Appendix A3.1: Mathematical statement of MAMS

Table A3.1 Sets, parameters, and variables for core CGE modules of MAMS model

SETS			
Symbol	Explanation	Symbol	Explanation
$a \in A$	activities	$f, f' \in F$	factors
$a \in ACES \ (\subset A)$	activities with CES function between Value Added and Intermediate inputs	$f \in FCAP (\subset F)$	capital factors
$a \in ALEO \ (\subset A)$	activities with Leontief function between value added and intermediate inputs	$f \in FCAPGOV (\subset FCAP)$	government capital factors
$c \in C$	commodities	$f \in FEXOG (\subset F)$	factors with exogenous growth rates
$c \in CD (\subset C)$	commodities with domestic sales of domestic output	$f \in FLABN (\subset F)$	non-labour factors
$c \in CDN (\subset C)$	commodities not in *CD*	$f \in FUEND (\subset F)$	factors with endogenous unemployment
$c \in CE (\subset C)$	exported commodities	$h \in H (\subset INSDNG)$	households (incl. NGOs)
$c \in CEN (\subset C)$	commodities not in *CE*	$i \in INS$	institutions (domestic and rest of world)
$c \in CECETN (\subset C)$	exported commodities without CET function	$i \in INSD (\subset INS)$	domestic institutions
$c \in CINF (\subset C)$	infrastructure commodity	$i \in INSDNG (\subset INSD)$	domestic non-government institutions
$c \in CM (\subset C)$	imported commodities	$i \in INSNG (\subset INS)$	non-government institutions
$c \in CMN (\subset C)$	commodities not in *CM*	$(f, a) \in MFA$	mapping showing that disaggregated factor f is used in activity a
$c \in CT (\subset C)$	transaction service commodities	$t \in T$	time periods

PARAMETERS—LATIN LETTERS			
$capcomp_{c,f}$	quantity of commodity c per unit of new capital f	$pwse_{c,t}$	world price for export substitutes (FCU)
$cwts_c$	weight of commodity c in the CPI	$qdst_{c,i,t}$	quantity of stock (inventory) change

$depr_f$	depreciation rate for factor *f*	$\overline{qe}_{c,t}$	export demand for *c* if *pwe* = *pwse* (world price for substitutes)
$dintrat_{i,t}$	interest rate on government bonds for domestic institution *i*	$qfhhtot_{f,t}$	total household stock of exogenous, non-labour factors
$dwts_c$	domestic sales price weights	$qfinsadj_{i,f,t}$	exogenous factor stock adjustment
$fdebtrelief_{i,t}$	foreign debt relief for domestic institution *i*	$qfpc_{i,f,t}$	per-capita quantity of exogenous-supply factor *f* by institution *i* and year *t*
$fdi_{i,t}$	foreign direct investment by institution *i* (rest of world) (FCU)	$rqgadj_{c,c',t}$	parameter linking government consumption growth across commodities
$fintrat_{i,t}$	interest rate on foreign debt for domestic institution *i* (paid)	$shii_{i,i'}$	share of net income of *i*' to *i* (*i*' ∈ INSDNG)
$fintratdue_{i,t}$	interest rate on foreign debt for domestic institution *i* (due)	$ta_{a,t}$	tax rate for activity *a*
$fprd_{f,a,t}$	productivity of factor *f* in activity *a*	$te_{c,t}$	export tax rate
$gbdist_i$	distortion factor for government borrowing from institution *i*	$tf_{f,t}$	direct tax rate for factor *f*
$gfcfshr_{f,i,t}$	share of gross fixed capital formation for institution *i* in capital factor *f*	$tfp01_{a,t}$	0-1 parameter for activities with endogenous TFP growth
$ica_{c,a}$	quantity of *c* as intermediate input per unit of aggregate intermediate in activity *a*	$tfpelasqg_{a,f,t}$	elasticity of TFP for activity *a* with respect to government capital stock *f*
$icd_{c,c',t}$	trade input of *c* per unit of commodity *c*' produced & sold domestically	$tfpelastrd_a$	elasticity of TFP for *a* with respect to GDP trade share
$ice_{c,c',t}$	trade input of *c* per unit of commodity *c*' exported	$tfptrdwt_{t,t'}$	weight of period *t*' in tfp-trade link in *t*
$icm_{c,c',t}$	trade input of *c* per unit of commodity *c*' imported	$tgap_{t,t'}$	gap between *t* and *t*' (years used for calculation of expected growth rate for *QA*)
$ifa_{f,a}$	quantity of capital f per unit of government activity *a*	$tins01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$igf_{c,f,t}$	quantity of gov consumption per unit of gov infrastructure capital stock f	$tinsbar_{i,t}$	exogenous component in direct tax rate for domestic institution *i*
$inta_a$	quantity of aggregate intermediate input per unit of activity *a*	$tm_{c,t}$	import tariff rate
iva_a	quantity of value-added per unit of activity *a*	$tq_{c,t}$	rate of sales tax

$mps01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates	$trnsfr_{i,i',t}$	Exogenous transfer from institution *i'* to institution *i*
$mpsbar_{i,t}$	Exogenous component in savings rate for domestic institution *i*	$trnsfr_{f,i',t}$	Exogenous transfer from institution *i'* to factor *f*
$poptot_t$	total population by year	$trnsfrpc_{i,i',t}$	per-capita transfers from institution *i'* to household institution *i*
$pwm_{c,t}$	import world price of *c* (FCU)	$tva_{a,t}$	rate of value-added tax for activity *a*

PARAMETERS—GREEK LETTERS			
α_{ac_c}	shift parameter for domestic commodity aggregation function	$\delta_{va_{f,a}}$	CES value-added function share parameter for factor *f* in activity *a*
$\alpha_{vag_{a,t}}$	exogenous component of efficiency (TFP) for activity *a*	$\gamma_{h_{a,c,h}}$	per capita household subsistence consumption of home commodity *c* from activity *a*
α_{q_c}	*Armington* function shift parameter	$\gamma_{m_{c,h}}$	per capita household subsistence cons of marketed commodity *c*
α_{t_c}	CET function shift parameter	ρ_{ac_c}	domestic commodity aggregation function exponent
$\beta_{h_{a,c,h}}$	marginal share of household consumption on home commodity *c* from activity *a*	ρ_{q_c}	*Armington* function exponent
$\beta_{m_{c,h}}$	marginal share of household consumption spending on marketed commodity *c*	ρ_{sav_i}	elasticity of savings rate with respect to per-capita income for institution (household) *h*
δ_{ac_a}	share parameter for domestic commodity aggregation function	ρ_{t_c}	CET function exponent
δ_{q_c}	*Armington* function share parameter	ρ_{va_a}	CES value-added function exponent
δ_{t_c}	CET function share parameter	$\theta_{a,c}$	yield of output *c* per unit of activity *a*

VARIABLES			
$ALPHAVA_{a,t}$	efficiency parameter in the CES value-added function	$PVA_{a,t}$	value-added price (factor income per unit of activity)
$ALPHAVA2_{a,t}$	endogenous TFP trend term by *a*	$PVAAVG_t$	average value-added price
$CALTFPG_t$	calibration factor for TFP growth	$PWE_{c,t}$	export world price of *c* (FCU)

CPI_t	consumer price index	$PX_{c,t}$	aggregate producer price for commodity
$GBORMS_{i,t}$	implicit government Central Bank borrowing (deficit monetization) from institution *i*	$PXAC_{a,c,t}$	price of commodity *c* from activity *a*
$GBORMSTOT_t$	total government Central Bank borrowing (deficit monetization)	$QD_{c,t}$	quantity sold domestically of domestically produced *c*
$GBOR_{i,t}$	change in holding of government bonds for domestic institution *i*	$QF_{f,a,t}$	quantity demanded of factor *f* by activity *a*
$GBORTOT_t$	total change in holding of government bonds	$QFINS_{i,f,t}$	real endowment of factor *f* for institution *i*
$DKGOV_{f,t}$	gross government investment in *f*	$QG_{c,t}$	quantity of government consumption of commodity *c*
$DKINS_{i,f,t}$	gross change in capital stock (investment in) *f* for institution *i*	$QH_{c,h,t}$	quantity consumed by household *h* of marketed commodity *c*
$DMPS_t$	uniform point change in savings rate of selected domestic institutions	$QHA_{a,c,h,t}$	quantity consumed of home commodity *c* from act *a* by hhd *h*
DPI_t	producer price index for non-traded output	$QINTA_{a,t}$	quantity of aggregate intermediate input used by activity *a*
$DTINS_t$	uniform point change in direct tax rate of selected domestic institutions	$QINT_{c,a,t}$	quantity of commodity *c* as intermediate input to activity *a*
EG_t	government expenditures	$QINV_{c,t}$	quantity of investment demand for commodity *c*
$EH_{h,t}$	consumption spending for household	$QM_{c,t}$	quantity of imports of commodity *c*
EXR_t	exchange rate (LCU per unit of FCU)	$QQ_{c,t}$	quantity of goods supplied to domestic market (composite supply)
$FBOR_{i,t}$	foreign borrowing for domestic institution *i*	$QT_{c,t}$	quantity of trade and transport demand for commodity *c*
$FDEBT_{i,t}$	foreign debt for domestic inst *i*	$QVA_{a,t}$	quantity of (aggregate) value-added
$FGRANT_{i,t}$	foreign grants to domestic institution *i* (FCU)	$QX_{c,t}$	aggregated quantity of domestic output of commodity
$GDEBT_{i,t}$	endowment of government bonds for *i*	$QXAC_{a,c,t}$	quantity of output of commodity *c* from activity *a*
$GDPREAL_t$	real GDP at market prices	$QGGRW_t$	real government consumption growth for all *c* in *t* relative to *t-1*
$GDPREALFC_t$	real GDP at factor cost	$QGGRWC_{c,t}$	real government consumption growth of *c* in *t* relative to *t-1*
$GSAV_t$	government savings	$SHIF_{i,f,t}$	share of institution *i* in income of factor *f*

