Synchronization of Unified Chaotic System by Robust H_2/ Sliding Mode Control

Omolbanin Yazdanbakhsh
Department of Electrical Engineering
Islamic Azad University of Najaf Abad
Isfahan, Iran
o.yazdanbakhsh@gmail.com

Saeid Hoseinia
Department of Electrical and Computer Engineering
Isfahan University of Technology
Isfahan, Iran
Hoseinia@cc.iut.ac.ir

***Abstract*— This paper investigates the chaos synchronization problem for a class of uncertain unified chaotic systems with external disturbances. Based on the proportional –integral (PI) switching surface, a sliding mode controller (SMC) is derived to not only guarantee the occurrence of a sliding motion of error states, but also reduce the effect of external disturbances via H_2 norm minimization. The parameters necessary for constructing both PI switching surface and the SMC can be found by the linear matrix inequality (LMI) optimization technique. Finally, a numerical simulation is presented to show the effectiveness of the proposed method.**

Unified chaotic system; synchronization; linear matrix inequality (LMI); H_2 norm.

I. INTRODUCTION

A chaotic system is a highly complex dynamic nonlinear system and its response exhibits a number of specific characteristics, including an excessive sensitivity to the initial conditions, broad Fourier transform spectra, and fractal properties of the motion in phase space. Since the pioneering work of Pecora and Carroll in 1990 [1], chaos synchronization has received increasing attention over the last few years. It can be applied in various fields such as power converters, biological systems, chemical reactor and information processing [2-4].

In 1963, Lorenz presented the first classical chaotic system [5] in a third-order autonomous system with only two multiplication-type quadratic terms but displays very complex dynamical behaviors. Chen and Ueta found another chaotic system in 1999, the Chen system, which is similar, but topologically non-equivalent to the Lorenz system [6]. In 2002 Lu and Chen found another critical system between the Lorenz and Chen systems, bearing the name of the Lu system [7]. To bridge the gap between Lorenz and Chen systems, Lu et al. introduced a unified chaotic system [8] in the same year, which contains the Lorenz and Chen systems as two extremes and the Lu system as a transition system between the Lorenz and Chen systems.

In this paper, the problem of synchronization for uncertain unified chaotic system with external disturbances is examined. Based on the lyapunov theory, the sliding mode control strategy, and the linear matrix inequality optimization technique, a robust sliding mode controller (SMC) design is proposed such that, the occurrence of sliding motion is guaranteed, and the H_2 norm from external disturbances to the controlled output is bounded by a given $\delta > 0$.

II. SYNCHRONIZATION FOR UNIFIED CHAOTIC SYSTEMS

Consider the unified chaotic system which is described by

$$\begin{aligned}
\dot{x} &= (25\alpha + 10)(y - x) \\
\dot{y} &= (28 - 35\alpha)x - xz + (29\alpha - 1)y \\
\dot{z} &= xy - \left(\frac{8+\alpha}{3}\right)z \\
X &= [x, y, z]' \\
y_e &= EX
\end{aligned} \tag{1}$$

where x, y, z are state variables and $\alpha \in [0,1]$. Obviously, system (1) becomes the original Lorenz system for $\alpha = 0$; while it becomes the original Chen system for $\alpha = 1$. When $\alpha = 0.8$, this system becomes Lu system. In particular, system (1) bridges the gap between Lorenz system and Chen system. Moreover, system (1) is always chaotic in the whole interval $\alpha \in [0,1]$.

For the unified chaotic system with uncertain parameters consider the master system and the slave system as:

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$$
\begin{aligned}
\dot{x}_m &= (25\alpha + 10)(y_m - x_m) \\
\dot{y}_m &= (28 - 35\alpha)x_m - x_m z_m + (29\alpha - 1)y_m \\
\dot{z}_m &= x_m y_m - (\frac{8+\alpha}{3})z_m \\
X_m &= [x_m, y_m, z_m]' \\
y_{em} &= EX_m
\end{aligned} \tag{2}
$$

