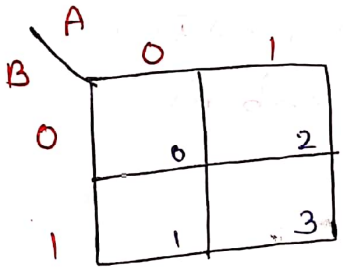
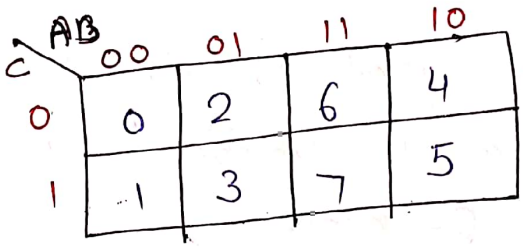
KARNAUGH MAP (k-map)

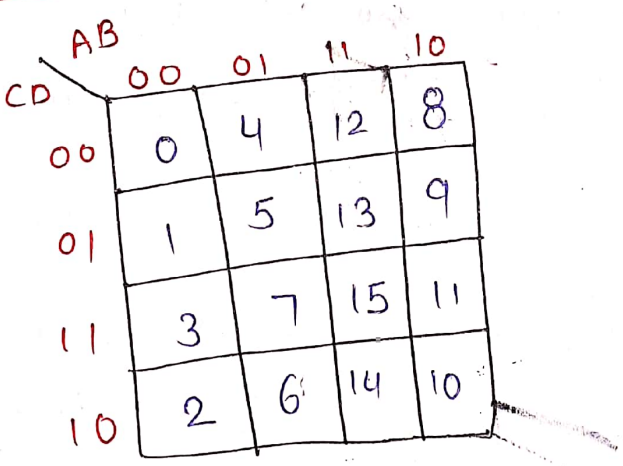
The simplification of the switching functions using boolean laws and theoram become complex with the increasing no of variable. k-map technique provides systematic method for simplifying and reducing the variable. The k-map can be designed for 2, 3 and 4 variables.



Two Variable



Three Variable

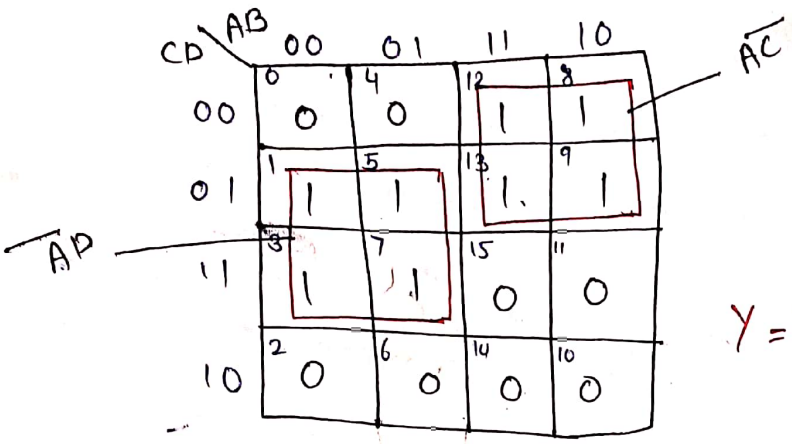


Four Variable

Q1 Simplify the following expressing Using the k-map for the 4 variable A, B, C and D

$Y = m_1 + m_3 + m_5 + m_7 + m_8 + m_9 + m_{12} + m_{13}$

Solution



$Y = \bar{A}D + A\bar{C}$

→ Construct k map and enter 1 in the cells corresponding to the minterm present in the expression and 0 in the other cells.

Q2 Plot the logical expression $AB\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}C + AB$ on a 4 variable k-map, obtain simplified expression from k-map

Ans $ABCD + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C + AB$

$= ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C(D+\bar{D}) + AB(C+\bar{C})(D+\bar{D})$

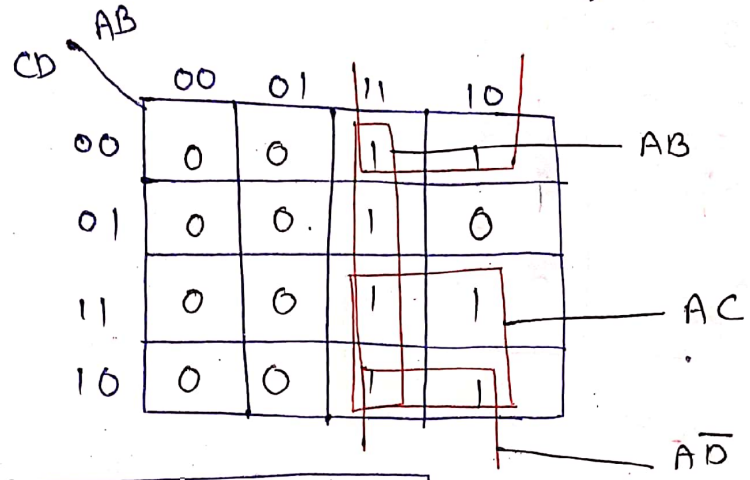
$= ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + (ABC + AB\bar{C})(D+\bar{D})$

$= ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D}$

$= ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D}$

$= m_{15} + m_8 + m_{11} + m_{10} + m_4 + m_{13} + m_{12}$

$= \Sigma_m (8, 10, 11, 12, 13, 14, 15)$ PAGE NO. .....

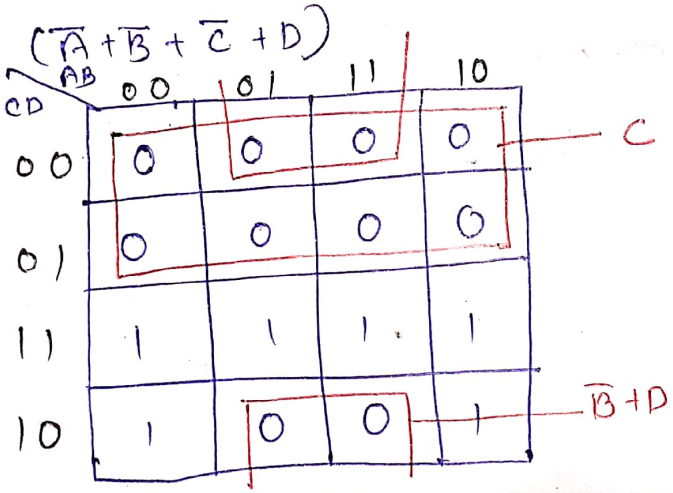


$Y = AB + AC + A\overline{D}$

Q2 Simplify the expression $Y = \Pi (0,1,4,5,6,8,9,12,13,14)$ using the k-map method

Also it can also be written in the POS form

$Y = (A+B+C+D)(A+B+C+\overline{D})(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$
$(\overline{A}+B+C+D)(\overline{A}+B+C+\overline{D})(\overline{A}+\overline{B}+C+D)(\overline{A}+\overline{B}+C+\overline{D})$
$(\overline{A}+\overline{B}+\overline{C}+D)$



$Y = C(\overline{B}+D)$

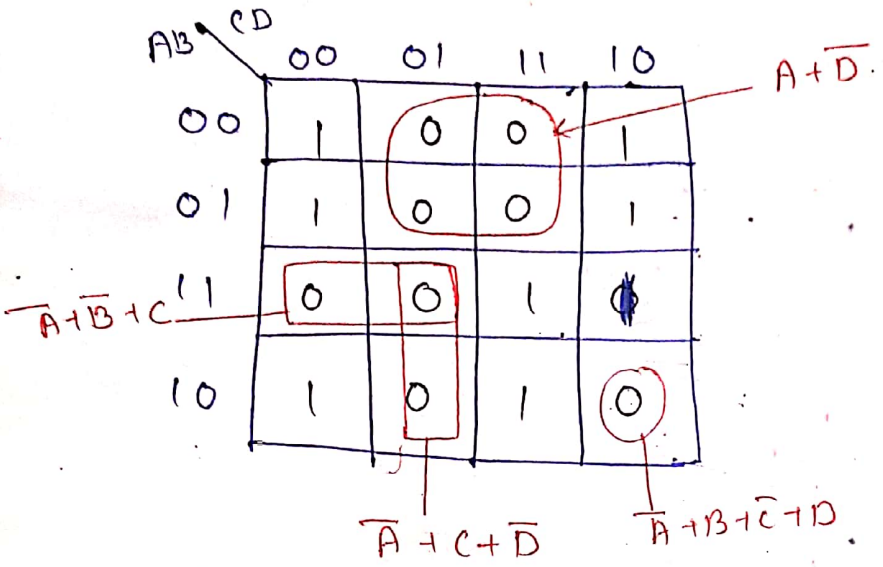
Q Minimize the given function using k-map and convert minimized function into pos form

$f = (A, B, C, D) = \Sigma(1, 3, 5, 7, 9, 10, 12, 13)$

Ans

$f(A, B, C, D) =$

$(A + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + C + \overline{D})(\overline{A} + \overline{B} + C)$



$Y = (A + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + C + \overline{D})(\overline{A} + \overline{B} + C)$

Don't Care Condition

Some input combination never occur during the normal input conditions. Such i/p are considered as don't care condition. It can be used for further simplification of the circuit.