$INSSAV_{i,t}$	savings of domestic non-government institution *i*	$TINS_{i,t}$	direct tax rate for domestic non-government institution *i*
$INVVAL_{i,t}$	investment value for institution *i*	$TINSADJ_{t}$	direct tax scaling factor
$MPS_{i,t}$	marginal propensity to save for domestic non-gov institution *i*	$TRDGDP_{t}$	foreign trade as share of GDP
$MPSADJ_{t}$	savings rate scaling factor	$TRII_{i,i',t}$	transfers from institution *i*' to *i* (both in the set INSDNG)
$PA_{a,t}$	activity price (unit gross revenue)	$WF_{f,t}$	average price of factor
$PDD_{c,t}$	demand price for commodity *c* produced & sold domestically	$WFDIST_{f,a,t}$	wage distortion factor for factor *f* in activity *a*
$PDS_{c,t}$	supply price for commodity *c* produced & sold domestically	$WFRES_{f,t}$	reservation wage for factor *f*
$PE_{c,t}$	export price (domestic currency)	$YF_{f,t}$	income of factor *f*
$PINTA_{a,t}$	aggregate intermediate input price for activity *a*	YG_{t}	government revenue
$PK_{f,t}$	price of new capital stock *f*	$YI_{i,t}$	income of domestic non-government institution
$PM_{c,t}$	import price (domestic currency)	$YIF_{i,f,t}$	income to domestic institution *i* from factor *f*
$POP_{i,t}$	population by household	$YIINT_{i,t}$	interest payment on government bonds to institution *i*
$PQ_{c,t}$	composite commodity price		

Table A3.2 Equations for the core CGE module of MAMS model

#	Equation	Domain	Description
Price Block			
(1)	$PM_{c,t} = pwm_{c,t} \cdot \left(1 + tm_{c,t}\right) \cdot EXR_t + \sum_{c' \in C} \left(PQ_{c',t} \cdot icm_{c',c,t}\right)$ $\left[\begin{matrix}\text{import price}\\(LCU)\end{matrix}\right] = \left[\begin{matrix}\text{import price}\\(FCU)\end{matrix}\right] \cdot \left[\begin{matrix}\text{tariff}\\\text{adjustment}\end{matrix}\right] \cdot \left[\begin{matrix}\text{exchange rate}\\(LCU \text{ per } FCU)\end{matrix}\right] + \left[\begin{matrix}\text{transaction}\\\text{costs}\end{matrix}\right]$	$c \in CM$ $t \in T$	Import price
(2)	$PE_{c,t} = \overline{PWE}_{c,t} \cdot \left(1 - te_{c,t}\right) \cdot EXR_t - \sum_{c' \in C} \left(PQ_{c',t} \cdot ice_{c',c,t}\right)$ $\left[\begin{matrix}\text{export price}\\(LCU)\end{matrix}\right] = \left[\begin{matrix}\text{export price}\\(FCU)\end{matrix}\right] \cdot \left[\begin{matrix}\text{tariff}\\\text{adjustment}\end{matrix}\right] \cdot \left[\begin{matrix}\text{exchange rate}\\(LCU \text{ per } FCU)\end{matrix}\right] - \left[\begin{matrix}\text{transaction}\\\text{costs}\end{matrix}\right]$	$c \in CE$ $t \in T$	Export price
(3)	$(a) \quad PDS_{c,t} \geq PE_{c,t} \qquad (b) \quad QE_{c,t} \geq 0$ $\left[\begin{matrix}\text{domestic supply}\\\text{price}\end{matrix}\right] \geq \left[\begin{matrix}\text{export price}\\(LCU)\end{matrix}\right] \qquad \left[\begin{matrix}\text{export}\\\text{quantity}\end{matrix}\right] \geq [0]$ ..... $(c) \quad \left(PDS_{c,t} - PE_{c,t}\right)\left(QE_{c,t} - 0\right) = 0$ $\left[\begin{matrix}\text{Complementary slackness relationship:}\\\text{1. If domestic price exceeds export price then export quantity is zero.}\\\text{2. If export quantity excceds zero, then domestic price equals export price}\end{matrix}\right]$	$c \in (CD \cap CECETN)$ $t \in T$	For non-CET exportables with domestic sales: (a) domestic floor price, (b) non-negative export quantity constraints; and (c) related complementary-slackness relationship.
(4)	$PDD_{c,t} = PDS_{c,t} + \sum_{c' \in C} \left(PQ_{c',t} \cdot icd_{c',c,t}\right)$ $\left[\begin{matrix}\text{domestic demander}\\\text{price}\end{matrix}\right] = \left[\begin{matrix}\text{domestic supplier}\\\text{price}\end{matrix}\right] + \left[\begin{matrix}\text{transaction}\\\text{costs}\end{matrix}\right]$	$c \in CD$ $t \in T$	Domestic demander price for domestic commodity
(5)	$PQ_{c,t} \cdot \left(1 - tq_{c,t}\right) \cdot QQ_{c,t} = PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t}$ $\left[\begin{matrix}\text{absorption}\\(\text{at demand prices}\\\text{net of sales tax})\end{matrix}\right] = \left[\begin{matrix}\text{domestic demander}\\\text{price times}\\\text{domestic sales quantity}\end{matrix}\right] + \left[\begin{matrix}\text{import price}\\\text{times}\\\text{import quantity}\end{matrix}\right]$	$c \in (CD \cup CM)$ $t \in T$	Absorption
(6)	$PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t}$ $\left[\begin{matrix}\text{producer price}\\\text{times marketed}\\\text{output quantity}\end{matrix}\right] = \left[\begin{matrix}\text{domestic supplier}\\\text{price times}\\\text{domestic sales quantity}\end{matrix}\right] + \left[\begin{matrix}\text{export price}\\\text{times}\\\text{export quantity}\end{matrix}\right]$	$c \in (CD \cup CE)$ $t \in T$	Marketed output value
(7)	$PA_{a,t} = \sum_{c \in C} PXAC_{a,c,t} \cdot \theta_{a,c}$ $\left[\begin{matrix}\text{activity}\\\text{price}\end{matrix}\right] = \left[\begin{matrix}\text{producer prices}\\\text{times yields}\end{matrix}\right]$	$a \in A$ $t \in T$	Activity price

(8)	$$PINTA_{a,t} = \sum_{c \in C} PQ_{c,t} \cdot ica_{c,a}$$ $$\begin{bmatrix} \text{aggregate intermediate input price} \end{bmatrix} = \begin{bmatrix} \text{intermediate input cost per unit of aggregate intermediate input} \end{bmatrix}$$	$a \in A$ $t \in T$	Aggregate intermediate input price
(9)	$$PA_{a,t} \cdot (1 - ta_{a,t}) \cdot QA_{a,t} = PVA_{a,t} \cdot QVA_{a,t} + PINTA_{a,t} \cdot QINTA_{a,t}$$ $$\begin{bmatrix} \text{activity price (net of taxes) times activity level} \end{bmatrix} = \begin{bmatrix} \text{value-added price times quantity} \end{bmatrix} + \begin{bmatrix} \text{aggregate intermediate input price times quantity} \end{bmatrix}$$	$a \in A$ $t \in T$	Activity revenue and costs
(10)	$$\overline{CPI}_t = \sum_{c \in C} PQ_{c,t} \cdot cwts_c$$ $$\begin{bmatrix} CPI \end{bmatrix} = \begin{bmatrix} \text{prices times weights} \end{bmatrix}$$	$t \in T$	Consumer price index
(11)	$$DPI_t = \sum_{c \in CD} PDS_{c,t} \cdot dwts_c$$ $$\begin{bmatrix} \text{price index for non-tradables} \end{bmatrix} = \begin{bmatrix} \text{supplier price for output marketed domestically times weights} \end{bmatrix}$$	$t \in T$	Price index for non-tradables

Production and trade block

(12)	$$QVA_{a,t} = iva_a \cdot QA_{a,t}$$ $$\begin{bmatrix} \text{demand for value-added} \end{bmatrix} = f \begin{bmatrix} \text{activity level} \end{bmatrix}$$	$a \in ALEO$ $t \in T$	Demand for aggregate value-added
(13)	$$QINTA_{a,t} = inta_a \cdot QA_{a,t}$$ $$\begin{bmatrix} \text{demand for aggregate intermediate input} \end{bmatrix} = f \begin{bmatrix} \text{activity level} \end{bmatrix}$$	$a \in ALEO$ $t \in T$	Demand for aggregate intermediate input
(14)	$$QVA_{a,t} = ALPHAVA_{a,t} \cdot \left(\sum_{f \in F} \delta va_{f,a} \cdot \left(fprd_{f,a,t} \cdot QF_{f,a,t} \right)^{-\rho va_a} \right)^{-\frac{1}{\rho va_a}}$$ $$\begin{bmatrix} \text{quantity of aggregate value-added} \end{bmatrix} = CES \begin{bmatrix} \text{factor inputs} \end{bmatrix}$$	$a \in A$ $t \in T$	Value-added
(15)	$$WF_{f,t} \cdot \overline{WFDIST}_{f,a,t} = PVA_{a,t} \cdot \left(1 - tva_{a,t}\right) \cdot QVA_{a,t} \cdot \left(\sum_{f' \in F} \delta va_{f',a} \cdot \left(fprd_{f',a,t} \cdot QF_{f',a,t} \right)^{-\rho va_a} \right)^{-1} \cdot \delta va_{f,a} \cdot fprd_{f,a,t}^{-\rho va_a} \cdot QF_{f,a,t}^{-\rho va_a - 1}$$ $$\begin{bmatrix} \text{marginal cost of factor } f \text{ in activity } a \end{bmatrix} = \begin{bmatrix} \text{marginal revenue product of factor } f \text{ in activity } a \end{bmatrix}$$	$a \in A$ $f \in F$ $t \in T$	Factor demand

