Appendix A3.1

Mathematical statement of MAMS

Table A3.1 Sets, parameters, and variables for core CGE modules of MAMS model

Symbol	Explanation
SETS	
$a \in A$	activities
$a \in ACES\ (\subset A)$	activities with CES function between Value Added and Intermediate inputs
$a \in ALEO\ (\subset A)$	activities with Leontief function between value added and intermediate inputs
$c \in C$	commodities
$c \in CD (\subset C)$	commodities with domestic sales of domestic output
$c \in CDN (\subset C)$	commodities not in CD
$c \in CE (\subset C)$	exported commodities
$c \in CEN (\subset C)$	commodities not in CE
$c \in CECETN (\subset C)$	exported commodities without CET function
$c \in CINF (\subset C)$	infrastructure commodity
$c \in CM (\subset C)$	imported commodities
$c \in CMN (\subset C)$	commodities not in CM
$c \in CT (\subset C)$	transaction service commodities
$f, f' \in F$	factors
$f \in FCAP (\subset F)$	capital factors
$f \in FCAPGOV (\subset FCAP)$	government capital factors
$f \in FEXOG (\subset F)$	factors with exogenous growth rates
$f \in FLABN (\subset F)$	non-labour factors
$f \in FUEND (\subset F)$	factors with endogenous unemployment
$h \in H (\subset INSDNG)$	households (incl. NGOs)
$i \in INS$	institutions (domestic and rest of world)
$i \in INSD (\subset INS)$	domestic institutions
$i \in INSDNG (\subset INSD)$	domestic non-government institutions

Table A3.1 (cont'd)

Symbol	Explanation
$i \in INSNG (\subset INS)$	non-government institutions
$(f,a) \in MFA$	mapping showing that disaggregated factor f is used in activity a
$t \in T$	time periods
PARAMETERS—LATIN LETTERS	
$capcomp_{c,f}$	quantity of commodity c per unit of new capital f
$cwts_c$	weight of commodity c in the CPI
$depr_f$	depreciation rate for factor f
$dintrat_{i,t}$	interest rate on government bonds for domestic institution i
$dwts_c$	domestic sales price weights
$fdebtrelief_{i,t}$	foreign debt relief for domestic institution i
$fdi_{i,t}$	foreign direct investment by institution i (rest of world) (FCU)
$fintrat_{i,t}$	interest rate on foreign debt for domestic institution i (paid)
$fintratdue_{i,t}$	interest rate on foreign debt for domestic institution i (due)
$fprd_{f,a,t}$	productivity of factor f in activity a
$gbdist_i$	distortion factor for government borrowing from institution i
$gfcfshr_{f,i,t}$	share of gross fixed capital formation for institution i in capital factor f
$ica_{c,a}$	quantity of c as intermediate input per unit of aggregate intermediate in activity a
$icd_{c,c',t}$	trade input of c per unit of commodity c' produced & sold domestically
$ice_{c,c',t}$	trade input of c per unit of commodity c' exported
$icm_{c,c',t}$	trade input of c per unit of commodity c' imported
$ifa_{f,a}$	quantity of capital f per unit of government activity a
$igf_{c,f,t}$	quantity of gov consumption per unit of gov infrastructure capital stock f
$inta_a$	quantity of aggregate intermediate input per unit of activity a

Table A3.1 (cont'd)

Symbol	Explanation
iva_{a}	quantity of value-added per unit of activity a
$mps01_{i}$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$mpsbar_{i,t}$	Exogenous component in savings rate for domestic institution i
$poptot_{t}$	total population by year
$pwm_{c,t}$	import world price of c (FCU)
$pwse_{c,t}$	world price for export substitutes (FCU)
$qdst_{c,i,t}$	quantity of stock (inventory) change
$\overline{qe}_{c,t}$	export demand for c if pwe = pwse (world price for substitutes)
$qfhhtot_{f,t}$	total household stock of exogenous, non-labour factors
$qfinsadj_{i,f,t}$	exogenous factor stock adjustment
$qfpc_{i,f,t}$	per-capita quantity of exogenous-supply factor f by institution i and year t
$rqgadj_{c,c',t}$	parameter linking government consumption growth across commodities
$shii_{i,i'}$	share of net income of i' to i (i'—INSDNG)
$ta_{a,t}$	tax rate for activity a
$te_{c,t}$	export tax rate
$tf_{f,t}$	direct tax rate for factor f
$tfp01_{a,t}$	0-1 parameter for activities with endogenous TFP growth
$tfpelasqg_{a,f,t}$	elasticity of TFP for activity a with respect to government capital stock f
$tfpelastrd_{a}$	elasticity of TFP for a with respect to GDP trade share
$tfptrdwt_{t,t'}$	weight of period t' in tfp-trade link in t
$tgap_{t,t'}$	gap between t and t' (years used for calculation of expected growth rate for QA)
$tins01_{i}$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates

Table A3.1 (cont'd)	
Symbol	**Explanation**
$tinsbar_{i,t}$	exogenous component in direct tax rate for domestic institution i
$tm_{c,t}$	import tariff rate
$tq_{c,t}$	rate of sales tax
$trnsfr_{i,i',t}$	Exogenous transfer from institution i' to institution i
$trnsfr_{f,i',t}$	Exogenous transfer from institution i' to factor f
$trnsfrpc_{i,i',t}$	per-capita transfers from institution i' to household institution i
$tva_{a,t}$	rate of value-added tax for activity a
PARAMETERS— GREEK LETTERS	
α_{ac_c}	shift parameter for domestic commodity aggregation function
$\alpha_{vag_{a,t}}$	exogenous component of efficiency (TFP) for activity a
α_{q_c}	Armington function shift parameter
α_{t_c}	CET function shift parameter
$\beta_{h_{a,c,h}}$	marginal share of household consumption on home commodity c from activity a
$\beta_{m_{c,h}}$	marginal share of household consumption spending on marketed commodity c
δ_{ac_a}	share parameter for domestic commodity aggregation function
δ_{q_c}	Armington function share parameter
δ_{t_c}	CET function share parameter
$\delta_{va_{f,a}}$	CES value-added function share parameter for factor f in activity a
$\gamma_{h_{a,c,h}}$	per capita household subsistence consumption of home commodity c from activity a
$\gamma_{m_{c,h}}$	per capita household subsistence cons of marketed commodity c
ρ_{ac_c}	domestic commodity aggregation function exponent
ρ_{q_c}	Armington function exponent