and

$$
\begin{aligned}
\dot{x}_s &= (25\alpha + 10)(y_s - x_s) \\
\dot{y}_s &= (28 - 35\alpha + \Delta_1)x_s - x_s z_s + (29\alpha - 1 + \Delta_2)y_s + u_1 \\
\dot{z}_s &= x_s y_s - (\frac{8+\alpha}{3} + \Delta_3)z_s + u_2 \\
X_s &= [x_s, y_s, z_s]' \\
y_{es} &= EX_s
\end{aligned} \tag{3}
$$

where the lower scripts 'm' and 's' stand for the master (or drive) system and the slave (or response) one, respectively. u_1 and u_2 are the sliding mode controllers such that two chaotic systems can be synchronized . $\Delta_i \;\; (i = 1,2,3)$ denote the bounded uncertain parameters. Note that input components appear only in those equations which include uncertainties.

Define the error signal as

$$
\begin{aligned}
e_1(t) &= x_s - x_m \\
e_2(t) &= y_s - y_m \\
e_3(t) &= z_s - z_m
\end{aligned} \tag{4}
$$

which gives

$$
\begin{aligned}
x_m z_m - x_s z_z &= -z_m e_1 - x_s e_3 \\
-x_m y_m + x_s y_s &= y_m e_1 + x_s e_2
\end{aligned} \tag{5}
$$

From (4) and (5), we have the following error dynamics

$$
\begin{aligned}
\dot{e}_1 &= -(25\alpha + 10)e_1 + (25\alpha + 10)e_2 \\
\dot{e}_2 &= (28 - 35\alpha)e_1 + (29\alpha - 1)e_2 - z_m e_1 \\
&- x_s e_3 + \Delta_1 x_s + \Delta_2 y_s + u_1 \\
\dot{e}_3 &= -(\frac{8+\alpha}{3})e_3 + y_m e_1 + x_s e_2 - \Delta_3 z_s + u_2 \\
e &= [e_1, e_2, e_3]' \\
y_e &= Ee
\end{aligned} \tag{6}
$$

or in the matrix form as

$$
\begin{aligned}
\dot{e} &= Ae + Bf + Bu + B\Delta f \\
y_e &= Ee
\end{aligned} \tag{7}
$$

In this paper, our problem is to examine the synchronization of uncertain chaotic system in the presence of disturbances. Hence, error dynamic is considered as follows

$$
\begin{aligned}
\dot{e} &= Ae + Bf + Bu + B\Delta f + Dw \\
y_e &= Ee
\end{aligned} \tag{8}
$$

where

$$
A = \begin{bmatrix} -(25\alpha + 10) & (25\alpha + 10) & 0 \\ (28 - 35\alpha) & (29\alpha - 1) & 0 \\ 0 & 0 & -(\frac{8+\alpha}{3}) \end{bmatrix}, e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}
$$

$$
B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, f = \begin{bmatrix} -z_m e_1 - x_s e_3 \\ y_m e_1 + x_s e_2 \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, D = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

$$
\Delta f = \begin{bmatrix} \Delta_1 & \Delta_2 & 0 \\ 0 & 0 & \Delta_3 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix}
$$

We assume there exist a positive number β such that $\|\Delta f\| \le \beta \|X_s\|$, and w is external disturbance.

III. SLIDING SURFACE AND CONTROLLER DESIGN

This paper aims at proposing a sliding mode controller to not only stabilize synchronization between master and slave, but also guarantee the H_2 norm from the external disturbances to controlled output is less than a given bound $\delta > 0$. The procedure involves two major steps. The first step is selecting a switching surface with appropriate parameter matrices such that, the sliding motion on sliding manifold is stable and has an H_2 cost bound constraint .Secondly, establishing a robust control law which guarantees the existence of the sliding manifold s(t)=0 even in the event of parameter uncertainty.