(16)	$QINT_{c,a,t} = ica_{c,a} \cdot QINTA_{a,t}$ $\begin{bmatrix} \text{intermediate demand} \\ \text{for commodity c} \\ \text{from activity a} \end{bmatrix} = f \begin{bmatrix} \text{aggregate intermediate} \\ \text{input quantity} \\ \text{for activity a} \end{bmatrix}$	$c \in C$ $a \in A$ $t \in T$	Disaggregated intermediate input demand
(17)	$QXAC_{a,c,t} + \sum_{h \in H} QHA_{a,c,h,t} = \theta_{a,c} \cdot QA_{a,t}$ $\begin{bmatrix} \text{quantity of output} \\ \text{of commodity c} \\ \text{from activity a} \end{bmatrix} + \begin{bmatrix} \text{quantity consumed of} \\ \text{home commodity c} \\ \text{from activity a in} \\ \text{all households} \end{bmatrix} = \begin{bmatrix} \text{activity-specific} \\ \text{marketed} \\ \text{production of} \\ \text{commodity c} \end{bmatrix}$	$a \in A$ $c \in C$ $t \in T$	Commodity production and allocation between market and home
(18)	$QX_{c,t} = \alpha ac_c \cdot \left(\sum_{a \in A} \delta ac_{a,c} \cdot QXAC_{a,c,t}^{-\rho ac_c} \right)^{-\frac{1}{\rho ac_c}}$ $\begin{bmatrix} \text{aggregate marketed} \\ \text{production of} \\ \text{commodity c} \end{bmatrix} = CES \begin{bmatrix} \text{output of commodity c} \\ \text{from activity a} \end{bmatrix}$	$c \in (CE \cup CD)$ $t \in T$	Output aggregation function
(19)	$\frac{PXAC_{a,c,t}}{PX_{c,t}} = QX_{c,t} \cdot \sum_{a' \in A} \left(\delta ac_{a',c} \cdot QXAC_{a',c,t}^{-\rho ac_c} \right)^{-1} \cdot \delta ac_{a,c} \cdot QXAC_{a,c,t}^{-\rho ac_c - 1}$ $\begin{bmatrix} \text{ratio of price of commodity c} \\ \text{from acitivty a to} \\ \text{average output price} \end{bmatrix} = f \begin{bmatrix} \text{aggregate marketed commodity} \\ \text{output and output of commodity c} \\ \text{from activity a} \end{bmatrix}$	$a \in A$ $c \in C$ $t \in T$	Ratio of prices for output aggregation function
(20)	$QX_{c,t} = \alpha t_c \cdot \left(\delta t_c \cdot QE_{c,t}^{\rho t_c} + (1 - \delta t_c) \cdot QD_{c,t}^{\rho t_c} \right)^{\frac{1}{\rho t_c}}$ $\begin{bmatrix} \text{aggregate marketed} \\ \text{domestic output} \end{bmatrix} = CET \begin{bmatrix} \text{export quantity, domestic} \\ \text{sales of domestic output} \end{bmatrix}$	$c \in (CD \cap CECET)$ $t \in T$	Output transformation (CET) function
(21)	$\frac{QE_{c,t}}{QD_{c,t}} = \left(\frac{PE_{c,t}}{PDS_{c,t}} \cdot \frac{1 - \delta t_c}{\delta t_c} \right)^{\frac{1}{\rho t_c - 1}}$ $\begin{bmatrix} \text{export-domestic} \\ \text{supply ratio} \end{bmatrix} = f \begin{bmatrix} \text{export-domestic} \\ \text{price ratio} \end{bmatrix}$	$c \in (CD \cap CECET)$ $t \in T$	Export-domestic supply ratio
(22)	$QX_{c,t} = QD_{c,t} + QE_{c,t}$ $\begin{bmatrix} \text{aggregate} \\ \text{marketed} \\ \text{domestic output} \end{bmatrix} = \begin{bmatrix} \text{domestic market} \\ \text{sales of domestic} \\ \text{output [for} \\ c \in (CD \cap CEN)] \end{bmatrix} + \begin{bmatrix} \text{exports [for} \\ c \in (CE \cap CDN)] \end{bmatrix}$	$c \in (CD \cap CEN) \cup (CE \cap CDN) \cup (CD \cap CECETN)$ $t \in T$	Output transformation for outputs without exports, exports without domestic sales, and non-CET exports with domestic sales

(23)	$QE_{c,t} = \overline{qe}_{c,t} \cdot \left(\frac{PWE_{c,t}}{pwse_{c,t}} \right)^{\rho e_c}$ $\left[\begin{matrix} export \\ demand \end{matrix} \right] = f \left[\begin{matrix} trend\ export\ quantity,\ world\ price \\ for\ exports\ relative\ to\ world \\ price\ for\ export\ substitutes \end{matrix} \right]$	$c \in CED$ $t \in T$	Export demand with CE demand function
(24)	$QQ_{c,t} = \alpha q_c \cdot \left(\delta q_c \cdot QM_{c,t}^{-\rho q_c} + (1 - \delta q_c) \cdot QD_{c,t}^{-\rho q_c} \right)^{-\frac{1}{\rho q_c}}$ $\left[\begin{matrix} composite \\ supply \end{matrix} \right] = f \left[\begin{matrix} import\ quantity,\ domestic \\ use\ of\ domestic\ output \end{matrix} \right]$	$c \in$ $(CM \cap CD)$ $t \in T$	Composite supply (*Armington*) function
(25)	$\frac{QM_{c,t}}{QD_{c,t}} = \left(\frac{PDD_{c,t}}{PM_{c,t}} \cdot \frac{\delta q_c}{1 - \delta q_c} \right)^{\frac{1}{1+\rho q_c}}$ $\left[\begin{matrix} import\text{-}domestic \\ demand\ ratio \end{matrix} \right] = f \left[\begin{matrix} domestic\text{-}import \\ price\ ratio \end{matrix} \right]$	$c \in$ $(CM \cap CD)$ $t \in T$	Import-domestic demand ratio
(26)	$QQ_{c,t} = QD_{c,t} + QM_{c,t}$ $\left[\begin{matrix} composite \\ supply \end{matrix} \right] = \left[\begin{matrix} domestic\ use\ of \\ marketed\ domestic \\ output\ [for \\ c \in (CD \cap CMN)] \end{matrix} \right] + \left[\begin{matrix} imports\ [for \\ c \in (CM \cap CDN)] \end{matrix} \right]$	$c \in$ $(CD \cap CMN)$ $\cup$ $(CM \cap CDN)$ $t \in T$	Composite supply for non-imported outputs and non-produced imports
(27)	$QT_{c,t} = \sum_{c' \in C'} \left(icm_{c,c',t} \cdot QM_{c',t} + ice_{c,c',t} \cdot QE_{c',t} + icd_{c,c',t} \cdot QD_{c',t} \right)$ $\left[\begin{matrix} trade\ and\ transport \\ demand\ for\ commodity\ c \end{matrix} \right] = \left[from\ imports \right] + \left[from\ exports \right] + \left[\begin{matrix} from\ marketed \\ domestic\ output \end{matrix} \right]$	$c \in CT$ $t \in T$	Demand for transaction services

Domestic institution block

(28)	$YF_{f,t} = \sum_{a \in A} WF_{f,t} \cdot \overline{WFDIST}_{f,a,t} \cdot QF_{f,a,t} + trnsfr_{f,row,t} \cdot EXR_t$ $\left[\begin{matrix} income\ of \\ factor\ f \end{matrix} \right] = \left[\begin{matrix} sum\ of\ activity\ payments \\ (activity\text{-}specific\ wages \\ times\ employment\ levels) \end{matrix} \right] + \left[\begin{matrix} income\ to\ factor\ f \\ from\ Rest\ of\ World \end{matrix} \right]$	$f \in F$ $t \in T$	Factor income
(29)	$SHIF_{i,f,t} = \frac{QFACINS_{i,f,t}}{\sum_{i' \in INS} QFACINS_{i',f,t}}$ $\left[\begin{matrix} share\ of\ institution\ i\ in \\ the\ income\ of\ factor\ f \end{matrix} \right] = \left[\begin{matrix} endowment\ of\ institution\ i\ of\ factor\ f \\ divided\ by\ total\ endowment\ of\ factor\ f \end{matrix} \right]$	$i \in INS$ $f \in F$ $t \in T$	Institutional shares in factor incomes
(30)	$YIF_{i,f,t} = SHIF_{i,f,t} \cdot \left[\left(1 - tf_{f,t} \right) \cdot YF_{f,t} \right]$ $\left[\begin{matrix} income\ of \\ institution\ i \\ from\ factor\ f \end{matrix} \right] = \left[\begin{matrix} share\ of\ income \\ of\ factor\ f\ to \\ institution\ i \end{matrix} \right] \cdot \left[\begin{matrix} income\ of\ factor\ f \\ (net\ of\ tax) \end{matrix} \right]$	$i \in INS$ $f \in F$ $t \in T$	Institutional factor incomes

(31)	$$YIINT_{i,t} = gintrat_{i,t} \cdot GDEBT_{i,t} - fintrat_{i,t} \cdot FDEBT_{i,t} \cdot EXR_t$$ $$\begin{bmatrix} net\ interest \\ income\ of \\ institution\ i \end{bmatrix} = \begin{bmatrix} interest\ earnings \\ on\ government \\ bonds \end{bmatrix} - \begin{bmatrix} interest \\ payments \\ on\ foreign\ debt \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Institutional net interest income
(32)	$$TRII_{i,i',t} = shii_{i,i'} \cdot (1 - MPS_{i',t}) \cdot (1 - TINS_{i',t}) \cdot YI_{i',t}$$ $$\begin{bmatrix} transfer\ from \\ institution\ i'\ to\ i \end{bmatrix} = \begin{bmatrix} share\ of\ net\ income \\ of\ institution\ i' \\ transfered\ to\ i \end{bmatrix} \cdot \begin{bmatrix} income\ of\ institution \\ i',\ net\ of\ savings\ and \\ direct\ taxes \end{bmatrix}$$	$i \in INS$ $i' \in INSDNG$ $t \in T$	Intra-institutional transfers
(33)	$$YI_{i,t} = \sum_{f \in F} YIF_{i,f,t} + \sum_{i' \in INSDNG'} TRII_{i,i',t} + YIINT_{i,t}$$ $$\begin{bmatrix} income\ of \\ institution\ i \end{bmatrix} = \begin{bmatrix} factor \\ income \end{bmatrix} + \begin{bmatrix} transfers\ from\ other \\ domestic\ non\text{-}government \\ institutions \end{bmatrix} + \begin{bmatrix} net \\ interest \\ income \end{bmatrix}$$ $$+ trnsfr_{i,gov,t} \cdot \overline{CPI_t} + trnsfrpc_{i,gov,t} \cdot POP_{i,t} \cdot \overline{CPI_t}$$ $$+ \begin{bmatrix} transfers\ from\ government \\ to\ non\text{-}household\ institutions \end{bmatrix} + \begin{bmatrix} transfers\ from \\ government\ to\ households \end{bmatrix}$$ $$+ trnsfr_{i,row,t} \cdot EXR_t + trnsfrpc_{i,row,t} \cdot POP_{i,t} \cdot EXR_t$$ $$+ \begin{bmatrix} transfers\ from\ Rest\ of\ World \\ to\ non\text{-}household\ institutions \end{bmatrix} + \begin{bmatrix} transfers\ from \\ Rest\ of\ World\ to\ households \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Income of domestic, non-government institutions
(34)	$$TINS_{i,t} = tinsbar_{i,t} \cdot \left(1 + \overline{TINSADJ}_t \cdot tins01_i\right) + DTINS_t \cdot tins01_i$$ $$\begin{bmatrix} direct\ tax \\ rate\ for \\ institution\ i \end{bmatrix} = \begin{bmatrix} exogenous\ rate\ adjusted \\ for\ scaling\ for \\ selected\ institutions \end{bmatrix} + \begin{bmatrix} point\ change \\ for\ selected \\ institutions \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Direct tax rates for domestic non-government institutions
(35)	$$MPS_{i,t} = mpsbar_{i,t} \cdot \left(\frac{\left(1 - TINS_{i,t}\right) \cdot YI_{i,t}}{\overline{POP}_{i,t}}\right)^{\rho_{sav_i} - 1} \cdot \left(1 + \overline{MPSADJ}_t \cdot mps01_i\right)$$ $$\begin{bmatrix} marginal \\ propensity \\ to\ save \end{bmatrix} = \begin{bmatrix} exogenous \\ term \end{bmatrix} \cdot \begin{bmatrix} adjustment\ for \\ per\text{-}capita \\ post\text{-}tax\ income \end{bmatrix} \cdot \begin{bmatrix} scaling\ adjustment \\ for\ selected \\ institutions \end{bmatrix}$$ $$+ \overline{DMPS}_t \cdot mps01_i$$ $$+ \begin{bmatrix} point\text{-}change \\ adjustment\ for \\ selected\ institutions \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Savings rates for domestic non-government institutions
(36)	$$INSSAV_{i,t} = MPS_{i,t} \cdot \left(1 - TINS_{i,t}\right) \cdot YI_{i,t}$$ $$\begin{bmatrix} savings\ for \\ institution\ i \end{bmatrix} = \begin{bmatrix} savings \\ rate\ for \\ institution\ i \end{bmatrix} \cdot \begin{bmatrix} income\ of \\ institution\ i \\ (net\ of\ direct\ taxes) \end{bmatrix}$$	$i \in INSDNG$	Savings for domestic non-government institutions