Table A3.1 (cont'd)	
Symbol	**Explanation**
ρsav_i	elasticity of savings rate with respect to per-capita income for institution (household) h
ρt_c	CET function exponent
ρva_a	CES value-added function exponent
$\theta_{a,c}$	yield of output c per unit of activity a
VARIABLES	
$ALPHAVA_{a,t}$	efficiency parameter in the CES value-added function
$ALPHAVA2_{a,t}$	endogenous TFP trend term by a
$CALTFPG_t$	calibration factor for TFP growth
CPI_t	consumer price index
$GBORMS_{i,t}$	implicit government Central Bank borrowing (deficit monetization) from institution i
$GBORMSTOT_t$	total government Central Bank borrowing (deficit monetization)
$GBOR_{i,t}$	change in holding of government bonds for domestic institution i
$GBORTOT_t$	total change in holding of government bonds
$DKGOV_{f,t}$	gross government investment in f
$DKINS_{i,f,t}$	gross change in capital stock (investment in) f for institution i
$DMPS_t$	uniform point change in savings rate of selected domestic institutions
DPI_t	producer price index for non-traded output
$DTINS_t$	uniform point change in direct tax rate of selected domestic institutions
EG_t	government expenditures
$EH_{h,t}$	consumption spending for household
EXR_t	exchange rate (LCU per unit of FCU)
$FBOR_{i,t}$	foreign borrowing for domestic institution i
$FDEBT_{i,t}$	foreign debt for domestic inst i

Table A3.1 (cont'd)	
Symbol	**Explanation**
$FGRANT_{i,t}$	foreign grants to domestic institution i (FCU)
$GDEBT_{i,t}$	endowment of government bonds for i
$GDPREAL_t$	real GDP at market prices
$GDPREALFC_t$	real GDP at factor cost
$GSAV_t$	government savings
$INSSAV_{i,t}$	savings of domestic non-government institution i
$INVVAL_{i,t}$	investment value for institution i
$MPS_{i,t}$	marginal propensity to save for domestic non-gov institution i
$MPSADJ_t$	savings rate scaling factor
$PA_{a,t}$	activity price (unit gross revenue)
$PDD_{c,t}$	demand price for commodity c produced & sold domestically
$PDS_{c,t}$	supply price for commodity c produced & sold domestically
$PE_{c,t}$	export price (domestic currency)
$PINTA_{a,t}$	aggregate intermediate input price for activity a
$PK_{f,t}$	price of new capital stock f
$PM_{c,t}$	import price (domestic currency)
$POP_{i,t}$	population by household
$PQ_{c,t}$	composite commodity price
$PVA_{a,t}$	value-added price (factor income per unit of activity)
$PVAAVG_t$	average value-added price
$PWE_{c,t}$	export world price of c (FCU)
$PX_{c,t}$	aggregate producer price for commodity
$PXAC_{a,c,t}$	price of commodity c from activity a

Table A3.1 (cont'd)

Symbol	Explanation
$QD_{c,t}$	quantity sold domestically of domestically produced c
$QF_{f,a,t}$	quantity demanded of factor f by activity a
$QFINS_{i,f,t}$	real endowment of factor f for institution i
$QG_{c,t}$	quantity of government consumption of commodity c
$QH_{c,h,t}$	quantity consumed by household h of marketed commodity c
$QHA_{a,c,h,t}$	quantity consumed of home commodity c from act a by hhd h
$QINTA_{a,t}$	quantity of aggregate intermediate input used by activity a
$QINT_{c,a,t}$	quantity of commodity c as intermediate input to activity a
$QINV_{c,t}$	quantity of investment demand for commodity c
$QM_{c,t}$	quantity of imports of commodity c
$QQ_{c,t}$	quantity of goods supplied to domestic market (composite supply)
$QT_{c,t}$	quantity of trade and transport demand for commodity c
$QVA_{a,t}$	quantity of (aggregate) value-added
$QX_{c,t}$	aggregated quantity of domestic output of commodity
$QXAC_{a,c,t}$	quantity of output of commodity c from activity a
$QGGRW_{t}$	real government consumption growth for all c in t relative to t-1
$QGGRWC_{c,t}$	real government consumption growth of c in t relative to t-1
$SHIF_{i,f,t}$	share of institution i in income of factor f
$TINS_{i,t}$	direct tax rate for domestic non-government institution i
$TINSADJ_{t}$	direct tax scaling factor
$TRDGDP_{t}$	foreign trade as share of GDP
$TRII_{i,i',t}$	transfers from institution i' to i (both in the set INSDNG)

Table A3.1 (cont'd)

Symbol	**Explanation**
$WF_{f,t}$	average price of factor
$WFDIST_{f,a,t}$	wage distortion factor for factor f in activity a
$WFRES_{f,t}$	reservation wage for factor f
$YF_{f,t}$	income of factor f
YG_{t}	government revenue
$YI_{i,t}$	income of domestic non-government institution
$YIF_{i,f,t}$	income to domestic institution i from factor f
$YIINT_{i,t}$	interest payment on government bonds to institution i

