only present input, not depends on previous input. or pass element.

Channel matrix:

$$\left[\frac{Y_k}{X_k}\right] = \begin{array}{c|ccccc} & Y_1 & Y_2 & Y_3 & \cdots & Y_n \\ \hline X_1 & Y_1/X_1 & Y_2/X_1 & Y_3/X_1 & \cdots & Y_n/X_1 \\ X_2 & \frac{Y_1}{X_2} & Y_2/X_2 & \frac{Y_3}{X_2} & \cdots & \frac{Y_n}{X_2} \\ X_3 & & & & & \\ \vdots & & & & & \\ X_m & \frac{Y_1}{X_m} & \frac{Y_2}{X_m} & \frac{Y_3}{X_m} & & \frac{Y_n}{Y_m} \end{array}$$

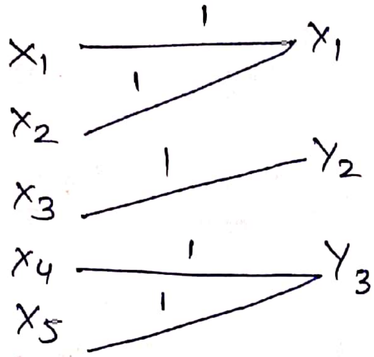
Lossless Channel



$$\begin{array}{c|ccccc} & y_1 & y_2 & y_3 & y_4 & y_5 \\ \hline X_1 & 1/3 & 2/3 & 0 & 0 & 0 \\ X_2 & 0 & 0 & 2/5 & 3/5 & 0 \\ X_3 & 0 & 0 & 0 & 0 & 1 \end{array}$$

$\Sigma R_i = 1$ → every column has one non zero element

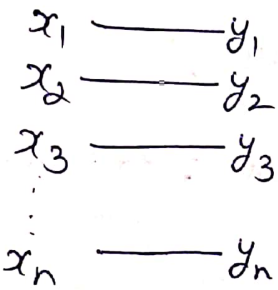
2. Deterministic Channel:



$$\begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & & & \\ x_2 & & & \\ x_3 & & & \\ x_4 & & & \\ x_5 & & & \end{array} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

every element is one. → Every Row has one non zero value.

3. Noiseless Channel

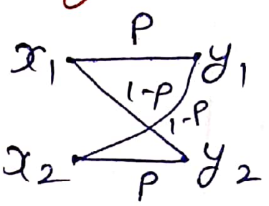


$$\begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & \cdots & y_n \end{array} \\ \begin{bmatrix} 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ & & \ddots & \\ 0 & \cdots & & 1 \end{bmatrix} \end{array}$$

→ Every Column has one non zero value.

Only diagonal terms are coming. So any intermediate signal is there.

4. Binary Symmetric Channel



$$\begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & p & 1-p \\ x_2 & 1-p & p \end{array}$$

$x_1 = x_2 = 0.5$

$$[0.5 \quad 0.5] \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

$$= \begin{bmatrix} 0.5p + 0.5(1-p) \\ 0.5(1-p) + 0.5p \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

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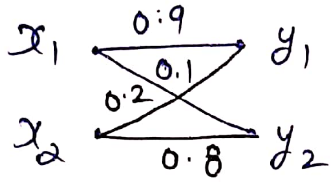


$$\begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

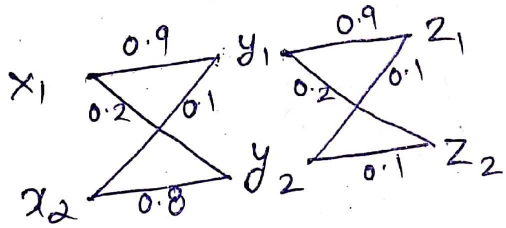
$$= \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 0.55 \\ 0.45 \end{bmatrix}$$

$$= \begin{bmatrix} 0.55 \\ 0.45 \end{bmatrix}$$

Joint probablity of Y_1 & Y_2

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

Cascading of Channels



$$\begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}_{2\times2} \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}_{2\times2}$$

$$\text{Channel matrix } (z/x) = \text{Channel matrix}\left(\frac{Y}{X}\right) \times \text{Channel matrix}\left(\frac{Z}{Y}\right)$$