(37)	$$EH_{h,t} = \left(1 - \sum_{i \in INSDNG} shii_{i,h}\right) \cdot \left(1 - MPS_{h,t}\right) \cdot (1 - TINS_{h,t}) \cdot YI_{h,t}$$ $$\begin{bmatrix} \text{household income} \\ \text{disposable for} \\ \text{consumption} \end{bmatrix} = \begin{bmatrix} \text{household income, net of direct} \\ \text{taxes, savings, and transfers to} \\ \text{other non-government institutions} \end{bmatrix}$$	$h \in H$ $t \in T$	Household consumption expenditure
(38)	$$QH_{c,h,t} = \overline{POP}_{h,t} \cdot \left(\gamma m_{c,h} + \frac{\beta m_{c,h} \cdot \left(\left[\frac{EH_{h,t}}{\overline{POP}_{h,t}} \right] - \sum_{c' \in C} PQ_{c',t} \cdot \gamma m_{c',h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a,c',t} \cdot \gamma h_{a,c',h} \right)}{PQ_{c,t}} \right)$$ $$\begin{bmatrix} \text{quantity of} \\ \text{household demand} \\ \text{for commodity } c \end{bmatrix} = f \begin{bmatrix} \text{household} \\ \text{consumption} \\ \text{spending, prices} \end{bmatrix}$$	$c \in C$ $h \in H$ $t \in T$	Household consumption demand for commodities from market
(39)	$$QHA_{a,c,h,t} = \overline{POP}_{h,t} \cdot \left(\gamma h_{a,c,h} + \frac{\beta h_{a,c,h} \cdot \left(\left[\frac{EH_{h,t}}{\overline{POP}_{h,t}} \right] - \sum_{c' \in C} PQ_{c',t} \cdot \gamma m_{c',h} - \sum_{a' \in A} \sum_{c' \in C} PXAC_{a',c',t} \cdot \gamma h_{a',c',h} \right)}{PXAC_{a,c,t}} \right)$$ $$\begin{bmatrix} \text{quantity of household demand} \\ \text{for commodity } c \text{ from activity } a \end{bmatrix} = f \begin{bmatrix} \text{household consumption} \\ \text{spending, prices} \end{bmatrix}$$	$a \in A$ $c \in C$ $h \in H$ $t \in T$	Household consumption demand for own production
(40)	$$YG_t = \sum_{i \in INSDNG} TINS_{i,t} \cdot YI_{i,t} + \sum_{f \in F} tf_{f,t} \cdot YF_{f,t} + \sum_{a \in A} ta_{a,t} \cdot PA_{a,t} \cdot QA_{a,t}$$ $$\begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = \begin{bmatrix} \text{direct taxes} \\ \text{from institutions} \end{bmatrix} + \begin{bmatrix} \text{direct taxes} \\ \text{from factors} \end{bmatrix} + [\text{activity tax}]$$ $$+ \sum_{a \in A} tva_{a,t} \cdot PVA_{a,t} \cdot QVA_{a,t} + \sum_{c \in CM} tm_{c,t} \cdot pwm_{c,t} \cdot QM_{c,t}$$ $$+ [\text{value-added tax}] + [\text{import tariffs}]$$ $$+ \sum_{c \in CE} te_{c,t} \cdot \overline{PWE}_{c,t} \cdot QE_{c,t} \cdot EXR_t + \sum_{c \in C} tq_{c,t} \cdot PQ_{c,t} \cdot QQ_{c,t}$$ $$+ [\text{export taxes}] + [\text{sales tax}]$$ $$+ \sum_{f \in F} YIF_{gov,f,t} + \sum_{i \in INSDNG} TRII_{gov,i,t} + trnsfr_{gov,row,t} \cdot EXR_t$$ $$+ [\text{factor income}] + \begin{bmatrix} \text{transfers from} \\ \text{domestic institutions} \end{bmatrix} + [\text{transfers from RoW}]$$	$t \in T$	Government recurrent revenue

(41)	$$EG_t = \sum_{c \in C} PQ_{c,t} \cdot QG_{c,t} + \sum_{i \in INSDNH} trnsfr_{i,gov,t} \cdot \overline{CPI}_t$$ $$\begin{bmatrix} government \\ spending \end{bmatrix} = \begin{bmatrix} government \\ consumption \end{bmatrix} + \begin{bmatrix} transfers\ to\ domestic \\ non\text{-}household\ institutions \end{bmatrix}$$ $$+ \sum_{h \in H} trnsfrpc_{h,gov,t} \cdot \overline{POP}_{h,t} \cdot \overline{CPI}_t + trnsfr_{row,gov,t} \cdot EXR_t$$ $$+ \begin{bmatrix} transfers\ to\ domestic \\ households \end{bmatrix} + \begin{bmatrix} transfers\ to \\ Rest\ of\ World \end{bmatrix}$$ $$+ \sum_{i \in INS} gintrat_{i,t} \cdot GDEBT_{i,t} + fintrat_{gov,t} \cdot FDEBT_{gov,t} \cdot EXR_t$$ $$+ \begin{bmatrix} interest\ payment \\ on\ domestic\ debt \end{bmatrix} + \begin{bmatrix} interest\ payment \\ on\ foreign\ debt \end{bmatrix}$$	$t \in T$	Government recurrent expenditures
(42)	$$QG_{c,t} = QG_{c,t-1} \cdot \left(1 + \overline{QGGRW}_t + \sum_{c' \in C} qg01_{c,c',t} \cdot \overline{QGGRWC}_{c',t}\right)$$ $$\begin{bmatrix} real\ government \\ consumption \\ of\ c\ in\ t \end{bmatrix} = \begin{bmatrix} real\ government \\ consumption \\ of\ c\ in\ t - 1 \end{bmatrix} \cdot \left[1 + \begin{bmatrix} adjustment\ for\ uniform \\ consumption\ growth, \\ e.g.\ absorption\ share \end{bmatrix} + \begin{bmatrix} adjustment\ for\ growth \\ specific\ to\ one\ or \\ more\ commodities \end{bmatrix}\right]$$	$c \in C$ $c \notin CINF$ $t \in T$ $t > 1$	Real government consumption (excluding infrastructure services)
(43)	$$QG_{c,t} = \sum_{\substack{i \in INS \\ f \in F}} igf_{c,f,t} \cdot QFINS_{i,f,t}$$ $$\begin{bmatrix} real\ government \\ consumption \\ of\ c\ in\ t \end{bmatrix} = \begin{bmatrix} quantity\ of\ gov\ consumption \\ per\ unit\ of\ gov\ infrastructure \\ capital\ stock\ f \end{bmatrix} \cdot \begin{bmatrix} real\ endowment\ of \\ factor\ f\ for \\ institution\ i \end{bmatrix}$$	$c \in CINF$ $t \in T$ $t > 1$	Real government consumption of infrastructure services
(44)	$$GSAV_t = YG_t - EG_t$$ $$\begin{bmatrix} government \\ savings \end{bmatrix} = \begin{bmatrix} government \\ recurrent\ revenue \end{bmatrix} - \begin{bmatrix} government \\ recurrent\ expenditures \end{bmatrix}$$	$t \in T$	Government savings
Investment block			

(45)	*(a)* $DKGOV_{f,t} \geq \sum_{a \in A \mid (f,a) \in MFA} ifa_{f,a,t} \cdot QA_{a,t} \cdot EXP\left(\ln\left(\frac{QA_{a,t}}{QA_{a,t-1}} \right) \right)\Big\|_{f \in FCAPGOVSER}$ $\left[\begin{matrix} \text{government investment} \\ \text{demand for capital } f \end{matrix} \right] \geq \left[\begin{matrix} \text{demand for government service capital in } t+1 : \\ \text{capital coefficient times expected activity level in } t+1 \end{matrix} \right]$ $+\left(\left(1+\sum_{c \in C} qg01_{f,c,t} \cdot \overline{QGGRWC}_{c,t} \right) \cdot QFINS_{gov,f,t} \right)\Big\|_{f \in FCAPGOVINF}$ $+\left[\begin{matrix} \text{demand for government infrastructure capital in } t+1 : \\ \text{growth rate times infra capital stock in } t \end{matrix} \right]$ $-QFINS_{gov,f,t} \cdot (1-depr_{f,t})$ $-\left[\begin{matrix} \text{remaining capital stock (after depreciation) in } t+1 \text{ if no investment in } t \end{matrix} \right]$ ..... *(b)* $DKGOV_{f,t} \geq 0$ $[\text{government investment}] \geq [\text{zero}]$ ..... *(c)* $\left(DKGOV_{f,t} - DKGOVDEM_{f,t} \right) \cdot \left(DKGOV_{f,t} - 0 \right) = 0$ where $DKGOVDEM_{f,t}$ = right-hand of part *(a)* of Equation 45 $\left[\begin{matrix} \text{Complementary slackness relationship :} \\ \text{1. If government investment exceeds its demand then this investment level is zero.} \\ \text{2. If the government investment level is above zero, then it equals its demand} \end{matrix} \right]$	$f \in FCAPGOV$ $t \in T$ $t > 1$	Real government demand for investment in capital stock f
(46)	$DKINS_{gov,f,t} = DKGOV_{f,t}$ $\left[\begin{matrix} \text{gross investment in } f \text{ of} \\ \text{institution ins (here "ins" = gov)} \end{matrix} \right] = \left[\begin{matrix} \text{gross government investment} \\ \text{demand for capital} \end{matrix} \right]$	$f \in FCAPGOV$ $t \in T$ $t > 1$	Real government investment in capital stock f (investment by destination)
(47)	$PK_{f,t} = \sum_{c \in C} capcomp_{c,f} \cdot PQ_{c,t}$ $\left[\begin{matrix} \text{price of new} \\ \text{capital stock} \end{matrix} \right] = \left[\begin{matrix} \text{total value of commodities } c \\ \text{per unit of new capital} \end{matrix} \right]$	$f \in FCAP$ $t \in T$	Price of new capital stock
(48)	$\sum_{f \in FCAPGOV} PK_{f,t} \cdot DKINS_{gov,f,t} = GSAV_t - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,gov,t} + \overline{GBORTOT}_t$ $\left[\begin{matrix} \text{government fixed} \\ \text{investment value} \end{matrix} \right] = \left[\begin{matrix} \text{government} \\ \text{savings} \end{matrix} \right] - \left[\begin{matrix} \text{spending on} \\ \text{stock changes} \end{matrix} \right] + \left[\begin{matrix} \text{total change in holdings} \\ \text{of government bonds} \end{matrix} \right]$ $+\overline{GBORMSTOT}_t + \left(\overline{FBOR}_{gov,t} + \overline{FGRANT}_{gov,t} \right) \cdot EXR_t$ $+\left[\begin{matrix} \text{Government Central Bank} \\ \text{borrowing (deficit monetization)} \end{matrix} \right] + \left[\begin{matrix} \text{foreign borrowing and} \\ \text{foreign grants (in LCU)} \end{matrix} \right]$	$t \in T$	Government investment value and financing