Table A3.2 Equations for the core CGE module of MAMS model

#	Equation	Domain
Price Block		
(1)	$PM_{c,t} = pwm_{c,t} \cdot \left(1 + tm_{c,t}\right) \cdot EXR_t + \sum_{c' \in C} \left(PQ_{c',t} \cdot icm_{c',c,t}\right)$	$c \in CM$ $t \in T$
(2)	$PE_{c,t} = \overline{PWE}_{c,t} \cdot \left(1 - te_{c,t}\right) \cdot EXR_t - \sum_{c' \in C} \left(PQ_{c',t} \cdot ice_{c',c,t}\right)$	$c \in CE$ $t \in T$
(3)	(a) $PDS_{c,t} \geq PE_{c,t}$ (b) $QE_{c,t} \geq 0$ (c) $\left(PDS_{c,t} - PE_{c,t}\right) \cdot \left(QE_{c,t} - 0\right) = 0$ [Complementary slackness relationship: 1. If domestic price exceeds export price then export quantity is zero. 2. If export quantity excceds zero, then domestic price equals export price]	$c \in (CD \cap CECETN)$ $t \in T$
(4)	$PDD_{c,t} = PDS_{c,t} + \sum_{c' \in C} \left(PQ_{c',t} \cdot icd_{c',c,t}\right)$	$c \in CD$ $t \in T$
(5)	$PQ_{c,t} \cdot \left(1 - tq_{c,t}\right) \cdot QQ_{c,t} = PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t}$	$c \in (CD \cup CM)$ $t \in T$
(6)	$PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t}$	$c \in (CD \cup CE)$ $t \in T$
(7)	$PA_{a,t} = \sum_{c \in C} PXAC_{a,c,t} \cdot \theta_{a,c}$	$a \in A$ $t \in T$
(8)	$PINTA_{a,t} = \sum_{c \in C} PQ_{c,t} \cdot ica_{c,a}$	$a \in A$ $t \in T$
(9)	$PA_{a,t} \cdot (1 - ta_{a,t}) \cdot QA_{a,t} = PVA_{a,t} \cdot QVA_{a,t} + PINTA_{a,t} \cdot QINTA_{a,t}$	$a \in A$ $t \in T$
(10)	$\overline{CPI}_t = \sum_{c \in C} PQ_{c,t} \cdot cwts_c$	$t \in T$
(11)	$DPI_t = \sum_{c \in CD} PDS_{c,t} \cdot dwts_c$	$t \in T$
Production and trade block		
(12)	$QVA_{a,t} = iva_a \cdot QA_{a,t}$	$a \in ALEO$ $t \in T$
(13)	$QINTA_{a,t} = inta_a \cdot QA_{a,t}$	$a \in ALEO$ $t \in T$

Table A3.2 (cont'd)

#	Equation	Domain
(14)	$QVA_{a,t} = ALPHAVA_{a,t} \cdot \left(\sum_{f \in F} \delta_{va\,f,a} \cdot \left(fprd_{f,a,t} \cdot QF_{f,a,t} \right)^{-\rho_{va_a}} \right)^{-\frac{1}{\rho_{va_a}}}$	$a \in A$ $t \in T$
(15)	$WF_{f,t} \cdot \overline{WFDIST}_{f,a,t} = PVA_{a,t} \cdot \left(1 - tva_{a,t}\right) \cdot QVA_{a,t} \cdot \left(\sum_{f' \in F} \delta_{va\,f',a} \cdot \left(fprd_{f',a,t} \cdot QF_{f',a,t} \right)^{-\rho_{va_a}} \right)^{-1} \cdot \delta_{va\,f,a} \cdot fprd_{f,a,t}^{-\rho_{va_a}} \cdot QF_{f,a,t}^{-\rho_{va_a}-1}$	$a \in A$ $f \in F$ $t \in T$
(16)	$QINT_{c,a,t} = ica_{c,a} \cdot QINTA_{a,t}$	$c \in C$ $a \in A$ $t \in T$
(17)	$QXAC_{a,c,t} + \sum_{h \in H} QHA_{a,c,h,t} = \theta_{a,c} \cdot QA_{a,t}$	$c \in C$ $a \in A$ $t \in T$
(18)	$QX_{c,t} = \alpha_{ac_c} \cdot \left(\sum_{a \in A} \delta_{ac\,a,c} \cdot QXAC_{a,c,t}^{-\rho_{ac_c}} \right)^{-\frac{1}{\rho_{ac_c}}}$	$c \in (CE \cup CD)$ $t \in T$
(19)	$\frac{PXAC_{a,c,t}}{PX_{c,t}} = QX_{c,t} \cdot \sum_{a \in A} \left(\delta_{ac\,a,c} \cdot QXAC_{a,c,t}^{-\rho_{ac_c}} \right)^{-1} \cdot \delta_{ac\,a,c} \cdot QXAC_{a,c,t}^{-\rho_{ac_c}-1}$	$a \in A$ $c \in C$ $t \in T$
(20)	$QX_{c,t} = \alpha_{t_c} \cdot \left(\delta_{t_c} \cdot QE_{c,t}^{\rho_{t_c}} + (1 - \delta_{t_c}) \cdot QD_{c,t}^{\rho_{t_c}} \right)^{\frac{1}{\rho_{t_c}}}$	$c \in (CD \cap CECET)$ $t \in T$
(21)	$\frac{QE_{c,t}}{QD_{c,t}} = \left(\frac{PE_{c,t}}{PDS_{c,t}} \cdot \frac{1 - \delta_{t_c}}{\delta_{t_c}} \right)^{\frac{1}{\rho_{t_c}-1}}$	$c \in (CD \cap CECET)$ $t \in T$
(22)	$QX_{c,t} = QD_{c,t} + QE_{c,t}$	$c \in (CD \cap CEN) \cup (CE \cap CDN) \cup (CD \cap CECETN)$ $t \in T$

Table A3.2 (cont'd)

#	Equation	Domain
(23)	$QE_{c,t} = \overline{qe}_{c,t} \cdot \left(\frac{PWE_{c,t}}{pwse_{c,t}} \right)^{\rho e_c}$	$c \in CED$ $t \in T$
(24)	$QQ_{c,t} = \alpha_{qc} \cdot \left(\delta_{qc} \cdot QM_{c,t}^{-\rho_{qc}} + (1 - \delta_{qc}) \cdot QD_{c,t}^{-\rho_{qc}} \right)^{-\frac{1}{\rho_{qc}}}$	$c \in (CM \cap CD)$ $t \in T$
(25)	$\frac{QM_{c,t}}{QD_{c,t}} = \left(\frac{PDD_{c,t}}{PM_{c,t}} \cdot \frac{\delta_{qc}}{1 - \delta_{qc}} \right)^{\frac{1}{1+\rho_{qc}}}$	$c \in (CM \cap CD)$ $t \in T$
(26)	$QQ_{c,t} = QD_{c,t} + QM_{c,t}$	$c \in (CD \cap CMN) \cup (CM \cap CDN)$ $t \in T$
(27)	$QT_{c,t} = \sum_{c' \in C'} \left(icm_{c,c',t} \cdot QM_{c',t} + ice_{c,c',t} \cdot QE_{c',t} + icd_{c,c',t} \cdot QD_{c',t} \right)$	$c \in CT$ $t \in T$
Domestic institution block		
(28)	$YF_{f,t} = \sum_{a \in A} WF_{f,t} \cdot \overline{WFDIST}_{f,a,t} \cdot QF_{f,a,t} + trnsfr_{f,row,t} \cdot EXR_t$	$f \in F$ $t \in T$
(29)	$SHIF_{i,f,t} = \frac{QFACINS_{i,f,t}}{\sum_{i' \in INS} QFACINS_{i',f,t}}$	$i \in INS$ $f \in F$ $t \in T$
(30)	$YIF_{i,f,t} = SHIF_{i,f,t} \cdot \left[\left(1 - tf_{f,t} \right) \cdot YF_{f,t} \right]$	$i \in$ $f \in F$ $t \in T$
(31)	$YIINT_{i,t} = gintrat_{i,t} \cdot GDEBT_{i,t} - fintrat_{i,t} \cdot FDEBT_{i,t} \cdot EXR_t$	$i \in$ $INSDNG$ $t \in T$
(32)	$TRII_{i,i',t} = shii_{i,i'} \cdot (1 - MPS_{i',t}) \cdot (1 - TINS_{i',t}) \cdot YI_{i',t}$	$i \in INS$ $i' \in$ $INSDNG$ $t \in T$