Now, a proportional-integral (PI) switching surface is suggested as

$$
s(t) = Ce(t) - \int_0^t (CA + CBK)e(s).ds \tag{10}
$$

where K is a designed parameter matrix, which will be clearly described later. C is chosen such that CB is nonsingular and $CD=0$. When the system operates in the sliding mode, it satisfies the following equations.

$$
s(t) = Ce(t) - \int_0^t (CA + CBK)e(s).ds = 0 \tag{11A}
$$

$$
\dot{s}(t) = C\dot{e}(t) - (CA + CBK)e(t) = 0 \tag{11B}
$$

Consequently, the equivalent control u_{eq} in the sliding manifold is obtained by differentiating (10) with respect to time and substituting from (8).

$$\begin{aligned}\dot{s} &= C\dot{e}-(CA+CBK)e = C(Ae+Bu+Bf \\ &+ B\Delta f + Dw) = -CAe - CBKe = 0\end{aligned} \tag{12}$$

Since CB is nonsingular and $CD=0$, the equivalent control u_{eq} in the sliding mode is given by

$$u_{eq} = Ke - f - \Delta f \tag{13}$$

Substituting u_{eq} into (8), the following sliding mode equation is obtained as

$$\dot{e} = (A+BK)e + Dw \tag{14}$$

In the following, the SMC which ensures the existence of the sliding motion will be derived.

Theorem1. Consider dynamics system (8). The reaching condition ($s^T\dot{s}<0$) of the sliding mode is satisfied if the controller $u(t)$ is given by

$$\begin{aligned}&u(t) = Ke - f - (CB)^{-1}(\mu + \eta e^{(-\lambda t)}\|s\|^{\rho-1} + \|CB\|\beta\|X_s\|.\|s\|^{-1})s \\ &0<\rho<1, \eta, \mu, \lambda > 0\end{aligned} \tag{15}$$

Proof: By substituting (8) and (15) into $s^T(t)\dot{s}(t)$, we get the following result:

$$\begin{aligned}&s^T\dot{s} = s^T\left[CBf + CBu + CB\Delta f - CBKe\right] \\ &= s^T(CBf + CBKe - CBf - (\mu + \eta e^{(-\lambda t)}\|s\|^{\rho-1} + \|CB\|\beta\|X_s\|.\|s\|^{-1})s \\ &+ CB\Delta f - CBKe) \le -s^T(\mu + \eta e^{(-\lambda t)}\|s\|^{\rho-1})s - s^T(\|CB\|\beta\|X_s\|.\|s\|^{-1})s \\ &+ \|s\|.\|CB\|\beta\|X_s\| = -(\mu + \eta e^{(-\lambda t)}\|s\|^{\rho-1})\|s\|^2 - \beta\|CB\|\|X_s\|.\|s\| + \|CB\|\beta\|X_s\|.\|s\| \\ &= -\mu\|s\|^2 - \eta e^{(-\lambda t)}\|s\|^{\rho+1}\end{aligned}$$

□.

Now, there still exists another problem that is how to solve for the state feedback gain K in PI switching surface via LMI formulation such that, the equivalent sliding surface is stable, and the H_2 norm performance is less than a prescribed bound $\delta>0$. The solution is given in theorem 2.

Theorem 2. Given system (14), its H_2 guaranteed cost is less than a prescribed level $\delta>0$, if there exist positive symmetric matrix x_2, semi-definite symmetric matrix Q, and matrix y_2 satisfying

$$\begin{aligned}&\begin{bmatrix} Ax_2 + x_2A^T + By_2 + y_2{}^TB^T & D \\ D^T & -I\end{bmatrix} < 0 \\ &\begin{bmatrix} Q & Ex_2 \\ x_2E^T & x_2\end{bmatrix} > 0 \\ &Trace(Q) < \delta^2\end{aligned} \tag{16}$$

and the state feedback gain is given by $K = y_2x_2{}^{-1}$.

Proof: The following relation holds for the H_2 norm of (14)

$$\|T_{y_ew}\|_2^2 = Trace(EL_cE^T) \tag{17}$$

where L_c solves the following lyapunov equation

$$(A+BK)L_c + L_c(A+BK)^T + DD^T = 0 \tag{18}$$

Consider a symmetric definite positive matrix x_2 such that

$$(A+BK)x_2 + x_2(A+BK)^T + DD^T \le 0 \tag{19}$$

which implies [9]

$$Trace\{Ex_2E^T\} \ge Trace\{EL_cE^T\} \tag{20}$$

By the schur complement [10] and denoting $Kx_2 = y_2$, (19) is equivalent to

$$\begin{bmatrix} Ax_2 + x_2A^T + By_2 + y_2{}^TB^T & D \\ D^T & -I\end{bmatrix} \le 0 \tag{21}$$

