Unit-2

Principal of Duality:

In two valued Boolean Algebra, dual of algebric expression can be obtained simply by interchanging 'OR' and 'AND' operators and by replacing 1's by 0's and vice versa.

OR $\longrightarrow$ AND 1 $\rightarrow$ 0

AND $\longrightarrow$ OR 0 $\rightarrow$ 1

Example

$A \cdot 0 = 0 \xrightarrow{\text{Dual}} A+1 = 1$

$(A+B)\cdot(A+C) \longrightarrow AB + AC$

$\longrightarrow A(B+C)$

$\Rightarrow$ $A + AB = A$

$\quad \hookrightarrow A\cdot(A+B) = A$

$\Rightarrow$ $A + \bar{A}B = A + B$

$\quad \hookrightarrow A\cdot(\bar{A}+B) = A\cdot B$

Minimization of Boolean Expression using Algebraic method

It can be simplified by applying appropriate law, theoram of boolean algebra.

Example

1. Prove that $A + A\cdot\bar{B} + \bar{A}\cdot B$

$$A + A\cdot\bar{B} + \bar{A}\cdot B$$

$$A(1+\bar{B}) + \bar{A}\cdot B$$

$$A\cdot 1 + \bar{A}\cdot B \qquad (1+\bar{B} = 1)$$

$$= A + B \qquad [(A + \bar{A}B) = A + B]$$

2. Simplify the Expression

$$\overline{A\bar{B} + ABC} + A(B + A\bar{B})$$

$$= \overline{\overline{A(\bar{B} + BC)} + A(B+A)}$$

$$= \overline{\overline{A(\bar{B} + C)} + AB + A\cdot A}$$

$$= \overline{\overline{A\bar{B} + AC} + AB + A}$$

$$= \overline{\overline{A\bar{B} + AC} + A(B+1)}$$

$$= \overline{\overline{A\bar{B} + AC} + A\cdot 1}$$

$$= \overline{(\overline{A\bar{B}})\cdot(\overline{AC}) + A}$$

$$= \overline{(\bar{A} + B)\cdot(\bar{A} + \bar{C}) + A} \qquad [\because (A+B)(A+C) = A + BC]$$

$$= \overline{(\bar{A} + B\bar{C}) + A}$$

$$= \overline{(\bar{A} + B\bar{C}) + A}$$

$$= \overline{\bar{A} + A + B\bar{C}}$$

$$= \overline{1 + B\bar{C}} \qquad [\because 1 + X = 1]$$

$$= \bar{1}$$

$$= 0$$

3. Simplify the Expression

$$Y = \overline{(AB + \bar{C})(\overline{A+B} + C)}$$

$$= \overline{(AB + \bar{C})(\bar{A}\cdot\bar{B} + C)}$$

$$= \overline{AB\cdot\bar{A}\bar{B} + ABC + \bar{A}\bar{B}\bar{C} + C\bar{C}}$$

$$= \overline{0 + ABC + \bar{A}\bar{B}\bar{C} + 0}$$

$$= \overline{ABC + \bar{A}\bar{B}\bar{C}}$$

$$= \overline{ABC}\cdot\overline{\bar{A}\bar{B}\bar{C}}$$

$$= (\bar{A} + \bar{B} + \bar{C})\cdot(\bar{\bar{A}} + \bar{\bar{B}} + \bar{\bar{C}})$$

$$= (\bar{A} + \bar{B} + \bar{C})\cdot(A + B + C)$$

4. If $\bar{A}B + C\bar{D} = 0$ then prove that

$$AB + \bar{C}(\bar{A}+\bar{D}) = AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D$$

Solution

L.H.S. $= AB + \bar{C}(\bar{A}+\bar{D}) + 0$

$= AB + \bar{C}(\bar{A}+\bar{D}) + \bar{A}B + C\bar{D}$ (as per given $\bar{A}B + C\bar{D} = 0$)

$= AB + \bar{A}\bar{C} + \bar{C}\bar{D} + \bar{A}B + C\bar{D}$

$= B(A+\bar{A}) + \bar{D}(C+\bar{C}) + \bar{A}\bar{C}$

$= B + \bar{D} + \bar{A}\bar{C}$

R.H.S $\quad AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D + 0$

$= AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D + \underbrace{\bar{A}B + C\bar{D}}_{\text{Given} = 0}$

$= B(A+\bar{A}) + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D + C\bar{D}$