Table A3.2 (cont'd)

#	Equation	Domain
(33)	$YI_{i,t} = \sum_{f \in F} YIF_{i,f,t} + \sum_{i' \in INSDNG'} TRII_{i,i',t} + YIINT_{i,t}$ $+trnsfri_{i,gov,t} \cdot \overline{CPI_t} + trnsfrpc_{i,gov,t} \cdot POP_{i,t} \cdot \overline{CPI_t}$ $+trnsfri_{i,row,t} \cdot EXR_t + trnsfrpc_{i,row,t} \cdot POP_{i,t} \cdot EXR_t$	$i \in INSDNG$ $t \in T$
(34)	$TINS_{i,t} = tinsbar_{i,t} \cdot \left(1 + \overline{TINSADJ_t} \cdot tins01_i\right) + DTINS_t \cdot tins01_i$	$i \in INSDNG$ $t \in T$
(35)	$MPS_{i,t} = mpsbar_{i,t} \cdot \left(\frac{\left(1 - TINS_{i,t}\right) \cdot YI_{i,t}}{\overline{POP_{i,t}}}\right)^{\rho sav_i - 1}$ $\cdot\left(1 + \overline{MPSADJ_t} \cdot mps01_i\right) + \overline{DMPS_t} \cdot mps01_i$	$i \in INSDNG$ $t \in T$
(36)	$INSSAV_{i,t} = MPS_{i,t} \cdot \left(1 - TINS_{i,t}\right) \cdot YI_{i,t}$	$i \in INSDNG$
(37)	$EH_{h,t} = \left(1 - \sum_{i \in INSDNG} shii_{i,h}\right) \cdot \left(1 - MPS_{h,t}\right) \cdot (1 - TINS_{h,t}) \cdot YI_{h,t}$	$h \in H$ $t \in T$
(38)	$QH_{c,h,t} = \overline{POP}h,t \cdot \left(\gamma_{m}c,h + \frac{\beta_{m}c,h \cdot \left(\left[\frac{EH_{h,t}}{\overline{POP}_{h,t}}\right] - \sum_{c' \in C} PQ_{c',t} \cdot \gamma_{m}c',h - \sum_{a \in A} \sum_{c' \in C} PXAC_{a,c',t} \cdot \gamma_{a,c',h}\right)}{PQ_{c,t}}\right)$	$c \in C$ $h \in H$ $t \in T$
(39)	$QHA_{a,c,h,t} = \overline{POP}h,t \cdot \left(\gamma_{a,c,h} + \frac{\beta_{a,c,h} \cdot \left(\left[\frac{EH_{h,t}}{\overline{POP}_{h,t}}\right] - \sum_{c' \in C} PQ_{c',t} \cdot \gamma_{m}c',h - \sum_{a' \in A} \sum_{c' \in C} PXAC_{a',c',t} \cdot \gamma_{a',c',h}\right)}{PXAC_{a,c,t}}\right)$	$a \in A$ $c \in C$ $h \in H$ $t \in T$

Table A3.2 (cont'd)

#	Equation	Domain
(40)	$YG_t = \sum_{i \in INSDNG} TINS_{i,t} \cdot YI_{i,t} + \sum_{f \in F} tf_{f,t} \cdot YF_{f,t} + \sum_{a \in A} ta_{a,t} \cdot PA_{a,t} \cdot QA_{a,t} + \sum_{a \in A} tva_{a,t} \cdot PVA_{a,t} \cdot QVA_{a,t} + \sum_{c \in CM} tm_{c,t} \cdot pwm_{c,t} \cdot QM_{c,t} + \sum_{c \in CE} te_{c,t} \cdot \overline{PWE}_{c,t} \cdot QE_{c,t} \cdot EXR_t + \sum_{c \in C} tq_{c,t} \cdot PQ_{c,t} \cdot QQ_{c,t} + \sum_{f \in F} YIF_{gov,f,t} + \sum_{i \in INSDNG} TRII_{gov,i,t} + trnsfr_{gov,row,t} \cdot EXR_t$	$t \in T$
(41)	$EG_t = \sum_{c \in C} PQ_{c,t} \cdot QG_{c,t} + \sum_{i \in INSDNH} trnsfr_{i,gov,t} \cdot \overline{CPI}_t + \sum_{h \in H} trnsfrpc_{h,gov,t} \cdot \overline{POP}_{h,t} \cdot \overline{CPI}_t + trnsfr_{row,gov,t} \cdot EXR_t + \sum_{i \in INS} gintrat_{i,t} \cdot GDEBT_{i,t} + fintrat_{gov,t} \cdot FDEBT_{gov,t} \cdot EXR_t$	$t \in T$
(42)	$QG_{c,t} = QG_{c,t-1} \cdot \left(1 + \overline{QGGRW}_t + \sum_{c' \in C} qg01_{c,c',t} \cdot \overline{QGGRWC}_{c',t} \right)$	$c \in C$ $c \notin CINF$ $t \in T$ $t > 1$
(43)	$QG_{c,t} = \sum_{\substack{i \in INS \\ f \in F}} igf_{c,f,t} \cdot QFINS_{i,f,t}$	$c \in CINF$ $t \in T$ $t > 1$
(44)	$GSAV_t = YG_t - EG_t$	$t \in T$