Symmetric Channel → Represented by channel ~~matrix~~ in which the second and subsequent Row contain the same element as that in the first Row. ~~but~~ in different order.

$$P(B/A) = \begin{array}{c|c} & \begin{array}{cccc} b_1 & b_2 & \cdots & b_s \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_r \end{array} & \begin{bmatrix} P_1 & P_2 & \cdots & P_s \\ P_2 & P_3 & \cdots & P_1 \\ \vdots & & & \\ P_3 & \cdots & \cdots & P_1 \end{bmatrix} \end{array}$$

Channel Capacity for all the channels:

1. Binary Symmetric Channel $C = 1 - h$

where $h = \bar{p}\log_2\frac{1}{\bar{p}} + p\log_2\frac{1}{p}$.

2. Noise less Channel = $\log_2 r$ bits/sec.

3. Deterministic channel = $\log_2 s$ bit/sy.

4. Symmetric Channel = $\log_2 s - h$

Noiseless Channel

$$I(A, B) = H(A) - H(A|B)$$

$H(A|B) = 0$

So $\boxed{I(A, B) = H(A)}$

Channel Capacity

$$C = \max(I(A, B))\, r_s$$

$$C = \max[H(A)]\, r_s$$

Deterministic Channel

$$I(A, B) = H(B) - H(B|A)$$

$H(B|A) = 0$

$$\boxed{I(A, B) = H(B)}$$

Channel Capacity $C = \max[I(A, B)] \times r_s$

$$\boxed{C = \max[H(B)]\, r_s}$$

Q for the channel ~~matrix~~ given below find the channel capacity

$$P(b_j|a_i) = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 1/3 & 1/6 & 1/2 \\ 1/6 & 1/2 & 1/3 \end{bmatrix} \end{array}$$

Channel capacity $= \boxed{\log_2 s - h}$

$$h = H(B|A) = \sum_{j=1}^{3} = \frac{1}{2}\log_2 2 + \frac{1}{3}\log_2 3 + \frac{1}{6}\log_2 6$$

$$\boxed{h = 1.459 \text{ b/sym}}$$

$$C = \log_2 3 - 1.4591$$

$\underline{s = 3}$ with 3 sequence.

$$\boxed{C = 0.1258 \text{ b/sec}}$$

Scanned by CamScanner

Channel Example :

Q4 A bsc has the following noise matrix with source probabilities of $p(x_1) = 2/3$ & $p(x_2) = 1/3$

$$P(y|x) = \begin{array}{c|cc} & y_1 & y_2 \\ x_1 & 3/4 & 1/4 \\ x_2 & \frac{1}{4} & 3/4 \end{array}$$

also represent by $\bar{P}$ and P

$$\begin{bmatrix} \bar{P} & P \\ P & \bar{P} \end{bmatrix}$$

(i) Determine $H(x)$, $H(y)$

(ii) find channel capacity c

(iii) find channel Efficiency and Redundancy

Ans

(i) $$H(x) = \sum_{i=1}^{2} P(x_i) \log_2 \left(\frac{1}{P(x_i)}\right)$$

$$= \frac{2}{3}\log_2\left(\frac{3}{2}\right) + \frac{1}{3}\log_2(3)$$

$$\boxed{H(x) = 0.9183}$$ bits

(ii) $H(y) = P(x,y) = \begin{array}{c|cc} & Y_1 & Y_2 \\ x_1 & 1/2 & 1/6 \\ x_2 & 1/12 & \frac{1}{4} \end{array} \Rightarrow P(Y) = $ $Y_1 = \frac{1}{2} + \frac{1}{12} = 7/12$

$Y_2 = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$

[for getting the probabablity of Y_1 when occur of

$x_1 = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$]