(49)	$$GBOR_{i,t} = \frac{gbdist_i \cdot INSSAV_{i,t}}{\sum_{i' \in INSDNG'} gbdist_{i'} \cdot INSSAV_{i',t}} \cdot \overline{GBORTOT}_t$$ $$\left[\begin{array}{c}\text{change in holdings of}\\ \text{government bonds}\\ \text{by institution i}\end{array}\right] = \frac{\left[\begin{array}{c}\text{savings by}\\ \text{by institution i}\end{array}\right]}{\left[\begin{array}{c}\text{total institution}\\ \text{savings value}\end{array}\right]} \cdot \left[\begin{array}{c}\text{(scaled) total change}\\ \text{in holdings of}\\ \text{government bonds}\end{array}\right]$$	$i \in INSDNG$ $t \in T$	Allocation of government bond borrowing across domestic non-government institutions
(50)	$$GBORMS_{i,t} = \frac{gbdist_i \cdot INSSAV_{i,t}}{\sum_{i' \in INSDNG'} gbdist_{i'} \cdot INSSAV_{i',t}} \cdot \overline{GBORMSTOT}_t$$ $$\left[\begin{array}{c}\text{Government Central Bank}\\ \text{borrowing by institution i}\end{array}\right] = \frac{\left[\begin{array}{c}\text{savings by}\\ \text{by institution i}\end{array}\right]}{\left[\begin{array}{c}\text{total institution}\\ \text{savings value}\end{array}\right]} \cdot \left[\begin{array}{c}\text{(scaled) total Government}\\ \text{Central Bank borrowing}\end{array}\right]$$	$i \in INSDNG$ $t \in T$	Allocation of the burden of Central Bank borrowing across domestic non-government institutions
(51)	$$INVVAL_{i,t} = INSSAV_{i,t} - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,i,t} - GBOR_{i,t}$$ $$\left[\begin{array}{c}\text{non-government fixed}\\ \text{investment value}\end{array}\right] = \left[\text{savings}\right] - \left[\begin{array}{c}\text{stock}\\ \text{changes}\end{array}\right] - \left[\begin{array}{c}\text{change in holdings of}\\ \text{government bonds}\end{array}\right]$$ $$-GBORMS_{i,t} + \left(\overline{FBOR}_{i,t} + \overline{FGRANT}_{i,t} + fdi_{i,t}\right) \cdot EXR_t$$ $$-\left[\begin{array}{c}\text{Government Central}\\ \text{Bank borrowing}\end{array}\right] + \left[\begin{array}{c}\text{foreign borrowing, grants,}\\ \text{and direct investment (in LCU)}\end{array}\right]$$	$i \in INSNG$ $t \in T$	Investment financing for non-government institutions
(52)	$$PK_{f,t} \cdot DKINS_{i,f,t} = gfcfshr_{f,i,t} \cdot INVVAL_{i,t}$$ $$\left[\begin{array}{c}\text{non-government spending}\\ \text{on capital stock f}\end{array}\right] = \left[\begin{array}{c}\text{total fixed investment value}\\ \text{times share for capital stock f}\end{array}\right]$$	$i \in INSNG$ $f \in FCAP$ $t \in T$	Non-government investment by capital stock (investment by destination)
(53)	$$QINV_{c,t} = \sum_{f \in FCAP} \left(capcomp_{c,f} \cdot \sum_{i \in INS} DKINS_{i,f,t} \right)$$ $$\left[\begin{array}{c}\text{real investment demand}\\ \text{for commodity c}\end{array}\right] = \left[\begin{array}{c}\text{demand for c for each type of capital,}\\ \text{summed over all institutions and capital types}\end{array}\right]$$	$c \in C$ $t \in T$	Total real investment demand by commodity (investment by origin or source)
Constraints for foreign exchange, factors, and commodities			

(54)	$$\sum_{c\in CM} pwm_{c,t}\cdot QM_{c,t} + \frac{\sum_{f\in F} YIF_{row,f,t}}{EXR_t} + \frac{\sum_{i\in INSDNG} TRII_{row,i,t}}{EXR_t}$$ $$\left[\begin{array}{c}import\\spending\end{array}\right] + \left[\begin{array}{c}factor\ income\\to\ Rest\ of\ World\end{array}\right] + \left[\begin{array}{c}transfers\ from\ domestic\\non\text{-}gov\ institutions\ to\ RoW\end{array}\right]$$ $$+trnsfr_{row,gov,t} + \sum_{i\in INSD} fintrat_{i,t}\cdot FDEBT_{i,t}$$ $$+\left[\begin{array}{c}transfers\ from\\government\ to\ RoW\end{array}\right] + \left[\begin{array}{c}interest\ payment\\on\ foreign\ debt\end{array}\right]$$ $$= \sum_{c\in CE} \overline{PWE}_{c,t}\cdot QE_{c,t} + \sum_{i\in INSDNH} trnsfr_{i,row,t} + \sum_{h\in H} trnsfrpc_{h,row,t}\cdot \overline{POP}_{h,t}$$ $$= \left[\begin{array}{c}export\\revenue\end{array}\right] + \left[\begin{array}{c}transfers\ from\ RoW\ to\ domestic\\non\text{-}household\ institutions\end{array}\right] + \left[\begin{array}{c}transfers\ from\ RoW\ to\\domestic\ households\end{array}\right]$$ $$+\sum_{f\in F} trnsfr_{f,row,t} + \sum_{i\in INSD}\left(\overline{FBOR}_{i,t} + \overline{FGRANT}_{i,t}\right) + fdi_{row,t}$$ $$+\left[\begin{array}{c}factor\ income\\from\ RoW\end{array}\right] + \left[\begin{array}{c}borrowing\\from\ RoW\end{array}\right] + \left[\begin{array}{c}grants\\from\ RoW\end{array}\right] + \left[\begin{array}{c}foreign\ direct\\investment\end{array}\right]$$	$t\in T$	Balance of payments (in FCU)
(55)	$$\sum_{a\in A} QF_{f,a,t} = \left(1-UERAT_{f,t}\right)\cdot \sum_{i\in INS} QFINS_{i,f,t}$$ $$\left[\begin{array}{c}demand\ for\\market\ factor\ f\end{array}\right] = \left[\begin{array}{c}1\text{ - }unemployment\ rate\\(i.e.,\ employment\ rate)\end{array}\right]\cdot\left[\begin{array}{c}sum\ of\ all\ institutional\\endowments\ of\ factor\ f\end{array}\right]$$	$f\in F$ $t\in T$	Factor markets
(56)	$$WFRES_{f,t} = WF_f^0\cdot\left(\frac{QHPC_t}{QHPC^0}\right)^{\varphi_f^{wfqhpc}}\cdot\left(\frac{\left(1-UERAT_{f,t}\right)}{\left(1-UERAT_f^0\right)}\right)^{\varphi_f^{wferat}}\cdot\left(\frac{CPI_t}{CPI^0}\right)^{\varphi_f^{wfcpi}}$$ $$\left[\begin{array}{c}reservation\ wage\\for\ factor\ f\\in\ year\ t\end{array}\right] = \left[\begin{array}{c}economy\text{-}wide\ wage\\for\ factor\ f\ in\\the\ base\ year\end{array}\right]\cdot\left[\begin{array}{c}adjustment\ due\ to\text{: }per\text{-}capita\ household\\consumption;\ employment\ rate;\ and\\CPI\ (all\ relative\ to\ base\ year\ values)\end{array}\right]$$	$f\in$ *FUEND* $t\in T$	Reservation wage
(57)	$$(a)\quad WF_{f,t}\geq WFRES_{f,t}\qquad (b)\quad UERAT_{f,t}\geq ueratmin_{f,t}$$ $$\left[\begin{array}{c}economy\text{-}wide\\wage\ for\ fac\text{-}\\tor\ f\ in\ year\ t\end{array}\right]\geq\left[\begin{array}{c}reservation\\wage\ for\ fac\text{-}\\tor\ f\ in\ year\ t\end{array}\right]\qquad \left[\begin{array}{c}unemployment\\rate\ for\ factor\\f\ \ in\ year\ t\end{array}\right]\geq\left[\begin{array}{c}minimum\ unem\text{-}\\ployment\ rate\ for\\factor\ f\ \ in\ year\ t\end{array}\right]$$ ..... $$(c)\quad \left(WF_{f,t}-WFRES_{f,t}\right)\cdot\left(UERAT_{f,t}-ueratmin_{f,t}\right)=0$$ $$\left[\begin{array}{l}Complementary\ slackness\ relationship:\\1.\ If\ wage\ exceeds\ reservation\ wage\ then\ unemployment\ rate\ is\ at\ its\ minimum.\\2.\ If\ unemployment\ rate\ excceds\ its\ minimum,\ then\ wage\ equals\ reservation\ wage\end{array}\right]$$	$f\in$ *FUEND* $t\in T$	For factors with endogenous unemployment: (a) Wage and (b) unemployment constraints; and (c) related complementary-slackness relationship