$= B(1+D) + \bar{B}\bar{D} + \bar{A}\bar{C}D + C\bar{D}$

$= B + \bar{B}\bar{D} + \bar{A}\bar{C}D + C\bar{D}$

$= B + \bar{D} + \bar{A}\bar{C}D + C\bar{D}$

$= B + \bar{D}(1+C) + \bar{A}\bar{C}D$ $\quad \{\because A + \bar{A}B = A + B\}$

$= B + \bar{D} + D\bar{A}\bar{C}$

$= B + \bar{D} + \bar{A}\bar{C}$

$$\boxed{\text{L.H.S} = \text{R.H.S.}}$$

5. Using Boolean Algebra Verify

(i) $(A+B)(B+C)(C+A) = AB + BC + CA$

(ii) $(A+B)(\bar{A}+C) = AC + \bar{A}B + BC$

Solution

(i) $(A+B)(B+C)(C+A) = AB+BC+CA$

$= (AB + AC + B \cdot B + BC)(C+A)$

$= [B(A+1) + AC + BC](C+A)$

$= (B + AC + BC)(C+A)$

$= BC + ACC + BCC + AB + AAC + ABC$

$= BC + AC + BC + AB + AC + ABC$

$= BC + AB(1+C) + AC$

$= BC + AB + AC$

Hence proved

(ii) $(A+B)(\bar{A}+C) = AC + \bar{A}B + BC$

L.H.S $= A\bar{A} + AC + \bar{A}B + BC$

$= 0 + AC + \bar{A}B + BC$

$= AC + \bar{A}B + BC$ Hence proved

6. Prove ~~that~~ $BCD + A\bar{C}\bar{D} + ABD = BCD + A\bar{C}\bar{D} + AB\bar{C}$

L.H.S. $= BCD + A\bar{C}\bar{D} + ABD \cdot 1$

$= BCD + A\bar{C}\bar{D} + ABD(C + \bar{C})$

$= BCD + A\bar{C}\bar{D} + ABCD + AB\bar{C}D$

$= BCD(1 + A) + A\bar{C}(\bar{D} + DB)$

$= BCD + A\bar{C}(\bar{D} + B)$

$= BCD + A\bar{C}\bar{D} + AB\bar{C}$

$\boxed{\text{L.H.S} = \text{R.H.S.}}$ Hence proved.

7. Prove ~~that~~ $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} = \bar{A} + \overline{B+C}$

L.H.S

$\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$

$= \bar{A}\bar{B}(\bar{C} + C) + \bar{A}B(\bar{C} + C) + A\bar{B}\bar{C}$

$= \bar{A}(\bar{B} + B) + A\bar{B}\bar{C}$

$= \bar{A} + A\bar{B}\bar{C}$

$= \bar{A} + \bar{B}\bar{C}$ $[\because A + \bar{A}B = A + B]$

$= A + \overline{B+C}$

$\boxed{\text{L.H.S} = \text{R.H.S}}$ Hence proved.

Scanned by CamScanner

Sum of Product and Product of Sum form

Logical functions are expressed in term of logical variables. It is expressed in the following form.

Product Term AND function is defined as the product term. It impliment the AND function. It can be appear either in complemented or uncomplemented form.

Sum Term OR function is generally used as the sum term. It can appear either in complemented or uncomplemented form.

Sum of Product form The logical sum of two or more logical product term is called sum of product form.

(i) $Y = AB + BC + AC$

(ii) $Y = \bar{A}B + A\bar{C} + BC$

Product of Sum form The logical product of two or more logical sum term. It is AND operation of OR operated variable

(ii) $Y = (A+B)(B+C)(C+\bar{A})$

(iii) $Y = (A+B+C)(A+\bar{C})$

Min term

~~A~~ product ~~term~~ containing all ~~the~~ k variable is either is complemented or non complemented form called as min term. The main property of minterm ~~that it~~ possess ~~the~~ value 1 for only 1 combination ~~of~~ k input variable.

Example

A	B	C	minterm
0	0	0	$\bar{A}\bar{B}\bar{C}$
0	0	1	$\bar{A}\bar{B}C$
0	1	0	$\bar{A}B\bar{C}$
0	1	1	$\bar{A}BC$
1	0	0	$A\bar{B}\bar{C}$
1	0	1	$A\bar{B}C$
1	1	0	$AB\bar{C}$
1	1	1	ABC

Canonical Sum of Product Expression

It is the logical sum ~~of~~ all minterms desired from ~~the~~ row ~~of truth~~ table whose value is

for example ~~if~~ $\bar{A}\bar{B}\bar{C}$, $A\bar{B}C$ and $AB\bar{C}$ than ~~it~~ can be expressed in

$$Y = \Sigma_m (0, 5, 6)$$

$$= m_0 + m_5 + m_6$$

$$= \bar{A}\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$$

Ques — Obtain the canonical sum of product form of the function

$$Y(A, B) = A + B$$

$$A \cdot 1 + B \cdot 1$$

$$A(B + \bar{B}) + B(A + \bar{A})$$

$$AB + A\bar{B} + AB + \bar{A}B$$

$$\boxed{Y(A, B) = A + B = AB + A\bar{B} + \bar{A}B}$$

Ques Canonical SOP of given function

$$Y = AB + ACD$$

$$= AB(C + \bar{C})(D + \bar{D}) + ACD(B + \bar{B})$$

$$= (ABC + AB\bar{C})(D + \bar{D}) + ABCD + A\bar{B}CD$$

$$= ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + ABCD + A\bar{B}CD$$

$$= ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD$$

Maxterm

The sum term containing all the k variable in the function either complemented or uncomplemented form is called the maxterm. it represented by $M_0, M_1, M_2, M_3, \ldots M_7$