Table A3.2 (cont'd)

#	Equation	Domain
Investment block		
(45)	$(a)\ DKGOV_{f,t} \geq \sum_{a \in A \mid (f,a) \in MFA} ifa_{f,a,t} \cdot QA_{a,t} \cdot EXP\left(\ln\left(\frac{QA_{a,t}}{QA_{a,t-1}}\right)\right)\Big\|_{f \in FCAPGOVSER} + \left(\left(1 + \sum_{c \in C} qg01_{f,c,t} \cdot \overline{QGGRWC}_{c,t}\right) \cdot QFINS_{gov,f,t}\right)\Big\|_{f \in FCAPGOVINF} - QFINS_{gov,f,t} \cdot (1 - depr_{f,t})$ $(b)\ DKGOV_{f,t} \geq 0$ $(c)\ \left(DKGOV_{f,t} - DKGOVDEM_{f,t}\right) \cdot \left(DKGOV_{f,t} - 0\right) = 0$ where $DKGOVDEM_{f,t}$ = right-hand of part *(a)* of Equation 45 [*Complementary slackness relationship*: 1. *If government investment exceeds its demand then this investment level is zero.* 2. *If the government investment level is above zero, then it equals its demand.*]	$f \in FCAPGOV$ $t \in T$ $t > 1$
(46)	$DKINS_{gov,f,t} = DKGOV_{f,t}$	$f \in FCAPGOV$ $t \in T$ $t > 1$
(47)	$PK_{f,t} = \sum_{c \in C} capcomp_{c,f} \cdot PQ_{c,t}$	$f \in FCAP$ $t \in T$
(48)	$\sum_{f \in FCAPGOV} PK_{f,t} \cdot DKINS_{gov,f,t} = GSAV_t - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,gov,t} + \overline{GBORTOT}_t + \overline{GBORMSTOT}_t + \left(\overline{FBOR}_{gov,t} + \overline{FGRANT}_{gov,t}\right) \cdot EXR_t$	$t \in T$
(49)	$GBOR_{i,t} = \frac{gbdist_i \cdot INSSAV_{i,t}}{\sum_{i' \in INSDNG'} gbdist_{i'} \cdot INSSAV_{i',t}} \cdot \overline{GBORTOT}_t$	$i \in INSDNG$ $t \in T$
(50)	$GBORMS_{i,t} = \frac{gbdist_i \cdot INSSAV_{i,t}}{\sum_{i' \in INSDNG'} gbdist_{i'} \cdot INSSAV_{i',t}} \cdot \overline{GBORMSTOT}_t$	$i \in INSDNG$ $t \in T$
(51)	$INVVAL_{i,t} = INSSAV_{i,t} - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,i,t} - GBOR_{i,t} - GBORMS_{i,t} + \left(\overline{FBOR}_{i,t} + \overline{FGRANT}_{i,t} + fdi_{i,t}\right) \cdot EXR_t$	$i \in INSNG$ $t \in T$

Table A3.2 (cont'd)

#	Equation	Domain
Investment block		
(52)	$PK_{f,t} \cdot DKINS_{i,f,t} = gfcfshr_{f,i,t} \cdot INVVAL_{i,t}$	$i \in INSNG$ $f \in FCAP$ $t \in T$
(53)	$QINV_{c,t} = \sum_{f \in FCAP} \left(capcomp_{c,f} \cdot \sum_{i \in INS} DKINS_{i,f,t} \right)$	$c \in C$ $t \in T$
(54)	$\sum_{c \in CM} pwm_{c,t} \cdot QM_{c,t} + \frac{\sum_{f \in F} YIF_{row,f,t}}{\text{EXR}_t} + \frac{\sum_{i \in INSDNG} TRII_{row,i,t}}{\text{EXR}_t} + trnsfr_{row,gov,t} + \sum_{i \in INSD} fintrat_{i,t} \cdot FDEBT_{i,t} = \sum_{c \in CE} \overline{PWE}_{c,t} \cdot QE_{c,t} + \sum_{i \in INSDNH} trnsfr_{i,row,t} + \sum_{h \in H} trnsfrpc_{h,row,t} \cdot \overline{POP}_{h,t} + \sum_{f \in F} trnsfr_{f,row,t} + \sum_{i \in INSD} \left(\overline{FBOR}_{i,t} + \overline{FGRANT}_{i,t} \right) + fdi_{row,t}$	$t \in T$
(55)	$\sum_{a \in A} QF_{f,a,t} = \left(1 - UERAT_{f,t}\right) \cdot \sum_{i \in INS} QFINS_{i,f,t}$	$f \in F$ $t \in T$
(56)	$WFRES_{f,t} = WF_f^0 \cdot \left(\frac{QHPC_t}{QHPC^0} \right)^{\varphi_f^{wfqhpc}} \cdot \left(\frac{\left(1 - UERAT_{f,t}\right)}{\left(1 - UERAT_f^0\right)} \right)^{\varphi_f^{wferat}} \cdot \left(\frac{CPI_t}{CPI^0} \right)^{\varphi_f^{wfcpi}}$	$f \in FUEND$ $t \in T$
(57)	$(a)\ WF_{f,t} \geq WFRES_{f,t}$ $(b)\ UERAT_{f,t} \geq ueratmin_{f,t}$ $(c)\ \left(WF_{f,t} - WFRES_{f,t}\right) \cdot \left(UERAT_{f,t} - ueratmin_{f,t}\right) = 0$ [*Complementary slackness relationship*: 1. *If wage exceeds reservation wage then unemployment rate is at its minimum.* 2. *If unemployment rate exceeds its minimum, then wage equals reservation wage.*]	$f \in FUEND$ $t \in T$
Asset stock updating and productivity block		
(58)	$QQ_{c\,t} = \sum_{a \in A} QINT_{c,a,t} + \sum_{h \in H} QH_{c,h,t} + QG_{c,t} + QINV_{c,t} + \sum_{i \in INS} qdst_{c,i,t} + QT_{c,t}$	$c \in C$ $t \in T$