(58)	$$QQ_{c\,t} = \sum_{a\in A} QINT_{c,a,t} + \sum_{h\in H} QH_{c,h,t} + QG_{c,t}$$ $$\begin{bmatrix} composite \\ supply \end{bmatrix} = \begin{bmatrix} intermediate \\ use \end{bmatrix} + \begin{bmatrix} household \\ consumption \end{bmatrix} + \begin{bmatrix} government \\ consumption \end{bmatrix}$$ $$+QINV_{c,t} + \sum_{i\in INS} qdst_{c,i,t} + QT_{c,t}$$ $$+\begin{bmatrix} fixed \\ investment \end{bmatrix} + \begin{bmatrix} stock \\ change \end{bmatrix} + \begin{bmatrix} trade\ and \\ transport \end{bmatrix}$$	$c \in C$ $t \in T$	Composite commodity markets

Asset stock updating and productivity block

(59)	$$QFINS_{i,f,t} = (1-depr_{f,t-1}) \cdot QFINS_{i,f,t-1} + DKINS_{i,f,t-1} + qfinsadj_{i,f,t-1}$$ $$\begin{bmatrix} stock\ of\ capital \\ type\ f\ held \\ by\ institution\ i \end{bmatrix} = \begin{bmatrix} non\text{-}depreciated \\ capital\ stock \end{bmatrix} + \begin{bmatrix} fixed\ invest\text{-} \\ ment\ in\ t\text{-}1 \end{bmatrix} + \begin{bmatrix} exogenous\ adjustment \\ in\ capital\ stock \end{bmatrix}$$	$i \in INS$ $f \in FCAP$ $t \in T$ $t>1$	Capital stocks by institution
(60)	$$FDEBT_{i,t} = FDEBT_{i,t-1} + FBOR_{i,t-1}$$ $$+\left(fintratdue_{i,t-1} - fintrat_{i,t-1}\right) \cdot FDEBT_{i,t-1} - fdebtrelief_{i,t-1}$$ $$\begin{bmatrix} foreign \\ debt\ in\ t \end{bmatrix} = \begin{bmatrix} foreign \\ debt\ in\ t\text{-}1 \end{bmatrix} + \begin{bmatrix} foreign\ bor\text{-} \\ rowing\ in\ \ t\text{-}1 \end{bmatrix} + \begin{bmatrix} unpaid\ interest\ on \\ foreign\ debt\ in\ t\text{-}1 \end{bmatrix} - \begin{bmatrix} foreign\ debt \\ relief\ in\ t\text{-}1 \end{bmatrix}$$	$i \in INSD$ $t \in T$ $t > 1$	Foreign debt of domestic institutions
(61)	$$GDEBT_{i,t} = GDEBT_{i,t-1} + GBOR_{i,t-1}$$ $$\begin{bmatrix} stock\ of\ government \\ bond\ held\ by \\ institution\ i \end{bmatrix} = \begin{bmatrix} redistributed\ holdings\ of \\ stock\ of\ government\ bond \\ held\ by\ institution\ i\ in\ t\text{-}1 \end{bmatrix} + \begin{bmatrix} government \\ borrowing \\ from\ i\ in\ t\text{-}1 \end{bmatrix}$$	$i \in INSDNG$ $t \in T$ $t > 1$	Government bond holdings of domestic institutions
(62)	$$GDPREAL_t = \sum_{c\in C}\sum_{h\in H} PQ_c^0 \cdot QH_{c,h,t} + \sum_{a\in A}\sum_{c\in C}\sum_{h\in H} PXAC_{a,c}^0 \cdot QHA_{a,c,h,t}$$ $$\left[real\ GDP\right] = \begin{bmatrix} household\ market \\ consumption \end{bmatrix} + \begin{bmatrix} household\ own \\ production\ consumption \end{bmatrix}$$ $$+\sum_{c\in C} PQ_c^0 \cdot QG_{c,t} + \sum_{c\in C} PQ_c^0 \cdot QINV_{c,t} + \sum_{c\in C}\sum_{i\in INS} PQ_c^0 \cdot qdst_{c,i,t}$$ $$+\begin{bmatrix} government \\ consumption \end{bmatrix} + \begin{bmatrix} fixed \\ investment \end{bmatrix} + \begin{bmatrix} stock \\ change \end{bmatrix}$$ $$+\sum_{c\in CE} EXR^0 \cdot PWE_c^0 \cdot QE_{c,t} - \sum_{c\in CM} EXR^0 \cdot PWM_c^{\,0} \cdot QM_{c,t}$$ $$+\left[exports\right] - \left[imports\right]$$	$t \in T$	Real GDP at market prices
(63)	$$TRDGDP_t = \frac{\sum_{c\in CE} EXR^0 \cdot PWE_c^0 \cdot QE_{c,t} + \sum_{c\in CM} EXR^0 \cdot PWM_c^{\,0} \cdot QM_{c,t}}{GDPREAL_t}$$ $$\begin{bmatrix} ratio\ of \\ trade\ to\ GDP \end{bmatrix} = \frac{\left[real\ trade\right]}{\left[real\ GDP\right]}$$	$t \in T$	Real Trade-GDP ratio

(64)	$$ALPHAVA_{a,t} = ALPHAVA2_{a,t} \cdot \prod_{f \in FCAP} \left[\frac{\sum_{i \in INS} QFINS_{i,f,t}}{\sum_{i \in INS} QFINS^{0}_{i,f}} \right]^{tfpelasqg_{a,f,t}} \cdot \left(\frac{\sum_{t' \in T} tfptrdwt_{t,t'} \cdot TRDGDP_{t'}}{TRDGDP^{0}} \right)^{tfpelastrd_a}$$ $$\begin{bmatrix} efficiency \\ term\ for \\ activity\ a \end{bmatrix} = \begin{bmatrix} trend \\ term\ for \\ activity\ a \end{bmatrix} \cdot \begin{bmatrix} product\ of: ratio\ of\ all \\ current\ real\ capital \\ endowment\ f\ to\ inital \\ value,\ raised \\ to\ the\ relevant\ elasticity \end{bmatrix} \cdot \begin{bmatrix} weighted\ avg.\ (over\ time) \\ of\ ratios\ of\ openness \\ to\ initial\ value,\ raised \\ to\ the\ relevant\ elasticity \end{bmatrix}$$	$a \in A$ $t \in T$ $t > 1$	Efficiency (TFP) by activity
(65)	$$ALPHAVA2_{a,t} = ALPHAVA2_{a,t-1} \cdot \left(1 + \alpha vag_{a,t} + \overline{CALTFPG_t} \cdot tfp01_{a,t}\right)$$ $$\begin{bmatrix} trend\ term\ for \\ activity\ a\ in\ t \end{bmatrix} = \begin{bmatrix} trend\ term\ for \\ activity\ a\ in\ t\text{-}1 \end{bmatrix} \cdot \begin{bmatrix} growth\ adjust\text{-} \\ ment\ factor \end{bmatrix}$$	$a \in A$ $t \in T$ $t > 1$	TFP trend term by activity
(66)	$$GDPREALFC_t = \sum_{a \in A} PVA^{0}_{a} \cdot \left(1 - tva^{0}_{a,t}\right) \cdot QVA_{a,t}$$ $$\begin{bmatrix} real\ GDP \\ at\ factor\ cost \end{bmatrix} = \begin{bmatrix} value\text{-}added \\ net\ of\ taxes \end{bmatrix}$$	$t \in T$	Real GDP at factor cost

Table A3.3 Notation for MDG module of MAMS model

SETS			
Symbol	Explanation	Symbol	Explanation
$a \in A$	activities	$i \in INSG$	government institution
$b \in B$	student behavioural characteristics ={*rep* = repeater; *dropout* = dropout; *pass* = pass; *grdcont* = continuing graduate; *grdexit* = exiting graduate; *g1entry* = entrant to grade 1; *grdcyc*= pass from last cycle-year; *contcyc* = pass within cycle}	$i \in INSNGAGG$	aggregate (domestic) non-government institution
$b \in BLOG$ $(\subset B)$	student behaviour determined by logistic function ={*pass, grdcont, g1entry*}	$b,b' \in MBB$	mapping between *b* (in *BRES*) and *b'* (in *BLOG*): ={(*rep, dropout).grd, grdexit.grdcont*}
$b \in BRES$ $(\subset B)$	student behaviour determined by residual scaling ={*rep* = repeater; *dropout* = dropout; *grdexit* = exiting graduate}	$b,b' \in MBB2$	mapping between *b* (in *BRES*) and all elements *b'* (also in *BRES)* that are related to the same element(s) in *BLOG*: ={*rep.*(*rep, dropout), dropout.*(*rep, dropout), grdexit.grdexit*}
$c \in C$	commodities	$c,c' \in MCE$	mapping private and public education into 1 education commodity, by cycle = {*c-edup.*(*c-edup*, *c-edupng*)} where *c-edupng* is private primary; similarly for *c-edus* and *c-edut*
$c \in CEDU$ $(\subset C)$	education services ={*c-edup* = primary; *c-edus* = secondary; *c-edut* = tertiary}; can include both private and public education	$c,c' \in MCHDC$	human development service *c* is aggregated to *c'*
$c \in CEDUT$ $(\subset C)$	tertiary education services = {*c-edut*}	$c,c' \in MCM$	mapping between aggregate (CMDG) and disaggregated MDG service commodities (CHLTH and CWTSN) = {*c-hlt.*(*c-hlt1g*, *c-hlt2g*, *c-hlt3g*, *c-hlt1ng*, *c-hlt2ng*, *c-hlt3ng*} and {*c-wtsn.*(*c-wtsn*)}
$c \in CELA$	educational cycle that corresponds to the age at which non-students would enter the labour force	$mdg \in MDG$	selected MDG indicators ={*mdg2, mdg4*, *mdg5, mdg7a*, *mdg7b*}
$c \in CHLTH$ $(\subset C)$	health services (public) ={*c-hlt1g* = low-tech; *c-hlt2g* = medium-tech; *c-hlt3g* = high-tech}; corresponding private health services labelled with "*ng*"	$mcyc(c,b,t',t)$	MDG2 in *t* is defined as the product over selected combinations of *b* and *t'* (where $t' \in T11$) = {*pass, g1entry*}
$cmdg$ $\in CMDG$	aggregate MDG (non-education) service commodities = {*c-hlt* = aggregate health in MDG functions, not in C; *c-wtsn* = water-sanitation services}	mdg $\in MDGSTD$	MDG indicators ={*mdg4* = under-5 mortality rate; *mdg5* = maternal mortality rate; *mdg7a* = access to safe water; *mdg7b* = access to basic sanitation}