Represented by

$$P(y) = \begin{bmatrix} \frac{7}{12} & \frac{5}{12} \end{bmatrix}$$

$$H(y) = \sum_{j=1}^{2} P(y_j) \log_2 \left(\frac{1}{P(y_j)}\right)$$

$$= \frac{7}{12} + \log_2 \frac{12}{7} + \frac{5}{12} \log_2 \frac{12}{5}$$

$$\boxed{H(y) = 0.9799} \text{ b/s}$$

2 Channel Capacity

$$\boxed{C = 1 - h}$$

where $h = \bar{p} \log_2 \frac{1}{\bar{p}} + p \log \frac{1}{p}$

$$C = 1 - \left[\frac{3}{4} \log_2 \frac{4}{3} + \frac{1}{4} \log_2 4\right]$$

$$C = 1 - 0.813$$

$$\boxed{C = 0.1887 \text{ b/s}}$$

(iii) $$\eta_{channel} = \frac{I(x,y)}{C}$$

$$I(x,y) = H(x) - H(X|y)$$

$$H(x|y) = H(x,y) - H(y)$$

↳ $H(x) + H(y/x)$

$H(x,y) \rightarrow$ check all the probabilities when x is applied

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or $H(x,y) = H(x) + H(y|x)$

$H(x,y) = 2 \times \frac{3}{4} \log_2 \frac{4}{3} + 2 \times \frac{1}{4} \log_2 4$

$= 1.623$

$H(x|y) = H(x,y) - H(y)$

$H(x|y) = .749$

$I(x,y) = .9183 - 0.749$

$\cancel{H}(x,y) = 0.1686$

(iii) $\eta_{ch} = I(x,y) / C$

$= 0.1686 / 0.1887$

$\boxed{\eta = 89.35\%}$

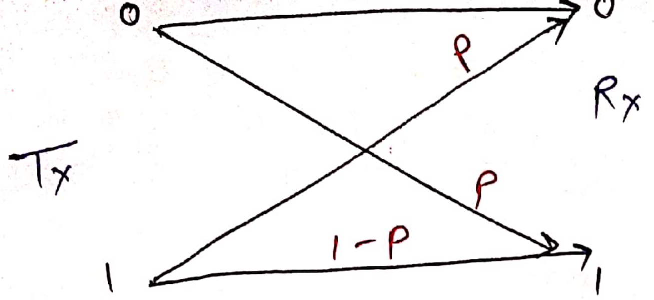
iv) Redundancy $= \gamma = 1 - \eta_{ch}$

$= 1 - .89$

$\boxed{\gamma = 10.65\%}$

Example of binary Symmetric Channel (BSC)

A channel can be visualized for transport 1's and 0's from the transmitter and Receiver. the defined input symbol shown in the figure. A BSC channel flips 1 to 0 and equal probablity.



A BSC channel

$$P(Y=0 \mid X=0) = 1-p$$
$$P(y=0 \mid X=1) = p$$
$$P(Y=1 \mid X=1) = 1-p$$
$$P(Y=1 \mid X=0) = p.$$

$$P(Y=0) = P(X=0) \cdot P(Y=0 \mid X=0) + P(X=1) \cdot P(Y=0 \mid X=1)$$
$$= 0.5(1-p) + 0.5(p)$$
$$= 0.5$$

$$P(Y=1) = P(X=0) \cdot P(Y=1 \mid X=0) + P(X=1) \cdot P(Y=1 \mid X=1)$$
$$0.5(p) + 0.5(1-p)$$
$$= 0.5$$

$$I(x_0; y_0) = I(0;0)$$
$$= \log_2 \left(\frac{P(Y=0 \mid X=0)}{P(Y=0)} \right) = \log_2 \left(\frac{1-p}{0.5} \right)$$

$$\boxed{I(x_0; y_0) = \log_2 2(1-p)}$$

$$I(x_1; y_0) = I(1;0)$$
$$\log_2 \left(\frac{P(Y=0 \mid X=1)}{P(Y=0)} \right) = \log_2 \left(\frac{p}{0.5} \right)$$

$$\boxed{I(x_1; y_0) = \log_2 2p}$$