Table A3.2 (cont'd)

#	Equation	Domain
(59)	$QFINS_{i,f,t} = (1 - depr_{f,t-1}) \cdot QFINS_{i,f,t-1} + DKINS_{i,f,t-1} + qfinsadj_{i,f,t-1}$	$i \in INS$ $f \in FCAP$ $t \in T$ t>1
(60)	$FDEBT_{i,t} = FDEBT_{i,t-1} + FBOR_{i,t-1}$ $+\left(fintratdue_{i,t-1} - fintrat_{i,t-1}\right) \cdot FDEBT_{i,t-1} - fdebtrelief_{i,t-1}$	$i \in INSD$ $t \in T$ $t > 1$
(61)	$GDEBT_{i,t} = GDEBT_{i,t-1} + GBOR_{i,t-1}$	$i \in INSDNG$ $t \in T$ $t > 1$
(63)	$GDPREAL_t = \sum_{c \in C} \sum_{h \in H} PQ_c^0 \cdot QH_{c,h,t} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{a,c}^0 \cdot QHA_{a,c,h,t}$ $+ \sum_{c \in C} PQ_c^0 \cdot QG_{c,t} + \sum_{c \in C} PQ_c^0 \cdot QINV_{c,t} + \sum_{c \in C} \sum_{i \in INS} PQ_c^0 \cdot qdst_{c,i,t}$ $+ \sum_{c \in CE} EXR^0 \cdot PWE_c^0 \cdot QE_{c,t} - \sum_{c \in CM} EXR^0 \cdot PWM_c^0 \cdot QM_{c,t}$	$t \in T$
(64)	$ALPHAVA_{a,t} = ALPHAVA2_{a,t} \cdot \prod_{f \in FCAP} \left[\frac{\sum_{i \in INS} QFINS_{i,f,t}}{\sum_{i \in INS} QFINS_{i,f}^0} \right]^{tfpelasqg_{a,f,t}}$ $\cdot \left(\frac{\sum_{t \in T} tfptrdwt_{t,t'} \cdot TRDGDP_{t'}}{TRDGDP^0} \right)^{tfpelastrd_a}$	$a \in A$ $t \in T$ $t > 1$
(65)	$ALPHAVA2_{a,t} = ALPHAVA2_{a,t-1} \cdot \left(1 + \alpha vag_{a,t} + \overline{CALTFPG_t} \cdot tfp01_{a,t}\right)$	$a \in A$ $t \in T$ $t > 1$
(66)	$GDPREALFC_t = \sum_{a \in A} PVA_a^0 \cdot \left(1 - tva_{a,t}^0\right) \cdot QVA_{a,t}$	$t \in T$

Table A3.3 Notation for MDG module of MAMS model

Symbol	Explanation
SETS	
$a \in A$	activities
$b \in B$	student behavioural characteristics ={rep = repeater; dropout = dropout; pass = pass; grdcont = continuing graduate; grdexit = exiting graduate; glentry = entrant to grade 1; grdcyc= pass from last cycle-year; contcyc = pass within cycle}
$b \in BLOG \ (\subset B)$	student behaviour determined by logistic function ={pass, grdcont, glentry}
$b \in BRES \ (\subset B)$	student behaviour determined by residual scaling ={rep = repeater; dropout = dropout; grdexit = exiting graduate}
$c \in C$	commodities
$c \in CEDU \ (\subset C)$	education services ={c-edup = primary; c-edus = secondary; c-edut = tertiary}; can include both private and public education
$c \in CEDUT \ (\subset C)$	tertiary education services = {c-edut}
$c \in CELA$	educational cycle that corresponds to the age at which non-students would enter the labour force
$c \in CHLTH \ (\subset C)$	health services (public) ={c-hlt1g = low-tech; c-hlt2g = medium-tech; c-hlt3g = high-tech}; corresponding private health services labelled with "ng"
$cmdg \in CMDG$	aggregate MDG (non-education) service commodities = {c-hlt = aggregate health in MDG functions, not in C; c-wtsn = water-sanitation services}
$c \in CWTSN \ (\subset C)$	water-sanitation service commodities{c-wtsn = water-sanitation services}
$eduarg \in EDUARG$	arguments in CE function for educational behaviour ={edu-qual = quantity of services per student; w-prem = semiskilled-unskilled wage ratio; w-prem2 = skilled-semiskilled wage ratio; mdg4 = under-five mortality rate; fcapinf = infrastructure capital stocks; qhpc = per-capita hhd consumption}
$f \in FEXOG$	factors with exogenous growth
$f \in FLAB$	labour factors {f-labn = less than completed secondary education; f-labs = complete secondary education (without completed tertiary); f-labt = completed tertiary education
$h \in H$	households (excl. NGOs) ={h = the single household}
$i \in INSG$	government institution
$i \in INSNGAGG$	aggregate (domestic) non-government institution

Table A3.3 (cont'd)

Symbol	Explanation
$b,b' \in MBB$	mapping between b (in BRES) and b' (in BLOG): ={(rep, dropout).grd, grdexit.grdcont}
$b,b' \in MBB2$	mapping between b (in BRES) and all elements b' (also in BRES) that are related to the same element(s) in BLOG: ={rep.(rep, dropout), dropout.(rep, dropout), grdexit.grdexit}
$c,c' \in MCE$	mapping private and public education into 1 education commodity, by cycle = {c-edup.(c-edup, c-edupng)} where c-edupng is private primary;similarly for c-edus and c-edut
$c,c' \in MCHDC$	human development service c is aggregated to c'
$c,c' \in MCM$	mapping between aggregate (CMDG) and disaggregated MDG service commodities (CHLTH and CWTSN) = {c-hlt.(c-hlt1g, c-hlt2g, c-hlt3g, c-hlt1ng, c-hlt2ng, c-hlt3ng} and {c-wtsn.(c-wtsn)}
$mdg \in MDG$	selected MDG indicators ={mdg2, mdg4, mdg5, mdg7a, mdg7b}
$mcyc(c,b,t',t)$	MDG2 in t is defined as the product over selected combinations of b and t' (where $t' \in T11$) = {pass, g1entry}
$mdg \in MDGSTD$	MDG indicators ={mdg4 = under-5 mortality rate; mdg5 = maternal mortality rate; mdg7a = access to safe water; mdg7b = access to basic sanitation}
$f,c \in MFC$	mapping indicating that students who have completed cycle c belong to labour type f ={f-labn.(c-edup); f-labs.(c-edus); f-labt.(c-edut)}
$mdgarg \in MDGARG$	arguments in CE function for MDGs ={cmdg = agg commodities; mdg = different MDGs; fcapinf = infrastructure capital stocks; hhdconspc = per-capita hhd consumption }
$t \in T$	time periods
$t \in T11$	time periods including preceding years for MDG2 calculation
PARAMETERS	
$\alpha.edu_{b,c}$	constant in logistic function for educational behaviour
$\alpha.educe_{b,c}$	constant in CE function for educational behaviour
$\alpha.mdg_{mdg}$	constant in logistic function for MDG achievement
$\alpha.mce_{mdg}$	constant in CE function for intermediate MDG variable
$\alpha.hd_{c}$	efficiency term in CES aggregation function for human development
$\beta_{edu_{b,c}}$	constant in logistic function for educational behaviour