A	B	C	maxterm.
0	0	0	$A+B+C$
0	0	1	$A+B+\overline{C}$
0	1	0	$A+\overline{B}+C$
0	1	1	$A+\overline{B}+\overline{C}$
1	0	0	$\overline{A}+B+C$
1	0	1	$\overline{A}+B+\overline{C}$
1	1	0	$\overline{A}+\overline{B}+C$
1	1	1	$\overline{A}+\overline{B}+\overline{C}$

∴ ⇒ it is clear that each maxterm is the complement of the corresponding minterm.

Canonial Product of Sum Expression

It is the logical product of all the maxterm derived from the row of truth table which function is 0. It can be expressed as

$$Y = \Pi(0, 2, 4, 7)$$

$$= M_0 . M_2 . M_4 . M_7$$

$$= (A+B+C)(A+\overline{B}+C)(\overline{A}+B+C)(\overline{A}+\overline{B}+\overline{C})$$

Q. Obtain the canonical POS expression of given equation

$$Y(ABC) = (A+\overline{B})(B+C)(A+\overline{C})$$

$Y = (A+\overline{B})(B+C)(A+\overline{C})$

$= (A+\overline{B}+0)(B+C+0)(A+\overline{C}+0)$

$= (A+\overline{B}+C\overline{C})(B+C+A\overline{A})(A+\overline{C}+B\overline{B})$

by using the Distributive Property

$Y = (A+\overline{B}+C)(A+\overline{B}+\overline{C})(A+B+C)(\overline{A}+B+C)$
$(A+\overline{C}+B)(A+\overline{B}+\overline{C})$

$Y = (A+\overline{B}+C)(A+\overline{B}+\overline{C})(A+B+C)(\overline{A}+B+C)$
$(A+B+\overline{C})$

Q Convert functions into Canonical form

$(A+\overline{B})(\overline{C}+\overline{D})(\overline{B}+\overline{C})$

$= (A+\overline{B})+(C\overline{C})+(D\overline{D})+(\overline{C}+\overline{D})+(A\overline{A})+(B\overline{B})$
$(\overline{B}+\overline{C})+A\overline{A}+D\overline{D}$

$= (A+\overline{B}+C)(A+\overline{B}+\overline{C})+D\overline{D}+(A+\overline{C}+\overline{D})(\overline{A}+\overline{C}+\overline{D})+B\overline{B}$
$+(A+\overline{B}+\overline{C})(\overline{A}+\overline{B}+\overline{C})+D\overline{D}$

$= (A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)(A+\overline{B}+\overline{C}+\overline{D})$
$(A+B+\overline{C}+\overline{D})(A+\overline{B}+\overline{C}+\overline{D})(\overline{A}+B+\overline{C}+\overline{D})(\overline{A}+\overline{B}+\overline{C}+\overline{D})$
$(A+\overline{B}+\overline{C}+D)(A+\overline{B}+\overline{C}+\overline{D})(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})$

$= (A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)(A + \overline{B} + \overline{C} + \overline{D})$
$(A + B + \overline{C} + \overline{D})(\overline{A} + B + \overline{C} + \overline{D})(\overline{A} + \overline{B} + \overline{C} + \overline{D})(\overline{A} + \overline{B} + \overline{C} + D)$

Deriving SOP and POS form from Truth table

The SOP expression from a boolean expression can be derived from its truth table by summing the product term.

The POS expression from a boolean function can be derived from the combination for the functions which value is 0.

In the product form the i/p variable appear in uncomplemented form if it posses the value 1 or in complemented form when it contains value 0.

In the sum form the i/p variable appear in uncomplemented form it has the value 0 and in the uncomplemented form if it has the value 1.

Truth table

Inputs A	B	C	Outputs	Product term	Sum term
0	0	0	0		$(A + B + C)$
0	0	1	0		$(A + B + \overline{C})$
0	1	0	1	$\overline{A} B \overline{C}$	
0	1	1	1	$\overline{A} B C$	
1	0	0	0		$(\overline{A} + B + C)$
1	0	1	1	$A \overline{B} C$	
1	1	0	0		$(\overline{A} + \overline{B} + C)$
1	1	1	1	ABC	