Q Simplify the Boolean equation

$F(A, B, C, D) = \Sigma_m(1, 3, 7, 11, 15) + \Sigma_d(0, 2, 5)$

DETAILED LECTURE NOTES

Ans

CD \ AB	00	01	11	10
00	d	0	0	0
01	1	d	0	0
11	1	1	1	1
10	d	0	0	0

CD

$\bar{A}\bar{B}$

$Y = \bar{A}\bar{B} + CD$

Quine-Mc Cluskey minimization Technique (Tabular method)

It is used when number of variable is very longe. because it is very difficult to minimize the expression

→ Prime Implicant longest possible group of 1

→ Essential Prime Implicant having one minterm that can not be combine in any other way

Ex $Y(A,B,C,D) = \sum_m (0,1,3,7,8,9,11,15)$

Step 1 Arrange the table in as per of the no of ones

→ 0 ones

→ 1 ones

→ 2 ones

→ 3 ones

→ 4 ones

Group no	minterm	Binary Representation A	B	C	D
0	m_0	0	0	0	0
1	m_1	0	0	0	1
	m_8	1	0	0	0
2	m_3	0	0	1	1
	m_9	1	0	0	1
3	m_7	0	1	1	1
	m_{11}	1	0	1	1
4	m_{15}	1	1	1	1

Step - 2

Group	Matched Pairs	Bin Rep A	B	C	D
0	$m_0 - m_1$	0	0	0	–
	$m_0 - m_8$	–	0	0	0
1	$m_1 - m_3$	0	0	–	1
	$m_1 - m_9$	–	0	0	1
	$m_8 - m_9$	1	0	0	–
2	$m_3 - m_7$	0	–	1	1
	$m_3 - m_{11}$	–	0	1	1
	$m_9 - m_{11}$	1	0	–	1
3	$m_7 - m_{15}$	–	1	1	1
	$m_{11} - m_{15}$	1	–	1	1

In second step the n group is compared with $(n+1)^{th}$ term. It is the matched variable if only one variable is in difference

Step 3 Again compare the nth term to (n+1)th term of the table 2.

Group	Mathred pair	Binary Rep A	B	C	D	
0	$m_0 - m_1$	—	0	0	—	$\bar{B}\bar{C}$
	$m_0 - m_8$	—	0	0	—	
	$m_8 - m_9$	1	0	0	—	
1	$m_1 - m_3$	—	0	—	1	$\bar{B}D$
	$m_1 - m_9$	—	0	—	1	
	$m_9 - m_{11}$	1	0	—	1	
2	$m_3 - m_7$	—	—	1	1	CD
	$m_3 - m_{11}$	—	—	1	1	
	$m_7 - m_{15}$	—	1	1	1	
	$m_{11} - m_{15}$	1	—	1	1	

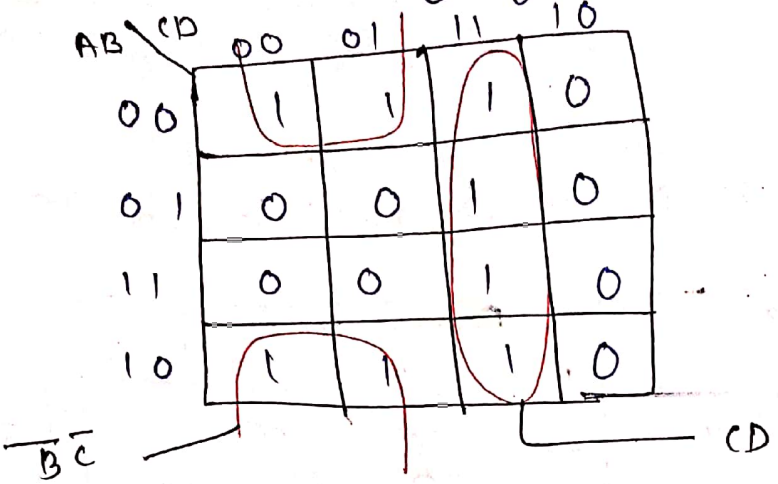
Step 4 Draw prime implicant table

P.I	min term involved	0	1	3	7	8	9	11	15
$\bar{B}\bar{C}$	0, 1, 8, 9	X	X			X	X		
$\bar{B}D$	1, 3, 9, 11		X	X			X	X	
CD	3, 7, 11, 15			X	X			X	X

→ Circle the cross which one single in a column
→ Consider prime implicant for the circle.

$$Y = \overline{B}\,\overline{C} + CD$$

It can also be simplify by the k-map method



$$Y = \overline{B}\,\overline{C} + CD$$