Table A3.3 (cont'd)

Symbol	Explanation
$\beta_{log_{mdg}}$	constant in logistic function for MDG achievement
$\delta_{hd_{c,i}}$	share parameter for HD CES function
$\varphi_{edu_{b,c,eduarg}}$	elasticity of behaviour b in cycle c with respect to argument eduarg in educational CE function
$\varphi_{m_{mdg,mdgarg}}$	elasticity of mdg with respect to argument mdgarg in CE function for MDG
$\gamma_{edu_{b,c}}$	parameter in logistic function for education
$\gamma_{mdg_{mdg}}$	parameter in logistic function for non-education MDGs
ρ_{hd_c}	exponent in CES aggregation function for human development
$depr_{f,t}$	depreciation rate for factor f
$discrat$	discount rate
$ext_{edu_{b,c}}$	maximum share for educational behaviour b in cycle c
$ext_{mdg_{mdg}}$	maximum value for MDG 7a and 7b; minimum value for MDG 4 and 5
$grdcont01_{c,c'}$	0-1 constant showing that for c' next cycle is c
ord_t	ordinal position of t in the set T
$popg1_t$	population in age cohort entering grade 1
$poplab_t$	population of labour force age
$poplabent_t$	population in age cohort entering labour force (age at end of a model education cycle)
$poptot_t$	total population in t
$qg1entncoh_{c,t}$	number of non-cohort (non-1st-year-primary) entrants to first cycle
$shif^0_{i,f,t}$	share of domestic institution i in income of factor f
$shr_{demot01_{c,c'}}$	0-1 parameter showing that for dropouts from c' the highest cycle is c
$shred^0_{b,c}$	base-year share for behavioural indicator behav in cycle c
shr_{grdcyc_c}	share of graduates (passing students) graduating from cycle c in base-year

Table A3.3 (cont'd)

Symbol	Explanation
$shrlabent_{c,t}$	share of drop-outs and leavers in cycle c that enter the labour force
$shrlabent2_{f,t}$	share of labour type f of labour force entrants without education
$yrcyc_{c}$	years in school cycle for each education cycle c
VARIABLES	
$EDUQUAL_{c,t}$	educational quality in cycle c in year t
EG_{t}	government expenditures
$INVVAL_{i,t}$	investment value for institution i
$MDGVAL_{mdg,t}$	value for MDG indicator mdg in t
$PQ_{c,t}$	price of commodity c in t
$PXAC_{a,c,t}$	price of commodity c from activity a
$QENR_{c,t}$	total number of students enrolled in cycle c in year t
$QENROLD_{c,t}$	number of old students enrolled in cycle c in year t
$QENRNEW_{c,t}$	number of new students enrolled in cycle c in year t
$QFACINS_{i,f,t}$	endowment of labour type f for institution i in t
$QH_{c,h,t}$	consumption of commodity c in t by household h
$QHA_{a,c,h,t}$	quantity consumed of home commodity c from activity a by household h
$QHPC_{t}$	Per-capita household consumption in t
$QQ_{c,t}$	quantity of goods supplied to domestic market (composite supply)
$QXHLTH_{mdg,t}$	government and NGO provision of aggregated health services related to health MDG
$SHREDU_{b,c,t}$	share of students in cycle c with behaviour b in t
$WF_{f,t}$	economy-wide wage for factor f in t
$ZEDU_{b,c,t}$	intermediate variable for educational outcome (defined by CE function; entering logistic function)
$ZMDG_{mdg,t}$	intermediate variable for standard MDGs (4-5-7a-7b) (defined by CE function; entering logistic function)