Also for any symmetric semi-definite Q ($Trace(Q) \le \delta^2$), we have

$$\begin{aligned}&Ex_2E^T \le Q \Rightarrow Trace\{Ex_2E^T\} \le Trace\{Q\} \\ &Ex_2E^T \le Q \Leftrightarrow \begin{bmatrix} x_2 & x_2E^T \\ Ex_2 & Q\end{bmatrix} \ge 0\end{aligned} \tag{22}$$

where the last relation is obtained by schur complements. Therefore, for x2 satisfying (19), we have

$$\begin{bmatrix} x_2 & x_2E^T \\ Ex_2 & Q\end{bmatrix} \ge 0 \Rightarrow \|T_{y_ew}\|_2^2 \le \delta^2 \tag{23}$$

□.

IV. NUMERICAL SIMULATION

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation results for the Lorenz system. When $\alpha = 0$, equations (2) and (3) are Lorenz system, and error dynamic system is obtained as follows

$$\begin{aligned} \dot{e} &= Ae + Bf + Bu + B\Delta f + Dw \\ y_e &= Ee \end{aligned} \tag{24}$$

where

$$A = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

The uncertainties are adopted as

$$\Delta_1 = \sin x_s, \Delta_2 = \cos y_s, \Delta_3 = \cos t$$

By choosing $\beta = 1$, $\|\Delta f\| \le \beta \|X_s\|$ is satisfied. w(t) is a Gaussian noise with mean 0 and variance 1, which is as shown in Fig. 1.

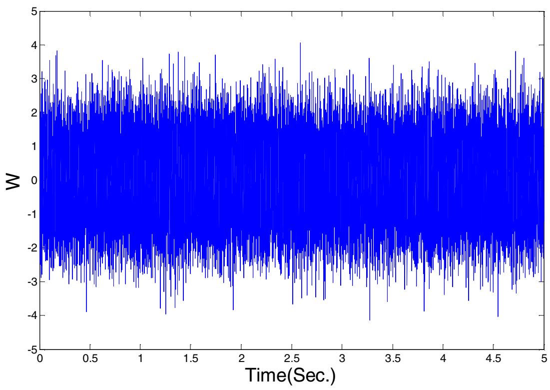

Figure 1. Gaussian noise with mean 0 and variance 1

We choose $C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ such that $CB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is nonsingular and $CD = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. By applying the condition in the Theorem 2 with the H_2 performance bound $\delta = 0.7071$, we can obtain the following matrixes

$$x_2 = \begin{bmatrix} 0.2298 & 0.1090 & 0.0000 \\ 0.1090 & 0.2266 & 0.0000 \\ 0.0000 & 0.0000 & 0.8787 \end{bmatrix}$$

$$y_2 = \begin{bmatrix} -7.9018 & -3.2254 & 0.0000 \\ 0.0000 & 0.0000 & 1.9039 \end{bmatrix}$$

and the state feedback gain is then given by

$$K = y_2 x_2^{-1} = \begin{bmatrix} -35.5388 & 2.4347 & 0.0000 \\ 0.0000 & 0.0000 & 2.1667 \end{bmatrix}$$

The parameters of the controller are $\mu = 10$, $\eta = 0.2$, $\lambda = 3$,and $\rho = 0.7$. Then the controller is obtained as the following

$$u = Ke - f - (10 + 0.2e^{(-3t)} \|s\|^{-0.3} + \beta \|X_s\| \|s\|^{-1}) s$$

The simulation results with initial conditions $(x_m(0), y_m(0), z_m(0)) = (1.5, 2, 1)$ and $(x_s(0), y_s(0), z_s(0)) = (-1, -5, -10)$ are shown in Figs. 2 and 3.