$c \in CWTSN \ (\subset C)$	water-sanitation service commodities {*c-wtsn* = water-sanitation services}	$f, c \in MFC$	mapping indicating that students who have completed cycle *c* belong to labour type *f* ={*f-labn.(c-edup); f-labs.(c-edus); f-labt.(c-edut)*}
$eduarg \in EDUARG$	arguments in CE function for educational behaviour ={*edu-qual* = quantity of services per student; *w-prem* = semiskilled-unskilled wage ratio; *w-prem2* = skilled-semiskilled wage ratio; *mdg4* = under-five mortality rate; *fcapinf* = infrastructure capital stocks; *qhpc* = per-capita hhd consumption}	$mdgarg \in MDGARG$	arguments in CE function for MDGs ={*cmdg* = agg commodities; *mdg* = different MDGs; *fcapinf* = infrastructure capital stocks; *hhdconspc* = per-capita hhd consumption }
$f \in FEXOG$	factors with exogenous growth	$t \in T$	time periods
$f \in FLAB$	labour factors {*f-labn* = less than completed secondary education; *f-labs* = complete secondary education (without completed tertiary); *f-labt* = completed tertiary education	$t \in T11$	time periods including preceding years for MDG2 calculation
$h \in H$	households (excl. NGOs) ={*h* = the single household}		

PARAMETERS			
$\alpha edu_{b,c}$	constant in logistic function for educational behaviour	$ext mdg_{mdg}$	maximum value for MDG 7a and 7b; minimum value for MDG 4 and 5
$\alpha educe_{b,c}$	constant in CE function for educational behaviour	$grdcont01_{c,c'}$	0-1 constant showing that for *c'* next cycle is *c*
αmdg_{mdg}	constant in logistic function for MDG achievement	ord_t	ordinal position of *t* in the set *T*
αmce_{mdg}	constant in CE function for intermediate MDG variable	$popg1_t$	population in age cohort entering grade 1
αhd_c	efficiency term in CES aggregation function for human development	$poplab_t$	population of labour force age
$\beta edu_{b,c}$	constant in logistic function for educational behaviour	$poplabent_t$	population in age cohort entering labour force (age at end of a model education cycle)
βlog_{mdg}	constant in logistic function for MDG achievement	$poptot_t$	total population in *t*
$\delta hd_{c,i}$	share parameter for HD CES function	$qg1entncoh_{c,t}$	number of non-cohort (non-1st-year-primary) entrants to first cycle
$\varphi edu_{b,c,eduarg}$	elasticity of behaviour *b* in cycle *c* with respect to argument *eduarg* in educational CE function	$shif^0_{i,f,t}$	share of domestic institution *i* in income of factor *f*
$\varphi m_{mdg,mdgarg}$	elasticity of *mdg* with respect to argument *mdgarg* in CE function for MDG	$shrdemot01_{c,c'}$	0-1 parameter showing that for dropouts from *c'* the highest cycle is *c*
$\gamma edu_{b,c}$	parameter in logistic function for education	$shred^0_{b,c}$	base-year share for behavioural indicator behav in cycle *c*
γmdg_{mdg}	parameter in logistic function for non-education MDGs	$shrgrdcyc_c$	share of graduates (passing students) graduating from cycle c in base-year
ρhd_c	exponent in CES aggregation function for human development	$shrlabent_{c,t}$	share of drop-outs and leavers in cycle *c* that enter the labour force

$depr_{f,t}$	depreciation rate for factor *f*	$shrlabent2_{f,t}$	share of labour type *f* of labour force entrants without education
discrat	discount rate	$yrcyc_{c}$	years in school cycle for each education cycle *c*
$extedu_{b,c}$	maximum share for educational behaviour *b* in cycle *c*		

VARIABLES			
$EDUQUAL_{c,t}$	educational quality in cycle *c* in year *t*	$QH_{c,h,t}$	consumption of commodity *c* in *t* by household *h*
EG_{t}	government expenditures	$QHA_{a,c,h,t}$	quantity consumed of home commodity *c* from activity *a* by household *h*
$INVVAL_{i,t}$	investment value for institution *i*	$QHPC_{t}$	Per-capita household consumption in *t*
$MDGVAL_{mdg,t}$	value for MDG indicator *mdg* in *t*	$QQ_{c,t}$	quantity of goods supplied to domestic market (composite supply)
$PQ_{c,t}$	price of commodity *c* in *t*	$QXHLTH_{mdg,t}$	government and NGO provision of aggregated health services related to health MDG
$PXAC_{a,c,t}$	price of commodity *c* from activity *a*	$SHREDU_{b,c,t}$	share of students in cycle *c* with behaviour *b* in *t*
$QENR_{c,t}$	total number of students enrolled in cycle *c* in year *t*	$WF_{f,t}$	economy-wide wage for factor *f* in *t*
$QENROLD_{c,t}$	number of old students enrolled in cycle *c* in year *t*	$ZEDU_{b,c,t}$	intermediate variable for educational outcome (defined by CE function; entering logistic function)
$QENRNEW_{c,t}$	number of new students enrolled in cycle *c* in year *t*	$ZMDG_{mdg,t}$	intermediate variable for standard MDGs (4-5-7a-7b) (defined by CE function; entering logistic function)
$QFACINS_{i,f,t}$	endowment of labour type *f* for institution *i* in *t*		

Table A3.4 Equations for MDG module of MAMS model

#	Equation	Domain	Description
(67)	$QHD_{c,i,t} = \sum_{\substack{c' \in C \\ \mid (c,c') \in MCHDC \\ \cup \mid i \in INSG}} QG_{c',t} + \sum_{\substack{c' \in C \\ \mid (c,c') \in MCHDC \\ \cup \mid i \in INSNGAGG}} (QQ_{c',t} - QG_{c',t})$ $\begin{bmatrix} \text{demand for HD (MDG or educ)} \\ \text{service c by aggregate demander i} \end{bmatrix} = \begin{bmatrix} \text{sum of gov and non-gov} \\ \text{demand for HD service} \end{bmatrix}$	$c \in C$ $i \in I$ $t \in T$	Separation of human development (HD services into government and non-government
(68)	$QHDAGG_{c,t} = \alpha hd_c \cdot \sum_{i \in INS} \left(\delta hd_{c,i} \cdot QHD_{c,i,t}^{-\rho hd_c} \right)^{-\frac{1}{\rho hd_c}} \Big\vert_{c \in CHDCES} + \sum_{i \in INS} QHD_{c,i,t} \Big\vert_{c \in CHDPRFSUB}$ $\begin{bmatrix} \text{aggregate demand for HD} \\ \text{(MDG or educ) service ac} \end{bmatrix} = \begin{bmatrix} \text{aggregation of HD demand as imperfect substit-} \\ \text{utes (CES) or as perfect substitutes (summed)} \end{bmatrix}$	$c \in C$ $i \in I$ $t \in T$	Aggregation of human development (HD) services (i.e., MDG and education)
(69)	$QHPC_t = \frac{\sum_{c \in C} \sum_{h \in H} PQ_c^0 \cdot QH_{c,h,t} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{a,c}^0 \cdot QHA_{a,c,h,t}}{poptot_t}$ $\begin{bmatrix} \text{real household} \\ \text{conumption per capita} \end{bmatrix} = \begin{bmatrix} \text{total household consumption at base -} \\ \text{year prices divided by total population} \end{bmatrix}$	$t \in T$	Real household consumption per capita
(70)	$EDUQUAL_{c,t} = \frac{QHDAGG_{c,t}}{QENR_{c,t}} \Big/ \frac{QHDAGG_c^0}{QENR_c^0}$ $\begin{bmatrix} \text{educational quality} \\ \text{in cycle c in year t} \end{bmatrix} = \begin{bmatrix} \text{real services per student} \\ \text{in cycle c in t} \end{bmatrix} \div \begin{bmatrix} \text{real services per student} \\ \text{in cycle c in base-year} \end{bmatrix}$	$c \in CEDU$ $t \in T$ $t > 1$	Educational quality
(71)	$QENROLD_{c,t} = SHREDU_{contcyc,c,t-1} \cdot QENR_{c,t-1} + SHREDU_{rep,c,t-1} \cdot QENR_{c,t-1}$ $\begin{bmatrix} \text{number old students} \\ \text{enrolled in cycle c in t} \end{bmatrix} = \begin{bmatrix} \text{enrolled in cycle c in t - 1} \\ \text{who continue in c} \end{bmatrix} + \begin{bmatrix} \text{enrolled in c in} \\ \text{t - 1 who repeated c} \end{bmatrix}$	$c \in CEDU$ $t \in T$ $t > 1$	Enrolment old students
(72)	$QENRNEW_{c,t} = SHREDU_{g1entry,c,t-1} \cdot popg1_t + qg1entncoh_{c,t} + \sum_{c' \in C} grdcont01_{c,c'} \cdot SHREDU_{grdcont,c,t-1} \cdot SHREDU_{grdcyc,c',t-1} \cdot QENR_{c',t-1}$ $\begin{bmatrix} \text{number new students} \\ \text{enrolled in cycle c in t} \end{bmatrix} = \begin{bmatrix} \text{(cohort) students entering} \\ \text{cycle c (c = primary)} \end{bmatrix}$ $+ \begin{bmatrix} \text{(non - cohort) students entering} \\ \text{c from outside school system} \end{bmatrix} + \begin{bmatrix} \text{enrolled in preceding cycle c' in} \\ \text{t - 1 who graduated and entered c} \end{bmatrix}$ $+$	$c \in CEDU$ $t \in T$ $t > 1$	Enrolment new students
(73)	$QENR_{c,t} = QENROLD_{c,t} + QENRNEW_{c,t}$ $\begin{bmatrix} \text{total number enrolled} \\ \text{in cycle c in t} \end{bmatrix} = \begin{bmatrix} \text{enrolled old students} \\ \text{in cycle c in t} \end{bmatrix} + \begin{bmatrix} \text{enrolled new students} \\ \text{in cycle c in t} \end{bmatrix}$	$c \in CEDU$ $t \in T$ $t > 1$	Total Enrolment