Table A3.4 Equations for MDG module of MAMS model

#	Equation	Domain
(67)	$QHD_{c,i,t} = \sum_{\substack{c' \in C \\ \mid (c,c') \in MCHDC \\ \cup \mid i \in INSG}} QGc',t + \sum_{\substack{c' \in C \\ \mid (c,c') \in MCHDC \\ \cup \mid i \in INSNGAGG}} \left(QQc',t - QGc',t\right)$	$c \in C$ $i \in I$ $t \in T$
(68)	$QHDAGG_{c,t} = \alpha_{hdc} \cdot \sum_{i \in INS} \left(\delta_{hdc,i} \cdot QHD_{c,i,t}^{-\rho_{hdc}}\right)^{-\frac{1}{\rho_{hdc}}} \Big\mid_{c \in CHDCES} + \sum_{i \in INS} QHDc,i,t \Big\mid_{c \in CHDPRFSUB}$	$c \in C$ $i \in I$ $t \in T$
(69)	$QHPC_t = \frac{\sum_{c \in C} \sum_{h \in H} PQ_c^0 \cdot QH_{c,h,t} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{a,c}^0 \cdot QHA_{a,c,h,t}}{poptot_t}$	$t \in T$
(70)	$EDUQUAL_{c,t} = \frac{QHDAGGc,t}{QENR_{c,t}} \Big/ \frac{QHDAGG_c^0}{QENR_c^0}$	$c \in CEDU$ $t \in T$ $t > 1$
(71)	$QENROLD_{c,t} = SHREDU_{contcyc,c,t-1} \cdot QENR_{c,t-1} + SHREDU_{rep,c,t-1} \cdot QENR_{c,t-1}$	$c \in CEDU$ $t \in T$ $t > 1$
(72)	$QENRNEW_{c,t} = SHREDU_{g1entry,c,t-1} \cdot popg1_t + qg1entncoh_{c,t} + \sum_{c' \in C} grdcont01_{c,c'} \cdot SHREDU_{grdcont,c,t-1} \cdot SHREDU_{grdcyc,c',t-1} \cdot QENR_{c',t-1}$	$c \in CEDU$ $t \in T$ $t > 1$
(73)	$QENR_{c,t} = QENROLD_{c,t} + QENRNEW_{c,t}$	$c \in CEDU$ $t \in T$ $t > 1$
(74)	$SHREDU_{b,c,t} = ext_{edb,c} + \frac{\alpha_{edub,c}}{1 + EXP\left(\gamma_{edub,c} + \beta_{edub,c} \cdot ZEDU_{b,c,t}\right)}$	$b \in BLOG$ $c \in CEDU$ $t \in T$
(75)	$ZEDU_{b,c,t} = \alpha_{educe b,c} \cdot \left(EDUQUAL_{c,t}\right)^{\varphi edu_{b,c,edu-qual}} \cdot \left(\frac{WF_{f-labs,t}}{WF_{f-labn,t}}\right)^{\varphi edu_{b,c,w-prem}} \cdot \left(\frac{WF_{f-labt,t}}{WF_{f-labs,t}}\right)^{\varphi edu_{b,c,w-prem}} \cdot MDGVAL_{mdg4,t}^{\varphi edu_{b,c,mdg4}} \cdot \prod_{f \in FCAPGOVINF} \left(\sum_{i \in INS} QFINS_{i,f,t}\right)^{\varphi edu_{b,c,f}} \cdot QHPC_t^{\varphi edu_{b,c,qhpc}}$	$b \in BLOG$ $c \in C$ $t \in T$

Table A3.4 (cont'd)

#	Equation	Domain
(76)	$SHREDU_{b,c,t} = \left(1 - \sum_{\substack{b' \in BLOG \\ \lvert (b,b') \in MBB}} SHREDU_{b',c,t}\right) \frac{SHREDU^0_{b,c}}{\sum_{\substack{b' \in BRES \\ \lvert (b,b') \in MBB2}} SHREDU^0_{b',c}}$	$b \in BRES$ $c \in CEDU$ $t \in T$
(77)	$SHREDU_{grdcyc,c,t} = \frac{SHREDU_{pass,c,t}}{yrcyc_c} \cdot \left(\frac{shr_{grdcyc_c}}{\frac{1}{yrcyc_c}}\right)^{\frac{1-SHREDU_{pass,c,t}}{1-SHREDU^0_{pass,c}}}$	$c \in CEDU$ $t \in T$
(78)	$SHREDU_{contcyc,c,t} = SHREDU_{pass,c,t} - SHREDU_{grdcyc,c,t}$	$c \in CEDU$ $t \in T$
(79)	$MDGVAL_{mdg2,t} = \prod_{\substack{b \in B, t' \in T11 \\ \lvert mcyc(c\text{-}edup1,b,t',t)}} SHREDU_{b,c\text{-}edup1,t'}$	$t \in T$
(80)	$LABPARTRAT_t = \frac{\sum_{i \in INS,\, f \in FLAB} QFINS_{i,f,t}}{\lvert shif^0_{i,f,t}} = \frac{}{poplabt - \sum_{c \in CELA} QENR_{c,t}} =$	$t \in T$ $t > 1$ $flab \notin FEXOG$
(81)	$QFINS_{i,f,t} = shif^0_{i,f,t} \cdot \left\{ \left(1 - depr_{f,t-1}\right) \cdot \sum_{i' \in INS} QFINS_{i',f,t-1} + \sum_{c,c' \in C} \left\lvert \begin{matrix} shrdemot01_{c,c'} \cdot shrlabent_{c,t} \\ (f,c) \in MFC \\ \cap c \notin CEDUT \end{matrix} \right. \cdot SHREDU_{grdexit,c',t-1} \cdot SHREDU_{grdcyc,c,t-1} \cdot QENR_{c,t-1}$ $+ \sum_{c \in C} \left\lvert \begin{matrix} (f,c) \in MFC \\ \cap c \in CEDUT \end{matrix} \right. \left(shr_{labent_{c,t}} \cdot SHREDU_{grdcyc,c,t-1} \cdot QENR_{c,t-1}\right)$ $+ \sum_{c \in C} \left\lvert \begin{matrix} (f,c) \in MFC \\ \cap c \in CEDUT \end{matrix} \right. \left(shrlabent_{c,t} \cdot SHREDU_{"grdcyc",c,t-1} \cdot QENR_{c,t-1}\right)$ $+ shrlabent2_{f,t} \cdot \left(poplabent_t - \sum_{c \in CELA} QENRNEW_{c,t} \right) \Big\}$	$i \in INS$ $f \in FLAB$ $t \in T$ $t > 1$

Table A3.4 (cont'd)

#	Equation	Domain
(82)	$MDGVAL_{mdg,t} = extmdg_{mdg} + \frac{\alpha mdg_{mdg}}{1 + EXP\left(\gamma mdg_{mdg} + \beta mdg_{mdg} \cdot ZMDG_{mdg,t}\right)}$	$mdg \in MDGSTD$ $t \in T$
(83)	$ZMDG_{mdg,t} = \alpha_{mce_{mdg}} \cdot \left(\prod_{cmdg \in CMDG} \left(\sum_{\substack{c \in C \\ (cmdg,c) \in MCM}} \frac{QQ_{c,t}}{poptot_t} \right)^{\varphi m_{mdg,cmdg}} \right) \cdot \prod_{f \in FCAPGOVINF} \left(\sum_{i \in INS} QFINS_{i,f,t} \right)^{\varphi m_{mdg,f}} \cdot \left(\prod_{mdg' \in MDGSTD} MDGVAL_{mdg',t}^{\varphi m_{mdg,mdg'}} \right) \cdot QHPC_t^{\varphi m_{mdg,hhdconspc}}$	$mdg \in MDGSTD$ $t \in T$