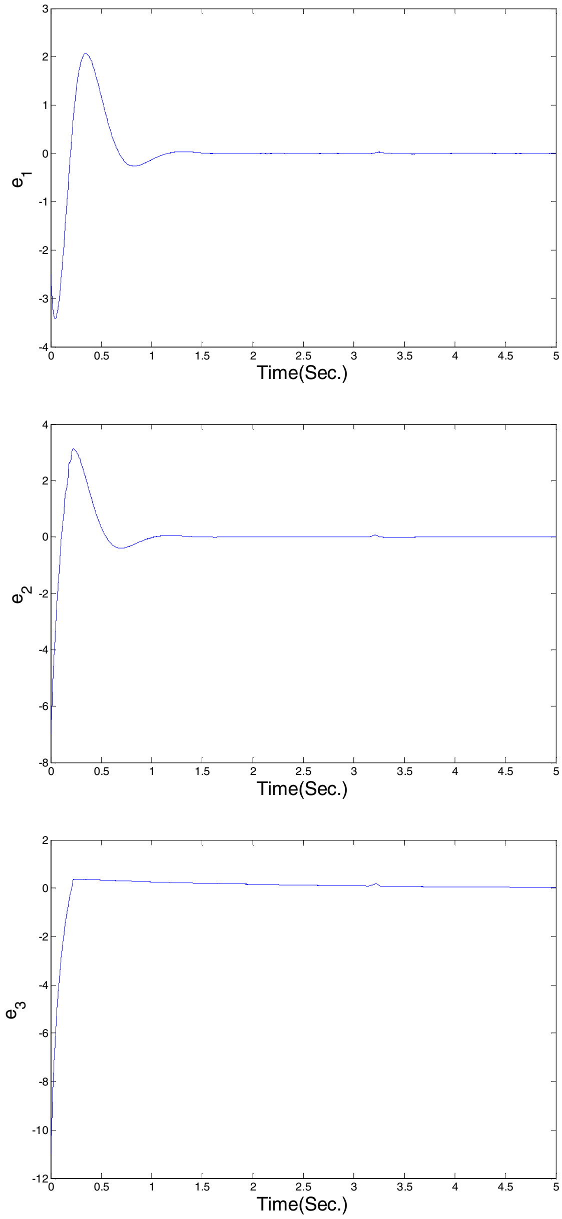

Figure 2. Time response of error synchronization

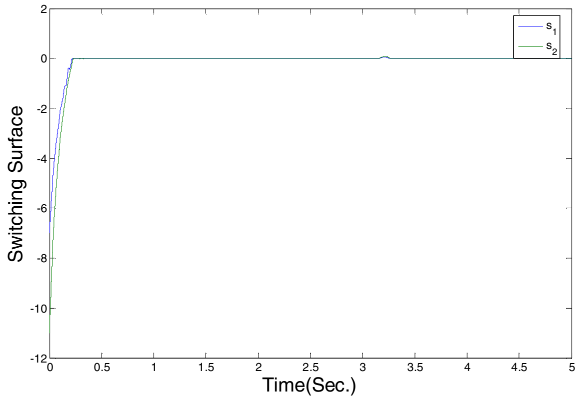

Figure 3. Time response of switching surface

To observe the noise effect, the response of the controlled output error $Y_e(t)$ is depicted in Fig. 4 for zero initial conditions.

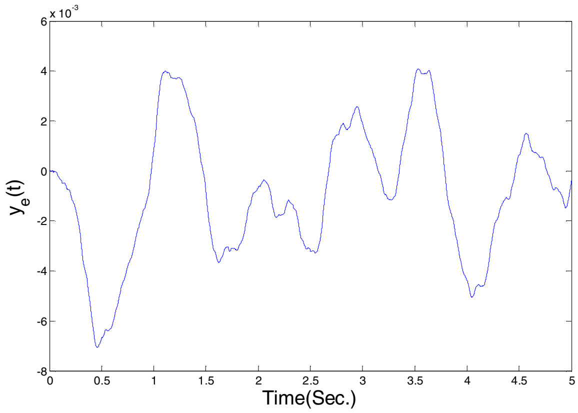

Figure 4. Time response of the controlled output with zero initial condition

V. CONCLUSION

In this paper we have presented the PI switching surface and robust SMC to stabilize the uncertain unified chaotic systems containing noise disturbance. By applying sliding mode control, and the linear matrix inequality optimization technique, a sliding mode controller is derived for not only guaranteeing occurrence of sliding motion, but also performing H_2 attenuation of external disturbance. Finally, a numerical simulation is provided to show the effectiveness of our method.

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