(74)	$$SHREDU_{b,c,t} = ext_{edu_{b,c}} + \frac{\alpha_{edu_{b,c}}}{1 + EXP\left(\gamma_{edu_{b,c}} + \beta_{edu_{b,c}} \cdot ZEDU_{b,c,t}\right)}$$ $$\begin{bmatrix} \text{student share with} \\ \text{behavior } b \text{ in cycle } c \end{bmatrix} = \begin{bmatrix} \text{logistic function of intermediate} \\ \text{behavior variable} \left(ZEDU_{b,c,t} \right) \end{bmatrix}$$	$b \in BLOG$ $c \in CEDU$ $t \in T$	Student behaviour (logistic function)[40]
(75)	$$ZEDU_{b,c,t} = \alpha_{educe_{b,c}} \cdot \left(EDUQUAL_{c,t} \right)^{\varphi_{edu_{b,c,edu-qual}}} \cdot \left(\frac{WF_{f-labs,t}}{WF_{f-labn,t}} \right)^{\varphi_{edu_{b,c,w-prem}}} \cdot \left(\frac{WF_{f-labt,t}}{WF_{f-labs,t}} \right)^{\varphi_{edu_{b,c,w-prem}}} \cdot MDGVAL_{mdg4,t}^{\varphi_{edu_{b,c,mdg4}}}$$ $$\cdot \prod_{f \in FCAPGOVINF} \left(\sum_{i \in INS} QFINS_{i,f,t} \right)^{\varphi_{edu_{b,c,f}}} \cdot QHPC_t^{\varphi_{edu_{b,c,qhpc}}}$$ $$\begin{bmatrix} \text{intermediate variable for student} \\ \text{share with behavior } b \text{ in cycle } c \end{bmatrix}$$ $$= \begin{bmatrix} \text{exogenous} \\ \text{trend value} \end{bmatrix} \cdot \begin{bmatrix} \text{influence of: education quality (service per student);} \\ \text{wage premia (for } c \le \text{secondary and } c \ge \text{tertiary, resp.);} \\ \text{student health (proxied by MDG4); level of infra-} \\ \text{structure; and per-capita household consumption} \end{bmatrix}$$	$b \in BLOG$ $c \in C$ $t \in T$	Student behaviour (CE function defining intermediate variable)[41]
(76)	$$SHREDU_{b,c,t} = \left(1 - \sum_{\substack{b' \in BLOG \\ \mid (b,b') \in MBB}} SHREDU_{b',c,t} \right) \frac{SHREDU^0_{b,c}}{\sum_{\substack{b' \in BRES \\ \mid (b,b') \in MBB2}} SHREDU^0_{b',c}}$$ $$\begin{bmatrix} \text{student share} \\ \text{with behavior} \\ b \text{ in cycle } c \end{bmatrix} = \begin{bmatrix} \text{residual value (1 less sum} \\ \text{of shares for related} \\ \text{elements in BLOG)} \end{bmatrix} \cdot \begin{bmatrix} \text{initial share of } b \text{ in} \\ \text{total shares for related} \\ \text{residual elements} \end{bmatrix}$$	$b \in BRES$ $c \in CEDU$ $t \in T$	Student behaviour (defined residually, given left-hand side of the logistic function for education).
(77)	$$SHREDU_{grdcyc,c,t} = \frac{SHREDU_{pass,c,t}}{yrcyc_c} \cdot \left(\frac{shr_{grdcyc_c}}{\frac{1}{yrcyc_c}} \right)^{\frac{1 - SHREDU_{pass,c,t}}{1 - SHREDU^0_{pass,c}}}$$ $$\begin{bmatrix} \text{student share that} \\ \text{graduates from} \\ \text{cycle } c \text{ in year } t \end{bmatrix} = \frac{\begin{bmatrix} \text{student share that passes} \\ \text{each grade within cycle } c \end{bmatrix}}{\begin{bmatrix} \text{number of years} \\ \text{in cycle } c \end{bmatrix}} \cdot \begin{bmatrix} \text{adjustment term : ratio between base - year} \\ \text{share of cycle graduates in total graduates} \\ \text{OVER the share of the last year in total} \\ \text{number of years in cycle } c \end{bmatrix}$$	$c \in CEDU$ $t \in T$	graduation rate by cycle (ratio cycle graduates over enrolment)

[40] The α and β parameters in the logistic functions (equations 74 and 82) have been calibrated so that (i) under base-year conditions, the left-hand side variables (showing student behaviour shares or MDG values) will replicate base-year values; and (ii) under conditions derived from supporting studies of health and education, the left-hand side variables will take on values indicative of or compatible with MDG achievement.

[41] In the computer program, equations 75 and 83 (constant-elasticity functions defining intermediate variables for educational behaviour or MDG achievement) are more complex in two respects. First, the terms that are raised to exponents, which represent elasticities, are all divided by base-year values. This formulation was preferred given our desire to simulate scenarios with changes in elasticities but without any changes in simulated base-year values for left-hand-side variables. Second, for the element *grdcont* ∈ BLOG, the decision to continue to the next education cycle depends on the values for the right-hand side variables that correspond to the next cycle.

(78)	$SHREDU_{contcyc,c,t} = SHREDU_{pass,c,t} - SHREDU_{grdcyc,c,t}$ $\begin{bmatrix} student\ share\ that \\ continues\ in\ cycle \\ c\ in\ year\ t \end{bmatrix} = \begin{bmatrix} student\ share\ that \\ passes\ each\ grade \\ within\ cycle\ c \end{bmatrix} - \begin{bmatrix} student\ share\ that \\ graduates\ from \\ cycle\ c\ in\ year\ t \end{bmatrix}$	$c \in CEDU$ $t \in T$	continuation rate by cycle
(79)	$MDGVAL_{mdg2,t} = \prod_{\substack{b \in B, t' \in T11 \\ \mid mcyc(c\text{-}edup1,b,t',t)}} SHREDU_{b,c\text{-}edup1,t'}$ $\begin{bmatrix} first\ cycle\ primary\ school \\ net\ completion\ rate \end{bmatrix} = \begin{bmatrix} product\ of\ student\ shares\ (g1entry \\ and\ pass)\ for\ first\ cycle\ primary \end{bmatrix}$	$t \in T$	MDG 2
(80)	$LABPARTRAT_t = \dfrac{\sum_{i \in INS,\, f \in FLAB} QFINS_{i,f,t}}{poplab_t - \sum_{c \in CELA} QENR_{c,t}}$ $\begin{bmatrix} labor\ force \\ participation\ rate \end{bmatrix} = \dfrac{[labor\ force]}{[population\ in\ labor\ force\ age\ -\ enrollment\ in\ secondary\ and\ terciary]}$	$t \in T$ $t > 1$ $flab \notin FEXOG$	Labour Force Participation Rate
(81)	$QFINS_{i,f,t} = shif^{0}_{i,f,t}$ $\begin{bmatrix} endowment\ of\ labor\ type \\ f\ for\ institution\ i\ in\ t \end{bmatrix} = \begin{bmatrix} share\ of\ i\ in \\ labor\ type\ f \end{bmatrix}$ $\cdot \Big\{ \big(1 - depr_{f,t-1}\big) \cdot \sum_{i' \in INS} QFINS_{i',f,t-1}$ $\cdot \{ [non\text{-}retired\ labor\ from\ previous\ year]$ $+ \sum_{c,c' \in C \mid \substack{(f,c) \in MFC \\ \cap c \notin CEDUT}} shrdemot01_{c,c'} \cdot shrlabent_{c,t}$ $\cdot SHREDU_{grdexit,c',t-1} \cdot SHREDU_{grdcyc,c,t-1} \cdot QENR_{c,t-1}$ $+ \begin{bmatrix} enrolled\ in\ non\text{-}tertiary\ cycle\ in\ t\text{-}1,\ who\ graduate, \\ exit\ the\ school\ system,\ and\ enter\ labor\ force\ in\ t \end{bmatrix}$ $+ \sum_{c \in C \mid \substack{(f,c) \in MFC \\ \cap c \in CEDUT}} \big(shrlabent_{c,t} \cdot SHREDU_{grdcyc,c,t-1} \cdot QENR_{c,t-1} \big)$ $+ [enrolled\ in\ tertiary\ cycle\ in\ t\text{-}1,\ who\ graduate\ and\ enter\ the\ labor\ force\ in\ t]$ $+ \sum_{c \in C \mid \substack{(f,c) \in MFC \\ \cap c \in CEDUT}} \big(shrlabent_{c,t} \cdot SHREDU_{"grdcyc",c,t-1} \cdot QENR_{c,t-1} \big)$ $+ [enrolled\ in\ school\ in\ t\text{-}1,\ who\ dropout + enter\ labor\ force\ in\ t\ at\ next\ lower\ level\ c]$ $+ shrlabent2_{f,t} \cdot \Big(poplabent_t - \sum_{c \in CELA} QENRNEW_{c,t} \Big) \Big\}$ $+ [entrants\ from\ outside\ educational\ system\ who\ are\ of\ labor\text{-}force\text{-}age] \}$	$i \in INS$ $f \in FLAB$ $t \in T$ $t > 1$	Labour supply

(82)	$$MDGVAL_{mdg,t} = extmdg_{mdg} + \frac{\alpha mdg_{mdg}}{1 + EXP\left(\gamma mdg_{mdg} + \beta mdg_{mdg} \cdot ZMDG_{mdg,t}\right)}$$ $$\begin{bmatrix} MDG \\ value \end{bmatrix} = \begin{bmatrix} logistic\ function\ of\ intermediate \\ MDG\ value\left(ZMDG_{mdg,t}\right) \end{bmatrix}$$	$mdg \in MDGSTD$ $t \in T$	MDGs 4, 5, 7a, and 7b (logistic function)
(83)	$$ZMDG_{mdg,t} = \alpha mce_{mdg} \cdot \left(\prod_{cmdg \in CMDG} \left(\sum_{\substack{c \in C \\ \mid (cmdg,c) \in MCM}} \frac{QQ_{c,t}}{poptot_t} \right)^{\varphi m_{mdg,cmdg}} \right)$$ $$\cdot \prod_{f \in FCAPGOVINF} \left(\sum_{i \in INS} QFINS_{i,f,t} \right)^{\varphi m_{mdg,f}}$$ $$\cdot \left(\prod_{mdg' \in MDGSTD} MDGVAL_{mdg',t}^{\varphi m_{mdg,mdg'}} \right) \cdot QHPC_t^{\varphi m_{mdg,hhdconspc}}$$ $$\begin{bmatrix} intermediate\ variable \\ for\ MDGs\ 4\ and\ 5 \end{bmatrix} = \begin{bmatrix} exogenous \\ parameter \end{bmatrix} \cdot \begin{bmatrix} influence\ of: real\ value\ for\ services\ per\ capita; \\ level\ of\ infrastructure;\ water\ and\ sanitation\ MDGs; \\ household\ consumption\ per\ capita \end{bmatrix}$$	$mdg \in MDGSTD$ $t \in T$	MDGs 4, 5, 7a, and 7b (CE function defining intermediate variable)